A Curriculum Project Using Interactive Notebooks in the Application of the Concrete - Representational - Abstract Model in the Inclusive Secondary Algebra II Mathematics Classroom.

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Abstract

Teachers can help support the learning of mathematical concepts of students with learning disabilities by incorporating specific teaching strategies. These strategies include concrete-to-representation-to-abstract model, advanced organizers, teacher demonstrations, guided problem-solving practice, and independent practice. This curriculum project applies these instructional methods to an Algebra II unit on trigonometry and the unit circle through the use of *interactive notebooks*. The curriculum provides a variety of hands on, interactive manipulatives with the intent of increasing the comprehension level of mathematical concepts for students with learning disabilities.

Table of content

Introduction

Many teachers read about best practices and up to date strategies but are often left wondering about actual classroom implementation. With an abundance of resources on specific instructional methods and strategies for teaching students with learning disabilities, there is yet a lack of resources for teaching upper level content such as the unit circle and trigonometry to students with learning disabilities.

The purpose of this curriculum is to provide educators with a unit containing detailed lesson plans that incorporate research based strategies to best support students with learning disabilities in upper level mathematics. Each lesson touches on a different aspect of trigonometry that individually links to the unit circle to develop students foundational understanding. The unit starts in lesson one by reviewing the basics of right triangle trigonometry. This recalls the needed prior knowledge to move forward with the rest of the unit. The sequence of instruction continues as follows: Lesson 2: Special Right Triangles and the First quadrant of the Unit Circle, Lesson 3: Radian Measure and Applications, Lesson 4: Paper Plate Unit Circle Project, and Lesson 5: More Work with the Unit Circle and the Pythagorean Identity. Each lesson is comprised of interactive notebook inserts referred to as *foldables* along with suggested instruction strategy and guided questioning in the lesson plan. Each lesson also includes an exit ticket to asses students comprehension and completion of objectives. The curriculum presented was developed with evidence based research supported best practices for teachers to meet the diverse need of their students with learning disabilities.

Literature Review

General Struggles

The increased efforts over the past decade to raise high school mathematics standards and performance as well as graduation requirements have placed greater emphasis on students with learning disabilities (LD) to complete more advanced levels of mathematics course work (Witzel, et al., 2008). With the current set of standards, 20 states and Washington D.C require Algebra II to graduate with a standard high school diploma (The Hechinger Report, 2010). As a result, many students with LD are at a greater risk of dropping out of school or not obtaining a high school diploma (Witzel, et al., 2008). Thus, students with LD face a unique set of obstacles. If these greater risks are not considered, then this group of students can face many academic struggles. While many of these students are of average to above average intelligence, it is their deficits in language, attention, memory, or metacognition that affect their development of mathematics skills (Miles & Forcht, 1995). The deficits impacting their performance in basic mathematics will then persist into struggles with more advanced courses such as algebra and trigonometry (Impecoven-Lind & Foegen, 2010).

Mathematical Struggles

The daily struggles that students with LD face are the start of a chain reaction that directly leads to their struggles in higher level mathematics. Impecoven-Lind and Foegen (2010) identify three areas of algebra difficulty that students with LD experience: cognitive processes, content foundations and algebra concepts. Cognitive process is where the chain starts. Students who experience attention problems may struggle to identify relevant information in a word problem due to irrelevant information overloading their cognitive systems (Impecoven-Lind et al., 2010). They may also experience difficulty learning the steps in an algorithm or problem-solving strategies (Miller & Mercer, 1997). Memory is another cognitive process that impacts learning, this can lead to retaining new facts, forgetting steps of an algorithm, and performing poorly on tasks involving mixed problems and multi-step word problems (Miller & Mercer 1997). Students with LD who face language difficulties will also struggle with word problems as symbols and syntax represent mathematical language and concepts (Miller and Mercer, 1997).

The struggle with cognitive processing effects content foundations, which is especially challenging in mathematics. There are three essential mathematical domains that should be masterd by students prior to taking algebra: (1) fluency with whole numbers; (2) fraction operations and concepts; and (3) aspects of geometry and measurement such as similar triangles and the ability to analyze properties of two and three dimensional shapes (NMAP, 2008). The cognitive processing difficulties required for mastering these domains often begins in elementary school and persist through middle school (Miller and Mercer, 1997) and are often the reason why students with LD do not have the content foundations needed to learn algebra.

The struggle with mastering content foundations is what leads into the last area of issue which is algebraic concepts. Since many students begin their study of algebra without the prerequisite skills necessary for success, they struggle to effectively apply higher level mathematical strategies (Impecoven-Lind & Foegen, 2010). Impecoven-Lind and Foegen (2010) reviewed a study that found common errors across a range of algebraic topics where students tended to use ineffective strategies when faced with algebraic concepts. For example, students with LD struggled to interpret the meaning of a variable so they often ignored it or tried to guess its value. They also used informal or intuitive methods to arrive at answers by avoiding the formal methods needed for advanced algebraic problems. This indicates that algebra teachers must thoughtfully and intentionally apply instructional strategies to purposefully help students with LD overcome such deficits (Impecoven-Lind & Foegen, 2010).

Evidence Based Best Practices

Between wide-ranging deficits in foundational mathematical skills and increased algebra II performance standards, students with LD at a greater risk for failure unless instructional supports are provided by mathematics teachers (Witzel et al., 2008, p. 271). One highly effective instructional strategy for teaching mathematical concepts is the *concrete to representational to abstract (CRA*) instructional approach (Witzel, et al., 2008). The concrete portion of the strategy starts students off exploring a mathematical concept with hands-on instruction using physical manipulatives. Next students move to learning through pictorial representations of previously used physical manipulatives. Finally, students then move to the abstract portion of the mathematical concepts. One way the CRA method can be applied in the mathematics class is with the use of *interactive notebooks*. When students use concrete materials to build interactive notebooks, they are more likely to make connections and recall information. In fact, Witzel (2005) stated that CRA can increase the likelihood that students remember procedural mathematical concepts because they are able to encode and retrieve information through a variety of sensory options (Witzel, 2005). The CRA approach supports learning of basic mathematical skills and algebraic concepts, but it can be more challenging to apply in higher level mathematics such as algebra II as many concepts are not easily represented in a concrete way (Miles and Forcht, 1995). Yet, whenever possible, algebra II teachers should apply the CRA model and gain the benefits of helping students with LD make connections with abstract concepts to deepen and solidify their understanding of higher-level mathematics.

Unfortunately, the body of research on specific algebra instructional strategies is limited, and this requires teachers to be creative (Impecoven-Lind & Foegen, 2010). For example, an instructional sequence can begin with an advanced organizer (i.e., review of prerequisite knowledge, communication of the lesson objective, and rationale for learning the content) followed by teacher demonstration, guided problem-solving practice, independent practice, and maintenance checks (Rosenshing, 1995). The demonstration and guided practice phases of instruction is a place where a CRA activity such as interactive notebooks can be integrated into instruction. Impecoven-Lind and Foegen (2010) summarize the benefits of using this strategy as it can support learning for students with disabilities. The following curriculum project was developed to provide algebra II teachers support in implementing the CRA instructional method.

Curriculum project

<table>
<tr><th>Lesson 1</th><th colspan="2">Right Triangle Trigonometry Review</th></tr>
<tr><td>Big Idea</td><td colspan="2">To active students’ prior knowledge of concepts and skills needed to form the unit circle.</td></tr>
<tr><td>Objectives</td><td colspan="2">Students will be able to:
1. Accurately identify the parts of a right triangle with respect to a specified angle
2. Calculate the sine, cosine, and tangent of an angle in a given right triangle
3. Create a sketch for a scenario and calculate the sine, cosine, or tangent of a specified angle.</td></tr>
<tr><td>Next Gen Standards</td><td colspan="2">Review and preparation for
AII-F.TF.2: Apply concepts of the unit circle in the coordinate plane to calculate the values of the six trigonometric functions given angles in radian measure.</td></tr>
<tr><td rowspan="2">Materials</td><td>Supplies</td><td>Handouts</td></tr>
<tr><td>• Notebook
• Markers/ color pencils/ highlighters
• Scissors
• Glue
• Pen/ pencil
• Smartboard to display notes</td><td>• Parts of a Right Triangle
• Basic Trigonometric Function
• Trig Ratio Practice
• Trig Ratio Applications
• Right Triangle Trigonometry Exit Ticket</td></tr>
</table>

Warm up (10 minutes)	1. Students are to cut and glue the "Parts of a Right Triangle" foldable into their interactive notebook. - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. 2. Class discussion: Have students share their thoughts about what they remember about right triangles. Then fill out the foldable together Question prompts: - What do you remember about right triangles? - What were some concepts that apply to right triangles? (Here you are looking for students to mention Pythagorean theorem or trigonometry) - Can anyone place any of the parts of the right triangle listed in our foldable? - What determines where the adjacent and opposite sides are?
Direct instruction/ Guided Practice (30 minutes)	1. Students are to cut and glue the "Basic Trigonometric Functions" foldable into their interactive notebook. - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example.

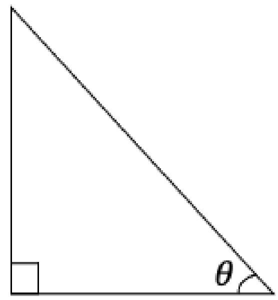
2. Class discussion: Have students share their thoughts about what they remember about sine, cosine, and tangent. Then fill in the foldable together.
    - Tip: instead of writing the ratio relationships for sine, cosine, and tangent as SOH, CAH, TOA horizontally, write it with a ratio $S\frac{O}{H}$, $C\frac{A}{H}$, $T\frac{O}{A}$. This gives a nice visual for students to remember where the parts go.
3. Students are to cut and glue “Trig Ratio Practice” and “Trig Ratio Applications” foldables.
    - Review over instructions and show a completed example before passing out the handout.
    - As students are working on cutting and gluing, circulate the room with your example.
4. Guided Practice: Fill out the foldables using an I do, we do, you do method.
    - First complete the Trig Ratio Practice foldable (I do). Model your solutions process for the first problem for the students on the board.
    - Next ask for students to help you do the second problem (we do). Here you could have a different student volunteer to calculate a specific trigonometric function for a specific angle. This would allow for six students to participate

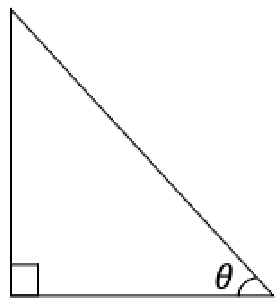
	- Then allow students to try the third problem on their own (you do). During this time circulate the room to check in on students individually. Those who finish quickly can be asked to put their answers on the board. This would allow for another six students to participate. - Check in: ask if students have any questions before moving onto the next foldable - Complete the "Trig Ratio Application" foldable following the same I do, we do, you do process. - Once finished, do another check in and ask if anyone has any questions before moving onto the closure portion of the lesson.
Closure (5 Minutes)	1. Students are to independently complete the Right Triangle Trigonometry Exit ticket. - You may circulate the room to assess how the students are doing and give any needed one on one help. - If students don't finish, they may take it home as homework

Parts of a Right Triangle
(with respect to θ)

Practice labeling the following parts, flip to check.

1. Right angle
2. Hypotenuse
3. Opposite side
4. Adjacent side





Directions:

Step 1: Cut along the dotted lines

Step 2: Fold the "Parts of a Right Triangle" along the solid line. Glue only the title portion to your interactive notebook. Should be able to flip bottom portion up.

Step 3: Label blank right triangle with vocab in correct positions. Glue this behind the Foldable flap in your interactive notebook.

Directions:

Step 1: Cut along the dotted lines

Step 2: Fold the "Basic Trigonometric Functions" along the solid line.

Step 3: Glue only the title portion to your interactive notebook. Should be able to flip bottom portion up.

Step 3: Behind each flap, fill in the corresponding definition

Basic Trigonometric Functions

Tangent

Cosine

Sine

Directions:

Step 1: Cut along the dotted lines

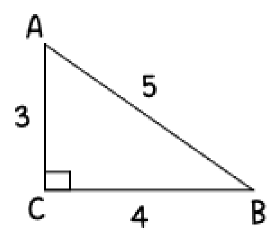
Step 2: Glue the "Trig Ratio Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

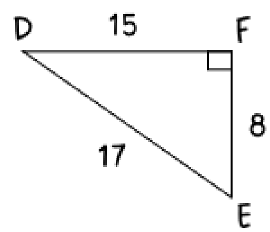
Trig Ratio Practice

Directions: Find the following trigonometric ratios for the identified angle.

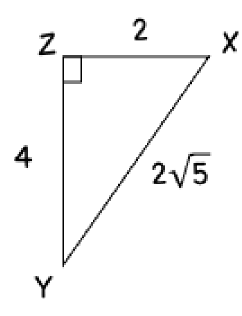
Sin A =	Sin B =
Cos A =	Cos B =
Tan A =	Tan B =



Sin D =	Sin E =
Cos D =	Cos E =
Tan D =	Tan E =



Sin X =	Sin Y =
Cos X =	Cos Y =
Tan X =	Tan Y =



Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Trig Ratio Applications" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Trig Ratio Applications

Directions: Draw a sketch and answer the following using trigonometric ratios.

1. A 25-foot ladder leaning against a vertical wall reaches a height of 24 feet and stretched 7 feet from the base of the wall. Find the sine, cosine, and tangent values of the angle that the ladder makes with the ground.

2. A 20-meter line is used to keep a hot air balloon in place. The sine of the angle that the line makes with the ground is $\frac{3}{4}$. How high is the balloon in the air?

3. From a point on the ground that is 100 feet from the base of a building, the tangent of the angle of elevation of the top of the building is $\frac{5}{4}$. To the nearest foot, how tall is the building?

Name ______________________ Date _________

Right Triangle Trigonometry Exit Ticket

Score: _____/_____

1. The bed of a truck is 5 feet above the ground. The driver of the truck uses a ramp 13 feet long to load and unload the truck and the ramp lands on the ground 12 feet from the truck. Find the sine, cosine, and tangent values of the triangle that the ramp makes with the ground.	2. From the top of a light house 75 feet high, the cosine of the angle of depression of a boat out at sea is $\frac{4}{5}$. To the nearest foot, how far is the boat from the base of the light house?

Name ______________________ Date _________

Right Triangle Trigonometry Exit Ticket

Score: _____/_____

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Lesson 2	Special Right Triangles and the First Quadrant of the Unit Circle
Big Idea	To introduce the foundation of what the unit circle is built off of and to make the connection between coordinates on the unit circle and sine and cosine of the corresponding angles.
Objectives	Students will be able to: 1. Apply the Pythagorean theorem to calculate the side lengths of the 45°-45°-90° and 30°-60°-90° special right triangles. 2. Reason with the 45°-45°-90° and 30°-60°-90° special right triangles to calculate the coordinates of the first quadrant of the unit circle 3. Apply right triangle trigonometry to compute the sine and cosine of specified angles 4. Use the unit circle to reason which quadrant an angle must lie in given the signs of a point on the terminal ray.
Next Gen Standards	Building towards: AII-F.TF.2: Apply concepts of the unit circle in the coordinate plane to calculate the values of the six trigonometric functions given angles in radian measure. AII-F.TF.8: Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1. Find the value of any of the six trigonometric functions given any other trigonometric function value and when necessary find the quadrant of the angle.

<table>
<tr><td rowspan="2">Materials</td><td>Supplies</td><td>Handouts</td></tr>
<tr><td>Notebook
Markers/ color pencils/ highlighters
Scissors
Glue
Pen/ pencil
Smartboard to display notes</td><td>Pythagorean Theorem
Special Triangles
First Quadrant of the Unit Circle
Unit Circle Practice
Special Triangles and the First Quadrant of the Unit Circle Exit Ticket</td></tr>
<tr><td>Warm up

(5 minutes)</td><td colspan="2">1. Students are to cut and glue the “Pythagorean Theorem” foldable into their interactive notebook.
- Review over instructions and show a completed example before passing out the handout.
- As students are working on cutting and gluing, circulate the room with your example.
2. Class discussion: Have students share their thoughts about what they remember about the Pythagorean Theorem. Then fill out the “when can you use it” section together
Question prompt:
- Can anyone tell me the formula for the Pythagorean Theorem?
Answer: $a^2 + b^2 = c^2$
- What kind of triangles can you use the Pythagorean Theorem on?
Answer: Right triangles.</td></tr>
</table>

	- What does the a, b, and c stand for? Answer: *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse. - How do you know what to plug in for a, b, and c? Answer: *c* is the hypotenuse which is located across from the right angle, *a* and *b* are the other side lengths. 3. Allow students to try to solve the example on their own, then review over the solution process together - You may decide to get them started by discussing which sides should be substituted for which variables.
Direct instruction/ Guided Practice (35 minutes)	1. Students are to cut and glue the "Special Triangle" foldable into their interactive notebook. - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example - Explain to the students that they are going to learn how to build the unit circle and that at the core it is comprised of two special triangles. 2. Direct instruction: Completed the 45°-45°-90° special right triangle first. Allow student participation to set up the Pythagorean Theorem and solve for the missing sides. Questions prompts:

	- What do we know about side lengths of isosceles triangles? Answer: Its has two sides that are the same length. - What should our set up for the Pythagorean Theorem look like? Answer: $x^2 + x^2 = 1$ - How should I begin to solve for x? Answer: Combine like terms. Tip: You will come to answer of $x = \frac{1}{\sqrt{2}}$, but this is not the value used in the unit circle. You should discuss rationalizing the denominator and show the process of obtaining a final answer of $x = \frac{\sqrt{2}}{2}$ (can be seen in the answer key). 3. Direct instruction: Complete the 30°-60°-90° special right triangle next. Allow student participation to set up the Pythagorean Theorem and solve for the missing sides. 4. Students are to cut and glue the "First Quadrant of the Unit Circle" foldable into their interactive notebook. - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. - Tip: Instruct students to not glue the individual triangles down yet as you will being completing that portion together as a class

	- Explain that the unit circle is a key tool in trigonometry and is a circle centered at the origin that has a radius of one. Introduce that they will be "building" the first quadrant of it today. 5. Guided Discovery: Start with the special right triangle that has 60° labeled on it. - Asking students if they can figure out where this triangle lines up on the unit circle. After allowing them to try on their own drag your triangle into the correct place on your smartboard. - The goal of this activity is to allow students to "discover" that the horizontal side length corresponds to the x coordinate and the vertical side length corresponds to the y coordinate of the point that the triangle touches on the unit circle. They are to then connect that the x coordinate equals the cosine of the angle and the y coordinate equals the sine of the angle. Question prompts: - How can our triangle be helpful in figuring out the coordinates of the point it touches on our unit circle? Answer: The horizontal side length will give the x coordinate and the vertical side length will give the y coordinate. If students do not initially make this connection, continue questioning.

	- What is an ordered pair made up of? Answer: x and y. - What does x represent in the coordinate plane? Answer: The horizontal distance. - How can we tell what our x coordinate might be? Answer: The horizontal side of the triangle. - What does y represent in the coordinate plane? Answer: The vertical distance. - How can we tell what our y coordinate might be? Answer: The vertical side of the triangle. After labeling the coordinates for the first triangle, then ask students to calculate sin(60°) and the cos(60°) using right triangle trigonometry. - Hint: remind them of $S\frac{O}{H}$, $C\frac{A}{H}$, $T\frac{O}{A}$ Go through a similar process with the special triangle labeled with 45° next an calculate sin(45°) and cos(45°). Then again with the special triangle labeled with 30°. 6. Answer question 3 on the foldable. By this point the students should have picked up on the relationship between the coordinates and the sine and cosine of the angle. 7. Students are to cut and glue the “Unit Circle Practice” foldable into their interactive notebook.

	- Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. 8. Guided Practice: Fill out the foldable together with a mix of teacher led discussion and student participation. - Tip: In question 1, fill out the unit circle diagram with the signs of the coordinates, then use that to fill out the table.
Closure (5 Minutes)	1. Students are to independently complete the Special Triangles and the First Quadrant of the Unit Circle Exit Ticket. - You may circulate the room to assess how the students are doing and give any needed one on one help. - If students don't finish, they may take it home as homework.

Directions:

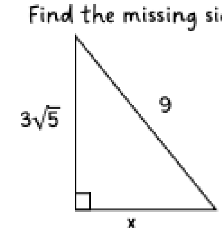
Step 1: Cut along the dotted lines

Step 2: Fold the "Pythagorean Theorem" along the solid lines.

Step 3: Glue only the back middle section into your interactive notebook. Should be able to flip bottom and top portion open. Text Should be visible when folded.

Step 3: Glue the right triangle cut out behind the "Example" flap in your interactive notebook.

Find the missing side, x.



Glue behind this flap

Pythagorean Theorem

When can you use it?

Example

Directions:

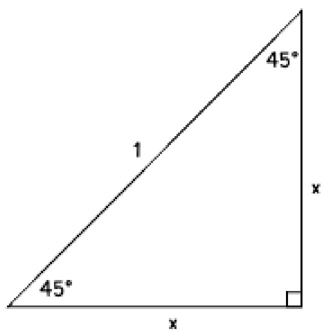
Step 1: Cut along the dotted lines

Step 2: Glue the "Special Triangles" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

45°-45°-90° Special Triangle

Consider the isosceles right triangle shown whose hypotenuse is equal to one and whose angles are both 45°. Solve for x and place your answer is simplest radical form.

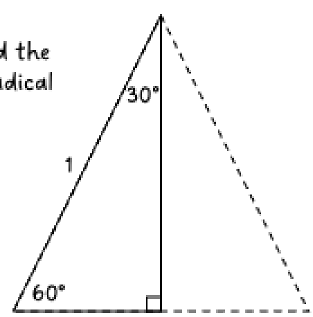


30°-60°-90° Special Triangle

Consider the 30°-60°-90° right triangle shown whose hypotenuse is equal to one and is half of an equilateral triangle.

a. What is the length of the shorter side of this right triangle?

b. Using the Pythagorean theorem, find the length of the longer side in simplest radical form



Directions:

Step 1: Cut the "First Quadrant of the Unit Circle" and special triangles along the dotted lines

Step 2: Glue the "First Quadrant of the Unit Circle" into your interactive notebook

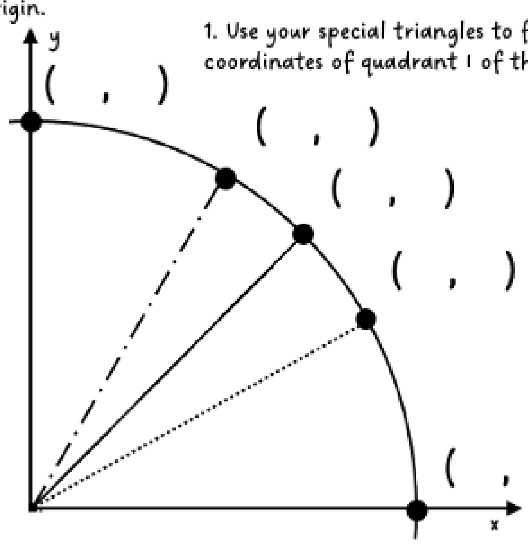
Step 3: Stack the 60° special right triangle onto the unit circle where it aligns with a coordinate point. Label the coordinates. Calculate the sine and cosine of 60°.

Step 4: Repeat step 3 with the 45° special triangle and the 30° special triangle. What do you notice?

First Quadrant of the Unit Circle

The basis of Trigonometry will be a very special circle know as the unit circle. This is a circle with a radius of 1 and a center at the origin.

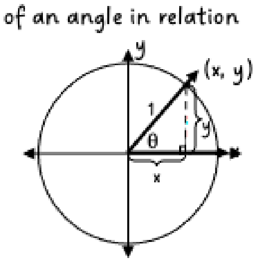
1. Use your special triangles to fill out the coordinates of quadrant I of the unit circle

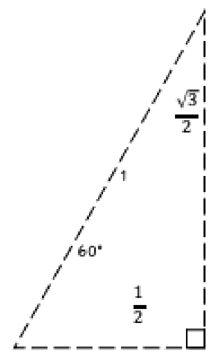


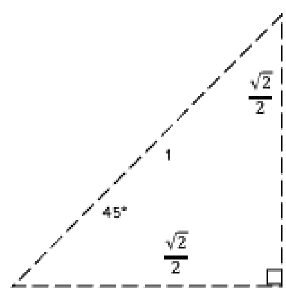
2. Find the following trigonometric ratios for the identified angle

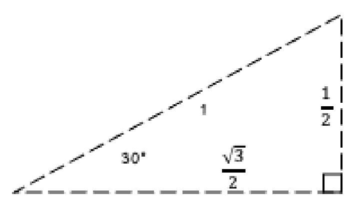
Sin(60°) =	Sin(45°) =	Sin(30°) =
Cos(60°) =	Cos(45°) =	Cos(30°) =

3. What can be said about the sin and cos of an angle in relation to the coordinates of the unit circle?









Directions:

Step 1: Cut along the dotted lines

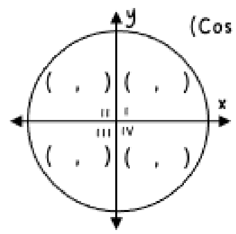
Step 2: Glue the "Unit Circle Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Unit Circle Practice

1. For each quadrant below, determine if the sine and cosine of an angle whole terminal ray falls in the quadrant would be positive or negative.

	I	II	III	IV
Sin				
Cos				



(Cos , Sin)

2. The terminal ray of an angle θ, draw in standard position passes through the point (-0.6, 0.8), which lies on the unit circle. What quadrant does this point lie and what is the value of cos(θ)?

3. For an angle drawn in standard position, it is known that its cosine is positive and its sine is negative. The terminal ray of this angle must land in which quadrant?

3. The point (-5, 12) lies on the circle whose equation is $x^2 + y^2 = 169$. What would represent the cosine of an angle drawn in standard position whose terminal rays passes through the point? (hint draw a sketch).

Name ______________________ Date _________

Special Triangles and the First Quadrant of the Unit Circle Exit Ticket

Score: _____/_____

1. What is the sin(60°)	2. If both the sine and cosine of an angle are less than zero, then when drawn in standard position in which quadrant would the terminal ray fall?
3. Which of the following values cannot be the sine of an angle? (Hint: think about the range of y-values on the unit circle). a. $\frac{5}{14}$ b. $\frac{-2}{3}$ c. $\frac{\sqrt{3}}{5}$ d. $\frac{4}{3}$	4. When drawn in standard position, an angle θ has a terminal ray that lies in the second quadrant and whose sine is equal to $\frac{9}{41}$. Find the cosine of θ in rational form (as a fraction).

Name ______________________ Date _________

Special Triangles and the First Quadrant of the Unit Circle Exit Ticket

Score: _____/_____

1. What is the sin(60°)	2. If both the sine and cosine of an angle are less than zero, then when drawn in standard position in which quadrant would the terminal ray fall?
3. Which of the following values cannot be the sine of an angle? (Hint: think about the range of y-values on the unit circle). a. $\frac{5}{14}$ b. $\frac{-2}{3}$ c. $\frac{\sqrt{3}}{5}$ d. $\frac{4}{3}$	4. When drawn in standard position, an angle θ has a terminal ray that lies in the second quadrant and whose sine is equal to $\frac{9}{41}$. Find the cosine of θ in rational form (as a fraction).

<table>
<tr><th>Lesson 3</th><th colspan="2">Radian Measure and Applications</th></tr>
<tr><td>Big Idea</td><td colspan="2">To introduce radian measure and add the degree and radian angles to all four quadrants of the unit circle.</td></tr>
<tr><td>Objectives</td><td colspan="2">Students will be able to:
1. Convert between degree and radian measurements
2. Reason with fractions to build the radian angle measures of the unit circle
3. Reason with rotation diagrams to calculate radian application problems</td></tr>
<tr><td>Next Gen Standards</td><td colspan="2">AII-F.TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</td></tr>
<tr><td rowspan="2">Materials</td><td>Supplies</td><td>Handouts</td></tr>
<tr><td>• Notebook
• Markers/ color pencils/ highlighters
• Scissors
• Glue
• Pen/ pencil
• Smartboard to display notes</td><td>• Radian Measure
• Angle Measure and the Unit circle
• Converting Between Degree and Radian Measure
• Radian Measure Exit Ticket</td></tr>
<tr><td>Warm up

(5 minutes)</td><td colspan="2">1. Class discussion: The goal is to get the students brains activated and to recall their prior knowledge of circles and angle measurement.</td></tr>
</table>

	Topics to be discussed can include but is not limited to circumference, area, and degrees. - Tip: Ask students the different ways a circle can be measured. Answers may include different metrics like feet, inches, hopefully a student mentions degrees and you can introduce that in this lesson they will be learning a new way to measure rotation and angles.
Direct instruction/ Guided Practice (35 Minutes)	1. Students are to cut and glue the "Radian Measure" foldable into their interactive notebook. - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. 2. Direct Instruction: Review over the definition insert and emphasize that radians measure angles as the ratio of their arc length to their radius. Then model your solution process for the example insert. 3. Students are to cut and glue the "Angle Measure and the Unit Circle" foldable into their interactive notebook - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example.

	- Explain to students that today they are going to piece together all the angle measurements on the unit circle in degrees and radians. - Tip: Challenge students to try and fill out the first quadrant from memory to assess how much they retained from lesson 2. 4. Guided Discovery: The goal of this foldable is for students to conclude that there are 2π radians in one full circle. Then they are to use that and their knowledge of fractions to figure out the radian measure of the angels on the unit circle. - Start by having students label all the degree measurements on the unit circle. - Then answer question a and question b. - Next use guiding questions to help them reason and figure out the radian angle measurements. Question prompts: - At $0°$ how many radians have we traveled? Answer: 0 radians - After one full rotation at $360°$ how many radians have we traveled? Answer 2π radians - If you travel half of the circle, how many radians would that be? Answer: π radians - If $180°$ is π radians, then what would $90°$ be? Hint: think about what fraction 90 is of 180?

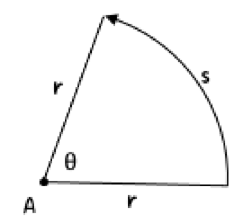
	Answer: $\frac{\pi}{2}$ radians - How many radians would you travel to hit the 45° angle? Answer: $\frac{\pi}{4}$ radians - If 45° is a quarter π radians then how many quarters is 135°? Answer: Three quarters - How many quarters is 225°? Answer: Five quarters - Continue with this questioning until students recognize the pattern and are able to fill out all the angles in radians 5. Students are to cut and glue the "Converting Between Degree and Radian Measure" foldable into their interactive notebook - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. - Explain to students that while it is nice to be able to think of fractions of a circle to convert to radians, we will need a more standard method to convert between any degree angle or radian angle. 6. Direct instruction: Complete the "To Radians" insert first. Then complete the "To Degrees" insert.

	- Tip: To help decided which ratio to multiply by, they will need the units of measure they are converting from to cancel with the denominator. i.e. if they are going from radians they will need π radians in the denominator, if they are going from degrees they will need 180° in the denominator. 7. Students are to cut and glue the "Radian Measure Practice" foldable into their interactive notebook - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. 8. Guided Practice: Complete this insert through a combination of teacher modeling solution process, students participation, and independent practice. - Have students try questions 1 and 2 on their own, circulate the room and ask for students to go to the board and write their answers. - Model the solution process for question 3a and 3b, then ask students to participate in answering question 3c and 3d. - Have students try question 4 on their own, encourage them to draw a diagram of the situation and refer back to their "Radian Measure" foldable. Then model the solution process on the board.

Closure (5 Minutes)	1. Students are to independently complete the Radian Measure Exit ticket. - You may circulate the room to assess how the students are doing and give any needed one on one help. - If students don't finish, they may take it home as homework.

The radian angle, θ, created by a rotation about a point A using a radius of r and passing through an arc length of s is defined as

$$\theta = \frac{s}{r} \quad \text{or} \quad s = \theta \cdot r$$



Directions:

Step 1: Cut along the dotted lines

Step 2: Fold the "Radian Measure" along the solid lines.

Step 3: Glue only the back middle section into your interactive notebook. Should be able to flip bottom and top portion open. Text Should be visible when folded.

Step 3: Glue the cut outs behind corresponding flap in your interactive notebook.

Determine the number of radians that the minute hand of a clock passes through if it has a length of 5 inches and its tip travels a total distance of 13 inches.

Glue behind this flap

Radian Measure

Definition

Example

Glue behind this flap

Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Angle Measure and the Unit Circle" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

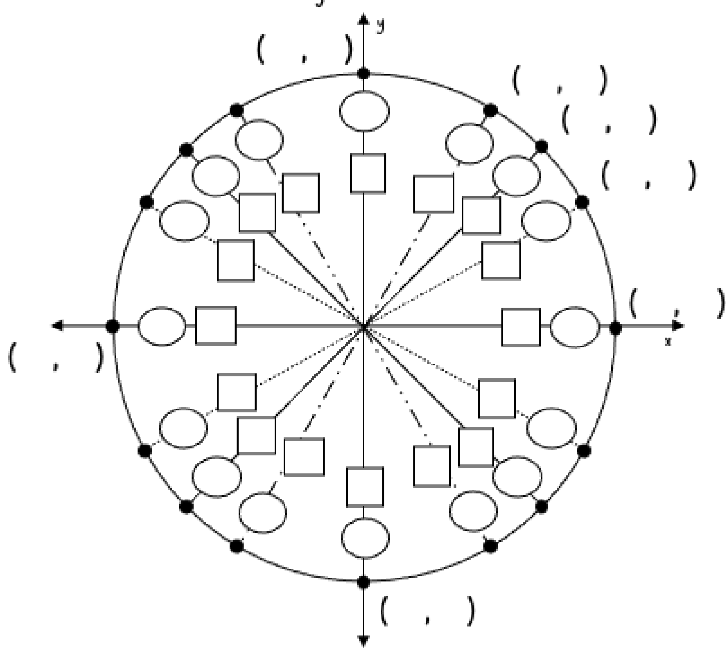
Angle Measure and the Unit Circle

Consider a full rotation around any point in the counter-clockwise (positive) direction

a. What would the arc length, s, of a circle be for one full rotation? Hint: think about circumference.

b. Based on the definition for radians and your answer to part (a), how many radians are there in one full rotation?

c. Now that we know how many radians are in one full rotation, lets piece together the radian angles in the unit circle, with what we already know. Fill the degree measure in the circles and the radian measure in the rectangles.



Multiply by a ratio with π and 180 so that **degrees cancel** out

$$\theta^\circ \cdot \frac{\pi}{180}$$

Convert each of the following common angles in degrees into radians. Express your answers in terms of pi.

a. $\theta = 90°$ b. $\theta = 120°$

Directions:

Step 1: Cut along the dotted lines

Step 2: Fold the "Converting" foldable along the solid lines.

Step 3: Glue only the back middle section into your interactive notebook. Should be able to flip bottom and top portion open. Text Should be visible when folded.

Step 3: Glue the cut outs behind corresponding flap in your interactive notebook.

Multiply by a ratio with π and 180 so that **radians cancel** out

$$\theta \text{ rad} \cdot \frac{180}{\pi}$$

Convert each of the following common radian angles into degrees.

a. $\theta = \frac{5\pi}{6}$ b. $\theta = \frac{3\pi}{4}$

Glue behind this flap

Converting

To Radians

To Degrees

Glue behind this flap

Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Radian Measure Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Radian Measure Practice

1. Convert each of the following degree angles to angles in radians. Express your answers in exact terms of pi.

a. 30° b. 45° c. 135° e. 270°

2. Convert each of the following angles given in radians into an equivalent measure in degrees.

a. $\frac{2\pi}{3}$ b. $\frac{\pi}{4}$ c. $\frac{11\pi}{4}$ e. $\frac{4\pi}{3}$

3. If an angle is drawn in standard position with each of the following radians angles, determine the quadrant its terminal ray lies in. (Hint: convert each angle into degrees)

a. 4.75 b. 5.28 c. 1.65 e. 2.9

4. A cat is attached to a 10 foot leash. He travels around an arc that has a length of 25 feet. What is the radian angle he has rotated through? What is the equivalent degree angle rounded to the nearest degree?

Name ______________________ Date _________

Radian Measure Exit Ticket

Score: _____/_____

1. The distance from the center of a Ferris wheel to a person who is riding is 38 feet. What distance does a person travel if the Ferris wheel rotates through an angle of 4.25 radians?	2. The pendulum of a grandfather clock makes an angle of 2.5 radians as its tip travels 18 feet. What is the length of the pendulum?
3. A wheel whose diameter is 30 feet rolls a distance of 45 feet without slipping. Through what degree angle did the wheel rotate? (Hint: find the radian angle first).	

Name ______________________ Date _________

Radian Measure Exit Ticket

Score: _____/_____

1. The distance from the center of a Ferris wheel to a person who is riding is 38 feet. What distance does a person travel if the Ferris wheel rotates through an angle of 4.25 radians?	2. The pendulum of a grandfather clock makes an angle of 2.5 radians as its tip travels 18 feet. What is the length of the pendulum?
3. A wheel whose diameter is 30 feet rolls a distance of 45 feet without slipping. Through what degree angle did the wheel rotate? (Hint: find the radian angle first).	

<table>
<tr><th>Lesson 4</th><th colspan="2">Paper Plate Unit Circle Project</th></tr>
<tr><td>Big Idea</td><td colspan="2">To piece together what students already understand about the unit circle, and physically build a model of the unit circle filling in the three remaining quadrants.</td></tr>
<tr><td>Objectives</td><td colspan="2">Students will be able to:
1. Reason with the 45°-45°-90° and 30°-60°-90° special right triangles to calculate the coordinates of all four quadrants of the unit circle
2. Use the unit circle to evaluate trigonometric functions.</td></tr>
<tr><td>Next Gen Standards</td><td colspan="2">AII-F.TF.2: Apply concepts of the unit circle in the coordinate plane to calculate the values of the six trigonometric functions given angles in radian measure.</td></tr>
<tr><td rowspan="2">Materials</td><td>Supplies</td><td>Handouts</td></tr>
<tr><td>• Notebook
• Markers/ color pencils/ highlighters
• Scissors
• Glue
• Paper fasteners
• Pen/ pencil
• Smartboard to display notes</td><td>• 45-45-90 Special Right Triangle
• 30-60-90 Special Right Triangle Version A
• 30-60-90 Special Right Triangle Version B
• Unit Circle and Trig Function Practice
• Unit Circle ad Trig Function Exit Ticket</td></tr>
</table>

Warm up (5 minutes)	1. Class discussion: The goal is to get the students brains activated and to recall their prior knowledge of the basics of the unit circle that they have learned thus far in this unit. Question Prompts - Who can tell me what the unit circle can be used for? Answer: To help calculate the sine and cosine of specific angles - Can anyone explain how they build the first quadrant of the unit circle? Answer: Explanation should include the use of special right triangles - What is radian measure? Answer: Another way to measure an angle, the ratio between arc length and radius - How are the signs of a coordinate determined? Answer: It depends what quadrant the point lies in Then introduce that they will be putting it all together and building a complete unit circle.
Direct instruction/ Guided Practice (35 minutes)	Suggested Prep work: Copy the 45-45-90, 30-60-90 Version A, and 30-60-90 Version B on three different colored pages. Make sure you have enough for each student to get one of each triangle (total of three triangles per student). Have coloring supplies to match the colors of the paper triangles.

	1. Follow the "Paper Plate Unit Circle Project Teacher Direction" handout to complete the unit circle with your students as a class. You will lead them through the process of building a paper plate unit circle from scratch. Suggested guided questioning at each step is listed below. This project will be completed using teacher modeling, guided questioning, and class discussion/ participation. Questions prompts for step 1: - How many radians are in one full rotation? Answer: 2π - How many radians are in a half rotation? - Answer: π - 90 is what fraction of 180? - Answer: One half - What is the radian angle measure at 90° - Answer: $\frac{\pi}{2}$ - Counting by halves, what is the fraction at 270°? - Answer: Three halves - What is the radian angle measure at 270°? - Answer: $\frac{3\pi}{2}$ Questions prompts for step 2: - 45 is what fraction of 180?

	Answer: A fourth - What is the radian angle measure at 45° Answer: $\frac{\pi}{4}$ - Counting by quarters, what is the fraction at 135°? Answer: Three quarters - What is the radian angle measure at 135°? Answer: $\frac{3\pi}{4}$ - Continuing counting by quarters, what is the fraction at 225°? Answer: Five quarters - What is the radian angle measure at 225°? Answer: $\frac{5\pi}{4}$ - Continuing counting by quarters, what is the fraction at 315°? Answer: Seven quarters - What is the radian angle measure at 315°? Answer: $\frac{7\pi}{4}$ Continue with this frame of questioning through the project. Tip: When starting to label the coordinates ask or remind students that in an ordered pair (x, y): x represents the horizontal distance and y represents the vertical distance.

	2. Students are to cut and glue the "Unit Circle and Trig Functions Practice" foldable into their interactive notebook - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. 5. Guided Practice: Fill out the foldables using a combination of independent practice and the I do, we do, you do method. - For questions 1 and 2 ask students to refer to their paper plate unit circle and try to answer on their own. Then review over the solution process. - For questions 3, 4, and 5 use the I do, we do, you do method as this is the first time they are seeing functions with sine and cosine. - Model your solutions process for question 3 for the students on the board. - Next ask for students to help you do the fourth problem. Here you could have a different student volunteer to calculate a specific trig function for a specific angle. This would allow for two students to participate - Then allow students to try the fifth problem on their own. During this time circulate the room to check in on students individually.

	Those who finish quickly can be asked to put their answers on the board.
Closure (5 minutes)	1. Students are to independently complete the Unit Circle and Trig Functions Exit ticket. - You may circulate the room to assess how the students are doing and give any needed one on one help. - If students don't finish, they may take it home as homework.

Paper Plate Unit Circle Project: Teacher Directions

Step 1: Creating the x and y axis.

1. fold plate into quarters by folding in half two times.
2. With a ruler and black marker trace creases and label.

Discussion:

- Have students help label coordinates (1,0), (0,1), (-1,0), and (0,-1)
- Have students help label degree measurements 0, 90, 180, 270, and 360.
- Have students help label radian degree measurements 2π, π, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$

Step 2: Creating angles of multiple of 45.

1. fold plate into eighths by folding in half three times.
2. With a ruler and blue marker trace creases and label.

Discussion:

- Have students help label degree measurements 45, 135, 225, and 315.
- Have students help label radian degree measurements $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, and $\frac{7\pi}{4}$

Step 3: Creating angles of multiple of 60.

1. fold plate into sixths by folding in half then folding in thirds.
2. With a ruler and red marker trace creases and label.

Discussion:

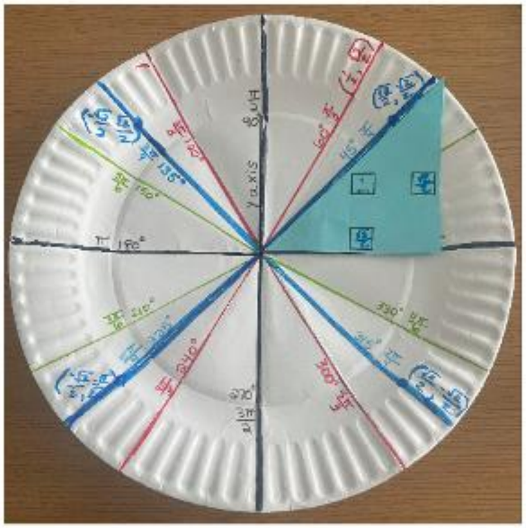
- Have students help label degree measurements 60, 120, 240, and 300.
- Have students help label radian degree measurements $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, and $\frac{5\pi}{3}$

Step 4: Creating angles of multiple of 30.

1. fold plate into twelfths by folding in half, then folding in thirds, then folding in half again.
2. With a ruler and green marker trace creases and label.

Discussion:

- Have students help label degree measurements 30, 150, 210, and 330.
- Have students help label radian degree measurements $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$



Step 5: Creating the coordinates for angles of multiple of 45.

1. Distribute the 45-45-90 special right triangle
2. Line up the triangle with the 45° angle in quadrant 1.

Discussion: Have students help label coordinates $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

3. Line up the triangle with the 135° angle in quadrant 2.

Discussion: Have students help label coordinates $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

4. Line up the triangle with the 225° angle in quadrant 3

Discussion: Have students help label coordinates $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

5. Line up the triangle with the 315° angle in quadrant 4.

Discussion: Have students help label coordinates $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

Step 6: Creating the coordinates for angles of multiple of 60.

1. Distribute the 30-60-90 special right triangle version A
2. Line up the triangle with the 60° angle in quadrant 1.

Discussion: Have students help label coordinates $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

3. Line up the triangle with the 120° angle in quadrant 2.

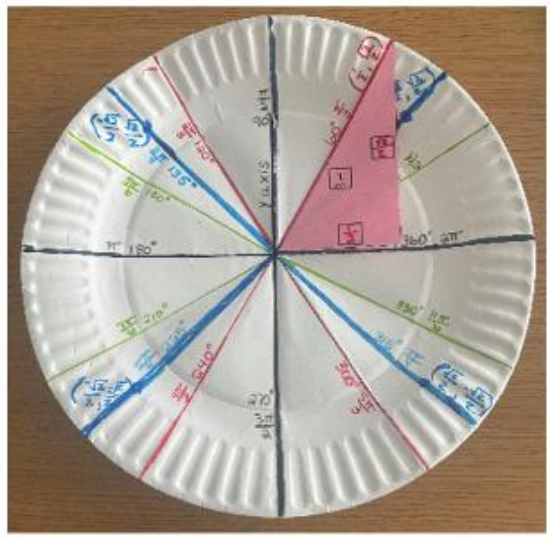
Discussion: Have students help label coordinates $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

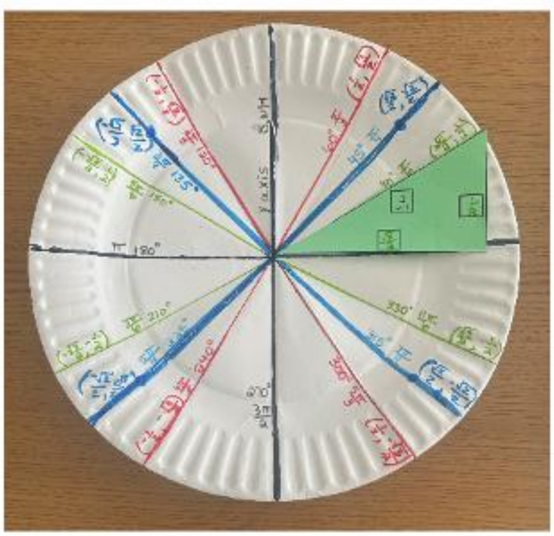
4. Line up the triangle with the 240° angle in quadrant 3

Discussion: Have students help label coordinates $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$

5. Line up the triangle with the 300° angle in quadrant 4.

Discussion: Have students help label coordinates $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$





Step 7: Creating the coordinates for angles of multiple of 30.

1. Distribute the 30-60-90 special right triangle version B
2. Line up the triangle with the 30° angle in quadrant 1.

Discussion: Have students help label coordinates $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

3. Line up the triangle with the 150° angle in quadrant 2.

Discussion: Have students help label coordinates $(-\frac{\sqrt{3}}{2}, \frac{1}{2})$

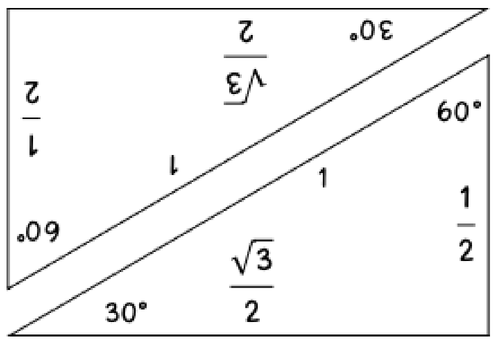
4. Line up the triangle with the 210° angle in quadrant 3

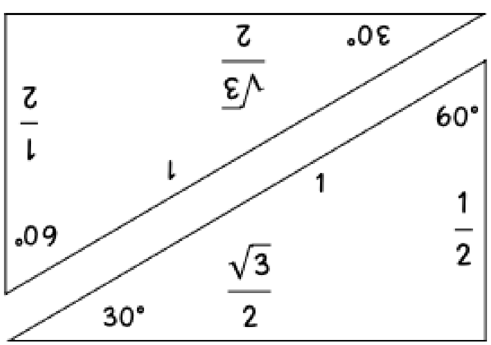
Discussion: Have students help label coordinates $(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

5. Line up the triangle with the 330° angle in quadrant 4.

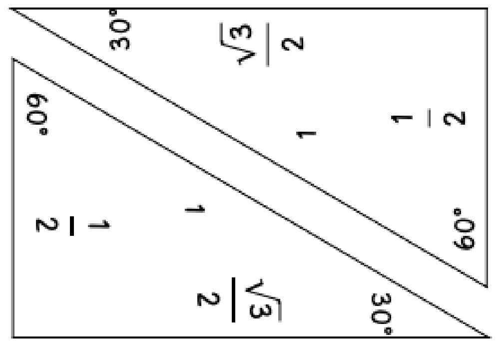
Discussion: Have students help label coordinates $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$

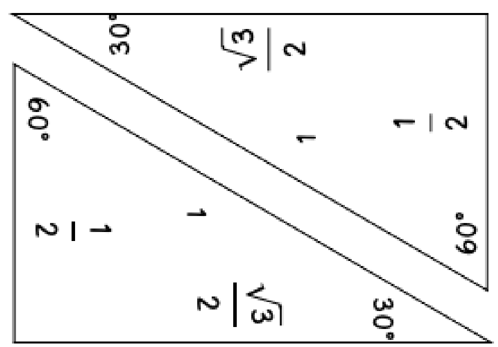
30-60-90 Special Right Triangle
Version B (used for multiples of 30°)



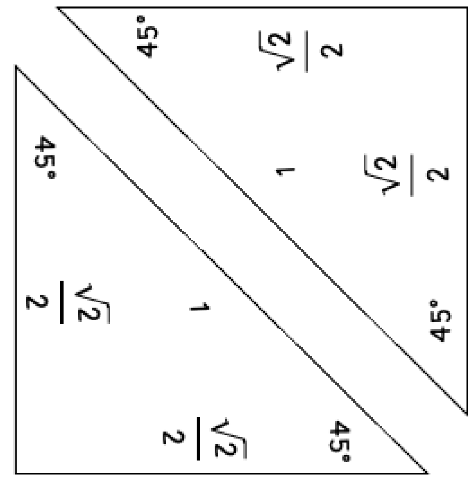


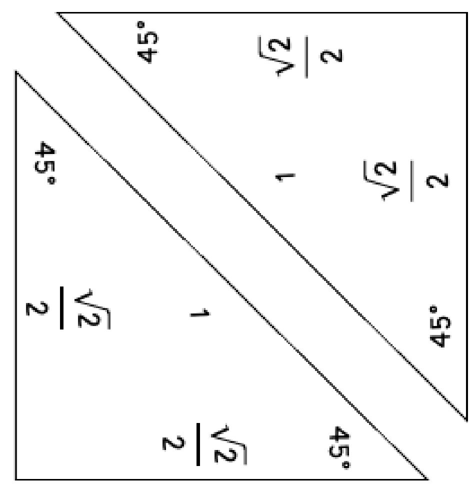
30-60-90 Special Right Triangle
Version A (used for multiples of 60°)





45-45-90 Special Right Triangle





Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Trig Ratio Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Unit Circle and Trig Functions Practice

1. Written in exact form, cos(135) =?

a. $-\frac{1}{2}$

b. $-\frac{\sqrt{3}}{2}$

c. $-\frac{\sqrt{2}}{2}$

d. $-\frac{\pi}{4}$

2. Which of the following is not equal to sin(30)?

a. cos(60°)

b. -sin(330°)

c. cos(300°)

d. sin(120°)

3. Given the function $f(x) = 6\sin(x)$ then what is the value of $f(60°)$?

4. Given the function $g(x) = 4\cos(x) - 2\sin(x)$ then what is the value of $g(\frac{11\pi}{6})$?

5. Given the function $h(x) = 3\sin(x) - \frac{1}{2}\cos(x)$ then what is the value of $h(\frac{4\pi}{3})$?

Name ______________________ Date _________

Unit Circle and Trig Functions Exit Ticket

Score: _____/_____

1. Which of the following values is not equal to the sin(270°)?. a. cos(180°) b. −sin(90°) c. −cos(0°) d. sin(360°)	2. Given the function $h(x) = \sin(x) \cdot \cos(x)$ then what is the value of $h(\frac{5\pi}{6})$?
3. For an angle drawn in standard position, it is known that its cosine is greater than zero and its sine is less than zero. The terminal ray of this angle must land in which quadrant?	

Name ______________________ Date _________

Unit Circle and Trig Functions Exit Ticket

Score: _____/_____

1. Which of the following values is not equal to the sin(270°)?. a. cos(180°) b. −sin(90°) c. −cos(0°) d. sin(360°)	2. Given the function $h(x) = \sin(x) \cdot \cos(x)$ then what is the value of $h(\frac{5\pi}{6})$?
3. For an angle drawn in standard position, it is known that its cosine is greater than zero and its sine is less than zero. The terminal ray of this angle must land in which quadrant?	

<table>
<tr><th>Lesson 5</th><th colspan="2">More work with the Unit Circle and the Pythagorean Identity</th></tr>
<tr><td>Big Idea</td><td colspan="2">To reinforce students understanding of the unit circle and to build procedural fluency with evaluating the sin and cosine of carious angles on the unit circle. Then to introduce some applications of the unit circle like Pythagorean identity.</td></tr>
<tr><td>Objectives</td><td colspan="2">Students will be able to:
1. Use their knowledge of the unit circle to evaluate the sine and cosine of various angles given in degrees and radians.
2. Use their knowledge of the unit circle to evaluate trigonometric functions with function composition.
3. Find a specified trigonometric function value given another trigonometric function value using the Pythagorean identity.</td></tr>
<tr><td>Next Gen Standards</td><td colspan="2">AII-F.TF.8: Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$. Find the value of any of the six trigonometric functions given any other trigonometric function value and when necessary find the quadrant of the angle.</td></tr>
<tr><td rowspan="2">Materials</td><td>Supplies</td><td>Handouts</td></tr>
<tr><td>• Notebook
• Markers/ color pencils/ highlighters
• Scissors
• Glue
• Pen/ pencil</td><td>• Unit Circle
• Pythagorean Trig Identity
• Unit Circle and Trig Identity Practice
• More Work with the Unit Circle Exit Ticket</td></tr>
</table>

<table>
<tr><td></td><td>• Smartboard to display notes</td><td></td></tr>
<tr><td>Warm up

(5 minutes)</td><td colspan="2">1. Pass out the “Unit Circle” foldable and challenge students to fill as much of the unit circle as they can from memory. Reassure them to try their best and there is no right or wrong amount to know, it’s just to test what they have learned so far in the unit.</td></tr>
<tr><td>Direct instruction/ Guided Practice

(35 minutes)</td><td colspan="2">3. Students are to cut and glue the “Unit Circle” foldable into their interactive notebook
- Review over instructions and show a completed example before passing out the handout.
- As students are working on cutting and gluing, circulate the room with your example.
4. Guided Practice: Ask students to fill in the remaining portion of the unit circle that they were not able to remember. This is a reinforcement activity to summarize the paper plate activity in Lesson 4.
- Instruct the students to rotate the triangles to figure out the coordinates, take into account what quadrant they are in and what to corresponding signs should be.
- If needed remind them that x = horizontal distance and y = vertical distance
- If students are struggling with radians, they can use the triangles to help count out the fraction of the circle they have rotated. The</td></tr>
</table>

	60° triangle can be used to count thirds, 45° triangle can be used to count fourths, and the 30° can be used to count sixths. 5. Students are to cut and glue the "Pythagorean Trig Identity" foldable into their interactive notebook - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example 6. Guided Practice: Answer Questions a, b, and c on the foldable. This leads you to deriving the Pythagorean trig identity $\sin^2(\theta) + \cos^2(\theta) = 1$. Encourage student participation as they have been exposed to the concepts individually in previous lessons in this unit. 7. Direct instruction: Model the solution process to question d. Ask for students participation to "help" you solve for the value of cosine. 8. Students are to cut and glue the "Unit Circle and Trig Identity Practice" foldable into their interactive notebook - Review over instructions and show a completed example before passing out the handout. - As students are working on cutting and gluing, circulate the room with your example. 9. Guided Practice: Complete this insert through a combination of teacher modeling solution process, students participation, and independent practice.

	- Have students try questions 1 and 2 on their own, circulate the room and ask for students to go to the board and write their answers. - Model the solution process for question 3 - Have students try question 4 on their own, encourage them to draw a diagram of the situation or refer back to their "Unit Circle" foldable. Then model the solution process on the board.
Closure (5 Minutes)	1. Students are to independently complete the More Work with the Unit Circle Exit ticket. - You may circulate the room to assess how the students are doing and give any needed one on one help. - If students don't finish, they may take it home as homework.

Directions:

Step 1: Cut "The Unit Circle" and special right triangles along the dotted lines

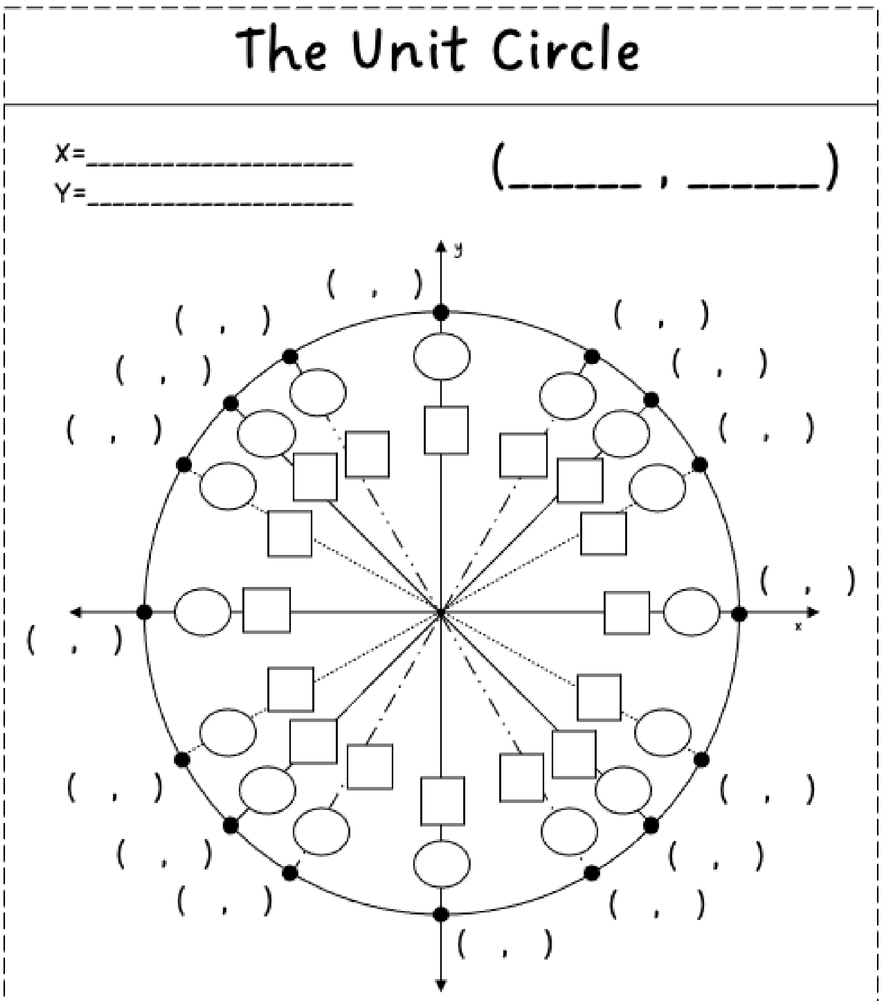
Step 2: With a paperclip, poke a hole through the center of the unit circle, and through the dot on each circle attached to each special right triangle.

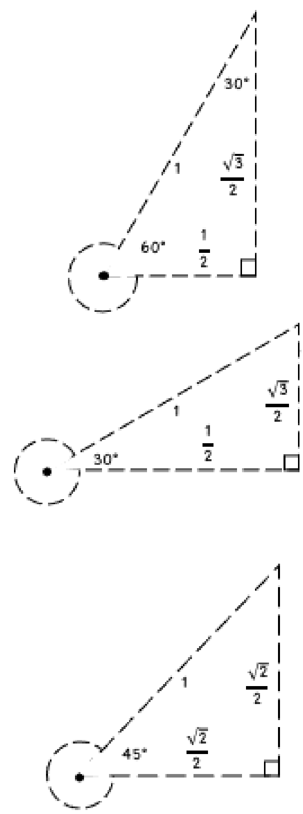
Step 3: Stack the triangles and align all holes with the hole made on the unit circle, fast with a paper fastener.

Step 4: Rotate the fastener a few times to make the holes a bigger, you want to be able to easily rotate each triangle around.

Step 5: Glue the unit circle flat in your interactive notebook centered on the page. Make sure to not glue the paper fastener in place!

Step 6: Use your special right triangles to fill out the coordinates of your unit circle.





Directions:

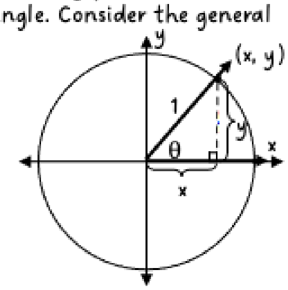
Step 1: Cut along the dotted lines

Step 2: Glue the "Pythagorean Trig Identity" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

The Pythagorean Trig Identity

A perpendicular line can be dropped from any point on the unit circle to the x axis to form a right triangle. Consider the general right triangle in the unit circle below.



a. Apply the Pythagorean theorem to the right triangle

b. What does x and y represent on the unit circle?

c. Substitute your answer from b into the equation you wrote in a. This is know as the Pythagorean trig identity.

d. An angle, θ , has a terminal ray that falls in the second quadrant. If it is known that $\sin(\theta) = \frac{3}{5}$, determine the value of $\cos(\theta)$.

Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Unit Circle and Trig Identity Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Unit Circle and Trig Identity Practice

1. An angle, θ , has a terminal ray that falls in the first quadrant and $\cos(\theta) = \frac{1}{3}$. Determine the value of $\sin(\theta)$ in simplest radical form.

2. If the terminal ray of θ lies in the fourth quadrant and $\sin(\theta) = -\frac{\sqrt{3}}{3}$ determine $\cos(\theta)$ in simplest form

3. If $f(x) = 2x$ and $g(x) = \cos(x)$ then evaluate $g(f(\frac{\pi}{2})$

4. Which of the following is equal to the cos(300)?

a. $\cos(60°)$ b. $-\cos(60°)$

c. $\cos(30°)$ d. $-\cos(30°)$

Name ________________________ Date _________

More Work with the Unit Circle Exit Ticket

Score: _____/_____

1. When drawn in standard position, an angle, θ , has a terminal ray that falls in the third quadrant and $\cos(\theta) = -\frac{8}{17}$. Determine the value of $\sin(\theta)$.	2. If $f(x) = 3x$ and $g(x) = \cos(x) + \sin(x)$ then evaluate $f(g(\frac{3\pi}{2}))$.

Name ________________________ Date _________

More Work with the Unit Circle Exit Ticket

Score: _____/_____

1. When drawn in standard position, an angle, θ , has a terminal ray that falls in the third quadrant and $\cos(\theta) = -\frac{8}{17}$. Determine the value of $\sin(\theta)$.	2. If $f(x) = 3x$ and $g(x) = \cos(x) + \sin(x)$ then evaluate $g(f(\frac{3\pi}{2}))$.

My Personal Suggestions for the Classroom

In my experience, I have seen teachers introduce the unit circle in one day and either instruct students to memorize it or give quick tips and tricks; both of which eliminate the conceptual understanding behind the unit circle. This was how I was taught as a student learning about the unit circle, and in my first year teaching I took this approach. I found that my students felt very overwhelmed with the diagram, lacked the understanding of what the unit circle is created from, and how and why it connects to trigonometric functions. The curriculum presented in the thesis incorporates scaffolding, guided questioning, and hands on interactive manipulatives to instruct students to develop the unit circle over several days and lessons. I have found that slowly introducing the unit circle in this manor allows time for each step to sink into the students mind and builds their confidence for when it comes time to piece the entire diagram together.

The needs of students with learning disabilities vary from student to student and classroom to classroom. It is the teacher's job to discern what their students need to be successful and provide them with that support. This curriculum is meant to be flexible and allow teachers to implement it in a way that will best support their current students. The use of interactive notebooks is a form of note taking that can being used all year or just for a hands on experience for a specific unit. I have found that the use of interactive notebooks has given an opportunity for my students to be creative and put effort into making something they can be proud of. If planning to incorporate interactive notebooks in the classroom, a fun way for students to buy into the idea and take ownership of their work is to give them a notebook and decorate the cover. I like to do this as a fun first day of school activity. I am sure to enthusiastically introduce the concept and try to get the students excited as well. If planning to implement interactive notebooks for just one unit, I suggest to have students create a section in their binder and give them a sheet of paper to decorate in place of a notebook cover. These handouts and manipulatives can be glued into notebooks, or loose-leaf and kept in a binder.

Guided practice and questioning strategies are implemented in this curriculum to help students discover specific concepts rather than directly presenting them with the final idea. This is to strengthen their critical thinking and deepen their conceptual understanding of the topics presented in this unit. I always tell my students that if they take the time and effort to understand the concepts, they save themselves from blatantly memorizing tons of facts. It is important for students with LD to not only learn how to complete a specific type of math problem, but to give them the tools to logically reason and problem solve to better support them in their future once they leave the classroom. In my classroom I like to facilitate a flowing class discussion with my guiding questions. I do not require students to raise hands to speak, but allow them to openly join the conversation when they choose. I create a classroom atmosphere where they know not to interrupt or talk over a peer and to be respectful. The suggested guided questioning presented in the lesson plans loosely follow a conversation pattern that fits this style of class discussion; but may be adapted to other styles such as hand raising or reflecting with a partner. While it would be great to guide students to discover every math concept and strategy on their own, some procedural methods are best taught through the teacher modeling the solution. During this time I explicitly verbalize my self-questioning throughout the entire solution process. This is to show students how to problem solve and what to think when they run into a problem that they do to automatically understand.

Conclusion

The number of children diagnosed with learning disabilities has increased over the last few decades, and while there is a wealth of information and strategies on how to teach students with LD, it is rarely applied to upper level mathematics. Considering the specific challenges that students with LD face such as deficits in language, attention, memory, or metacognition along with the raised mathematics standards and graduation requirements, it’s more important now for mathematics teachers to have resources to meet the needs of their students. Students with LD can thrive in the general education classroom and successfully make their way through the upper level mathematics when given the tools they need to learn. The goal of this curriculum project is to take evidence based best practices such as concrete to representational to abstract approach and create an interactive hands-on sequence of lessons on upper level mathematics topics such as trigonometry and the unit circle through the use of interactive notebooks. It is the authors hope that educators can take the curriculum presented to support the instruction and learning of their students with LD in the upper level mathematics classroom.

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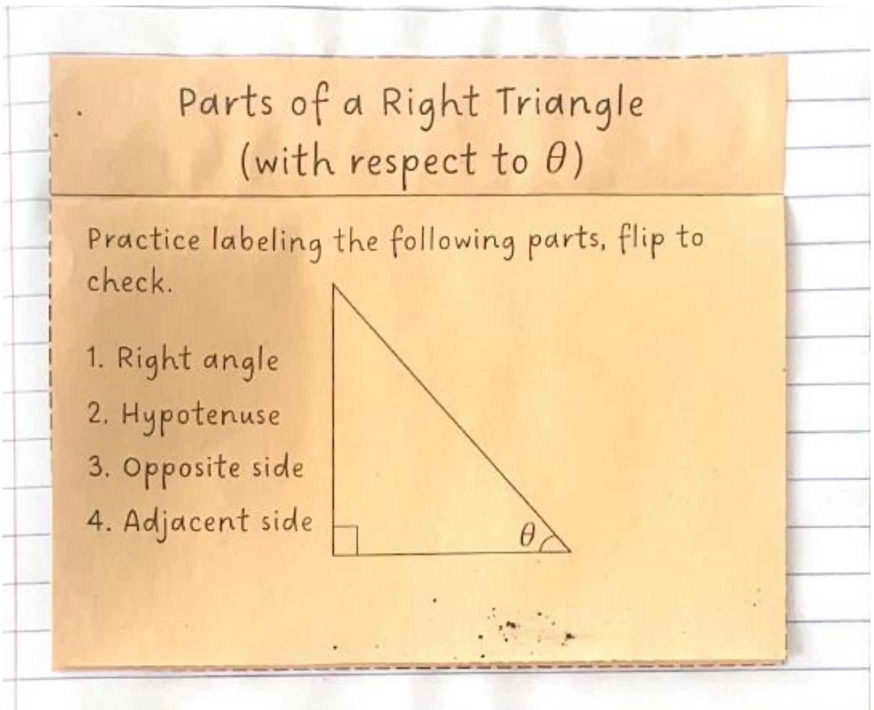
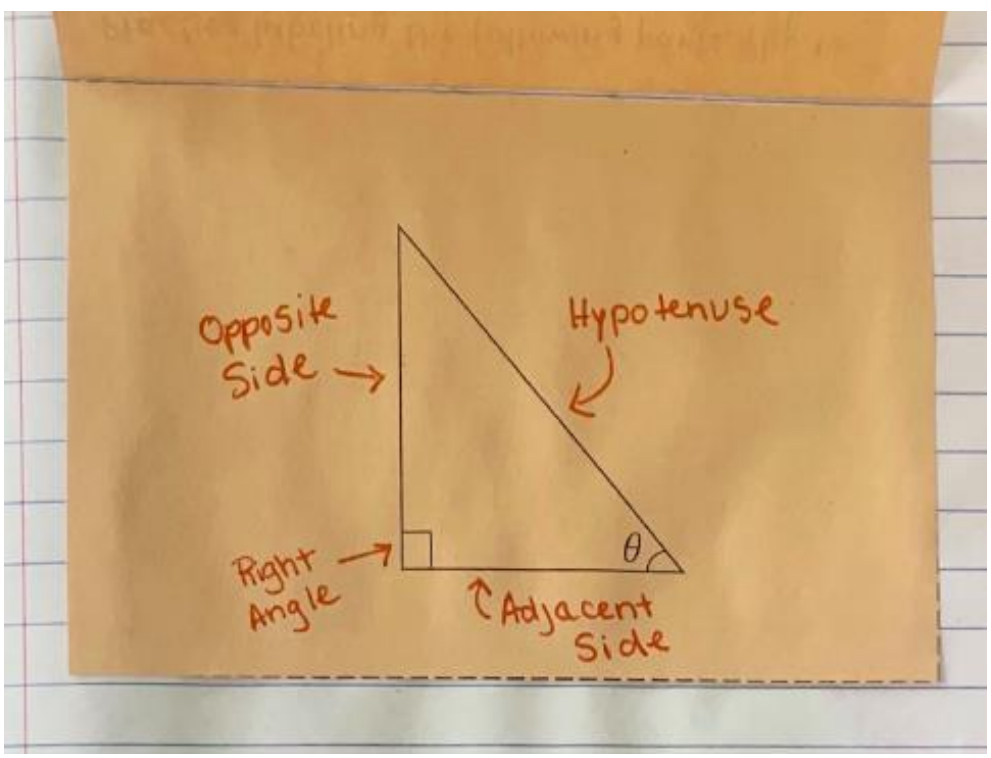
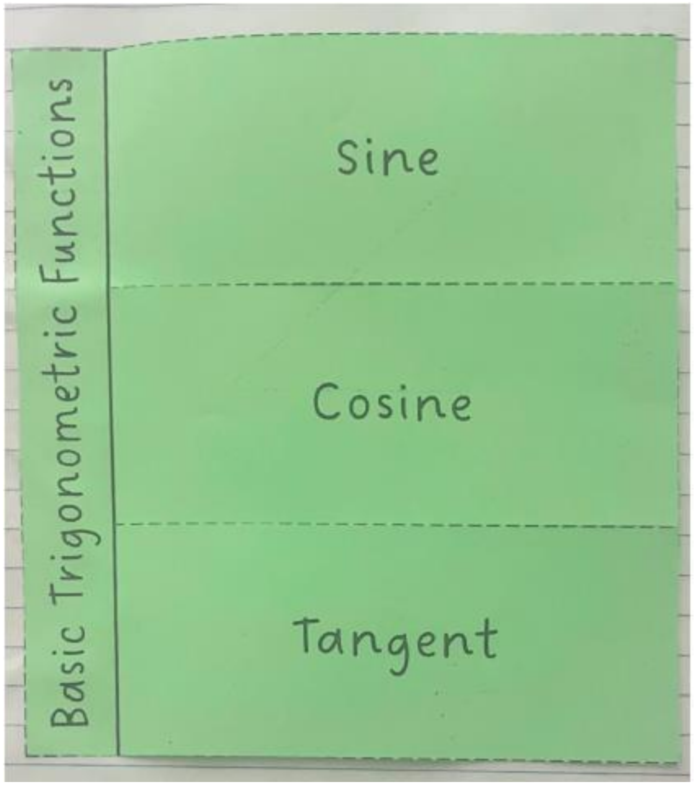
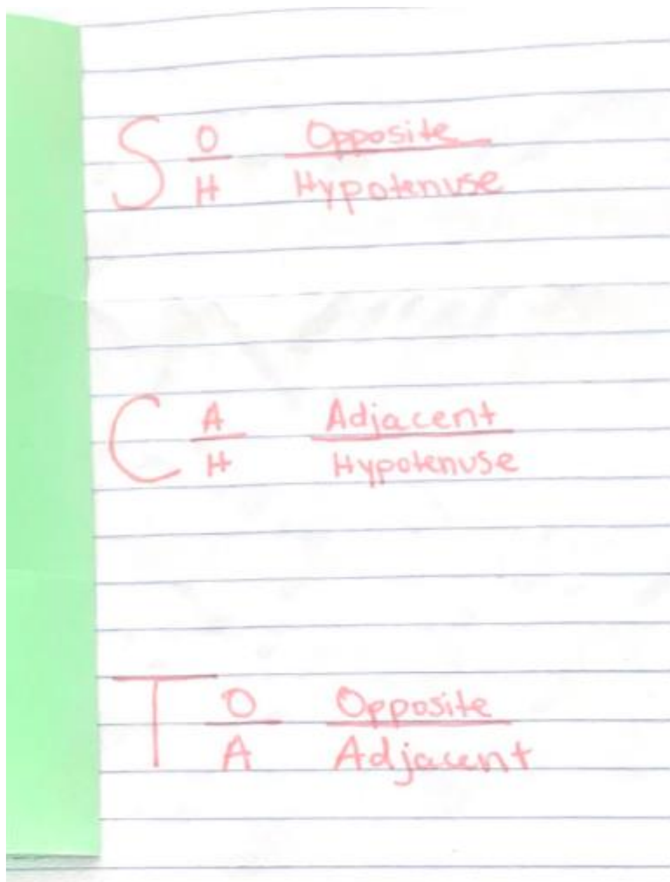
Appendix

Lesson 1 Answer Keys






Basic Trigonometric Functions
Sine
Cosine
Tangent


S O/H Opposite/Hypotenuse
C A/H Adjacent/Hypotenuse
T O/A Opposite/Adjacent

Directions:

Step 1: Cut along the dotted lines

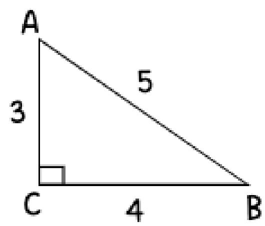
Step 2: Glue the "Trig Ratio Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

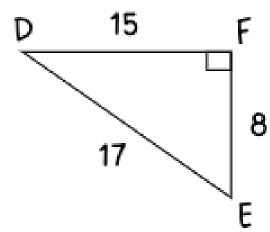
Trig Ratio Practice

Directions: Find the following trigonometric ratios for the identified angle. $S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

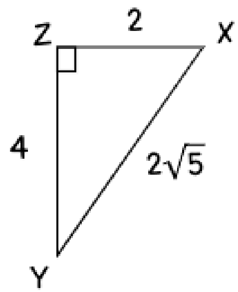
Sin A = $\frac{4}{5}$	Sin B = $\frac{3}{5}$
Cos A = $\frac{3}{5}$	Cos B = $\frac{4}{5}$
Tan A = $\frac{4}{3}$	Tan B = $\frac{3}{4}$



Sin D = $\frac{8}{17}$	Sin E = $\frac{15}{17}$
Cos D = $\frac{15}{17}$	Cos E = $\frac{8}{17}$
Tan D = $\frac{8}{15}$	Tan E = $\frac{15}{8}$



Sin X = $\frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$	Sin Y = $\frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$
Cos X = $\frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$	Cos Y = $\frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$
Tan X = $\frac{4}{2} = 2$	Tan Y = $\frac{2}{4} = \frac{1}{2}$



Directions:

Step 1: Cut along the dotted lines

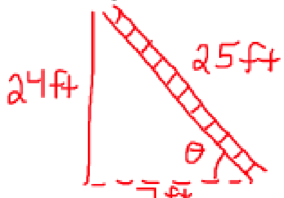
Step 2: Glue the "Trig Ratio Applications" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Trig Ratio Applications

Directions: Draw a sketch and answer the following using trigonometric ratios.

1. A 25-foot ladder leaning against a vertical wall reaches a height of 24 feet and stretched 7 feet from the base of the wall. Find the sine, cosine, and tangent values of the angle that the ladder makes with the ground.

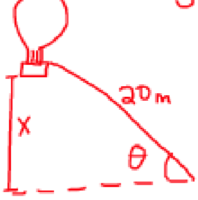


$$\sin(\theta) = \frac{24}{25}$$

$$\cos(\theta) = \frac{7}{25}$$

$$\tan(\theta) = \frac{24}{7}$$

2. A 20-meter line is used to keep a hot air balloon in place. The sine of the angle that the line makes with the ground is $\frac{3}{4}$. How high is the balloon in the air?



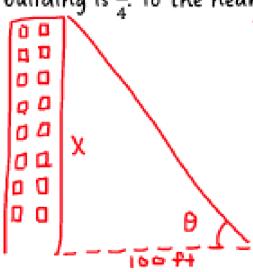
$$\sin(\theta) = \frac{3}{4}$$

$$\frac{3}{4} = \frac{x}{20}$$

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{60}{4}$$

$$x = 15m$$

3. From a point on the ground that is 100 feet from the base of a building, the tangent of the angle of elevation of the top of the building is $\frac{5}{4}$. To the nearest foot, how tall is the building?



$$\tan(\theta) = \frac{5}{4}$$

$$\frac{5}{4} = \frac{x}{100}$$

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{500}{4}$$

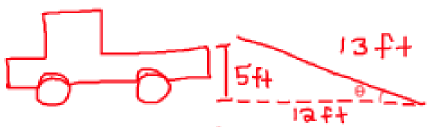
$$x = 125ft$$

Name ______________________ Date _________

Right Triangle Trigonometry Exit Ticket

Score: _____/_____

1. The bed of a truck is 5 feet above the ground. The driver of the truck uses a ramp 13 feet long to load and unload the truck and the ramp lands on the ground 12 feet from the truck. Find the sine, cosine, and tangent values of the triangle that the ramp makes with the ground.

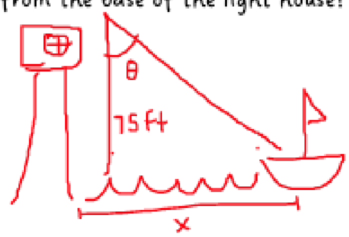


$\sin(\theta) = \frac{5}{13}$

$\cos(\theta) = \frac{12}{13}$

$\tan(\theta) = \frac{5}{12}$

2. From the top of a light house 75 feet high, the cosine of the angle of depression of a boat out at sea is $\frac{4}{5}$. To the nearest foot, how far is the boat from the base of the light house?



T $\frac{O}{A}$

$\tan(\theta) = \frac{4}{5}$

$\frac{4}{5} = \frac{x}{75}$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{300}{5}$

$X = 60ft$

Name ______________________ Date _________

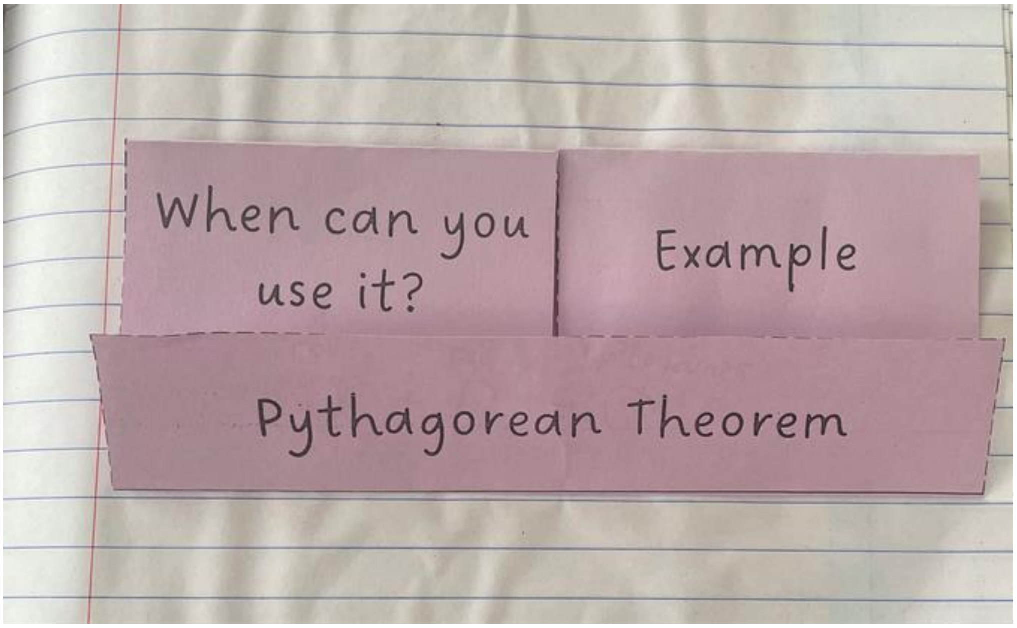
Right Triangle Trigonometry Exit Ticket

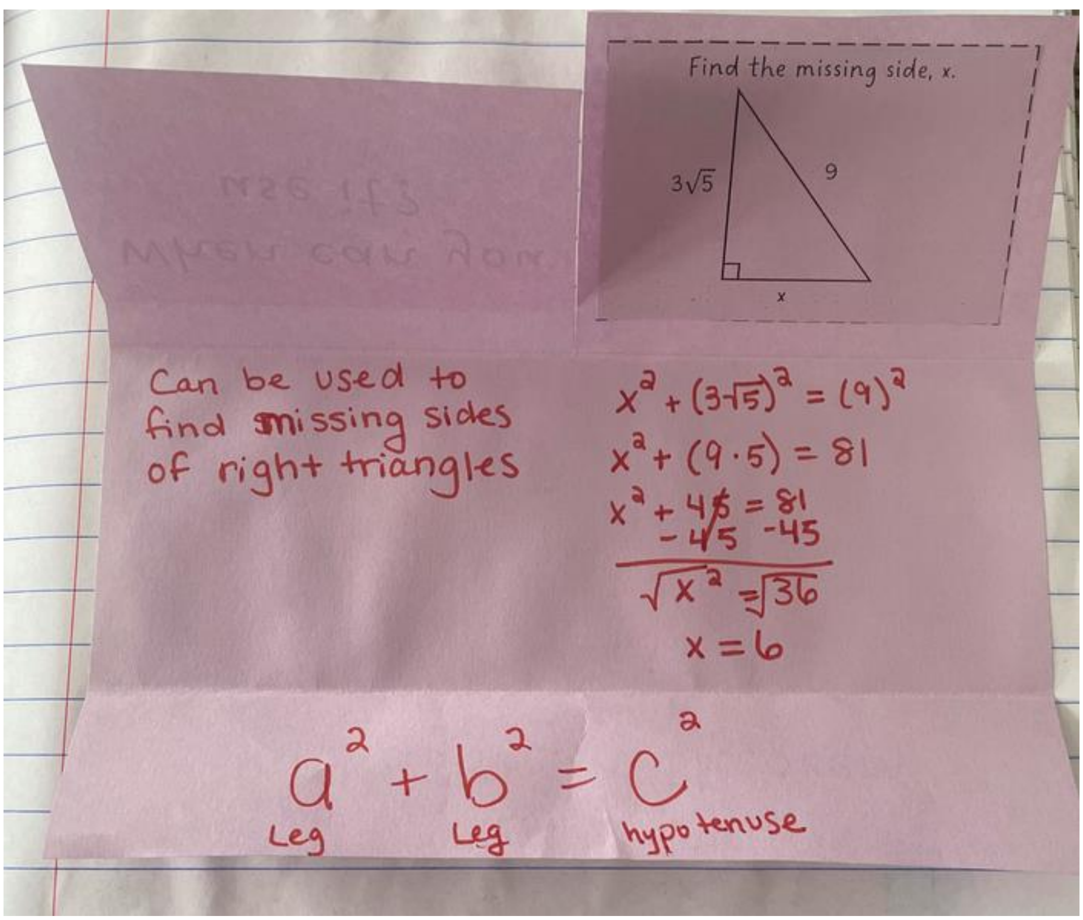
Score: _____/_____

1. The bed of a truck is 5 feet above the ground. The driver of the truck uses a ramp 13 feet long to load and unload the truck and the ramp lands on the ground 12 feet from the truck. Find the sine, cosine, and tangent values of the triangle that the ramp makes with the ground.

2. From the top of a light house 75 feet high, the cosine of the angle of depression of a boat out at sea is $\frac{4}{5}$. To the nearest foot, how far is the boat from the base of the light house?

Lesson 2 Answer Keys





Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Special Triangles" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

45°-45°-90° Special Triangle

Consider the isosceles right triangle shown whose hypotenuse is equal to one and whose angles are both 45°. Solve for x and place your answer is simplest radical form.

$x^2 + x^2 = 1$

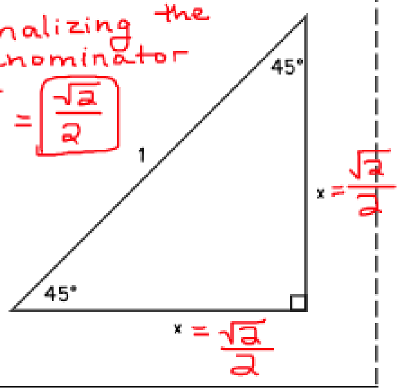
$\frac{2x^2}{2} = \frac{1}{2}$

$\sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \frac{1}{\sqrt{2}}$

Rationalizing the denominator

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$



30°-60°-90° Special Triangle

Consider the 30°-60°-90° right triangle shown whose hypotenuse is equal to one and is half of an equilateral triangle.

a. What is the length of the shorter side of this right triangle?

Short side = $\frac{1}{2}$

b. Using the Pythagorean theorem, find the length of the longer side in simplest radical form

$\left(\frac{1}{2}\right)^2 + (x)^2 = (1)^2$

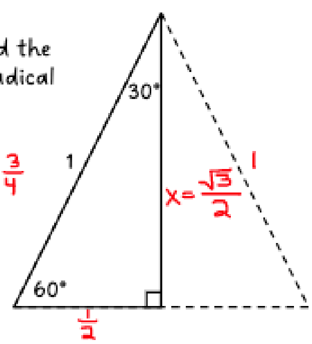
$\cancel{\frac{1}{4}} + x^2 = 1$

$-\cancel{\frac{1}{4}} \qquad -\frac{1}{4}$

$\sqrt{x^2} = \sqrt{\frac{3}{4}}$

$x = \frac{\sqrt{3}}{2}$

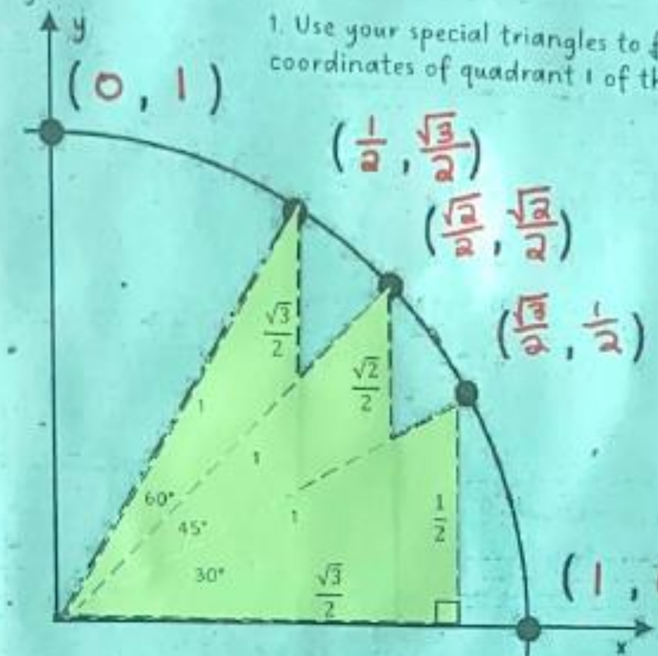
Think: $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$



First Quadrant of the Unit Circle

The basis of Trigonometry will be a very special circle know as the unit circle. This is a circle with a radius of 1 and a center at the origin.

1. Use your special triangles to fill out the coordinates of quadrant I of the unit circle



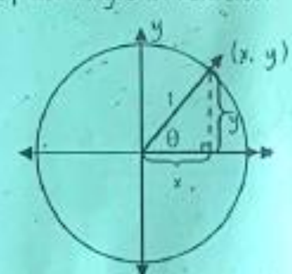
2. Find the following trigonometric ratios for the identified angle

$\sin(60°) = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$	$\sin(45°) = \frac{\sqrt{2}}{2}$	$\sin(30°) = \frac{1}{2}$
$\cos(60°) = \frac{1}{2}$	$\cos(45°) = \frac{\sqrt{2}}{2}$	$\cos(30°) = \frac{\sqrt{3}}{2}$

3. What can be said about the sin and cos of an angle in relation to the coordinates of the unit circle?

$\sin(\theta) = y$

$\cos(\theta) = x$



Directions:

Step 1: Cut along the dotted lines

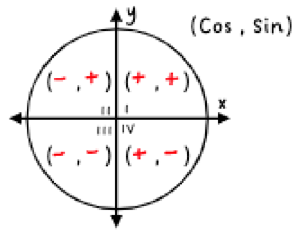
Step 2: Glue the "Unit Circle Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

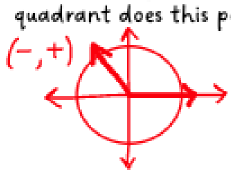
Unit Circle Practice

1. For each quadrant below, determine if the sine and cosine of an angle whole terminal ray falls in the quadrant would be positive or negative.

	I	II	III	IV
Sin	+	+	−	−
Cos	+	−	−	+



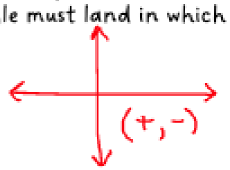
2. The terminal ray of an angle θ, draw in standard position passes through the point (-0.6, 0.8), which lies on the unit circle. What quadrant does this point lie and what is the value of cos(θ)?



This point lies in quadrant II

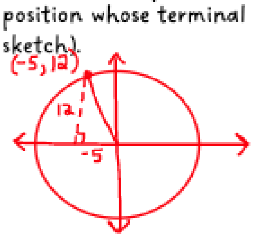
$\cos(\theta) = -0.6$

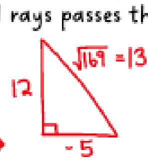
3. For an angle drawn in standard position, it is known that its cosine is positive and its sine is negative. The terminal ray of this angle must land in which quadrant?



quadrant IV

3. The point (-5, 12) lies on the circle whose equation is $x^2 + y^2 = 169$. What would represent the cosine of an angle drawn in standard position whose terminal rays passes through the point? (hint draw a sketch).





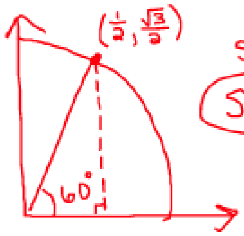
$\cos(\theta) = \frac{-5}{13}$

Name ________________________ Date _________

Special Triangles and the First Quadrant of the Unit Circle Exit Ticket

Score: _____/_____

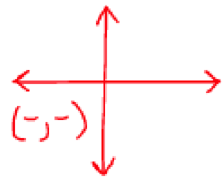
2. What is the sin(60°)



$Sin(\theta) = y$

$Sin(60°) = \frac{\sqrt{3}}{2}$

2. If both the sine and cosine of an angle are less than zero, then when drawn in standard position in which quadrant would the terminal ray fall?



quadrant III

3. Which of the following values cannot be the sine of an angle? (Hint: think about the range of y-values on the unit circle).

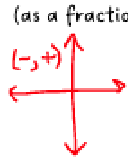
a. $\frac{5}{14} \approx .36$ b. $\frac{-2}{3} \approx -.67$

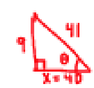
c. $\frac{\sqrt{3}}{5} \approx .35$ d. $\frac{4}{3} \approx 1.33$

Unit Circle ranges from $[-1, 1]$

4. When drawn in standard position, an angle θ has a terminal ray that lies in the second quadrant and whose sine is equal to $\frac{9}{41}$. Find the cosine of θ in rational form (as a fraction).

$Sin(\theta) = \frac{9}{41}$





$(9)^2 + (x)^2 = (41)^2$

$81 + x^2 = 1681$

$-81 \qquad -81$

$\sqrt{x^2} = \sqrt{1600}$

$x = 40$

$Cos(\theta) = \frac{40}{41}$

Name ________________________ Date _________

Special Triangles and the First Quadrant of the Unit Circle Exit Ticket

Score: _____/_____

2. What is the sin(60°)

2. If both the sine and cosine of an angle are less than zero, then when drawn in standard position in which quadrant would the terminal ray fall?

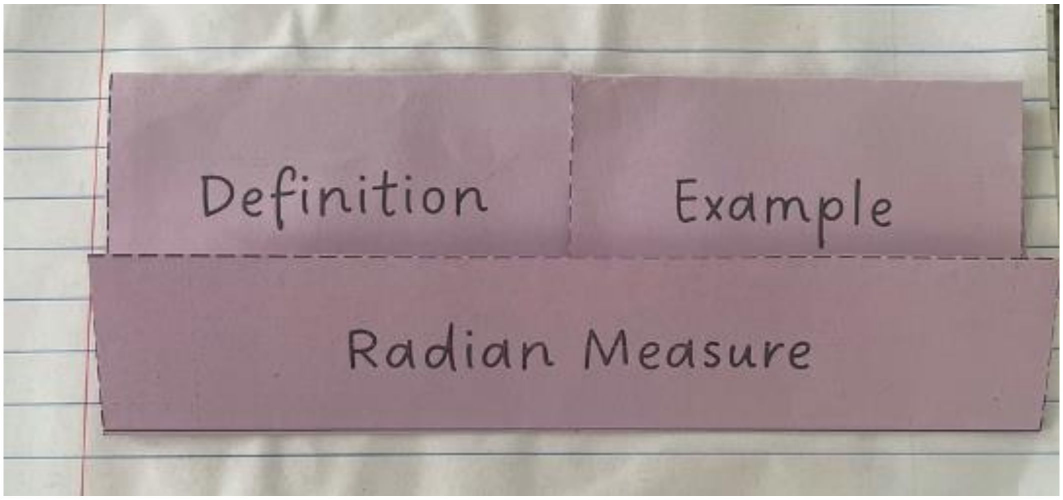
3. Which of the following values cannot be the sine of an angle? (Hint: think about the range of y-values on the unit circle).

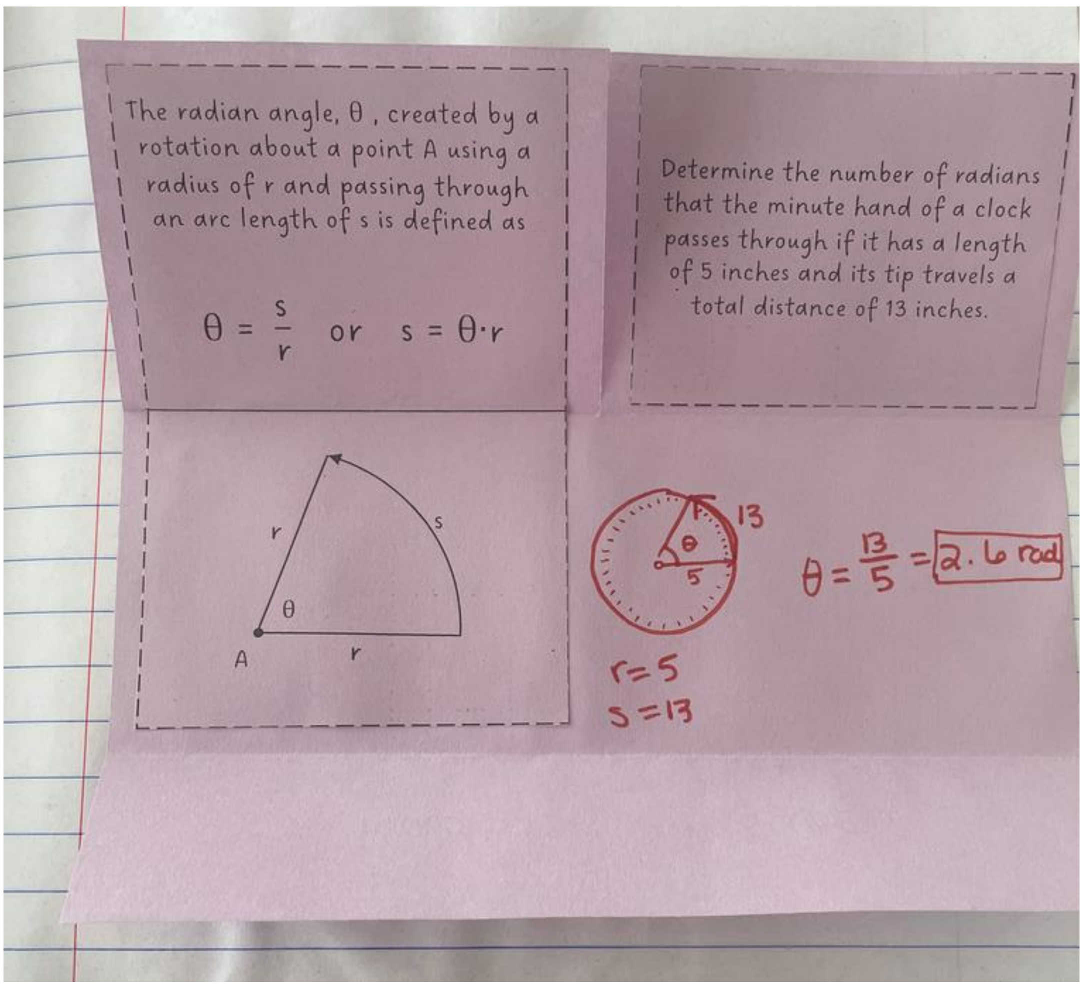
a. $\frac{5}{14}$ b. $\frac{-2}{3}$

c. $\frac{\sqrt{3}}{5}$ d. $\frac{4}{3}$

4. When drawn in standard position, an angle θ has a terminal ray that lies in the second quadrant and whose sine is equal to $\frac{9}{41}$. Find the cosine of θ in rational form (as a fraction).

Lesson 3 Answer Keys





Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Angle Measure and the Unit Circle" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Angle Measure and the Unit Circle

Consider a full rotation around any point in the counter-clockwise (positive) direction

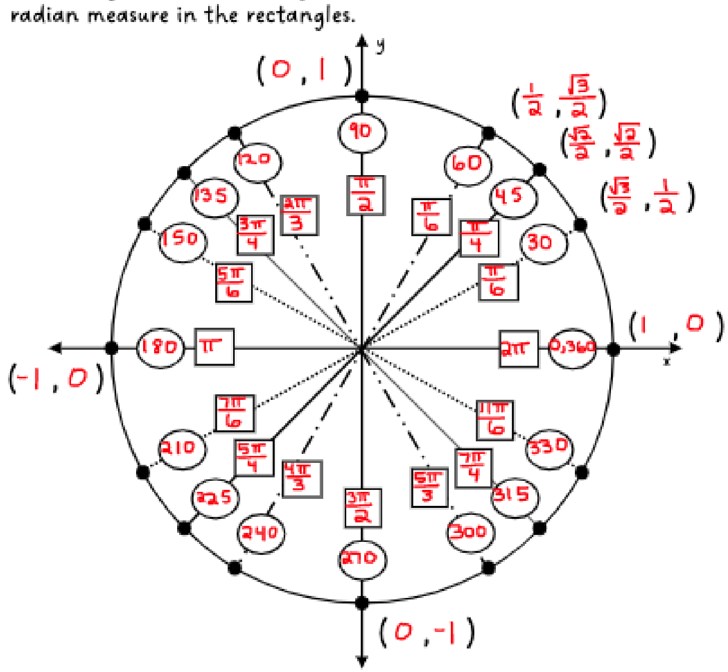
a. What would the arc length, s, of a circle be for one full rotation? Hint: think about circumference.

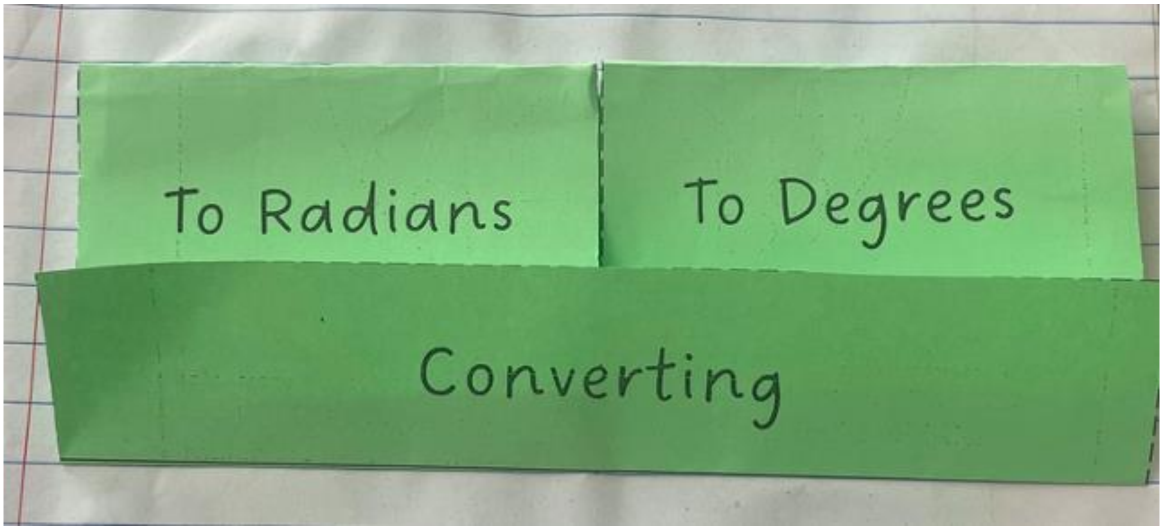
$S = 2\pi r$

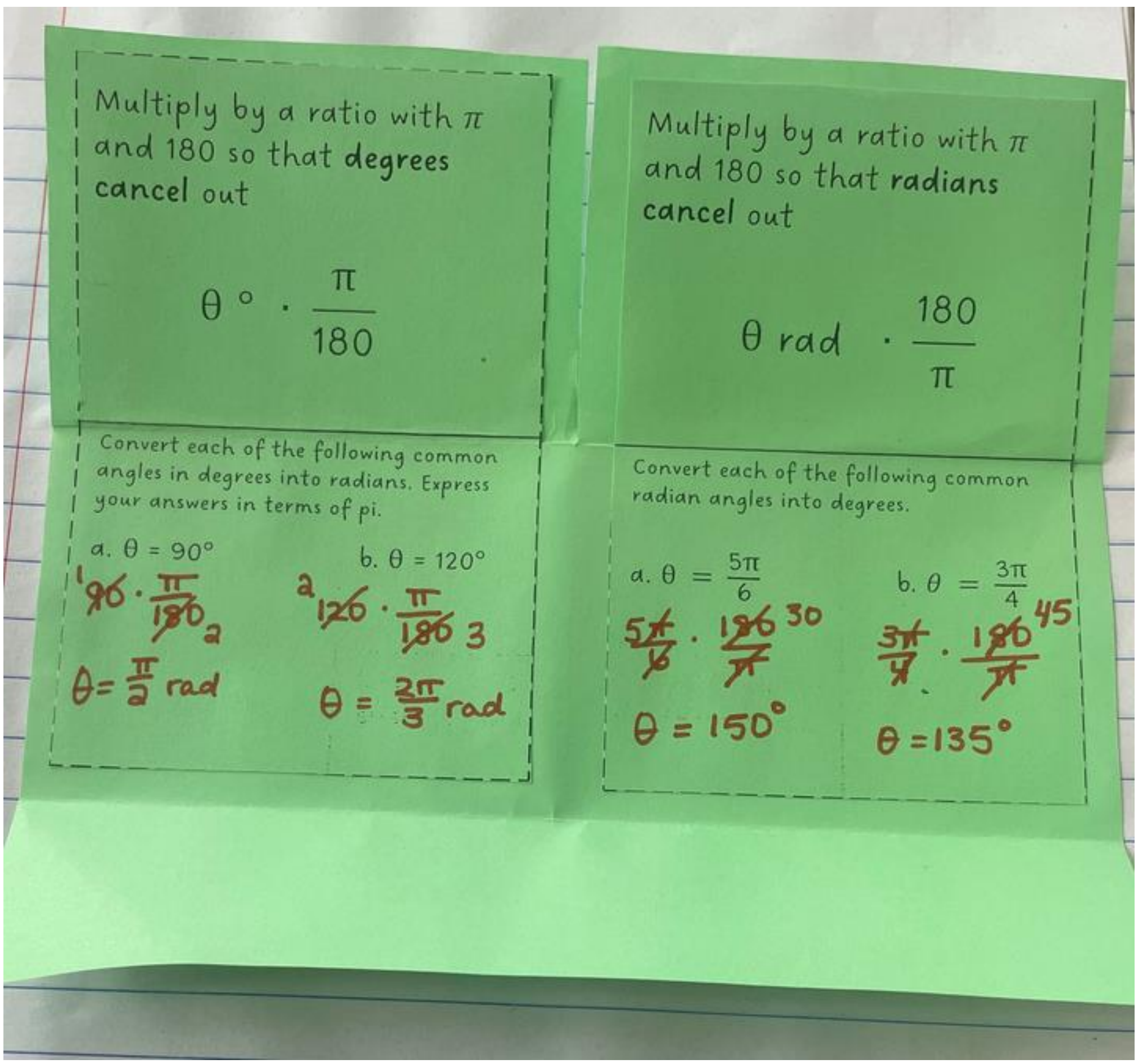
b. Based on the definition for radians and your answer to part (a), how many radians are there in one full rotation?

$\theta = \frac{s}{r} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi$

c. Now that we know how many radians are in one full rotation, lets piece together the radian angles in the unit circle, with what we already know. Fill the degree measure in the circles and the radian measure in the rectangles.




To Radians
To Degrees
Converting


Multiply by a ratio with π and 180 so that **degrees cancel** out
$\theta^\circ \cdot \frac{\pi}{180}$
Convert each of the following common angles in degrees into radians. Express your answers in terms of pi.
a. θ = 90°
$\theta = \frac{\pi}{2}$ rad
b. θ = 120°
$\theta = \frac{2\pi}{3}$ rad
Multiply by a ratio with π and 180 so that **radians cancel** out
$\theta \text{ rad} \cdot \frac{180}{\pi}$
Convert each of the following common radian angles into degrees.
a. $\theta = \frac{5\pi}{6}$
θ = 150°
b. $\theta = \frac{3\pi}{4}$
θ = 135°

Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Radian Measure Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Radian Measure Practice

1. Convert each of the following degree angles to angles in radians. Express your answers in exact terms of pi.

a. 30°

$\cancel{30}^{1} \cdot \frac{\pi}{\cancel{180}_{6}}$

$\theta = \frac{\pi}{6}$ rad

b. 45°

$\cancel{45}^{1} \cdot \frac{\pi}{\cancel{180}_{4}}$

$\theta = \frac{\pi}{4}$ rad

c. 135°

$\cancel{135}^{3} \cdot \frac{\pi}{\cancel{180}_{4}}$

$\theta = \frac{3\pi}{4}$ rad

e. 270°

$\cancel{270}^{3} \cdot \frac{\pi}{\cancel{180}_{2}}$

$\theta = \frac{3\pi}{2}$ rad

2. Convert each of the following angles given in radians into an equivalent measure in degrees.

a. $\frac{2\pi}{3}$

$\frac{2\cancel{\pi}}{\cancel{3}_{1}} \cdot \frac{\cancel{180}^{60}}{\cancel{\pi}}$

$\theta = 120°$

b. $\frac{\pi}{4}$

$\frac{\cancel{\pi}}{\cancel{4}} \cdot \frac{\cancel{180}^{45}}{\cancel{\pi}}$

$\theta = 45°$

c. $\frac{11\pi}{4}$

$\frac{11\cancel{\pi}}{\cancel{4}} \cdot \frac{\cancel{180}^{45}}{\cancel{\pi}}$

$\theta = 495°$

e. $\frac{4\pi}{3}$

$\frac{4\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{180}^{60}}{\cancel{\pi}}$

$\theta = 240$

3. If an angle is drawn in standard position with each of the following radians angles, determine the quadrant its terminal ray lies in. (Hint: convert each angle into degrees)

a. 4.75

$4.75 \cdot \frac{180}{\pi}$

$\theta \approx 272°$

Q IV

b. 5.28

$5.28 \cdot \frac{180}{\pi}$

$\theta = 303°$

Q IV

c. 1.65

$1.65 \cdot \frac{180}{\pi}$

$\theta = 95$

Q II

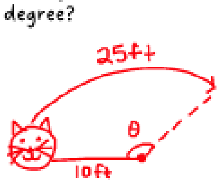
e. 2.9

$2.9 \cdot \frac{180}{\pi}$

$\theta = 166$

Q II

4. A cat is attached to a 10 foot leash. He travels around an arc that has a length of 25 feet. What is the radian angle he has rotated through? What is the equivalent degree angle rounded to the nearest degree?



$\theta = \frac{s}{r}$

$\theta = \frac{25}{10}$

$\theta = 2.5$ rad

$2.5 \cdot \frac{180}{\pi}$

$\theta \approx 143°$

Name ________________________ Date _________

Radian Measure Exit Ticket

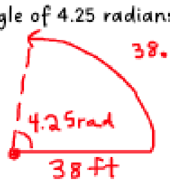
Score: _____/_____

1. The distance from the center of a Ferris wheel to a person who is riding is 38 feet. What distance does a person travel if the Ferris wheel rotates through an angle of 4.25 radians?

$\theta = \frac{s}{r}$

$38 \cdot 4.25 = \frac{s}{\cancel{38}} \cdot \cancel{38}$

$161.5 \text{ ft} = s$



2. The pendulum of a grandfather clock makes an angle of 2.5 radians as its tip travels 18 feet. What is the length of the pendulum?

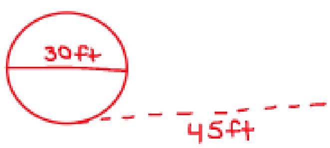
$\theta = \frac{s}{r}$

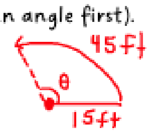
$r \cdot 2.5 = \frac{18}{r} \cdot r$

$\frac{\cancel{2.5}r}{\cancel{2.5}} = \frac{18}{2.5}$

$r = 7.2 \text{ ft}$

3. A wheel whose diameter is 30 feet rolls a distance of 45 feet without slipping. Through what degree angle did the wheel rotate? (Hint: find the radian angle first).





$\theta = \frac{s}{r}$

$\theta = \frac{45}{30}$

$\theta = 1.5 \text{ rad}$

$1.5 \cdot \frac{180}{\pi}$

$\theta = 85.9... \approx 86^\circ$

Name ________________________ Date _________

Radian Measure Exit Ticket

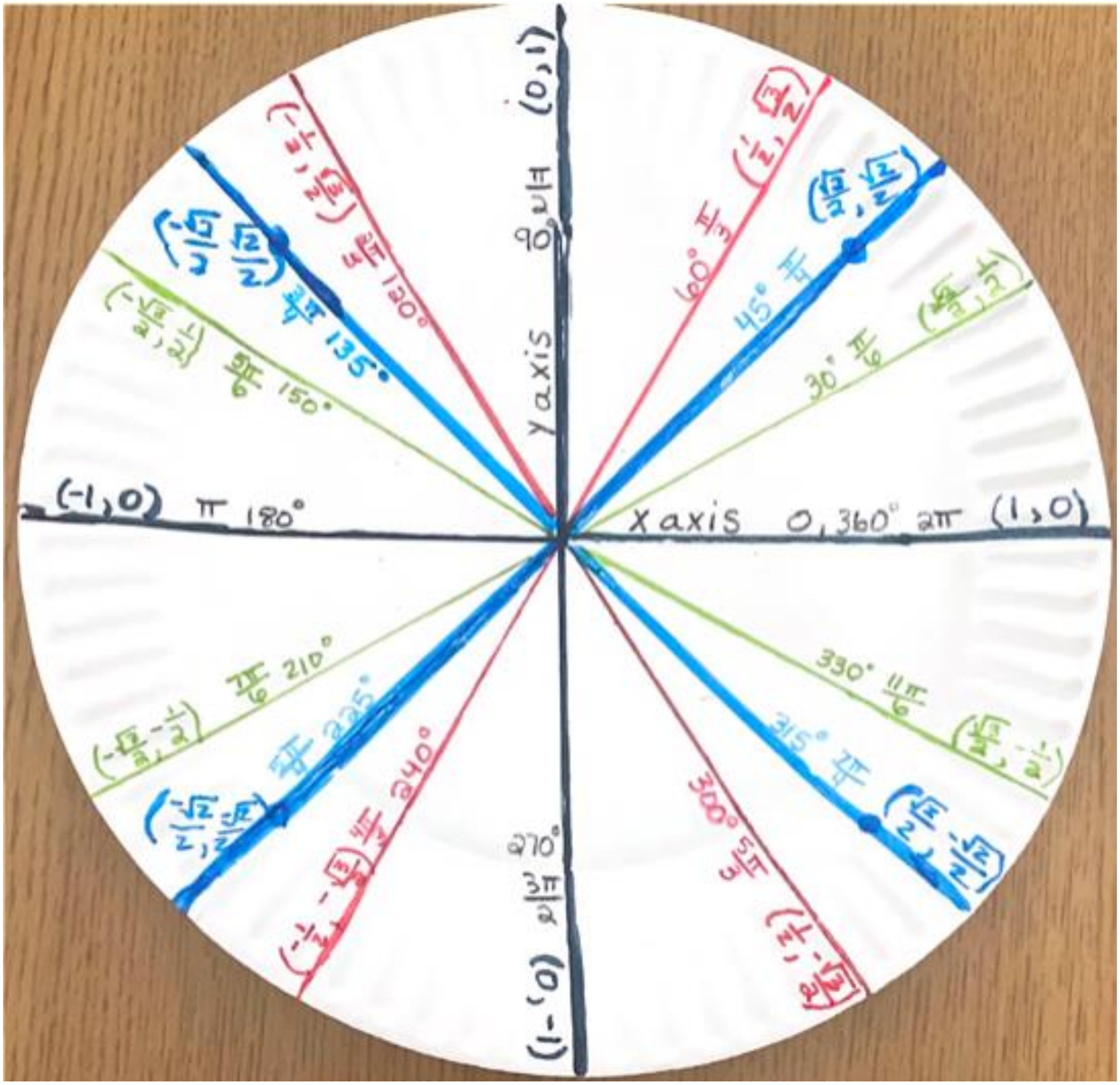
Score: _____/_____

1. The distance from the center of a Ferris wheel to a person who is riding is 38 feet. What distance does a person travel if the Ferris wheel rotates through an angle of 4.25 radians?

2. The pendulum of a grandfather clock makes an angle of 2.5 radians as its tip travels 18 feet. What is the length of the pendulum?

3. A wheel whose diameter is 30 feet rolls a distance of 45 feet without slipping. Through what degree angle did the wheel rotate? (Hint: find the radian angle first).

Lesson 4 Answer Keys



Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Trig Ratio Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Unit Circle and Trig Functions Practice

1. Written in exact form, cos(135) =?

a. $-\frac{1}{2}$ b. $-\frac{\sqrt{3}}{2}$

c. $-\frac{\sqrt{2}}{2}$ d. $-\frac{\pi}{4}$

2. Which of the following is not equal to sin(30)? $= \frac{1}{2}$

a. cos(60°) $= \frac{1}{2}$ b. -sin(330°) $= -\left(-\frac{1}{2}\right) = \frac{1}{2}$

c. cos(300°) $= \frac{1}{2}$ d. sin(120°) $= \frac{\sqrt{3}}{2}$

3. Given the function f(x) = 6sin(x) then what is the value of f(60°)?

$$f(60^\circ) = 6\sin(60^\circ) = \cancel{6}^{3}\left(\frac{\sqrt{3}}{\cancel{2}}\right) = 3\sqrt{3}$$

4. Given the function g(x) = 4cos(x)−2sin(x) then what is the value of $g(\frac{11\pi}{6})$?

$$g\left(\frac{11\pi}{6}\right) = 4\cos\left(\frac{11\pi}{6}\right) - 2\sin\left(\frac{11\pi}{6}\right) = \cancel{4}^{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right) - \cancel{2}^{1}\left(-\frac{1}{\cancel{2}}\right) = 2\sqrt{3}+1$$

5. Given the function $h(x) = 3\sin(x) - \frac{1}{2}\cos(x)$ then what is the value of $h(\frac{4\pi}{3})$?

$$h\left(\frac{4\pi}{3}\right) = 3\sin\left(\frac{4\pi}{3}\right) - \frac{1}{2}\cos\left(\frac{4\pi}{3}\right) = 3\left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(\frac{1}{2}\right) = -\frac{3\sqrt{3}}{2} - \frac{1}{4}$$

Name ________________________ Date _________

Unit Circle and Trig Functions Exit Ticket

Score: _____/_____

1. Which of the following values is not equal to the sin(270°)?. $= -1$

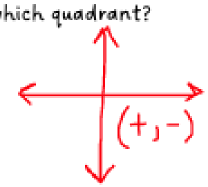
a. cos(180°) $= -1$ b. −sin(90°) $= -1$

c. −cos(0°) $= -1$ d. sin(360°) $= 1$

2. Given the function h(x) = sin(x) · cos(x) then what is the value of $h(\frac{5\pi}{6})$?

$$h\left(\frac{5\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) \cdot \cos\left(\frac{5\pi}{6}\right)$$
$$= \frac{1}{2} \cdot \frac{-\sqrt{3}}{2}$$
$$= \frac{-\sqrt{3}}{4}$$

3. For an angle drawn in standard position, it is known that its cosine is greater than zero and its sine is less than zero. The terminal ray of this angle must land in which quadrant?



quadrant IV

Name ________________________ Date _________

Unit Circle and Trig Functions Exit Ticket

Score: _____/_____

1. Which of the following values is not equal to the sin(270°)?.

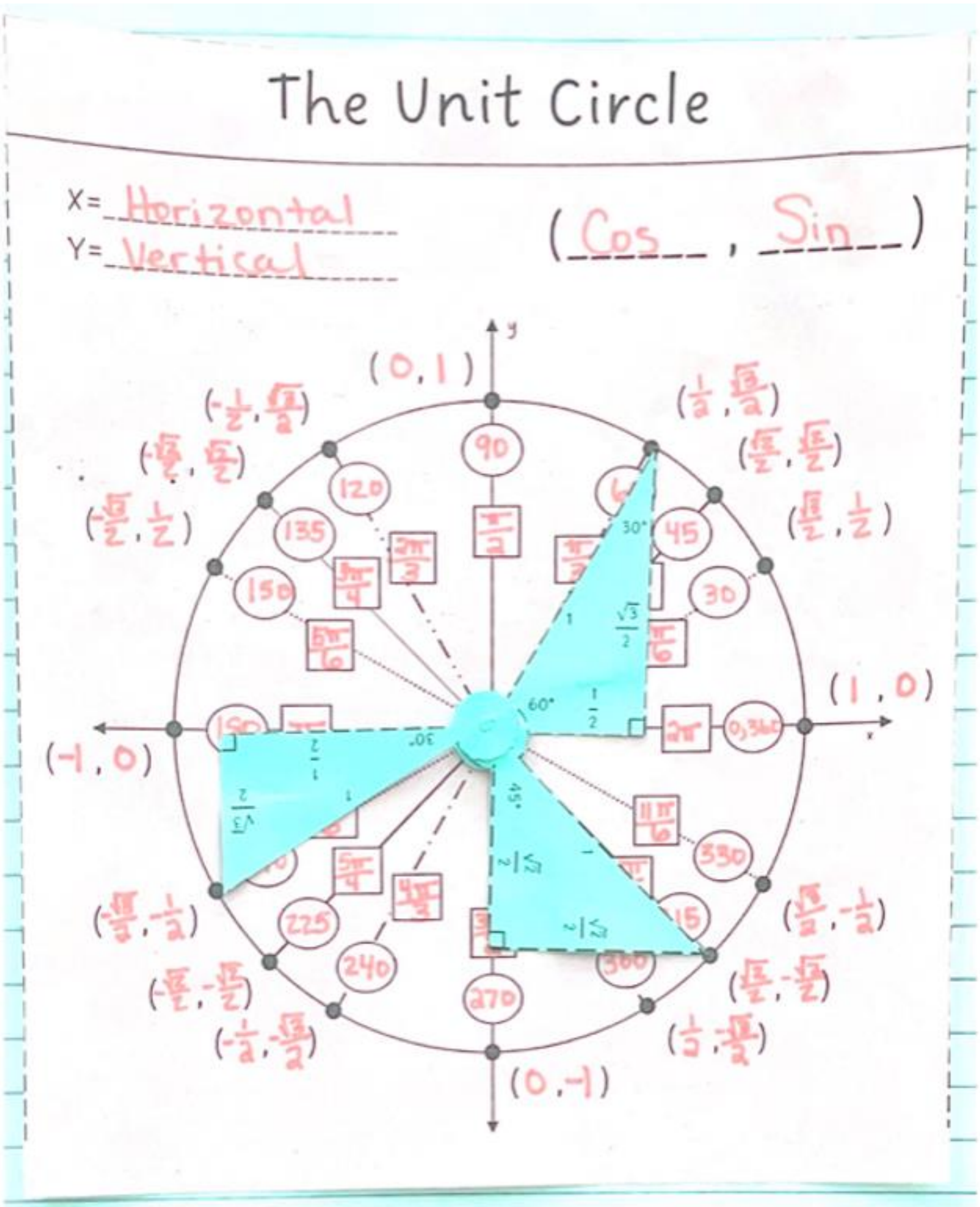
a. cos(180°) b. −sin(90°)

c. −cos(0°) d. sin(360°)

2. Given the function h(x) = sin(x) · cos(x) then what is the value of $h(\frac{5\pi}{6})$?

3. For an angle drawn in standard position, it is known that its cosine is greater than zero and its sine is less than zero. The terminal ray of this angle must land in which quadrant?

Lesson 5 Answer Keys



Directions:

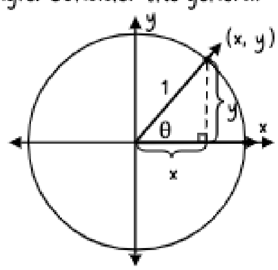
Step 1: Cut along the dotted lines

Step 2: Glue the "Pythagorean Trig Identity" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

The Pythagorean Trig Identity

A perpendicular line can be dropped from any point on the unit circle to the x axis to form a right triangle. Consider the general right triangle in the unit circle below.



a. Apply the Pythagorean theorem to the right triangle

$x^2 + y^2 = 1$

b. What does x and y represent on the unit circle?

$x = \cos(\theta)$

$y = \sin(\theta)$

c. Substitute your answer from b into the equation you wrote in a. This is know as the Pythagorean trig identity.

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

or

$\sin^2\theta + \cos^2\theta = 1$

d. An angle, θ , has a terminal ray that falls in the second quadrant. If it is known that $\sin(\theta) = \frac{3}{5}$, determine the value of $\cos(\theta)$.

(−,+)

cosine is negative

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$

$\frac{9}{25} + \cos^2\theta = 1$

$-\frac{9}{25} \qquad -\frac{9}{25}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{16}{25}}$

$\cos\theta = \pm\frac{4}{5}$

Think: $\frac{25}{25} - \frac{9}{25} = \frac{16}{25}$

Since θ is in QII, $\cos\theta = -\frac{4}{5}$

Directions:

Step 1: Cut along the dotted lines

Step 2: Glue the "Unit Circle and Trig Identity Practice" flat in your interactive notebook.

Step 3: Answer questions along with the teacher.

Unit Circle and Trig Identity Practice

1. An angle, θ, has a terminal ray that falls in the first quadrant and $\cos(\theta) = \frac{1}{3}$. Determine the value of $\sin(\theta)$ in simplest radical form

$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$

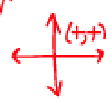
$\sin^2\theta + \cancel{\frac{1}{9}} = 1$

$\quad - \cancel{\frac{1}{9}} \quad - \frac{1}{9}$

$\sqrt{\sin^2\theta} = \sqrt{\frac{8}{9}}$

$\sin^2\theta = \pm\frac{\sqrt{8}}{3}$

$\pm\frac{\sqrt{8}}{3} = \pm\frac{\sqrt{4}\sqrt{2}}{3} = \pm\frac{2\sqrt{2}}{3}$



since θ is in QI

$\sin\theta = \frac{2\sqrt{2}}{3}$

2. If the terminal ray of θ lies in the fourth quadrant and $\sin(\theta) = -\frac{\sqrt{3}}{3}$ determine $\cos(\theta)$ in simplest form

Think: $\left(\frac{-\sqrt{3}}{3}\right)^2 = \frac{3}{9} = \frac{1}{3}$

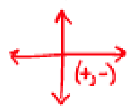
$\left(\frac{-\sqrt{3}}{3}\right)^2 + \cos^2\theta = 1$

$\cancel{\frac{1}{3}} + \cos^2\theta = 1$

$-\cancel{\frac{1}{3}} \quad - \frac{1}{3}$

$\sqrt{\cos^2\theta} = \sqrt{\frac{2}{3}}$

$\cos\theta = \pm\frac{\sqrt{2}}{\sqrt{3}}$



since θ is in QIV

$\cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$

3. If $f(x) = 2x$ and $g(x) = \cos(x)$ then evaluate $g(f(\frac{\pi}{2}))$

$f\left(\frac{\pi}{2}\right) = 2\left(\frac{\pi}{2}\right) = \pi$

$g(\pi) = \cos(\pi) = -1$

$g\left(f\left(\frac{\pi}{2}\right)\right) = -1$

4. Which of the following is equal to the cos(300)? $= \frac{1}{2}$

a. $\cos(60°) = \frac{1}{2}$ b. $-\cos(60°) = -\frac{1}{2}$

c. $\cos(30°) = \frac{\sqrt{3}}{2}$ d. $-\cos(30°)$ $-\frac{\sqrt{3}}{2}$

Name ______________________ Date _________

More Work with the Unit Circle Exit Ticket

Score: _____/_____

1. When drawn in standard position, an angle, θ, has a terminal ray that falls in the third quadrant and $\cos(\theta) = -\frac{8}{17}$. Determine the value of $\sin(\theta)$.

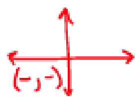
$\sin^2\theta + \left(-\frac{8}{17}\right)^2 = 1$

$\sin^2 + \frac{64}{289} = 1$

$-\frac{64}{289} \quad -\frac{64}{289}$

$\sqrt{\sin^2\theta} = \sqrt{\frac{225}{289}}$

$\sin\theta = \pm\frac{15}{17}$



Since θ lies in QIII

$\sin\theta = -\frac{15}{17}$

2. If $f(x) = 3x$ and $g(x) = \cos(x) + \sin(x)$ then evaluate $f(g(\frac{3\pi}{2}))$.

$g\left(\frac{3\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) + \sin\left(\frac{3\pi}{2}\right)$

$= 0 - 1$

$= -1$

$f(-1) = 3(-1)$

$= -3$

$f\left(g\left(\frac{3\pi}{2}\right)\right) = -3$

Name ______________________ Date _________

More Work with the Unit Circle Exit Ticket

Score: _____/_____

1. When drawn in standard position, an angle, θ, has a terminal ray that falls in the third quadrant and $\cos(\theta) = -\frac{8}{17}$. Determine the value of $\sin(\theta)$.

2. If $f(x) = 3x$ and $g(x) = \cos(x) + \sin(x)$ then evaluate $g(f(\frac{3\pi}{2}))$.