Exploring Quadratic Expressions through the 5E Model

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Abstract

With the recent pedagogical wave, there has been a transition towards student-centered learning methodologies within mathematics classrooms. Recognizing the evolving landscape of education and the need for personalized learning experiences, a departure from traditional teacher-centered instruction is imperative to prioritize active student engagement through inquiry-based learning and collaborative problem-solving. Empowering students through these approaches fosters deeper conceptual understanding, enhances critical thinking skills, and promotes mathematical fluency. This curriculum project presents four sequential Algebra I lessons exploring quadratic expressions using the 5E Instructional Model. This model, rooted in constructivist principles, provides a structured framework intended to address the new pedagogical wave. The 5E Model can promote a stronger foundation for student knowledge in quadratic expressions and lead to stronger conceptual understanding in Algebra II. Answer keys for all student materials are provided in the appendix.

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Introduction

The shift to student-centered learning in mathematics classrooms holds promise for addressing the diverse needs and learning styles of students while also promoting equity and inclusivity. The struggle lies within the shift from teacher-centered instruction to student-centered learning. It can be daunting having the responsibility of learning placed more on the students. The 5E instructional model allows for a controlled way that students take charge of their learning, which can be helpful for teachers transitioning to this pedagogical approach. The following curriculum project explores quadratic expressions at the Algebra I level through the 5E Model. The four lessons include:

- Lesson 1 – Creating equivalent expressions to represent a situation.
- Lesson 2 – Creating expanded/factored binomials to represent a situation.
- Lesson 3 – Expanding factored trinomials to represent a situation.
- Lesson 4 – Factoring expanded trinomials to create equivalent expressions.

Student-centered learning lends well to collaborative learning, characterized by group work and cooperative problem-solving. Through peer interactions, students can develop a deeper understanding of concepts by sharing and hearing from different perspectives, as well as promoting peer teaching. Collaborative learning refines communication skills, deepens conceptual understanding, and fosters a sense of ownership and responsibility, empowering learners to actively participate in their own academic progress.

Literature Review

Understanding, Fluency, and Reasoning

Mathematics education presents learning and problem-solving processes through the lens of conceptual understanding, procedural fluency, and mathematical reasoning. Conceptual understanding, or the ability to comprehend the principles of mathematics and recognize the relationships between concepts, is essential. Likewise, procedural fluency, or the ability to execute processes accurately and with precision, is equally essential. Mathematical reasoning and critical thinking skills is generated through both conceptual understanding and procedural fluency. Conceptual understanding and procedural fluency are positively correlated and work in tandem rather than independently (Star, 2013)

To develop conceptual understanding, teachers need to promote mathematical awareness, independence, and personally constructed ideas through effective instruction (D'Ambrosio, 1995; Manouchehri & Enderson, 1999). Such instruction includes a focus on mathematical discourse and ensures that teachers use practices where students use inquiry and reasoning to justify their thinking (McClain et al., 2000). To foster collaboration that supports mathematical discourse, it is important to create appropriate tasks for students to participate in conversations that lead to meaningful thinking and reasoning. Kilic et al., (2010) stated teachers should keep in mind the strategies that require student explanations of their thinking and that encourage listening to their peers during problem-solving. In conjunction with the 5E Instructional Model, such instruction can deepen students' conceptual understanding in the mathematics classroom.

The 5E Model

The 5E Instructional Model emerged as a pedagogical framework that offers a structured yet flexible approach that can create unique learning experiences to promote active student learning and lead to conceptual understanding and critical mathematical thinking skills. The 5E Instructional Model has five stages: instruction Engagement, Exploration, Explanation, Elaboration, and Evaluation. The Engagement stage introduces the instructional task to students as teachers assess their previous knowledge to make connections to the current task (Omotayo, 2017, p. 18). In Lesson 1 of this curriculum project, students are introduced to the pool tile problem and are asked to engage in determining how many tiles are around the border of a ten-by-ten pool. This is an open-ended task that has a low barrier of entry, making it accessible for all students. It is designed to foster collaboration and the sharing of ideas and provides the opportunity for teachers to practice implementing tasks that promote mathematical discourse. Students should be asked to recall certain mathematical properties, and connections can be made between how those properties can be used to create their equivalent expressions to represent the pool tile situation. This real-world problem can be effectively implemented with the 5E instructional model (Omotayo, 2017, p. 18).

In the next stage, Exploration, students are provided with opportunities to apply prior knowledge to generate new ideas/concepts through the curriculum materials (Omotayo, 2017). This stage is reflected in the first section of Lesson 2 where students are asked to create two equivalent expressions for the combined area of a swimming pool with two sections. They may be able to generate the expanded form expression and not the other, but by allowing them to collaborate with peers and providing them with the definition of a factored form expression, they may be able to expand their thinking and generate other expressions.

Explanation, the third stage, provides the opportunity for students to demonstrate their conceptual understanding in Lesson 2 while teachers address misconceptions to facilitate deeper understanding (Omotayo, 2017). After students discuss and solidify how the factored and expanded forms can be extrapolated from the swimming pool area, they are then asked to create a diagram to represent a given expression.

Lesson 3 will connect prior learning and represent the Elaboration Stage of the 5E Instructional Model. In this lesson, students are asked to consider how the area model would change if both dimensions were changed to create a four-section swimming pool. By making connections and constructing and reconstructing information (Llewellyn, 2007), Lesson 4 challenges their thinking by using the swimming pool area model as a method for factoring quadratic expressions. Lesson 4 represents the culmination of their learning so students complete a formative assessment (Ticket out the Door) where they must factor quadratic expressions.

Each lesson below details which part of the 5E model is being applied.

LESSON 1 PLAN

Grade: 9	Subject: Algebra I	Date:

Topic:

Quadratic Functions visualized through Pool Border Problem

Lesson 1: Engagement

Learning Objectives:

Students will be able to recognize patterns in a context and create an expression(s) to represent it.

Materials Needed:

- Pre-Investigation
- Whiteboards
- Dry Erase Markers/Erasers

NYS Math Standard:

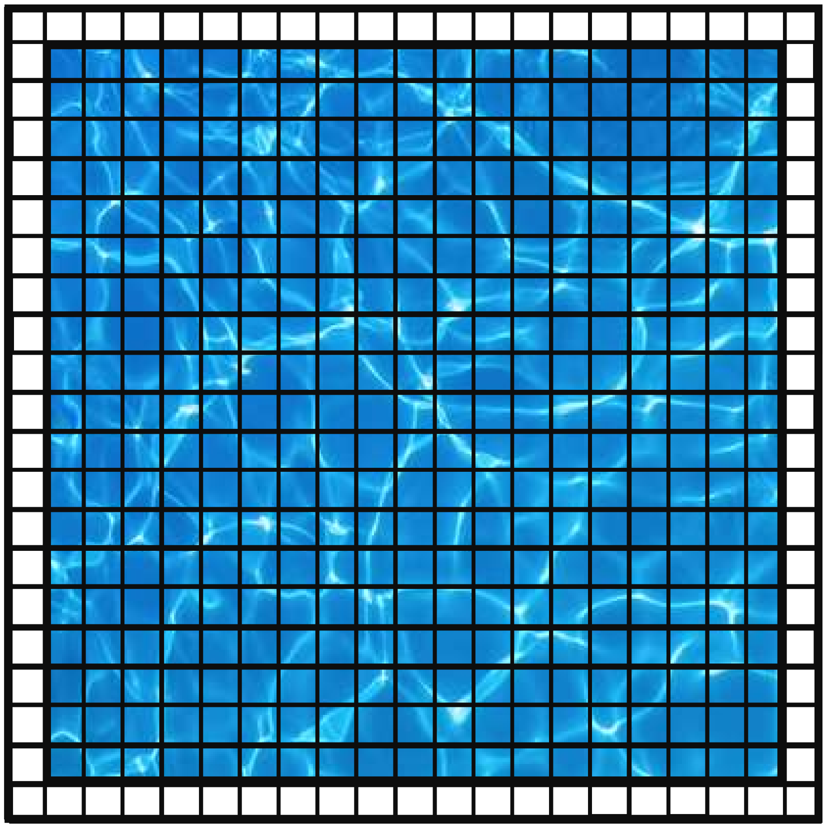
AI-A.CED: Create equations that describe numbers or relationships

Structure/Activity:

Students will work in their table groups. Each group will receive Pre-Investigation packet, dry-erase markers, and one large whiteboard. The scribe of the group will write the group's answers to the questions on the whiteboards. The teacher will observe and facilitate discussions with the groups. Since each member of the group can have different perspectives on the pattern, it is important that students be allowed the opportunity to share their perspectives and have it shown on the whiteboards. After all groups finish their questions, they will do a gallery walk to look at other groups' whiteboards. They will make notice of which groups had different expressions than their own, and we will discuss as a class how they derived those expressions.

Pre-Investigation

Notice the swimming pool below.



The swimming pool is tiled with the same sized squares on the inside and around the border of the pool.

How many tiles are around the border of a 10 x 10 square pool?

How many tiles are around the border of a 5 x 5 square pool?

Around a 3 x 3 square pool?

Suppose there is a square pool with dimensions n x n. How many tiles would be around the border of said pool?

In the pool border problem, you produced different but equivalent expressions to represent the area. You can confirm that the expressions are equivalent by using properties of numbers and operations.

An important property is the ***Distributive Property:***

For any real numbers, a, b, and c:

$$\boldsymbol{a(b+c)=ab+ac} \;\; \textbf{\textit{and}} \;\; \boldsymbol{a(b-c)=ab-ac}$$

For example, this property guarantees that $4(x+2)=4x+8$ for any x.

We say that $4(x+2)$ is in ***factored form*** and that $4x+8$ is in ***expanded form.***

1. How is the Distributive Property illustrated in the expressions?

Along with the Distributive Property, the ***Commutative Property*** is also useful for writing equivalent expressions. The Commutative Property states that for any real numbers a and b:

$$\boldsymbol{a+b=b+a}$$

LESSON 2 PLAN

Grade: 9

Subject: Algebra I

Date:

Topic:

Quadratic Functions visualized through pool tile areas

Lesson 2: Exploration and Explanation

Learning Objectives:

Students will be able to recognize patterns in a context and create an expression(s) to represent it.

Materials Needed:

- Investigation 1
- Algebra Tiles

NYS Math Standard:

AI-F.IF.8: Write a function in different but equivalent forms to reveal and explain different properties of the function

Structure/Activity:

Students will work in their table groups. Each group will receive Investigation 1 packets, as well as a box of Algebra Tiles.

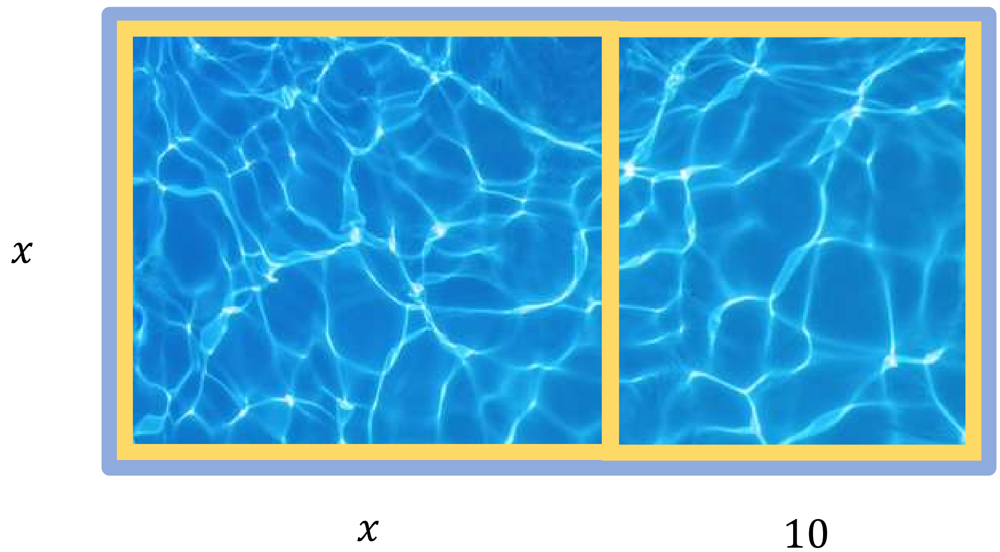
The teacher will instruct the first page of Investigation 1, introducing the vocabulary terms. Students will observe and note their observations about the rectangles on the first page and will create expressions to represent the areas. The teacher should guide students to discovering the factored forms. After 3 minutes of group discussion, the teacher should ask students to share their expressions and compare to other groups. Then the teacher will summarize the class findings by denoting which expression is in factored form and which is in expanded form.

Students will then work in their groups to complete the investigation. The teacher will supervise and troubleshoot groups when they need help.

Investigation 1: Expanding

Recall the pool border problem. Ignoring the border, it is still necessary for those building the pool to know the number of tiles needed to tile the bottom. Imagine we begin with a square pool with a length of x tiles, and we want to build upon that pool with a separate section for diving with a length of ten tiles.

Below is a diagram of a pool with swimming and diving sections outlined in yellow.



Write two expressions for the total amount of tiles for the pool. How do you know that the expressions are equivalent?

Vocabulary:

The expression $n(n-4)$ is in ***factored form*** because it is written as a *product of factors*.

The expression $n^2 - 4n$ is in ***expanded form*** because it is written as the *sum or difference of terms*.

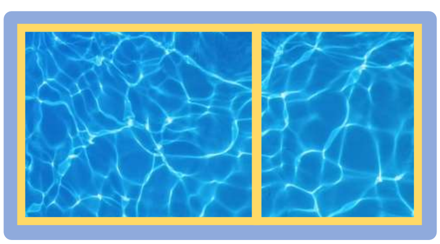
> A ***term*** is an expression that consists of variables and/or numbers multiplied together.

Of the two expressions you created for the problem on the previous page, was one of them in factored form? Expanded form?

Factored: ______________ **Expanded:** _______________

1. Each diagram below shows a large pool divided into two sections – diving and swimming. Write two expressions, ***factored form,*** and ***expanded form***, for the total number of tiles for the pools.

x

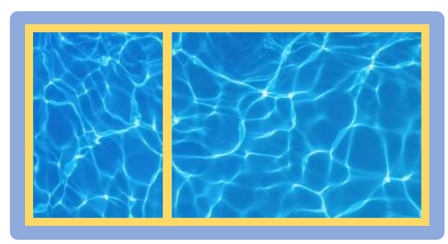


x 6

Factored: ______________

Expanded: _____________

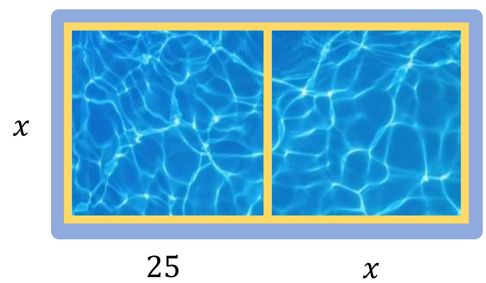
a

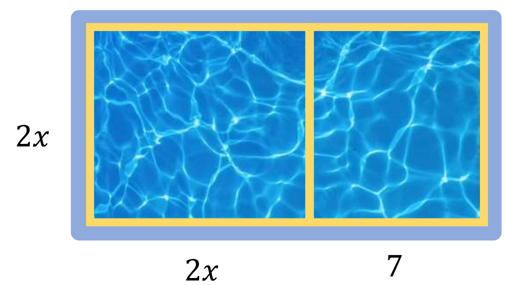


b c

Factored: ______________

Expanded: _____________





Factored: ______________

Expanded: _____________

Factored: ______________

Expanded: _____________

a. What do the factors in the factored form represent about the pool?

b. What do the terms in the expanded form represent about the pool?

2. Complete the following steps for each of the factored expressions below.
    - Using the algebra tiles, create a pool whose area is represented by the expression.
    - Draw the pool. Label the dimensions and the area of each section.
    - Write an equivalent expression in ***expanded form.***

a. $x(x + 4)$

<u>Pool diagram</u>

<u>Equivalent Expression</u>

b. $x(x + 2)$

<u>Pool Diagram</u>

<u>Equivalent Expression</u>

c. $x(5 + 2)$

<u>Pool Diagram</u>

<u>Equivalent Expression</u>

When an expanded form of a quadratic is specifically written in the form,

$$ax^2 + bx + c$$

That is, in the form where the x^2 term comes first, followed by the x term, followed by the constant, the expression is said to be in ***standard form.***

3. Go back and rewrite all the expanded forms from #2, so that they are in standard form.

LESSON 3 PLAN

Grade: 9	Subject: Algebra I	Date:

Topic:

Expanding Quadratic Functions through pool tile areas

Lesson 3: Elaboration

Learning Objectives:

Students will be able to recognize patterns in areas of rectangles and create an equivalent expression to represent them.

Materials Needed:

- Investigation 2
- Algebra Tiles

NYS Math Standard:

AI-F.IF.8: Write a function in different but equivalent forms to reveal and explain different properties of the function

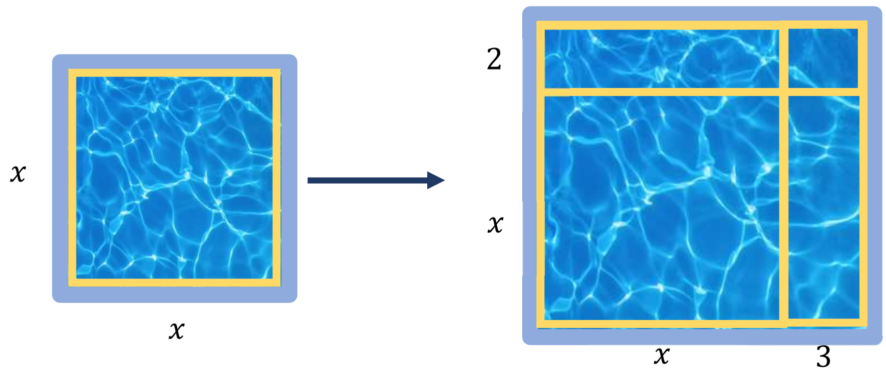
Structure/Activity:

Students will work in their table groups. Each group will receive Investigation 2 packets, as well as a box of Algebra Tiles.

The teacher will instruct the first page of Investigation 2. Students will observe and note their observations about the rectangles on the first page and will create expressions to represent the areas. Students will then work in their groups to complete the investigation. The teacher will supervise and troubleshoot groups when they need help.

Investigation 2: Changing Both Dimensions

Suppose that we begin with a square swimming pool with a dimension of x tiles. One dimensions of the pool will be increased by 3 tiles to create a diving section, and the other dimension will be increased by 2 tiles to create a children's section and a jacuzzi section.

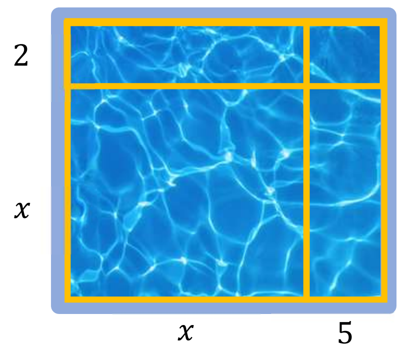


- How do the areas (number of tiles) of the square pool and the new rectangular pool compare?

- How can you represent the area of the new rectangular pool?

1. Each pool has been subdivided into four smaller sections. Write two expressions for the total area of the pool. One expression should be in ***factored form***, and one should be in ***expanded form***.

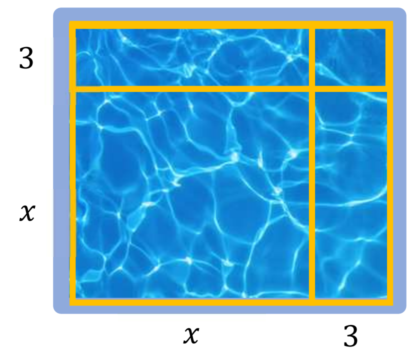
a.



Factored: _______________

Expanded: ______________

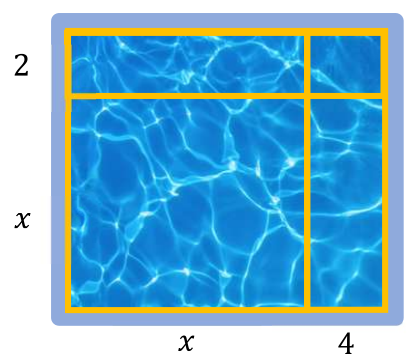
b.



Factored: _____________

Expanded: ___________

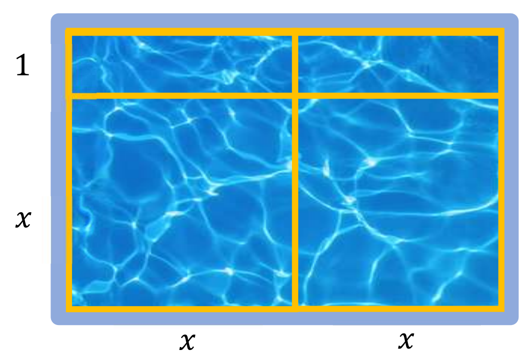
c.



Factored: _______________

Expanded: ______________

d.



Factored: _____________

Expanded: ___________

2. Rewrite the expanded forms in standard form where necessary.

3. a. What information do the factors in each factored form provide about the pool? Use an example to justify your thinking.

b. What information do the terms in each expanded form provide about the pool? Use an example to justify your thinking.

4. A pool has sides lengths of x tiles. One dimension is doubled and then increased by two tiles. The other dimension is increased by three tiles.

a. Use the Algebra Tiles to create a new pool.

b. Make a sketch of the new pool. Label the dimensions and the area of each section.

c. Write two expressions for the area of the new pool, one in ***factored form*** and the other in ***standard form***.

5. Use the pool model to help write each expression in equivalent *standard form.*

a. $(x + 3)(x + 5)$ b. $3x(x + 2)$ c. $(2x + 1)(x + 4)$

LESSON 4 PLAN

Grade: 9

Subject: Algebra I

Date:

Topic:

Factoring Quadratic Functions through pool tile areas

Lesson 4: Evaluation

Learning Objectives:

Students will be able to recognize patterns in a context and create an expression(s) to represent it.

Materials Needed:

- Investigation 3
- Algebra Tiles

NYS Math Standard:

AI-F.IF.8: Write a function in different but equivalent forms to reveal and explain different properties of the function

Structure/Activity:

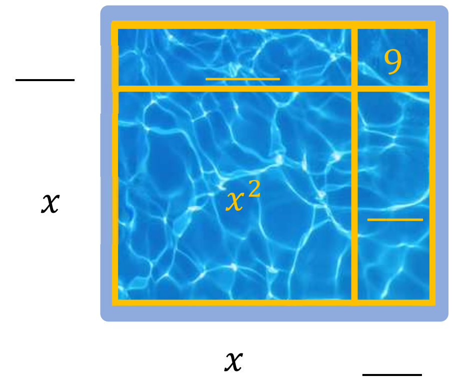
Students will work in their table groups. Each group will receive Investigation 3 packets, as well as a box of Algebra Tiles.

The teacher will introduce the first page of Investigation 3. Students will observe and note their observations about the rectangles on the first page and will create expressions to represent the areas. The students will then share their expressions with a partner and compare. After 3 minutes of partner discussion, the teacher should ask students to share their expressions and compare to other groups.

Students will then work in their groups to complete the investigation. The teacher will supervise and troubleshoot groups when they need help.

Investigation 3: Factoring

1. In the diagram to the right, fill in each blank with an appropriate length or area.



2. Write two expressions, one in ***factored*** and one in ***standard form***, for the area of the pool.

Factored: ________________________

Standard: _______________________

3. Consider the expression:

$$x^2 + bx + 8$$

   a) What are some values for b that you can obtain from factoring? Write the expression in factored form.

b) Compare your work to your group members. Were there other solutions that you did not have?

4. For the following questions, find values of r and s that make the equations true.

   a) $x^2 + 10x + 24 = (x + 6)(x + r)$

   b) $x^2 + 11x + 24 = (x + s)(x + r)$

   c) $x^2 + 25x + 24 = (x + r)(x + s)$

   d) Describe the strategies you used to factor the expressions in parts (a)-(c).

Tickets Out of the Door

5. Factor each expression using the methods from Question 4.

a) $x^2 + 5x + 2x + 10$

b) $x^2 + 11x + 10$

c) $x^2 + 3x - 10$

d) $x^2 + 16x + 15$

e) $x^2 - 8x + 15$

f) $x^2 - 12x + 36$

Classroom Implementation

For the 5E Instructional Model to effectively deepen conceptual understanding, students must buy into the Engagement stage. Students must have a safe learning environment in which they are comfortable taking risks. This requires students to provide explanations behind their reasoning, offer feedback about their thinking without revealing answers, share perspectives and critique others' reasoning, and work with familiar contexts to think conceptually (Kilic et al., 2010). These lessons allow students to generate multiple equivalent expressions; it is thus crucial that students are not told if their expressions are correct. The idea of equivalence should be revealed when all groups share their expressions and demonstrate that each expression generates equivalent outputs. Lesson 2 begins with the swimming pool area context, which should be modeled similarly to Lesson 1. Most students will be able to generate the expanded form expression for the situation, but it may require teacher facilitation for students to generate the factored form expression. A helpful line of questioning for this may be, "What do the terms in your expression represent in this situation?" at which students may explain that the two terms represent the two different sections of the swimming pool. Building off their thinking, the teacher may ask, "Is there a way to find the area of the swimming pool as a whole rather than the two separate sections?" Having students ground the expressions in the context of the area will develop their conceptual understanding, as they should have strong previous knowledge of areas before the lesson. Developing their conceptual understanding in the first two lessons will pay off in the subsequent lessons and will allow you to effectively facilitate learning.

Conclusion

The 5E Instructional Model is a helpful framework in building student-driven classrooms that promote collaboration and autonomy. This shift may present a challenge to teachers who were informed in the traditional instructional methods. Teachers who have expressed difficulties with implementing this model had to learn to embrace the student-centered approach or abandon the 5E Model. Students must drive their own learning to develop deep conceptual understanding, and teachers must be facilitators in that process. As Turan and Matteson (2021) recall, regarding teachers who found difficulties with the model, “the teachers were facilitators in the exploration stage, but in the explanation stage, we only heard the teachers’ voices, not those of the students” (2021). It is thus key for teachers to hear student voices through all the 5E Model phases. It is hoped that teachers who access this curriculum project find it beneficial for themselves and can use the materials in their own classrooms.

References

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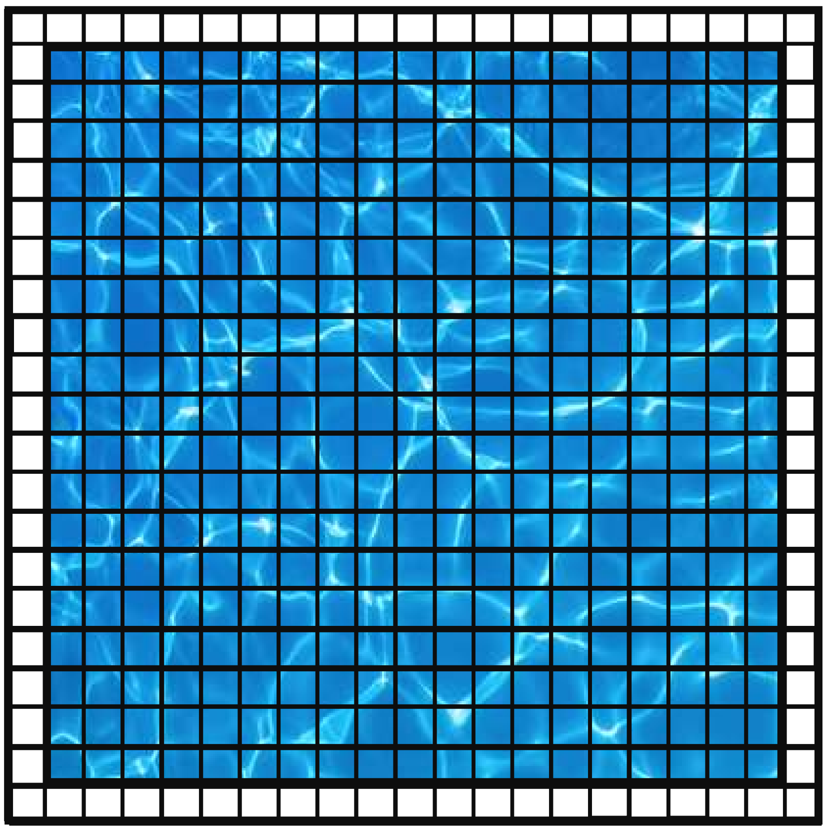
Pre-Investigation

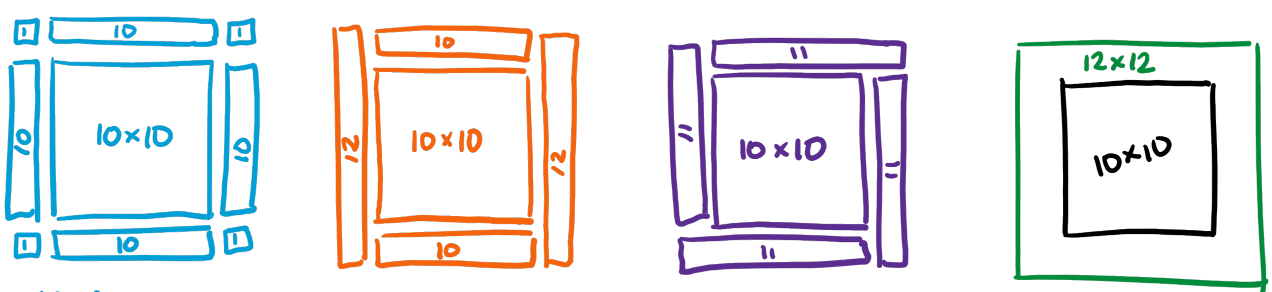
Appendix

Notice the swimming pool below.



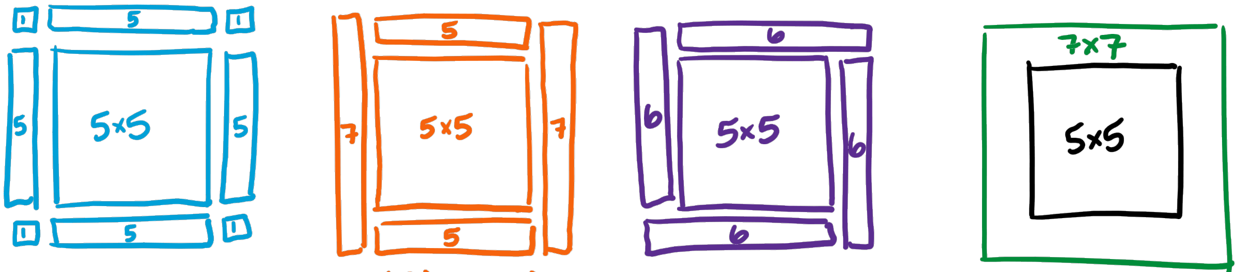
The swimming pool is tiled with the same sized squares on the inside and around the border of the pool.

How many tiles are around the border of a 10 x 10 square pool?



4(10) + 4(1)	2(10) + 2(12)	4(11)	12(12) − 10(10)
40 + 4	20 + 24	44	144 − 100
44	44		44

How many tiles are around the border of a 5 x 5 square pool?



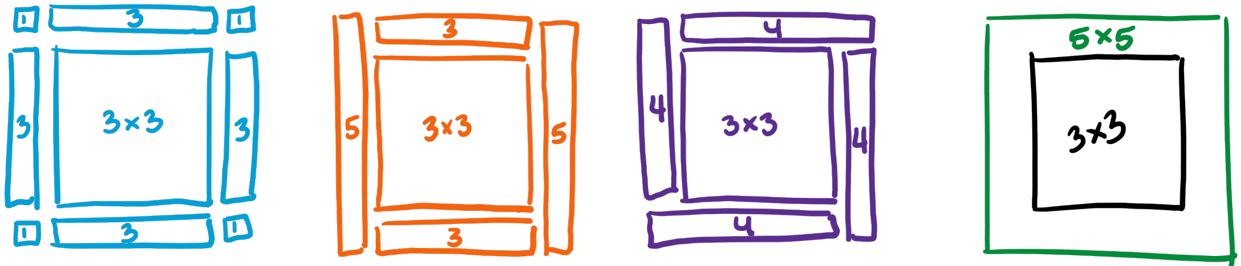
$4(5)+4(1)$
$20+4$
24

$2(5)+2(7)$
$10+14$
24

$4(6)$
24

$7(7)-5(5)$
$49-25$
24

Around a 3 x 3 square pool?



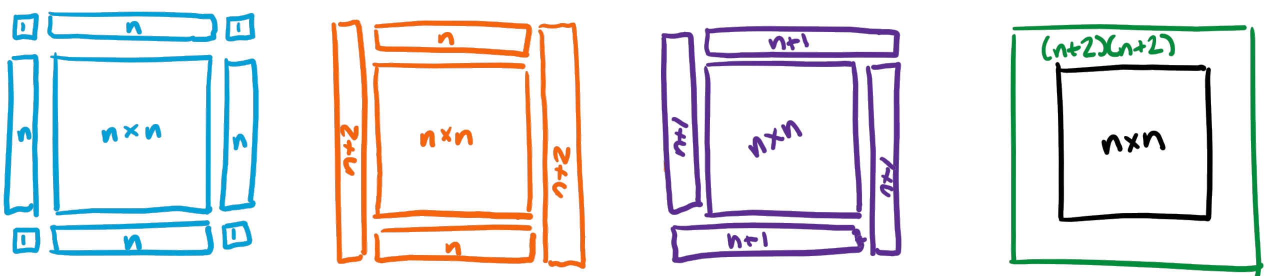
$4(3)+4(1)$
$12+4$
16

$2(3)+2(5)$
$6+10$
16

$4(4)$
16

$5(5)-3(3)$
$25-9$
16

Suppose there is a square pool with dimensions n x n. How many tiles would be around the border of said pool?



$4(n)+4(1)$

$2(n)+2(n+2)$

$4(n+1)$

$(n+2)(n+2)-n^2$

In the pool border problem, you produced different but equivalent expressions to represent the area. You can confirm that the expressions are equivalent by using properties of numbers and operations.

An important property is the ***Distributive Property:***

For any real numbers, a, b, and c:

$$\boldsymbol{a(b+c) = ab + ac} \text{ and } \boldsymbol{a(b-c) = ab - ac}$$

For example, this property guarantees that $4(x+2) = 4x + 8$ for any x.

We say that $4(x+2)$ is in ***factored form*** and that $4x + 8$ is in ***expanded form.***

2. How is the Distributive Property illustrated in the expressions?

$2(n) + 2(n+2)$
$2n + 2n + 4$
$4n + 4$

$4(n+1)$
$4n + 4$

Along with the Distributive Property, the ***Commutative Property*** is also useful for writing equivalent expressions. The Commutative Property states that for any real numbers a and b:

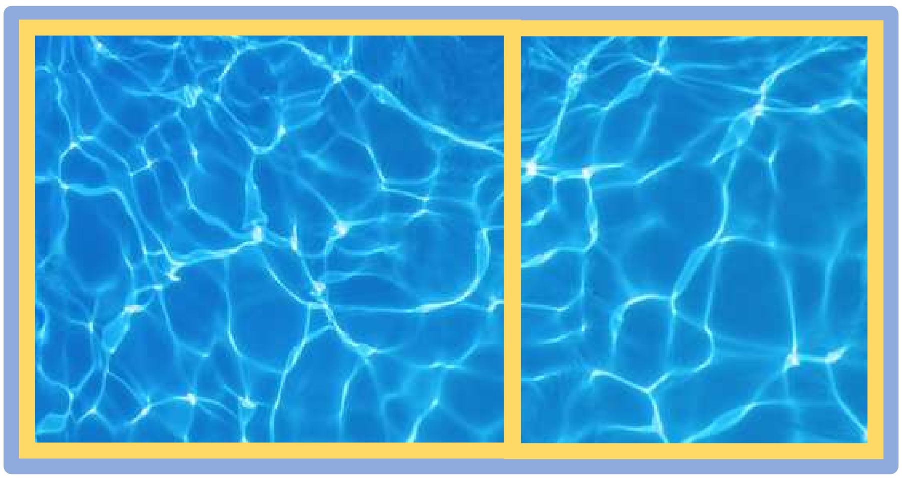
$$\boldsymbol{a + b = b + a}$$

Investigation 1: Expanding

Recall the pool border problem. Ignoring the border, it is still necessary for those building the pool to know the number of tiles needed to tile the bottom. Imagine we begin with a square pool with a length of x tiles, and we want to build upon that pool with a separate section for diving with a length of ten tiles.

Below is a diagram of a pool with swimming and diving sections outlined in yellow.

x



x 10

Write two expressions for the total amount of tiles for the pool. How do you know that the expressions are equivalent?

Vocabulary:

The expression $n(n-4)$ is in ***factored form*** because it is written as a *product of factors*.

The expression $n^2 - 4n$ is in ***expanded form*** because it is written as the *sum or difference of terms*.

> A ***term*** is an expression that consists of variables and/or numbers multiplied together.

Of the two expressions you created for the problem on the previous page, was one of them in factored form? Expanded form?

Factored: $x(x+10)$ **Expanded:** x^2+10x

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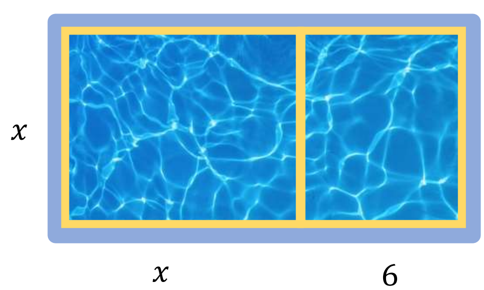
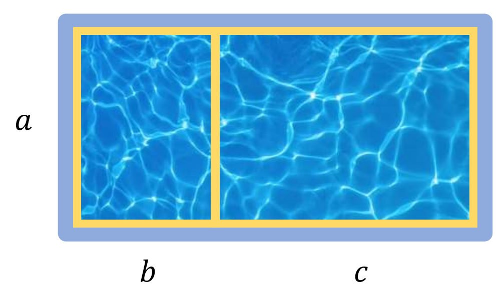
2. Each diagram below shows a large pool divided into two sections – diving and swimming. Write two expressions, ***factored form,*** and ***expanded form***, for the total number of tiles for the pools.

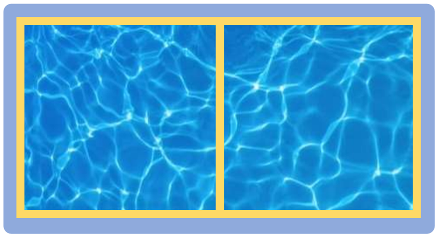




Factored: $x(x+6)$ **Factored:** $a(b+c)$

Expanded: x^2+6x **Expanded:** $ab+ac$

x

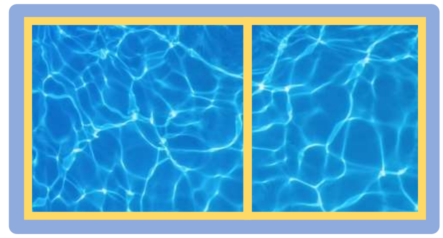


25 $\qquad$ x

Factored: $x(25+x)$

Expanded: $25x + x^2$

$2x$



$2x$ $\qquad$ 7

Factored: $2x(2x+7)$

Expanded: $4x^2 + 14x$

c. What do the factors in the factored form represent about the pool?

the factors represent the dimensions of the total pool

$x(x+6)$

length × width

d. What do the terms in the expanded form represent about the pool?

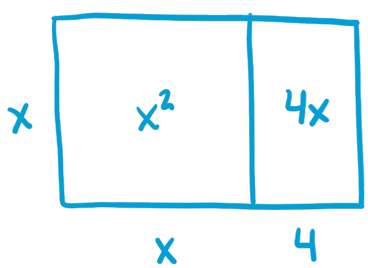
the terms represent the areas of the smaller pool sections

2. Complete the following steps for each of the factored expressions below.

- Using the algebra tiles, create a pool whose area is represented by the expression.
- Draw the pool. Label the dimensions and the area of each section.
- Write an equivalent expression in ***expanded form.***

d. $x(x + 4)$

<u>Pool diagram</u>

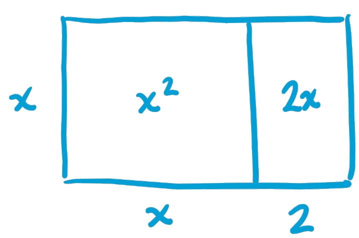


<u>Equivalent Expression</u>

x^2+4x

e. $x(x + 2)$

<u>Pool Diagram</u>

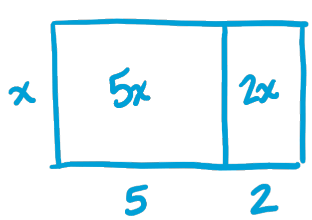


<u>Equivalent Expression</u>

x^2+2x

f. $x(5 + 2)$

<u>Pool Diagram</u>



<u>Equivalent Expression</u>

$5x+2x$

When an expanded form of a quadratic is specifically written in the form,

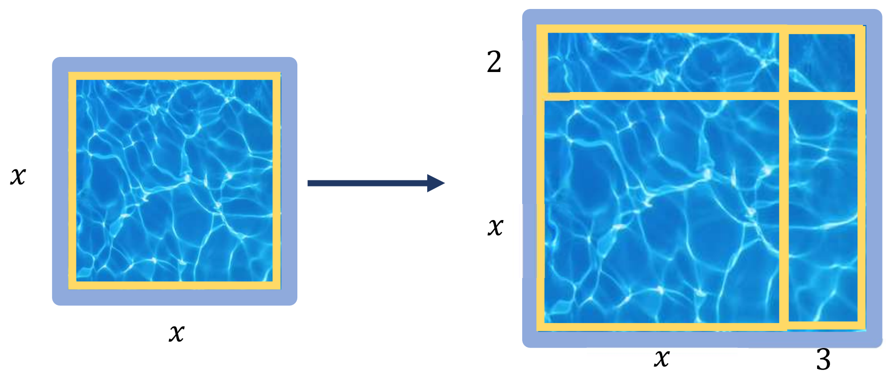
$$ax^2 + bx + c$$

That is, in the form where the x^2 term comes first, followed by the x term, followed by the constant, the expression is said to be in ***standard form.***

3. Go back and rewrite all the expanded forms from #2, so that they are in standard form.

Investigation 2: Changing Both Dimensions

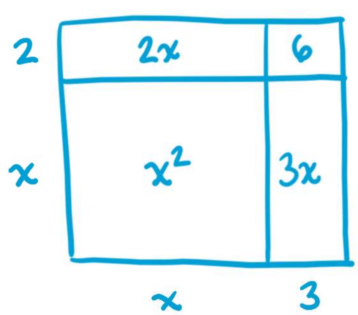
Suppose that we begin with a square swimming pool with a dimension of x tiles. One dimensions of the pool will be increased by 3 tiles to create a diving section, and the other dimension will be increased by 2 tiles to create a children's section and a jacuzzi section.



- How do the areas (number of tiles) of the square pool and the new rectangular pool compare?

area is larger, with 3 new areas added

- How can you represent the area of the new rectangular pool?



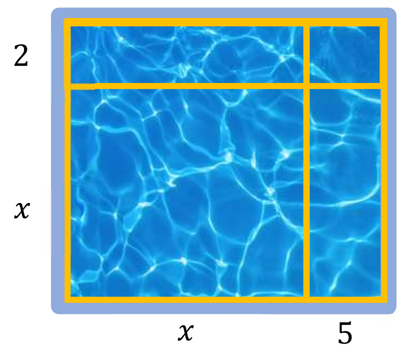
$x^2+2x+3x+6$

x^2+5x+6 ← expanded form

$(x+2)(x+3)$ ← factored form

1. Each pool has been subdivided into four smaller sections. Write two expressions for the total area of the pool. One expression should be in ***factored form***, and one should be in ***expanded form***.

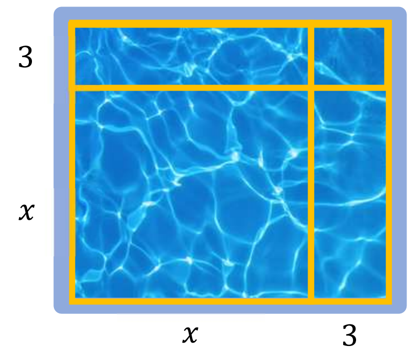
a.



Factored: $(x+2)(x+5)$

Expanded: $x^2+7x+10$

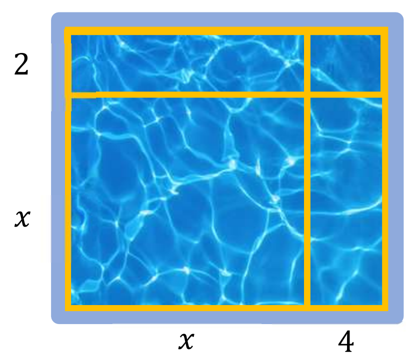
b.



Factored: $(x+3)(x+3)$

Expanded: x^2+6x+9

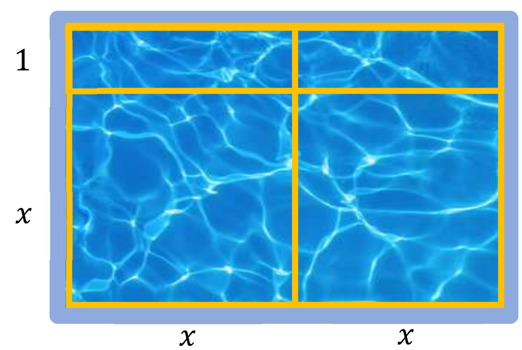
c.



Factored: $(x+2)(x+4)$

Expanded: x^2+6x+8

d.



Factored: $(x+1)(x+x)$

Expanded: $2x^2+2x$

2. Rewrite the expanded forms in standard form where necessary.

3. a. What information do the factors in each factored form provide about the pool? Use an example to justify your thinking.

factors represent dimensions of total pool

$$\overbrace{(x+2)}^{l} \cdot \overbrace{(x+4)}^{w}$$

b. What information do the terms in each expanded form provide about the pool? Use an example to justify your thinking.

terms represent areas of smaller pool sections

4. A pool has sides lengths of x tiles. One dimension is doubled and then increased by two tiles. The other dimension is increased by three tiles.

d. Use the Algebra Tiles to create a new pool.

e. Make a sketch of the new pool. Label the dimensions and the area of each section.

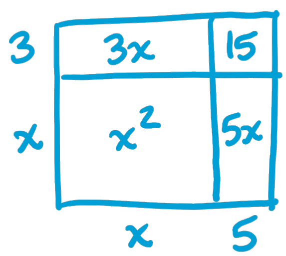
3	$3x$	$3x$	6
x	x^2	x^2	$2x$
	x	x	2

f. Write two expressions for the area of the new pool, one in ***factored form*** and the other in ***standard form***.

factored : $(2x+2)(x+3)$ standard : $2x^2+8x+6$

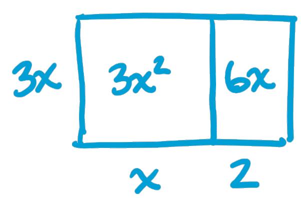
5. Use the pool model to help write each expression in equivalent *standard form*.

a. $(x+3)(x+5)$



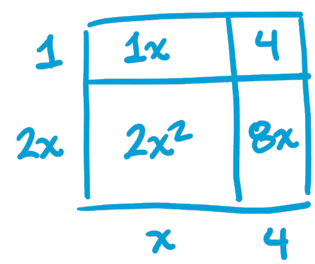
$x^2+8x+15$

b. $3x(x+2)$

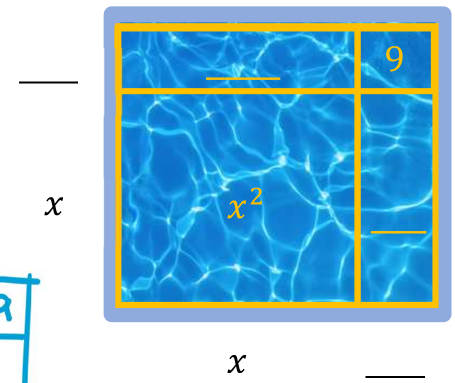


$3x^2+6x$

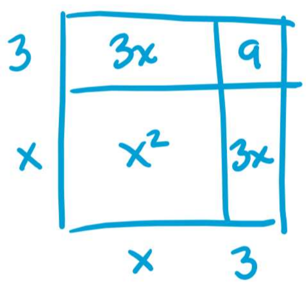
c. $(2x+1)(x+4)$

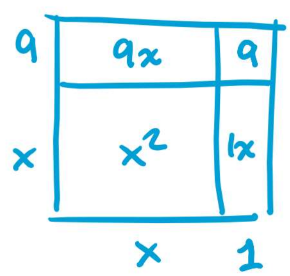


$2x^2+9x+4$



6. In the diagram to the right, fill in each blank with an appropriate length or area.





7. Write two expressions, one in ***factored*** and one in ***standard form***, for the area of the pool.

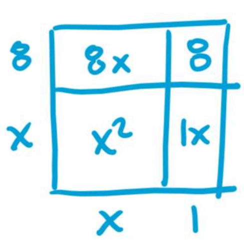
Factored: $(x+3)(x+3)$ OR $(x+9)(x+1)$

Standard: x^2+6x+9 OR $x^2+10x+9$

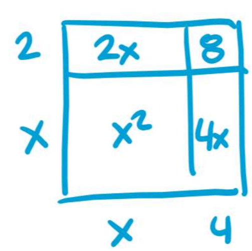
8. Consider the expression:

$$x^2 + bx + 8$$

c) What are some values for b that you can obtain from factoring? Write the expression in factored form.



OR



x^2+9x+8

$b=9$

x^2+6x+8

$b=6$

d) Compare your work to your group members. Were there other solutions that you did not have?

9. For the following questions, find values of r and s that make the equations true.

e) $x^2 + 10x + 24 = (x + 6)(x + r)$

6	6x	24
x	x^2	4x
	x	4

$(x+6)(x+4)$

$\boxed{r = 4}$

24
1 24
2 12
3 8
4 6

f) $x^2 + 11x + 24 = (x + s)(x + r)$

3	3x	24
x	x^2	8x
	x	8

$(x+3)(x+8)$

$\boxed{s = 3}$ $\boxed{r = 8}$

g) $x^2 + 25x + 24 = (x + r)(x + s)$

1	1x	24
x	x^2	24x
	x	24

$(x+1)(x+24)$

$\boxed{r = 1}$ $\boxed{s = 24}$

h) Describe the strategies you used to factor the expressions in parts (a)-(c).

two values that add to b value and multiply to c value

Tickets Out of the Door

10. Factor each expression using the methods from Question 4

g) $x^2 + 5x + 2x + 10$

$x^2+7x+10$

5	5x	10
x	x^2	2x
	x	2

$(x+5)(x+2)$

h) $x^2 + 11x + 10$

10	10x	10
x	x^2	1x
	x	1

$(x+10)(x+1)$

i) $x^2 + 3x - 10$

-10

1 -10
2 -5
5 -2
10 -1

5	5x	-10
x	x^2	-2x
	x	-2

$(x+5)(x-2)$

j) $x^2 + 16x + 15$

15	15x	15
x	x^2	1x
	x	1

$(x+15)(x+1)$

k) $x^2 - 8x + 15$

15

1 15
3 5
-1 -15
-3 -5

-3	-3x	15
x	x^2	-5x
	x	-5

$(x-3)(x-5)$

l) $x^2 - 12x + 36$

-6	-6x	36
x	x^2	-6x
	x	-6

$(x-6)(x-6)$