A Curriculum Project on Project-based Learning Supported by Scaffolding and Collaboration in an Algebra I Classroom.

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Abstract

This curriculum project was designed to engage students in an Algebra classroom by providing hands-on learning opportunities that can positively impact the world around them. For this curriculum project students will investigate the world issue of pollution to design multiple mathematical representations of this issue and theorize practical solutions. This experience is meant to be collaborative, and student driven using scaffolding, reciprocal learning, and a project-based learning approach. The curriculum presents sequential lesson plans, activities, classwork, and homework on the topic of functions. The project-based learning lessons are aligned to the New York State (NYS) Next Generation Standards**.** The goal is to deepen students mathematical reasoning, procedural fluency, and conceptual understanding and ultimately to provide a solid foundation of high school mathematics that prepares students to join a STEM college or career field after high school.

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Introduction

Mathematics educators are confronted with questions of "When will we ever use this? Why are we learning this? Why do I need to know this? Why is this important?" Teachers can share how important learning mathematics is, but students often struggle to connect what they are learning in the classroom to their everyday lives. Project Based Learning (PBL) can be used to help teachers make this connection between our students and the content. The purpose of this PBL curriculum project is to connect algebra to real world scenarios. Through building connections, we hope to get students more involved, engaged and interested in learning mathematics which can potentially increase the number of students that continue their education in a science, technology, engineering, or mathematics (STEM) field after high school.

Connecting classroom instruction to real world topics that interest students can be challenging and time consuming. PBL requires teachers to coach more, use direct instruct less, to embrace interdisciplinary learning instead of remaining locked in single-subject silos, and to be more comfortable with uncertainty and discovery during the learning process (Intel, 2012). Using local issues and concerns such as homelessness, global warming, food waste and what is done with it, overflowing landfills, etc. can be great topics for PBL, but these are big issues. Instead of planning a massive project, the learning process can be made more manageable by chunking the project into smaller parts, with frequent checkpoints built into the timeline. Instead of a traditional summative exam, authentic assessments can be developed. PBL and scaffolded assessment allow students to work from a base line of knowledge to build a deeper understanding, while working on something much bigger than the classroom. Something that may even change the world around them all while deepening their conceptual understanding of the mathematical topic at hand.

Literature Review

PBL is a current instructional strategy that is student driven, interdisciplinary, collaborative, technology based, and has been implemented into the K-12 classroom to encourage students' deep understanding. Aligning with PBL is performance-based assessment (PBA), which allows students to engage in activities that require various performance skills towards producing outcomes that meet expected quality levels. When using PBA teachers must determine specific tasks, provide learners a chance to observe and apply learning. As soon as students begin to build new mathematical perceptions and relationships, they can make decisions of how to overcome task-related problems. At the procedural level, PBA can determine the extent to which students have developed the necessary skills to solve the task while showing proficiency of educational objectives in mathematics (Mohammad, 2018). The main goal of this type of instruction is for the students to meet the learning objectives in smaller groups to support engagement in the learning, as well as gain a greater understanding of the information that they are being taught (Trach, 2018). Instead of short-term memorization and summative regurgitation, PBL provides an opportunity for students to engage deeply with the target content, bringing about a focus on long-term retention. (Traci, 2018).

There is a subset of outcomes and benefits from using PBL such as increased enthusiasm, engagement, motivation, and the use of collaboration and discourse (Traci, 2018). The PBL structure lends itself to building intrinsic motivation because it centers student learning around a central question or problem and a meaningful outcome. After being provided with a real-world problem then the need to know can drive learning. Students then want to seek and understand the answer or solution as much or more than the teacher wants to know what they know, understand, and can do (Traci, 2018). Duke (2017) presents a recent study that suggests PBL correlated positively with student achievement, particularly in schools serving high-poverty communities. Especially relevant for this population, PBL supports the ability to communicate effectively with others and develop teamwork and critical thinking skills (Cult of Pedagogy, 2018). Likewise, Han et al. (2015) investigated whether participating in STEM PBL positively affected students' mathematical achievement. Over the three-year study, STEM PBL instruction influenced student achievement in mathematics by both student demographic backgrounds and performance levels. Students who were typically underperforming showed significantly higher growth rates on mathematics scores than high and middle performing students. Results of the study implied that STEM PBL can decrease the achievement gap. (Han et al., 2015).

The collaborative nature of PBL projects reinforces the Social and Emotional Learning (SEL) programs being implemented at progressive schools around the world (Traci. 2018). Many educators have turned to using this type of instruction to support learning because traditional methods were not providing desired results. Olympic Algebra, Seeing the Sequence, and Owl Box Project are all ways that PBL can be implemented into an algebra classroom. This curriculum project was designed to support teachers in developing algebra lessons that build up to PBL. It includes three scaffolded lessons before the PBL activity is provided. Each lesson begins with a detailed lesson plan. Some teachers may need and want the level of detail provided and some may not. Each teacher can modify the plans and lessons to fit in their own classroom.

The Curriculum

Algebra 1 Functions

Lesson Title: Characterizing Functions Day1

- **Content Standard(s): (**NYS CCLS or Content Standards) High School: Functions » Linear, Quadratic, & Exponential Models* | Common Core State Standards Initiative (corestandards.org)
- CCSS.MATH.CONTENT.HSF.BF. A.1
  Write a function that describes a relationship between two quantities
- CCSS.MATH.CONTENT.HSF.BF. A.1.A
  Determine an explicit expression, a recursive process, or steps for calculation from a context
- CCSS.MATH.CONTENT.HSF.LE. A.1
  Distinguish between situations that can be modeled with linear functions and with exponential functions.

- **Learning Objectives**
- Students will create functions and use them to solve problems.
- Students will determine an explicit expression, a recursive process, or steps for calculation from a context.

Instructional Resources and Materials
writing utensil
graph paper
note packets
worksheets
Chromebook (if available)
graphing calculator

Instructional Strategies and Learning Tasks (times provided are based on an 80-minute class)
The Teacher will:
Provide the Warm-up: (10 Minutes)

- Display three sets of data on the board. Ask students to study each data set and make notes about what they observe.
- Have a discussion where students can share what they observed. Write down all suggestions on the board so that the class can refer to them later in the lesson.
- Use guiding questions such a "what do you notice about the data sets? Can you identify the type of function they represent?" Have students make inferences about the type of function they believe each data set to represent and support their inferences with evidence.
- Provide a graph of each data set so that the students can see a visual representation of the functions. Ask the students if they can now see a trend.
- Have students examine the table of values and compare it to the graph of the function to find any patterns that may help determine the type of function it represents.

Reflection Activity: (10 Minutes)

- Have students complete the reflection activity in their math journals. (Students keeping a math journal helps to: solidify procedural knowledge base; provide more opportunities to deepen conceptual understanding; and improves their involvement in discussions.)

- Ask students if anyone is willing to share the response to the questions in the activity. The questions should be summary questions of the class discussion and may include: *How can we identify a linear function from its data? How can we identify a quadratic function from its data? How can we identify an exponential function from its data? How can we identify an absolute value function from its data? How might we recognize a square root and a cube root function from a set of data?*
- Allow enough time for students to complete this activity before moving on to the classroom activity problems.

Class Activity: (15 Minutes)

- Have students read the provided problems and analyze the data sets for the problems. Allow students to work as partners or in small groups to complete the example as a guided group activity.
- Through informal assessment of student work provide additional support to students that may need it, answer questions, and address misconceptions.
- Ask students to walk you through their thinking and how they completed the problem. Ask if any other students used a different method for solving the problem.

Group Work: (25 minutes)

- Have students complete provided exercises. Allow students to collaborate with a partner.
- Have students complete remaining exercises for homework.

Closure: (5 Minutes)

- Ask students "*How do we determine the type of function from a given set of data by analyzing the y-values when the x-values are increasing by a constant increment?*"
- Summarize key points/ Take aways from the lesson.

Exit Ticket: (5 Minutes)

- Ask students to complete the exit ticket and hand it in before exit the classroom.

Differentiation and planned universal supports Please indicate all Universal Design of Learning (UDL) aspects (Engagement, Representation, and Action/Expression) of your lesson here.

- **Engagement:** The students will work collaboratively to explore their surface level understanding functions, methods for writing and solving functions, and how to determine functions based on key features and characteristics represented by data sets, ordered pairs, graphs, etc. The students will be able to discuss as a class what methods for solving and what functions they believe are being represented. If one student understands the material, they may be able to explain the material in a way that their peers may be able to make a connection with.
- **Representation:** The students will complete graphs of functions, analyze data sets, tables, and symbolic representations to evaluate functions, classify functions, and to solve problems. Encourage students to: participate in discussions about classifying functions; how to write functions; and walk the teacher through how to solve the given problems.
- **Action/Expression:** The students will be working as a class to compose their own meaning, expression, and representations of the various types of functions. For those who need it, an audio recording can be made of the lesson, as well as videos related to functions can be provided for students. Students are asked to provide examples that engage them in the development of their understanding of each type of function and how to represent functions in diverse ways. The students are asked to rank their understanding on a scale of 1 to 4 using their fingers (4: I understand it and could teach it to someone else, 3: I understand it, but I could not teach it to someone else, 2: I need more practice/ I do not fully understand, 1: I am completely lost) as a method of a quick check in for the teacher.

- **Cooperative Learning:** The students will be working together to form their own understanding of functions. Cooperative learning provides opportunities for students to interact in ways that enhance and deepen their learning. By intentionally incorporating the elements of positive interdependence and individual accountability, teachers set the stage for students to be responsible for their own learning; the learning of those in their group; and the ability to demonstrate what they know, understand, and can do. Studies show that well-organized cooperative learning opportunities positively affect academic as well as socioemotional achievement, self-esteem, motivation, and engagement with school all while helping to minimize feelings of social isolation (Dean, 2013).

Type of Student Assessments and what is being assessed:

- **Informal Assessment:** Through observation support students that may be struggling. If students are having trouble understanding, then a whole group discussion on the issue may be beneficial. Different methods of finding answers can also be discussed and evaluated.
- **Formal Assessment**: A homework sheet, such as the remainder of the questions in their classwork packet, based on the lesson will be provided. This allows the teacher to determine if students understand how to write a function that describes a relationship between two quantities; how to characterize and describe a function; and how to use the function to solve a problem in context. Using graphs, tables, and other visuals creates a representation of the problem and can provide an example of a real-world problem.
- **Modifications to the Assessments** (for diverse learners)**:** For those who may need it, videos or audio recordings can be made or found for students. The worksheets that are handed out will also have visual representations of what the lesson includes, and for those who are English language learners a translation of the directions can be provided. Any worksheets or packets will be handed out during in-person classes and a digital copy will be provided in google classroom.

Evaluation Criteria How will you know if your students achieved the learning objective? Make the connections between the objective and assessments explicit.: Students will have achieved understanding when they are able to create functions from a given context and use them to solve problems

Relevant theories and/or research best practices (cite all sources, in APA format, that you used to develop this lesson- including theories, pedagogy, UDL, Co-teaching etc.)**:**

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 3) Engagement Strategies*. YouTube. youtube.com/watch?v=cmBsE4LX8NM

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 4): Representation Strategies*. YouTube. Youtube.com/watch?v=9TVKGvnGPoa

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 5): action and expression strategies*. YouTube. youtube.com/watch?v=H66jHEiGNLQ

Mathematics Learning Standards. (n.d.). http://www.nysed.gov/common/nysed/files/programs/curriculum-instruction/nys-next-generation-mathematics-p-12-standards.pdf

Dean, Ceri B., and Robert J. Marzano. *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement*. Pearson Education, 2013.

Materials for this lesson follow below.

Day 1 Warm-Up

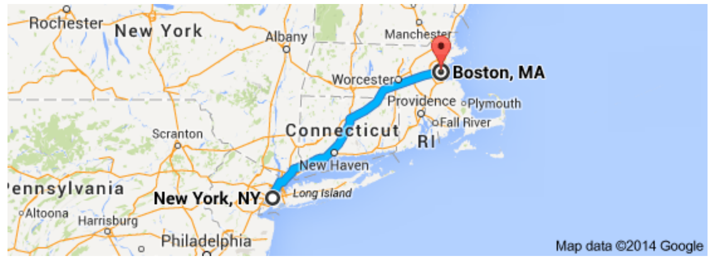
When tables are used to model functions, we typically have just a few sample values of the function and therefore must do some detective work to figure out what the function might be. Look at these three tables:

x	$f(x)$
0	6
1	12
2	18
3	24
4	30
5	36

x	$g(x)$
0	0
1	14
2	24
3	30
4	32
5	30

x	$h(x)$
0	1
1	3
2	9
3	27
4	81
5	243

Class Activity Day 1



Noam and Athena had an argument about whether it would take longer to get from NYC to Boston and back by car or by train. To settle their differences, they made separate, nonstop round trips from NYC to Boston. On the trip, at the end of each hour, both recorded the number of miles they had traveled from their starting points in NYC. The tables below show their travel times, in hours, and the distances from their starting points, in miles. The first table shows Noam's travel time and distance from the starting point, and the second represents Athena's. Use both data sets to justify your answers to the questions below.

Time in Hours	Noam's Distance
0	0
1	55
2	110
3	165
4	220
5	165
6	110
7	55
8	0

Time in Hours	Athena's Distance
0	0
1	81
2	144
3	189
4	216
5	225
6	216
7	189
8	144
9	81
10	0

a. Who do you think is driving, and who is riding the train? Explain your answer in the context of the problem.

b. According to the data, how far apart are Boston and New York City? Explain mathematically.

c. How long did it take each of them to make the round trip?

d. According to their collected data, which method of travel was faster?

e. What was the average rate of change for Athena for the interval from 3 to 4 hours? How might you explain that in the context of the problem?

f. Noam believes a quadratic function can be used as a model for both data sets. Do you agree? Use and describe the key features of the functions represented by the data sets to support your answer.

Day 1 Group Work

Explain why each function can or cannot be used to model the given data set.

x	$f(x)$
0	5
1	8
2	9
3	8
4	5
5	0
6	-7

a. $f(x) = 3x + 5$

b. $f(x) = -(x-2)^2 + 9$

c. $f(x) = -x^2 + 4x - 5$

d. $f(x) = 3^x + 4$

e. $f(x) = (x-2)^2 + 9$

f. $f(x) = -(x+1)(x-5)$

Match each table below to the function and the context and explain how you made your decision.

A

x	y
1	9
2	18
3	27
4	18
5	9

Equation____

Context_____

B

x	y
1	12
2	24
3	36
4	48
5	60

Equation ____

Context ____

C

x	y
0	160
1	174
2	156
3	106
4	24

Equation ___

Context____

D

x	y
1	2
2	4
3	8
4	16
5	32

Equation____

Context ____

E

x	y
2	8
3	9
4	8
5	5
6	0

Equation____

Context____

Matching Equations to Real World Scenarios

Match the function to the context that it describes

Equations:

1. F(x)=12x
2. H(x) = -9|x - 3| +27
3. G(x) = - (x)(x – 6)
4. P(x) = 2^{x}
5. Q(x) = $-16x^2 + 30x + 160$

Contexts:

_________A. The population of bacterial doubles every month and the total population vs time is recorded

_________ B. A ball was launched upward from the top of a building, and the vertical distance of the ball from the ground vs. time was recorded.

________ C. The height of a certain animal`s vertical leap was recorded at regular time intervals of one second; the animal returned to ground level after six seconds.

________ D. Melvin saves the same amount of money every month. the total amount saved after each month was recorded.

________ E. Chris ran at a constant rate on a straight-line path and then returned at the same rate. His distance from his starting point was recorded at regular time intervals.

Day 1 Exit Ticket

Analyze these data sets, recognizing the unique pattern and key feature(s) for each relationship. Then use your findings to fill in the missing data, match to the correct function from the list on the right and describe the key feature(s) that helped you choose the function.

Table A

x	y
0	6
1	10
2	14
3	
4	22
5	

Table B

x	y
0	6
1	15
2	18
3	15
4	
5	

Table C

x	y
-1	$\frac{1}{6}$
0	1
1	
2	36
3	
4	1296

Table D

x	y
-1	
0	6
1	8
2	6
3	0
4	
5	-24

Equations:

$f(x) = 6^x$

$h(x) = -3(x-2)^2 + 18$

$g(x) = -2(x+1)(x-3)$

$r(x) = 4x + 6$

Table A: ___________________ Key Feature(s): ____

Table B: ___________________ Key Feature(s): ____

Table C: ___________________ Key Feature(s): ____

Table D: ___________________ Key Feature(s): ____

Homework Day 1

1.

a. Determine the function type that could be used to model the data set at the right and explain why.

b. Complete the data set using the special pattern of the function you described above.

c. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

x	y
0	
1	10
2	0
3	-6
4	-8
5	
6	

d. Determine the function type that could be used to model the data set and explain why.

e. Complete the data set using the special pattern of the function you described above.

f. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

x	y
-1	
0	
1	
2	16
3	64
4	256
5	1024

g. Determine the function type that could be used to model the data set and explain why.

h. Complete the data set using the special pattern of the function you described above.

i. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

x	y
-1	
0	12
1	
2	24
3	
4	36
5	

Circle all the function types that could be used to model a context if the given statement applies.

a. When x-values are at regular intervals, the first difference of y-values are not constant.

Linear Function | Quadratic Function | Exponential Function | Absolute Value Function

b. When x-values are at regular intervals, the second difference of y-values are not constant.

Linear Function | Quadratic Function | Exponential Function | Absolute Value Function

c. When x-values are at regular intervals, the quotient of any two consecutive y-values are a constant that is not equal to 0 or 1.

Linear Function Quadratic Function Exponential Function Absolute Value Function

d. There may be up to two different x-values for $y = 0$.

Linear Function Quadratic Function Exponential Function Absolute Value Function

Algebra 1 Functions

Lesson Title: Constructing functions Day 2

Content Standard(s): High School: Functions » Linear, Quadratic, & Exponential Models* | Common Core State Standards Initiative (corestandards.org)

- CCSS.MATH.CONTENT.HSF.LE. A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Learning Objectives associated with the content standards:

- Students will create functions given a graph, a description of a relationship, or two input-output pairs.
- Students will interpret solutions of functions to determine if the solutions are viable or nonviable.

Instructional Resources and Materials:

- writing utensil
- graph
- note packets
- worksheets
- Chromebook (if available)
- graphing calculator

Instructional Strategies and Learning Tasks (The times provided are based on an 80- minute class.):

The Teacher will:

Provide the warm-up: (10 Minutes)

- Divide the class into three sections. Have each section analyze one of the provided tables and answer the questions that follow.
- Have one student from each section share the work of their group and explain the group`s approach.
- Discuss the strategies of each group and how they are similar/different.

Reflection Activity: (10 Minutes)

- Have students create a Frayer model* in their math journals for the different functions with information for how to identify them; their key characteristics; symbolic representations, graphic representations, and examples. (Students keeping a math journal helps to: solidify procedural knowledge base; provide more opportunities to deepen conceptual understanding; and improves their involvement in discussions.)
- Ask students if anyone is willing to share the examples they provided for this activity.
- Allow enough time for students to complete this activity before moving on to the classroom activity problems

Class Activity: (15 minutes)

- Begin with this example as a guided activity to help students get started.
- Start by plotting the pairs in the table using the values in the second and third columns
- The graph appears to be linear, but it is important to remember the window we are using on the calculator might be hiding some notable features of the graph.

- Use the table to investigate the data. Look for differences and key features in the data to give us clues about the graph.
- Students may decide to extend the table. Make sure that they create the function, verify their results, and discuss the limitations of their model.
- Through informal assessment of student work provide additional support to students that may need it, answer questions, and address misconceptions.
- Ask students to walk you through their thinking and how they completed the problem. Ask if any other students used a different method for solving the problem.
- Ask Students "*What limitations do we have in this problem? What information do we need so that we can provide more specific and accurate answers? What are pieces of information that are missing that may affect our model?*"

Group work: (25 Minutes)

- Have students complete provided exercises. Allow students to collaborate with a partner.
- Have students complete remaining exercises for homework.

Closing: (10 Minutes)

- Ask students "*what are limitations when modeling from a set of data? When given a data set, what strategies can be used to determine the type of function that describes the relationship between the data?*"
- Summarize key points/ Take aways from the lesson.

Exit Ticket: (5 Minutes)

- Have students complete the exit ticket and turn it in before leaving class.

* From (*Vocabulary Strategy*, n.d.)

Differentiation and planned universal supports: Universal Design of Learning (UDL) aspects (Engagement, Representation, and Action/Expression) of your lesson here.

- **Engagement:** The students will work collaboratively to explore their surface level understanding functions, methods for writing and solving functions, and how to determine functions based on key features and characteristics represented by data sets, ordered pairs, graphs, etc. The students will be able to discuss as a class what methods for solving and what functions they believe are being represented. If one student understands the material, they may be able to explain the material in a way that their peers may be able to make a connection with.
- **Representation:** The students will construct functions through the use of data sets, tables, and graphic representations to evaluate functions, classify functions, and to solve problems. Encourage students to participate in discussions about constructing functions; how to write functions; and walk the teacher through how to solve the given problems.
- **Action/Expression:** The students will be working as a class to compose their own meaning and expression using the various representations of functions. For those who need it, an audio recording can be made of the lesson, as well as videos related to functions can be provided for students. The students are asked to rank their understanding on a scale of 1 to 4 using their fingers (4: I understand it and could teach it to someone else, 3: I understand it, but I could not teach it to someone else, 2: I need more practice/ I do not fully understand, 1: I am completely lost) as a method of a quick check in for the teacher.
- **Cooperative Learning:** The students will be working together to form their own understanding of functions. Cooperative learning provides opportunities for students to interact in ways that enhance and deepen their learning. By intentionally incorporating the elements of positive interdependence and individual accountability, teachers set the stage for students to be responsible for their own learning; the learning of those in their group; and the ability to

demonstrate what they know, understand, and can do. Studies show that well-organized cooperative learning opportunities positively affect academic as well as socioemotional achievement, self-esteem, motivation, and engagement with school all while helping to minimize feelings of social isolation (Dean, 2013).

Type of Student Assessments and what is being assessed:

- **Informal Assessment:** Through observation support students that may be struggling. If students are having trouble understanding, then a whole group discussion on the issue may be beneficial. Different methods of finding answers can also be discussed and evaluated. The teacher can ask the students to rank their understanding on a scale of 1 to 4.
- **Formal Assessment**: A homework sheet, such as the remainder of the questions in their classwork packet, based on the lesson will be provided. This allows the teacher to determine if the students understand how to write a function when provided with the graph, ordered pair, a description of the relationship, etc. and how to use the function to solve a problem in context. Using graphs, tables, and other visuals creates a representation of the problem and can provide an example of a real-world problem.
- **Modifications to the Assessments** (for diverse learners): For those who may need it, videos or audio recordings can be made or found for students. The worksheets that are handed out will also have visual representations of what the lesson includes, and for those who are English language learners a translation of the directions can be provided. Any worksheets or packets will be handed out during in-person classes and a digital copy will be provided in google classroom.

Evaluation Criteria: How will you know if your students achieved the learning objective? Make the connections between the objective and assessments explicit.: Students will have achieved understanding when they are able to create functions given a graph, a description of a relationship, or two input-output pairs.

Relevant theories and/or research best practices:

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 3) Engagement Strategies*. YouTube. youtube.com/watch?v=cmBsE4LX8NM

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 4): Representation Strategies*. YouTube. Youtube.com/watch?v=9TVKGvnGPoa

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 5): action and expression strategies*. YouTube. youtube.com/watch?v=H66jHEiGNLQ

Mathematics Learning Standards. (n.d.). http://www.nysed.gov/common/nysed/files/programs/curriculum-instruction/nys-next-generation-mathematics-p-12-standards.pdf

Dean, Ceri B., and Robert J. Marzano. *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement*. Pearson Education, 2013.

Vocabulary Strategy. (n.d.). https://www.nbss.ie/sites/default/files/publications/frayer_model_-

_vocbulary_strategy_handout__copy_3.pdf

Day 2 Warm-Up

a.) Identify the type of function that each table appears to represent (e.g., quadratic, linear, exponential, square root).

b.) Explain how you were able to identify the function.

c.) Find the symbolic representation of the function.

d.) Plot the graphs of your data.

A

x	y
1	5
2	7
3	9
4	11
5	13

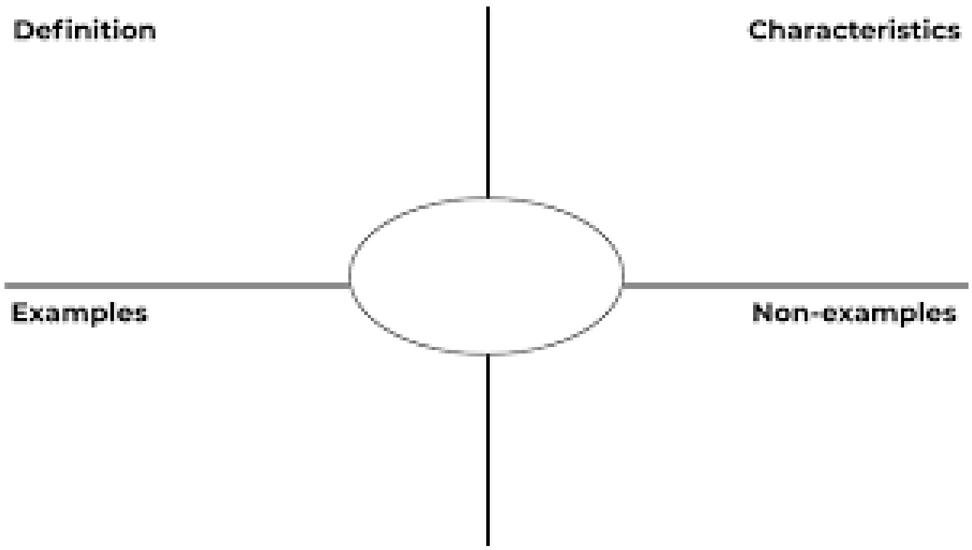
B

x	y
1	6
2	9
3	13.5
4	20.25
5	30.375

C

x	y
1	3
2	12
3	27
4	48
5	75

Frayer Model Template for Math Journal



Day 2 Classwork

Enrique is a biologist who has been monitoring the population of a rare fish in Lake Placid. He has tracked the population for 5 years and has produced the following estimates:

Year Tracked	Year Since 2002	Estimated Fish Population
2002	0	1,000
2003	1	899
2004	2	796
2005	3	691
2006	4	584

Create a graph and a function to model this situation and use it to predict (assuming the trend continues) when the fish population will be gone from the Lake Placid ecosystem. Verify your results and explain the limitations of each model.

Day 2 Group Work

Bella is a BMX bike racer and wants to identify the relationship between her bike's weight and the height of jumps (a category she gets judged on when racing). On a practice course, she tests out 7 bike models with different weights and produces the following data.

Weight (lb.)	Height of Jump (ft)
20	8.9
21	8.82
22	8.74
23	8.66
24	8.58
25	8.5
26	8.42
27	8.34

a. Bella is sponsored by Twilight Bikes and must ride a 32 lb bike. What can she expect her jump height to be?

b. Bella asks the bike engineers at Twilight to make the lightest bike possible. They tell her the lightest functional bike they could make is 10 lb. Based on this data, what is the highest she should expect to jump if she only uses Twilight bikes?

c. What is the maximum weight of a bike if Bella's jumps must be at least 2 ft high during a race?

e.) The concentration of medicine in a patient's blood as time passes is recorded in the table below.

Time (hours)	Concentration of Medicine (ml)
0	0
0.5	55.5
1	83
1.5	82.5
2	54

a. The patient cannot be active while the medicine is in his blood. How long, to the nearest minute, must the patient remain inactive? What are the limitations of your model(s)?

b. What is the highest concentration of medicine in the patient's blood?

f.) A student is conducting an experiment, and as time passes, the number of cells in the experiment decreases. How many cells will there be after 16 minutes?

Time (minutes)	Cells
0	5,000,000
1	2,750,000
2	1,512,500
3	831,875
4	457,531
5	251,642
6	138,403

Day 2 Homework

Research linear, quadratic, and exponential functions using the Internet. For each of the three types of functions, provide an example of a problem/situation you found on the Internet where that function was used to model the situation or answer the problem. Include the actual function used in the example and web page where you found the example.

Algebra 1 Functions

Lesson Title: Functions Research Day 3

Content Standard(s): High School: Functions » Linear, Quadratic, & Exponential Models* | Common Core State Standards Initiative (corestandards.org) . High School: Modeling | Common Core State Standards Initiative (corestandards.org)

- Identifying variables in the situation and selecting those that represent essential features.
- CCSS.MATH.CONTENT.HSF.IF. B.4
  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- CCSS.MATH.CONTENT.HSF.BF. A.1.B
  Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
- CCSS.MATH.CONTENT.HSF.IF. B.5
  Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*

Learning Objectives associated with the content standards:

- The students will identify essential features of their topic and translate them into variables.
- Students will formulate a model by creating and selecting representations that describe relationships between the variables.

Instructional Resources and Materials:

- writing utensil
- graph
- note packets
- worksheets
- Chromebook (if available)
- graphing calculator

Instructional Strategies and Learning Tasks (times provided are based on an 80-minute class)

Note: The teacher could adjust the lesson plan so that the students can try to calculate the estimated environmental impact they and their family have on the environment. The students could collect the data for the to look at a small-scale glimpse of contributing factors to pollution the data collection can be a homework assignment that can be put together in a warm-up discussion. This could be a chance to demonstrate what is expected of the students for this project. With the data accumulated the teacher can use it to create an algebraic representation, create a visual representation, etc.

The Teacher will:

Warm-up discussion and homework check: (10 Minutes)

- Have a discussion where students can share what real world functions they discovered in their homework. Write down all examples with their accompanying functions on the board so that the class can refer to them later in the lesson.

- Use guiding questions such a "What topics did you discover while doing your research? What drew you to this topic? What functions did you derive from these scenarios? Did anyone have a different function for this scenario? Can you identify the type of function the example is?" Have students make inferences about the type of function they believe each data set to represent and support their inferences with evidence.

Class Activity: (15 Minutes)

- "Pick" a topic from the student examples. The students may have used pollution or carbon-foot print in the warm-up discussion.
- Discuss the topic of pollution in further depth. Ask prompting questions such as "What is pollution? What do you think of when you hear the word "pollution"? What factors play into a carbon footprint? Where does pollution come from? Who is responsible for pollution? What type of pollution is there? How could we "fix" pollution?"
- Lead the discussion into the contributing factors or levels of pollution. For example, pollution on a micro level, macro level, on land, in the sea, etc. Responses may include but are not limited to individuals/people, families, communities, schools, state, country, factories, and the world/ global level.
- Introduce the project that the students will be completing. State that, "As a group you will research one of the factors that contribute to pollution."

Group Work: (45 minutes)

- Depending on the class: assign the groups that students will be in or allow the students to make their own group.
- Allow students to research their topic, complete the brainstorming worksheet, and answer questions they created from the brainstorming worksheet.
- Provide guidance and further prompting where necessary
- Walk the room to check student progress of their research and project development.

Closure: (5 Minutes)

- Summarize key points/ Take aways from the lesson.
- Inform students that work will need to be done outside of class to complete the project. Have students assign roles and work to the members in their group so that everyone can contribute to the project even if they cannot meet up outside of class to work on the project.
- Inform students when presentations will be done. Remind the students to use the provided rubrics and worksheets to assist in the completion of the project and presentation.

Reflection Activity: (10 Minutes)

- Have students complete the reflection activity in their math journals. (Students keeping a math journal helps to: solidify procedural knowledge base; provide more opportunities to deepen conceptual understanding; and improves their involvement in discussions.)
- The questions should be summary questions of the class discussion and may include: *Who creates pollution? What Interests you about this research topic? What contributing factor of pollution is the topic of research for your group? Is there anything you are curious about or look forward to learning about this topic? Did you find anything during your research that surprised you? How do you feel about working in a group on a project? What other notes can you make about your research from today?*
- Allow enough time for students to complete this activity before the completion of the class.

Differentiation and planned universal supports:

- **Engagement:** The students will work collaboratively to explore one factor of a real-world scenario by creating functions, representations, and by developing a plan of action for the scenario. The students will be able to discuss as a class what key characteristics are involved in the real-world scenario of pollution and the garbage island in the ocean. This lesson explores collaborative learning. If one student understands the material, they may be able to explain the material in a way that their peers may be able to make a connection with. All students are asked to provide examples of how to solve the given problem by engaging them in the development of examples they are able to build a deeper understanding of the real-world scenario and the application of mathematics in everyday life.
- **Representation:** The students will construct functions through the use of data sets, tables, and graphic representations to evaluate a real-world scenario and problem. Each student is asked to participate in discussions about the problem and how to solve the problem in a logically obtainable way. The students will be prompted for discussion by the teacher.
- **Action/Expression:** The students will be working as a class to compose the beginning ideas for the project. For those who need it, an audio recording can be made of the lesson, as well as videos related to the real-world scenario of pollution can be provided for students.
- **Cooperative Learning:** The students will be working together to form a base line for the project. Students will be working in groups to develop and research a contributing factor of the real-word scenario to be presented to the whole class on a later date. Cooperative learning provides opportunities for students to interact in ways that enhance and deepen their learning. By intentionally incorporating the elements of positive interdependence and individual accountability, teachers set the stage for students to be responsible for their own learning; the learning of those in their group; and the ability to demonstrate what they know, understand, and can do. Studies show that well-organized cooperative learning opportunities positively affect academic as well as socioemotional achievement, self-esteem, motivation, and engagement with school all while helping to minimize feelings of social isolation (Dean, 2013).

Type of Student Assessments and what is being assessed:

- **Informal Assessment:** As the teacher walks around, the teacher can talk through any struggles that students may be facing, and if there is a reoccurring misconception and students are having trouble understanding, there can then be a full group discussion on the issue.

- **Formal Assessment**: For homework, the students may continue their in-class research with their group, or individually based on group assignment of tasks. Students will be asked to answer any questions their group developed or wanted to ponder during the class brainstorming. Students will be asked to present their research and discoveries as a group on Day 5. The presentations may vary depending on student comfort level.

- **Modifications to the Assessments:** For those who may need it, videos or audio recordings can be made or found for students. The worksheets that are handed out will also have visual representations of what the lesson includes, and for those who are English language learners a translation of the directions can be provided. Any worksheets or packets will be handed out during in-person classes and a digital copy will be provided in google classroom. Students will be asked to present their project to the class. Based on student comfort level this could be a PowerPoint, video recording, etc. as long as it is approved by the teacher and meets the requirements set forth by the rubric and guiding worksheets.

Evaluation Criteria: Students will have achieved understanding when they are able to create functions and representations of a real-world scenario.
Relevant theories and/or research best practices: Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 3) Engagement Strategies*. YouTube. youtube.com/watch?v=cmBsE4LX8NM Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 4): Representation Strategies*. YouTube. Youtube.com/watch?v=9TVKGvnGPoa Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 5): action and expression strategies*. YouTube. youtube.com/watch?v=H66jHEiGNLQ *Cooperative Learning: 7 Free PDF Assessment Instruments*. (n.d.). Www.dailyteachingtools.com. https://www.dailyteachingtools.com/cooperative-learning-evaluate.html *Group Presentation Rubric*. (n.d.). Retrieved April 18, 2022, from https://pbl101.weebly.com/uploads/3/1/3/1/31318861/group_presentation_rubric.pdf *Mathematics Learning Standards*. (n.d.). http://www.nysed.gov/common/nysed/files/programs/curriculum-instruction/nys-next-generation-mathematics-p-12-standards.pdf Dean, Ceri B., and Robert J. Marzano. *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement*. Pearson Education, 2013.

Project Introduction

As a group you will research one of the contributing factors to pollution to create a presentation and plan of action for the contributing factor you group was assigned/choose. You will be responsible for discussing how your contributing factor connects, leads to, or is related to the next contributing factor. Your finished project will be presented to the class beginning on __________.

Your presentation should include:

- Algebraic representations of the scenario, solution, impact of the solution
- Visual representations for the scenario, solution, impact of the solution
- A description of the real-world scenario your group is researching.
- Discoveries
- Contributions
- Practical solutions
- Best solution to the scenario
- Constraints for solution(s)
- Plan of action
- Cost of action
- How your plan could be implemented
- What you would have to do to implement your plan
- The impact of your plan translated in terms of the real-world scenario
- How your contributing factor connects to the next level of the pollution cycle
- A list of related questions that could not be or were not solved.

Brainstorming Worksheet

Part 1: Answer the following prompts. Try to provide multiple responses. In part b provide solutions to the questions you created in that section.

1.) What are questions we need to ask ourselves?

b.)

2.) What constraints are there?

b.)

3.) What are possible solutions?

b.)

4.) What resources are necessary to implement a plan of action?

b.)

Additional notes/comments:

Part 2: To the best of your ability answer the following questions.

1.) How would you describe the problem in your own words?

2.) What pieces of the problem make sense to you and which parts are confusing?

3.) Who cares about this problem and why?

4.) How can the problem be varied and extended?

5.) What have others done to address this problem? Why have previous efforts to solve this problem fallen short?

6.) How to you plan to demonstrate that your solution is a good one?

7.) What has been the most interesting aspect of the material that you have gathered so far?

8.) After reviewing your data or sources, what do you see as the latest issues in the field of your topic?

9.) What do you think are the vital facts of the matter?

10.) What new insight can you contribute?

11.) What is the most meaningful action you can take in response to this problem?

12.) Who is the audience? How can you best reach them?

13.) How can you best package your work so that others can understand it and be moved by it?

Group Presentation Rubric

The teacher will use this rubric to evaluate each group's presentation. Look at this rubric to understand what you will be graded on. The Group Presentation Rubric will be combined with the Peer Participation Rubric to determine your final grade for the project.

Trait	Criteria				Points
	1	2	3	4	
Content Did the presentation have valuable material?	Presentation contained little to no valuable material.	Presentation had moments when valuable material was present but as a whole content was lacking.	Presentation had a good amount of material and benefited the class.	Presentation had an exceptional amount of valuable material and was extremely beneficial to the class.	____
Collaboration *Did everyone contribute to the presentation?* Did everyone seem well versed in the material?	The teammates never worked from others' ideas. It appears only a few people worked on the presentation.	The teammates sometimes worked from others' ideas. However, it seems as though certain people did not do as much work as others.	The teammates worked from others' ideas most of the time. And it seems like every did some work, but some people are carrying the presentation.	The teammates always worked from others' ideas. It was evident that all the group members contributed equally to the presentation.	____
Organization Was the presentation well organized and easy to follow?	The presentation lacked organization and had little evidence of preparation.	There were minimal signs of organization or preparation.	The presentation had organizing ideas but could have been much stronger with better preparation.	The presentation was well organized, well prepared and easy to follow.	____
Presentation *Did the presenters Speak clearly?* Did the engage the audience? Was it obvious the material had been rehearsed?	Presenters were unconfident and demonstrated little evidence of planning prior to presentation.	Presenters were not consistent with the level of confidence/ preparedness they showed the classroom but had some strong moments.	Presenters were occasionally confident with their presentation however the presentation was not as engaging as it could have been for the class.	Presenters were all very confident in delivery and they did an excellent job of engaging the class. Preparation is very evident.	____

*From (*Group Presentation Rubric*, n.d.)

Cooperative Learning Self Evaluation

Name______________________________Team_______________________Date__________

Project Topic or Title:

Briefly describe your contribution to the cooperative learning project:

If you were doing this project again, what would you do differently to improve your work?

How could your team work together more effectively next time?

Your Teacher's Comments:

Your Grade for Yourself:

Your Teacher's Grade for You:

*From (Cooperative Learning: 7 Free PDF Assessment Instruments, n.d.)

Peer Evaluation Form

Name____Class Period______Date_________________

Write the names of your group members in the numbered boxes. Then, assign yourself a value for each listed attribute. Finally, do the same for each of your group members and total all of the values.

Values: 1=Strongly Agree 2=Agree 3=Disagree 4=Strongly Disagree

Attribute	Yourself	1.	2.	3.
Was dependable in attending group meetings.				
Willingly accepted assigned tasks.				
Contributed positively to group discussions.				
Completed work on time or made alternative arrangements.				
Helped others with their work when needed.				
Did work accurately and completely.				
Contributed a fair share to weekly papers.				
Worked well with other group members.				
Overall was a valuable member of the team.				
Column Totals ▷				

*From (Cooperative Learning: 7 Free PDF Assessment Instruments, n.d.)

Grading Rubric for Teacher use

Group Presentation Rubric

Trait	Criteria				Points
	1	2	3	4	
Content Did the presentation have valuable material?	Presentation contained little to no valuable material.	Presentation had moments when valuable material was present but as a whole content was lacking.	Presentation had a good amount of material and benefited the class.	Presentation had an exceptional amount of valuable material and was extremely beneficial to the class.	_____
Collaboration *Did everyone contribute to the presentation?* Did everyone seem well versed in the material?	The teammates never worked from others' ideas. It appears only a few people worked on the presentation.	The teammates sometimes worked from others' ideas. However, it seems as though certain people did not do as much work as others.	The teammates worked from others' ideas most of the time. And it seems like every did some work, but some people are carrying the presentation.	The teammates always worked from others' ideas. It was evident that all the group members contributed equally to the presentation.	_____
Organization Was the presentation well organized and easy to follow?	The presentation lacked organization and had little evidence of preparation.	There were minimal signs of organization or preparation.	The presentation had organizing ideas but could have been much stronger with better preparation.	The presentation was well organized, well prepared and easy to follow.	_____
Presentation *Did the presenters Speak clearly?* Did the engage the audience? Was it obvious the material had been rehearsed?	Presenters were unconfident and demonstrated little evidence of planning prior to presentation.	Presenters were not consistent with the level of confidence/ preparedness they showed the classroom but had some strong moments.	Presenters were occasionally confident with their presentation however the presentation was not as engaging as it could have been for the class.	Presenters were all very confident in delivery and they did an excellent job of engaging the class. Preparation is very evident.	_____

Total Points: ____________

*From (*Group Presentation Rubric*, n.d.)

Algebra 1 Functions

Lesson Title: Functions Research Day 4

Content Standard(s): High School: Functions » Linear, Quadratic, & Exponential Models* | Common Core State Standards Initiative (corestandards.org) . High School: Modeling | Common Core State Standards Initiative (corestandards.org) • Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables. • Analyzing and performing operations on the relationships to draw conclusions • Validating the conclusions by comparing them with the situation, and then improving the model if necessary • CCSS.MATH.CONTENT.HSF.BF. B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
Learning Objectives associated with the content standards: • Students will formulate a model by creating and selecting representations that describe relationships between the variables. • Students will analyze the relationships between the variables to draw conclusions. • Students will experiment with cases and illustrate an explanation of the effects.
Instructional Resources and Materials: • writing utensil • graph • note packets • worksheets • Chromebook (if available) • graphing calculator
Instructional Strategies and Learning Tasks that support diverse student needs. (Include what you and students will be doing.)**:** The Teacher will: Warm-up discussion: (10 Minutes) • Have a discussion where students can share their findings with the class. This will hopefully decrease repletion between the groups. A whole class discussion allows each group to contribute to other groups. Each group can inspire, support, and inform each other. • Use guiding questions such a "What did you discover while doing your research? What functions did you derive? What obstacles did you come across while doing your research? Where you able to produce a solution to these problems? Did you find anything interesting while doing your research that you would like to share? Is there any information that you found that would fit with another group's topic? What sources did you find most helpful?" Group Work and Teacher Check-Ins: (55 minutes)

- Allow students to research their topic, answer questions they created from the brainstorming worksheet, and develop their presentation.
- Remind students the importance of citing their sources. Ask students to complete the sources worksheet to stay organized.
- Provide guidance and further prompting where necessary
- Have a meeting with each group individually while the other groups continue their research and development of their presentation. Meet with each group to check their progress, understanding, and plan of execution.
- After meeting with each group continue to walk the room to check student progress of their research and project development.

Closure: (5 Minutes)

- Inform students that work will need to be done outside of class to complete the project. Have students assign roles and work to the members in their group so that everyone can contribute to the project even if they cannot meet up outside of class to work on the project.
- Inform students when presentations will be done and the order in which groups will be presenting. Remind the students to use the provided rubrics and worksheets to assist in the completion of the project and presentation.

Reflection Activity: (10 Minutes)

- Have students complete the reflection activity in their math journals. (Students keeping a math journal helps to: solidify procedural knowledge base; provide more opportunities to deepen conceptual understanding; and improves their involvement in discussions.)
- The questions should be summary questions of the class and may include: *Is there anything you are excited about or look forward to presenting to the class? Did you find anything during your research that surprised you? What does your group still have to do to be ready for presentation? How can you contribute to the group to assure that the remaining tasks are completed in a timely manner? What other notes can you make about your research and project development from today?*
- Allow enough time for students to complete this activity before the completion of the class.

Differentiation and planned universal supports:

- **Engagement:** The students will work collaboratively to explore one stage of a real-world scenario by creating functions, representations, and by developing a plan of action for the scenario. This lesson explores collaborative learning. If one student understands the material, they may be able to explain the material in a way that their peers may be able to make a connection with. All students are asked to provide examples of how to solve the given problem by engaging them in the development of examples they can build a deeper understanding of the real-world scenario and the application of mathematics in everyday life.
- **Representation:** The students will construct functions, data sets, tables, and graphic representations to represent and evaluate a real-world scenario. Each student is asked to participate in discussions about the problem and how to solve the problem in a logically obtainable way. The students will be prompted for discussion by the teacher.
- **Action/Expression:** The students will be working in small groups to research one of the contributing factors to pollution. The groups will be asked to discuss their findings, struggles, and ideas with each other to prevent any repetition in the presentations. For those who need it, an audio recording can be made of the lesson, as well as videos related to the real-world scenario of littering/carbon-footprint can be provided for students.

- **Cooperative Learning:** Students will be working in groups to develop and research a stage of the real-word scenario to be presented to the whole class on a later date. Cooperative learning provides opportunities for students to interact in ways that enhance and deepen their learning. By intentionally incorporating the elements of positive interdependence and individual accountability, teachers set the stage for students to be responsible for their own learning; the learning of those in their group; and the ability to demonstrate what they know, understand, and can do. Studies show that well-organized cooperative learning opportunities positively affect academic as well as socioemotional achievement, self-esteem, motivation, and engagement with school all while helping to minimize feelings of social isolation (Dean, 2013).

Type of Student Assessments and what is being assessed:

- **Informal Assessment:** As the teacher walks around, the teacher can talk through any struggles that students may be facing, and if there is a reoccurring misconception and students are having trouble understanding, there can then be a full group discussion on the issue.

- **Formal Assessment**: For homework, the students may continue their in-class research with their group, or individually based on group assignment of tasks. Students will be asked to answer any questions their group developed during their research phase. Students will be asked to present their research and discoveries as a group. The presentations may vary depending on student comfort level.

- **Modifications to the Assessments:** For those who may need it, videos or audio recordings can be made or found for students. The worksheets that are handed out will also have visual representations of what the lesson includes, and for those who are English language learners a translation of the directions can be provided. Any worksheets or packets will be handed out during in-person classes and a digital copy will be provided in google classroom. Students will be asked to present their project to the class. Based on student comfort level this could be a PowerPoint, video recording, etc. if it is approved by the teacher and meets the requirements set forth by the rubric and guiding worksheets

Evaluation Criteria: Students will have achieved understanding when they are able to create functions to model the real-world scenario and the impact the selected plan of action would have on the scenario as well as representations of the real-world scenario.

Relevant theories and/or research best practices:

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 3) Engagement Strategies*. YouTube. youtube.com/watch?v=cmBsE4LX8NM

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 4): Representation Strategies*. YouTube. Youtube.com/watch?v=9TVKGvnGPoa

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 5): action and expression strategies*. YouTube. youtube.com/watch?v=H66jHEiGNLQ

Mathematics Learning Standards. (n.d.). http://www.nysed.gov/common/nysed/files/programs/curriculum-instruction/nys-next-generation-mathematics-p-12-standards.pdf

Dean, Ceri B., and Robert J. Marzano. *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement*. Pearson Education, 2013.

Day 4 Project Checklist

- ☐ Have a brief introduction to your topic
- ☐ Formed a hypothesis and included it in presentation
- ☐ Presentation includes a description of methods you used for solving and investigating solutions
- ☐ Includes tables, functions and graphs related to your findings
- ☐ Presentation includes an interpretation of your data
- ☐ Have four or more sources to support work
- ☐ Created a reference page
- ☐ Have multiple in text citations
- ☐ Have multiple options that serve as a solution
- ☐ Investigated “best” solution to determine constraints, effects, etc.
- ☐ Finalize slides, including references
- ☐ Decide who will speak for each slide
- ☐ Meet with group to practice presentation
- ☐ Presentation is integrated so that every person speaks several times
- ☐ Found ways to keep the audience interested
- ☐ Avoided including too much content
- ☐ Developed clear and appropriate visuals (PowerPoint slides, diagrams etc.) and handouts
- ☐ Worked out ways to involve the audience (e.g., invited comments, encouraged questions)
- ☐ Allowed time for questions, and thought through what people might ask
- ☐ Summarized appropriately, outlining key points
- ☐ Prepared index cards or skeleton notes to refer to (rather than reading notes in full)
- ☐ On the day of your presentation provide a hardcopy set of slides for your instructor.

Validity and Conclusion

The provided activities allow the teacher to act more as a facilitator and to be able to work more one-on-one with students that may be struggling, allowing the needs of all students to be met in a way that promotes engagement and discourse. These activities offer the opportunity for students to engage in mathematical discourse and create a classroom culture that offers a comfortable low-risk environment. One challenge of this curriculum project may be finding a research topic that the students can connect or relate to. A second challenge might be considering the depth of the topic to which the teacher should dive into the selected issue. Some issues can get insipid or overwhelming when researching or discussing, they can be topics that are massive in size and thus difficult to focus on and narrow in on. A third challenge may be how to group the students for collaboration and group projects. The answer keys for the materials are provided in the appendix. It is the author's hope that these materials will be used in other teachers' classrooms and even inspire them to create other PBL materials to be implemented in their own classrooms.

References

Beswick, K. (2011). Putting context in context: An examination of the evidence for the benefits of ‘contextualized’ tasks. *International Journal of Science and Mathematics Education*, *9*(2), 367–390. https://doi.org/10.1007/s10763-010-9270-z

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 3) Engagement Strategies*. YouTube. youtube.com/watch?v=cmBsE4LX8NM

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 4): Representation Strategies*. YouTube. Youtube.com/watch?v=9TVKGvnGPoa

Center for Excellence in Teaching and Learning at OU. (2017, July 25) *Universal Design for Learning (part 5): action and expression strategies*. YouTube. youtube.com/watch?v=H66jHEiGNLQ

Cooperative Learning: 7 Free PDF Assessment Instruments. (n.d.). Www.dailyteachingtools.com. https://www.dailyteachingtools.com/cooperative-learning-evaluate.html

Cult of Pedagogy. (2018, April 6). *Project based learning.* https://www.cultofpedagogy.com/project-based-learning/

Dean, Ceri B., and Robert J. Marzano. *Classroom Instruction That Works: Research-Based Strategies for Increasing Student Achievement*. Pearson Education, 2013.

Duke, N. K. (2017, June 20). New Study Shows the Impact of PBL on Student Achievement. Edutopia; George Lucas Educational Foundation. https://www.edutopia.org/article/new-study-shows-impact-pbl-student-achievement-nell-duke-anne-lise-halvorsen

Group Presentation Rubric. (n.d.). Retrieved April 18, 2022, from https://pbl101.weebly.com/uploads/3/1/3/1/31318861/group_presentation_rubric.pdf

Han, S., Capraro, R., & Capraro, M. (2015). How science, technology, engineering, and mathematics (stem) project-based learning (pbl) affects high, middle, and low achievers differently: The impact of student factors on achievement. *International Journal of Science & Mathematics Education*, *13*(5), 1089–1113. https://doi.org/10.1007/s10763-014-9526-0

High school: Modeling | common core state standards initiative. (n.d.). Retrieved November 18, 2020, from http://www.corestandards.org/Math/Content/HSM/

Intel Corporation. (2012). *Overview and benefits of project-based learning.* https://www.intel.com/content/dam/www/program/education/us/en/documents/project-design/projectdesign/benefits-of-projectbased-learning.pdf

Mathematics Learning Standards. (n.d.). http://www.nysed.gov/common/nysed/files/programs/curriculum-instruction/nys-next-generation-mathematics-p-12-standards.pdf

Mohammad Ahmad Alkhateeb. (2018). The effect of using performance-based assessment strategies to tenth-grade students' achievement and self-efficacy in Jordan. *Cypriot Journal of Educational Sciences*, *13*(4). https://doi.org/10.18844/cjes.v13i4.2815

Project-based learning in your algebra 1 classroom – stem jobs. (n.d.). Retrieved December 1, 2020, from https://edu.stemjobs.com/project-based-learning-in-your-algebra-i-classroom/

Project-Based Learning: Why and How? | Education World. (n.d.). Www.educationworld.com. https://www.educationworld.com/a_curr/project-based-learning-benefits-best-practices.shtml

Standards for mathematical practice | common core state standards initiative. (n.d.). Retrieved November 18, 2020, from http://www.corestandards.org/Math/Practice/

Trach, E. (2018, May 18). How Students Benefit From Project-Based Assessments (With Examples). Schoology.com; Schoology. https://www.schoology.com/blog/how-students-benefit-project-based-assessments-examples

Vocabulary Strategy. (n.d.). https://www.nbss.ie/sites/default/files/publications/frayer_model_-_vocbulary_strategy_handout__copy_3.pdf

Walshaw, M. (2017). Understanding mathematical development through Vygotsky. *Research in Mathematics Education*, *19*(3), 293–309. https://doi.org/10.1080/14794802.2017.1379728

Appendix

Day 1 Class Activity

Noam and Athena had an argument about whether it would take longer to get from NYC to Boston and back by car or by train. To settle their differences, they made separate, nonstop round trips from NYC to Boston. On the trip, at the end of each hour, both recorded the number of miles they had traveled from their starting points in NYC. The tables below show their travel times, in hours, and the distances from their starting points, in miles. The first table shows Noam's travel time and distance from the starting point, and the second represents Athena's. Use both data sets to justify your answers to the questions below.

Time in Hours	Noam's Distance
0	0
1	55
2	110
3	165
4	220
5	165
6	110
7	55
8	0

Time in Hours	Athena's Distance
0	0
1	81
2	144
3	189
4	216
5	225
6	216
7	189
8	144
9	81
10	0

a. **Who do you think is driving, and who is riding the train? Explain your answer in the context of the problem.**

It seems Noam is driving and Athena is riding the train because Noam traveled 55 miles in the first hour and Athena traveled 81 miles in the first hour.

b. **According to the data, how far apart are Boston and New York City? Explain mathematically.**

Noam's maximum distance was 220 miles, Athena's maximum distance was 225 miles → They may have departed or ended at different points. So I would say that Boston and New York City are 220 miles apart as traveling by car may give a more accurate measurement.

c. **How long did it take each of them to make the round trip?**

Noam traveled for 8 hours and Athena traveled for 10 hours.

Noam	Athena
(0,0)	(0,0)
(8,0)	(10,0)

d. **According to their collected data, which method of travel was faster?**

Noam traveled 2 hours faster than Athena. Assuming Noam traveled by car we can conclude that traveling by car was faster.

e. **What was the average rate of change for Athena for the interval from 3 to 4 hours? How might you explain that in the context of the problem?**

$$\frac{216-189}{4-3} = \frac{27}{1} = 27 \text{ miles per hour.}$$

The Average rate of change for this time interval was 27 miles per hour

Since Athena was likely on a train, the train may have slowed down to complete stops during this time period.

f. **Noam believes a quadratic function can be used as a model for both data sets. Do you agree? Use and describe the key features of the functions represented by the data sets to support your answer.**

* The two data sets have some similarities. Both are symmetric, have a vertex, have (0,0) as an x-intercept and a y-intercept.

* Noam has (8,0) as an x-intercept and Athena has (10,0) as an x-intercept

* Noam's data set has a constant difference of ± 55 depending on the side of the vertex we are analyzing.
  - Best modeled through an absolute value function or piece wise function)

* Athena's data set does not have a constant first difference (Not linear) but has a constant second difference of -18
  - Best modeled by a quadratic function.

Day 1 Group Work

Explain why each function can or cannot be used to model the given data set.

x	$f(x)$
0	5
1	8
2	9
3	8
4	5
5	0
6	−7

3, 1, −1, −3, −5, −7

−2, −2, −2, −2, −2

The data set is quadratic

cannot a. $f(x) = 3x + 5$

The function is linear but the data set is not.

$3(2)+5=9$

$6+5=9$

$11 \neq 9$

Can b. $f(x) = -(x-2)^2 + 9$

$f(x) = -(x-2)(x-2)+9$

$f(x) = -[x^2-2x-2x+4]+9$

$f(x) = -[x^2-4x+4]+9$

$f(x) = -x^2+4x-4+9$ $= \boxed{-x^2+4x+5}$

cannot c. $f(x) = -x^2 + 4x - 5$

$-0^2+4(0)-5=5$

$0+0-5=5$

$-5 \neq 5$

Cannot d. $f(x) = 3^x + 4$ exponential

$3^1+4=8$

$3+4=8$

$7 \neq 8$

Cannot e. $f(x) = (x-2)^2 + 9$

$(0-2)^2+9=5$

$(-2)^2+9=5$

$4+9=5$

$13 \neq 5$

Can f. $f(x) = -(x+1)(x-5)$

$f(x) = -[x^2-5x+x-5]$

$f(x) = -[x^2-4x-5]$

$f(x) = -x^2+4x+5$

$f(x) = ax^2+bx+c$

$f(x) = ax^2+bx+5$

$8 = a(1)^2+b(1)+5$

$3 = a+b$

$9 = a(2)^2+2b+5$

$9 = 4a+2b+5$

$4 = 4a+2b$

$2 = 2a+b$

$-1(3=a+b)$

$-3=-a-b$

$2=2a+b$

$-3=-a-b$

$-1=a$

$3=-1+b$

$4=b$

$\boxed{f(x) = -x^2+4x+5}$

Exit Ticket Day 1

Analyze these data sets, recognizing the unique pattern and key feature(s) for each relationship. Then use your findings to fill in the missing data, match to the correct function from the list on the right and describe the key feature(s) that helped you choose the function.

Table A

x	y
0	6
1	10
2	14
3	18
4	22
5	26

Table B

x	y
0	6
1	15
2	18
3	15
4	6
5	0

Table C

x	y
-1	$\frac{1}{6}$
0	1
1	6
2	36
3	216
4	1296

Table D

x	y
-1	0
0	6
1	8
2	6
3	0
4	-10
5	-24

Equations:

$f(x) = 6^x$

$h(x) = -3(x-2)^2 + 18$

$g(x) = -2(x+1)(x-3)$

$r(x) = 4x + 6$

Table A: r(x) **Key Feature(s):** constant slope of 4

Table B: h(x) **Key Feature(s):** (2,18) is max. Second difference is -6

Table C: f(x) **Key Feature(s):** exponential growth, common ratio of 6

Table D: g(x) **Key Feature(s):** x intercepts at x=-1, x=3 Second difference is -4

Homework Day 1

6.

a. Determine the function type that could be used to model the data set at the right and explain why. Quadratic - the second difference is 4

b. Complete the data set using the special pattern of the function you described above.

c. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

Minimum at (4,-8)

x	y
0	24
1	10
2	0
3	−6
4	−8
5	-6
6	0

-10
-6
-2

d. Determine the function type that could be used to model the data set and explain why.

e. Complete the data set using the special pattern of the function you described above.

f. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

d.) exponential - y value is multiplied by a constant value of 4

Since the function is exponential as x increases so does y, as x decreases y approaches zero. Thus there is no max or min value.

x	y
−1	1/4
0	1
1	4
2	16
3	64
4	256
5	1024

g. Determine the function type that could be used to model the data set and explain why.

h. Complete the data set using the special pattern of the function you described above.

i. If it exists, find the minimum or maximum value for the function model. If there is no minimum or maximum, explain why.

g.) Linear - the y values differ by a constant rate of 6.

There is no max or min value, as x increases y increases, as x decreases y decreases.

x	y
−1	6
0	12
1	18
2	24
3	30
4	36
5	42

Circle all the function types that could be used to model a context if the given statement applies.

j. When x-values are at regular intervals, the first difference of y-values are not constant.

Linear Function   Quadratic Function   Exponential Function   Absolute Value Function

k. When x-values are at regular intervals, the second difference of y-values are not constant.

Linear Function   Quadratic Function   Exponential Function   Absolute Value Function

l. When x-values are at regular intervals, the quotient of any two consecutive y-values are a constant that is not equal to 0 or 1.

Linear Function | Quadratic Function | Exponential Function | Absolute Value Function

m. There may be up to two different x-values for $y = 0$.

Linear Function | Quadratic Function | Exponential Function | Absolute Value Function

Day 2 Warm-Up

a.) Identify the type of function that each table appears to represent (e.g., quadratic, linear, exponential, square root).

A : Linear
B : Exponential
C : Quadratic

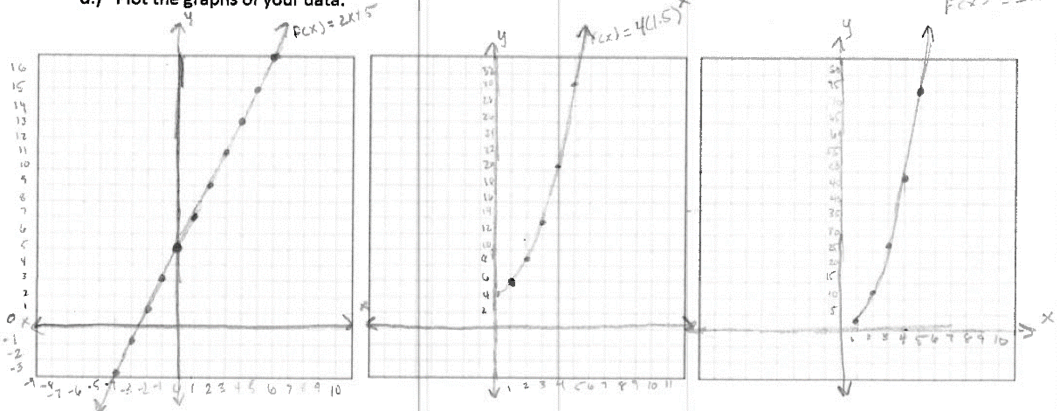
b.) Explain how you were able to identify the function.

I analyzed the difference between one output and the next.
A is a constant growth (+2)
B is a geometric sequence (1.5)
C the differences between the outputs differ by 6

c.) Find the symbolic representation of the function.

A : $f(x) = 2x + 5$
B : $f(x) = 4(1.5)^x$
C : $f(x) = 3x^2$

d.) Plot the graphs of your data.



A

x	y
1	5
2	7
3	9
4	11
5	13

B

x	y
1	6
2	9
3	13.5
4	20.25
5	30.375

C

x	y
1	3
2	12
3	27
4	48
5	75

Day 2 Classwork

Enrique is a biologist who has been monitoring the population of a rare fish in Lake Placid. He has tracked the population for 5 years and has produced the following estimates:

Year Tracked	Year Since 2002	Estimated Fish Population
2002	0	1,000
2003	1	899
2004	2	796
2005	3	691
2006	4	584

-101, -103, -105, -107
2, 2, 2
Quadratic!

Create a graph and a function to model this situation and use it to predict (assuming the trend continues) when the fish population will be gone from the Lake Placid ecosystem. Verify your results and explain the limitations of each model. y intercept : $(0, 1000)$

$f(x) = ax^2 + bx + c$

$f(x) = ax^2 + bx + 1000$

Substitute $(1, 899)$

$899 = a + b + 1000$
$-1000 \quad -1000$

$-101 = a + b$

Substitute $(2, 796)$

$796 = a(2^2) + 2b + 1000$

$796 = 4a + 2b + 1000$
$-1000 \quad -1000$

$-204 = 4a + 2b$

$-2 \times (-101 = a + b)$

$202 = -2a - 2b$
$-204 = 4a + 2b$

$\frac{-2}{2} = \frac{2a}{2}$

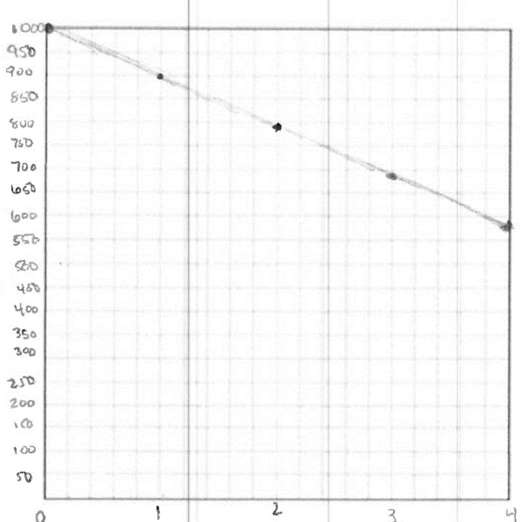
$-1 = a$

$-1 + b = -101$
$+1 \quad +1$

$b = -100$

$$\boxed{f(x) = -x^2 - 100x + 1000}$$

By extending the table we can conclude that the fish population will be gone from the lake Placid ecosystem by 2012



limitations

- The viewing window on the graph can hide key features of the function
- The results achieved by using the function must be interpreted in context to the problem.
  → Depending on the time of year / month the measurements were taken the estimatation on when the fish would disappear from lake Placid would vary.

$0 = -x^2 - 100x + 1000$

$x = \frac{100 \pm \sqrt{(-100)^2 - 4(-1)(1000)}}{2(-1)}$

$x = \frac{100 \pm \sqrt{1000 + 4000}}{-2}$

$x = \frac{100 \pm \sqrt{5000}}{-2}$

Not reasonable → $x \approx -109.16$ years after 2002

$x \approx 9.16$ years after 2002

∴ Between 2011 and 2012 the fish population will be gone.

Day 2 Group Work

Bella is a BMX bike racer and wants to identify the relationship between her bike's weight and the height of jumps (a category she gets judged on when racing). On a practice course, she tests out 7 bike models with different weights and produces the following data.

Weight (lb.)	Height of Jump (ft)
20	8.9
21	8.82
22	8.74
23	8.66
24	8.58
25	8.5
26	8.42
27	8.34

-0.08
-0.08
-0.08
-0.08
-0.08
-0.08
-0.08

a. Bella is sponsored by Twilight Bikes and must ride a 32 lb bike. What can she expect her jump height to be?

$8.9 = a_0 - 0.08(20)$
$8.9 = a_0 - 1.6$
$+1.6 \quad +1.6$
$10.5 = a_0$

$h(w) = 10.5 - 0.08w$
$h(32) = 10.5 - 0.08(32)$
$h(32) = 7.94$

Bella can expect her jump height to be 7.94 feet for a 32 lb bike

b. Bella asks the bike engineers at Twilight to make the lightest bike possible. They tell her the lightest functional bike they could make is 10 lb. Based on this data, what is the highest she should expect to jump if she only uses Twilight bikes?

$h(w) = 10.5 - 0.08w$
$h(10) = 10.5 - 0.08(10)$
$h(10) = 10.5 - 0.8$
$h(10) = 9.7$

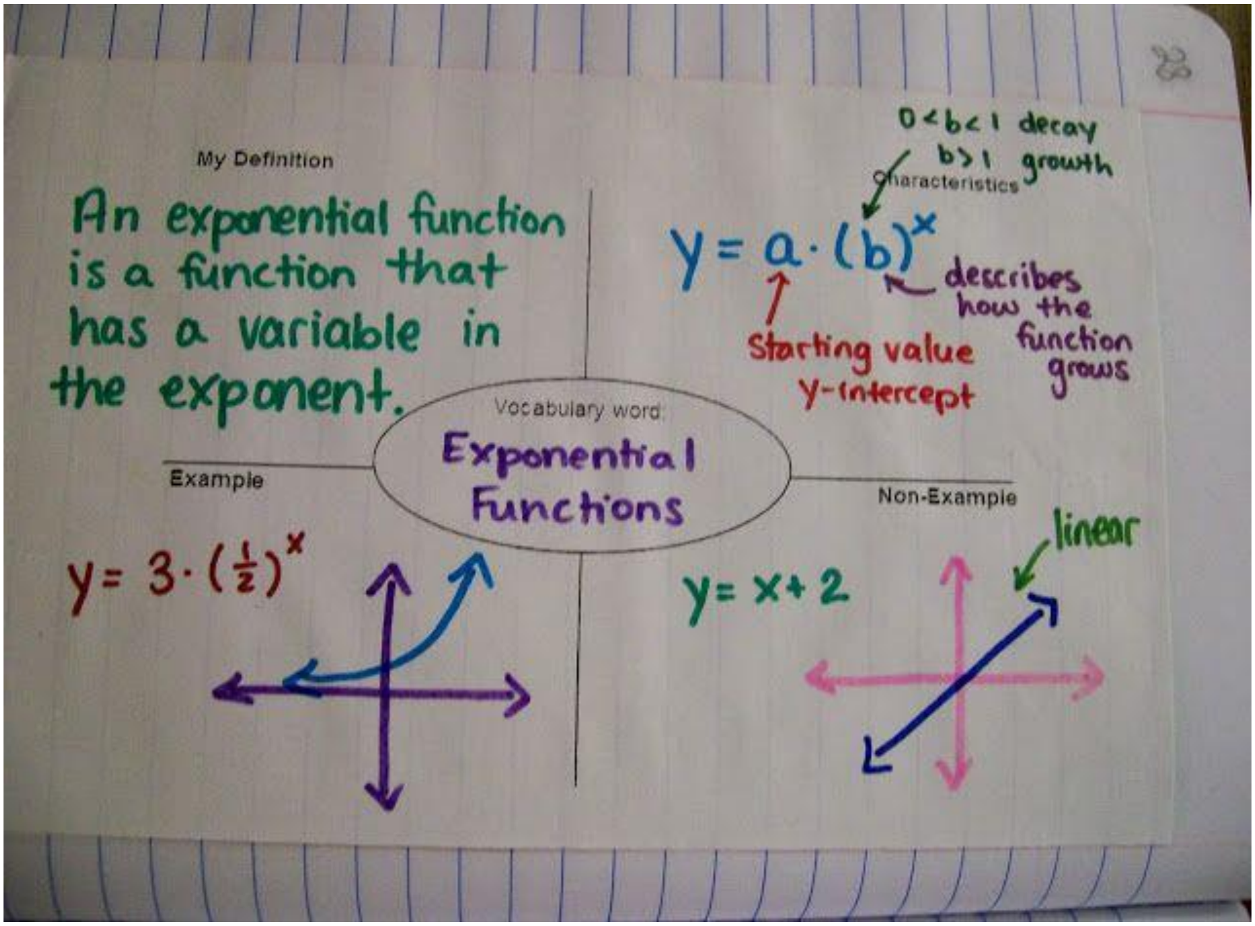
The heighest she should expect to jump if she only uses Twilight bikes is 9.7 feet.

c. What is the maximum weight of a bike if Bella's jumps must be at least 2 ft high during a race?

$h(w) = 10.5 - 0.08w$
$2 = 10.5 - 0.08w$
$106.25 = w$

The maximum bike weight that will jump at least 2 feet high during a race is 106.25 pounds

Frayer Model Example for Math Journal (Answers will vary)



Day 2 Homework

Research linear, quadratic, and exponential functions using the Internet. For each of the three types of functions, provide an example of a problem/situation you found on the Internet where that function was used to model the situation or answer the problem. Include the actual function used in the example and web page where you found the example.

(Answers will vary)

Linear Function: Example Budgeting

"When it comes to budgeting, a lot of individuals use linear equations. Likewise, many large corporations use linear equations to estimate their budgets and product costs. Budgeting with linear equations allows these businesses to provide better prices to their customers, allowing them to compete successfully. A party planner has a limited budget for an upcoming event. She will have to calculate how much it will cost her customer to hire a location and pay for meals per participant. You may write a linear equation to illustrate the total cost, expressed as y, for any number of people in attendance, or x if the rental space is $780 and food costs $9.75 per person. In this case, the linear equation would be $y = 9.75x + 780$. The party planner can use this equation to substitute any number of party participants and tell her client the total cost of the event, including food and rental costs."

From Linear Functions in Real-life Examples of Linear Systems (byjusfutureschool.com)

Quadratic Function: Example Throwing a Ball

A ball is thrown straight up, from 3 m above the ground, with a velocity of 14 m/s. When does it hit the ground?

$h = 3 + 14t - 5t^2$

From Real World Examples of Quadratic Equations (mathsisfun.com)

Exponential Function: Example miles walked per hour

"An exponential model is a model that does not increase or decrease at a steady rate; the rate of the increase or decrease will change. For example, pretend a person walked 1 mile in the first hour, 2 miles the second hour, 4 miles the third hour, and 8 miles the fourth hour. The distance the person walks is not increasing at a steady, linear rate. Instead, it is increasing at an exponential rate. The exponential model could be expressed as $y = 2^x - 1$, where x is the number of hours and y is the number of miles.

From Exponential & Linear Models | Function, Real Life Problems & Graph | Study.com

Project Introduction

As a group you will research one of the contributing factors to pollution to create a presentation and plan of action for the contributing factor you group was assigned/choose. You will be responsible for discussing how your contributing factor connects, leads to, or is related to the next contributing factor. Your finished project will be presented to the class beginning on __________.

Your presentation should include:

- Algebraic representations of the scenario, solution, impact of the solution
- Visual representations for the scenario, solution, impact of the solution
- A description of the real-world scenario your group is researching.
- Discoveries
- Contributions
- Practical solutions
- best solution to the scenario
- constraints for solution(s)
- plan of action
- cost of action
- how your plan could be implemented
- what you would have to do to implement your plan
- The impact of your plan translated in terms of the real-world scenario
- How your contributing factor connects to the next level of the pollution cycle
- A list of related questions that could not be or were not solved.

Brainstorming Worksheet *(Answers will vary)*

Part 1: Answer the following prompts. Try to provide multiple responses. In part b provide solutions to the questions you created in that section.

1.) What are questions we need to ask ourselves?

What can we do/implement to solve this issue?

How can we make our plan work?

b.) *New laws can solve this issue. We would have to talk to people in charge of making or passing laws to make our plan work*

2.) What constraints are there?

Resources are a constraint when it comes to implementing our plan

Predicting how much waste there is and how quickly it is growing, or the impact on the growth of the garbage on a daily basis can be hard to calculate

b.) *We could ask for help when we find we need resources we do not have access to. We can conduct experiments and research to determine how much each individual, state, industry, country, etc contributes to this issue.*

3.) What are possible solutions?

We could implement a law that requires proper recycling and disposal of garbage by people, businesses, and garbage companies.

We could create a program that encourages the clean-up and collection of garbage to be disposed of properly.

4.) What resources are necessary to implement a plan of action?

Government involvement would be necessary to implement some plans of action.
A local plan could be implemented easily with the help of peers and families

Additional notes/comments:

People outside of the classroom may react better to images and graphs. Images of landfills and the island of garbage may encourage people to help our cause.

Part 2: To the best of your ability answer the following questions.

14.) How would you describe the problem in your own words?

There is an abundance of garbage in the environment, on land and in the oceans, which was not disposed of properly creating a negative impact.

15.) What pieces of the problem make sense to you and which parts are confusing?

It makes sense that people litter and it impacts the environment. It is confusing that animals contribute to the carbon-foot print as well. I do not understand how the land fills have become so full and why people have not started to do something about it already.

16.) Who cares about this problem and why?

Individuals care about this problem because it is dirty. Environmentalists care about this issue because of its effect on nature.

17.) How can the problem be varied and extended?

We could ask other students to participate in the research for this project. We could try to implement our plan throughout the state, country, and world.

18.) What have others done to address this problem? Why have previous efforts to solve this problem fallen short?

Others have implemented the go green initiative, garbage companies have offered free recycling, schools encourage recycling, Reduce Reuse and Recycle moto. Other efforts have failed because individuals do not feel like they need to be held accountable for recycling. Many companies are not monitored to make sure that they are using environmentally friendly tactics.

19.) How to you plan to demonstrate that your solution is a good one?

I plan to demonstrate that my solution is a good one by proving that it is cost efficient, time-efficient, and effective.

20.) What has been the most interesting aspect of the material that you have gathered so far?

It is interesting how much garbage is out in the environment where it does not belong.

21.) After reviewing your data or sources, what do you see as the latest issues in the field of your topic?

Holding people accountable for environmentally friendly actions is an important issue in this field of topic as well as making sure that once garbage is collected that it is disposed of properly by all parties involved.

22.)What do you think are the vital facts of the matter?

Our actions whether we are aware of it or not are negatively impact the environment and animals around us. If everyone would do their part, we could stop this from getting worse. If people were encouraged to clean up the communities, they live in we could decrease the garbage that is not disposed of properly and that is floating in the ocean or blowing through the woods. By implementing a plan whether it seems big or small we could start to chip away at our negative impact until there is no longer an island made of garbage.

23.) What new insight can you contribute?

If everyone took an hour out of their week to clean up garbage they see outside and dispose of it properly we could start to decrease the animals that are hurt by our actions.

24.)What is the most meaningful action you can take in response to this problem?

I can start small and work toward the bigger picture. I plan to recycle and do my part to decrease our carbon-foot print and hopefully our environmental impact.

25.)Who is the audience? How can you best reach them?

My audience is the town officials. I feel I can best reach them through visual representations of the mathematics and the predictions of where we will be if we continue on this path. I think that by adding current images of the impact that garbage has had on the environment will motivate my audience.

26.) How can you best package your work so that others can understand it and be moved by it?

I will use images and visual representations; I will also translate my work from the mathematical language that we use inside the classroom to something more relatable and understandable to people outside of the classroom.