Alternative Assessments:

Building Success in Geometry Through Culture, Language, and Identity

Kevin L. Murphy

State University of New York (SUNY) Brockport

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Abstract

Culturally relevant pedagogy is a great concept, but to make it a reality for your students you have to create a climate in your classroom that let's students know that their humanity is valued just as much as their academic ability. The inability of students to see themselves in the curriculum leads to low student engagement. Lower levels of student engagement have demonstrated a correlation with living in poverty. Mathematics achievement scores for the historically underserved populations often found in urban school settings - African American, Latinx, Native American, and poor students shows that they continue to be underserved by mathematics education. Teachers who create contexts in their classroom for students to exhibit high involvement in the learning of geometry cultivate students that are more engaged and as a result achieve higher levels of academic achievement. This work aims to improve academic achievement in the study of geometry using culturally relevant pedagogy and teacher enthusiasm to foster higher levels of student engagement.

Keywords: urban education, culturally relevant pedagogy, culturally relevant teaching, authentic exercises, academic achievement, urban scholars, underserved student population, teacher enthusiasm, student engagement

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Introduction

This curriculum project connects culturally relevant pedagogy (CRP) with the teaching of Geometry by integrating content with culture, language, and identity. The curriculum includes ideas and recommendations for where to fit the assessment within the geometry curriculum. Specifically, the three assessments in this curriculum project are: Connecting Cultural and Personal Identity to Rigid Motions, Tangrams, and Tessellations; Making Proofs Familiar and Accessible to all students; and Mathematical Literacy. In addition to the assessments, this curriculum provides: (a) an explanation of the assessment type; (b) sample responses that teachers can expect from students; and (c) grading tools with examples of feedback. These assessments are designed to make the Geometry curriculum accessible to all high school students with the goal of providing the opportunity for students to demonstrate mastery of concepts, especially the student population found in urban school districts. The first two alternative assessments (Connecting Cultural and Personal Identity to Rigid Motions, Tangrams, and Tessellations, and Making Proofs Familiar and Accessible to All Students) contain pre- and post-assessments. The purpose of these assessments is two-fold: (a) to assist the teacher in driving instruction based on the students' prior knowledge; and (b) to allow the student to see the growth in their content knowledge from the beginning of the unit to the end of the unit. The latter purpose is designed to help transfer responsibility for learning from the teacher to the student.

Homework is noticeably absent from these assessments. Resources for homework can be found in your textbook, from on-line sources, departmental or district colleagues, as well as the teacher themselves. The assigning of homework is therefore left up to the professional judgment of the teacher. Whether you are a novice teacher or an experienced teacher, using homework to assess student learning is a skill that you will develop as a novice and that you will continue to develop as an experienced teacher. Teaching colleagues are a great resource if and when you are in need of assistance.

This curriculum project was created during the COVID-19 pandemic that resulted in the large-scale shutdown of schools. As a result, its use for face to face teaching versus remote instruction depends on the teacher utilizing it and the policies of the district that guide instructional decisions. The resources provided are designed to work in either scenario, either face to face or remote instruction. However, as in most cases, remote instruction may be more challenging. Many on-line learning management systems (LMS) were built upon a textual framework that does not always lend itself well to geometry as well as other mathematical topics. A list of programs/applications for showing mathematics digitally, along with a description of what and how each app can be used, can be found in Appendix A.

Literature Review

Culturally Relevant Pedagogy

As teachers, we want our students to learn. Ladson-Billings (1995) states that culturally relevant pedagogy (CRP) rests on three criteria that students must experience: "(a) Students must achieve academic success; (b) students must develop and/or maintain cultural competence; and (c) students must develop a critical consciousness through which they challenge the status quo of the current social order" (p. 160). This curriculum project focuses primarily on the first two criteria of achieving academic success, which aligns with the goal of students learning, developing, and maintaining cultural competence.

Teachers developing CRP to support student academic success is important. Martin et al. (2010) explains that mathematics achievement scores for the historically underserved populations are often found in urban school settings. African American, Latinx, Native American, and students within lower socioeconomic status (SES) systems continue to be underserved by mathematics education. However, Gay (2018) says that CRP can be used to address the needs of these students.

The success of CRP can be found in a few of its key components. One of the components of CRP is the ability for students to see themselves in the curriculum. Emdin (2016) refers to students in the previously mentioned underserved populations as "neoindigenous", describing the way they "look, act, and engage in the classroom in ways that are inconsistent with traditional school norms" (p. 9). Teachers should be aware of how their own implicit bias factors into how they engage with and characterize these students. Otherwise, they may inadvertently view these students through the lens of "deficit syndrome" (Gay, 2018, p. 31). This view may cause a perception that limits student mathematical ability by what Valencia (2010) referred to as “deficit thinking” (p. 2). Both of these characterizations refer to the notion that students fail in school because of things that either they or their family lack, including but not limited to intelligence, motivation, socialization, and support. Teachers must be able to internalize that their cultural lens is different from that of their students, and recognize the difference between the student’s lens and the teacher’s lens (Goldenberg, 2014). Kohl (1991) informs us that what teachers often characterize as a failure to learn, or a lack of desire and motivation to learn, is really a phenomenon he referred to as “not-learning” (p. 16). Not-learning is when students are not lacking any character traits or skills to learn, but rather in an attempt to maintain their cultural and personal identities they make decisions not to engage in the learning process. Not-learning is the students’ way of declaring their ability to maintain their sense of who they are. This sense is often more important than whatever content the teacher is attempting to teach them. *It is for this reason that students must be able to see themselves in the curriculum.*

One of the ways that teachers can ensure that students see themselves in the curriculum is by engaging students using authentic exercises. Authentic exercises are an extension of what are typically referred to as “real-world” exercises, except these exercises tap into the lived experiences that the students are taught. This may be achieved using what Emdin (2016) refers to as “cultural artifacts”, or things such as clothing, music, speech, language, food, and even media that are relevant to students' experiences. The successful implementation of CRP has the potential to lead to our second criteria that students will experience which is what Ladson-Billings (year) referred to as “cultural competence”. Ladson-Billings states that “culturally relevant teaching requires that students maintain some cultural integrity as well as academic excellence" (Ladson-Billings, 1995, p. 160) Ladson-Billings goes on to say that teachers who practice culturally relevant pedagogy incorporate "culture as a vehicle for learning" (p. 161). Instead of using cultural artifacts as ways to identify perceived deficits in students' mathematical abilities, teachers are encouraged to use these same artifacts as a tool to build connections between students and the content. In a study of 37 at-risk students, Hubert (2014) found that students had positive feelings toward mathematics instruction rooted in CRP. Students stated they preferred being taught using CRP, as one student expressed, CRP "made the class feel so alive" (Hubert, 2014, p. 329).

Teacher Enthusiasm

One of the factors that has been studied for its positive impact on student academic performance is teacher enthusiasm. "A teacher who is perceived to have a dynamic, enthusiastic style...tends to have students who report being highly intrinsically motivated regarding the subject matter as well as feeling energized in class" (Patrick, 2000, p. 225) In addition, teachers who exhibit greater evidence of their enthusiasm have "students are more likely to be interested, energetic, curious, and excited about learning" (Patrick et al. 2000, p. 233) When looking for evidence of the positive impact of teacher enthusiasm on student performance, one must first drill down into what is meant by teacher enthusiasm. Teacher enthusiasm among teachers of mathematics in particular has been studied from the perspective of enthusiasm for the subject matter itself as well as enthusiasm for teaching the subject matter. Kunter et al. (2008) found teacher enthusiasm for teaching to be a greater predictor of student motivation than teacher enthusiasm for mathematics subject matter. Mahler et al. (2018) found no relationship between teacher enthusiasm for teaching mathematics and student performance, but that there was a positive relationship between student performance and teacher's subject-specific enthusiasm It is important to note that Patrick et al. (2000) found "the evidence does NOT suggest that a steady diet of teacher enthusiasm can act as a panacea for the motivational ills of students" and that it seems to be "highly unlikely that a teacher's enthusiasm could maintain a student's intrinsic motivation to learn in the absence of a learning context that is also actively supportive of the student's growing interests and needs" (p. 233). The development and implementation of CRP, whose importance has already been shown previously in this work, may serve as that panacea.

Student Engagement

While teacher enthusiasm has been connected to student performance, CRP is the pathway to increased student engagement. Ideally all students would be intrinsically motivated, but since that is not the case teachers must play an active role in seeking how to engage all students in learning. Student engagement is critical because lower levels of engagement have demonstrated a correlation with living in poverty, while high levels of engagement have been associated with future college attendance and career success (Kearney et al. 2013). Having the expectation that students will be motivated to learn can increase teacher enthusiasm (Cobb & Foehler, 1992) and teacher enthusiasm leads to increased student performance (Patrick et al. 2000).

Jussim (1989) speaks of teacher expectations leading to self-fulfilling prophecies by taking us through three sources of expectancy confirmation, two with negative implications for students that will be addressed here. The first states that even when teacher expectations are initially erroneous, they may evoke performance levels from students consistent with those expectations. The second states teachers expectations may lead to perceptual biases that can cause them to interpret student actions in ways consistent with these perceptual biases, similar to the way that implicit bias can lead to “deficit syndrome” (Gay, 2018, p. 31) or “deficit thinking” (Valencia, 2010, p. 2). It is easy to see why having positive expectations of students is very important.

With this in mind we will shift our focus to strategies that generate positive outcomes in regard to student engagement. The first is a paradigm shift from *student engagement* to *student involvement in learning mathematics*. The difference between student engagement and student involvement in learning mathematics is that involvement is a psychological state, it does not assume a long term nor preexisting positive association with mathematics, and it attempts to describe a quality of experience that students seek to repeat (Turner et al. 1998). “During involvement, attention is wholly concentrated, time passes quickly, and there is deep comprehension, focused emotional investment and a motivational drive to continue” (Turner et al. 1998, p. 731).

In studying the instructional patterns of high- and low-involvement teachers, Turner et al. (1998) found that with high-involvement teachers there was a higher-press for understanding and more provision of autonomy. These teachers also created a climate where error was viewed constructively and demonstrated that it was ok to not know something. Additionally, they exemplified for their students a respect for and an interest in the mathematics to be learned, which aligns with being enthusiastic about the subject matter of mathematics. This work aspires to help teachers become high-involvement teachers that create students that are highly-involved in learning mathematics.

The Curriculum

This curriculum project is aligned with the Common Core State Standards (CCSS) in Mathematics and the New York State (NYS) Next Generation Mathematics Standards, which makes it user friendly across the United States (US). The curriculum presents lesson and assessment ideas that can support and foster success in the Geometry classroom. Each assessment idea provides recommendations for where to fit the assessment within the geometry curriculum. In addition to the assessment itself, this curriculum provides: (a) an explanation of the assessment type; (b) sample responses that teachers can expect from students; and (c) grading tools with examples of feedback. These assessments are designed to make the curriculum accessible to all levels of geometry students, and to provide them the opportunity to demonstrate mastery of concepts and content. The assessments presented are as follows:

Alternative Assessment 1: Connecting Cultural and Personal Identity to Rigid Motions, Tangrams, and Tessellations

Alternative Assessment 2: Making Proofs Familiar and Accessible to All Students

Alternative Assessment 3: Mathematical Literacy – The “Lit” Project

All images, solutions, and answer keys to the assessments will be provided in the appendix.

Group Work Norms

The group roles in the table below are provided if group roles have not previously been established: (Source: Daily Teaching Tools)

GROUP MEMBER ROLE	RESPONSIBILITIES
The Facilitator	- Provides leadership and direction for the group - leads discussions - suggests solutions to team problems - focuses work around the learning task - makes sure that every voice is heard
The Recorder	- Keeps a public record of the team's ideas and progress - Checks to be sure that ideas are clear and accurate - Uses charts, multiple colors, and other techniques to highlight and summarize the ideas of the team
The Summarizer	- Restates the group's conclusions and responses - Prepares a summary of the group's efforts - Checks for clarity of understanding
The Presenter	- Presents the group's finished work to the class - Regularly contributes to the team's efforts
The Materials Manager (optional)	- Gets and returns supplies and materials
The Timekeeper (optional)	- Monitors time and helps to keep the group on task

The last two optional group member roles have been added if the teacher determines they are necessary and/or the size of the group needs to be larger than four people.

Alternative Assessment 1:

Connecting Cultural and Personal Identity to

Rigid Motions, Tangrams, and Tessellations

This assessment is best used in a high school geometry classroom, with possible applications in an 8th grade mathematics classroom as well. As teachers, part of our job is to show our students how enthusiastic we are about the subjects we teach. Nowhere is this more important than in the mathematics classroom. When asked the question, "Mister, why do you like math?" without delay my answer is "because I look at math as solving a bunch of puzzles, and I love puzzles!" We are going to share this puzzle loving philosophy with our students by teaching them how to solve geometric puzzles, get them vested in this work by relating these puzzles to their culture and experiences, and showing them how the ability to solve these puzzles leads to success in mathematics. The introduction to this alternative assessment will be implemented with tangrams because tangrams have the benefit of being useful in developing problem solving and logical thinking skills, nonverbal thinking skills, visual-spatial awareness, creativity and many other mathematical concepts including congruency, which will be very important to learning geometry, particularly when students engage the topic of proofs (Hallissey, 2012).

Unit	**Rigid Motions, Tangrams, and Tessellations**
Objectives	Designed for a unit on Transformations, including but not limited to the following unit objectives: 1. Students **can** perform reflections, translations, and rotations on two-dimensional figures. 2. Students **can** identify lines of symmetry and rotational symmetry. 3. Students **can** observe how transformations map a preimage onto an image. 4. Students **can** define the vector that describes a translation. 5. Students **can** identify corresponding parts of two-dimensional figures. 6. Students **can** apply a sequence of rigid motions from one figure to another figure to demonstrate that the figures are congruent.
Common Core Standards	**Experiment with transformations in the plane.** **HSG.CO.A.5.** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions.** **HSG.CO.B.6.** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **HSG.CO.B.7.** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
Next Gen Standards	**Experiment with transformations in the plane.** **GEO-G.CO.5.** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions.**

	GEO-G.CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure. Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **GEO-G.CO.7.** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
Standards of Mathematical Practice	**MP1.** Make sense of problems and persevere in solving them. **MP3.** Construct viable arguments and critique the reasoning of others. **MP4.** Model with mathematics. **MP5.** Use appropriate tools strategically. **MP7.** Look for and make use of structure.

Connecting Cultural and Personal Identity to Transformations

The following assessment should be given as a pre-assessment to allow students to self-evaluate their knowledge of rigid motions before being formally introduced to the unit. The pre-assessment will also serve as a tool for teachers to provide insight into what students' perceptions are of their knowledge of rigid motions. Students will be given the same assessment as a post-assessment to measure their growth at the end of the unit.

Name ______________________________ Geometry

Date ______________________________ Period: ____________

Rigid Motions Pre/Post Assessment

Answer all questions.

1.	Name the transformations that are rigid motions. ______________________________
2.	What characteristics must be true in order for transformations to be rigid motions? ______________________________ ______________________________ ______________________________
3.	Identify your knowledge of rigid motions. (Circle one) 1. Say what? I am not knowledgeable at all about rigid motions. 2. Meh. I've heard the term before but I don't really know what rigid motions are. 3. So-so. I know a little bit about rigid motions, but I forgot a lot too. 4. It's lit! I think I know EVERYTHING about rigid motions!

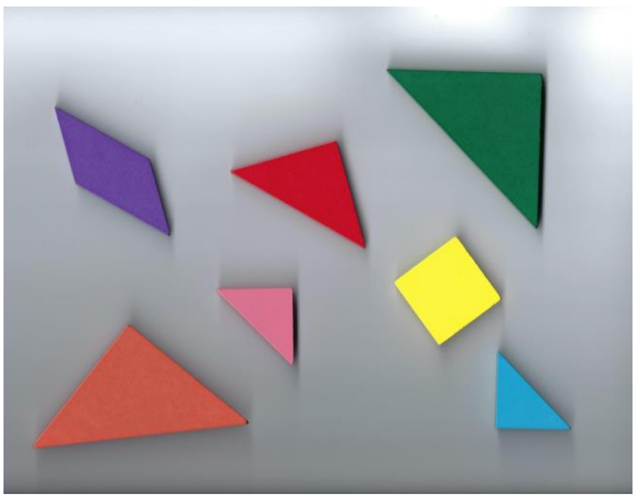
Rigid Motions Pre/Post-Assessment Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Knowledge of Each Rigid Motion Transformation	Student correctly identified all 3 rigid motions	Student correctly identified 2 rigid motions	Student correctly identified 1 rigid motions	Student didn't correctly identify any of the rigid motions	
Knowledge of What Makes A Transformation a Rigid Motion	Student correctly identified both characteristics	Student correctly identified 1 characteristic	Student correctly identified 1 characteristic but all other statements were incorrect	Student didn't correctly identify any characteristics	
Rigid Motions Knowledge Self-Assessment (4=very knowledgeable, 1=not knowledgeable at all)	Student identified themselves as a 4	Student identified themselves as a 3	Student identified themselves as a 2	Students identified themselves as a 1	
SCORE				TOTAL SCORE:	_____ / 12

Opening Exercise: Connecting Fun to Academic Growth

This exercise is a student-led, student-centered exercise. Students should be separated into groups of 4 members maximum, using previously established group roles and group norms for completing group work. The teacher's role is to: assign groups; provide the packets of tangram pieces; circulate around the room to make sure that students are staying on task; and answer any questions that students may have. Students should be encouraged to work collaboratively in their groups and only seek teacher guidance when necessary. After assigning groups and handing out the tangram packets, student groups should be given approximately 10 minutes to solve the puzzle challenge.

Lastly, the teacher will ask follow-up questions to further help students understand the process and retain the steps. The lesson should take approximately one 45-minute class period. We are going to start by giving them a pack of 7 tangram pieces in a food storage bag, and challenging them with the task of arranging the pieces into one large rectangle. The students should be instructed to dump the bag of tangram pieces out onto one of the desks in their group and spread them out. They will see a spread of the pieces that looks something like the picture below:



The students will work in groups to limit the number of sets of tangram packets needed, as well as provide additional students to spread out the required work. Students will be instructed that they cannot lift the pieces off of the desk, except to flip a piece over if necessary.

The recorder should record the steps that the group uses to arrange the tangram pieces into the square, with the focus being on how the group members manipulate the tangram pieces - what “types” of movements are necessary but not each individual movement , and any patterns they recognized while trying to solve the puzzle challenge.

Once they have solved the puzzle challenge, the recorder will draw a picture of their solution so there is a record of it. The groups will first create a square from their tangram pieces (task 1). When the students have solved the puzzle, they will have something that looks like the picture below:

Figure 5- Tangram Rectangle Reassembled

Students will then disassemble the square, and see if they can find a different way to solve the puzzle challenge (task 2). These two tasks are really going to test the mathematical standards of practice listed for this unit, especially MP1! The goal of this exercise is for the students to have fun solving this 7-piece puzzle, while they begin to discover the properties of the transformations that are rigid motions!

After assigning groups and handing out the tangram packets, student groups should be given approximately 15 minutes to complete this assignment. Once appropriate time has elapsed (when it is clear that most groups have completed the two tasks), a presenter from each group should be prompted to present the picture of their solution, the types of movements (motions) of the pieces, any patterns they recognized, ways they got stuck, and how they overcame their challenges to the class. These unique pictures, motions, and patterns should be recorded where they are visible to the entire class. Additional presenters will be called until there are no new solutions, movements, or patterns to present.

Connecting Fun to Academic Growth: Closure

The teacher will then guide students through a reflection process by revisiting the process used to solve the challenge puzzle. Some questions to facilitate the reflection process may include but are not limited to the following:

1. How did we start?
2. How would you describe each of the different movements that were applied to the tangram pieces? Identify the names of the transformations described.
    a. This is where we want to formally introduce the terms “rotation”, “reflection”, and “translation”.
3. Was there anything special about any of the tangram pieces that made manipulating the pieces easy, or that put limitations on manipulating any of the pieces?
4. What were the challenges that the groups encountered while trying to solve the puzzle?
5. How did the groups overcome their challenges?

The lesson should take approximately one 45-minute class period. If there is time left over prior to the end of the period., instruct them to go to the website https://mathigon.org/tangram and play around with creating other shapes with the tangram pieces other than a “boring old square!”

Rotations: Round and Round We Go Exercise

In the first exercise students were put into groups where they discovered the three transformations that are rigid motions - rotations, reflections, and translations. In the second exercise, "Rotations: Round and Round We Go", students will delve deeper into rotations. This exercise is a teacher-led, student-centered exercise. Students will explore the properties of rotations and why a rotation is a rigid motion. The teacher's role is to: provide graph-ruled index cards; demonstrate the process for creating a rotation tessellation. If you don't have a document camera, or an Elmo, this YouTube video can be used - https://www.youtube.com/watch?v=WvtfS9pQvhM. Teachers will first need to watch the entire video and make notes of the timestamps that students need to pay close attention to. You can then step through the video so that students can follow along. Otherwise the video may be too long to watch and students may lose attention. Teachers should circulate around the room to make sure that students are staying on task and answer any questions that students may have. After verifying that all students have successfully created their rotation tessellation, the teacher will guide students through a reflection process by revisiting the steps for creating the tessellation, and pointing out the steps that revealed the properties of rotations and why a rotation is a rigid motion. This will help students understand rotations and retain the properties and their importance.

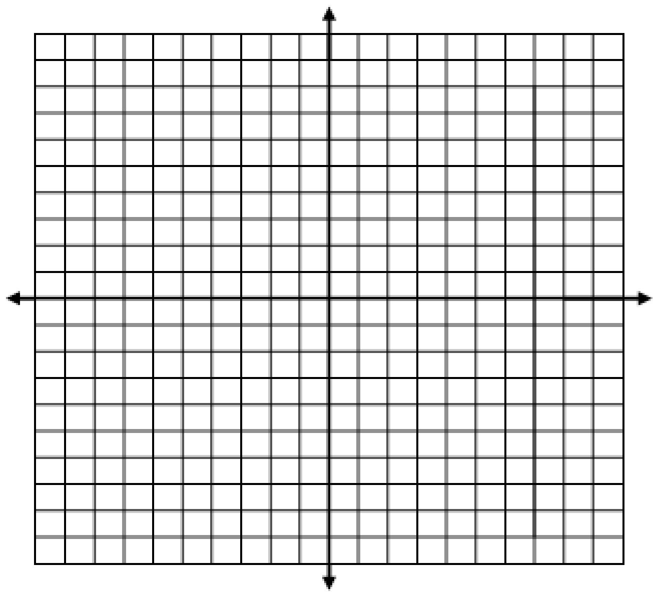
Rotations: Cartesian Coordinates Exercise

In this exercise students will demonstrate the properties of rotations and why a rotation is a rigid motion using the coordinate grid. The teacher's role is to: provide a copy of the exercise; guide the students through the exercise, and then instruct them how to verify that the second triangle is the exact same size and shape as the first triangle using the patty-paper; circulate around the room to make sure that students are following the procedure; and answer any questions that students may have. After verifying that all students have successfully demonstrated the congruency of the preimage and the image, the teacher will inform the students that they have just performed a 90-degree counterclockwise rotation and bring the lesson to closure. An answer key for the "Rotations: Cartesian Coordinates" Exercise can be found in Appendix C.

Name ____ Geometry
Date ____ Period: _______________

Rigid Motions – Rotations: Cartesian Coordinates Exercise

1. Using the graph below, graph ΔABC whose vertices are $A(2, 1)$, $B(6, 1)$, and $C(6, 4)$.
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices
2. On the same graph, graph $\Delta A'B'C'$ whose vertices are $A'(-1, 2)$, $B'(-1, 6)$, and $C'(-4, 6)$
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices



OBSERVATIONS

A. What do you notice about the relationship between the vertices of ΔABC and $\Delta A'B'C'$?
B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

Rotations Closure

Some questions to facilitate the reflection process may include but are not limited to the following:

1. What does the word "rotation" mean?
2. How did we verify that both triangles were the same size and shape?
3. What did you notice about the orientation of the two triangles?
4. What would happen if we rotated in the opposite direction?
5. If no direction is specified, what direction does a rotation go in?
6. How do we write a 90-degree rotation using symbolic notation?
7. What point did we rotate around? (Where was our **center of rotation**?)
8. What is one way you can identify a tessellation is based on a rotation?
9. Why is a rotation considered to be a rigid motion?

The lesson should take approximately one 45-minute class period.

Translations: Slide Like Drake Exercise

In this exercise, "Translations: Slide Like Drake", students will delve deeper into translations. This exercise is a teacher-led, student-centered exercise. Students will explore the properties of translations and why a translation is a rigid motion.

The teacher's role is to: provide graph-ruled index cards; demonstrate the process for creating a rotation tessellation (if you don't have a document camera, or an Elmo, this YouTube video can be used - https://www.youtube.com/watch?v=WZCTLsQNEdU. You will need to step through it so that students can follow along with the work in the video. Otherwise the video will be too long to watch and you will lose the students' attention quickly! Keep it interactive!); circulate around the room to make sure that students are staying on task; and answer any questions that students may have. After verifying that all students have successfully created their translation tessellation, the teacher will guide students through a reflection process by revisiting the steps for creating the tessellation, and pointing out the steps that revealed the properties of translations and why a translation is a rigid motion. This will help students understand translations and retain the properties and their importance.

Translations: Cartesian Coordinates Exercise

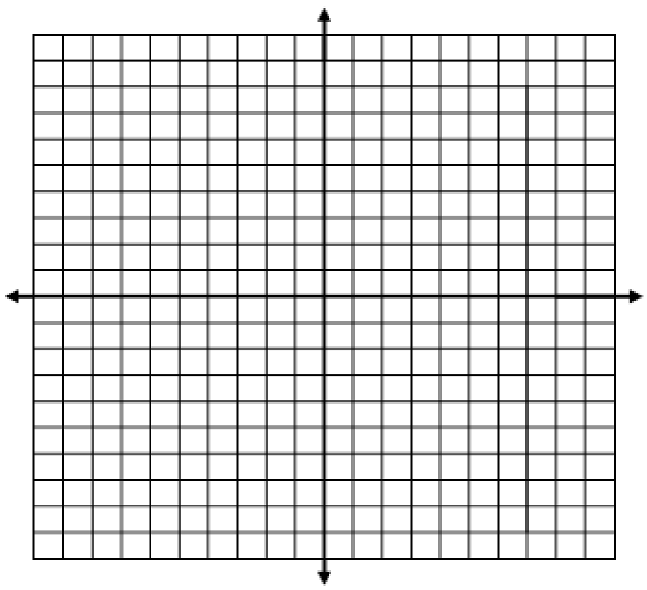
In this exercise students will demonstrate the properties of translations and why a translation is a rigid motion using the coordinate grid. The teacher's role is to: provide a copy of the exercise; guide the students through the exercise, and then instruct them how to verify that the second triangle is the exact same size and shape as the first triangle using the patty-paper; circulate around the room to make sure that students are following the procedure; and answer any questions that students may have. After verifying that all students have successfully demonstrated the congruency of the preimage and the image, the teacher will inform the students that they have just performed a translation (slide) and bring the lesson to closure. An answer key for the "Translations: Cartesian Coordinates" Exercise can be found in Appendix D.

Name ____ Geometry

Date ____ Period: _______________

Rigid Motions – Translation: Cartesian Coordinates Exercise

1. Using the graph below, graph ΔABC whose vertices are A(-2, 1), B(-6, 1), and C(-6, 4).
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices
2. On the same graph, graph $\Delta A'B'C'$ whose vertices are A'(2, 4), B'(-2, 4), and C'(-2, 7)
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices



OBSERVATIONS

A. What do you notice about the relationship between the vertices of ΔABC and $\Delta A'B'C'$?
B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

Translations Closure

Some questions to facilitate the reflection process may include but are not limited to the following:

1. What does the word “translation” mean?
2. How did we verify that both triangles were the same size and shape?
3. What did you notice about the orientation of the two triangles?
4. How do we write a translation +5 units in the x-direction and -3 units in the y-direction using symbolic notation?
5. What is one way you can identify a tessellation is based on a translation?
6. Why is a translation considered to be a rigid motion?

The lesson should take approximately one 45-minute class period.

Reflections: Who is that in the Mirror Exercise

In this reflections exercise students will delve deeper into reflections. This exercise is a teacher-led, student-centered exercise. Students will explore the properties of reflections and why a reflection is a rigid motion. The motivating factor for the students in this exercise is that everyone's favorite word is their own name, so let's use the power of reflections and art to highlight the students and their names! The teacher's role is to: provide paper/card-stock; demonstrate the process for creating a reflection tessellation (if you don't have a document camera, or an Elmo, draw your name using some sort of bubble letters using whatever display technology you have at your disposal. Make sure you connect the letters, or else when you cut out the letters they will not look right. Practice doing this before you model it for the students. Tips for creating the tessellation and sample images can be found in Appendix L; circulate around the room to make sure that students are staying on task; answer any questions that students may have. After verifying that all students have successfully created their reflection tessellation, the teacher will guide students through a reflection process by revisiting the steps for creating the reflection, and pointing out the steps that revealed the properties of reflections and why a reflection is a rigid motion. This will help students understand reflections and retain the properties of reflections and their importance.

Reflections: Cartesian Coordinates Exercise

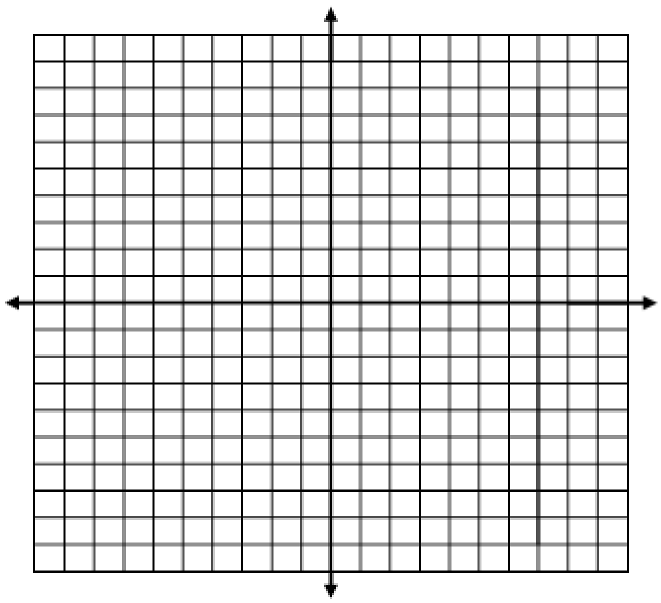
In this exercise students will demonstrate the properties of reflections and why a reflection is a rigid motion using the coordinate grid. The teacher's role is to: provide a copy of the exercise; guide the students through the exercise, and then instruct them how to verify that the second triangle is the exact same size and shape as the first triangle using the patty-paper; circulate around the room to make sure that students are following the procedure; and answer any questions that students may have. After verifying that all students have successfully demonstrated the congruency of the preimage and the image, the teacher will inform the students that they have just performed a reflection and bring the lesson to closure. An answer key for the "Reflections: Cartesian Coordinates" Exercise can be found in Appendix E.

Name ____________________________________ Geometry

Date ____________________________________ Period: ______________

Rigid Motions – Reflection: *Cartesian Coordinates* Exercise

1. Using the graph below, graph ΔLIT whose vertices are L(3, 2), I(9, 4), and T(6, 8).
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices
2. On the same graph, graph $\Delta L'I'T'$ whose vertices are L'(3, -2), I'(9, -4), and T'(6, -8)
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices



OBSERVATIONS

A. What do you notice about the relationship between the vertices of ΔLIT and $\Delta L'I'T'$?

B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

Reflections: Closure

Some questions to facilitate the reflection process may include but are not limited to the following:

1. What does the word "reflection" mean?
2. How did we verify that both triangles were the same size and shape?
3. What did you notice about the orientation of the two triangles?
4. How do we write a reflection over the y-axis using symbolic notation?
5. What happens to the value of the x-coordinate when a reflection over the x-axis is performed? (r_{x-axis})
6. What happens to the value of the y-coordinate when a reflection over the line y=x is performed? $(r_{y=x})$
7. Why is a reflection considered to be a rigid motion?

The lesson should take approximately one 45-minute class period.

Sequence of Rigid Motions: Two+ Chains Exercise

In this exercise, *Sequence of Rigid Motions; Two+ Chains*, students will delve deeper into sequences of rigid motions. This exercise is a teacher-led, student-centered exercise. Students will explore the effect of sequencing two or more transformations, and why the result is still a rigid motion. The teacher's role is to: provide graph-ruled index cards; demonstrate the process for creating a glide reflection tessellation (if you don't have a document camera, or an Elmo, this YouTube video can be used - https://www.youtube.com/watch?v=ZHDkBJP7OlQ. This video isn't as long as the other videos, but you may still need to create points to skip to in the video); circulate around the room to make sure that students are staying on task; and answer any questions that students may have. After verifying that all students have successfully created their glide reflection tessellation, the teacher will guide students through a reflection process by revisiting the steps for creating the tessellation, and pointing out the steps that revealed the properties of glide reflections and why a glide reflection is a rigid motion. This will help students understand glide reflections and retain the properties of glide reflections and their importance.

Sequence of Rigid Motions: Cartesian Grid Exercise

In this exercise students will demonstrate sequences of rigid motions using the coordinate grid. The teacher's role is to: provide a copy of the exercise; guide the students through the exercise, and then instruct them how to verify that the final triangle is the exact same size and shape as the first triangle using the patty-paper; circulate around the room to make sure that students are following the procedure; and answer any questions that students may have. After verifying that all students have successfully demonstrated the congruency of the first preimage and the last image, the teacher will inform the students that they have just performed a sequence of rigid motions and bring the lesson to closure.

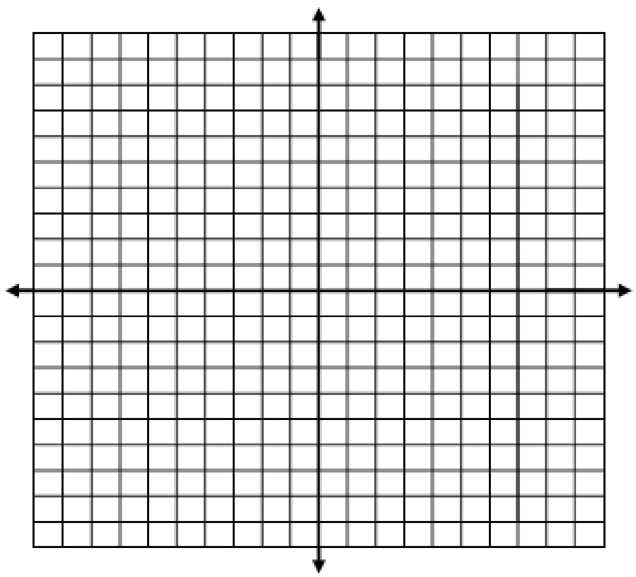
An answer key for the "Sequence of Rigid Motions: Cartesian Grid" Exercise can be found in Appendix F.

Name ___________________________________ Geometry

Date ___________________________________ Period: ______________

Rigid Motions – Sequence of Rigid Motions: Cartesian Grid Exercise

1. Using the graph below, graph ΔABC whose vertices are $A(2, 1)$, $B(7, 2)$, and $C(7, 8)$.
2. On the same graph, graph $\Delta A'B'C'$ whose vertices are $A'(-2, 1)$, $B'(-7, 2)$, and $C'(-7, 8)$
3. On the same graph, graph $\Delta''B''C''$ whose vertices are $A''(1, -4)$, $B''(-4, -3)$, and $C''(-4, 3)$
   (Be sure to label each vertex of each triangles with its letter identifier, and to connect the 3 vertices of each triangle.)



OBSERVATIONS

A. What relationship, if any, exists between the vertices of ΔABC and $\Delta A''B''C''$?
B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

Sequence of Rigid Motions: Closure

Some questions to facilitate the reflection process may include but are not limited to the following:

1. What is a sequence of rigid motions?
2. How did we verify that both triangles were the same size and shape?
3. What did you notice about the orientation of the two triangles?
4. How do we write a translation +5 units in the x-direction and -3 units in the y-direction followers by a reflection over the line y=x using symbolic notation?
5. Does the order matter when we write a sequence of rigid motions?

The lesson should take approximately one 45-minute class period.

Rigid Motions: Student Choice Project

To close out this unit, students will be given a choice of four projects to complete:

1. A translation tessellation
2. A rotation tessellation
3. A glide reflection tessellation
4. A tangram story book

This project has the potential to include cross-disciplinary collaboration between the art teacher and the math teacher. Students will receive grades in each class for parts of the project, with different deliverables for each class, and rubrics for each deliverable. Students should tap into their cultural roots, family lineage, personal interests and personal identity to create the tiles that will be used to make the tessellations, or to write the story they will tell with the tangram story book.

A description of the required components for each of the project options can be found below, followed by the rubrics that will be used to score the different student choice project options.

Tessellations

For the tessellation project option, students may choose between doing a rotation, translation, or guide reflection tessellation. Each of these tessellations uses different transformations to create beautiful pieces of art while making use of the properties of transformations and rigid motions.

Tessellations Artwork Deliverables

The student will submit the following deliverables for the tessellations project:

A. The first deliverable will be to craft the tile students will use to create the tessellation artwork. Students must submit the tile for a grade and approval before beginning to make copies of the tile on their large piece of paper/poster board. The student should write the type of transformation used for the tessellation on the back of the tile, as well as a description of what the tile image looks like to them.

B. The second deliverable will be a copy of the tessellation using four tessellated copies of their tile on an 8 ½ x 11 inch piece of paper. This will demonstrate that they know how to properly lay out the copies of the tile with no gaps. This will also be useful in identifying what type of tessellation it is using a smaller section of the artwork than a full page. The student should decorate one of the copies of the tile in order to demonstrate what the tile image is.

C. The third deliverable will be to complete a piece of artwork using a larger than 8 ½ x 11 sheet of paper as specified by their art teacher. The student should create a margin around the inside the paper, and tile from margin to margin with tiles. The student must include a title for the artwork, their name (they can use a pseudonym if so desired), and include a very brief paragraph describing to the viewer what they are looking at or should remember about the work when they see it. The tessellation tile should be included with the submission. The art teacher will specify the grading criteria for coloring, shading, etc. for the tessellation for the grade earned in the art class.

<u>Tessellations Artwork Reflection</u>**:**

Once the student completes their artwork in their art class, they will write a reflection essay addressing the following five prompts:

A. What was the cultural relevance, family lineage, personal interest or personal identity connection between them and their artwork?
B. How would you describe the process of creating your artwork?
C. What type of transformation was used in your tessellation, and why is that transformation a rigid motion?
D. What observations did you make about the properties of your transformation?
E. What could you have done differently, or what did you think about doing differently while creating your artwork?

Tangrams

For the tangram project option, students will generate figures using the 7 tangram pieces and then write a story about the figures they created. The story is to be well written and connected to the figures.

Tangrams Story Book Deliverables:

The student will submit the following deliverables for the tangram story book project:

A. The first deliverable will be a picture of the perfect square, with each of the 7 tans shown using dashed lines to indicate the borders, as well as cut-outs of the tans that fit over the dashed-line bordered pieces, the silhouette from one of the figures in their story, with dashed lines indicating the borders of the tans that make up the figure, with a list of the sequence of rigid motions that move the tans from the perfect square to the silhouette of the given figure. This page will not be included with the story book. Students should label tans in the square and in the tangram to show correspondence and movement of the tan pieces.

B. The second deliverable will be the completed tangram story book, with a minimum of 10 sets of tangrams. The art teacher will specify the grading criteria for coloring, shading, etc. for the tangram story book for the grade earned in the art class.

C. The third deliverable will be the visual answer key for the tangram puzzles, with the tan dividing lines identified. Solution tangrams need not be the same size as the originals. They can be reduction dilations.

Tangrams Story Book Reflection:

Once the student completes their story book, they will write a reflection essay addressing the following five prompts:

a. What was the cultural relevance, family lineage, personal interest or personal identity connection between them and your story book?

b. What were your biggest challenges in creating the story book?

c. What observations did you make about the properties of your transformations when you created the tangram figures, and why were those transformations rigid motions?
d. What strategy/strategies would you recommend to a player/participant who struggles as they try to solve your tangram puzzles?
e. What could you have done differently, or what did you consider doing differently while creating your story book?

Grand Finale

For the final step of the tessellation projects, artwork will go on display on the wall outside of the students' math class for a week, followed by going on display on the wall outside of the students' art class for a week.

For the final step of the story book project, the story books will go on display in the school library. Students will receive extra credit for participating in one of the following two story sharing options:

A. Students will participate in a community service activity where they will partner with a class of grade-level or developmentally appropriate students to read their stories to the students, and allow the students the opportunity to "solve" the puzzles of their tangram figures. Pencils should be used to "solve" the puzzles so that they can be used more than once.

B. Students will participate in an exchange program with a class of foreign students who will make similar tangram story books, swapping books with students from another country (or state within the U.S.). Additional credit will be given for establishing pen pal relationships with their student counterparts. Participation is contingent upon the ability of the teacher to establish a relationship with a teacher colleague in a foreign country (or state).

The rubrics for the project choices, as well as the reflection essay, are found on the following page.

Rigid Motions Student Choice Project: Tessellation Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Creativity, Completeness, and Attention to Detail	The artwork shows creativity, the tiles demonstrate the connectedness of the transformations, and the tiles fit together well.	The artwork lacks one of the following: creativity, the tiles demonstrate the connectedness of the transformations, the tiles fit together well.	The artwork lacks one of the following: creativity, the tiles demonstrate the connectedness of the transformations, the tiles fit together well.	The artwork does not successfully demonstrate any of the following: creativity, the tiles demonstrate the connectedness of the transformations, the tiles fit together well.	
Grammar and Spelling	The artwork description contains no spelling and no grammar errors.	The artwork description contains 1-2 spelling or grammar errors.	The artwork description contains 3-4 spelling or grammar errors.	The artwork description contains 5+ spelling or grammar errors.	
Deliverable 1 - Tessellation Tile	The tessellation tile has been correctly crafted, the type of transformation is labeled, and a description of the image is provided.	The tessellation tile lacks one of the following: the tile has been correctly crafted, the type of transformation is labeled, a description of the image is provided.	The tessellation tile lacks two of the following: the tile has been correctly crafted, the type of transformation is labeled, a description of the image is provided.	The tessellation tile does not successfully demonstrate any of the following: the tile has been correctly crafted, the type of transformation is labeled, a description of the image is provided.	
Deliverable 2 - Four Copy Tessellation Artwork	The submission should have 4 copies of the tessellation tile, all of the copies of the tile fit together properly, and one of the tiles is decorated.	The submission lacks one of the following: 4 copies of the tessellation tile, all of the copies of the tile fit together properly, one of the tiles is decorated.	The submission lacks two of the following: 4 copies of the tessellation tile, all of the copies of the tile fit together properly, one of the tiles is decorated.	The submission does not successfully demonstrate any of the following: 4 copies of the tessellation tile, all of the copies of the tile should fit together properly, one of	

				the tiles is decorated.	
Deliverable 3 - Completed Tessellation Artwork	The artwork is tiled from margin to margin, includes the student's name, a title, and a brief description.	The artwork lacks one of the following: tiled from margin to margin, includes the student's name, a title, and a brief description.	The artwork lacks any two of the following: tiled from margin to margin, includes the student's name, a title, and a brief description.	The artwork lacks three or more of the following: tiled from margin to margin, includes the student's name, a title, and a brief description.	
SCORE				TOTAL SCORE:	_____ / 20

Rigid Motions Student Choice Project: Tangram Story Book Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Grammar and Spelling	The story book contains no spelling and no grammar errors.	The story book contains 1-2 spelling or grammar errors.	The story book contains 3-4 spelling or grammar errors.	The story book contains 5+ spelling or grammar errors.	
Creativity and Completeness	The story book demonstrates artistic detail, words and other illustrations complement the tangrams, and each tangram uses all 7 tans.	The story book lacks one of the following: artistic detail, words and other illustrations to complement the tangrams, each tangram uses all 7 tans.	The story book lacks two of the following: artistic detail, words and other illustrations complement the tangrams, and each tangram uses all 7 tans.	The story book lacks any attempt at artistic detail, words and other illustrations to complement the tangrams, and one or more tangrams does not use all 7 tans.	
Deliverable 1 - Perfect Square to Tangram Write-up	The write-up contains a picture of the perfect square with tan cut-outs and dashed outlines for each tan, a picture of one of the tangram silhouettes from the story book, both pictures have their tans labeled, and the sequences of rigid motions that move the tans from the square to the tangram figure picture.	The write-up lacks one of the following: a picture of the perfect square with tan cut-outs and dashed outlines for each tan, a picture of one of the tangram silhouettes from the story book, both pictures have their tans labeled, and the sequences of rigid motions that move the tans from the square to the tangram figure picture.	The write-up lacks two of the following: a picture of the perfect square with tan cut-outs and dashed outlines for each tan, a picture of one of the tangram silhouettes from the story book, both pictures have their tans labeled, and the sequences of rigid motions that move the tans from the square to the tangram figure picture.	The write-up lacks three or more of the following: a picture of the perfect square with tan cut-outs and dashed outlines for each tan, a picture of one of the tangram silhouettes from the story book, both pictures have their tans labeled, and the sequences of rigid motions that move the tans from the square to the tangram figure picture.	

Deliverable 2 - Completed Story Book	The story book contains 10 sets of tangrams.	The story book contains 7-9 sets of tangrams.	The story book contains 4-6 sets of tangrams.	The story book contains 3 sets of tangrams or less.	
Deliverable 3 - Answer Key	The answer key contains solutions for all 10 sets of tangrams.	The answer key contains solutions for 7-9 sets of tangrams.	The answer key contains solutions for 4-6 sets of tangrams.	The answer key contains solutions for 3 sets of tangrams or less.	
SCORE				TOTAL SCORE:	_____ / 20

Rigid Motions Student Choice Project Reflection Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Organization and Structure	The student reflection is arranged in a logical order, with a clear introduction, body, and conclusion.	The student reflection is arranged in a somewhat logical order, with a clear introduction and body, but the conclusion isn't supported by the proceeding content.	The student reflection has a clear introduction, but follows no logical order within the body, and the conclusion is illogical.	The student reflection follows no clear nor logical order, does not communicate any clear statements of reflection, learning nor understanding, preventing the reader from gleaming anything from the reflection.	
Grammar and Spelling	The student response contains no spelling and no grammar errors.	The student response contains 1-2 spelling or grammar errors.	The student response contains 3-4 spelling or grammar errors.	The student response contains 5+ spelling or grammar errors, making the reflection difficult to read, or the student neglected to submit a reflection.	
Reflection Prompts Addressed	The student reflection addresses all 5 of the prompts.	The student reflection addresses 4 of the prompts.	The student reflection addresses 2-3 of the prompts.	The student reflection addresses 1 or fewer of the prompts.	
SCORE				TOTAL SCORE:	_____ / 12

Alternative Assessment 2:

Making Proofs Familiar and Accessible to All Students

This assessment is best used in a high school geometry classroom, with limited applications in a high school algebra classroom as well. In order to make the connection between the content and student life experiences, each proof starts with a story that may be common to the lives of many of the students that will engage with the content.

Unit	**Proofs**
Objectives	Designed for a unit on Congruence, Proof, and Constructions, or a unit on Similarity, Proof, and Trigonometry, including but not limited to the following unit objectives: 1. Students **can** prove two triangles are congruent using SSS, ASA, AAS, SAS, and HL. 2. Students **can** prove corresponding parts of congruent triangles are congruent. 3. Students **can** prove two triangles similar using AA, SAS, or SSS.
Common Core Standards	**Prove theorems about lines, angles, triangles, and parallelograms.** **HSG.CO.C.9.** Prove theorems about lines and angles. **HSG.CO.C.10.** Prove theorems about triangles. **HSG.CO.C.11.** Prove theorems about parallelograms. **Prove theorems involving similarity.** **HSG.SRT.B.4.** Prove theorems about triangles. **HSG.SRT.B.5.** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Next Gen Standards	**Prove and apply theorems about lines, angles, triangles, and parallelograms.** **GEO-G.CO.9.** Prove and apply theorems about lines and angles. **GEO-G.CO.10.** Prove and apply theorems about triangles. **GEO-G.CO.11.** Prove and apply theorems about parallelograms.

	Prove theorems involving similarity. **GEO-G.SRT.4.** Prove and apply similarity theorems about triangles. **GEO-G.SRT.5b.** Use congruence and similarity criteria for triangles to prove relationships in geometric figures.
Standards of Mathematical Practice	**MP1.** Make sense of problems and persevere in solving them. **MP3.** Construct viable arguments and critique the reasoning of others. **MP4.** Model with mathematics. **MP5.** Use appropriate tools strategically. **MP6.** Attend to precision.

Making Proofs Familiar and Accessible to All Students

The following assessment should be given as a pre-assessment to allow students to self-evaluate their knowledge of geometric proofs and how to develop and write them, before being formally introduced to the unit. The pre-assessment will also serve as a tool for teachers to provide insight into what students' perceptions are of their knowledge of proofs. Students will be given the same assessment as a post-assessment to measure their growth at the end of the unit.

It is important to note that one of the examples and one of the exercises in this assessment are predicated on the assumption that most students like cupcakes and ice cream. As the types of sweet treats may differ for your students, the teacher should modify the foods used in these stories to make them more culturally relevant to students in their region.

Name ______________________________ Geometry

Date ______________________________ Period: ____________

Proof Writing Pre/Post Assessment

Answer all questions.

1.	List the steps in the process of writing a proof. ________________________________ ________________________________ ________________________________ ________________________________ ________________________________ ________________________________ ________________________________ ________________________________ ________________________________
2.	List the 5 triangle congruence theorems/criteria. 1. ____________ 2. ____________ 3. ____________ 4. ____________ 5. ____________
3.	Identify your level of comfort writing proofs. (Circle one) 1. Say what? I am not comfortable at all writing proofs. 2. Meh. I am a little comfortable writing proofs. 3. So-so. I'm fairly comfortable writing proofs. 4. Holla! I am extremely comfortable writing proofs!

Proofs Pre/Post-Assessment Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Knowledge of the process	Student correctly identified all 6 steps	Student correctly identified 4-5 steps	Student correctly identified 2-3 steps	Student correctly identified 1 step or less	
Knowledge of Triangle Congruence Theorems	Student correctly identified all 5 theorems	Student correctly identified 3-4 theorems	Student correctly identified 2 theorems	Student correctly identified 1 theorem or less	
Proof Writing Comfortability Self-Assessment (4=extremely comfortable, 1=not comfortable at all)	Student identified themselves as a 4	Student identified themselves as a 3	Student identified themselves as a 2	Students identified themselves as a 1	
SCORE				TOTAL SCORE:	_____ / 12

Cupcakes

This geometric proof curriculum was designed to familiarize students with the process of writing geometric proofs by connecting the academic activity with an authentic life experience. The first example, “The Curious Case of the Missing Cupcakes”, is designed to be a teacher-led, student-centered exercise. The teacher should facilitate the process of guiding students through the steps, starting by twice reading the story to the students, the first time to get the students familiar with the story, and the second time instructing students to look for context clues in the story that might help students solve the case. The teacher might even ask the students what they think is going to happen in the story simply based on the title of the story. The enthusiasm of the teacher should be clearly visible to the students, as well as the level of fun and engaged interest in the story/activity. Once the teacher has read the story to the students two times, the teacher should guide the students through answering the questions proceeding the story. (Responses to the questions have been provided.) Once the questions have all been answered, it is time to pull all of the data together and present the case. The teacher is invited to get animated as they are presenting the case to the students. This will further display the level of excitement and interest the teacher has in the activity, and model engaged interest in solving proofs for the students. The language in the presentation has been written to engage the students using the language/dialect many students may experience in their own homes, not to model proper English. This is meant to provide the students with an authentic experience that they can relate to, making the activity more personal for them (Thomas & Berry III, 2019). Once the presentation of the case has been concluded, the teacher will guide students through a reflection process where students will revisit the steps for proving something, in this case *what happened to the cupcakes*? The steps will be identified as the same steps that students will use when writing geometric proofs. Lastly, the teacher will ask follow-up questions to further help the students understand the process and retain the steps. The lesson should take approximately one 45-minute class period. A handout to be provided to students can be found in Appendix G.

THE CURIOUS CASE OF THE MISSING CUPCAKES

The story…

Darrell is going over to his friend Crystal's house for a birthday party. He likes Crystal, so he wants to make a good impression on her. He irons his best fit and lays it out for the party the next day, and asks his mother to make a pan of chocolate cupcakes, Crystal's favorite! His mother wakes up early and makes the cupcakes before she leaves for her 6 AM shift at her first job. She texts Darrell from work to let Darrell know that she made the cupcakes and they are in the fridge. There is also a box of candles, next to the cupcakes. Darrell was up playing Fortnite until 3:00 in the morning, so he doesn't see the text until he wakes up at noon. He rushes to the fridge to look at the cupcakes but they aren't there! He finds a clean cupcake pan drying on a dish towel, the open box of candles on the kitchen table next to some chocolate cupcake crumbs and a half empty strawberry melon Brisk bottle, his younger brother Nathan's favorite drink! And it was still a little cold! Darrell thinks he knows what happened, but his suspect (Nathan), who never sleeps late unless he eats too much, was asleep! What happened to the cupcakes? How can Darrell prove it?

Let's start off by identifying our goal – "What are we trying to prove?"

- Nathan ate all of the cupcakes!

What do we know? (What evidence do we have?)

1. Darrell's mother made the cupcakes before she left for work.
2. She put the cupcakes in the refrigerator, along with the candles.
3. The cupcake pan was drying on a towel.

4. There was a half empty Brisk on the kitchen table next to some chocolate cupcake crumbs, and it was still cold.
5. Nathan only sleeps late when he eats too much, and he was asleep!

What else do we need to know, or would help us prove our case?

A. A confession! (Nathan is **not** going to just confess! He doesn't want to get in trouble.)
B. Proof that Nathan ate the cupcakes
    a. Crumbs around his mouth
    b. Chocolate on his fingers
    c. Candles in his possession

How do we prove what we don't know?

A. Sneak into Nathan's room and
    a. Look for the missing candles – found them on the floor in Nathan's room next to his bed, with chocolate on them!
    b. Check his fingers for chocolate (he's sloppy. He wouldn't have washed them. Take a picture with your phone)
    c. Check his mouth for cupcake crumbs. Nope, too nasty! Let mama do it!

Present our case!

Darrell's and Nathan's mother comes home from work at 3:00 to grab something to eat and check on the boys before going to her second job from 4-midnight. As soon as she walks in the door, she asks Darrell if he thinks Crystal will like the cupcakes she made before work, and Darrell tells her that Nathan ate the cupcakes! Of course, in typical Nathan fashion, he always tries to use a lie, Nathan says, "I ain't eat no chocolate cupcakes!" (notice Darrell didn't SAY they were chocolate…), so now Darrell has to prove it. Darrell presents his evidence:

1. The cupcakes are gone and his mother said she made them. You don't call mama a liar.
2. The cupcake pan was on the towel drying. Nathan is the only one that washes the dishes. That's one of his chores for the house. He does it without even thinking about it because he's been doing it for so long. It's the only thing he's clean about, "washing them dishes"!
3. There were chocolate cupcake crumbs next to a cold Brisk on the kitchen table. Nathan is the only one in the house that drinks strawberry melon Brisk. Darrell and his mother like the sweet tea Brisk.
4. I found these 7 candles on the floor next to Nathan's bed with chocolate on them. 7 is Nathan's favorite number!
5. Nathan also had chocolate on his fingers. Look! I took a picture with my phone.
6. "And I bet he still has chocolate crumbs on his mouth, cuz he nasty!"

Mama says to Nathan, "com'ere boy!" She wipes his mouth and when she looks at her hand, she sees chocolate! "Let me see those nasty-behind hands of yours! I done told you about washing your hands!" He had chocolate on his fingers too! "I'm going to give you one last chance to tell the truth, or else you're going to get in even MORE trouble! Did you eat your brother's cupcakes?" Nathan lowers his head and says, "yes mama." She tells Nathan, "this what we gon' do. I'm going to take $20 out of YOUR school clothes money, and give to your brother so he can go to the store and buy that girl some cupcakes and a birthday card. You gon have to get one less pair of pants this year. Also, you are grounded for two weeks for lying, and for touching what don't belong to you – that means no video games, and no cell phone. Give it here! And I better not see you nowhere near no controller! Last but not least, you are going to apologize to your brother for eating his little girlfriend's cupcakes". "Mama, she's not my girl..." "Shut up boy! Apologize Nathan!" "I'm sorry Darrell. I shouldn't have ate your STUPID cupcakes!" "Say it like you mean it and don't add nothin' else to it before I whoop your tail!" "I'm sorry Darrell." Darrell, go into my room and get the shoebox that says "Nathan" on it, and bring me $20, and it better just be $20 you take. Y'all know your mama don't like no liars and I don't like no thieves!" Darrell goes and gets the money and leaves for the party, Nathan goes in the living room to watch Netflix, and mama heats up some leftovers and rushes to eat so she won't be late for her second job.

Let's reflect on our story, and our process for proving something. What were our steps?

1. Identify our goal. (What are we trying to prove?)
2. What do we know? (What evidence are we given?)
3. What else do we need to know? (What other evidence do we need? How are we going to get that evidence?)
4. What is our plan for how we are going to prove our claim? (What is the path from what we know to what we want to prove? What are the steps?)
5. Present our case. (Give our statements and their reasons. When we have enough that there is logic to get us from what we were given to what we want to prove, we have *enough* statements (and reasons)
6. Assert our conclusion. (State our claim, our last statement, with the one final reason that ties everything else together)

Follow-up questions:

What did we need to start our case?

- We needed to know what we ultimately wanted to prove in the first place
- We needed some evidence that was given to us to start us down our path

Once we knew what we wanted to prove, and we had some information that was given to us, what did we do next?

- We had to figure out a plan to get us from what we were given, to having enough information to prove our conclusion.

How did we come up with the other statements we needed to prove our claim?

- We had to figure out what statements would have to be true if the reason for our claim was true.
- We found the reasons that all of those statements were true

Once we had all of our statements and reasons that made our statements true, what did we do?

- We listed all of our statements with all of the reasons that made them true.

What would have NOT been enough information to backup/substantiate our claim?

- How many of those steps were really required?
- Was there any information that we were given, or could have found, that while it would have been true, would not have helped us prove our case?

What was our last step?

- We presented our claim/conclusion and the reason that our claim was ultimately true.

Ice Cream

In the first example the teacher modeled the entire process of "writing" a proof that logically and systematically leads to a conclusion. In the second exercise, "The Incredible Issue of India's Ice Cream", students will engage in a similar story to the cupcake story, but this time students will work together in groups to produce their own proofs. This exercise is a student-led, student-centered exercise. Students should be separated into groups of 4 members maximum, using previously established group roles and group norms for completing group work. The teacher's role is to: assign groups; provide copies of the story and the steps for proving something; circulate around the room to make sure that students are staying on task; and answer any questions that students may have. Students should be encouraged to work collaboratively in their groups and only seek teacher guidance when necessary. Students are encouraged to use their own creativity in generating questions and responses needed throughout the proof process. After assigning groups and handing out the copies of the materials, the handout and highlighters, student groups should be given approximately 15 minutes to produce their proof. Once appropriate time has elapsed, the presenter from each group should be prompted to present the findings and conclusions of the group to the class. After the presentation of the cases is concluded, the teacher will guide students through a reflection process by revisiting the steps for proving something, and in this case "what happened to the whipped cream?" The steps will be identified as the same steps that students will use when writing geometric proofs. Lastly, the teacher will ask follow-up questions to further help students understand the process and retain the steps. The lesson should take approximately one 45-minute class period. A handout to be provided to students can be found in Appendix H. Sample responses to the questions answered during the process can be found in Appendix I. Lastly, teachers are encouraged, if possible both financially and logistically, to provide little ice cream cups to the students as a prize for completing their first proof independent of the teacher.

THE INCREDIBLE ISSUE OF INDIA'S ICE CREAM

The story…

India LOVES ice cream! India loves ice cream like she loves breathing! Nothing in the world makes her happier than to have some ice cream! She loves ice cream in the summer. She loves ice cream in the winter. She loves ice cream in the spring. She loves ice cream in the fall. India LOOOOOVES ice cream! Whenever she goes shopping with her dad and her brother, she makes sure to get 3 different flavors of ice cream. Because she loves ice cream so much, it is the one thing her dad never says "no" to. Her favorite flavors are strawberry, chocolate, and cookie dough. One day India had a really bad day at school. She got a math test back and found out that she got a D, even though she really thought she got like a B on it. When she got home she really needed something to make her feel better, and she knew exactly what worked for her every time...ice cream. Her plan when she got home was to make a banana split with all three of her favorite flavors, a banana, some chocolate syrup and some whipped cream. When her dad got home from work, she was going to be honest with him and tell him everything that happened that day. Her and her dad had built a good relationship since her mom passed away the year before, and Christmas break was coming up soon. When she got home, she took off her coat, took off her boots, washed her hands, and headed to the kitchen. When she opened up the fridge, there was no more whipped cream, the most important topping on her banana split! She could smell hot chocolate though, and there was a mug in the kitchen sink. Her brother Miguel was in the living room asleep on the couch. He got home from school before India, because his school lets out earlier, and he only has to take one bus. India always stays after school so she takes the late bus. India thinks she knows what happened to the whipped cream, but she knows Miguel won't tell the truth if she confronts him about it. What happened to the whipped cream? How can India prove it?

This is your time to shine on your own!

Your steps are:

1. Identify your goal (What are you trying to **prove**?)
2. What do you know? (What evidence are you **given**?)
3. What else do you need to know? (What other evidence do you need? How are you going to get that evidence?)
4. What is your **plan** for how you are going to prove your claim? (What is the path from what you know to what you want to prove? What are the steps/statements?)
5. Present your case. (Give your **statements** and your **reasons**. When you have enough that there is logic to get you from what you were given to what you want to prove, you have *enough* statements (and reasons)).
6. Assert your conclusion. (State your claim, your last statement, with the one final reason that ties everything else together.)

"Cupcakes and Ice Cream": Closure

The one huge difference between the "Cupcakes" and the "Ice Cream" is that there was no negative consequence at the end of "Ice Cream" story, because there was nothing implied that said Miguel **couldn't** use the last of the whipped cream! We did prove what we were trying to prove however. The process was still EXACTLY THE SAME! Because this is the second exercise, the students are working more independently, so the teacher may need to provide more support for this exercise, if needed.

When we prove something in real life, we are not always trying to work towards some consequence, or to restore some balance to the world. Sometimes we are just looking for information. We just want to find something out!

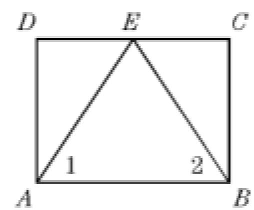
The next step is to do a geometric proof. The goal is to show students that the steps between the real-life examples, and the geometric proof are EXACTLY THE SAME, with the possible addition of drawing a picture. It's all about deductive reasoning!

SAS Triangle Congruence Example 1

We will now guide our students through their first geometric proof. The first example, "Side-Angle-Side Triangle Congruence Example 1", is designed to be a teacher-led, student-centered exercise. The teacher should facilitate the process of guiding students through the same steps we used with "Cupcakes" and "Ice Cream"

Given: rectangle $ABCD$ with E, the midpoint of $\overline{DC}$.

Prove: $\angle 1 \cong \angle 2$



Our steps are:

1. Identify your goal (What are you trying to **prove**?)
    a. We want to prove that $\angle 1 \cong \angle 2$. In order to do this, we have to define what it means for two angles to be "congruent". This means that the $m\angle 1 = m\angle 2$.
2. What do you know? (What evidence are you **given**? LWYK! DAP!)
    a. We have been provided with two pieces of information
        i. ABCD is a rectangle
            1. Since ABCD is a rectangle we know that all pairs of opposite sides are the same length, so $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$.
            2. We also know that the four angles of the rectangle are all right angles, and therefore they are all congruent (equal in measure), so $m\angle ABC = m\angle BCD = m\angle CDA = m\angle DAB = 90°$.

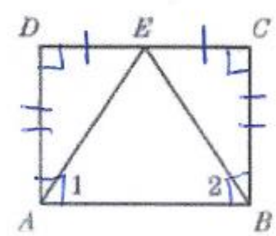
iii. E is the midpoint of line segment DC.

1. A “midpoint” of a line segment divides the line segment into two congruent segments, meaning they have the same length. (Make sure to be very clear here because many students have trouble differentiating between sides and angles, as well as how to write the length of a side, refer to a line segment, the measure of an angle, and how to refer to a specific angle in a picture using the correct vertices.) This means that $\overline{DE} \cong \overline{EC}$.

iv. Now that we have all of our evidence, we want to do something very special with what we know, and that is to **Label What You Know (LWYK)**, and if possible, **Draw A Picture (DAP)**. We are going to take our picture and mark it up with our evidence.

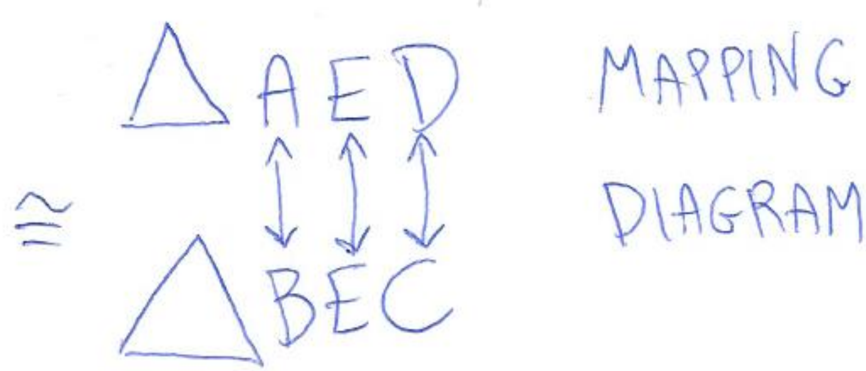
Given: rectangle $ABCD$ with E, the midpoint of $\overline{DC}$.

Prove: $\angle 1 \cong \angle 2$



We now have all of our congruent sides, congruent line segments, and congruent angles marked up on our picture.

3. What else do you need to know? (What other evidence do you need? How are you going to get that evidence?)
    a. In this case if we want to prove that $\angle 1 \cong \angle 2$ we have to **show** that $\triangle AEB$ is the kind of triangle that has two angles that are congruent. What type of triangle has two congruent angles, and as a result the sides opposite those angles are congruent? An "isosceles" triangle! This means that we have to **show** $\overline{AE} \cong \overline{EB}$.
4. What is your **plan** for how you are going to prove your claim? (What is the path from what you know to what you want to prove? What are the steps/statements?)
    a. Our plan is to show that $\triangle AED \cong \triangle BEC$. Remind students that the order of the vertices matters. In order to do that we are going to use the first of five triangle congruence criteria that we will study - Side-Angle-Side Congruence. We start with a mapping diagram, which establishes a relationship between the corresponding parts of the triangles we want to prove congruent.

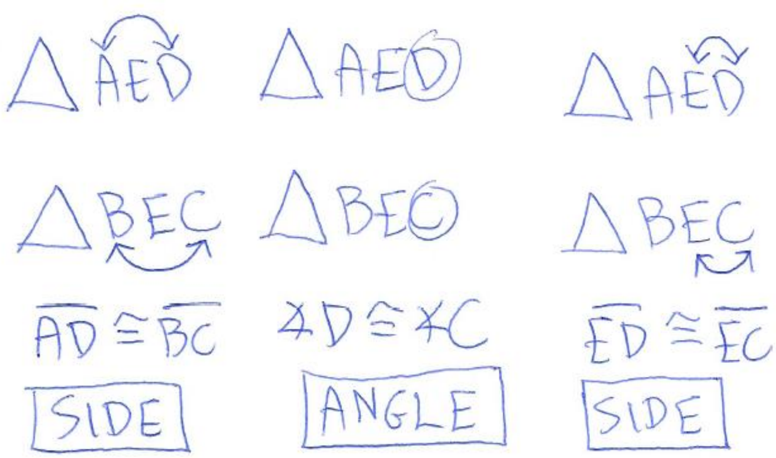


The mapping diagram is important because it gives us the corresponding parts of each of the triangles, which will be VERY useful in our proof.

CORRESPONDING SIDES	CORRESPONDING ANGLES
$\overline{AE}$ corresponds to $\overline{BE}$	$\angle DAE$ corresponds to $\angle CBE$
$\overline{ED}$ corresponds to $\overline{EC}$	$\angle D$ corresponds to $\angle C$
$\overline{AD}$ corresponds to $\overline{BC}$	$\angle DEA$ corresponds to $\angle CEB$

These corresponding sides and corresponding angles are important to us because we will use them to demonstrate our Side-Angle-Side congruence theorem (SAS).

With SAS it is important to note that the pair of corresponding angles have to connect the pairs of corresponding sides. That is why we show the correspondence the way we do in the following picture:



We are now ready for the next steps in writing our proof...

5. Present your case. (Give your **statements** and your **reasons**. When you have enough that there is logic to get you from what you were given to what you want to prove, you have *enough* statements (and reasons)).
6. Assert your conclusion. (State your claim, your last statement, with the one final reason that ties everything else together.)

We will combine steps 5 and 6, making asserting our conclusion the last step of presenting our case. We will provide two different representations of presenting our case - the more popular t-chart, and the paragraph format more commonly found in college proofs courses.

T-Chart Proof

STATEMENTS	REASONS
1. Rectangle ABCD, E is the midpoint of $\overline{DC}$	1. Given
2. $\overline{AD} \cong \overline{BC}$	2. Properties of rectangles (opposite sides of rectangles are congruent)
3. $\angle D$ is a right angle $\angle C$ is a right angle	3. Properties of rectangles (all interior angles of a rectangle are right angles)
4. $\angle D \cong \angle C$	4. All right angles are congruent
5. $\overline{DE} \cong \overline{EC}$	5. Definition of a midpoint (a midpoint divides a segment into two congruent segments)
6. $\triangle AED \cong \triangle BEC$	6. Side-Angle-Side is congruent to Side-Angle-Side (SAS)
7. $\overline{AE} \cong \overline{EB}$	7. Corresponding parts of congruent triangles are congruent. (CPCTC)
8. $\triangle AEB$ is an isosceles triangle	8. Definition of an isosceles triangle (a triangle with two congruent sides is isosceles)
9. $\angle 1 \cong \angle 2$	9. Base angles (opposite congruent sides) of an isosceles triangle are congruent.

Statements and reasons 1 through 8 present our case (step 5), and step 9 asserts our conclusion (step 6).

Paragraph Proof (tell the story that leads to the desired conclusion)

We were given the statements that ABCD is a rectangle, and E is the midpoint of $\overline{DC}$. Since E is the midpoint of $\overline{DC}$, E divides $\overline{DC}$ into two congruent pieces, which means that the $\overline{DE} \cong \overline{EC}$. Since ABCD is a rectangle, and opposite sides of a rectangle are congruent, that means $\overline{AD} \cong \overline{BC}$ because they are opposite sides of a rectangle. It also means that $\angle D$ and $\angle C$ are right angles, because all of the interior angles of a rectangle are right angles. Since $\angle D$ and $\angle C$are both right angles, it also means that $\angle D \cong \angle C$ because all right angles are congruent. We can now say that $\triangle AED \cong \triangle BEC$ using the Side-Angle-Side (SAS) triangle congruence theorem. Since $\triangle AED \cong \triangle BEC$, we know that $\overline{AE} \cong \overline{BE}$ because corresponding parts of congruent triangles are congruent. This means that $\triangle AEB$ is an isosceles triangle. Since $\angle 1$ and $\angle 2$ are base angles across from the two congruent sides of an isosceles triangle, $\angle 1 \cong \angle 2$. //

Sentences 1 through 8 of our paragraph present our case (step 5), and sentence 9 asserts our conclusion (step 6).

Let's now reflect on our process for writing a geometric proof to show that two angles are congruent. What were our steps?

A. Identify our goal. (What are we trying to prove?)
    a. $\angle 1 \cong \angle 2$
B. What do we know? (What evidence are we given? LWYK! DAP!)
    a. ABCD is a rectangle, and E is the midpoint of $\overline{DC}$.
C. What else do we need to know? (What other evidence do we need? How are we going to get that evidence?)
    a. We need to know that triangle $\triangle AED \cong \triangle BEC$ so that we can show that $\overline{AE} \cong \overline{BE}$, and that $\triangle AEB$ is an isosceles triangle.
D. What is our plan for how we are going to prove our claim? (What is the path from what we know to what we want to prove? What are the steps?)
    a. We are going to show that $\triangle AED \cong \triangle BEC$ using SAS, show that $\triangle AEB$ is isosceles, and then conclude that $\angle 1 \cong \angle 2$ because they are base angles of an isosceles triangle.
E. Present our case. (Give our statements and their reasons. When we have enough that there is logic to get us from what we were given to what we want to prove, we have *enough* statements (and reasons)
    a. We did this using our Statement-Reasons t-chart, and using a paragraph proof.

F. Assert our conclusion. (State our claim, our last statement, with the one final reason that ties everything else together)
    a. Both our Statements-Reasons t-chart and our paragraph proof lead us to the conclusion we were trying to prove.

Follow-up questions: (notice that these follow-up questions and the answers to them are EXACTLY the same as the follow-up questions and answers we gave for the "Cupcakes" example!)

What did we need to start our case?

- We needed to know what we ultimately wanted to prove in the first place
- We needed some evidence that was given to us to start us down our path

Once we knew what we wanted to prove, and we had some information that was given to us, what did we do next?

- We had to figure out a plan to get us from what we were given, to having enough information to prove our conclusion.

How did we come up with the other statements we needed to prove our claim?

- We had to figure out what statements would have to be true if the reason for our claim was true.
- We found the reasons that all of those statements were true

Once we had all of our statements and reasons that made our statements true, what did we do?

- We listed all of our statements with all of the reasons that made them true.

What would have NOT been enough information to backup/substantiate our claim?

- How many of those steps were really required?
- Was there any information that we were given, or could have found, that while it would have been true, would not have helped us prove our case?

What was our last step?

- We presented our claim/conclusion and the reason that our claim was ultimately true.

SAS Triangle Congruence Example 1: Closure

In our first geometric proof example, we wanted to show students how to write a geometric proof, but the goal was to show them that the steps between the real-life examples, and the geometric proof are EXACTLY THE SAME, with the possible addition of drawing a picture. We did not draw a picture for this example, but we did draw a mapping diagram and created a correspondence table. The correspondence table was a critical component of writing our proof. Not only did it allow us to visually improve the conceptualization of the corresponding sides and angles, key components to showing the triangles congruent using SAS, but it also allowed us to simultaneously introduce the concept of CPCTC (Corresponding Parts of Congruent Triangles are Congruent). CPCTC is typically taught as a separate lesson, but incorporating it into the teaching of the SAS theorem shows students where it comes from and how to use it. This will help to prevent future exercises where students know that at the end of their proof they want to show that two corresponding parts of congruent triangles are congruent, but don’t understand the logic behind the concept so they draw a t-chart, state the conclusion, and write "CPCTC" as the reason without including the logical statements and reasons to arrive at that conclusion.

SAS Triangle Congruence Proof Exercise 1

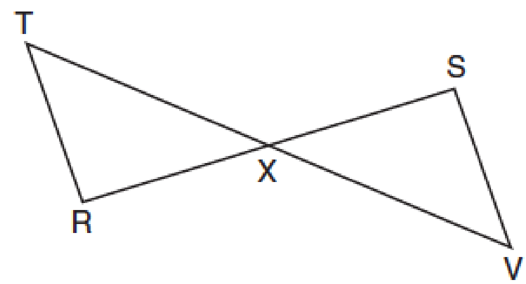
In the first example the teacher modeled the entire process of writing a proof that logically and systematically leads to a conclusion. In the first exercise, SAS Triangle Congruence Proof Example 1, students will engage in writing a similar proof to Example 1, but this time students will work together in groups to produce their own proofs. This exercise is a student-led, student-centered exercise. Students should be separated into groups of 4 members maximum, using previously established group roles and group norms for completing group work. The teacher's role is to: assign groups; provide copies of the example, the scoring rubric, and the steps for proving something; to circulate around the room to make sure that students are staying on task; and answer any questions that students may have. Students should be encouraged to work collaboratively in their groups and rely on the teacher only when necessary. Students are encouraged to use their knowledge of geometric theorems and postulates in coming up with some of the responses they need to the questions asked throughout the proof process. Student groups should be given approximately 10 minutes to produce their proof. Once appropriate time has elapsed, the presenter from each group should be prompted to present the findings and conclusions of the group to the class. Once the presentation of the proofs has been concluded, the teacher should guide students through a reflection process where students will revisit the steps for writing a Euclidean geometry proof. Lastly, the teacher will ask follow-up questions to further help the students understand the process and retain the information. Students will then complete one more exercise, where they will write a Euclidean geometry proof on their own. The lesson should take approximately one 45-minute class period. An answer key for Proofs Exercise 1 and Exercise 2 can be found in Appendix J and Appendix K, respectively. A copy of the student scoring rubric is provided in the table below. The rubric has been created generic enough for either type of proof. Teachers should use their own professional judgment to modify the rubrics as needed.

Student Proofs Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Knowledge of the Process	Student work shows evidence of all of the steps of the proof writing process	Student work lacks evidence of 1-2 steps of the proof writing process	Student work lacks evidence of 3-4 steps of the proof writing process	Student work lacks evidence of 5+steps of the proof writing process	
Logical Structure	Student work follows a logical flow from beginning to end	Student work contains 1 statement that cannot be reached based on preceding statements	Student work contains 2-3 statements that cannot be reached based on preceding statements	Student work contains 3+ statements that cannot be reached based on preceding statements	
Completeness	Student work reaches the desired conclusion, without leaving out any necessary statements and/or reasons, and contains no incorrect statements and/or reasons.	Student work does not reach the desired conclusion, leaves out necessary statements and/or reasons, or contains incorrect statements and/or reasons.	Student work results in the desired conclusion, but the conclusion is not supported with the necessary correct statements and/or reasons.	Student work does not reach the desired conclusion, leaves out necessary statements and/or reasons, and contains incorrect statements and/or reasons.	
SCORE				TOTAL SCORE:	_____ / 12

SAS Triangle Congruence Proof Exercise 1 - Group Work (June 2017 New York State Geometry Regents Exam question # 33)

33 Given: $\overline{RS}$ and $\overline{TV}$ bisect each other at point X
$\overline{TR}$ and $\overline{SV}$ are drawn



Prove: $\overline{TR} \parallel \overline{SV}$

Teamwork makes the dream work!

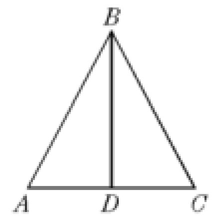
Your steps are:

1. Identify your goal (What are you trying to **prove**?)
2. What do you know? (What evidence are you **given**? **LWYK! DAP!**)
3. What else do you need to know? (What other evidence do you need? How are you going to get that evidence?)
4. What is your **plan** for how you are going to prove your claim? (What is the path from what you know to what you want to prove? What are the steps/statements?)
5. Present your case. (Give your **statements** and your **reasons**. When you have enough that there is logic to get you from what you were given to what you want to prove, you have *enough* statements (and reasons)).
6. Assert your conclusion. (State your claim, your last statement, with the one final reason that ties everything else together.)

SAS Triangle Congruence Proof Exercise 2 - Independent Work

Given: $\triangle ABC$, $\overline{BD}$ is both the median and the altitude of $\overline{AC}$.

Prove: $\overline{BA} \cong \overline{BC}$



This is your time to shine on your own!

Your steps are:

1. Identify your goal (What are you trying to **prove**?)
2. What do you know? (What evidence are you **given**? **LWYK! DAP!**)
3. What else do you need to know? (What other evidence do you need? How are you going to get that evidence?)
4. What is your **plan** for how you are going to prove your claim? (What is the path from what you know to what you want to prove? What are the steps/statements?)
5. Present your case. (Give your **statements** and your **reasons**. When you have enough that there is logic to get you from what you were given to what you want to prove, you have *enough* statements (and reasons)).
6. Assert your conclusion. (State your claim, your last statement, with the one final reason that ties everything else together.)

Answer keys for Proofs Exercise 1 and Proofs Exercise 2 can be found in Appendix J and Appendix K respectively.

Alternative Assessment 3:

Mathematical Literacy – The "Lit" Project

While the focus of this curriculum project is geometry, some of the assessments will fit nicely with other areas of mathematics, such as this mathematical literacy project. Every mathematics course requires students to understand mathematical language used in the discussion of the content. This assessment was designed for grades 7-12 but can be used in any mathematics classroom. Mathematical literacy and its associated fluency can be improved using the pedagogical practices at any grade level.

Unit	The Lit Project
Objectives	Designed to supplement a year of mathematics instruction, including but not limited to the following yearly objectives: 1. Students **will** improve mathematical fluency, and their ability to communicate mathematical concepts 2. Students **will** improve their ability to create mathematical models of real-world problems. 3. Students **can** find an entry point for the solution to mathematical problems by translating mathematical concepts and language into ideas and language that are familiar to them. Students **will** create a tool that will help them solve mathematical problems independently Designed to supplement a year of mathematics instruction, including but not limited to the following yearly objectives: 4. Students will improve mathematical fluency, and their ability to communicate mathematical concepts 5. Students will improve their ability to create mathematical models of real-world problems. 6. Students can find an entry point for the solution to mathematical problems by translating mathematical concepts and language into ideas and language that are familiar to them. 7. Students will create a tool that will help them solve mathematical problems independently.

Standards of Mathematical Practice	**MP1.** Make sense of problems and persevere in solving them. **MP4.** Model with mathematics. **MP5.** Use appropriate tools strategically. **MP6.** Attend to precision.

Mathematical Literacy – The "Lit" Project

The following rubric should be used as a guide for students to use to help them as they develop each chapter of the "Lit" project. The goal is to provide them with direction as well as a tool for students to self-evaluate the progress they are making throughout the chapters. The rubric will also be used as a tool by teachers to share feedback with students on the comprehensiveness of the work done on the "Lit" project. In this manner, the project will be a learning collaboration between teacher and student. The project will be a "living document", designed to continually be added to during the course of the school year, and may serve as a model for students to use in subsequent school years, emphasizing the importance of student note taking skills, attention to detail, synthesis of topics, and as a reference when students are completing independent and/or group work. Students should be encouraged to use the project and the feedback received on it as a way to take ownership of their education. Teachers should especially highlight the fact that the quality and thoughtfulness put into the project will determine its usefulness to the student. Students need to understand that the amount of work put into the project will determine how useful the project proves to be to them. As a result of using the rubric, students should see an improvement in the assessment of their work with each chapter submission and teacher feedback. Students should be encouraged to use the rubric to self-evaluate prior to submitting each iteration of the project to their teacher. After each chapter is returned students should compare their self-evaluation with teacher feedback using direct comparison of performance ratings or through dialogue with the teacher. To facilitate this process, teachers should distribute a copy of the rubric to students prior to students submitting the project for the teacher to score. A copy of the rubric should be placed inside each students' binder so that it is always available for them to reference. The rubric should be used in cooperation with a unit test as an alternative assessment to evaluate student academic growth during that unit. The chapter and the unit test should be equally weighted. This will provide students who do not typically do well on pen-and-paper assessments with an opportunity to still perform well on the unit. The chapter should also serve as a study guide to the student, and as such should be scored and returned to the student prior to the unit test. Students should be encouraged to include pictures/images when applicable, to help visualize and demonstrate concepts or vocabulary. Students should also create sample exercises, perhaps even sharing them with classmates, to demonstrate mastery of the content.

The Rubric

The *Lit Project* is designed to be used throughout the school year. The *Student Chapter Scoring Rubric* is designed to be used for each chapter or unit of content. The *Comprehensive Project Scoring Rubric* is designed to be used for each reporting term of the school year, as well as part of the final class grade for the school year. A sample submission of the *Lit Project* can be found in Appendix B.

Student Chapter Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Chapter topic is well-defined	Chapter has a title, title reflects the content, and appears in the TOC	Chapter title is missing, title doesn't reflect content, or does not appear in the TOC. (1 of 3)	Chapter title is missing, doesn't reflect content, or does not appear in the TOC. (2 of 3)	Chapter title is missing, and nothing appears in the TOC.	
Sections within the chapter are well-defined	All sections have a title that reflects the content, and section title appears in the TOC	1 or more section titles missing, not reflective of content, or does not appear in the TOC. (1 of 3)	1 or more section titles missing, does not reflect the content, or does not appear in the TOC. (2 of 3)	All section titles missing, and nothing appears in the TOC for any section.	
Pages are clearly labeled	All pages are numbered, including chapter number. Both appear consistently in the same section of the document (header/footer).	1 or more pages missing numbers, or chapter name not listed in the same section of document (header/footer). (1 out of 3)	1 or more pages missing page numbers and chapter name does not appear in the same section of document (header/footer). (2 out of 3)	All pages missing page numbers, name of chapter does not appear in the header/footer	
Mathematical vocabulary is clearly denoted	Vocabulary is clearly marked, has a reference number, and is defined in the glossary	Vocabulary is not clearly marked, does not have a reference number, or is not defined in the glossary. (1 out of 3)	Vocabulary is not clearly marked, does not have a reference number, or is not defined in the glossary. (2 out of 3)	No vocabulary denoted.	
Pictures and sample exercises are included (BONUS POINTS)		Pictures, sample exercises, and solutions are included.	Pictures and sample exercises, or sample exercises with solutions included.	Pictures or sample exercises included (1 of 2)	**Pictures and sample exercises are included (BONUS POINTS)**
SCORE				TOTAL SCORE:	_____ / 16

Comprehensive Project Scoring Rubric

PERFORMANCE LEVELS					
Performance Ratings	**4 Points - Proficient**	**3 Points – Some signs of proficiency**	**2 Points – Limited signs of proficiency**	**1 Point – Remediation or Assistance required**	**Points Awarded Per Performance Rating**
Table of Contents	Every section of the project is documented in the TOC with a section name, section number, and starting page number.	All major sections of the project are documented in the TOC with a section name, section number, and starting page. Minor sections are missing.	All major sections of the project are documented with a section name, but the section numbers or starting page numbers are missing.	One or more major sections of the project are missing.	
Appendix	All supporting answer keys and other supplemental information referenced are included in the appendix.	75% or more of answer keys and other supplemental information referenced are included in the appendix.	25-74% of answer keys and other supplemental information referenced are included in the appendix.	Less than 25% of answer keys and other supplemental information referenced are included in the appendix.	
Glossary	All words and phrases that have been footnoted are included in the glossary, in alphabetical order, including their definitions.	75% or more of words and phrases that have been footnoted are included in the glossary, in alphabetical order, including their definitions.	25 - 74% of words and phrases that have been footnoted are included in the glossary, in alphabetical order, including definitions.	Less than 25% of words and phrases that have been footnoted are included in the glossary, no visible attempt to put the words and terms in alphabetical order, or definitions missing.	
Growth	Student scores per chapter maintained the same level or improved with each chapter submission	Student scores per chapter submission are trending upwards, with a few scores that decreased.	Students scores per chapter submission are trending downwards, with a few scores that increased.	Student scores per chapter decreased with each chapter submission.	
SCORE				TOTAL SCORE:	_____ / 16

My Personal Suggestions for the Classroom

Culturally relevant pedagogy is a great concept, but to make it a reality for your students you have to create a climate in your classroom that let's students know that their humanity is valued just as much as their academic ability, if not more. Even if you teach a specific subject as is typical for the urban high school classroom, it will help you and your students to help them acquire the soft skills that don't directly translate to better test scores. One thing I try to do at least once a month is have "Mindset Mondays". These are days when we have lessons that relate to topics that go beyond mathematics content. Some Mindset Monday topics I've used are "Goal Setting", "Bringing Your A Game", "Never Give Up", and "Knock It Out (Four Fours)". If you think about the titles of these lessons, many of them are simply spending an entire class period having some fun working on the Standards for Mathematical Practice in a non-mathematical context. This is also an excellent opportunity to allow students to suggest topics that connect to their cultural and personal interests. This can generate data that the teacher can use to drive instruction.

The teacher may also elect to have a "Culture Day" in the classroom with some frequency. Every Thursday is "Culture Day" in my classroom, where I would wear African attire, or the attire of some other culture known to me, and the music played in the classroom that day was the music of another culture. This was a great way to connect with students of Caribbean or Latinx descent. On the remaining days of the week other genres of music would get played, with the occasional student guest DJ selecting the songs played during independent work time.

Teacher enthusiasm can come in the form of enthusiasm about the subject of mathematics or enthusiasm about teaching mathematics. When trying to decide which of these two types of enthusiasm to display in the classroom choose both! Have fun teaching, have fun with the math, have fun with the students! Cooper (2014, p. 363) reported that in a survey of 275,000 U.S. high school students conducted from 2006 to 2009, 65% reported being bored at least once a day in school. Don't let that be your class! Teachers can achieve this by simply being ourselves and sharing our passion with our students. They want and need to see that we are human, just like them. Showing our passion, showing what attracted us to math, showing what attracts us to teaching, is an excellent way to do that. For more ideas on connective instructive practices and involvement in learning mathematics, read the articles by Cooper (2014) and Turner et al. (1998) in the references section.

It is also important to allow students to show their passion, and to share that with each other as well as the teacher. The student output in this work dealing with transformations is a great way to allow students to do that. The tangram story in particular is a great way for students to share their work with other students in their classes, as well as younger students or students of limited ability in other grades, programs, buildings, or even sister-city schools and programs. A tangram story exchange with students from another city, state, or even country would be a great way for students to interact with students from a culture different than theirs, as well as expand their worldview! I am currently beginning the process of establishing a connection with students in a school in Kenya where my students will exchange tangram stories with those students. There are some logistical issues to work out, but I am excited about the possibilities for my students!

Conclusion

Geometry is a unique area of mathematics that lends itself to many visual representations of its content. This can result in many natural connections between the content and the lived experiences of students. By connecting the content to the culture, language, and identities of students, learning the content becomes more personal for the students, which has a positive effect on student engagement. The enthusiasm for the content displayed by the teacher can bridge the gap between geometry existing as a required subject and geometry being a subject that students pursue with vigor. That is when we can move from student engagement to student involvement in learning mathematics. It is the author's hope that sharing this work with other teachers provides them with valuable resources they can use in their classrooms, with their students, and that helps their students achieve greater success in geometry and mathematics as a whole. We want students to learn, and to receive all of the benefits that the world and life has to offer as a result of that learning.

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Appendix

Appendix A - List of Programs/Apps for Showing Math work Digitally

LIST OF PROGRAMS/APPS FOR SHOWING MATH WORK DIGITALLY

- Bakpak
- Capture Thought
- Classkick
- DESMOS
- Educreation
- Equatio
- Explain Everything
- Flipgrid
- Floop
- GoFormative
- Google Drawing
- Google Forms
- Google Slides/Docs
- Jamboard
- Kami
- Keynote
- Microsoft Word
- Nearpod
- Notability
- OneNote
- Paint Program
- Peardeck
- Screencastify
- SeeSaw
- Showbie
- Smart Learning Suite
- Vittle
- Webpaint
- Whiteboard.fi
- WootMath
- Zoom

PROGRAM/APP DESCRIPTIONS

Bakpak

Bakpax uses artificial intelligence to read handwriting and grade assignments in seconds. It saves you time, provides students with instant feedback, and gives you deeper insights into class performance.

Capture Thought

Capture Thought uses your own smartphone to record videos and turn any place into a virtual classroom.

Classkick

In 1:1 or group settings, students input drawings, text, images, and audio or answer fill-the-blank or multiple choice in response to teacher-created material.

DESMOS

Help every student learn math and love learning math. With that in mind, we've assembled a collection of unique and engaging digital activities.

Educreation

Record your voice and iPad® screen to create dynamic video lessons that students and colleagues can access any time, as needed.

Equatio

Easy-to-use extension for Google Chrome. It's a perfect partner for Google Docs plus Sheets, Forms, Slides and Drawings' - letting you add mathematical equations, formulas and more to documents with a click.

Explain Everything

The online interactive whiteboard app where people share and learn without boundaries. Join from any device and collaborate in real-time

Flipgrid

Flipgrid is a website that allows teachers to create "grids" to facilitate video discussions. Each grid is lik a message board where teachers can pose questions, called "topics," and their students can pos video responses that appear in a tiled grid display.

Floop

Floop is a cloud-based website where students can receive annotated feedback from teachers and peers.

GoFormative

GoFormative, is a web-based tool that allows teachers to create digital formative assessments, tasks, or assignments that are easily accessible from any electronic device

Google Drawing

Google Drawings allows users to collaborate and work together in real time to create flowcharts, organisational charts, website wireframes, mind maps, concept maps, and other types of diagrams.

Google Forms

Create a required question and choose file upload for work

Google Slides/Docs

Have students take a picture of their work and insert into slide or doc

Jamboard

Jamboard is one smart display. Quickly pull in images from a Google search, use the easy-to-read handwriting and shape recognition tool, and draw with a stylus but erase with your finger – just like a whiteboard.

PROGRAM/APP DESCRIPTIONS

↣ Kami

Kami is an online document annotation and markup tool. You can highlight, underline, and strikethrough text in PDF and other document formats.

↣ Keynote

Keynote is slideshow presentation software developed by Apple. You can use it to create a well-designed presentation.

↣ Microsoft Word

Use the inking tool within Microsoft Word

↣ Nearpod

Nearpod is a student engagement platform that can be used to amazing effect in the classroom. The concept is simple. A teacher can create presentations that can contain Quiz's, Polls, Videos, Images, Drawing-Boards, Web Content and so on.

↣ Notability

Notability: powerful, yet wonderfully simple note-taking and PDF annotation.

↣ OneNote

Microsoft OneNote is a note-taking program for free-form information gathering and multi-user collaboration. It gathers users' notes, drawings, screen clippings, and audio commentaries. Notes can be shared with other OneNote users over the Internet or a network.

↣ Paint Program

A graphics program that enables you to draw pictures on the display screen which are represented as bit maps. In addition to these tools, paint programs also provide easy ways to draw common shapes such as straight lines, rectangles, circles, and ovals.

↣ Peardeck

Pear Deck is an interactive presentation tool used to actively engage students in individual and social learning. Teachers create presentations using their Google Drive account.

↣ Screencastify

Screencastify is a Chrome browser extension that records your screen, face, voice, and more. To use Screencastify, find its icon in the Chrome toolbar and choose among the recording options: record a single tab in your web browser, capture all screen activity, or use your webcam to record or insert a video of yourself.

↣ SeeSaw

Seesaw is a platform for student engagement. Teachers can empower students to create, reflect, share, and collaborate. Students "show what they know" using photos, videos, drawings, text, PDFs, and links. It's simple to get student work in one place and share with families, and nothing is shared without teacher approval.

↣ Showbie

Showbie offers many tools that you can use to utilize your classroom iPads to their maximum potential. Once you have created an assignment, students can submit their work to you via Showbie. You can also return it to your eager young minds complete with feedback.

↣ Smart Learning Suite

SMART Learning Suite Online is a tool that transforms static lesson delivery with game-based activities, formative assessments, and student collaboration to enhance learning experiences on any device.

PROGRAM/APP DESCRIPTIONS

↣ Vittle

Use Vittle to create video lectures and flip your classroom. Get your point across exactly and in an impactful way. Collaborate across time and space like never before. Videos that you own and control. Vittle produces HD video files that you own and fully control.

↣ Webpaint

Web Paint provides the following easy to use drawing tools that let you draw shapes, lines, and add text to live web pages and take screenshot (touch screen supported): Pencil tool - draw a custom line with the selected line width and color.

↣ Whiteboard.fi

Whiteboard.fi is a simple tool that can be used instantly. By creating a class and letting your students join, everyone will get a digital whiteboard. Whiteboard.fi is an instant formative assessment tool for your classroom, providing you with live feedback and immediate overview over your students.

↣ WootMath

Woot Math provides engaging, research-based tools to help teachers reach more students and help all students deeply understand mathematics.

↣ Zoom

Use the whiteboard feature within the zoom app.

Appendix B - The Lit Project (Sample)

KEVIN'S LIT GEOMETRY PROJECT

BY

Kevin Murphy

Kevin Murphy

655 Colfax Street

585-317-9038

2012790@rcsd121.org

www.rcsdk12.org/Domain/9098

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Glossary G-1

CHAPTER 1 - INTRODUCTION

1.1 What is this?

In this project you will be creating a study-guide, a dictionary, and a reference book, all of the things that you should need to be successful in geometry this year. This is where you will highlight important notes, define vocabulary terms that will be important this year, yet may not be part of your everyday language, pictures that visually describe concepts, terms, theorems[1], and **postulates**[2]. All of these will be used to help make you a better geometry student and achieve greater success than you would without them.

1.2 Introductory Vocabulary

Below you will find some vocabulary terms that are provided to help start you off with your **glossary**[3], as well as to introduce you to some of the terms and concepts we will be covering this year. You do not need to limit your glossary to mathematical vocabulary. Your glossary can and should be used to define any word that you see, hear, read, or use this school year.

<u>Vocabulary Words</u>

Mean

Congruent[4]

Similar[5]

Rigid motion[6]

Median (statistics)[7]

Median (geometry)[8]

---

[1] Theorems - a statement that can be demonstrated to be true using accepted mathematical operations and arguments.
[2] Postulate - a statement that is taken to be true, to serve as a starting point for further reasoning and arguments.
[3] Glossary - an alphabetical list of words/terms found in or relating to a specific subject or text.
[4] Congruent - two figures or objects that have the same size and shape.
[5] Similar - two figures or objects that have the same shape and corresponding sides are proportional.
[6] Rigid motion - a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations, which preserve distances and angle measures.
[7] Median (statistics) - a measure of the center in a set of numerical data. The numbers in the set are listed in ascending order. If there are an odd number of values, the median is the middle number. If there are an even number of values, the median is the average of the two middle numbers.
[8] Median (geometry) - a line segment that starts at a vertex of a triangle and extends to the midpoint of the side opposite that vertex.

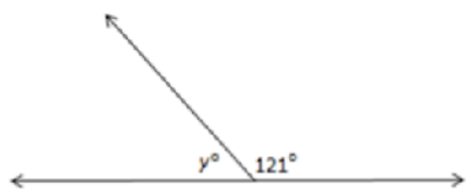
1.3 Introductory Images

Below you will find some images that are provided to help start you off with visualizing some of the terms and concepts we will be covering in geometry. Do not worry if any or all of them are foreign to you right now. They will not be by the time the school year is over!

SUPPLEMENTARY ANGLES/LINEAR PAIRS

The sum of the angles on a straight line is 180° and two such angles are called a **linear pair.** Two angles are called ***supplementary*** if the sum of their measures is ____________.

2. **Determine the measure of the missing angle in the diagram.**



In the picture above, the angle labeled y^o and the angle labeled 121^o form a **linear pair**[9] because the two angles form a straight line. Since a straight line has 180^o in it, the two angles are **supplementary**[10]. We will master these two concepts later on in the year. (BUT, if you were to solve for *y* now, how do you think you would do it? Think back to 1-step equations in algebra…)

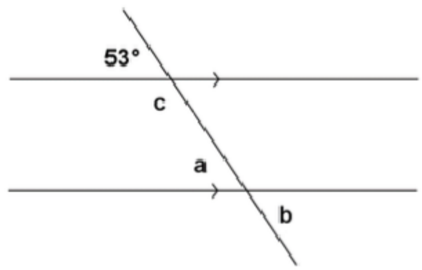
[9] Linear pair - two adjacent angles that form a straight line (180 degrees).
[10] Supplementary - two angles that form a linear pair.

1.4 Introductory Sample Exercises

In this section you will be shown two sample exercises, and you are invited to create a sample exercise of your own dealing with each concept.

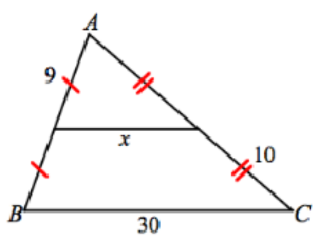
1.4.1 Transversals[11] Sample

In the exercise below, find the unknown (labeled) angle. Give reasons for your solution.



1.4.2 Midsegments[12] Sample

Find the value of *x* and the perimeter of triangle ABC.



[11] Transversal - a line that cuts across two or more parallel lines.
[12] Midsegment - a line segment connecting the midpoints of two sides of a triangle.

1.5 Making It Plain/Translating Vocabulary

In this section we will look at some mathematical terminology, and then some everyday ways of expressing the same idea/concept. While it is important for you to know the mathematical terminology, it is more important for you to know the concepts the terminology describes.

MATHEMATICAL TERMINOLOGY	EVERYDAY LANGUAGE
Vertical angles[13]	Two equal angles across from each other when two lines make an X
Parallel Lines[14]	Two lines that never meet and go like this (makes a motion with fingers)
Dilation[15]	Makes a shape grow or shrink, like when you go to the eye doctor and the put the drops in your eyes your eyeballs get big

[13] Vertical angles - each of the pairs of opposite angles made by two intersecting lines.
[14] Parallel lines - two straight lines in a plane that do not intersect at any point.
[15] Dilation - a transformation that produces an image that is the same shape as the original but a different size.

GLOSSARY

WORD/PHRASE	DEFINITION
Congruent	two figures or objects that have the same size and shape.
Dilation	a transformation that produces an image that is the same shape as the original but a different size
Glossary	an alphabetical list of words/terms found in or relating to a specific subject or text.
Linear Pair	two adjacent angles that form a straight line (180 degrees).
Median (geometry)	a line segment that starts at a vertex of a triangle and extends to the midpoint of the side opposite that vertex.
Median (statistics)	a measure of the center in a set of numerical data. The numbers in the set are listed in ascending order. If there are an odd number of values, the median is the middle number. If there are an even number of values, the median is the average of the two middle numbers.
Midsegment	a line segment connecting the midpoints of two sides of a trian
Parallel Lines	two straight lines in a plane that do not intersect at any point
Postulate	a statement that is taken to be true, to serve as a starting point for further reasoning and arguments.
Rigid Motion	a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations, which preserve distances and angle measures.
Similar	two figures or objects that have the same shape and corresponding sides are proportional.
Supplementary (angles)	two angles that form a linear pair.
Theorems	a statement that can be demonstrated to be true using accepted mathematical operations and arguments.
Transversal	a line that cuts across two or more parallel lines.
Vertical Angles	each of the pairs of opposite angles made by two intersecting line

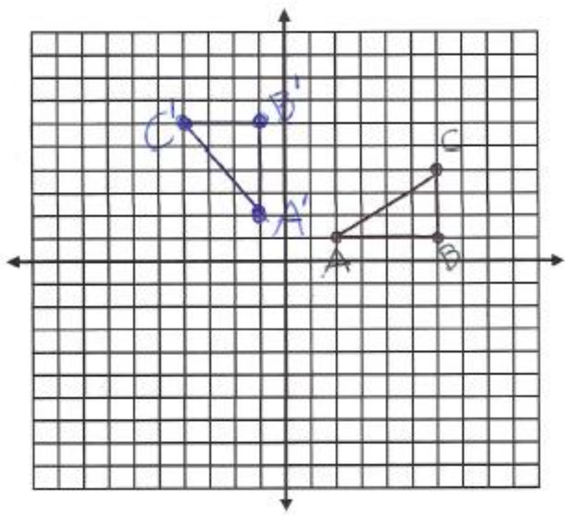
Appendix C - "Rigid Motions Rotations Cartesian Coordinates Answer Key"

Name TEACHER ANSWER KEY Geometry

Date ______________________ Period: __________

Rigid Motions – Rotations: Cartesian Coordinates Exercise

1. Using the graph below, graph ΔABC whose vertices are $A(2, 1)$, $B(6, 1)$, and $C(6, 4)$.
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices
2. On the same graph, graph $\Delta A'B'C'$ whose vertices are $A'(-1, 2)$, $B'(-1, 6)$, and $C'(-4, 6)$
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices



OBSERVATIONS

A. What do you notice about the relationship between the vertices of ΔABC and $\Delta A'B'C'$?

A(2,1) $(x, y) \rightarrow (-y, x)$

A'(-1,2) numbers swapped and then sign of 1st # changed

DO NOT TEACH MEMORIZATION OF THIS. TEACH ROTATION OF PAPER.

B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

YES. IF YOU TRACE ΔABC ON THE PATTY PAPER AND ROTATE 1/4 TURN (90°) COUNTERCLOCKWISE, ΔABC WILL REST ON TOP OF ΔA'B'C'.

1

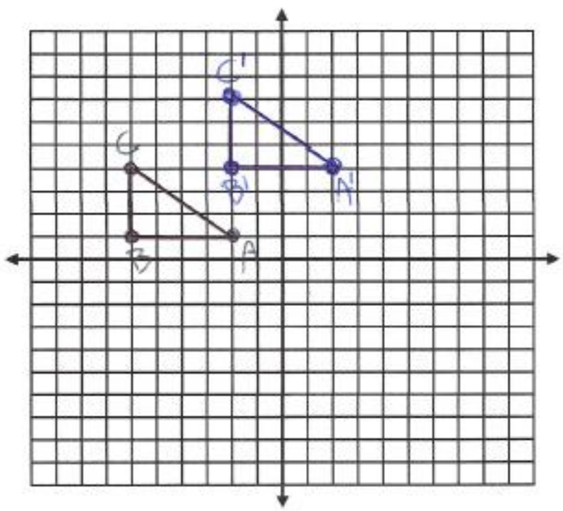
Appendix D - "Rigid Motions Translations Cartesian Coordinates Answer Key"

Name TEACHER ANSWER KEY Geometry

Date ____________________ Period: __________

Rigid Motions – Translation: Cartesian Coordinates Exercise

1. Using the graph below, graph ΔABC whose vertices are A(-2, 1), B(-6, 1), and C(-6, 4).
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices
2. On the same graph, graph $\Delta A'B'C'$ whose vertices are A'(2, 4), B'(-2, 4), and C'(-2, 7)
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices



OBSERVATIONS

A. What do you notice about the relationship between the vertices of ΔABC and $\Delta A'B'C'$?

A(-2,1) (x+4, y+3) A'(2,4)
B(-6,1) (x+4, y+3) B'(-2,4)
C(-6,4) (x+4, y+3) C'(-2,7)

each vertex is shifted right 4 units and up 3 units

B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

YES. IF YOU TRACE ΔABC ON THE PATTY PAPER, AND SLIDE IT UP AND TO THE RIGHT, ΔABC WILL REST ON TOP OF $\Delta A'B'C'$.

1

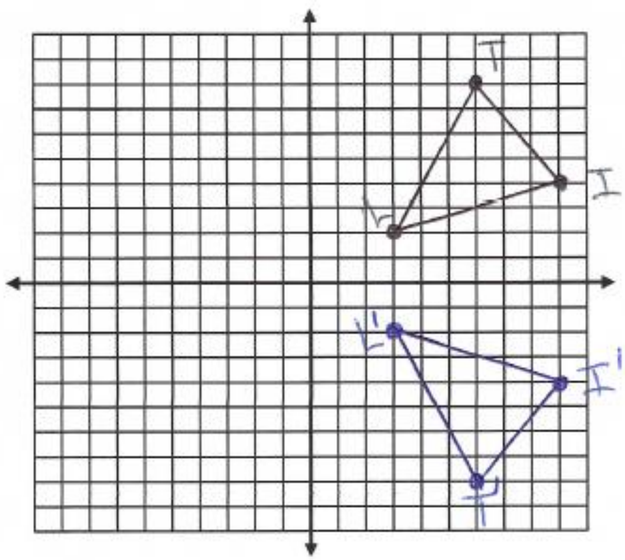
Appendix E - "Rigid Motions Reflections Cartesian Coordinates Answer Key"

Name TEACHER ANSWER KEY Geometry

Date ______ Period: ______

Rigid Motions – Reflection: Cartesian Coordinates Exercise

1. Using the graph below, graph ΔLIT whose vertices are L(3, 2), I(9, 4), and T(6, 8).
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices
2. On the same graph, graph $\Delta L'I'T'$ whose vertices are L'(3, -2), I'(9, -4), and T'(6, -8)
   a. Be sure to label each vertex with its letter identifier, and to connect the 3 vertices



OBSERVATIONS

A. What do you notice about the relationship between the vertices of ΔLIT and $\Delta L'I'T'$?

L(3,2) - THE SIGN OF THE Y-COORDINATE SWITCHED.
L'(3,-2) - L AND L' ARE THE SAME NUMBER OF BOXES FROM THE X-AXIS.

B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape?

YES! IF YOU TRACE ΔLIT ON THE PATTY PAPER, AND FOLD THE PATTY PAPER AT THE X-AXIS, ΔLIT WILL REST ON TOP OF $\Delta L'I'T'$.

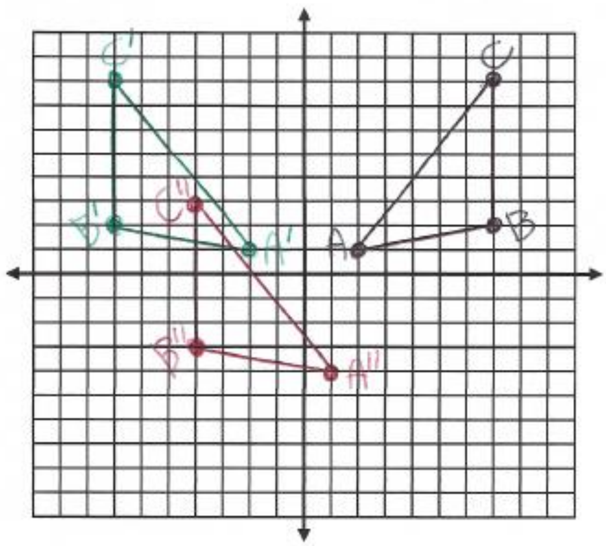
Appendix F - "Sequences of Rigid Motions: Cartesian Grid Answer Key"

Name TEACHER ANSWER KEY Geometry

Date ______________________ Period: ______________

Rigid Motions – Sequence of Rigid Motions: Cartesian Grid Exercise

1. Using the graph below, graph ΔABC whose vertices are A(2, 1), B(7, 2), and C(7, 8).
2. On the same graph, graph $\Delta A'B'C'$ whose vertices are A'(-2, 1), B'(-7, 2), and C'(-7, 8)
3. On the same graph, graph $\Delta''B''C''$ whose vertices are A"(1, -4), B"(-4, -3), and C"(-4, 3)
   (Be sure to label each vertex of each triangles with its letter identifier, and to connect the 3 vertices of each triangle.)



OBSERVATIONS

A. What relationship, if any, exists between the vertices of ΔABC and $\Delta A''B''C''$? THERE IS NO OBVIOUS NUMERICAL RELATIONSHIP BETWEEN THE TRIANGLES. A(2,1) ↓??? A''(1,-4)
B. Are both triangles the same size and shape? How do you think you could prove they are or are not the same size and shape? YES. IF YOU TRACE ΔABC ON THE PATTY PAPER, FOLD THE PATTY PAPER AT THE Y-AXIS, AND THEN SLIDE ΔABC DOWN AND TO THE RIGHT, YOU CAN GET ΔABC TO REST ON TOP OF $\Delta A''B''C''$.

1

Appendix G - "The Curious Case of the Missing Cupcakes" (Student Handout)

"THE CURIOUS CASE OF THE MISSING CUPCAKES"

The story…

Darrell is going over to his friend Crystal's house for a birthday party. He likes Crystal, so he wants to make a good impression on her. He irons his best fit and lays it out for the party the next day, and asks his mother to make a pan of chocolate cupcakes, Crystal's favorite! His mother wakes up early and makes the cupcakes before she leaves for her 6am shift at her first job. She texts Darrell from work to let Darrell know that she made the cupcakes and they are in the fridge. There is also a box of candles, next to the cupcakes. Darrell was up playing Fortnite until 3:00 in the morning, so he doesn't see the text until he wakes up at noon. He rushes to the fridge to look at the cupcakes but they aren't there! He finds a clean cupcake pan drying on a dish towel, the open box of candles on the kitchen table next to some chocolate cupcake crumbs and a half empty strawberry melon Brisk bottle, his younger brother Nathan's favorite drink! And it was still a little cold! Darrell thinks he knows what happened, but his suspect (Nathan), who never sleeps late unless he eats too much, was asleep! What happened to the cupcakes? How can Darrell prove it?

Let's reflect on our story, and our process for proving something. What were our steps?

1. Identify our goal. (What are we trying to prove?)
2. What do we know? (What evidence are we given?)
3. What else do we need to know? (What other evidence do we need? How are we going to get that evidence?)
4. What is our plan for how we are going to prove our claim? (What is the path from what we know to what we want to prove? What are the steps?)
5. Present our case. (Give our statements and their reasons. When we have enough that there is logic to get us from what we were given to what we want to prove, we have *enough* statements (and reasons)
6. Assert our conclusion. (State our claim, our last statement, with the one final reason that ties everything else together)

Appendix H - "The Incredible Issue of India's Ice Cream" (Student Handout)

"THE INCREDIBLE ISSUE OF INDIA'S ICE CREAM"

The story…

India LOVES ice cream! India loves ice cream like she loves breathing! Nothing in the world makes her happier than to have some ice cream! She loves ice cream in the summer. She loves ice cream in the winter. She loves ice cream in the spring. She loves ice cream in the fall. India LOOOOOVES ice cream! Whenever she goes shopping with her dad and her brother, she makes sure to get 3 different flavors of ice cream. Because she loves ice cream so much, it is the one thing her dad never says "no" to. Her favorite flavors are strawberry, chocolate, and cookie dough. One day India had a really bad day at school. She got a math test back and found out that she got a D, even though she really thought she got like a B on it. When she got home she really needed something to make her feel better, and she knew exactly what worked for her every time...ice cream. Her plan was when she got home to make a banana split with all three of her favorite flavors, a banana, some chocolate syrup and some whipped cream, and when her dad got home from work, she was going to be honest with him and tell him everything that happened that day. Her and her dad had built a good relationship since her mom passed away the year before, and Christmas break was coming up soon. When she got home, she took off her coat, took off her boots, washed her hands, and headed to the kitchen. When she opened up the fridge, there was no more whipped cream, the most important topping on her banana split! She could smell hot chocolate though, and there was a mug in the kitchen sink. Her brother Miguel was in the living room asleep on the couch. He got home from school before India, because his school lets out earlier, and he only has to take one bus. India always stays after school so she takes the late bus. India thinks she knows what happened to the whipped cream, but she knows Miguel won't tell the truth if she confronts him about it. What happened to the whipped cream? How can India prove it?

This is your time to shine on your own!

Your steps are:

1. Identify your goal (What are you trying to **prove**?)
2. What do you know? (What evidence are you **given**?)
3. What else do you need to know? (What other evidence do you need? How are you going to get that evidence?)
4. What is your **plan** for how you are going to prove your claim? (What is the path from what you know to what you want to prove? What are the steps/statements?
5. Present your case. (Give your **statements** and your **reasons**. When you have enough that there is logic to get you from what you were given to what you want to prove, you have *enough* statements (and reasons)).
6. Assert your conclusion. (State your claim, your last statement, with the one final reason that ties everything else together.)

Appendix I - "Ice Cream" Sample Responses

1. What are you trying to prove?

    a. Miguel used the whipped cream on his hot chocolate.

2. What do you know?

    a. India smelled hot chocolate when she came home.

    b. There was a mug in the sink.

    c. Miguel gets home before India.

    d. India has to stay(?) after school.

    e. It's a cold time of the year, because Christmas break was coming up soon, and India was wearing boots and a coat!

3. What else do you need to know? How are you going to get this evidence?

    a. Was there whipped cream in the refrigerator?

    b. Does Miguel use whipped cream in his hot cocoa?

    c. Ask Miguel? But he's a liar!

    d. Check around the cup (mug) for a whipped cream ring!

4. What is your plan for proving your claim? How do you get from what you know to what you want to prove?

    a. Find out if the father still has the receipt, to see if they bought whipped cream.

    b. See if Miguel has a whipped cream moustache!

    c. When her father gets home, ask him if HE used the whipped cream!

5. Present your case.

   a. When her father gets home from work, India asks him if he has the receipt from grocery shopping. Ask him if HE used the last of whipped cream. If her father used the whipped cream, there is no reason to get Miguel involved, or find any other evidence.

      i. No receipt

         1. Do you remember buying whipped cream when we went grocery shopping?

      ii. Has the receipt

         1. Check the receipt to see if they bought whipped cream.

      iii. They definitely bought whipped cream.

         1. Ask her father if he used the last of the whipped cream.

         2. What size was the whipped cream, and when did they buy it?

      iv. Her father says he didn't use the last of the whipped cream.

         1. Now she tells her father that Miguel used the last of the whipped cream!

            a. State that Miguel used the last of the whipped cream in his hot chocolate.

            b. She knows this because there is a mug in the sink.

         2. What if Miguel DID use the last of the whipped cream?

            a. They just gotta buy some more.

         3. They still haven't confronted Miguel yet!

a. They ask Miguel if he ate the last of the whipped cream.

b. Miguel says yes.

i. This is what we were trying to prove!

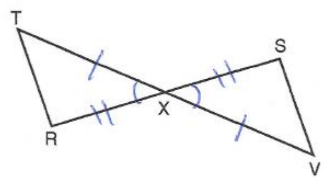
Appendix J - Proofs Exercise 1 Answer Key

Proofs Exercise 1 T-Chart

33 Given: $\overline{RS}$ and $\overline{TV}$ bisect each other at point X

$\overline{TR}$ and $\overline{SV}$ are drawn

LWYK!



Prove: $\overline{TR} \parallel \overline{SV}$

PLAN: SHOW · 1) $\triangle TXR \cong \triangle VXS$ 2) $\angle T \cong \angle V$

$\triangle TXR \cong \triangle VXS$

STATEMENTS	REASONS
1) $\overline{RS}$ and $\overline{TV}$ bisect each other at point X	1) GIVEN
2) $\overline{TX} \cong \overline{XV}$	2) DEFINITION OF BISECT.
3) $\angle TXR \cong \angle VXS$	3) VERTICAL ANGLES ARE $\cong$.
4) $\overline{RX} \cong \overline{XS}$	4) DEFINITION OF BISECT
5) $\triangle TXR \cong \triangle VXS$	5) SIDE-ANGLE-SIDE
6) $\angle T \cong \angle V$	6) CPCTC
7) $\overline{TR} \parallel \overline{SV}$	7) IF TWO LINES ARE CUT BY A TRANSVERSAL SUCH THAT ALTERNATE INTERIOR $\angle$s ARE $\cong$, THE LINES ARE $\parallel$.

Geometry (Common Core) – June '17 [20]

Proofs Exercise 1 Paragraph Proof

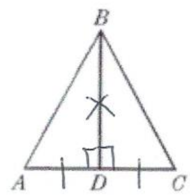
We are given that $\overline{RS}$ and $\overline{TV}$ bisect each other at point X. Since $\overline{RS}$ bisects $\overline{TV}$, $\overline{TX} \cong \overline{XV}$ by the definition of bisect. $\angle TXR$ and $\angle VXS$ are vertical angles because segments $\overline{RS}$ and $\overline{TV}$ intersect at point X, which means that $\angle TXR \cong \angle VXS$. Since $\overline{TV}$ bisects $\overline{RS}$, $\overline{RX} \cong \overline{XS}$ by the definition of bisect. This gives us $\triangle TXR \cong \triangle VXS$ by the Side-Angle-Side congruence theorem. $\angle T \cong \angle V$are congruent because corresponding parts of congruent triangles are congruent. If two lines are cut by a transversal such that alternate interior angles are congruent, then the two lines are parallel, therefore $\overline{TR} \parallel \overline{SV}$. //

Appendix K - Proofs Exercise 2 Answer Key

Proofs Exercise 2 T-Chart

Given: $\triangle ABC$, $\overline{BD}$ is both the median and the altitude of $\overline{AC}$.

Prove: $\overline{BA} \cong \overline{BC}$



LWYK!

PLAN: SHOW 1) $\triangle BDA \cong \triangle BDC$

$\triangle BDA \cong \triangle BDC$

STATEMENTS	REASONS
1) $\triangle ABC$, $\overline{BD}$ is both the median and altitude of $\overline{AC}$	1) GIVEN
2) $\overline{BD} \cong \overline{BD}$	2) REFLEXIVE PROPERTY (BOTH $\triangle$s SHARE THE SAME SIDE)
3) $\angle BDA$ is a right $\angle$ $\angle BDC$ is a right $\angle$	3) DEFINITION OF ALTITUDE (PERPENDICULAR TO A SEGMENT)
4) $\angle BDA \cong \angle BDC$	4) ALL RIGHT $\angle$s ARE $\cong$.
5) $\overline{AD} \cong \overline{DC}$	5) DEFINITION OF MEDIAN (LINE SEGMENT JOINING A VERTEX TO THE MIDPOINT OF A LINE SEG.)
6) $\triangle BDA \cong \triangle BDC$	6) SIDE-ANGLE-SIDE
7) $\overline{BA} \cong \overline{BC}$	7) CPCTC

Proofs Exercise 2 Paragraph Proof

We are given the ABC is a triangle with $\overline{BD}$ as a median and an altitude of $\overline{AC}$. Since $\triangle BDA$ and $\triangle BDC$ BDC both share $\overline{BD}$, $\overline{BD} \cong \overline{BD}$ (Reflexive property). $\angle BDA$ and $\angle BDC$ both right angles because of the definition of an altitude. Since $\angle BDA$ and $\angle BDC$ are both right angles, $\angle BDA \cong \angle BDC$ because all right angles are congruent. Given that $\overline{BD}$ is also a median of $\overline{AC}$, it divides $\overline{AC}$ into $\overline{AD}$ and $\overline{DC}$, and $\overline{AD} \cong \overline{DC}$ by the definition of a median. We then arrive at $\triangle BDA \cong \triangle BDC$ by the Side-Angle-Side congruence theorem. Since at $\triangle BDA \cong \triangle BDC$, $\overline{BA} \cong \overline{BC}$ because corresponding parts of congruent triangles are congruent (CPCTC). //

Appendix L - Reflection Tessellation Sample Images

Name Plate Pre-Image (Original)

- Don't worry about cutting exactly on the lines. Draw the lines in pencil and you can always erase the lines later if you don't cut exactly on the line.
- If you have a letter with a closed loop, like a "D", "P", "Q", or "R", fold the loop in half and then cut the loop out).
- Don't be afraid to mess up a little. It's good to model for students that they can make mistakes.

Name Plate Image (Copy/Reflection of the Original)

Name Plate Unfolded (Preimage reflected across line of symmetry)