Teaching Quadratic Functions Using Realistic Mathematics Education

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Abstract

This curriculum project offers a sequence of four lessons for teaching quadratic functions in an Algebra I classroom. The lessons make use of the Realistic Mathematics Education (RME) teaching model. The RME model suggests that students should be introduced to mathematical concepts within realistic contexts and given time to explore, observe, discuss, and inquire about a concept to build understanding before gradually learning the abstract definitions and details. This represents a reversal of traditional teaching methods where students learn the abstract information first, then apply it to realistic concepts. RME has been shown to increase mathematical performance, elicit multiple pathways of student thinking, and improve engagement and overall enjoyment of mathematics. The lessons provided in this curriculum project progress through the key topics learned in a quadratic functions unit. Each new topic is anchored to a visualization presented in the first lesson of a ball being tossed into the air. All materials, activities, notes, and assignments are included, and answer keys can be found in the Appendix.

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Introduction

Mathematics teachers today typically agree that traditional teaching methods such as lecturing and direct instruction are not effective for engaging students in understanding mathematical concepts. Skemp (1976) presented two different types of understanding in mathematics, relational and instrumental. Relational understanding is knowing what to do and why while instrumental is rules without reason. The result of the latter is a disconnected collection of rules with a reliance on memorization. This typically leaves students unable to explain their thinking, recognize concepts in new contexts, or see how concepts interrelate (Di Muro, 2006). Student thinking becomes inflexible, and knowledge is strictly limited to that which can be reproduced, rather than created (Van den Huevel-Panhuizen & Drijvers, 2020). Instruction that supports relational understanding makes use of conceptual models, encourages students to speak and write about their thinking, employs a multitude of problem solving methods, and asks questions that challenge student thinking (Di Muro, 2006). Creating an instructional design that follows these criteria can be daunting, so it is helpful to have a model to guide instruction. Realistic Mathematics Education (RME) is an instructional model that does precisely this. RME was pioneered in the Netherlands in the second half of the 20th century as a response to the increasingly mechanistic approach to teaching mathematics (Van den Huevel-Panhuizen & Drijvers, 2020). The RME model seeks to develop deep, interconnected understanding of mathematical concepts by introducing them within concrete, realistic contexts, before gradually revealing the abstract details and definitions. Visual models, collaborative explorations, and multi-faceted reasoning are promoted as students build up their relational understanding. The quadratic functions unit in Algebra 1 is challenging for many students and presents an opportunity to employ RME because of the preponderance of visual models that can be linked with abstract concepts and definitions to build relational understanding. This curriculum project offers a sequence of lessons that follow the RME model of instruction to introduce quadratic functions to Algebra students.

Literature Review

What is Realistic Mathematics Education?

Realistic Mathematics Education (RME) was designed to improve student understanding of mathematical concepts by engaging students in realistic problem solving. RME instruction includes a gradual realization of the concrete definitions and details as they are needed. RME proponents Gravemeijer and Doorman (1999) state that students should first experience a global introduction to a mathematical concept in a way that creates the need for formal descriptions of the concepts involved. In other words, they state that the need-to-know drives learning. There are 6 principles of RME outlined by Van den Huevel-Panhuizen and Drijvers (2020). The *activity principle* enlists students as active participants in the learning process and emphasizes that mathematics is best learned by doing. The *reality principle* is the crux of the entire model, and is where learning is grounded within realistic contexts, specifically with problems that are meaningful to students. *Realistic*, as clarified by Van den Huevel-Panhuizen & Drijvers, does not necessarily mean *real world*, but rather that a student must be able to attach meaning to the problem in some fashion. The *l*evel *principle* advises that learning will progress through a sequence of levels, beginning with informal, context-embedded understanding and ending with formal, abstract understanding. The *intertwinement principle* advocates for the inclusion of multiple domains of mathematics (geometry, measurement, number theory, etc.) when learning a given concept, rather than isolating concepts into single domains. The *interactivity principle* represents the importance of discourse and collaboration in learning mathematics. Finally, the *guidance principle* refers to the role of the teacher in RME, which is that of a proactive guide who monitors student learning and leverages the realistic scenarios to reach a variety of students.

Evidence of the Effectiveness of RME

Numerous studies and curriculum evaluations have concluded that the RME model is effective for teaching mathematics. In the United Kingdon, two schools that piloted RME found that students were more successful with mathematics assessment questions and were more capable of explaining their understanding than students who received traditional mathematics teaching (Dickinson & Hough, 2012). Moreover, the students and teachers overwhelmingly spoke positively about their experiences teaching and learning mathematics through RME, with many citing increased engagement with content, discussion, understanding of concepts, and overall enjoyment of mathematics (Dickinson & Hough, 2012). Additionally, Barnes (2004) found evidence that after learning through RME, students were better able to consider multiple or alternative representations of a concept. The mechanism behind this result was the apparent symbiosis between vertical and horizontal mathematization spurred by the environment of RME (Barnes, 2004). Horizontal mathematization is the predominant factor in RME and refers to the linking of abstract mathematical concepts to a concrete context, while vertical mathematization is more traditional and describes the organization of techniques and strategies strictly within the abstract mathematical realm (Van den Huevel-Panhuizen & Drijvers, 2020). Through RME, students employ both horizontal and vertical mathematization, leading to nuanced understandings of concepts and the ability to use a variety of strategies and representations.

Design Considerations

There are some considerations for those designing an RME-based learning experience. Cobb et al. (2008) suggested that classroom activities be grounded in opportunities for mathematical discourse, both in whole-group and peer-to-peer settings, in which the teacher serves as a facilitator guiding the discussion towards important mathematical concepts. They also suggest that students be encouraged to accompany numeric explanations with conceptual explanations whenever possible. Finally, Van den Huevel-Panhuizen (1996) discusses considerations for assessment when using RME. She suggests that, because of the active nature of RME, assessment should feature a more observational approach so that teachers can observe student thinking and actions in the moment when grappling with a problem. The observational assessment can take place informally while students work, or in a more formal test setting, just so long as the teacher can evaluate *how* a student is thinking and what future activities may promote improved mathematical outcomes for that student. This curriculum project provides a sequence of lessons for teaching quadratic functions using RME, including all materials, notes, activities, and assignments with answer keys.

Curriculum

Overview and Timeline

The following sequence of lessons utilizes the Realistic Mathematics Education model of instruction to introduce and teach the concept of quadratic functions in an Algebra I class. In keeping with the tenets of the RME model, students are introduced to quadratics through a visual, real-world context: tossing a ball up into the air. This visualization introduces the concept and is returned to throughout each lesson as new ideas relating to quadratics are introduced and explored. This curriculum consists of four 60-minute lessons and would take place over the course of four class meetings: Day 1 -- Lesson 1: Graphs of Quadratics; Day 2 -- Lesson 2: x-intercepts of Quadratics; Day 3 -- Lesson 3: Completing the Square; Day 4 -- Lesson 4: The Quadratic Formula.

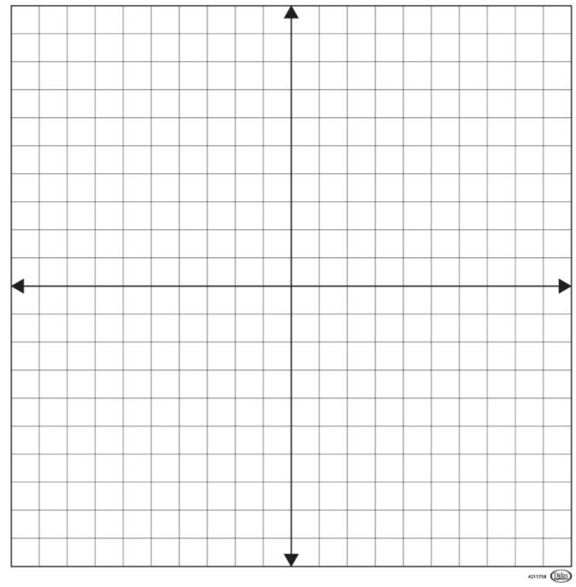
Lesson 1: Graphs of Quadratics (60 minutes)	
Objectives	- Students will be able to describe what a quadratic function is - Students will be able to determine the key features of a parabola graphically and algebraically. - Students will be able to sketch graphs of quadratic functions using tables and key features.
Next Generation Learning Standards	F.IF - Interpret functions that arise in applications in terms of the context. F.IF.4i- Interpret key features of graphs and tables in terms of the quantities, and F.IF.4ii - sketch graphs showing key features given a verbal description of the relationship. F.IF.7 - Graph functions and show key features of the graph by hand and by using technology where appropriate.
Materials	- Ball or object for tossing - Opening Discussion Question Slide - Desmos Quadratics Introduction Activity (on a Chromebook) - Independent Practice Questions - Closing Discussion Slide
Introduction (15 min)	To introduce the lesson, the teacher will toss an object (e.g. baseball, tennis ball, etc.) up into the air. This simple display will serve as a reoccurring scenario to which concepts will be linked throughout this unit. **Discussion:** Students will be asked to consider the time-height relationship of the object. What would a table of values look like? What would a graph of this look like? Can we make a sketch of this graph? Students will work in pairs to discuss these questions and make conjectures. As students discuss, the teacher will circulate and act as a facilitator of conversation, while listening for mathematical discourse. Once discussions have finished, pairs will share their thoughts with the class and give feedback to one another. To wrap up the introduction, the teacher will reveal that this scenario can be modeled by a quadratic function. Students will then launch into the Desmos activity to explore quadratic functions.
Instructional (20 min) Component	Students will complete an asynchronous Desmos activity that introduces the concept of quadratics and some key terminology. The activity will walk students through the definition of a quadratic and some key features (y-intercept, max/min, vertex, axis of symmetry). Following the Desmos activity, students will go back into their pairs and explain to each other how to graph a quadratic using a table, and how find each of the key features of a quadratic and use them to graph. The teacher will once again circulate to listen for understanding and math discourse.
Independent Problem Solving (20 min)	Students will practice graphing quadratics using tables and key features. The independent practice will be collected and used as formative assessment.
Conclusion (5 min)	To close the lesson, the teacher and students will reconvene and return to the opening ball-toss scenario. This time, the teacher will pose the question of how long the ball is in the air. This question should get students thinking about the x-intercepts and zeros, which is the focus of the next lesson.
Exit Ticket	Students will complete an exit ticket before they leave in which they are asked to identify the key features of three quadratics. One graph, one table, and one equation.

Independent Problem Solving: **Graphing Quadratics Practice**

1. Fill in the table and plot the points to graph the quadratic. Then answer questions (a)-(d).

$$y = x^2 - 4x + 2$$

x	y
-1	
0	
1	
2	
3	
4	
5	



a. What is the *y*-intercept for this quadratic?

b. Does this quadratic have a minimum or maximum value? How can you tell? What is the value?

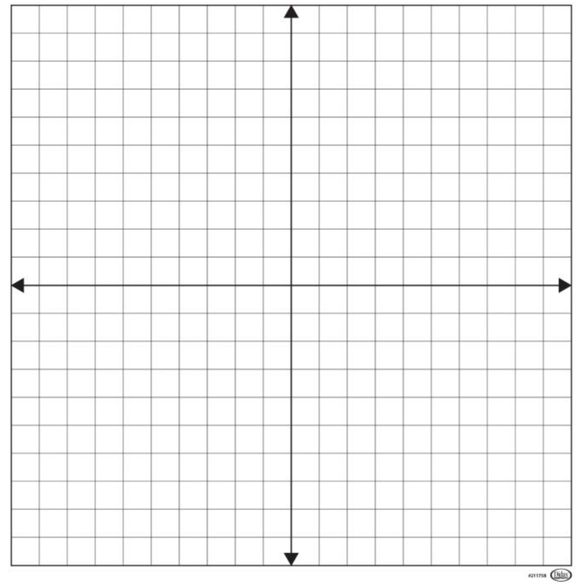
c. What is the equation for the axis of symmetry?

d. What is the vertex?

2. Fill in the table and plot the points to graph the quadratic. Then answer the questions (a)-(d).

$$y = -x^2 - 6x - 3$$

x	y
-6	
-5	
-4	
-3	
-2	
-1	
0	



a. What is the y-intercept for this quadratic?

b. Does this quadratic have a minimum or maximum value? How can you tell? What is the value?

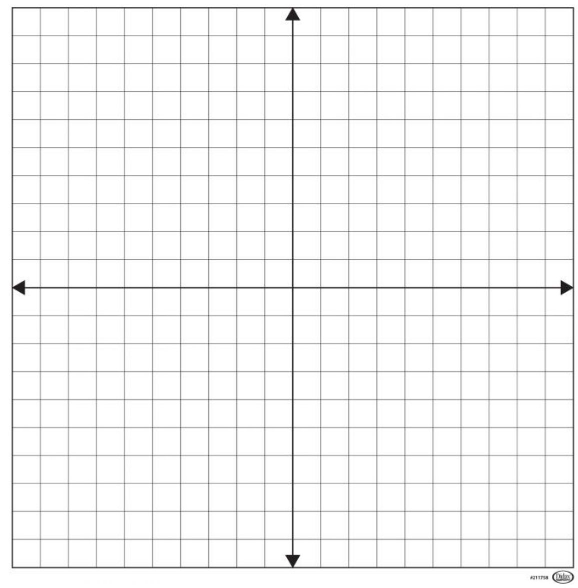
c. What is the equation for the axis of symmetry?

d. What is the vertex?

3. Find the key features listed for the quadratic below *algebraically* and use them to sketch its graph.

$$y = x^2 + 2x - 3$$

a. y-intercept:



b. Opening up or down?

c. Axis of symmetry:

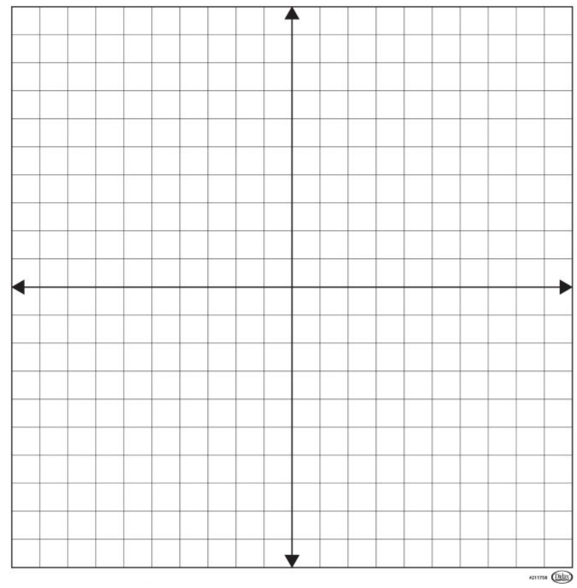
d. Vertex:

e. Does this quadratic have a maximum or minimum? How can you tell? What is the value?

4. Find the key features listed for the quadratic below *algebraically* and use them to sketch its graph.

$$y = x^2 + 8x + 7$$

a. *y*-intercept:



b. Opening up or down?

c. Axis of symmetry:

d. Vertex:

5. The function $h(t) = -16t^2 + 16x + 5$ can be used to model the relationship between time and height for a ball thrown into the air.

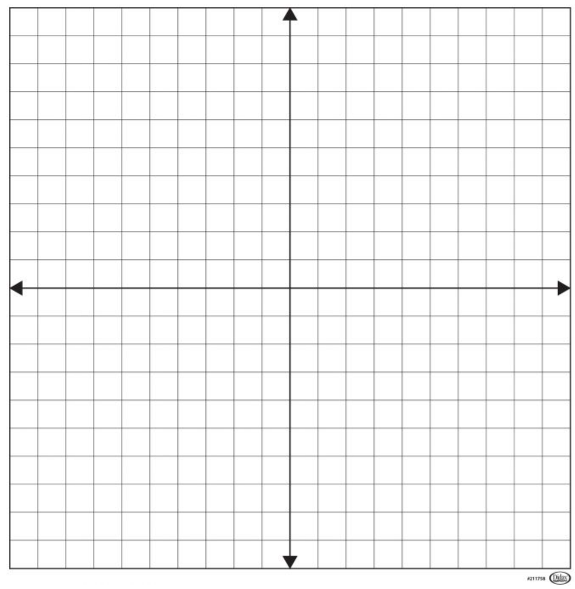
a) Find the key features of this graph *algebraically*, then sketch the graph of the function.

y-intercept:

Opening up or down?:

Axis of Symmetry:

Vertex:



b) What does the *y*-intercept represent in the context of this scenario?

c) How high was the ball thrown? Explain how you know.

d) At what time did the ball begin falling back toward the ground? Explain how you know.

Exit Ticket:

Identify the key features for each quadratic. There is a quadratic graph, table, and equation.

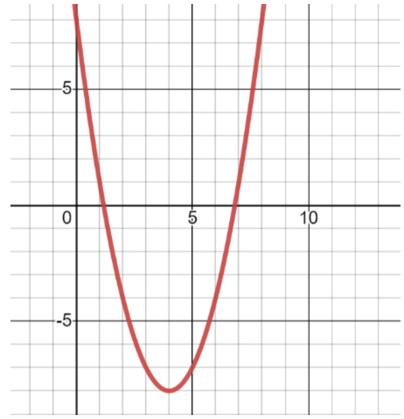
1. y-intercept: ____________

   Axis of Symmetry: ____________

   Vertex: ______________

   Maximum/Minimum: _______________

Graph:



2. y-intercept: ____________

   Axis of Symmetry: ____________

   Vertex: ______________

   Maximum/Minimum: _______________

Table:

x	y
-1	-5
0	-8
1	-7
2	-8
3	-5
4	0

3. Find the key features for this quadratic *algebraically*.

   y-intercept: ____________

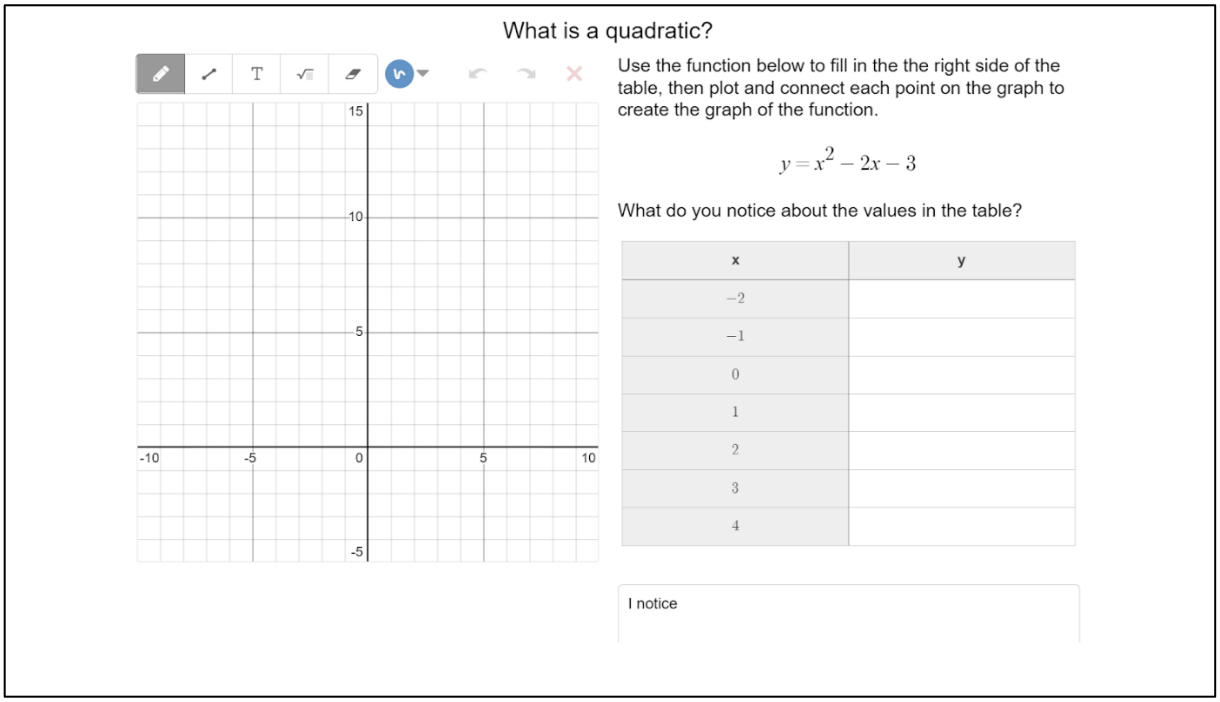
Equation: $y = -x^2 + 4x - 3$

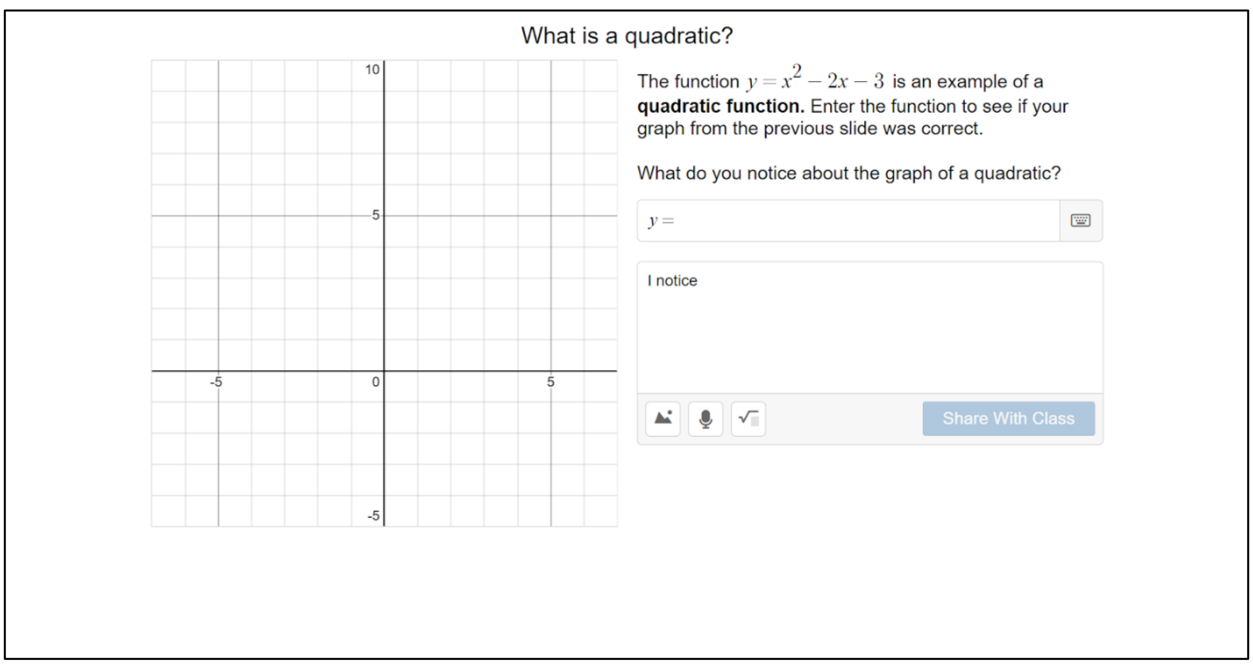
   Axis of Symmetry: ____________

   Vertex: ______________

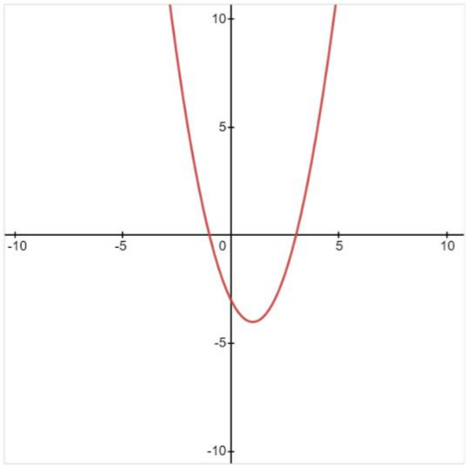
   Maximum/Minimum: ______________

Desmos Activity Slides





What is a quadratic?



A **quadratic function** is a polynomial function in which x^2 is the largest exponent.

Click me

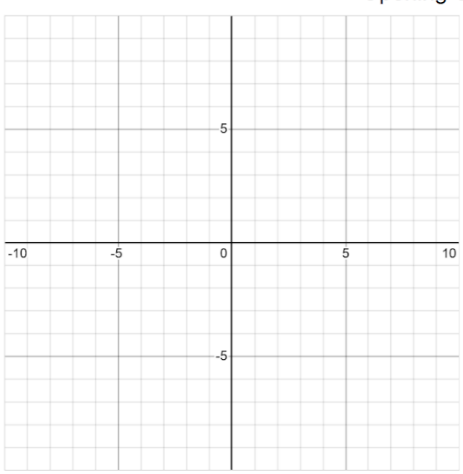
The standard form of a quadratic is $y = ax^2 + bx + c$.

Click me

Quadratics form a "U-shaped" graph called a **parabola**.

Click me

Opening Up vs Down



Enter some values for a in the function below.

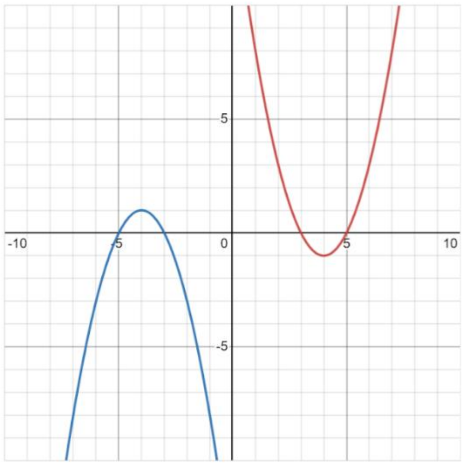
$$y = ax^2 - 1$$

$a =$

What kind of value will make the parabola flip over?

Share With Class

Maximum and Minimum



The graph of $y = x^2 + 8x + 15$ is shown in red.

The graph of $y = -x^2 + 8x - 15$ is shown in blue.

Which graph has a **maximum** point?

Red

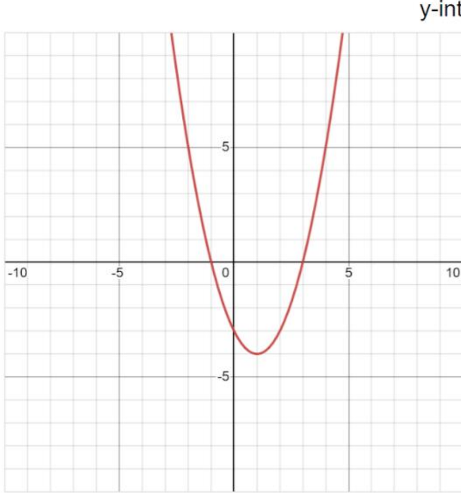
Blue

Which graph has a minimum point?

Red

Blue

y-intercept



Where does this graph pass through the y-axis?

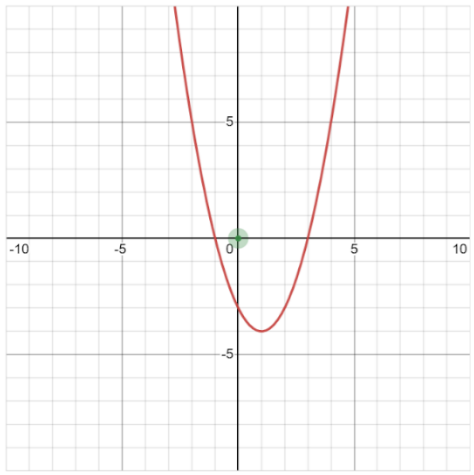
$y =$

The equation for this parabola is $y = x^2 - 2x - 3$

Explain how the y-intercept is related to the equation.

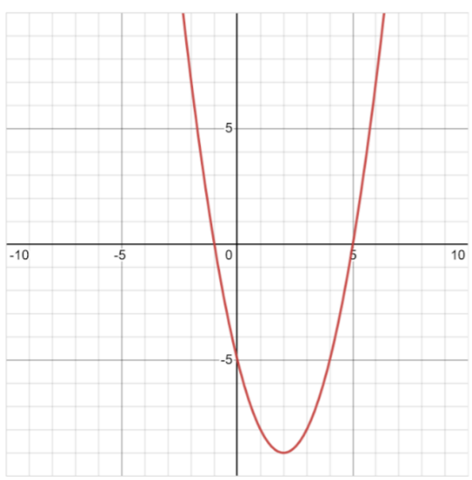
Share With Class

Vertex



Where does the parabola "turn around"? Drag the point to that location.

Axis of Symmetry



Enter a value for x that will create a line that cuts the parabola in half symmetrically.

$x =$

Submit

Axis of Symmetry and Vertex

Axis of Symmetry: $x = \#$

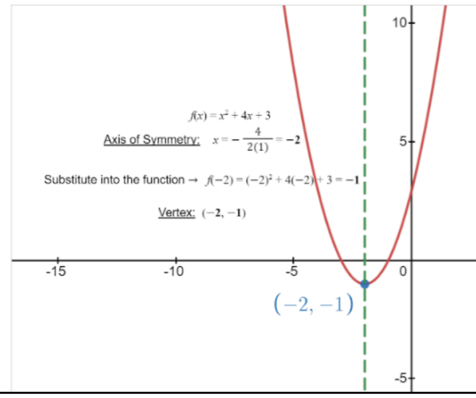
$\downarrow$

Vertex: $(\#, y)$

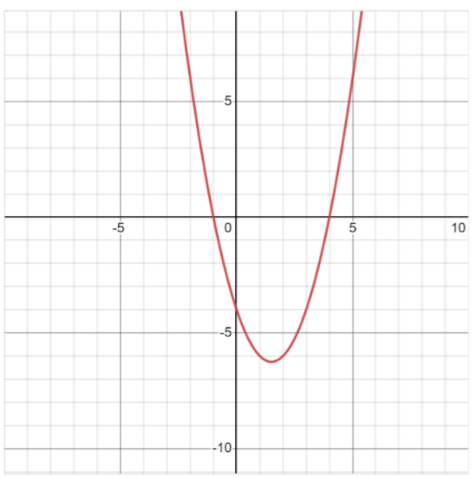
To find the Axis of Symmetry and Vertex without a graph, we start with this formula.

Axis of symmetry: $x = -\dfrac{b}{2a}$

Once we have the axis of symmetry, we find the vertex by substituting that x-value into the function to get the y-value.



Key Features Summary



Let's recap all of the key features of quadratics we have learned so far. The function $y = x^2 - 3x - 4$ is shown on the graph.

1. **y-intercept**: The c-value in the function.

Click me

2. **Opening up or down?**: a > 1, so the parabola opens up.

3. **Axis of Symmetry**: $x = -\dfrac{b}{2a} = -\dfrac{(-3)}{2(1)} = \dfrac{3}{2}$

Click me

4. **Vertex**: Found by substituting the axis of symmetry value into the function.

$f(1.5) = (1.5)^2 - 3(1.5) - 4 = -6.25$

Click me

5. **Maximum** or **Minimum**

Click me

Discussion Slides

Opening Discussion:

Consider the relationship between the height of the ball and time.

- What would a table of values look like for this relationship?
- What would the graph look like for this relationship? Sketch some possibilities!

Closing Discussion:

How long will the ball be in the air?

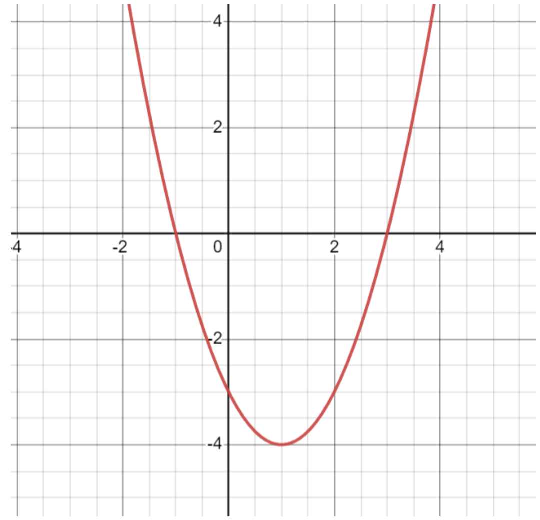
Can we use the graph to figure this out?

Lesson 2: *x*-intercepts of Quadratics (60 minutes)	
Objectives	- Students will be able find *x*-intercepts of quadratic functions graphically and algebraically. - Students will be able to solve real world problems involving *x*-intercepts of quadratic functions.
Next Generation Learning [Standards]	- A.APR.3 – Identify zeros when suitable factorizations are available. - A.REI.4.b.iii – Solve quadratic equations by factoring. - F.IF.4 – Interpret functions that arise in applications in terms of the context.
Materials	- Ball or object for tossing - Opening Discussion Slide - Guided note sheet - Gallery walk questions - Closing Discussion Slide - Exit Ticket
Introduction (10 min)	To introduce the lesson, the teacher will revisit the real-world scenario from the first lesson by tossing a ball into the air. This time the teacher will also have a graph of the function displayed. **Discussion:** Students will be asked to consider when the ball will hit the ground. Where can we find this on the graph? Where could we find this in a table? Once students determine the *x*-intercepts to be the point of importance, the last question posed will be how to find the *x*-values? Note: It will be assumed that students already know how to solve polynomial equations by factoring from previous instruction. It will not be assumed that they have linked those solutions to *x*-intercepts, that will be realized through this lesson.
Instructional (25 min) Component	In pairs, students will work through a scaffolded, guided notes sheet that walks them through the process of finding *x*-intercepts. The notes will build off of their prior knowledge of solving polynomial equations by factoring and link the solutions (zeros) to *x*-intercepts.
Independent Problem Solving (20 min)	**Gallery Walk**: Students will walk around the room and solve practice problems posted on the walls relating to finding *x*-intercepts of quadratic functions.
Conclusion (5 min)	To close the lesson, the teacher and students will reconvene for a brief discussion. The teacher will ask, “What happens if we can’t factor and graphing is inconclusive? Typically, a real-world example of a quadratic will not be nicely factorable. What do we do for those examples?” After some discussion, the teacher will reveal that there are special methods and techniques for solving quadratic equations that are not factorable. This will prepare students for the next lesson’s content, which is finding zeros using the quadratic formula and completing the square.
Exit Ticket	Students will complete an exit ticket before they leave in which they must select the correct function that matches a given quadratic graph.

Guided Notes:

Finding *x*-intercepts of Quadratics

1. The function $y = x^2 - 2x - 3$ is shown on the graph.



a) What are the *x*-intercepts of this function? Circle them on the graph and write the coordinates for each point.

b) What do you notice about the *y*-value for each of these points?

c) Complete the statement below:

To find the *x*-intercepts, I need to figure out the *x*-value when *y* = ______.

In other words, we are given the equation $y = x^2 - 2x - 3$, so we simply need to substitute 0 in for *y* and solve.

d) Fill in the blank to set up an equation that will help you find the *x*-intercept.

$$x^2 - 2x - 3 = \_\_\_\_\_\_\_$$

e) Now solve by factoring ==>

Now you can find the *x*-intercepts for a quadratic function algebraically! We also call these values "zeros". These terms are interchangeable.

Steps:

1. Get the equation into standard form and set equal to 0.

2. Factor.

3. The solutions are the *x*-intercepts.

Example: Find the *x*-intercepts for the function $f(x) = x^2 + 4x - 12$ algebraically.

Example: Find the zeros for the function $y = 2x^2 + 7x + 3$ algebraically.

Example: The function $h(t) = -16t^2 - 16t + 32$ models the height of an acorn *h* as it falls from a tree branch, where *t* is the elapsed time in seconds.

a) If we want to figure out when the acorn will hit the ground, which key feature of this quadratic must we find?

b) When will the acorn hit the ground?

c) Sketch a graph that illustrates this scenario and label the *x*-intercept with the value you found.

Gallery Walk Questions: Post each question around the room in no particular order. Have students complete their work on a piece of paper to be turned in at the end. Students may work in pairs to complete the questions.

1. Find the x-intercepts for the function $y = x^2 - 3 - 28$ algebraically.

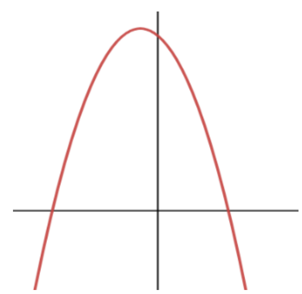
2. Find the zeros of the function $y = x^2 + 3x$ by factoring.

3. Determine the x-intercepts of the function $g(x) = 2x^2 + 7x + 6$.

4. Determine the zeros of the function $f(x) = 3x^2 + 10x - 8$.

5. The function $a(x) = x^2 - 8x + c$ has a zero at $x = 3$. Determine the other zero *and* the value of c.

6. Write a quadratic function in standard form that could represent the function shown on the graph. Explain how you determined your function.



7. A football throw can be modeled by the function $h(t) = -16t^2 + 32t$ where t is the time passed since the ball left the thrower's hand. Determine when the ball will reach the receiver.

8. A school is designing a new soccer field. The length of the field must be 40 yards longer than the width, and the total area of the field will be 6000 square yards.

   a) Write a quadratic equation that represents the area of the soccer field.

   b) Solve the equation to determine the dimensions of the soccer field.

Exit Ticket

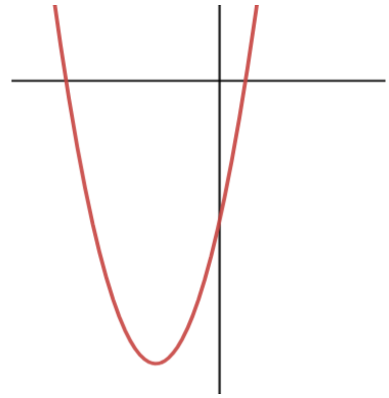
Which of the functions below is shown on the graph? Explain your choice.

a) $y = -x^2 - 5x + 6$

b) $y = x^2 + 5x - 6$

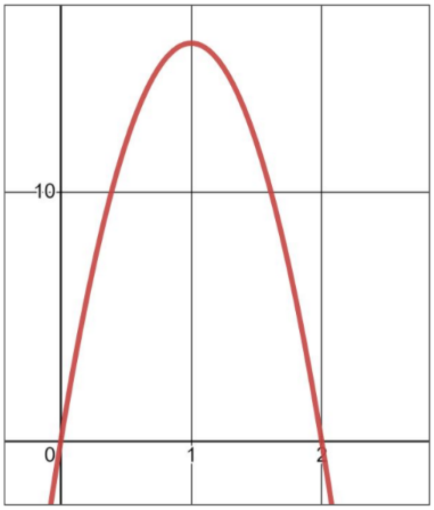
c) $y = -(x - 5)(x + 1)$

d) $y = (x + 1)(x - 5)$



Explanation:

Discussion Slides



Opening Discussion:

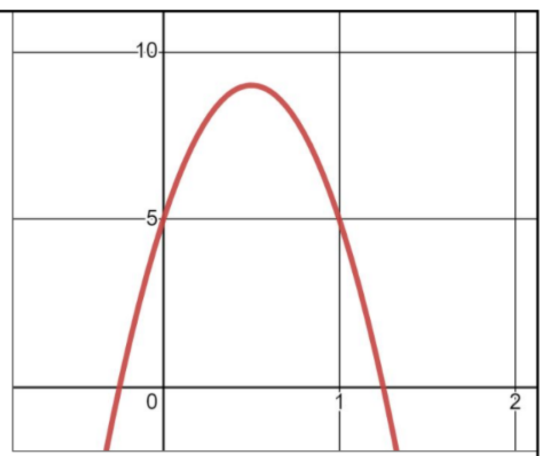
Let’s revisit the scenario from last class. The graph shows the time-height relationship of the ball.

- When will the ball return to the hand?
- Where can we see this on the graph?
- Where could we find this in a table?

Closing Discussion:

Real-world examples of quadratic functions rarely can be represented by nice, factorable equations.

What happens if we have an example like this one, when the graph is inconclusive and we can’t factor?



$$h(t) = -16t^2 + 16t + 5$$

Lesson 3: Completing the Square (60 minutes)	
Objectives	- Students will be able to solve quadratic equations algebraically using the method of completing the square.
Next Generation Learning Standards	- A.REI.4a - Use the method of completing the square to transform and quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Understand that the quadratic formula is a derivative of this process. - A.REI.4b – Solve quadratic equations by (iv) completing the square.
Materials	- Ball or object for tossing - Opening Discussion Question Slide - Completing the Square lesson notes - Polypad teacher demonstration
Introduction (15 min)	To introduce the lesson, the teacher will return to the ball-tossing scenario and this time provide a function to model the time-height relationship that cannot be factored. This will prompt the need for other algebraic methods for solving quadratics. **Discussion:** Students will be presented with a worked example of a quadratic equation solved using the method of completing the square. They will be given several minutes to analyze the work and generate things they notice or wonder. Students will discuss their noticings and wonderings in pairs, then as a group.
Instructional (10 min) Component	After students have analyzed and discussed the worked example of completing the square, the teacher will present a Polypad demonstration visualizing completing the square geometrically. Teacher Demo Instructions: - Open the Polypad link - Show students that the Algebra tiles on one side of the square represent the given quadratic ($x^2 + 6x + 3$). - Copy the Algebra tiles or drag new tiles over to the other side of the scale and begin rearranging them into a perfect square. - Pause before filling in the bottom right corner and ask students how many [1] tiles will be needed to "complete the square", then fill in the square. This will make the scale unbalanced. - Ask students how they can rebalance the scale. They should realize that you need to add [-1] tiles to balance-out the scale. - Below the scale, slowly reveal the algebraic representation and relate each step to the visualization above.
Independent Problem Solving (20 min)	Following the Polypad demonstration, students will practice solving quadratic equations using the method of completing the square both geometrically and algebraically. The final question will ask students to consider how we may use completing the square to solve the standard form of a quadratic for x. This will ultimately result in the quadratic formula, which is the focus of lesson 4.
Conclusion (15 min)	Students will be asked to consider how they may use completing the square to solve the standard form of a quadratic for x. This will ultimately result in the quadratic formula, which is the focus of lesson 4. Exit Ticket: Students will complete an exit ticket relating to completing the square.

Guided Notes:

Completing the Square

Up to this point we have learned how to solve quadratic equations by graphing and by factoring. How can we solve a quadratic equation that is not factorable or that has an inconclusive graph?

Completing the square is a clever method for solving quadratic equations of this type. The goal is to create something that looks like $x^2 = a$, so that all we need to do to find x is take the square root of each side of the equation.

The process of completing the square is shown below in symbol form. Analyze this worked example. What do you notice? What do you wonder?

<u>I notice:</u> | <u>I wonder:</u>

$$x^2 + 6x + 3 = 0$$

$$x^2 + 6x + 9 + 3 - 9 = 0$$

$$(x^2 + 6x + 9) + 3 - 9 = 0$$

$$(x + 3)(x + 3) - 6 = 0$$

$$(x + 3)^2 - 6 = 0$$

$$(x + 3)^2 = 6$$

$$x + 3 = \pm\sqrt{6}$$

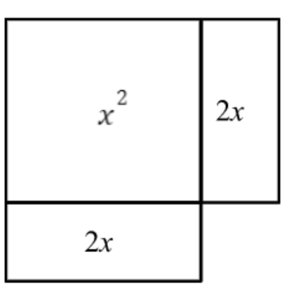
$$x = -3 \pm \sqrt{6}$$

Let's consider this process geometrically. [Polypad Demonstration]

Try some yourself!

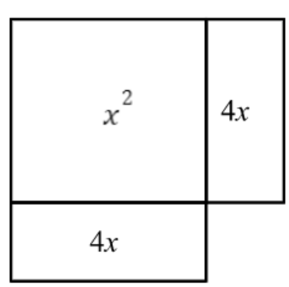
Use the area models to complete the square for each quadratic equation. Once you have filled in the area model, represent your area algebraically and solve. The first few are started for you.

1. $x^2 + 4x + 7 = 0$



$+ 7 -$ _____

2. $x^2 + 8x - 3 = 0$

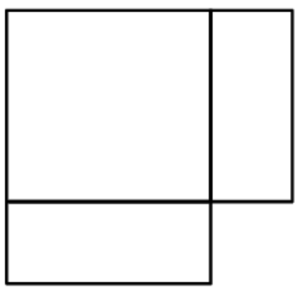


$- 3 -$ _____

Algebraic representation:

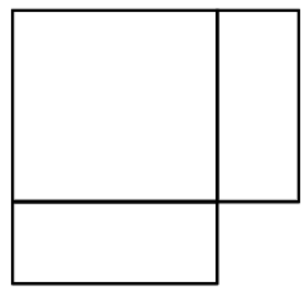
Algebraic representation:

3. $x^2 + 10x + 4 = 0$



Algebraic representation:

4. $x^2 - 4x + 10 = 0$



Algebraic representation:

6. How would this process change for a quadratic in which $a \neq 1$? Try the example below. What initial step must be taken in order for the process to work?

$$2x^2 + 4x - 10 = 0$$

Solve each quadratic using the method of completing the square. You may choose whether to use a geometric or algebraic approach.

7. $x^2 - 8x = 11$

8. $4x^2 + 16x = 24$

Conclusion

Consider the standard form of a quadratic $ax^2 + bx + c = 0$. Now that we know the method of completing the square, we can create a general formula that can be used to solve *any* quadratic equation. Use completing the square to solve the equation for *x*.

$$ax^2 + bx + c = 0$$

Exit Ticket:

Which equation is equivalent to $y = x^2 + 24x - 18$? Show the work that leads to your answer.

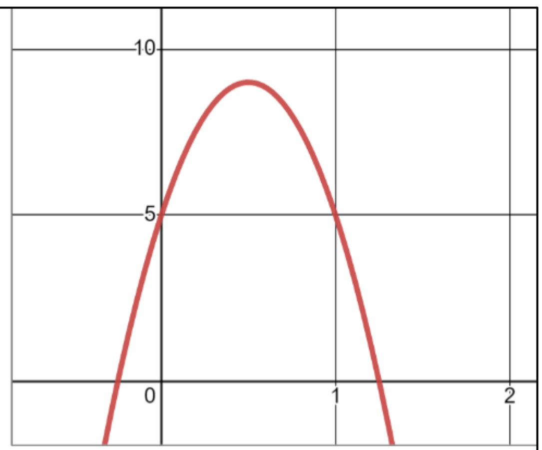
1) $y = (x+12)^2 - 162$
2) $y = (x+12)^2 + 126$
3) $y = (x-12)^2 - 162$
4) $y = (x-12)^2 + 126$

Discussion Slide

Opening Discussion:

Recall our closing discussion from the last lesson.

What happens if we have a quadratic equation whose graph is inconclusive and that we can't factor?



$$h(t) = -16t^2 + 16t + 5$$

Lesson 4: The Quadratic Formula (60 minutes)	
Objectives	- Students will be able to solve quadratic equations algebraically using the quadratic formula. - Students will be able to solve real-world problems involving quadratics.
Next Generation Learning Standards	- A.REI.4b(v) – Solve quadratics by the quadratic formula. - F.IF.4i – Interpret key features of graphs and tables in terms of the quantities - F.IF.8a – For a quadratic function, use an algebraic process to find zeros, maxima, minima, and symmetry of the graph, and interpret these in terms of context.
Materials	- Ball or object for tossing - Opening Discussion Question Slide - Guided notes - Independent Practice questions - Exit Ticket
Introduction (10 min)	To introduce the lesson, the teacher will return to the ball-tossing scenario and the function from the previous lesson that was not factorable. **Discussion:** Students will be asked how they can proceed with solving a quadratic equation of this form. Students may offer up completing the square as a method, but ultimately realize it is not the most feasible option. Students should recall the quadratic formula that was derived through completing the square at the end of the previous lesson. They will be asked to use the formula to determine when the ball will hit the ground.
Instructional (25 min) Component	Students will work through guided notes with the teacher. Students will relate the quadratic formula to the axis of symmetry formula from lesson one and explore several real-world problems modeled by quadratic equations. The instructional component will be predominantly student driven. The teacher will ask open-ended questions that foster discussion and exploration.
Independent Problem Solving (20 min)	Students will work in pairs to solve several multi-part real-world quadratic problems.
Conclusion (5 min)	Students will complete an exit ticket question in which they are asked to explain the significance of key features of quadratics in the context of real-world problems.

Guided Notes:

The Quadratic Formula

In the last lesson, we ended by deriving the quadratic formula shown below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

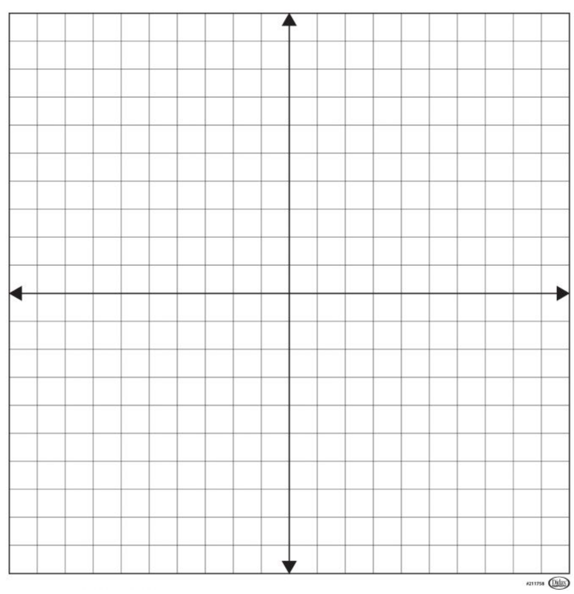
This formula is immensely useful as it allows us to solve real-world problems involving quadratics that often do not have nice graphs or factorable expressions.

Recall from lesson one, we have a formula (shown below) to calculate the axis of symmetry for a quadratic.

$$x = -\frac{b}{2a}$$

What do you notice about the axis of symmetry formula and the quadratic formula?

Use the quadratic function $y = x^2 + 2x - 8$ and the grid to explore the relationship between the formulas.



Real World Problems

1.Consider the following scenario.

A lighting technician is preparing her lighting system for an upcoming concert. The drummer in the band is well-known for throwing a drumstick high into the air at the end of their hit song. The band has asked that a pulse of light be emitted when the stick reaches its apex, and for all of the lights to be temporarily shut off when the stick hits the ground. The function below models the height of the stick as time passes.

$$h(t) = -16t^2 + 24x + 10$$

a) The drummer is on a raised platform. Based on the function and context, how high would you predict the platform to be? Explain.

b) The technician can program the lighting system to flash precisely as directed using time codes. She will trigger the lighting cue by pushing a button once the drummer throws his stick. When programing this code, how much time does she need to allow to pass between triggering the cue and the pulse of light?

c) How much time does she need to allow between triggering the cue and shutting the lights off?

2. A man is building a rectangular garden out of wood onto the side of a shed in his backyard. The garden will have three sides with the fourth side being against the shed.

a) Draw a picture to represent this scenario.

b) The function $A = w(36 - w)$ models the area of the garden for a given width, w. What width will produce the maximum possible area for the man's garden, and what is that area? Show your work or explain your reasoning.

c) The man purchased enough soil to cover 256 square feet of garden. Determine how much wood he will need to enclose the garden.

Independent Practice: **Real-world Quadratics Practice**

1. A last-second miracle basketball shot is taken from deep down court, almost 60 feet away from the hoop. The height of the ball can be modeled by the function $h(t) = -16t^2 + 22t + 8$.

a) Interpret the value of *c* in this function with regards to the context of this problem.

a) The height of the gymnasium is 24 feet. Is this shot low enough to make it to the hoop unobstructed? Show your work.

b) After how many seconds will the ball reach its maximum height? Round to the nearest hundredth.

c) The hoop is 10 feet high. At what time(s) will the ball reach the height of the hoop?

d) If the ball misses the hoop, at what time will it hit the ground?

2. A football kicker is attempting a 45-yard kick to win the game. The path of the ball can be modeled by the function $f(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$, where x is the horizontal distance from the kick, and $f(x)$ is the height of the football above the ground, both measured in feet.

a) What is the maximum height the football will reach and at what distance will it reach this height?

b) The goal post is 10 feet high; will the kick win the game?

c) From how many yards away could the kicker have made the kick?

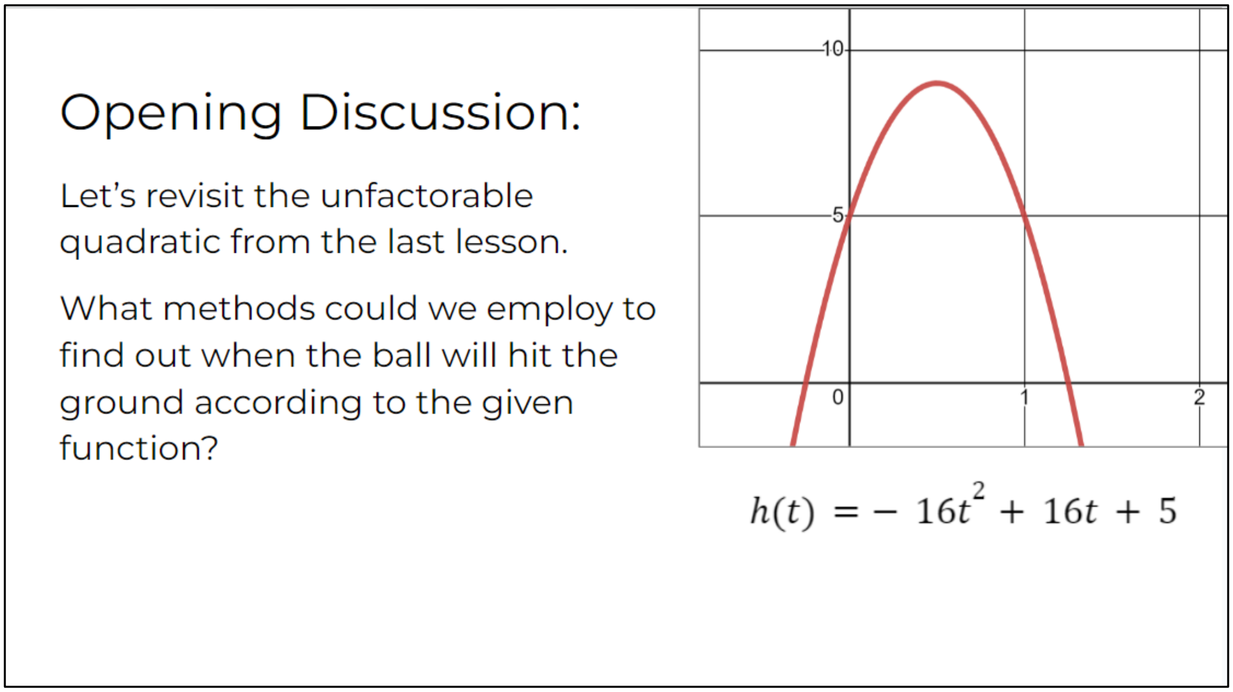
3. A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by x, and the area of the garden is 108 square meters. Determine, algebraically, the dimensions of the garden in meters.

Exit Ticket:

a) Explain what the vertex can tell you about a real-world problem modeled by a quadratic that is a function of a rectangle's area with respect to its width.

b) Explain what the x-intercept can tell you about a real-world problem modeled by a quadratic that is a function of an object's height with respect to time.

Discussion Slide



Validity

This curriculum was given to a teacher with 6 years of experience teaching Algebra I for analysis and feedback. In bold are the questions asked and the responses from the teacher follow.

After reading through the curriculum plan, what are some strengths you identified?

Lesson 1: I love the introduction of the ball being thrown in the air. I think it gives students a visual of the function that they can refer back to later in the unit. I also like that you are using Lesson 1 as time to identify and define most of the important vocabulary in the unit. I like the lead-in to x-intercepts in Lesson 2.

Lesson 2: I think a gallery walk is a great way to assess individual student understanding, and allow students who are excelling to complete more problems, while students who may be struggling can complete less problems.

Lesson 3: I love the interactivity of this lesson. I think students really grasp the concept of completing the square when they use the box model!

Lesson 4: Real world problems look great! I like that it looks like you're encouraging students to draw pictures that represent the scenarios.

What suggestions do you have for improving the curriculum?

Lesson 1: This lesson is jam packed with vocabulary. I am wondering how you will have students keep track of the words. Will they be posted around the room? Will they receive a printout of the last page of the Desmos activity? I think it might be helpful to have a reference when completing the independent practice.

Lesson 2: I would consider a warm-up activity checking their understanding of vocabulary from the previous lesson. I might do a group work activity where they have a quadratic and a separate equation and have to identify the key features. You could add some comparing questions such as, “Which quadratic opens up and how do you know?”, “Which quadratic will have a larger y-intercept, and how do you know?”, “Which quadratic has a positive axis of symmetry, and how do you know?”. I would scaffold the gallery walk questions with level 1 and level 2 questions. Level 1 questions would be to assess vocabulary and basic factoring, while level 2 would assess solving and more applications.

Lesson 3: I would talk about the size of the single squares having the same width as the rectangular “x” pieces. I would have physical pieces in case a student does not have a charged Chromebook or is having trouble with the technology. Add the vocabulary term “Vertex Form”. I would add a section of the lesson where students can use vertex form to identify the vertex in an equation/graph.

Lesson 4: Is there a way to add vocabulary to the practice problems? Yes, students should be using words like vertex and y-intercept when solving but students often are confused on “what” they are looking for. For example, Question 1d is asking students to find the x-intercept and they may be able to use the methods but are unsure how to start because they haven’t made the connection between “hitting the ground” and the “x-intercept”. I would also talk about why the negative x-intercept is not a viable option in this case.

Conclusion

When the traditional methods for teaching mathematical concepts fall short, Realistic Mathematics Education can be the solution that not only achieves results, but also awakens mathematical engagement and enjoyment in students. This curriculum project offers a sequence of lessons for teaching quadratic functions that leverages RME’s innovative design by exposing students to complex mathematical concepts within rich, imaginable contexts. Answer keys for all materials needed are provided in the Appendix section of this document. I hope that teachers accessing this curriculum project can find use for it in their classroom and yield positive student learning outcomes.

References

Barnes, H. (2004). Realistic mathematics education: Eliciting alternative mathematical conceptions of learners. *African Journal of Research in Mathematics, Science and Technology Education*, *8*(1), 53-64.

Cobb, P., Zhao, Q., & Visnovska, J. (2008). Learning from and adapting the theory of realistic mathematics education. *Éducation et didactique*, (2-1), 105-124.

Dickinson, P., & Hough, S. (2012). Using realistic mathematics education in UK classrooms. *Centre for Mathematics Education, Manchester Metropolitan University, Manchester, UK*.

Di Muro, P. (2006). Best Practices in Mathematics Instruction: Teaching for Understanding. *NADE Digest*, *2*(1), 1-8.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational studies in mathematics*, *39*(1-3), 111-129.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. Mathematics teaching, 77(1), 20-26.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic mathematics education. *Encyclopedia of mathematics education*, 713-717.

Wubbels, T., Korthagen, F., & Broekman, H. (1997). Preparing teachers for realistic mathematics education. *Educational studies in mathematics*, *32*(1), 1-28.

Appendix

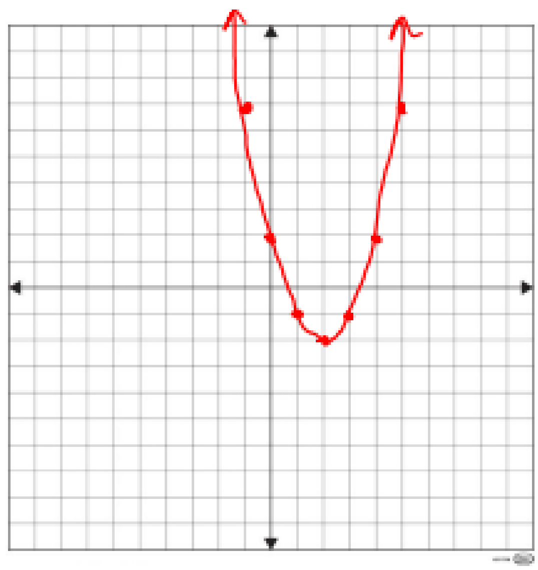
Lesson 1 Answer Key

Graphing Quadratics Practice

1. Fill in the table and plot the points to graph the quadratic. Then answer questions (a)-(d).

$$y = x^2 - 4x + 2$$

x	y
-1	7
0	2
1	-1
2	-2
3	-1
4	2
5	7



a. What is the y-intercept for this quadratic?

(0, 2)

b. Does this quadratic have a minimum or maximum value? How can you tell? What is the value?

Minimum because the parabola opens up. The minimum value is $y = -2$.

c. What is the equation for the axis of symmetry?

$\boxed{x = 2}$

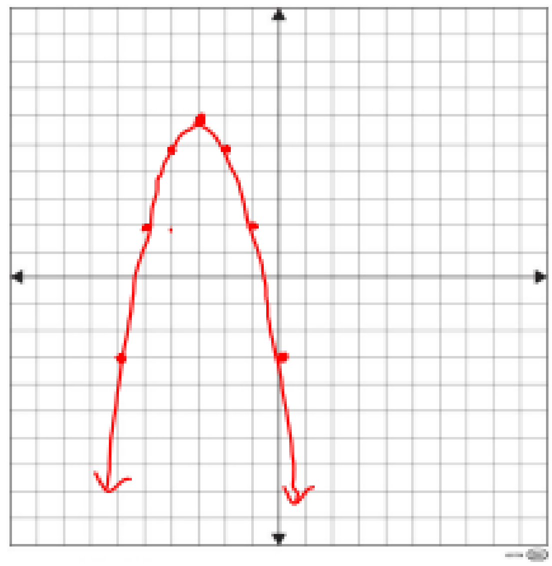
d. What is the vertex?

(2, -2)

2. Fill in the table and plot the points to graph the quadratic. Then answer the questions (a)-(d).

$$y = -x^2 - 6x - 3$$

x	y
-6	-3
-5	2
-4	5
-3	6
-2	5
-1	2
0	-3



a. What is the y-intercept for this quadratic?

(0, −3)

b. Does this quadratic have a minimum or maximum value? How can you tell? What is the value?

Maximum because the parabola opens down.
The maximum value is $y = 6$.

c. What is the equation for the axis of symmetry?

$x = -3$

d. What is the vertex?

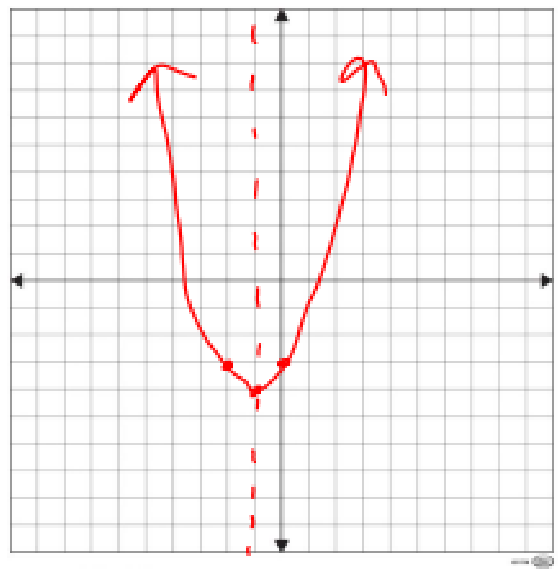
(-3, 6)

3. Find the key features listed for the quadratic below *algebraically* and use them to sketch its graph.

$$y = x^2 + 2x - 3$$

a. y-intercept:

-3



b. Opening up or down?

Opening up

c. Axis of symmetry:

$x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)} = -1$

$\boxed{x = -1}$

d. Vertex:

$x = -1$ $y = (-1)^2 + 2(-1) - 3$

$y = 1 - 2 - 3$

$y = -4$

$(-1, -4)$

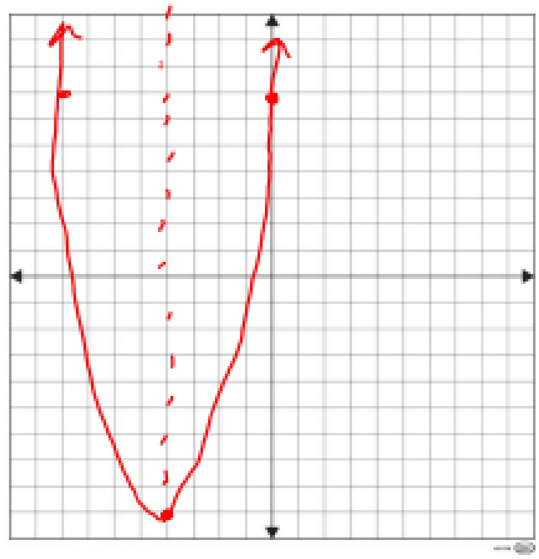
e. Does this quadratic have a maximum or minimum? How can you tell? What is the value?

Minimum because the parabola opens up.

The minimum value is $y = -4$.

4. Find the key features listed for the quadratic below *algebraically* and use them to sketch its graph.

$$y = x^2 + 8x + 7$$



a. y-intercept:

7

b. Opening up or down?

up

c. Axis of symmetry:

$x = -\frac{b}{2a}$

$x = -\frac{8}{2(1)} = -4$

$\boxed{x = -4}$

d. Vertex:

$x = -4$ $\quad y = (-4)^2 + 8(-4) + 7$

$y = 16 - 32 + 7$

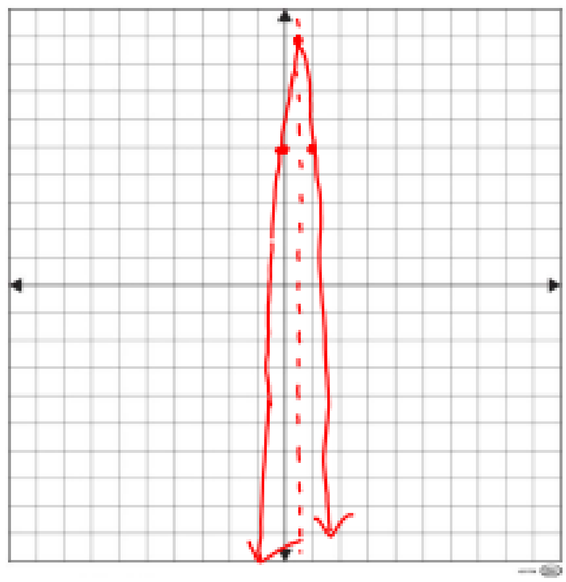
$y = -16 + 7$

$y = -9$

$(-4, -9)$

5. The function $h(t) = -16t^2 + 16x + 5$ can be used to model the relationship between time and height for a ball thrown into the air.

a) Find the key features of this graph *algebraically*, then sketch the graph of the function.



y-intercept: 5

Opening up or down?: Down

Axis of Symmetry:

$x = \frac{-b}{2a}$

$x = \frac{-16}{2(-16)} = \frac{1}{2}$ $\boxed{x = \frac{1}{2}}$

Vertex:

$x = \frac{1}{2}$ $y = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 5$

$y = -4 + 8 + 5$ $\left(\frac{1}{2}, 9\right)$

$y = 9$

b) What does the *y*-intercept represent in the context of this scenario?

The ball was thrown from a height of 5 feet.

c) How high was the ball thrown? Explain how you know.

9 feet because the maximum value of the function is 9.

d) At what time did the ball begin falling back toward the ground? Explain how you know.

t= 0.5 seconds

The turning point of the function occurs at t = 0.5.

Exit Ticket:

Identify the key features for each quadratic. There is a quadratic graph, table, and equation.

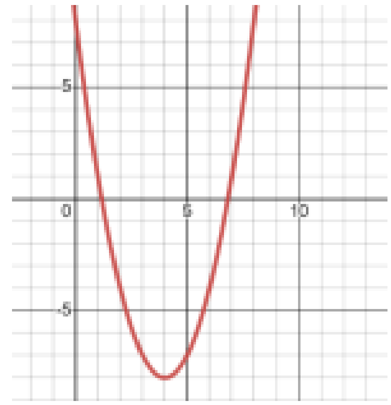
1. y-intercept: 8

Axis of Symmetry: $x = 4$

Vertex: $(4, -8)$

Maximum/Minimum: minimum

Graph:



2. y-intercept: -8

Axis of Symmetry: $x = 1$

Vertex: $(1, -7)$

Maximum/Minimum: minimum

Table:

x	y
-1	-5
0	-8
1	-7
2	-8
3	-5
4	0

3. Find the key features for this quadratic *algebraically*.

y-intercept: -3

Equation: $y = -x^2 + 4x - 3$

Axis of Symmetry: $x = 2$

$x = \frac{-b}{2a}$ $x = \frac{-4}{2(-1)} = 2$

Vertex: $(2, 1)$

$y = -(2)^2 + 4(2) - 3$

$y = -4 + 8 - 3 = 1$

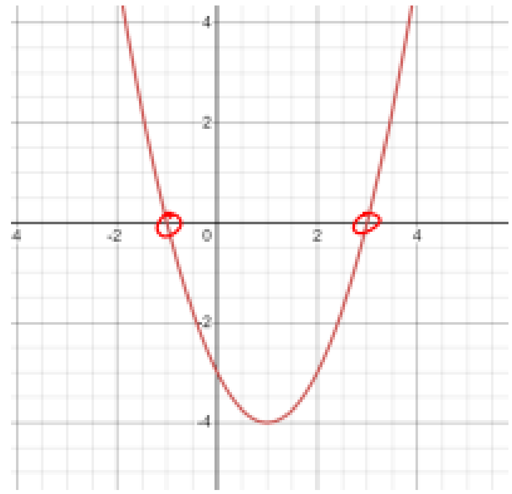
Maximum/Minimum: maximum

Lesson 2 Answer Key

Guided Notes:

Finding *x*-intercepts of Quadratics

1. The function $y = x^2 - 2x - 3$ is shown on the graph.



a) What are the *x*-intercepts of this function? Circle them on the graph and write the coordinates for each point.

(-1, 0) and (3, 0)

b) What do you notice about the *y*-value for each of these points?

The y-value for both points is 0.

c) Complete the statement below:

To find the *x*-intercepts, I need to figure out the *x*-value when $y =$ 0.

In other words, we are given the equation $y = x^2 - 2x - 3$, so we simply need to substitute 0 in for *y* and solve.

d) Fill in the blank to set up an equation that will help you find the *x*-intercept.

$$x^2 - 2x - 3 = 0$$

e) Now solve by factoring ==> $(x - 3)(x + 1) = 0$

$x - 3 = 0$ $\quad$ $x + 1 = 0$

$\boxed{x = 3}$ $\quad$ $\boxed{x = -1}$

Now you can find the *x*-intercepts for a quadratic function algebraically! We also call these values "zeros". These terms are interchangeable.

Steps:

1. Get the equation into standard form and set equal to 0.

2. Factor.

3. The solutions are the *x*-intercepts.

Example: Find the x-intercepts for the function $f(x) = x^2 + 4x - 12$ algebraically.

$$0 = x^2 + 4x - 12$$
$$0 = (x+6)(x-2)$$
$$x+6=0 \qquad x-2=0$$
$$\boxed{x=-6} \qquad \boxed{x=2}$$

Example: Find the zeros for the function $y = 2x^2 + 7x + 3$ algebraically.

$ac = 6$

$1 \quad 6$

$$0 = 2x^2 + 7x + 3$$
$$0 = 2x^2 + 6x + x + 3$$
$$0 = 2x(x+3) + (x+3)$$
$$0 = (2x+1)(x+3)$$

$2x+1=0$

$2x=-1$

$\boxed{x=-\frac{1}{2}}$

$x+3=0$

$\boxed{x=-3}$

Example: The function $h(t) = -16t^2 - 16t + 32$ models the height of an acorn h as it falls from a tree branch, where t is the elapsed time in seconds.

a) If we want to figure out when the acorn will hit the ground, which key feature of this quadratic must we find? The x-intercept.

b) When will the acorn hit the ground?

$$0 = -16t^2 - 16t + 32$$
$$0 = -16(t^2 + t - 2)$$
$$0 = -16(t+2)(t-1)$$
$$t+2=0 \qquad t-1=0$$
$$\boxed{t=-2} \qquad \boxed{t=1}$$

Disregard

The acorn will hit the ground after 1 second.

c) Sketch a graph that illustrates this scenario and label the x-intercept with the value you found.

Gallery Walk Questions: Post each question around the room in no particular order. Have students complete their work on a piece of paper to be turned in at the end. Students may work in pairs to complete the questions.

1. Find the x-intercepts for the function $y = x^2 - 3 - 28$ algebraically.

$0 = (x-7)(x+4)$
$x = -7, -4$

2. Find the zeros of the function $y = x^2 + 3x$ by factoring.

$0 = x(x+3)$
$x = 0, -3$

3. Determine the x-intercepts of the function $g(x) = 2x^2 + 7x + 6$.

$0 = (2x+3)(x+2)$
$x = -\frac{3}{2}$ $x = -2$

4. Determine the zeros of the function $f(x) = 3x^2 + 10x - 8$.

$0 = (3x-2)(x+4)$
$x = \frac{2}{3}$ $x = -4$

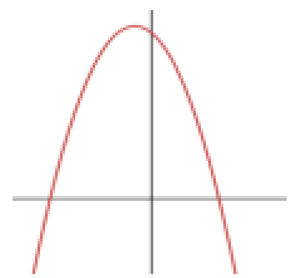
5. The function $a(x) = x^2 - 8x + c$ has a zero at $x = 3$. Determine the other zero *and* the value of c.

$0 = (x-3)(x-5)$
zero: $x = 5$, $c = 15$

6. Write a quadratic function in standard form that could represent the function shown on the graph. Explain how you determined your function.

(multiple options)

$y = (x-2)(x+3)$



7. A football throw can be modeled by the function $h(t) = -16t^2 + 32t$ where t is the time passed since the ball left the thrower's hand. Determine when the ball will reach the receiver.

$0 = -16t(t-2)$
$t = 0, t = 2$ 2 seconds

8. A school is designing a new soccer field. The length of the field must be 40 yards longer than the width, and the total area of the field will be 6000 square yards.

a) Write a quadratic equation that represents the area of the soccer field.

$A = x(x+40)$

b) Solve the equation to determine the dimensions of the soccer field.

$6000 = x^2 + 40x$
$0 = x^2 + 40x - 6000$
$0 = (x+100)(x-60)$
$x = -100, 60$

$\boxed{w = 60 \text{ yds}, \ l = 100 \text{ yds}}$

Exit Ticket

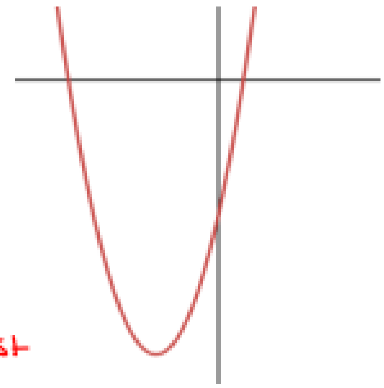
Which of the functions below is shown on the graph? Explain your choice.

a) $y = -x^2 - 5x + 6$

b) $y = x^2 + 5x - 6$

c) $y = -(x-5)(x+1)$

d) $y = (x+1)(x-5)$



Explanation:

The parabola opens up, so the value of a must be positive, which rules out a and c.

d has zeros at -1 and 5, which does not match the function on the graph, so the answer must be b.

Lesson 3 Answer Key

Completing the Square

Up to this point we have learned how to solve quadratic equations by graphing and by factoring. How can we solve a quadratic equation that is not factorable or that has an inconclusive graph?

Completing the square is a clever method for solving quadratic equations of this type. The goal is to create something that looks like $x^2 = a$, so that all we need to do to find x is take the square root of each side of the equation.

The process of completing the square is shown below in symbol form. Analyze this worked example. What do you notice? What do you wonder?

<u>I notice:</u> <u>I wonder:</u>

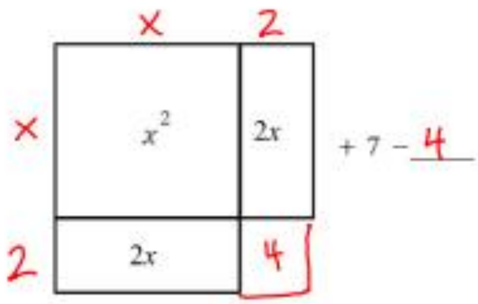
$$x^2 + 6x + 3 = 0$$
$$x^2 + 6x + 9 + 3 - 9 = 0$$
$$(x^2 + 6x + 9) + 3 - 9 = 0$$
$$(x + 3)(x + 3) - 6 = 0$$
$$(x + 3)^2 - 6 = 0$$
$$(x + 3)^2 = 6$$
$$x + 3 = \pm\sqrt{6}$$
$$x = -3 \pm \sqrt{6}$$

Let's consider this process geometrically. [Polypad Demonstration]

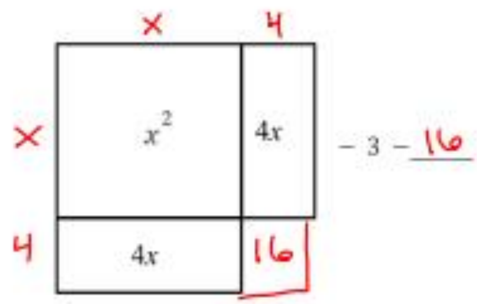
Try some yourself!

Use the area models to complete the square for each quadratic equation. Once you have filled in the area model, represent your area algebraically and solve. The first few are started for you.

1. $x^2 + 4x + 7 = 0$



2. $x^2 + 8x - 3 = 0$



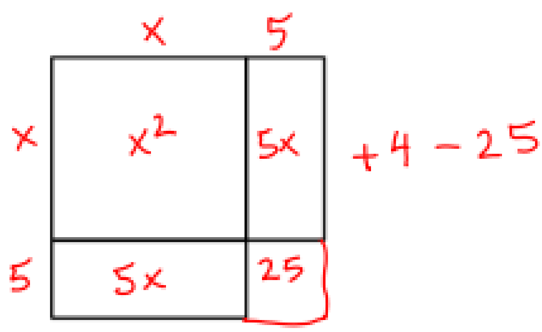
Algebraic representation:

$$(x^2+4x+4)+7-4=0$$
$$(x+2)^2+3=0$$
$$x+2=\pm\sqrt{-3}$$
$$x=-2\pm\sqrt{-3}$$
Non real solutions

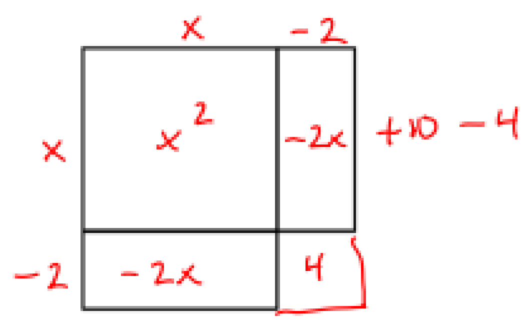
Algebraic representation:

$$(x^2+8x+16)-3-16=0$$
$$(x+4)^2-19=0$$
$$x+4=\pm\sqrt{19}$$
$$x=-4\pm\sqrt{19}$$

3. $x^2+10x+4=0$



4. $x^2-4x+10=0$



Algebraic representation:

$$(x^2+10x+25)+4-25=0$$
$$(x+5)^2-21=0$$
$$x+5=\pm\sqrt{21}$$
$$x=-5\pm\sqrt{21}$$

Algebraic representation:

$$(x^2-4x+4)+10-4=0$$
$$(x-2)^2+6=0$$
$$x-2=\pm\sqrt{-6}$$
$$x=2\pm\sqrt{-6}$$
Non real solutions

6. How would this process change for a quadratic in which $a \neq 1$? Try the example below. What initial step must be taken in order for the process to work?

Factor out the GCF.

$$2x^2+4x-10=0$$
$$2(x^2+2x-5)=0$$
$$2[(x^2+2x+1)-5-1]=0$$
$$2[(x+1)^2-6]=0$$
$$2(x+1)^2-12=0$$
$$x+1=\pm\sqrt{6}$$
$$x=-1\pm\sqrt{6}$$

Solve each quadratic using the method of completing the square. You may choose whether to use a geometric or algebraic approach.

7. $x^2 - 8x = 11$

$-11 \quad -11$

$x^2 - 8x - 11 = 0$

	x	-4	
x	x^2	$-4x$	$-11-16$
-4	$-4x$	16	

$(x-4)^2 - 27 = 0$

$x - 4 = \pm\sqrt{27}$

$x = 4 \pm 3\sqrt{3}$

8. $4x^2 + 16x = 24$

$-24 \quad -24$

$4x^2 + 16x - 24 = 0$

$4(x^2 + 4x - 6) = 0$

$4[(x+2)^2 - 6 - 4] = 0$

$4[(x+2)^2 - 10] = 0$

$4(x+2)^2 - 40 = 0$

$x + 2 = \pm\sqrt{10}$

$x = -2 \pm \sqrt{10}$

Conclusion

Consider the standard form of a quadratic $ax^2 + bx + c = 0$. Now that we know the method of completing the square, we can create a general formula that can be used to solve *any* quadratic equation. Use completing the square to solve the equation for x.

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2 = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Exit Ticket:

Which equation is equivalent to $y = x^2 + 24x - 18$? Show the work that leads to your answer.

1) $y = (x + 12)^2 - 162$
2) $y = (x + 12)^2 + 126$
3) $y = (x - 12)^2 - 162$
4) $y = (x - 12)^2 + 126$

$(x+12)^2 - 18 - 144$

$(x+12)^2 - 162$

Lesson 4 Answer Key

Guided Notes:

The Quadratic Formula

In the last lesson, we ended by deriving the quadratic formula shown below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula is immensely useful as it allows us to solve real-world problems involving quadratics that often do not have nice graphs or factorable expressions.

Recall from lesson one, we have a formula (shown below) to calculate the axis of symmetry for a quadratic.

$$x = -\frac{b}{2a}$$

What do you notice about the axis of symmetry formula and the quadratic formula?

The axis of symmetry formula is contained within the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic function $y = x^2 + 2x - 8$ and the grid to explore the relationship between the formulas.

$$x = \frac{-b}{2a}$$

$$x = \frac{-2}{2} = -1$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-8)}}{2(1)}$$

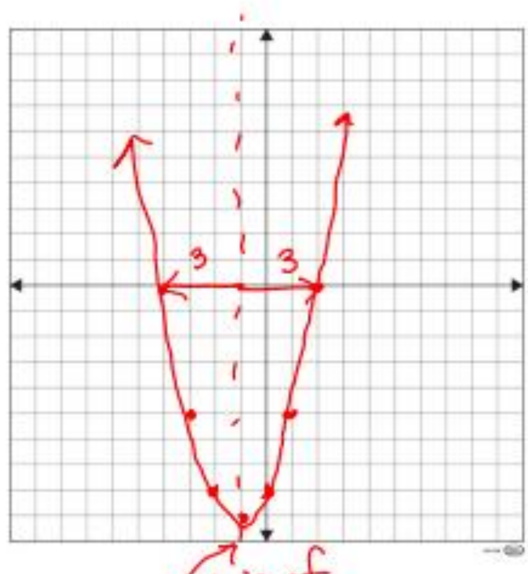
$$x = \frac{-2 \pm \sqrt{4 + 32}}{2}$$

$$x = \frac{-2 \pm \sqrt{36}}{2}$$

$$x = \frac{-2 \pm 6}{2}$$

$$x = -1 \pm 3$$

Zeros are 3 away from the axis of symmetry on either side.



Axis of Symmetry

Real World Problems

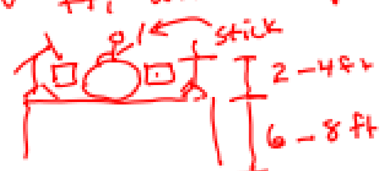
1.Consider the following scenario.

A lighting technician is preparing her lighting system for an upcoming concert. The drummer in the band is well-known for throwing a drumstick high into the air at the end of their hit song. The band has asked that a pulse of light be emitted when the stick reaches its apex, and for all of the lights to be temporarily shut off when the stick hits the ground. The function below models the height of the stick as time passes.

$$h(t) = -16t^2 + 24x + 10$$

a) The drummer is on a raised platform. Based on the function and context, how high would you predict the platform to be? Explain.

6 – 8 feet. The starting height of the stick (AKA the drummer's hand) is 10 ft, which is probably 2 – 4 feet above platform.



b) The technician can program the lighting system to flash precisely as directed using time codes. She will trigger the lighting cue by pushing a button once the drummer throws his stick. When programing this code, how much time does she need to allow to pass between triggering the cue and the pulse of light? Need to find the vertex.

$t = \frac{-b}{2a}$

$t = \frac{-24}{2(-16)} = \frac{-24}{-32} = \frac{3}{4}$

$h(\frac{3}{4}) = -16(\frac{3}{4})^2 + 24(\frac{3}{4}) + 10$

$= -9 + 18 + 10$

$= 19$

$(\frac{3}{4}, 19)$

At $t = \frac{3}{4}$ seconds the stick will reach its maximum height, so she will need to allow .75 seconds to pass.

c) How much time does she need to allow between triggering the cue and shutting the lights off?

Need to find the zeros.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-24 \pm \sqrt{24^2 - 4(-16)(10)}}{2(-16)}$

$x = \frac{-24 \pm \sqrt{576 + 640}}{-32}$

$x = \frac{-24 \pm \sqrt{1216}}{-32}$

$x = -.34$

$x = 1.84$

The stick will hit the ground at 1.84 seconds.

2. A man is building a rectangular garden out of wood onto the side of a shed in his backyard. The garden will have three sides with the fourth side being against the shed.

a) Draw a picture to represent this scenario.

b) The function $A = w(36 - w)$ models the area of the garden for a given width, w. What width will produce the maximum possible area for the man's garden, and what is that area? Show your work or explain your reasoning. Need to find the vertex.

$A = w(36-w)$

$A = -w^2 + 36w$

$w = \frac{-b}{2a}$

$w = \frac{-36}{-2} = 18$

$A = 18(36-18)$

$A = 324$

$(18, 324)$

The maximum possible area is 324 ft^2 when the width is 18 ft.

c) The man purchased enough soil to cover 256 square feet of garden. Determine how much wood he will need to enclose the garden.

$256 = w(36-w)$

$0 = -w^2 + 36w - 256$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \frac{-36 \pm \sqrt{36^2 - 4(-1)(-256)}}{2(-1)}$

$w = \frac{-36 \pm \sqrt{1296 - 1024}}{-2}$

$w = \frac{-36 \pm \sqrt{272}}{-2}$

$w = 9.75$

$w = 9.75$, $l = 26.25$

$9.75 + 9.75 + 26.25$

$= 45.75$ feet of wood.

Independent Practice: **Real-world Quadratics Practice**

1. A last-second miracle basketball shot is taken from deep down court, almost 60 feet away from the hoop. The height of the ball can be modeled by the function $h(t) = -16t^2 + 22t + 8$.

a) Interpret the value of *c* in this function with regards to the context of this problem.

The shot was released from a height of 8ft.

a) The height of the gymnasium is 24 feet. Is this shot low enough to make it to the hoop unobstructed? Show your work.

Vertex: $t = \frac{-b}{2a}$

$t = \frac{-22}{2(-16)}$

$t = \frac{-22}{-32} = \frac{2}{3}$

$h(\frac{2}{3}) = -16(\frac{2}{3})^2 + 22(\frac{2}{3}) + 8$

$h(\frac{2}{3}) = 15.6$

The shot reaches a maximum height of 15.6 ft, so it will not hit the ceiling.

b) After how many seconds will the ball reach its maximum height? Round to the nearest hundredth.

The ball reaches its maximum height after .67 seconds.

c) The hoop is 10 feet high. At what time(s) will the ball reach the height of the hoop?

$10 = -16t^2 + 22t + 8$

$-10 \qquad\qquad -10$

$0 = -16t^2 + 22t - 2$

$t = \frac{-22 \pm \sqrt{22^2 - 4(-16)(-2)}}{2(-16)}$

$t = .098, 1.28$

.098 and 1.28 seconds

d) If the ball misses the hoop, at what time will it hit the ground?

$0 = -16t^2 + 22t + 8$

$t = \frac{-22 \pm \sqrt{22^2 - 4(-16)(8)}}{2(-16)}$

$t = 1.67$ seconds

2. A football kicker is attempting a 45 yard kick to win the game. The path of the ball can be modeled by the function $f(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$, where x is the horizontal distance from the kick, and $f(x)$ is the height of the football above the ground, both measured in feet.

a) What is the maximum height the football will reach and at what distance will it reach this height?

$x = \frac{-b}{2a}$

$x = \frac{-(2/3)}{2(-1/225)} = 75$

$f(75) = -\frac{1}{225}(75)^2 + \frac{2}{3}(75)$

$= 25$

The ball reaches a maximum height of 75 feet after 25 horizontal feet.

b) The goal post is 10 feet high; will the kick win the game?

Goal post is 45 yards = 135 feet away.

$f(135) = -\frac{1}{225}(135)^2 + \frac{2}{3}(135)$

$= 9$ No, the kick will fall short.

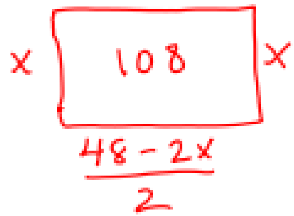
c) From how many yards away could the kicker have made the kick?

44 yards = 132 feet

$f(132) = -\frac{1}{225}(132)^2 + \frac{2}{3}(132)$

$= 10.56$ 44 yards would have went through.

3. A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by *x*, and the area of the garden is 108 square meters. Determine, algebraically, the dimensions of the garden in meters.



$w = x$

$l = \frac{48-2x}{2} = 24 - x$

$A = x(24-x)$

$A = -x^2 + 24x$

$108 = -x^2 + 24x$

$0 = -x^2 + 24x - 108$

$0 = -(x^2 - 24x + 108)$

$0 = -(x-18)(x-6)$

$x - 18 = 0 \quad x - 6 = 0$

$x = 18 \quad x = 6$

$w = 6$
$l = 18$

Exit Ticket:

a) Explain what the vertex can tell you about a real-world problem modeled by a quadratic that is a function of a rectangle's area with respect to its width.

The vertex tells you the width that will produce the maximum possible area.

b) Explain what the *x*-intercept can tell you about a real-world problem modeled by a quadratic that is a function of an object's height with respect to time.

The x-intercept tells you when the object hits the ground.