Using Choice to Engage Students in Problem Solving:

7th and 8th Grade Mathematics

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Abstract

This curriculum project was designed to increase levels of student autonomy by providing choice between tasks and techniques a student can use when studying algebraic equations at the 7th and 8th grade levels. This curriculum was designed for teachers to offer students a choice between two or three alternatives when practicing new skills. By providing students with a range of different contexts in which they can practice and develop algebraic concepts and skills, students can experience a sense of autonomy, which increases intrinsic motivation and learning outcomes. The curriculum provides three lessons with three student choices for learning each lesson. The lessons cover methods and procedures for generating equations with a single variable and using the distributive property to simplify expressions. The keys for all instructional materials are included in the appendix.

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Introduction

A difficult challenge for teachers of 7th Grade Mathematics is maintaining student interest in content they are to learn. To promote interest, teachers can design lessons to incorporate student choice while meeting lesson objectives. Such instruction can "increase student motivation by appealing to students' needs for autonomy and competence" (Beymer & Thomson, 2015, p. 105). Williams (1998) supports the idea of student choice as it has been shown to improve learning and task performance.

In the first lesson, students select between real-world problem sets to model different scenarios with variables. Students can choose between three categories of problems and select a context in which they are most interested. The choice problems are grouped into three broad categories, *Sports and Games*, *Careers and Consumers*, and *Culture and the Environment*. These activities allow students to choose sets of questions based on their interest. The second lesson provides students the option of demonstrating their understanding of the distributive property in three different ways. Students can choose between three alternative approaches for finding the simplified expression of a constant multiplied by a binomial (multiplying using the distributive property). The first approach will have students use the area model to find an equivalent expression of a single term multiplied by a binomial. The area model method utilizes a rectangle that has a length dimension equal to one factor of the distributive expression and a width dimension equal to the other factor of the expression. When computing area for the pieces of the rectangle, students will arrive at a solution that is equivalent to applying the distributive algorithm (prior knowledge of how to use area models, algebra tiles and the algorithm for simplifying expressions with distribution is critical). The second approach includes use of algebra tiles to evaluate multiplication of polynomials using the distributive method. Using the tiles, students can compute each term separately (using tiles to represent a unit term) and then combine these terms to provide a simplified expression. The third approach requires that the students use the distributive algorithm by multiplying a single term in one expression with every term in the other expression. The last lesson provides a review where students will revisit what they have learned and demonstrate their proficiency using either a group game or a standard worksheet. Allowing for an option between these two tasks increases student engagement and the likelihood of positive learning outcomes.

Literature Review

The use of choice in creating favorable outcomes is supported by the *self-determination theory* (SDT). Originally developed by Decci and Richard Ryan to explain intrinsically motivated behaviors, SDT theory addressed concerns with the drive-theory principle. The drivetheory sought to explain behaviors of organisms but had difficulty explaining exploratory behaviors that would lead an organism to investigate its environment, even though doing so would not necessarily allow pain avoidance. Drive theory states that organisms (humans included) react to their environment to reduce tension felt when the primary drives for food, water, oxygen and pain avoidance are not satisfied. To account for this, Decci suggested that a person's intrinsic motivation could be described as a, "need to feel personal causation" (Oxford Library of Psychology, 2014, p. 2). SDT implies that a person who is given the freedom to act autonomously is more likely to be acting under the effects of intrinsic motivation. Thus, when a student is acting under intrinsic motivation they will, "yield more positive behavioral and affective outcomes than controlled types of motivation" (Oxford Library of Psychology, 2014, p.

4). Beymer and Thomas (2015) stated that offering choice appeals to students' needs for autonomy and competence in the classroom is one way to increase student motivation. When students are offered a choice, they are more intrinsically motivated compared to students who are assigned tasks (Deci & Vallerand, 1991).

Though choice has been shown to be a powerful motivator in student engagement, there are studies indicating that too much choice can be associated with negative effects. Chock and Brozyna (2011) state that increased choice can lead to a deterioration of, "performance during numerical calculations" which would be detrimental in a mathematics class (p. 548). This deterioration is a result of increased cognitive processes that a person experiences when deciding between one of many options. These are referred to as search costs and include the amount of time and mental energy that a person, especially a *maximizer*, spends to find the best option. Once an option has been selected, people with a high degree of choice can feel remorse at their decision. When designing lessons with choice, then, one needs to balance the intrinsic motivation a person experiences from choice while not overloading the individual and generating negative outcomes.

Beyond the number of choices provided to the student, a designer of a mathematics curriculum must consider whether the options provided to the student are sets of tasks students believe they are capable of accomplishing. Denissen, Zarrett, and Eccles (2007) showed that students seek "challenging tasks when they believe that they are capable of completing such tasks" (Denissen, Zarrett, & Eccles, 2007, p. 431). Also, they note that when given a choice of differing tasks, students choice should be influenced by their interests and the choice based on this interest will improve achievement. The design of any lesson incorporating choice must assure choices reflect student interests and tasks associated with these choices are something the student believes they can accomplish.

Curriculum

This curriculum project was designed using SDT and Drive Theory criteria to support student intrinsic motivation and engage them with learning content by having the option to choose between different learning activities. In this way, the curriculum should satisfy students' needs for autonomy leading to an environment that is, "sufficient to promote human growth and functioning" (Oxford Library of Psychology, 2014). The lessons provided below are intended for implementation in a Grade 7 mathematics course. Each set of materials for Lesson 1, Lesson 2 and Lesson 3 are types of practice problems that can be presented to students following a lesson on the topic.

Lesson 1: Options A, B and C

Overview

Students will be provided problem sets from 3 different categories (Option A, Option B, and Option C), each of which will assess the same set of skills and will require the learner to use equations to arrive at arithmetic and algebraic solutions to various real-world problems. Allowing the student to select a category provides a layer of autonomy in the lesson and encourages motivation and engagement. These activities should be provided to the student following lessons which build the foundation for manipulating equations and expressions to achieve desired results.

Objective

Students can generate equations of one variable from a real-world situation. **Standards**

http://www.nysed.gov/common/nysed/files/programs/curriculum-instruction/nys-nextgeneration-mathematics-p-12-standards.pdf

NY-7.EE.4-Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities

NY-7.EE.4.a - Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Name: Lesson 1 – Date: Class:

Option A

A person is playing a video game where they score 20 points for each elimination and 10 points for each assist. e = number of eliminations a = number of assists Expression: (20)(e)+(10)(a)
What would a person's score be if he had six (6) eliminations and four (4) assists?
Write an expression for the number of points a person would have if they played 3 games. (Use variables and do not solve.)
In a football game, a team can score 6 points by getting a touchdown, 1 point for an extra point conversion and 3 points for a field goal. Write an expression for the points in a game using these values. t = number of touchdowns e = number of extra point conversions f = number of field goals made
Write an expression for how many points a team would have after two games.

Option B

Each contractor at a company can construct framing for a room in 8 hours and install sheetrock in that room in 6 hours. f = number of rooms to frame s = number of rooms to sheetrock t = total time Expression: (8)(f)+(6)(s)=t
How much time would the contractor need to frame 3 rooms and install sheetrock in 2 rooms?
Write an expression for the amount of time it would take 2 contractors to complete the work. (Use variables, don't try to solve.)
When Sandy gets her allowance, she likes to buy bags of gummy bears for 2 dollars each, games for her phone for 1 dollar each and car magazines for 4 dollars each. b = number of bags of gummy bears g = number of games for phone m = number of magazines Write an expression for the amount of gummy bears, games and magazines she could buy with her allowance.

Name: Lesson 1 – Date: Class:

Option C

On average, each time a student opens the Tik Tok application, she will spend 20 minutes watching videos and every time she opens Instagram, she will spend 10 minutes scrolling through feeds. t = # of times a student opens Tik Tok i = # of times a student opens Instagram An expression for the amount of time one student spends on social media is shown below: (20)(t)+(10)(i)
How many minutes would this person spend on social media if she opened Tik Tok 6 times and Instagram 4 times?
Write an expression for the amount of time 3 students would spend on social media. (Use variables and do not solve.)
A zookeeper knows that each fox she cares for needs 1 pound of food per day, that each coyote needs 2 pounds of food per day and that each deer needs 4 pounds of food per day. Write and expression for the amount of food a zoo uses in a day. (Use variables and do not solve.) d = # of deer f = # of foxes c = # of coyotes
Write an expression for the amount of food the zoo uses in 3 days.

Lesson 2 Options A, B, and C

Overview

In assignments provided for this lesson, students will begin to practice skills related to simplifying expressions using the distributive property. Again, these materials are meant to supplement a lesson in which students have been instructed in methods and procedures for applying the distributive property. To promote student interest, each person in the class will be given a selection of how they will demonstrate their knowledge using one of three skills. Students will either apply the algorithm of multiplying a single term by each element in an expression, by using an area model to arrive at the solution or by manipulating a series of virtual elements in a program like PowerPoint.

- Standard Algorithm: student will multiply one term by each of the terms in the expression, combine like terms and present a solution.
- Area Model: students will use the method of representing terms as lengths on a rectangular shape (similar to the approach used by students to solve multiplication problems in 3rd grade NY-3.MD.7.c). They will find the area in terms of a variable, combine like terms and report a solution.
- Virtual Manipulatives: students will use virtual manipulatives to create representations of multiplication problems where factors are located on the outside of the model and the product is shown in the middle of the model. This lesson provides an example of how this may look in a virtual program such as PowerPoint. When creating this file, be sure to include many copies of each type of factor term (copy, paste and stack on one another) so that a student does not run out of virtual tiles when completing the multiplication.

Objective

Students can use a variety of techniques to simplify multiplication of a single term by a polynomial.

Students recognize that distribution can be represented with area models.

Standards

NY-8.EE.b- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and combining like terms.

Name: Lesson 2 – Date: Class:

Option A

1.) $2(x + 2)$ 5.) $3(3x + 1)$

2.) $3 (2x + 3)$ 6.) $4(1x + 3)$

3.) $4(2x + 5)$ 7.) $5(2x + 1)$

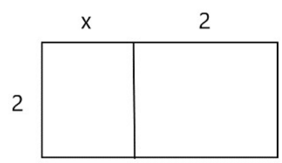
4.) $5(4x + 1)$ 8.) $2x(x + 3)$

Name: Lesson 2 – Date: Class:

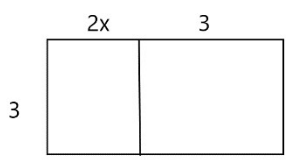
Option B

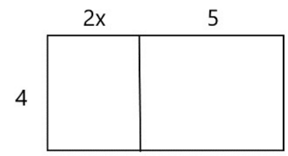
Use area models and distribution 3.) 4(2x+5) to simplify the following expr.

1.) 2(x+2)

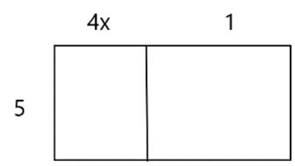


2.) 3(2x+3)

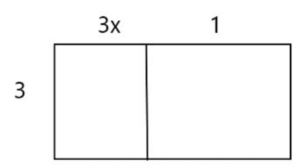




4.) 5(4x+1)



5.) 3(3x+1)



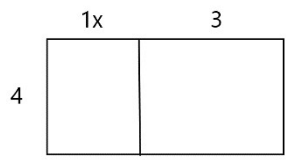
Option B

6.) 4(1x+3) 9.) 2x(2x+4)

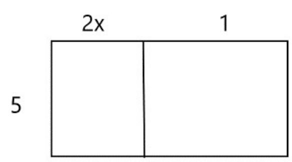
Name: Lesson 2 – Option C

Class:

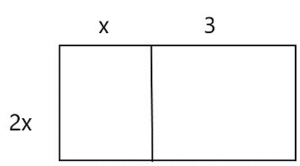
Date:

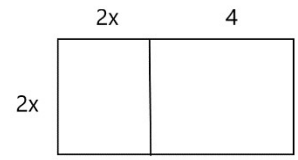


7.) 5(2x+1)

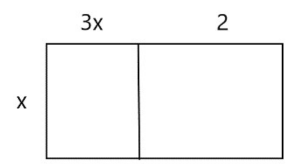


8.) 2x(x+3)

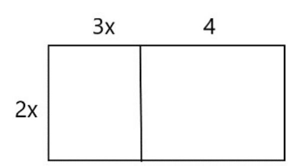




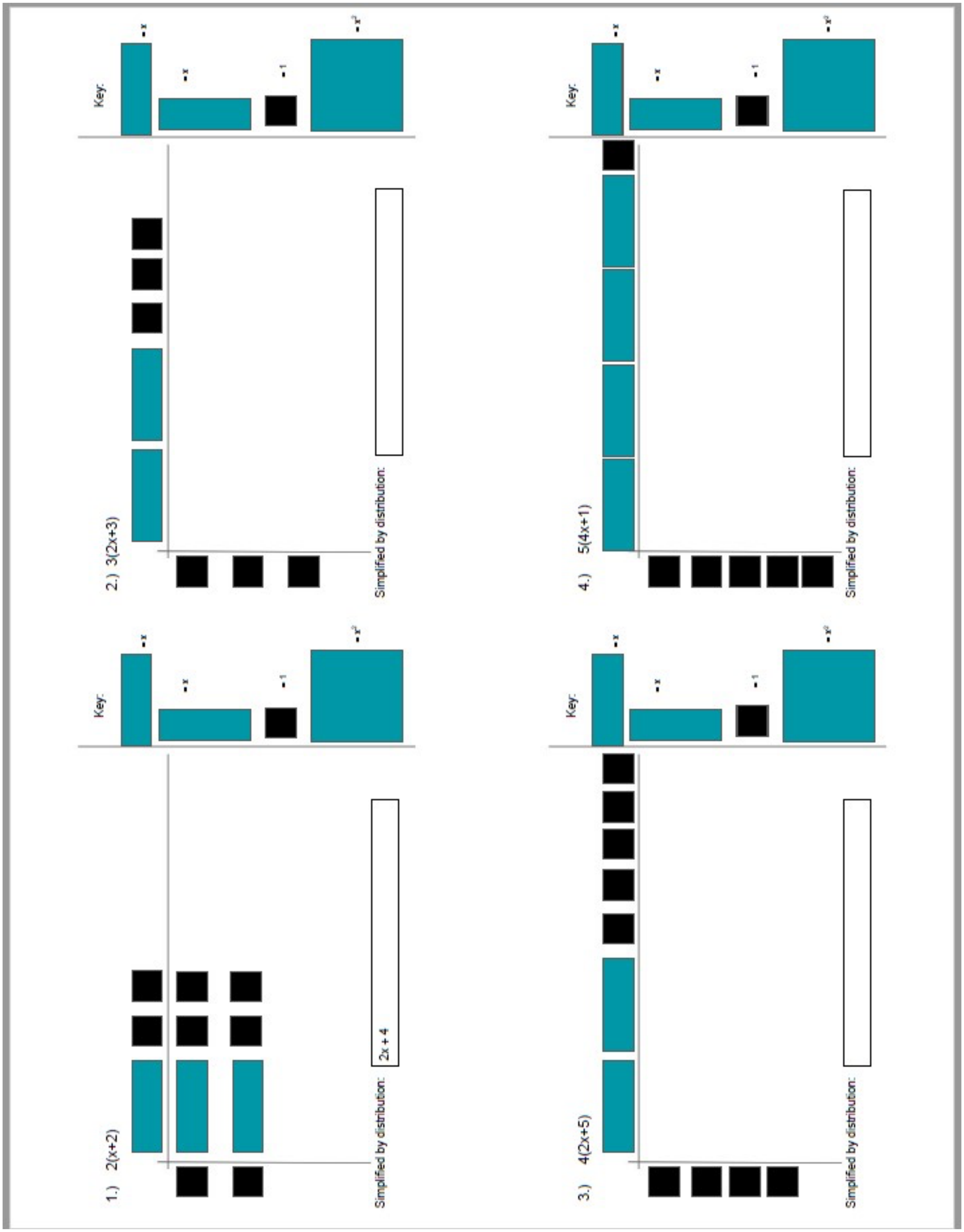
10.) $x(3x+2)$



11.) $2x(3x+4)$



Name: Lesson 2 – Date: Class:



Name: Lesson 2 – Option C

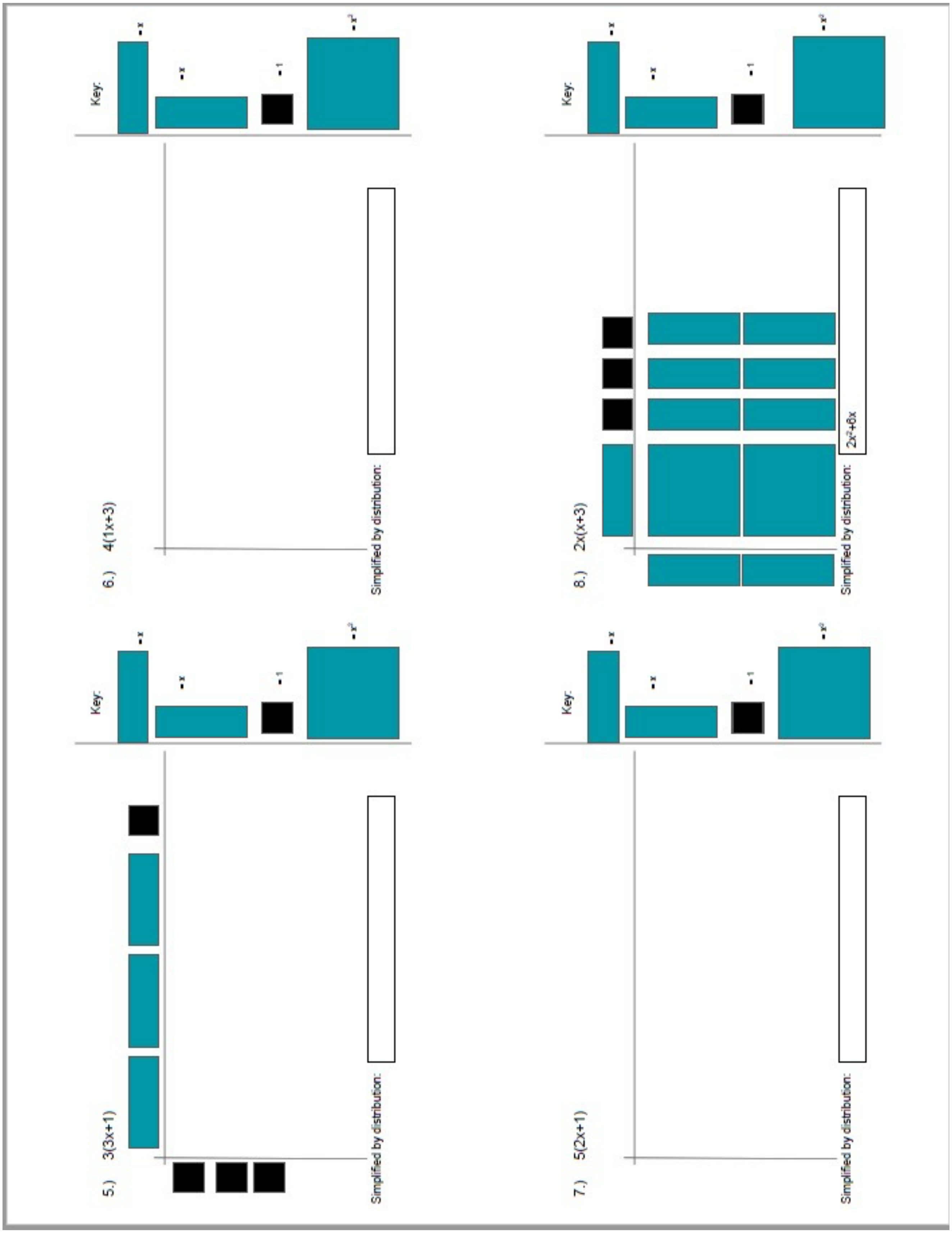
Class:

Date:

Name: Lesson 2 – Option C

Class:

Date:



Lesson 3 Options A and B

Overview

Students will be provided two options for demonstrating their understanding of techniques to simplify multiplication using the distributive property.

- One choice involves students working in pairs to complete a "Connect Four" type of game where players alternate turns rolling two standard die and answering questions found at the coordinates of the die roll (rolling a 6 and a 2, would allow the student to choose a question located on the 6th row and 2nd column or the 2nd column and 6th row). The student who successfully answers four adjacent questions wins the game.
- The second option is for students to complete a worksheet independently that assesses their ability to utilize procedures to simplify polynomial multiplication.

Objective

Students can generate equations of one variable from a real-world situation.

Standards

NY-7.EE.4-Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities

NY-7.EE.4.a - Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Name: –

Date:

	1	2	3	4	5	6
1	2(3x+7)	3(5x+8)	3(2x+10)	9x(4-3x)	4x(6x-2)	x(2x-7)
2	5(3x-9)	3(3x+5)	6(6x+1)	6x(5x+1)	3(5x+8)	5(6x+8)
3	2(4x-7)	5(7x-9)	4x(3+x)	3(7x+1)	4(5x+9)	9(7x+2)
4	5(3x+9)	8x(7x+3)	9(4x+7)	10x(x+2)	7x(5x+2)	3x(2x-8)
5	7x(3x-2)	2(9+2x)	4x(6x-7)	3x(9+5x)	2(2x+10)	4(4x-4)
6	5x(x+1)	9(2x+4)	2(8x+3)	8(3x-5)	8x(6x+7)	9(6x+4)

Name: –

Date:

Lesson 3 Option B

Class:

1.) $9(4x + 7)$

2.) $5(3x - 9)$

3.) $2(4x - 7)$

4.) $5(3x + 9)$

5.) $7x(3x - 2)$

6.) $5x(x + 1)$

7.) $3(5x + 8)$

8.) $3(3x + 5)$

9.) $5(7x - 9)$

10.) $8x(7x + 3)$

11.) $2(9 + 2x)$

12.) $9(2x + 4)$

13.) $3(2x + 10)$

14.) $6(6x + 1)$

15.) $4x(3 + x)$

16.) $9(4x + 7)$

Classroom Application

The lessons in this curriculum project were presented to two 7th Grade Mathematics classes in upstate NY. Comparative analysis revealed the class that received curricular choices had a general level of excitement about the material compared to the class that did not have curricular choice. In particular, students were interested in knowing what the questions were about and they chose methods or problems that used more of their prior learning. A higher average summative assessment score seems to indicate that the class whose curriculum included choice, had a greater level of proficiency working with expressions and the distributive property. Alternatively, the class that wasn't allowed to choose seemed indifferent about the assignments they were assigned. Most of the points missed on the summative assessment by both groups of students related to distribution. Thus, teachers are encouraged to include a section on repeated addition as it relates to distribution. While presenting the curriculum, students asked if they could work in groups, and this could easily be incorporated into the lesson along with choices about due dates and preferred exit ticket methods.

Conclusion

Maintaining student interest in a topic can be challenging, but providing opportunities for students to exercise autonomy through activity selection can lead to increased levels of intrinsic motivation and consequently improved learning outcomes. When designing a series of activities for a lesson that will offer students choice between different tasks, it is necessary that students feel that they can accomplish each task and that these tasks reflect the interests of the class. It is the author's hope that activities from these lessons can be used to promote increased levels of intrinsic motivation for students who are learning how to simplify algebraic expressions.

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Appendix – Keys for Lessons

Lesson 1

Name:

Class:

A person is playing a video game where they score 20 points for each elimination and 10 points for each assist.

e = number of eliminations
a = number of assists

Expression:
(20)(e)+(10)(a)

What would a person's score be if he had six (6) eliminations and four (4) assists?

$20(6)+10(4)=$
$120+40=$ 160 points

Write an expression for the number of points a person would have if they played 3 games. (Use variables and do not solve.)

$3(20e+10a)=60e+30a$

In a football game, a team can score 6 points by getting a touchdown, 1 point for an extra point conversion and 3 points for a field goal. Write an expression for the points in a game using these values.

t = number of touchdowns
e = number of extra point conversions
f = number of field goals made

$6(t)+1(e)+3(f)=6t+e+3f$

Write an expression for how many points a team would have after two games.

$2(6t+e+3f)=12t+2e+6f$

Name:

Class:

Each contractor at a company can construct framing for a room in 8 hours and install sheetrock in that room in 6 hours. f = number of rooms to frame s = number of rooms to sheetrock t = total time Expression: (8)(f)+(6)(s)=t
How much time would the contractor need to frame 3 rooms and install sheetrock in 2 rooms?
$8(3)+6(2)=$ $24+12=36$ hours
Write an expression for the amount of time it would take 2 contractors to complete the work. (Use variables, don't try to solve.)
$2(8f+6s)=16f+12s$
When Sandy gets her allowance, she likes to buy bags of gummie bears for 2 dollars each, games for her phone for 1 dollar each and car magazines for 4 dollars each. b = number of bags of gummy bears g = number of games for phone m = number of magazines Write an expression for the amount of gummy bears, games and magazines she could buy with her allowance.
$2b+1g+4m$

Name:

Class:

On average, each time a student opens the Tik Tok application, she will spend 20 minutes watching videos and every time she opens Instagram, she will spend 10 minutes scrolling through feeds.

t = # of times a student opens Tik Tok
i = # of times a student opens Instagram

An expression for the amount of time one student spends on social media is shown below:
(20)(t)+(10)(i)

How many minutes would this person spend on social media if she opened Tik Tok 6 times and Instagram 4 times?

$20(6)+10(4)$
$120+40=160$ minutes

Write an expression for the amount of time 3 students would spend on social media. (Use variables and do not solve.)

$3(20t+10i)=60t+30i$

A zookeeper knows that each fox she cares for needs 1 pound of food per day, that each coyote needs 2 pounds of food per day and that each deer needs 4 pounds of food per day. Write and expression for the amount of food a zoo uses in a day. (Use variables and do not solve.)

d = # of deer
f = # of foxes
c = # of coyotes

$1f+2c+4d$

Write an expression for the amount of food the zoo uses in 3 days.

$3(1f+2c+4d)=3f+6c+12d$

Lesson 2

Name:

Date: Class:

Use area models and distribution to simplify the following expr.

1.) 2(x+2) = $2x+4$

	x	2
2	$2x$	4

2.) 3(2x+3) = $6x+9$

	2x	3
3	$6x$	9

3.) 4(2x+5) = $8x+20$

	2x	5
4	$8x$	20

4.) 5(4x+1) = $20x+5$

	4x	1
5	$20x$	5

5.) 3(3x+1) = $9x+3$

	3x	1
3	$9x$	3

Name:

Date: Class:

6.) $4(1x+3) = 4x + 12$

	1x	3
4	$4x$	12

7.) $5(2x+1) = 10x + 5$

	2x	1
5	$10x$	5

8.) $2x(x+3) = 2x^2 + 6x$

	x	3
2x	$2x^2$	$6x$

9.) $2x(2x+4) = 4x^2 + 8x$

	2x	4
2x	$4x^2$	$8x$

10.) $x(3x+2) = 3x^2 + 2x$

	3x	2
x	$3x^2$	$2x$

11.) $2x(3x+4) = 6x^2 + 8x$

	3x	4
2x	$6x^2$	$8x$

Name:

Date: Class:

Use the distribution property to simplify the following expressions.

1.) $2(x+2)$

$2x+4$

5.) $3(3x+1)=$

$9x+3$

2.) $3(2x+3)=6x+9$

6.) $4(1x+3)=$

$4x+12$

3.) $4(2x+5)=8x+20$

7.) $5(2x+1)=$

$10x+5$

4.) $5(4x+1)=20x+5$

8.) $2x(x+3)=$

$2x^2+6x$

Name:

Date: Class:

9.) $2x(2x+4) =$

$4x^2 + 8x$

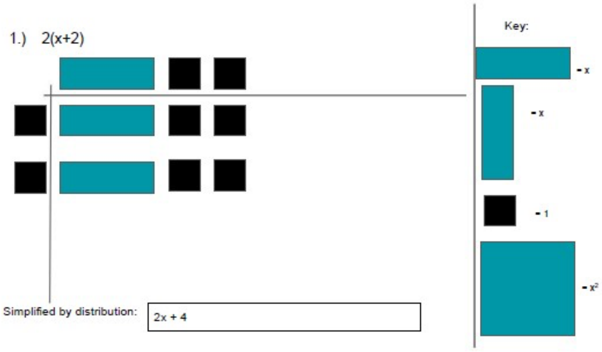
10.) $x(3x+2) =$

$3x^2 + 2x$

11.) $2x(3x+4) =$

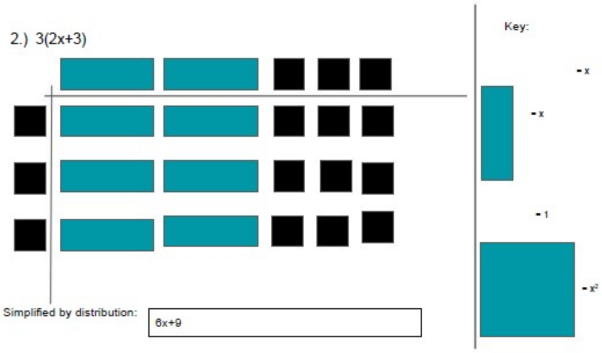
$6x^2 + 8x$

1.) 2(x+2)



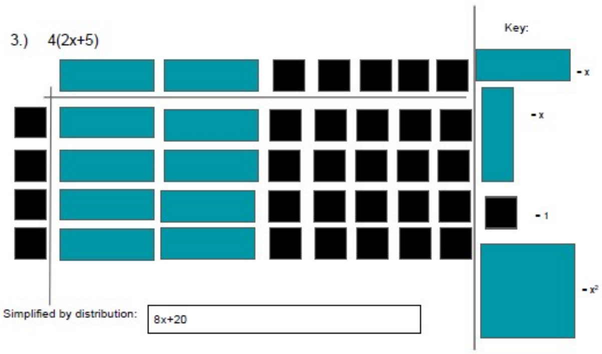
Simplified by distribution: 2x + 4

2.) 3(2x+3)



Simplified by distribution: 6x+9

3.) 4(2x+5)

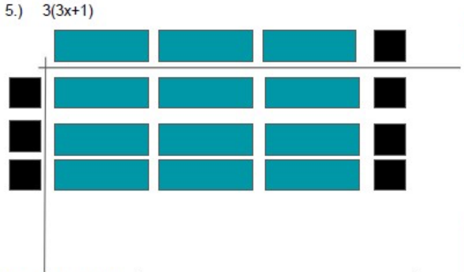


Simplified by distribution: 8x+20

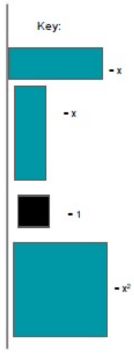
4.) 5(4x+1)

Simplified by distribution: 20x+5

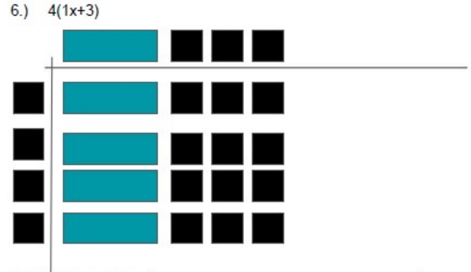
5.) 3(3x+1)



Simplified by distribution: 9x+3



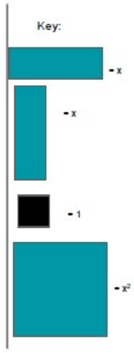
6.) 4(1x+3)



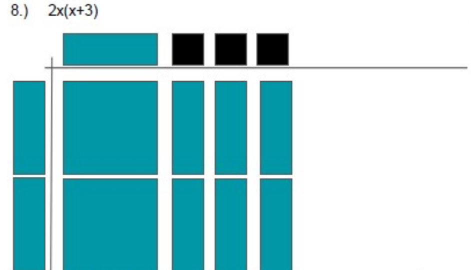
Simplified by distribution: 4x+12

7.) 5(2x+1)

Simplified by distribution: 10x+5



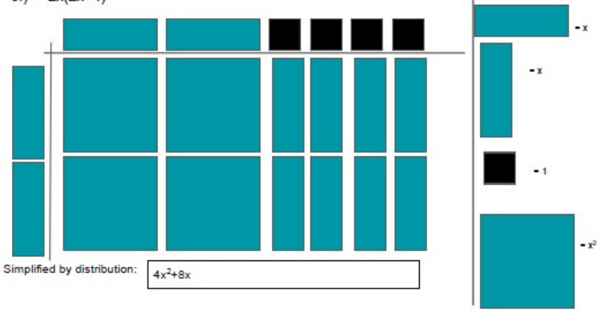
8.) 2x(x+3)



Simplified by distribution: $2x^2+6x$

8.) $2x(2x+4)$

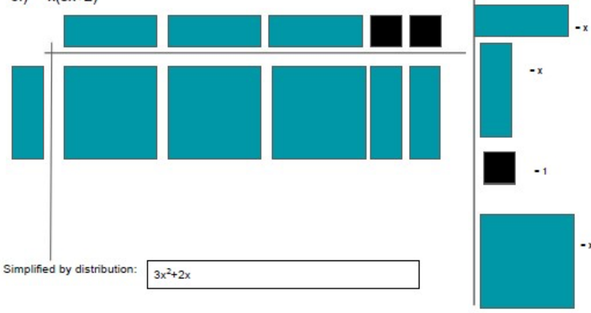
Key:



Simplified by distribution: $4x^2+8x$

8.) $x(3x+2)$

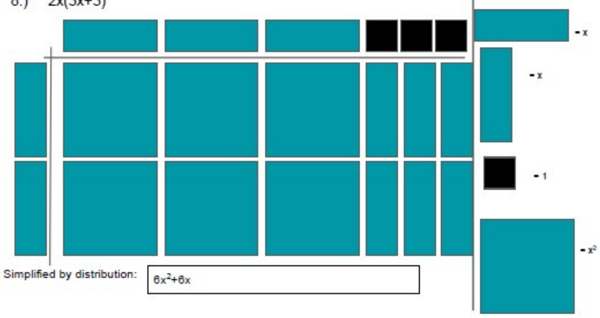
Key:



Simplified by distribution: $3x^2+2x$

8.) $2x(3x+3)$

Key:



Simplified by distribution: $6x^2+6x$

Lesson 3

	1	2	3	4	5	6
1	2(3x+7) $6x+14$	3(5x+8) $15x+24$	3(2x+10) $6x+30$	9x(4-3x) $-27x^2+36x$	4x(6x-2) $24x^2-8x$	x(2x-7) $2x^2-7x$
2	5(3x-9) $15x-45$	3(3x+5) $9x+15$	6(6x+1) $36x+6$	6x(5x+1) $30x^2+6x$	3(5x+8) $15x+24$	5(6x+8) $30x+40$
3	2(4x-7) $8x-14$	5(7x-9) $35x-45$	4x(3+x) $12x+4x^2$	3(7x+1) $21x+3$	4(5x+9) $20x+36$	9(7x+2) $63x+18$
4	5(3x+9) $15x+45$	8x(7x+3) $56x^2+24x$	9(4x+7) $36x+63$	10x(x+2) $10x^2+20$	7x(5x+2) $35x^2+14x$	3x(2x-8) $6x^2-24x$
5	7x(3x-2) $21x^2-14x$	2(9+2x) $4x+18$	4x(6x-7) $24x^2-28x$	3x(9+5x) $15x^2+27x$	2(2x+10) $4x+20$	4(4x-4) $16x-16$
6	5x(x+1) $5x^2+5x$	9(2x+4) $18x+36$	2(8x+3) $16x+6$	8(3x-5) $24x-40$	8x(6x+7) $48x^2+56x$	9(6x+4) $54x+36$

Name:

Date: Class:

1.) $2(3x+7)$

$6x+14$

2.) $5(3x-9)$

$15x+(-45)$

3.) $2(4x-7)$

$8x+(-14)$

4.) $5(3x+9)$

$15x+45$

5.) $7x(3x-2)$

$21x+(-14)$

6.) $5x(x+1)$

$5x^2+5x$

7.) 3(5x+8)

$15x+24$

8.) 3(3x+5)

$9x+15$

9.) $5(7x-9)$

$35x+(-45)$

10.) $8x(7x+3)$

$56x^2+24x$

11.) $2(9+2x)$

$18+4x$

12.) $9(2x+4)$

$18x+36$

13.) $3(2x+10)$

$6x+30$

14.) $6(6x+1)$

$36x+6$

15.) $4x(3+x)$

$12x+4x^2$

16.) $9(4x+7)$

$36x+63$