Blended Learning in the Geometry Classroom

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Abstract

This curriculum project contains four lessons aligned with New York State Common Core Geometry Standards designed with the goal to increase student engagement. The lessons were constructed by using the blended learning station rotation model. This consists of four stations: mini-lesson, independent practice, technology, and a hands-on activity. Each station serves a different purpose in student learning and caters to all learning styles.

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Introduction

Teachers are often faced with how to engage students in the mathematics classroom. Traditional instruction of notetaking with independent practice does not meet the needs of all learners. Blended learning can be used to engage students and it allows for group instruction, independent practice, technology, and hands-on learning experiences. The blended learning station rotation model provides a different activity either completed individually, in groups, or with a teacher (Ayob et al., 2020). Since students are constantly rotating what they are working on, students stay engaged throughout the entire lesson. According to the National Council of Mathematics, technology plays a special role in mathematics education and can be used to deepen understanding (Bos, 2009). In many New York schools, students have one-to-one devices, so technology can and must be incorporated in a meaningful way.

This curriculum project was developed using blended learning materials that align with Common Core Geometry Standards for teachers to use in their own classroom. The blended learning station rotation model allows for active student engagement and individualized instruction to meet the needs of all students. The four stations are small group instruction, independent practice, technology, and hands-on learning. Each station has a different activity designed to target a specific learning objective. Each teacher who uses this curriculum will need to determine the most beneficial way to group students.

The four specific Geometry topics the lessons cover are reflections, centroids and orthocenters, trigonometry, and three-dimensional shapes. The mini-lesson station is used to teach the more difficult topics within these lessons that students may need more assistance with. The technology stations use either Delta Math or GeoGebra where students can either receive immediate feedback or manipulate shapes. Finally, the hands-on station allows students to see the topics in a unique way, extending student knowledge while increasing engagement.

Literature Review

Blended Learning Station Rotation Model

Most teachers work diligently to keep students engaged and motivated in the classroom. One instructional method that supports such learning is the station rotation model of blended learning. Blended learning is generally described as various types of instructional styles using a variety of academic approaches (Ayob et al., 2020). Specifically, in the station rotation model of blended learning, students rotate between two, three, or even four different learning stations all using different modalities, including technology and small group instruction (Ayok et al., 2020). Information at each station is presented in a slightly different way, whether that be orally, visually, or through demonstrations (Tomlinson & Kalbfleisch, 1998). This curriculum project presents four lessons that are all outlined the same way. During the first part of class, students learn about new content in a whole class lesson. Then, the class breaks up into four different stations; Station 1 is a mini-lesson, Station 2 is independent practice, Station 3 is technology, and Station 4 is hands on. It has been proved that "Integration of blended station rotation model in teaching mathematics is more effective than the traditional method of teaching in improving the performance in and attitude toward mathematics of Grade 10 students" (Mondragon et al., 2018, p. 7). Therefore, blended learning can increase student engagement and motivation in the classroom.

Small Group Instruction

The lessons in this curriculum include mini-lesson stations of blended learning. When teachers instruct small groups (6 or fewer students), blended learning model works well as the station rotation model allows teachers to spend more time with individual students (Ayob et al., 2020). Taking it one step further, groups can be developed to cater to each group's needs. Tomlinson and Kalbfleisch (1998) stated that all students require an appropriate level of challenge to learn. These blended learning stations are designed to do just that. In addition, small group instruction is designed to allow for student discussion and initiate student conversation about mathematical ideas,

which allows students to develop metacognitive control skills (Goos et al., 2002). Working in small groups, students can dive deeper into topics to deepen their level of thinking and understanding.

Technology

Technology in the mathematics classroom can be used to deepen student understanding of new concepts. Boss (2009) showed that "virtual manipulatives are a great way to demonstrate a conceptual understanding of a mathematical idea…Manipulatives help in making abstract concepts more concrete and are a steppingstone to a deeper conceptual understanding" (p. 110). Technology in the classroom can also be used to give students instant feedback as to how they are performing. The website used in these lesson plans, Delta Math, gives students a certain number of questions based on how well they perform, and gives video examples of how to complete the problems. This is an excellent way for students to use a self-check for understanding, especially with the link provided to the correct response (Bos, 2009).

Hands On Collaboration

Hands on collaboration at Station 4 benefits different types of learners who struggle with direct instruction. Hands on learning proved to be beneficial to tactile learners, kinesthetic learners, and to other students in the classroom as well. (Rodrigues, 2012, p. 29) The unique hands-on learning experience fits perfect into the blended learning style of lesson design that may not work best in other lessons.

Curriculum

Lesson 1: Reflections

A. The Big Idea

A reflection is a type of transformation that can be applied to points as well as shapes to create congruent figures.

B. NYS Next Generation Standards

GEO-G.CO.3 Given a regular or irregular polygon, describe the rotations and reflections (symmetries) that carry the polygon onto itself.

GEO-G.CO.4 Develop definitions of rotations, reflections, and translations in terms of points, angles, circles, perpendicular lines, parallel lines, and line segments.

GEO-G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another

C. Student Learning Targets

- I can reflect points and shapes over lines on a graph.
- I can apply definitions of reflections in terms of points not on a coordinate grid.
- I can describe reflections that map shapes back onto themselves.

D. Lesson Timeline (80 minute class period)

➔ Extra 2 minutes included for transition time between lesson/stations

Anticipatory set (5 minutes)

- Students will use their prior knowledge of transformations to decide "Which Doesn't Belong" with a partner using white boards

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Whole class lesson (20 minutes)

- Teacher will walk the students through the lesson on reflections (first 4 pages in lesson)
- Hand out lesson notes separate from station packet- students should keep notes

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Stations (48 minutes total)

Station 1 – Mini-Lesson (12 minutes)

- Small groups will work with the teacher to identify lines and angles of symmetry of shapes not on the coordinate grid.
- Clean up any misconceptions about the group lesson

Station 2 – At your desk (12 minutes)

- Students will independently practice reflecting shapes on a coordinate grid. Students will be able to reflect across the $x-axis, y-axis, x=a, y=a, y=x, and\ y=-x$ lines.

Station 3 – Technology (12 minutes)

- Students will work through practice problems on Delta Math. They will be asked to visually identify the line of reflection for one skill. The other skill has students identify the negative reciprocal slope to find the equation of the line of reflection.

- Delta Math Skills: “Find Line of Reflection Experimentally”, “Find Line of Reflection Methodically”
  - Students must answer 5 questions correctly from each skill. Students can continue to practice after they complete the requirement.

Station 4 – Hands On (12 minutes)

- Students will color a mosaic such that it contains reflexive symmetry.

Exit pass (5 minutes)

- The students will complete “which doesn’t belong” exit pass on their own.

E. Assessment

- The student station packet will be collected. The teacher will assess individual student learning from the Delta Math data as well as their work shown in the station packet.

F. Materials

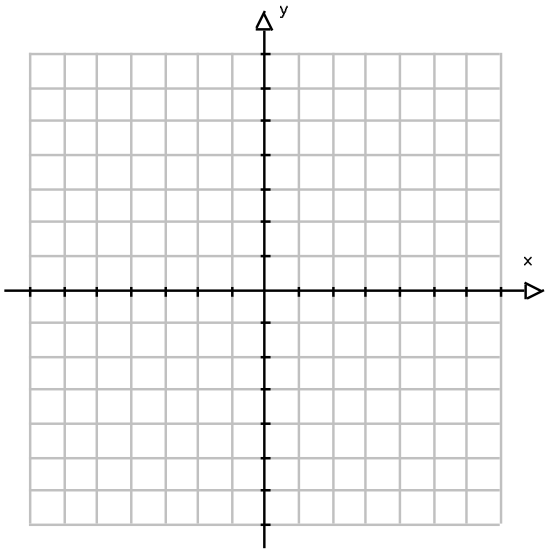
- Laptops, colored pencils, student note packets, rulers, highlighters (optional)

Name _____

Reflection Class Notes

Coordinate Geometry Given points *M* (2, 2), *N* (4, 5), and *O* (6, -1), graph △*MNO* and its reflection image as indicated. State the image point coordinates.

1. r_{y-axis}

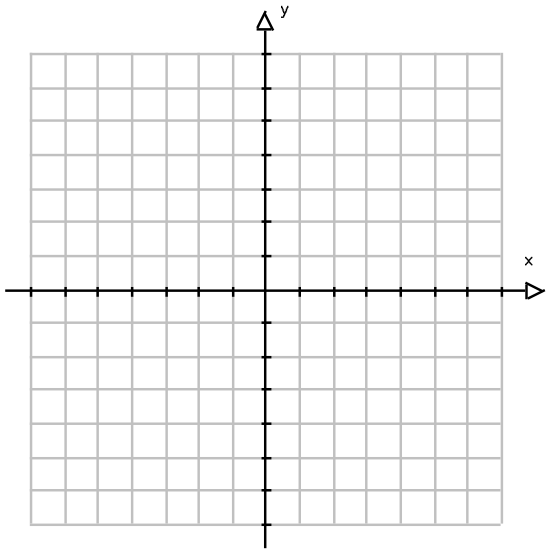


$M(2, 2) \rightarrow M'$: _____

$N(4, 5) \rightarrow N'$: _____

$O(6, -1) \rightarrow O'$: _____

2. r_{x-axis}



$M(2, 2) \rightarrow M'$: _____

$N(4, 5) \rightarrow N'$: _____

$O(6, -1) \rightarrow O'$: _____

Is there a pattern?

RULES:

To reflect over the *y*-axis:

To reflect over the x-axis:

Is a line reflection considered a *rigid motion*? Why or why not?

Is orientation preserved in a line reflection? Why or why not?

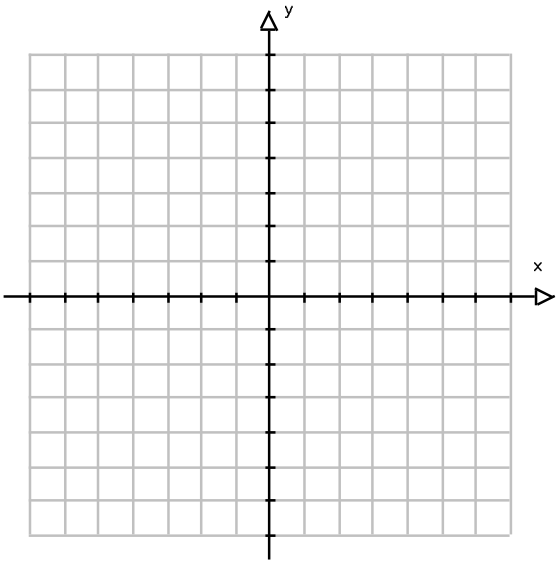
Reflecting over a Line that's Not an Axis

Steps:

1. Locate and plot the given point.
2. Locate and graph the line of reflection.
3. Move along the line PERPENDICULAR to the line of reflection until the given point and its image are equidistance from the given line. (Count the boxes away from the line of reflection, count the same number on the other side of the line.)

Coordinate Geometry Given points *M*(2, 2), *N*(4, 5), and *O*(6, -1), graph △*MNO* and its reflection image as indicated. State the image point coordinates.

3. $r_{x=1}$

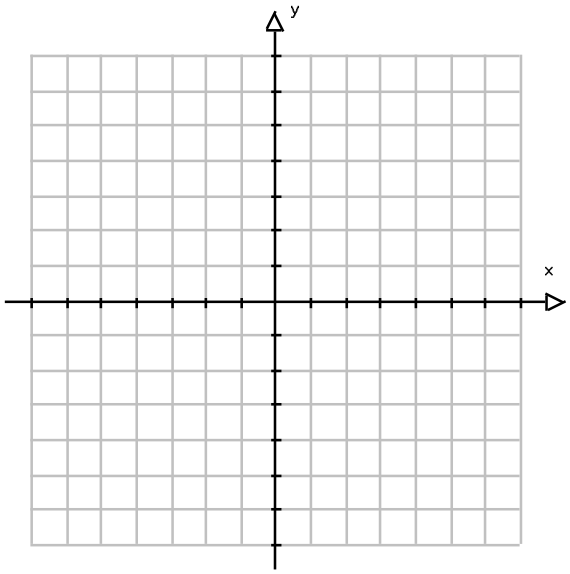


$M(2, 2) \rightarrow M'$: _____

$N(4, 5) \rightarrow N'$: _____

$O(6, -1) \rightarrow O'$: _____

4. $r_{y=-2}$



$M(2, 2) \rightarrow M'$: _____

$N(4, 5) \rightarrow N'$: _____

$O(6, -1) \rightarrow O'$: _____

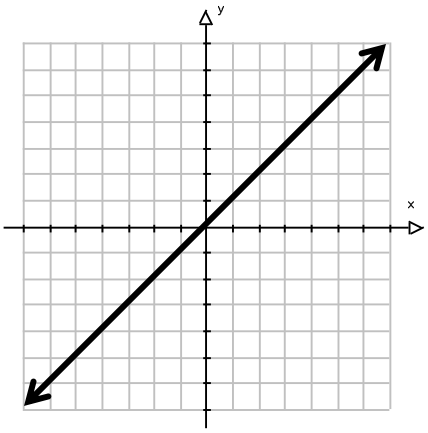
In all the above examples, is… a) distance preserved?

b) orientation preserved?

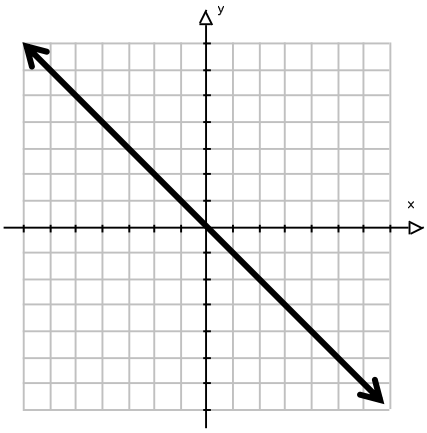
Reflecting Over Lines That Aren't Horizontal or Vertical

The two most common lines that are slanted and reflected over are:

$y = x$

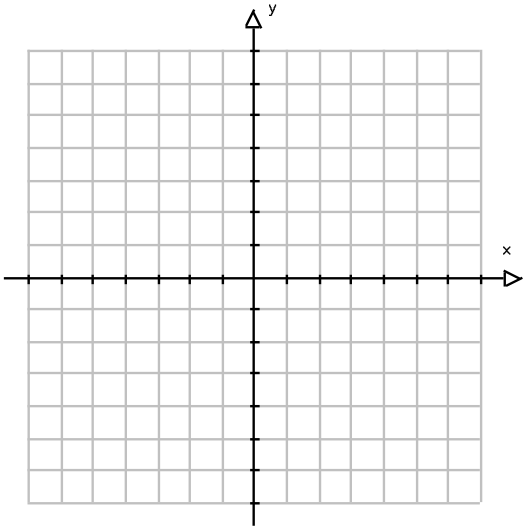


$y = -x$



Coordinate Geometry Given points *M*(2, 2), *N*(4, 5), and *O*(6, -1), graph △*MNO* and its reflection image as indicated. State the image point coordinates.

5. $r_{y=x}$



$M(2, 2) \rightarrow M'$: _____

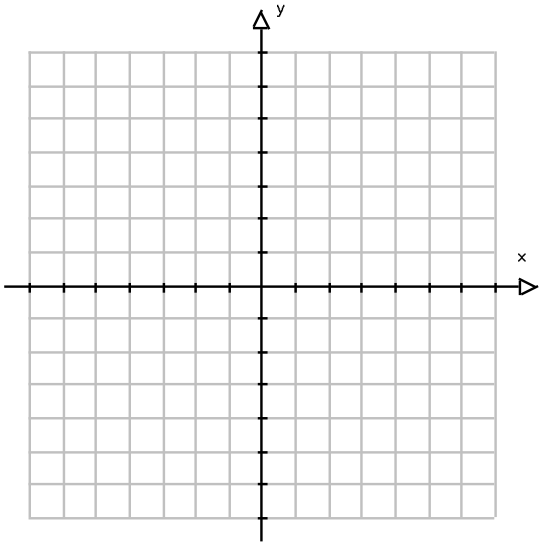
$N(4, 5) \rightarrow N'$: _____

$O(6, -1) \rightarrow O'$: _____

RULES:

To reflect over the line $y = x$,

6. $r_{y=-x}$



$M(2, 2) \rightarrow M'$: _____

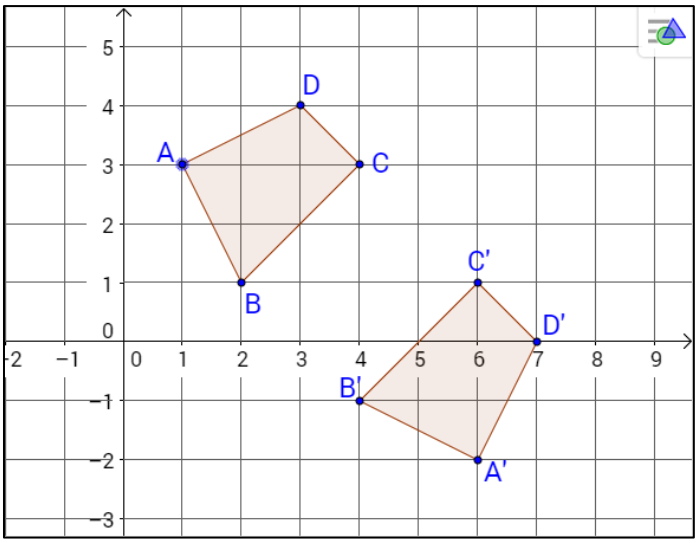
$N(4, 5) \rightarrow N'$: _____

$O(6, -1) \rightarrow O'$: _____

To reflect over the line $y = -x$,

Writing Lines of Reflection

7. Write the equation of the line of reflection that maps *ABCD* onto *A'B'C'D'*.

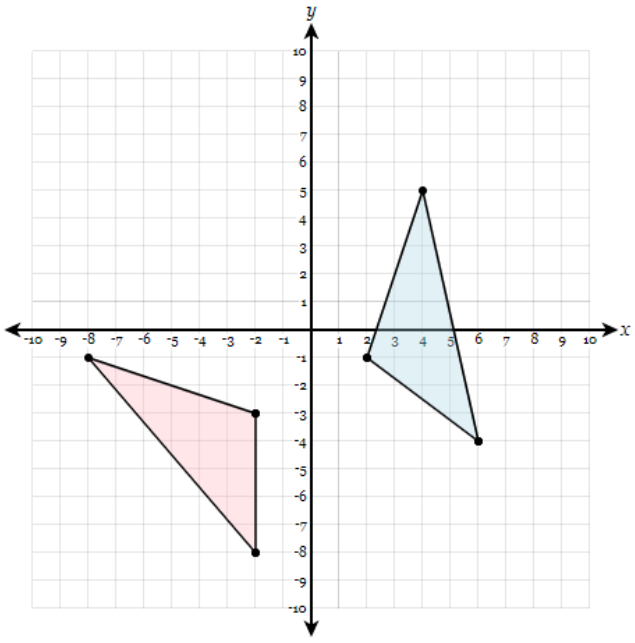


Slope: _____

Y-intercept: _____

Equation: _____

8.



Slope: _____

Y-intercept: _____

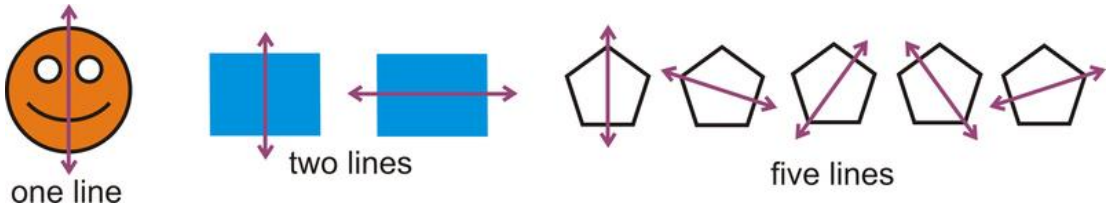
Equation: _____

Name _____

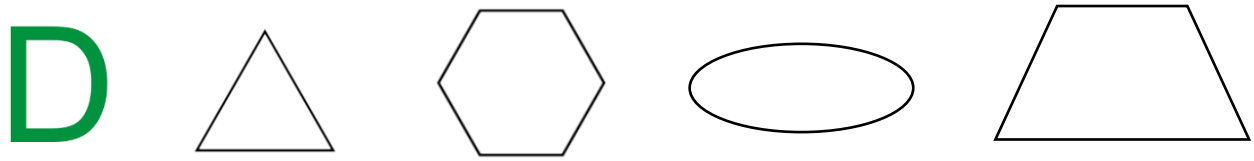
Station 1 – Mini Lesson

Symmetry: When a rigid motion maps a figure onto itself

Line Symmetry / Reflection Symmetry: If there is a reflection for which the figure is its own image.



Find all lines of symmetry for the shapes/letters below:

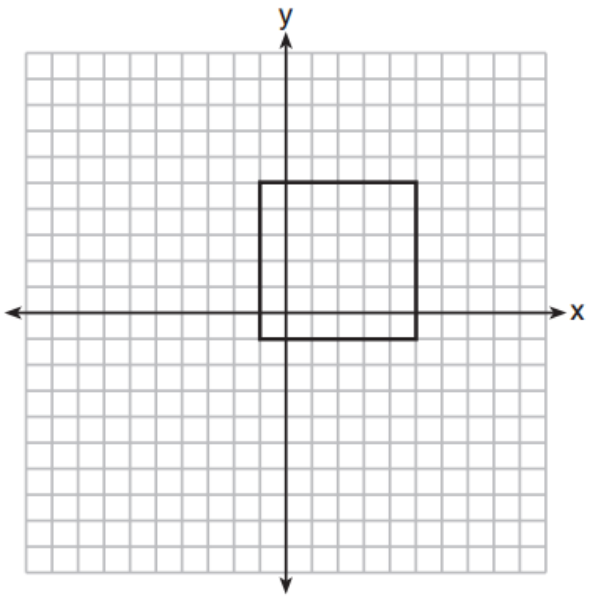


Draw your own shapes below! One with horizontal symmetry and the other with vertical symmetry:

Station 2 – At your desk

Practice:

1. In the diagram below, a square is graphed in the coordinate plane.



A reflection over which line does *not* carry the square onto itself?

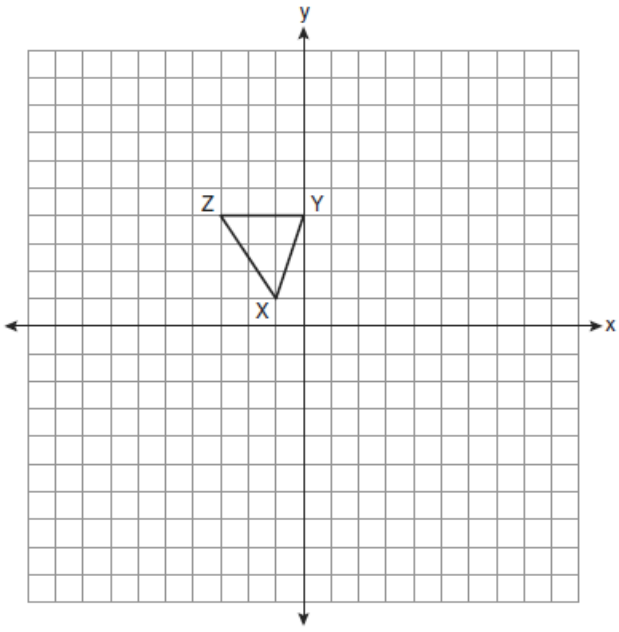
(1) $x = 5$

(2) $y = 2$

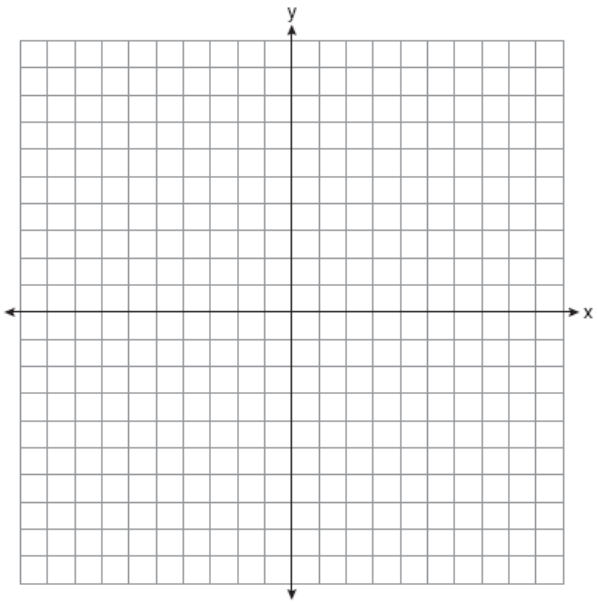
(3) $y = x$

(4) $y = -x + 4$

2. Triangle *XYZ*, shown in the diagram below, is reflected over the line $x = 2$. State the coordinates of $\triangle X'Y'Z'$, the image of $\triangle XYZ$.



3. Triangle *ABC* has vertices $A(-1,1)$, $B(1,3)$, and $C(4,1)$. The image of $\triangle ABC$ after a reflection over the line $y = x$ is $\triangle A'B'C'$. State and label the coordinates of $\triangle A'B'C'$.



4. Point $(-2,3)$ is reflected in the x-axis. In which quadrant does its image lie?

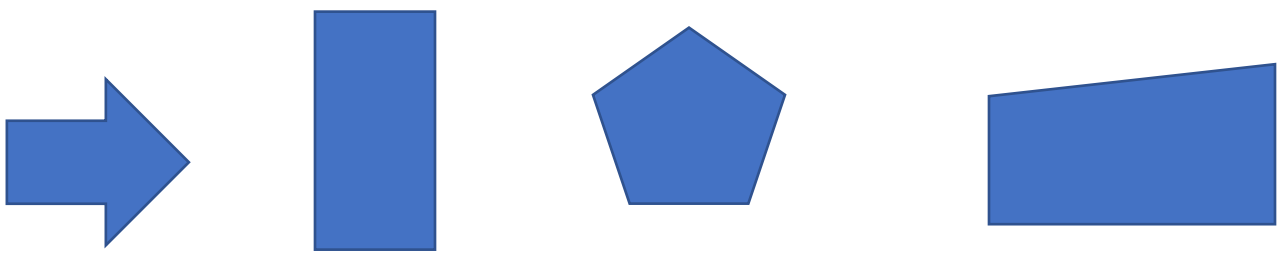
1) I
2) II
3) III
4) IV

5. Triangle *ABC* has coordinates $A(2,0)$, $B(0,7)$ and $C(5,1)$. Under a reflection in the *y*-axis, which point remains invariant (does not move)?

1) A
2) B
3) C
4) None of the above

<u>If you already did the mini-lesson, try the question below:</u>

6. Draw all lines of symmetry on the shapes below:



Station 3 – Technology

Directions:

1. Login to Delta Math

2. Complete the "Reflections Practice"

    - Find Line of Reflection Experimentally

    - Find Line of Reflection Methodically

3. You must answer 5 questions from each skill correct

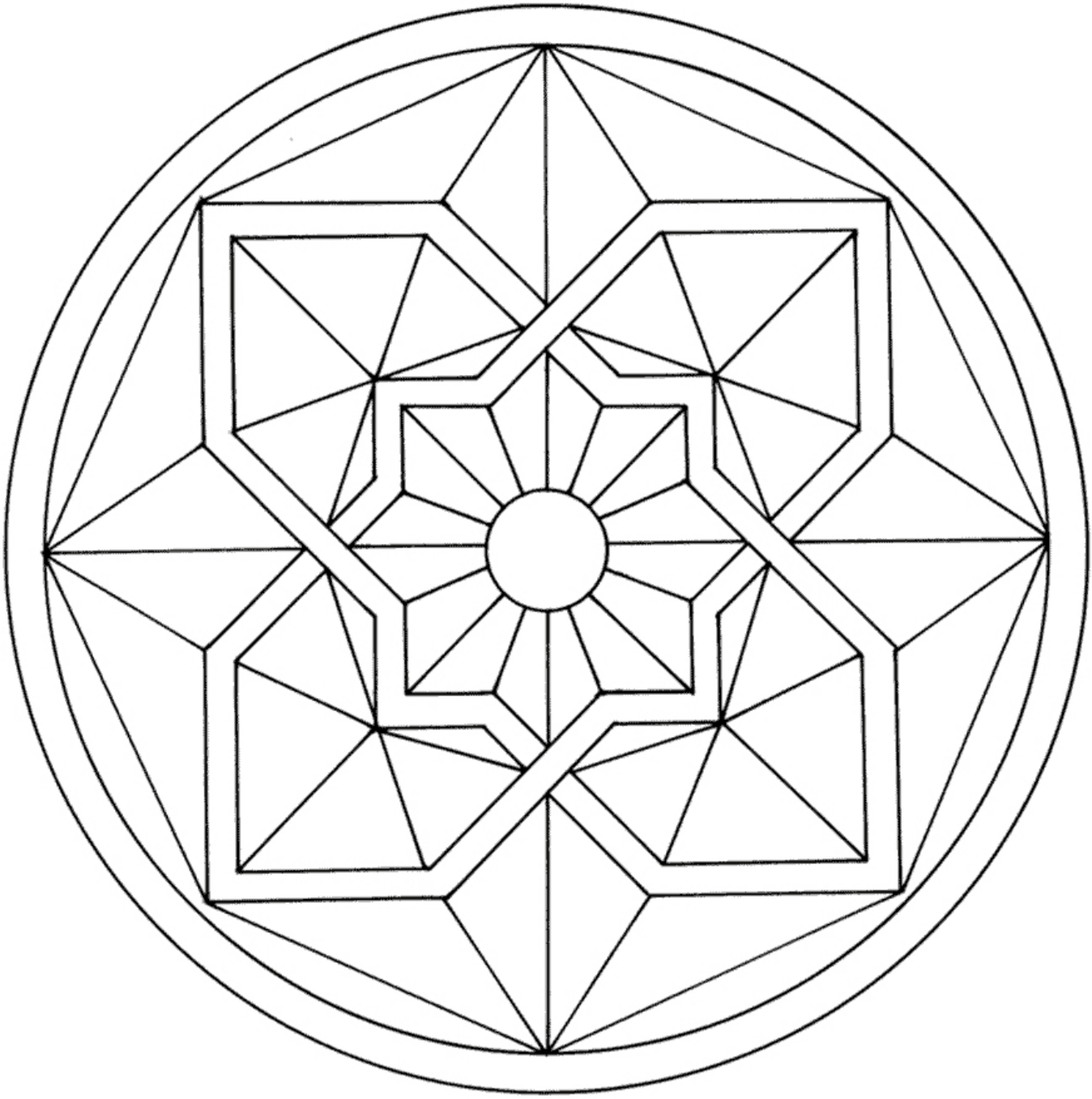
4. Finish Early? Keep trying more problems! Or go back and practice a skill from a previous unit

Checklist:

- ☐ Find Line of Reflection Experimentally- 5 questions correct
- ☐ Find Line of Reflections Methodically – 5 questions correct

Station 4 – Hands On

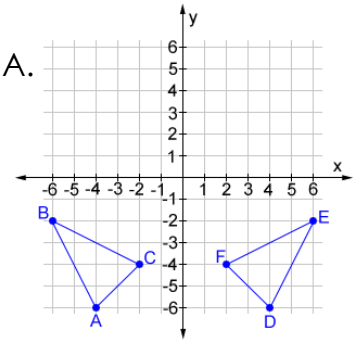
Directions: Color the mosaic below, making sure to create **reflexive symmetry.** Colored pencils are on the table!



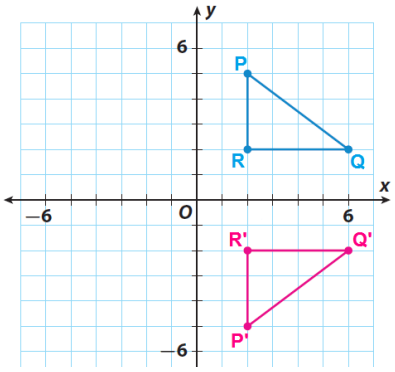
Teacher Materials

Anticipatory Set: Students will work in pairs with whiteboards. They will have 30 seconds to look at the images on the board and decide which transformation doesn't belong. Sample image sets are below:

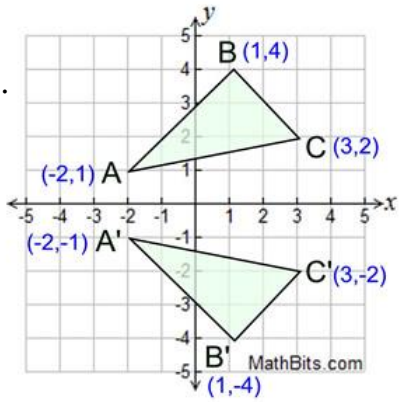
A.



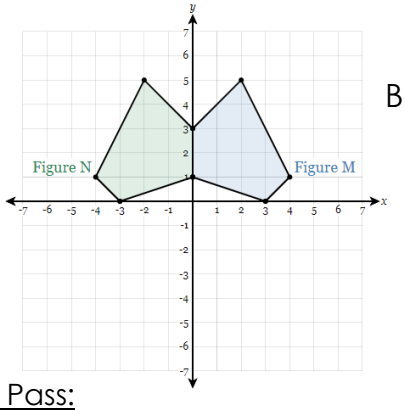
B.



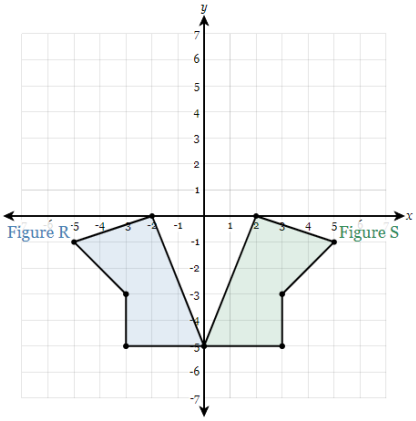
C.



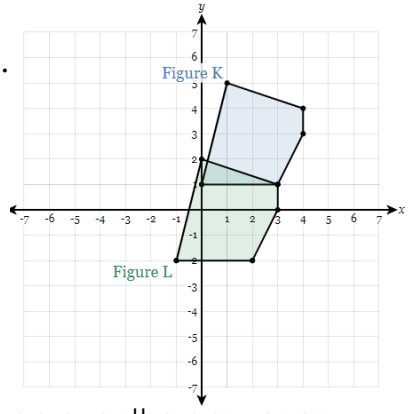
A.



B.



C.



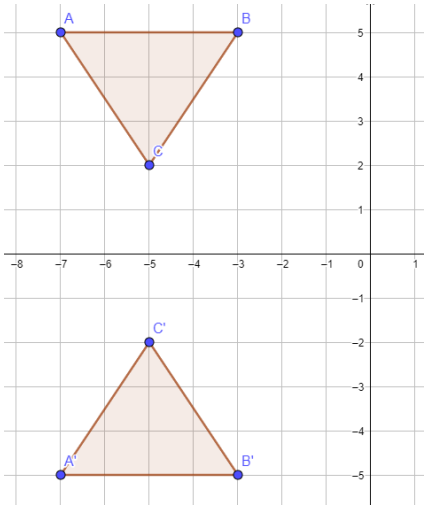
Exit Pass:

Students will complete this independently. Collect their papers, then go over the answer as a class.

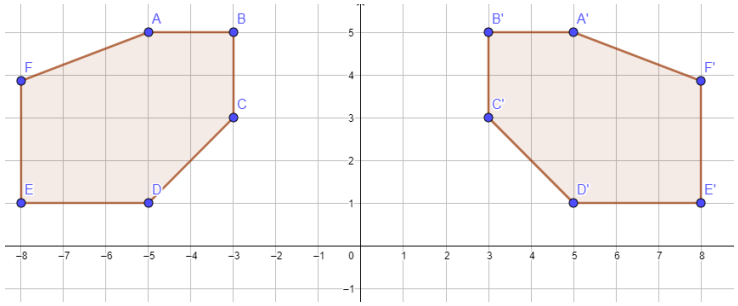
Name _____

Reflection Exit Ticket

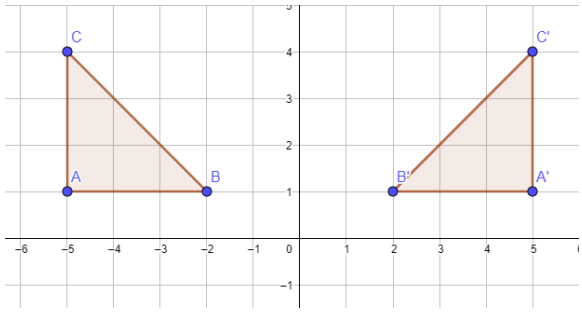
B.



A.



C.



Which doesn't belong? (A, B, or C). Explain your answer.

Lesson 2: Centroids and Orthocenters

A. The Big Idea

Medians and altitudes are two different line segments inside of triangles used to find centroids and orthocenters, respectively.

B. NYS Next Generation Standards

GEO-G.SRT.4 Prove and apply similarity theorems about triangles (The centroid of the triangle divides each median in the ratio 2:1)

GEO-G.CO.12 Make, justify, and apply formal geometric constructions (Construct points of concurrency of a triangle (centroid, circumcenter, incenter, and orthocenter))

C. Student Learning Targets

- I can identify the difference between medians and altitudes and construct both.
- I can apply my knowledge of medians and altitudes to geometric proofs.

D. Lesson Timeline (80 minute class period)

➔ Extra 2 minutes included for transition time between lesson/stations

Anticipatory set (5 minutes)

- Students will use their prior knowledge of midpoints and perpendicular lines to come up with their own definitions of median and altitude during the partner warm-up. Students will be able to identify the visual differences between medians and altitudes.

Whole class lesson (20 minutes)

- Teacher will walk the students through the lesson on medians and altitudes (first 3 pages in lesson)
- Hand out lesson notes separate from station packet- students should keep notes

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Stations (48 minutes total)

Station 1 – Mini-Lesson (12 minutes)

- Small groups will work with the teacher to construct Medians and Altitudes to find the Circumcenter and Orthocenter in a triangle respectively
- Clean up any misconceptions about the group lesson

Station 2 – At your desk (12 minutes)

- Students will independently practice using the 2:1 ratio for medians of a triangle.
- Students will independently practice finding the orthocenter of a triangle on a coordinate plane.

Station 3 – Technology (12 minutes)

- Students will use laptops to open GeoGebra to observe the Altitude File and the Median File. Students will answer 3 questions about observations made when manipulating the vertices of the triangle in both files.

Station 4 – Hands On (12 minutes)

- Students will rearrange statements and reasons to create proofs involving altitudes and medians
- If you have a co-teacher, they can sign off on the students' work. If you do not have a co-teacher, have students take a picture of proof after it is rearranged and submit online.

Exit pass (5 minutes)

- The class will complete a matching activity sorting properties of Medians and Altitudes.

E. Assessment

- The student station packet will be collected. The teacher will assess individual student learning from their work shown in the station packet and exit ticket.

F. Materials

- Laptops, student note packets, rulers, compasses

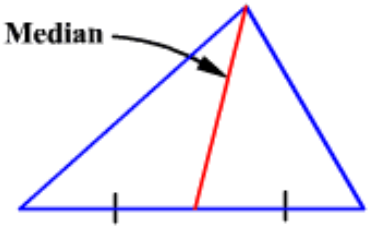
Lesson 2

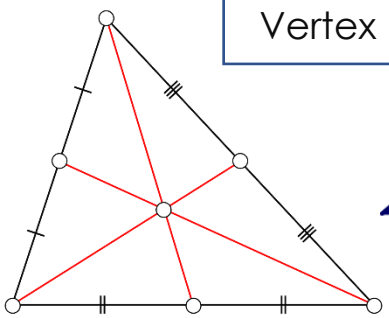
Medians and Altitudes

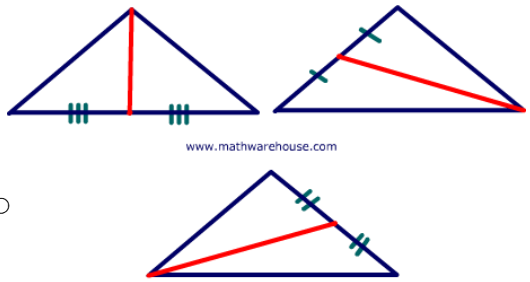
Warm-up: Come up with your own definition of median and altitude using the pictures provided and the word bank

Median:

Word Bank:		
Vertex	Midpoint	Perpendicularly

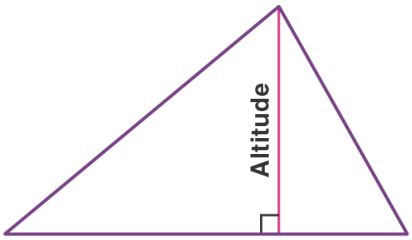


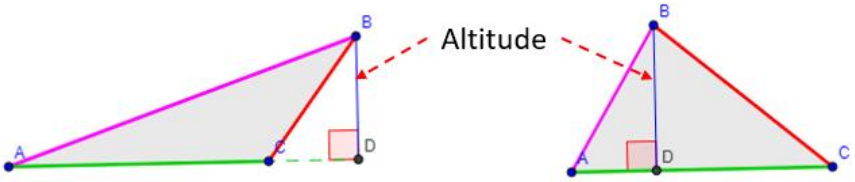




Definition: The median connects the _____ of a triangle to the _____ of the opposite side.

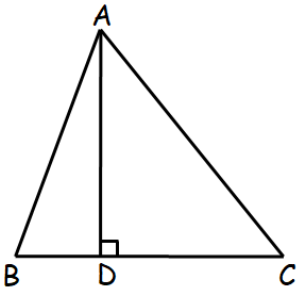
Altitude:

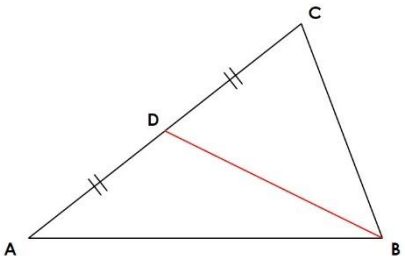


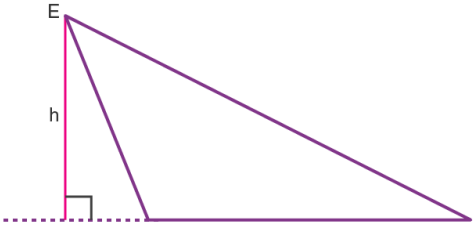


Definition: The altitude connects the _____ of a triangle to the opposite side _____.

Directions: Label the images below as altitudes or medians:



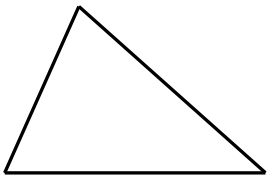


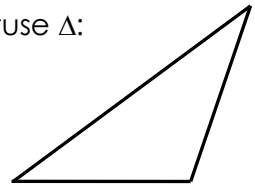


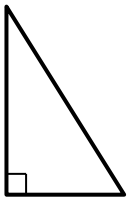
_____ _____ _____

Median: A line segment that connects the vertex of a triangle to the midpoint of the opposite side.

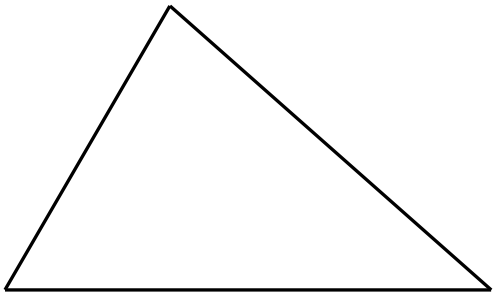
Draw a *median* for each type of triangle, starting at the upper vertex.

Acute Δ: 

Obtuse Δ: 

Right Δ: 

Every Triangle has 3 medians. Draw one from each vertex:



The concurrence of the three medians is called the _____.

Concurrency of Medians Theorem: The medians of a triangle are concurrent at a point that is *two thirds* the distance from each vertex to the midpoint of the opposite side.

$DC = \frac{2}{3} DJ$ $EC = \frac{2}{3} EG$ $FC = \frac{2}{3} FH$

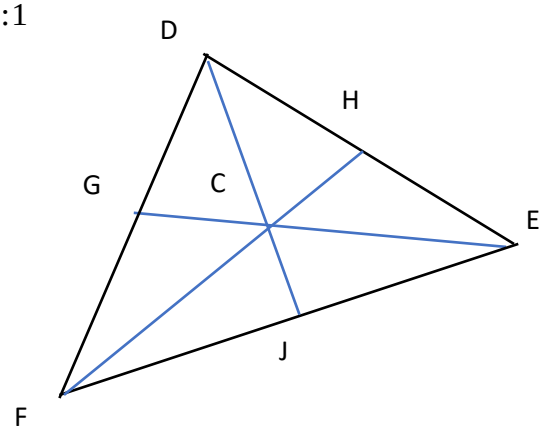
The ratio of the parts of each median is 2:1. The section of the median nearest the vertex is twice as long as the section near the midpoint of the triangle's side.

$DC : CJ = 2:1$ $CE : GC = 2:1$ $FC : CH = 2:1$

(Ex1) If DC = 8, what is the length of $\overline{DJ}$?

(Ex2) If EG = 9, what is the length of $\overline{GC}$?

(Ex3) If FH = 27, what is the length of $\overline{FC}$?

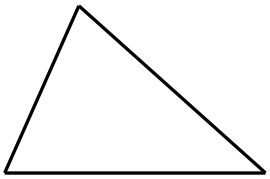


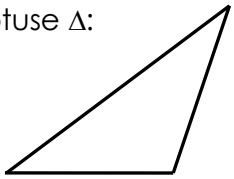
Remember, the median hits the side of the triangle at its **midpoint**

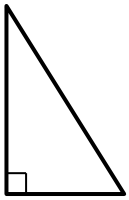
If $\overline{DH} = 3$, then $\overline{HE} =$ _____ and $\overline{DE} =$ _____

Altitude: A line segment that connects the vertex of a triangle to the opposite side <u>perpendicularly</u>.

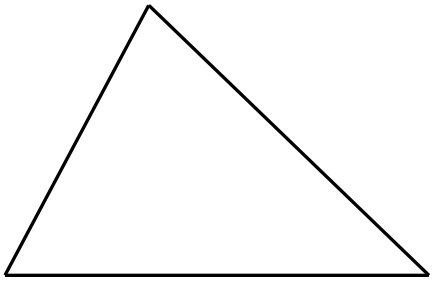
Draw an *altitude* for each type of triangle, starting at the upper vertex:

Acute Δ: 

Obtuse Δ: 

Right Δ: 

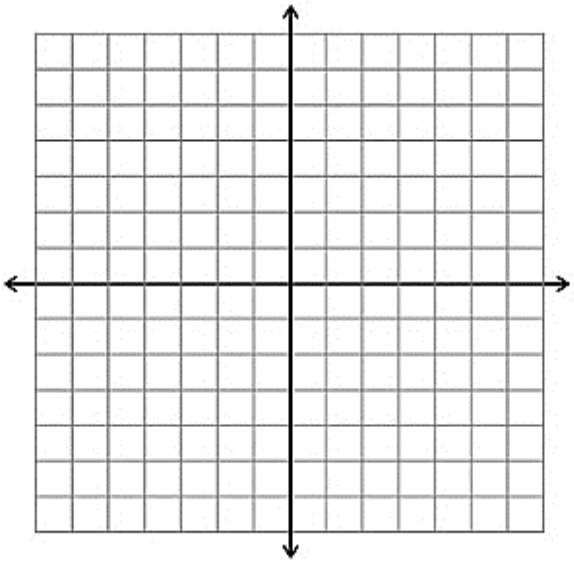
Every Triangle has 3 altitudes. Draw one from each vertex:



The concurrence of the three altitudes is called the _____.

<u>Finding the orthocenter on the coordinate plane:</u>

Triangle ABC has vertices A(1,3) B(2,7) and C(6,3). What are the coordinates of the orthocenter?



Hint:

Perpendicular lines have

_____ _____

slopes

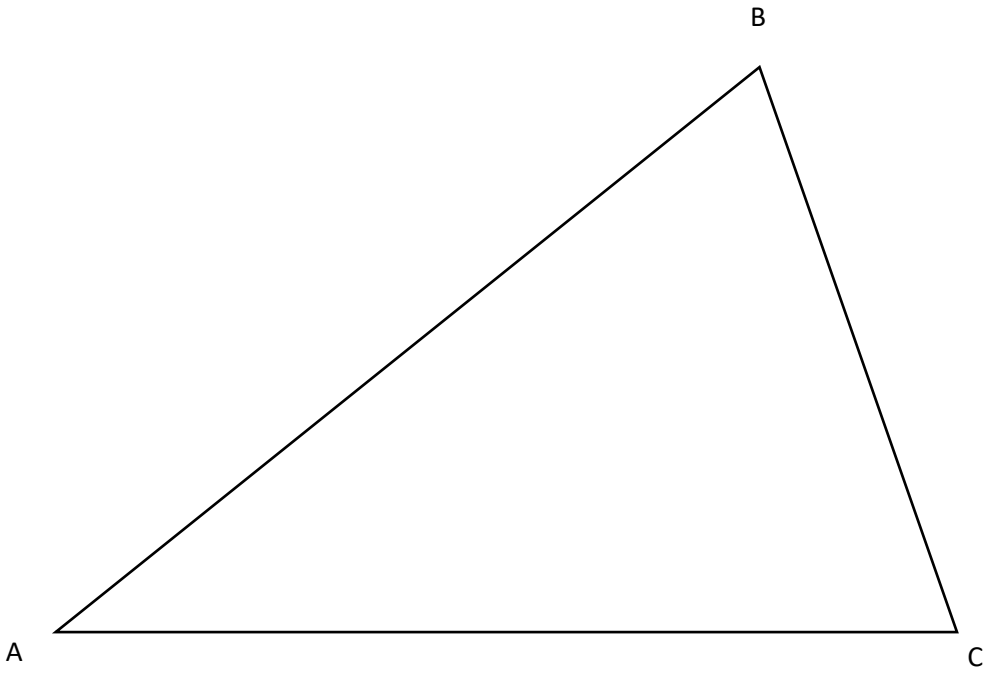
Name _____

Station 1 – Mini Lesson (Constructions)

Directions: Construct a median of ΔABC to side $\overline{AC}$ below.

Steps:

1. Construct the perpendicular bisector to find the midpoint, label it M.
2. Draw a line from the vertex B to the midpoint, M.



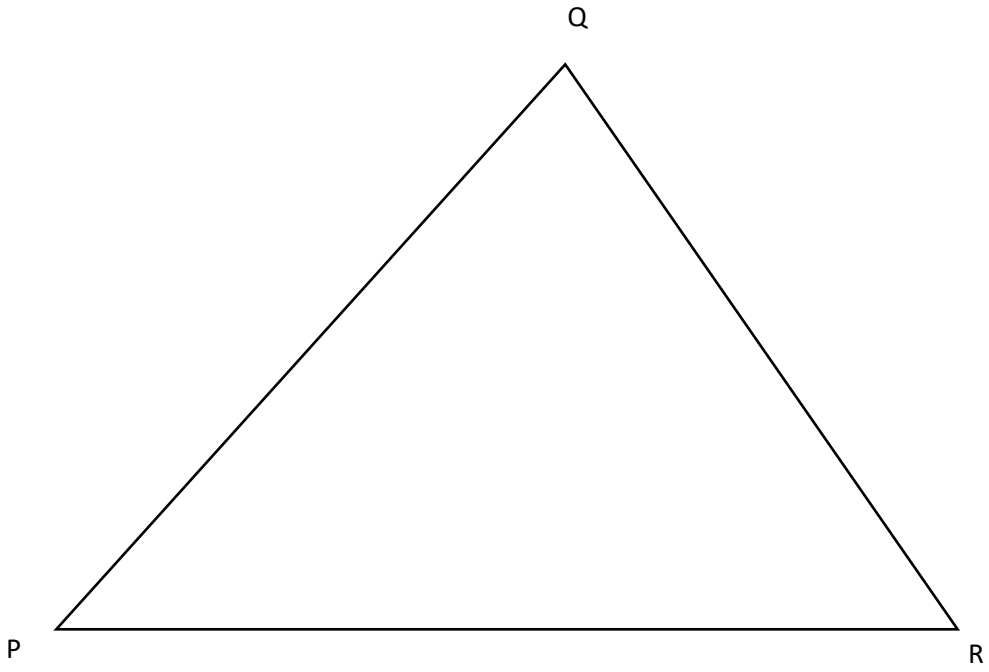
What do we know about $\overline{AM}$ and $\overline{MC}$?

Directions: Construct the centroid of the triangle below with your compass and a straightedge. The centroid is where all 3 _____ meet.

Steps:

1. Construct the medians

2. The point of intersection is the centroid, label it with a C.



Do we need to construct all 3 medians to find the centroid?

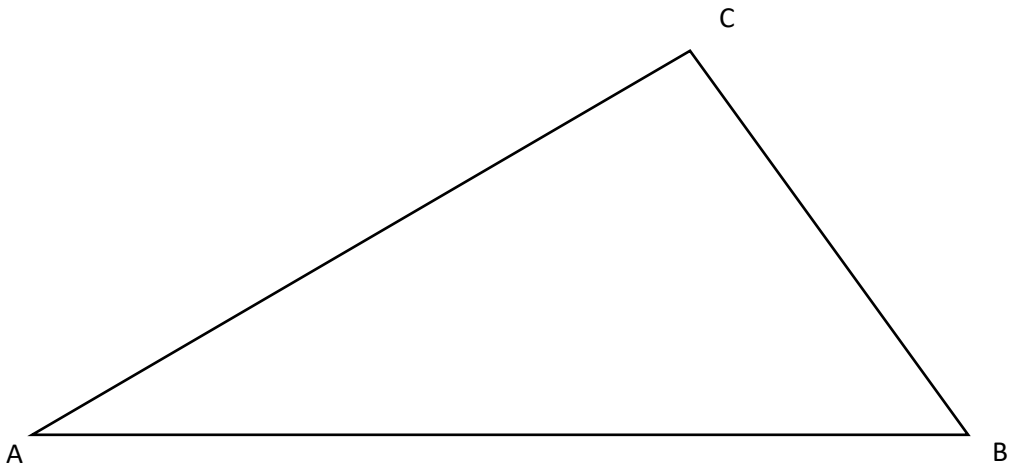
Directions: Construct an altitude of ΔABC to side $\overline{AB}$ below.

Steps:

1. Put your pointer on C, and draw an arc through side $\overline{AB}$

2. Put your pointer on the intersection points of $\overline{AB}$, then make a curvy arc below the triangle.

3. Connect C with your curvy arc below the triangle to create the altitude.



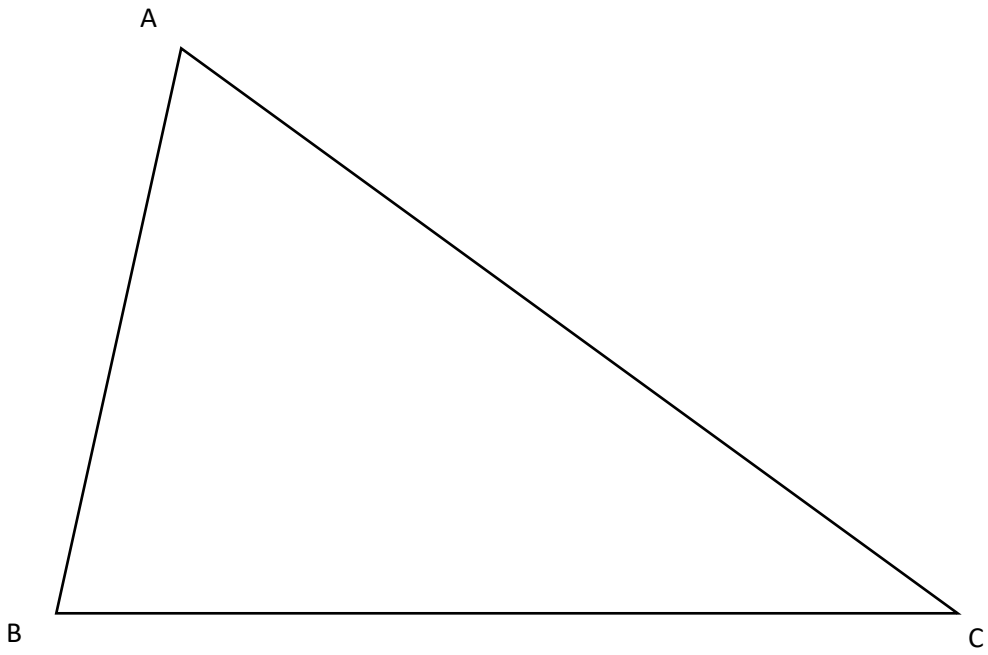
What do we know about the angle of intersection of the altitude with $\overline{AB}$?

Directions: Construct the orthocenter of the triangle below with your compass and a straightedge. The orthocenter is where all 3 _____ meet.

Steps:

1. Construct the altitudes

2. The point of intersection is the orthocenter, label it with a H

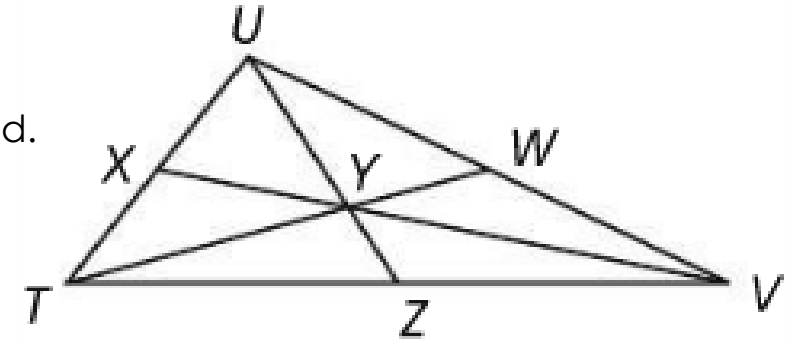


Do we need to construct all 3 altitudes to find the orthocenter?

Station 2 – At your Desk (Independent Practice)

Directions: Answer the following questions using your notes from the class lesson

1. In ΔTUV, Y is the centroid.



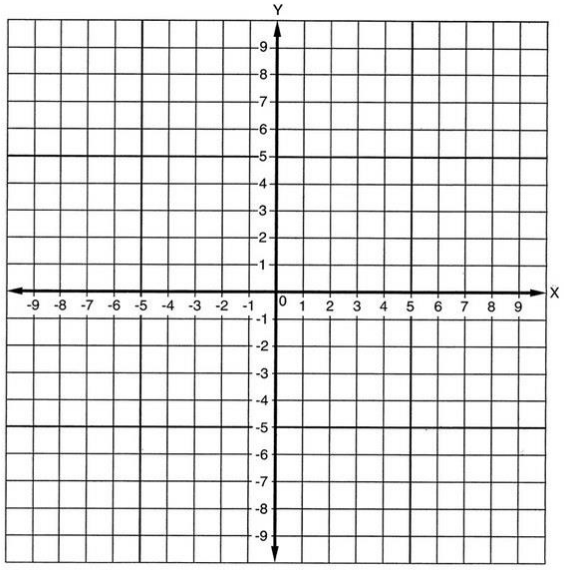
a. If $YW = 9$, find TY and TW.

b. If $YU = 8$, find ZU and ZY.

c. If $VX = 9$, find VY and XY.

d. A centroid separates a median into two segments. What is the ratio of the length of the shorter segment to the length of the longer segment?

2. Find the coordinates of the orthocenter of ΔABC with the coordinates $A(0,-2)$, $B(4,-2)$, and $C(-2,-8)$.



Station 3 – Technology

Directions: Open the GeoGebra Files and answer the questions below.

Altitude Link

1. The intersection of the altitudes in a triangle is called the _____.

2. Move vertex B left or right until the Orthocenter appears outside of the triangle. What type of triangle did you create?

(Acute, Right, or Obtuse)

3. Move vertex B left or right until the Orthocenter lines up directly over vertex A or C. What type of triangle did you create?

(Acute, Right, or Obtuse)

Median Link

1. The intersection of the medians in a triangle is called the _____.

2. What is true about the diagram?

a) $XC = CE$ b) $DC = CZ$

c) $XF = FZ$ d) $YE = \frac{2}{3}EZ$

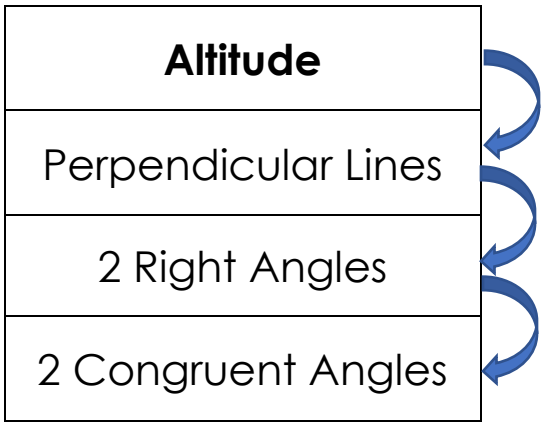
3. Move around the vertex of the triangle (X, Y, and Z). Can you make the centroid fall outside of the triangle? Why or why not?

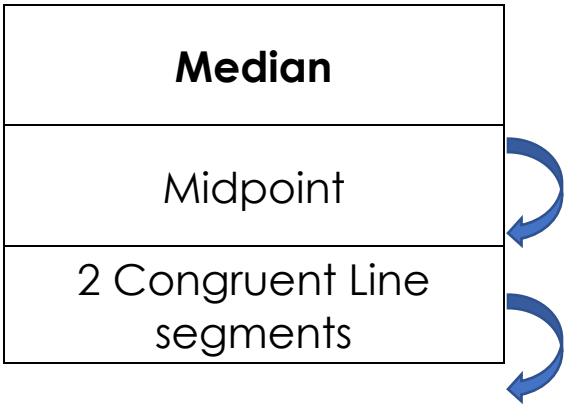
_____

_____

Station 4 – Hands on

Directions: Use the sequences provided on this sheet to arrange the proofs in the correct order. You may work with a partner.





Once you believe you have put your proof in the correct order, have the teacher check. You must correctly assemble at least one proof.

Easy: Blue Proof Teacher signature: _____

Easy: Green Proof Teacher signature: _____

Hard: Red Proof Teacher signature: _____

Teacher Materials

Station 3 - GeoGebra Files

Post these links on your class website, or turn them into a QR code! You do not need a GeoGebra account to view the link.

Altitude Link:

https://www.geogebra.org/m/zndnrdan

Median Link:

https://www.geogebra.org/classic/tbvqdabx

Station 4 – Proof Cut-Outs

Leave copies of the blank templates (first page of each proof) out for the students to use. Cut out the proof statements from the key and put them next to the corresponding proof. You can either keep the statements and reasons paired together, or cut them separately for more of a challenge.

It helps if you laminate the pieces you need to cut out, but it is not necessary.

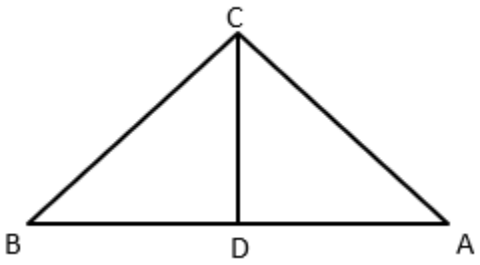
The red proof is the hardest since it involves both Median and Altitude.

Red Proof

Given: $\overline{CD}$ is a median of ΔABC

$\overline{CD}$ is an altitude of ΔABC

Prove: $\Delta ADC \cong \Delta BDC$



Statement	Reason

Red Proof Key

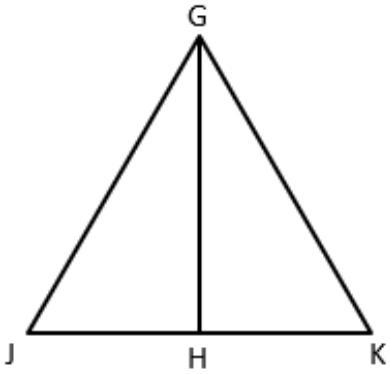
S- $\overline{CD}$ is a median of ΔABC	R- Given
S- D is the midpoint of $\overline{BA}$	R- Definition of Median
S- $\overline{BD} \cong \overline{DA}$	R- Definition of Midpoint
S- $\overline{CD}$ is an altitude of ΔABC	R- Given
S- $\overline{CD} \perp \overline{BA}$	R- Definition of Altitude
S- $\angle CDB$ *and* $\angle CDA$ are right angles	R- Definition of ⊥ lines
S- $\angle CDB \cong \angle CDA$	R- All right angles are ≅
S- $\overline{CD} \cong \overline{CD}$	R- Reflexive Postulate
S- $\Delta ADC \cong \Delta BDC$	R- SAS Theorem

Blue Proof

Given: $\overline{GH}$ is an altitude of ΔGJK

$\overline{JG} \cong \overline{KG}$

Prove: $\Delta JGH \cong \Delta KGH$



Statement	Reason

Blue Proof Key

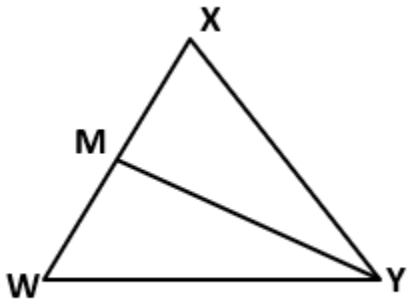
S- $\overline{GH}$ is an altitude of ΔGJK	R- Given
S- $\overline{GH} \perp \overline{JK}$	R- Definition of Altitude
S- $\angle GHJ \; and \; \angle GHK$ are right angles	R- Definition of ⊥ lines
S- $\angle GHJ \cong \angle GHK$	R- All right angles are ≅
S- $\overline{JG} \cong \overline{KG}$	R- Given
S- $\overline{GH} \cong \overline{GH}$	R- Reflexive Postulate
S- $\Delta JGH \; and \; \Delta KGH$ are right triangles	R- Right triangles have a right angle
S- $\Delta JGH \cong \Delta KGH$	R- HL Theorem

Green Proof

Given: $\overline{YM}$ is a median of ΔXYW

$\overline{YW} \cong \overline{XY}$

Prove: $\angle W \cong \angle X$



Statement	Reason

Green Proof Key

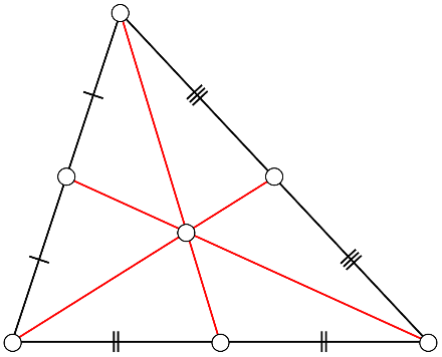
S- $\overline{YM}$ is a median of ΔXYW	R- Given
S- M is the midpoint of $\overline{XW}$	R- Definition of Median
S- $\overline{XM} \cong \overline{WM}$	R- Definition of Midpoint
S- $\overline{YW} \cong \overline{XY}$	R- Given
S- $\overline{MY} \cong \overline{MY}$	R- Reflexive Postulate
S- $\Delta YMW \cong \Delta YMX$	R- SSS Theorem
S- $\angle W \cong \angle X$	R- CPCTC

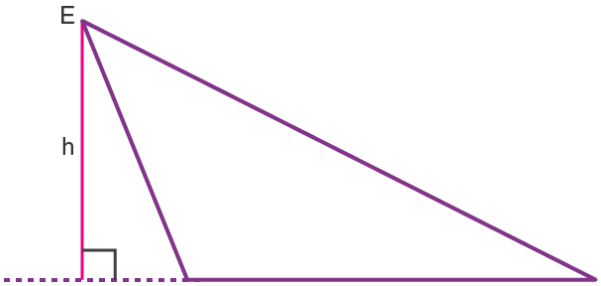
Exit Pass:

Students can discuss in pairs, then the class will complete the sorting activity together. This can be done on a SmartBoard, asking volunteers to drag and place an item into the correct box.

_____
Perpendicular Lines
2 Right Angles
2 Congruent Angles

_____
Midpoint
2 Congruent Line segments





Point of concurrency is the Orthocenter

Point of concurrency is the Centroid

A line segment that connects the vertex of a triangle to the <u>midpoint</u> of the opposite side.

A line segment that connects the vertex of a triangle to the opposite side <u>perpendicularly</u>.

Median

Altitude

Lesson 3: Trigonometry

A. The Big Idea

Trigonometry can be used to find a missing side or angle of a right triangle. It can also be applied to real world problems involving Angle of Elevation and Angle of Depression.

B. NYS Next Generation Standards

GEO-G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles
GEO-G.SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right traingles in applied prolems

C. Student Learning Targets

- I can define trigonometric ratios and solve problems involving right triangles.
- I can apply my knowledge of trigonometry to solve real world problems, including angles of elevation and angles of depression.
- I can explain the relationship between cofunctions.

D. Lesson Timeline (80 minute class period)
➔ Extra 2 minutes included for transition time between lesson/stations

Anticipatory set (5 minutes)

- Students will use their prior knowledge to find the missing side of a right triangle using the Pythagorean Theorem.

Whole class lesson (20 minutes)

- Teacher will walk the students through the lesson on trigonometric functions (first 4 pages in lesson)
- Hand out lesson notes separate from station packet- students should keep notes

---

Stations (48 minutes total)

Station 1 – Mini-Lesson (12 minutes)

- Clean up any misconceptions about the group lesson
- Small groups will work with the teacher to apply their knowledge of trigonometry to Angle of Elevation and Angle of Depression questions

Station 2 – At your desk (12 minutes)

- Students will independently practice finding a trigonometric ratio of a right triangle
- Students will independently practice finding a missing side or angle of a right triangle
- Students will independently practice Angle of Elevation and Depression (if already visited Station 1)

Station 3 – Technology (12 minutes)

- Students will work through practice problems on Delta Math. They will be asked to find missing angle measures using their knowledge of cofunctions.
- Delta Math Skills: “Cofunction Equations”
  - Students must answer 5 questions correctly from the skill. Students can continue to practice after they complete the requirement.

Station 4 – Hands On (12 minutes)

- Students will use a trigonometry table to compare the sine and cosine values of complementary angles.
- Students will calculate angle measures that will make sine and cosine values equal.

Exit pass (5 minutes)

- The class will complete a true/false activity on trigonometry questions.

E. Assessment

- The student station packet will be collected. The teacher will assess individual student learning from the Delta Math data as well as their work shown in the station packet.

F. Materials

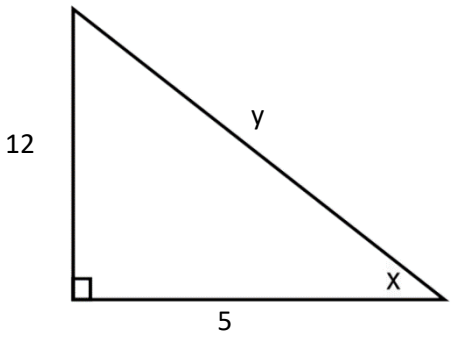
- Laptops, student note packets, calculators, highlighters

Lesson 3

Trigonometry

Warm-up: Find the missing side of the right triangle (y) using the Pythagorean Theorem.

$a^2 + b^2 = c^2$



How do you know when to use trigonometry in a problem? (Hint: Can we use Pythagorean Theorem to find the value of x?)

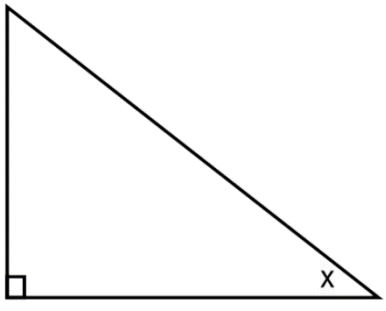
3 Sides of Right Triangles

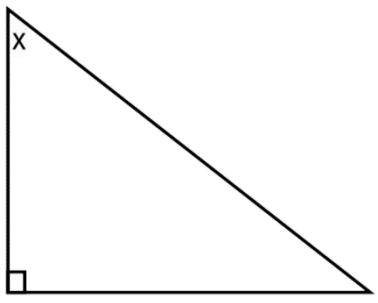
Hypotenuse: This is the longest side that is always opposite the right angle.

Opposite: This side is opposite the angle involved in the problem.

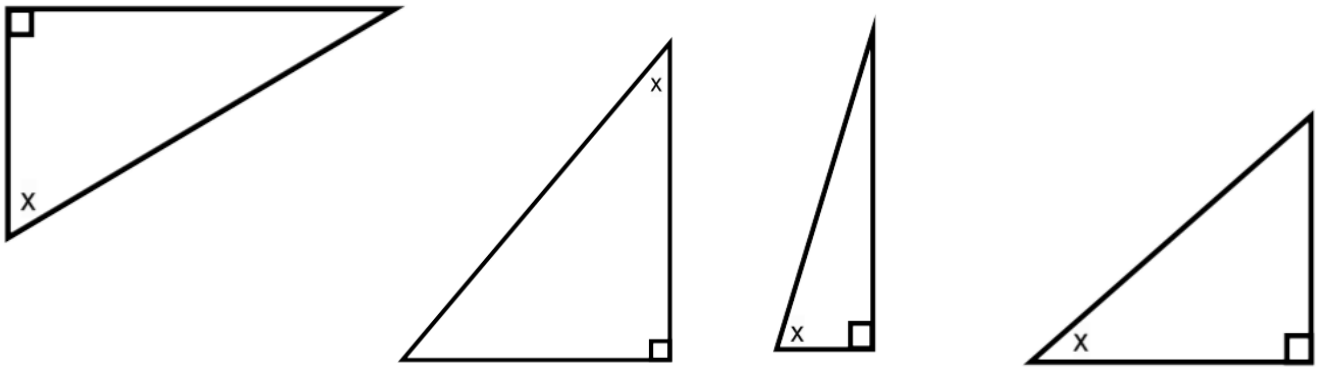
Adjacent: This is the last side, next to the angle.

Label the sides of the right triangle using the words opposite, adjacent, and hypotenuse based off angle x.





You try! Label the three sides of each triangle using H, O and A.



3 Trig Functions:

SOH-CAH-TOA

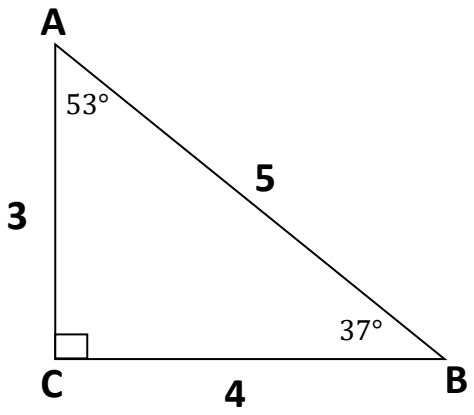
$$S = \frac{O}{H} \qquad C = \frac{A}{H} \qquad T = \frac{O}{A}$$

$$\text{Sin } x = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cos } x = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tan } x = \frac{\text{opposite}}{\text{adjacent}}$$

1. Label H (Hypotenuse is always across from the right angle)
2. Label A
3. Label O
4. Cross out unused side
5. Choose correct formula with the remaining two letters.
6. Plug in values



Find the following trigonometric ratios:

sin ∠A:

sin ∠B:

cos ∠A:

cos ∠B:

tan ∠A:

tan ∠B:

What do you notice about sin ∠A and cos ∠B?

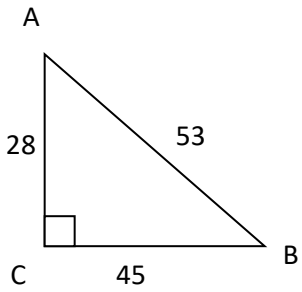
$\boldsymbol{\sin \angle A = \cos \angle B}$

$\boldsymbol{\angle A + \angle B = 90°}$

Finding Trigonometric Ratios:

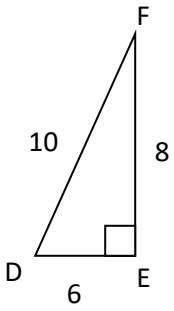
Find the appropriate trig ratio for each example. Keep each ratio as a fraction.

1) Find the sin of $\angle A$

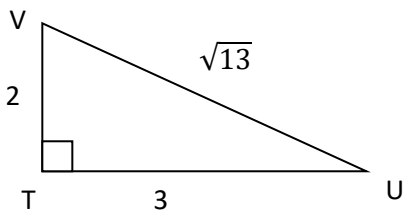


2) Find the cos of $\angle D$

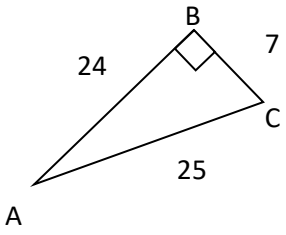
Label H, A, O



3) What is the tan of $\angle V$?



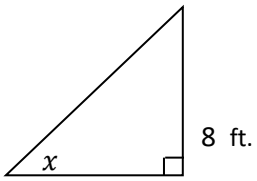
4) Find the cos of $\angle A$



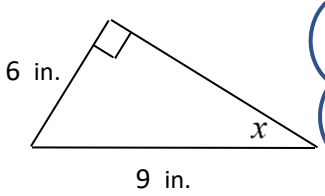
Finding Angle Measures:

Find the value of the indicated angle. Round each angle to the nearest degree.

1)



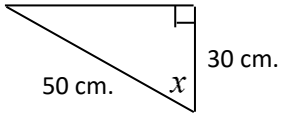
2)



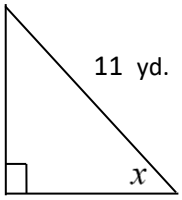
-Put calculator in **"Degree Mode"**

-Use **inverse** trig functions!

3)



4)



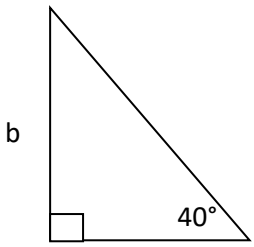
Finding Side Lengths:

Find the length of each missing side to the **nearest hundredth**:

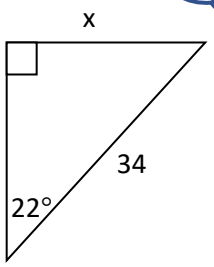
-Put trig function over 1

-Cross Multiply

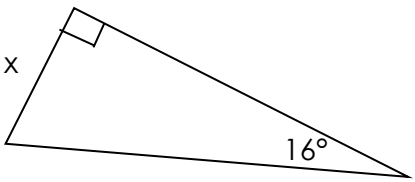
1)



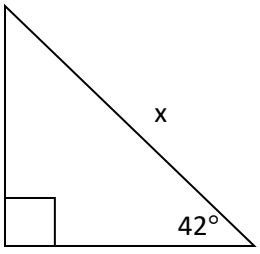
2)



3)



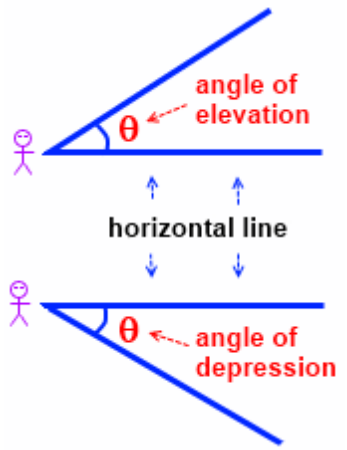
4)

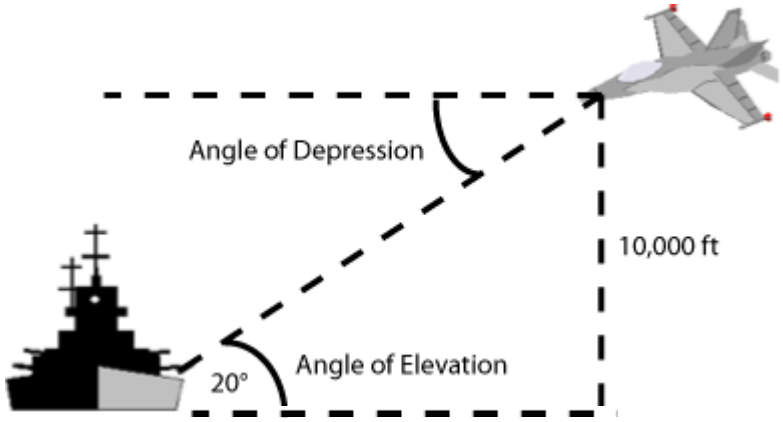


Station 1 – Mini Lesson

Angle of Elevation vs. Angle of Depression

Angles of Elevation and Depression are always formed by a horizontal line and the line of sight.





Angle of Elevation: Measured from the _____. Always _____

Angle of Depression: Measured from the _____. Always _____

Angle of Elevation = Angle of Depression

Why?

Practice Problems:

1) A tree casts a shadow that is 15 feet long. The angle of elevation from the end of the shadow to the top of the tree is 66°. Determine the height of the tree, to the *nearest foot*.

2) A bird sits on top of a lamppost and spots a worm on the ground. The angle of depression from the bird to the worm is 35°. The distance from the bird to the worm is 25 meters. How tall is the lamppost to the nearest meter?

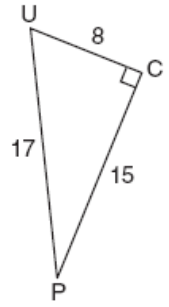
3) A boy flying a kite lets out 392 feet of string, which makes an angle of elevation of 52° with the ground. Assuming the string is tied to the ground, find to the nearest foot, how high the kite is above the ground.

Challenge:

4) A person stands 52 feet from the base of a tree. The angle of elevation from his eye, which is 6 feet above the ground, to the top of the tree is 48°. Find the height of the tree to the nearest foot.

Station 2 – At your Desk (Independent Practice)

1) The diagram at the right shows right triangle *UPC*.

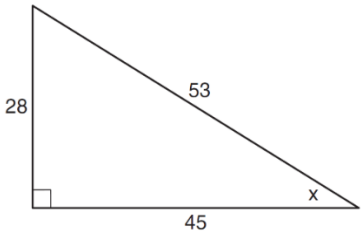


Which ratio represents the sine of $\angle U$?

1) $\frac{15}{8}$ 2) $\frac{15}{17}$ 3) $\frac{8}{15}$ 4) $\frac{8}{17}$

2) Which ratio represents $\cos x$ in the right triangle shown below?

Label H, A, O

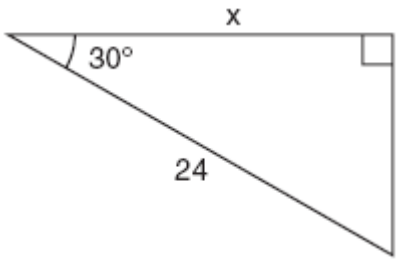


1) $\frac{28}{53}$ 2) $\frac{28}{45}$ 3) $\frac{45}{53}$ 4) $\frac{53}{28}$

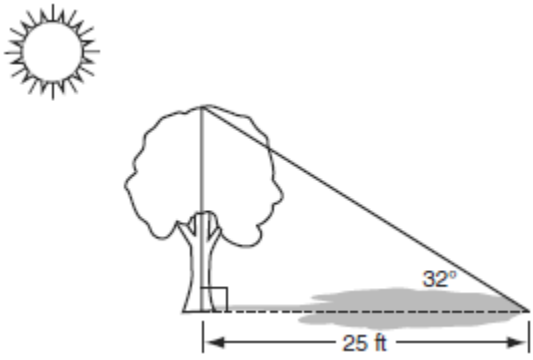
3) Find the value of x to the *nearest tenth of a foot.*

-Put trig function over 1

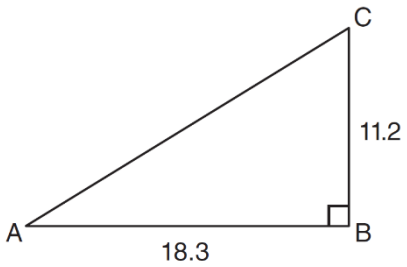
-Cross Multiply



4) Find the height of the tree to the *nearest hundredth of a foot.*



5) What is the measure of $\angle A$, to the *nearest tenth of a degree*?

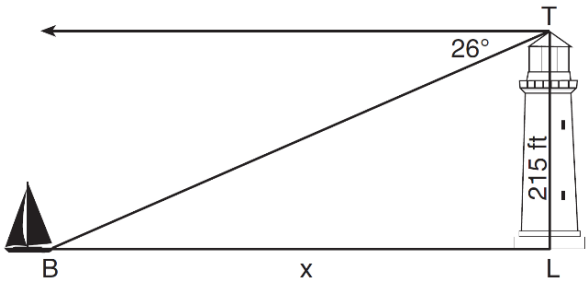


-Put calculator in **"Degree Mode"**

-Use **inverse** trig functions!

<u>If you already did the mini lesson, try the question below:</u>

6) The top of a lighthouse, T, is 215 feet above sea level, L, as shown in the diagram below. The angle of depression from the top of the lighthouse to a boat, B, at sea is 26°. Determine, to the nearest foot, the horizontal distance, x, from the boat to the base of the lighthouse.



Station 3 – Technology

Directions:

1. Login to Delta Math

2. Complete the "Cofunctions Practice"

    -Cofunction Equations

3. You must answer 5 questions correct

4. Finish Early? Keep trying more problems! Or go back and practice a skill from a previous unit

Checklist:

☐ Cofunction Equations – 5 questions

$\sin \angle A = \cos \angle B$

$\angle A + \angle B = 90°$

Stuck? Look at an example with the answer below:

If the angles are represented in degrees, find both angles:

$$\sin(x + 39) = \cos(6x - 12)$$

Angle 1: ☐° Angle 2: ☐° Submit Answer

When Sine and Cosine are equal → Angles add to 90

$$(x + 39) + (6x - 12) = 90$$
$$7x + 27 = 90$$
$$7x = 63$$
$$x = 9$$

Angle 1: $x + 39 = 9 + 39 = 48°$ Angle 2: $6x - 12 = 6(9) - 12 = 42°$

Station 4 – Hands On

Let's Explore Co-Functions without the use of a calculator! (Crazy!)

The chart to the right shows the approximate values of trig functions for different angle measures.

Use the chart to the right to find the following values. Highlight as you go!

1. sin (20°) = .3420 cos (70°) = .3420

2. sin (80°) = _____ cos (10°) = _____

3. cos (15°) = _____ sin (75°) = _____

4. cos (37°) = _____ sin (53°) = _____

What pattern did you notice? (Hint: add up the angle measures in each problem…)

If two angles are **complementary**, then:

sin (angle) = cos (90° – angle)

$$\sin \angle A = \cos \angle B$$

$$\angle A + \angle B = 90°$$

Practice: *find the acute angle that makes the equation true.*

1. sin(15°) = cos (75°) 2. sin(40°) = cos (_____)

$90 - 15 = 75°$

3. cos(83°) = sin(_____) 4. cos(21°) = sin (_____)

5. sin(90°) = cos(_____) 6. cos(45°) = sin (_____)

Deg	Sine (sin)	Cosine (cos)	Tangent (tan)	Deg	Sine (sin)	Cosine (cos)	Tangent (tan)
0	0.0000	1.0000	0.0000	45	0.7071	0.7071	1.0000
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.5000	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1.0000	90	1.0000	0.0000	undefined

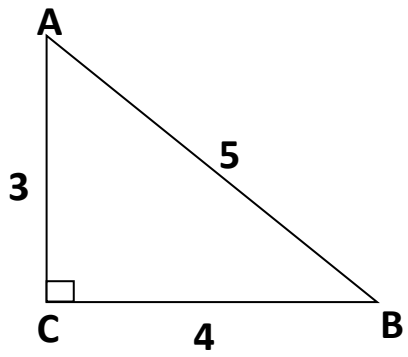
Teacher Materials

Exit Pass: Project the statements on a screen and have the class show thumbs up or thumbs down for true or false. After each question, discuss the correct answer as a class. If false, correct the statement to make it true!

1. The Angle of Depression equals the Angle of Elevation

2. If $\sin \angle A = \cos \angle B$, then $\angle A = \angle B$

Use the triangle to the right for questions 3 and 4.



3. $\sin \angle A = \frac{4}{5}$

4. $\cos \angle B = \frac{5}{4}$

Lesson 4: Three-Dimensional Shapes

G. The Big Idea

3-dimensional shapes can be explored using cross sections, revolutions, volume, and Cavalieri's Principle.

H. NYS Next Generation Standards

GEO-G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
GEO-G.GMD.4 Identify the shapes of plane sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
GEO-G.GMD.2 Give an informal argument using Cavalieri's principle for the formulas for volume of a sphere and other solid figures.

I. Student Learning Targets

- I can apply volume formulas to three-dimensional shapes to find different values.
- I can identify the shape created by a cross section of a three-dimensional object.
- I can identify a three-dimensional object that is created by revolving a two-dimensional shape around a line.
- I can understand Cavalieri's principle and how it applies to volumes of shapes with different bases.

J. Lesson Timeline (80 minute class period)
➔ Extra 2 minutes included for transition time between lesson/stations

Anticipatory set (5 minutes)

- Students will use their prior life experiences to examine cross sections of every day food items

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Whole class lesson (20 minutes)

- Teacher will walk the students through the lesson on cross sections, volumes, and revolving 2-dimensional shapes (first 5 pages in lesson)
- Hand out lesson notes separate from station packet- students should keep notes

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Stations (48 minutes total)

Station 1 – Mini-Lesson (12 minutes)

- Clean up any misconceptions about the group lesson
- Small groups will work with the teacher to apply their knowledge of volume to see how it relates to Cavalieri's Principle

Station 2 – At your desk (12 minutes)

- Students will independently practice finding the volume of a cylinder, triangular prism, and rectangular prism
- Students will independently practice Cavalieri's Principle (if already visited Station 1)

Station 3 – Technology (12 minutes)

- Students will work through practice problems on Delta Math. They will be asked to identify the cross section created by cutting a plane through a 3-dimensional shape.
- Delta Math Skills: "Identify Cross Sections of Solids"

- Students must answer 10 questions correctly from the skill. Students can continue to practice after they complete the requirement.

Station 4 – Hands On (12 minutes)

- Students will create their own manipulative by taping a 2-dimensional shape to the side of a pencil
- When students spin the pencil fast between their palms, they will see the 3-dimensional shape created

---

Exit pass (5 minutes)

- Have one person in each group demonstrate their 2-dimensional shape revolving around the pencil. Have the rest of the class decide what 3-dimensional shape is being created.

K. Assessment

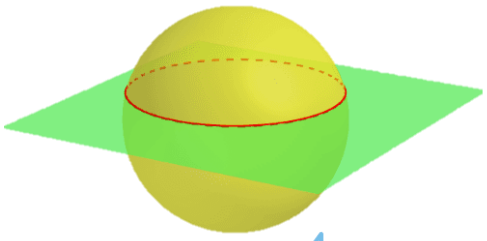
- The student station packet will be collected. The teacher will assess individual student learning from the Delta Math data as well as their work shown in the station packet.

L. Materials

- Laptops, student note packets, calculators, pencils, scissors, blank cardstock paper, tape

Lesson 4

3D Geometry



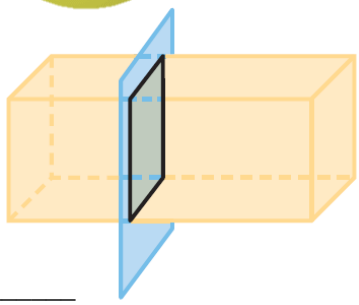
Warm-up: Think about it!

1. What shape would you see if you sliced an orange?

_____

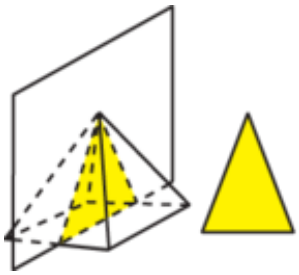
2. What shape would you see if you sliced a loaf of bread?

_____



Cross Section: _____

Different Cross Sections of a Rectangular Pyramid:



_____

Perpendicular to the base

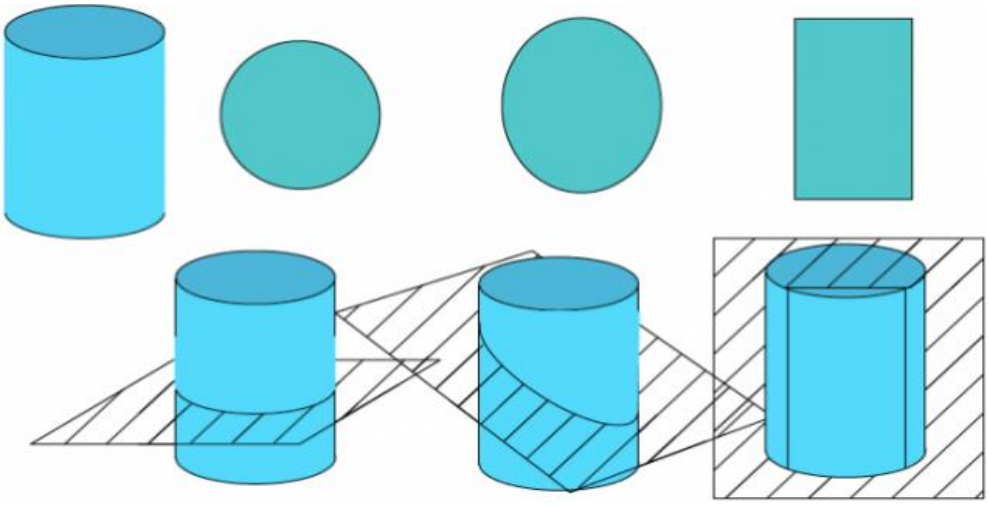
_____

Parallel to the base

_____

Random Cross Section

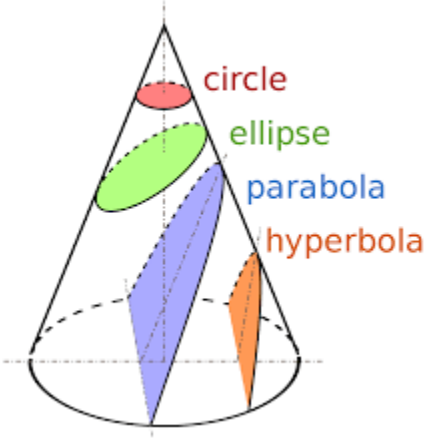
Different Cross Sections of a Cylinder:

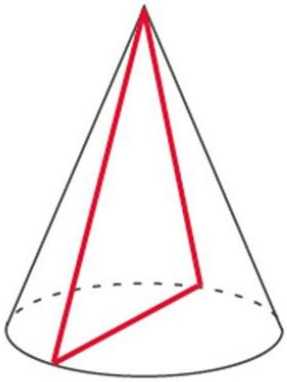


Parallel to the base

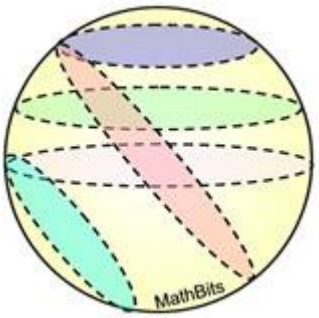
Perpendicular to the base

Different Cross Sections of a Cone:





Cross Sections of a Sphere
(all circles)



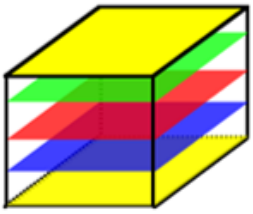
1) A plane intersects an octagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

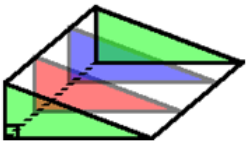
1) triangle
2) trapezoid
3) octagon
4) rectangle

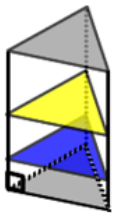
2) A two-dimensional cross section is taken of a rectangular prism. Which of the following cannot be the two-dimensional cross section?

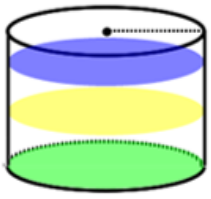
1) rectangle
2) triangle
3) square
4) circle

Cross Sections and Volume









Base area times ***height*** is a direct result of STACKING congruent cross sections on top of each other to "fill" in the shape. Above are a few visual examples of how cross sections are used to fill the shape and create volume. Try to think about the volume formulas for each.

General Prism Volume Formula:

$$V = Bh$$

$$B = area\ of\ Base, \qquad h = height\ of\ prism$$

Volumes of Prisms and Cylinders

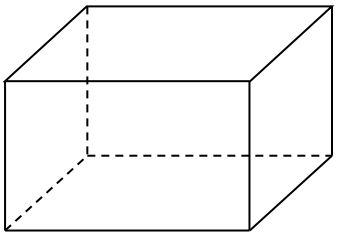
Volume: _____

Rectangular Prism:

Volume Formula

$V = Bh$

where *B = AREA of the base*



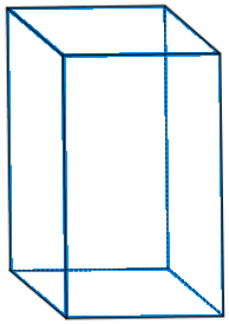
The base of a rectangular prism is a _____

so the formula is _____.

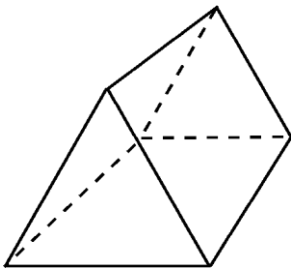
Practice: Find the volume of each rectangular prism below.

1) A rectangular prism with the dimensions 4 in., 6 in., 8 in.

2)



Triangular Prism:



Volume Formula

$V = Bh$

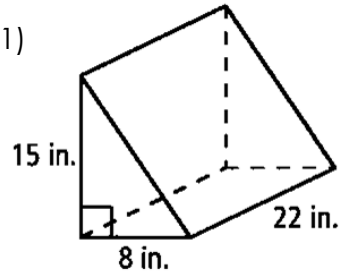
where *B = AREA of the base*

The base of a triangular prism is a _____

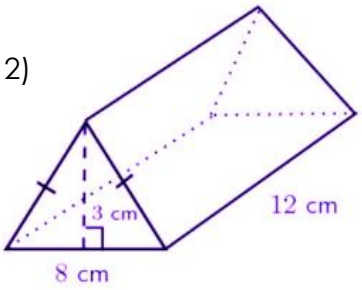
so, the formula is _____.

Practice: Find the Volume of each triangular prism below:

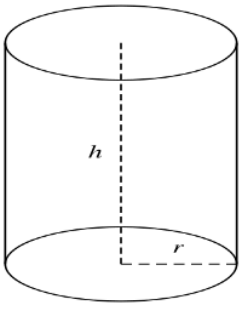
1)



2)



Cylinder:



Volume Formula

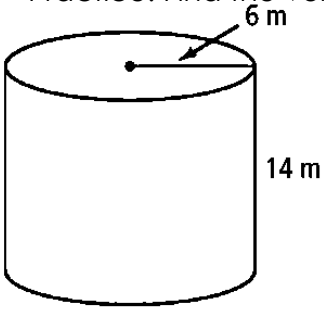
$V = Bh$

where *B = AREA of the base*

The base of a cylinder is a _____

so, the formula is _____.

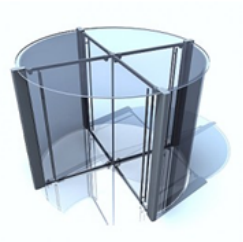
Practice: Find the volume of the cylinder in terms of π and to the nearest hundredth



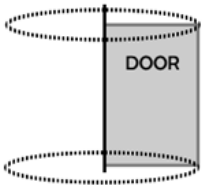
Revolving 2D Shapes to get volume

Instead of stacking congruent cross sections to form the volume of a solid, another technique is to rotate it about an axis. Thinking about a revolving door and how it swings (rotates) about the center creating a cylindrical shape.

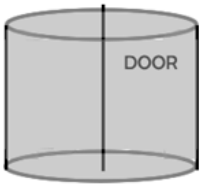
Revolving Door



Cross Section of Door



Imagine as the door revolves it filling the entire space of the cylinder



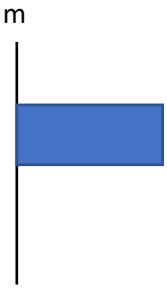
Describe the solid that is formed by rotating each of these figures about line m!

1.

Name:

Sketch:

2.



Name:

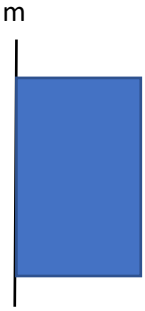
Sketch:

3.

Name:

Sketch:

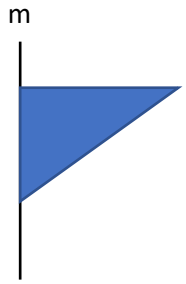
4.



Name:

Sketch:

5.



Name:

Sketch:

6.

Name:

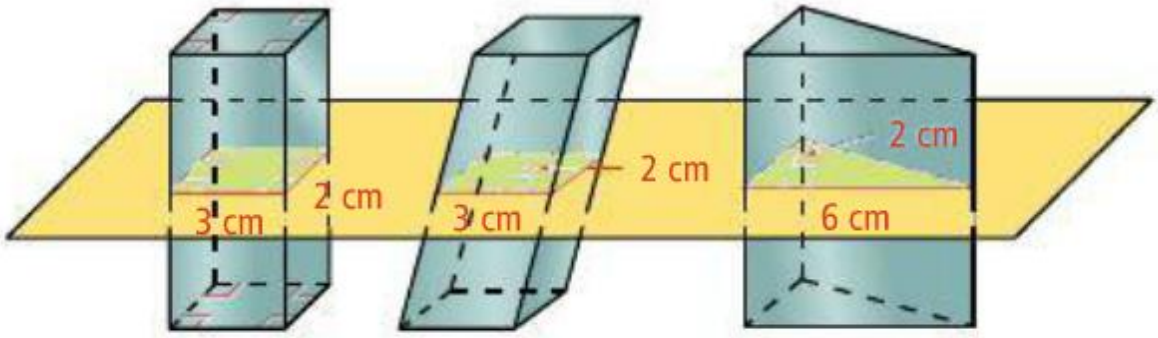
Sketch:

Station 1 – Mini Lesson

take note

Theorem 11-5 Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.



A = bh

A = (3)(2)

A = bh

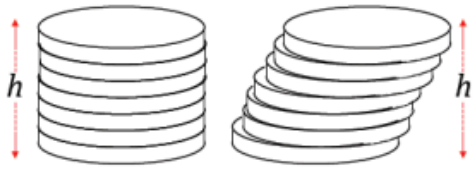
A = (3)(2)

A = bh/2

A = ((6)(2))/2

A = 12/2

1) Explain how Cavalieri's Principle applies to this stack of nickels.



Regents Questions:

1. A rectangular prism has a base with a length of 25, a width of 9, and a height of 12. A second prism has a square base with a side of 15. If the volumes of the two prisms are equal, what is the height of the second prism?

2. Two prisms have equal heights and equal volumes. The base of one is a pentagon and the base of the other is a square. If the area of the pentagonal base is 36 square inches, how many inches are in the length of each side of the square base?

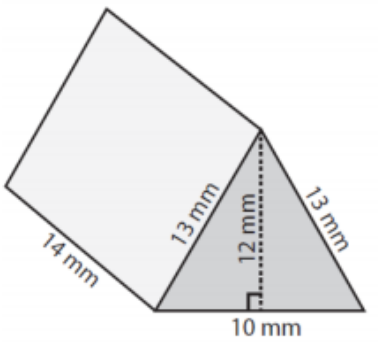
3. Tim has a rectangular prism with a length of 10 centimeters, a width of 2 centimeters, and an unknown height. He needs to build another rectangular prism with a length of 5 centimeters and the same height as the original prism. The volume of the two prisms will be the same. Find the width, in centimeters, of the new prism.

4. Two prisms with equal altitudes have equal volumes. The base of one prism is a square with a side length of 5 inches. The base of the second prism is a rectangle with a side length of 10 inches. Determine and state, in inches, the measure of the width of the rectangle.

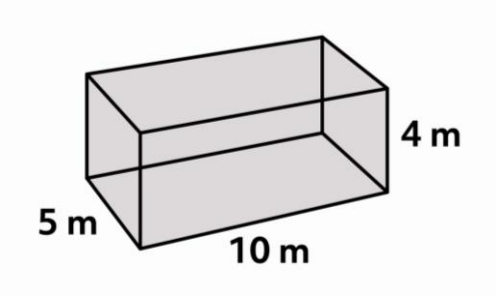
Station 2 – At your desk

1. A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of 6 feet and a height of 1 foot. The pool is going to be filled to the top with water. Determine the volume of the water in the pool, to the nearest cubic foot.

2. What is the volume, in cubic feet of the prism, rounded to the nearest cubic foot?



3. What is the volume of the rectangular prism?



<u>If you already did the mini-lesson, try the question below:</u>

4. Two prisms with equal altitudes have equal volumes. The base of one prism is a square with a side length of 5 inches. The base of the second prism is a rectangle with a side length of 10 inches. Determine and state, in inches, the measure of the width of the rectangle.

Station 3 – Technology

Directions:

1. Login to Delta Math

2. Complete the "Cross Sections Practice"

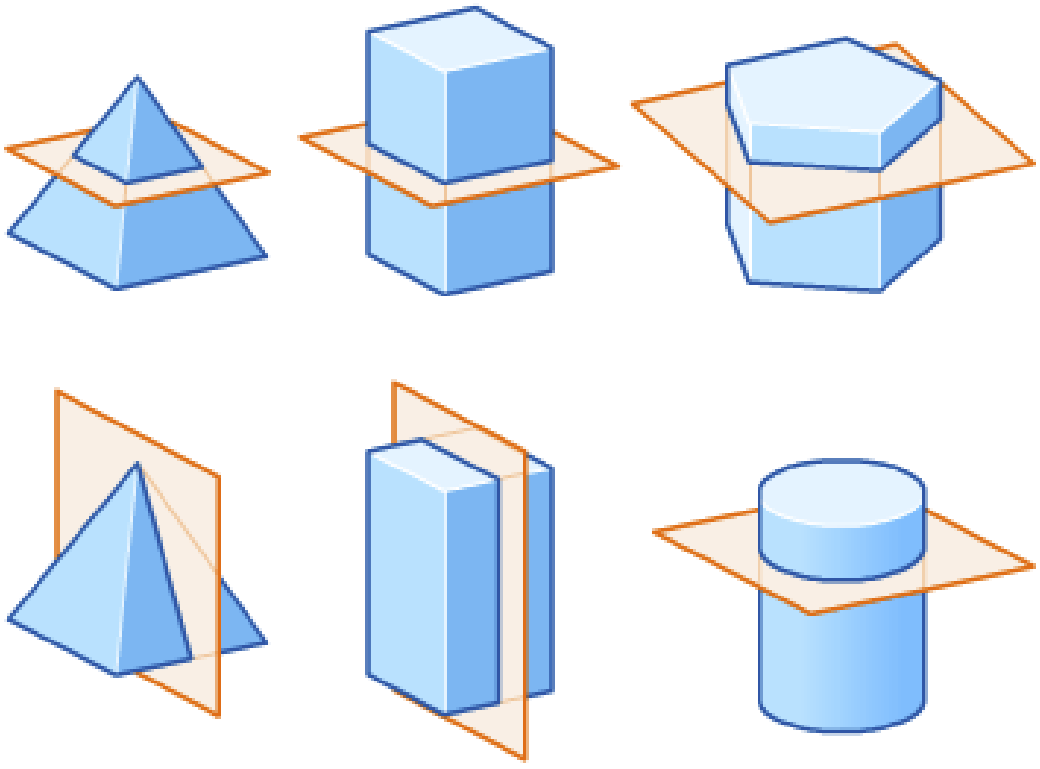
    -Identify Cross Sections of Solids

3. You must answer 10 questions correct

4. Finish Early? Keep trying more problems! Or go back and practice a skill from a previous unit

Checklist:

☐ Identify Cross Sections of Solids – 10 questions



Station 4 – Hands On

We are going to revolve 2D shapes to create 3D objects!

Directions:

1. Pick one shape to revolve (half circle, rectangle, or right triangle)
➔ Try to have each person in your group pick a different shape!

2. Cut out your shape on cardboard (the smaller the better)

3. Tape your shape to the side of a pencil toward the top

Answer the following questions:

1. The 2-dimensional shape I picked was _____

2. When I spin the pencil in my hands very fast, I see a _____ (3-dimensional shape)

3. Upload a picture of your pencil to the drop box online!

Classroom Implementation

To ensure student success with this curriculum, these lessons are to be taught at a high school level after students have successfully completed some background in Geometry from middle school. These four lessons do not need to be taught in any specific order, as they are all stand-alone lessons from different units in Common Core Geometry. For each lesson, you need to split your class into four groups. It works best if you have students of similar ability level working at the same station so you can provide appropriate scaffolding during the mini lesson. In addition, starting your lowest-level group at the Mini-Lesson and your highest-level group at the Independent Practice will allow students to be appropriately challenged.

Although the blended learning station rotation model seems like a rigid schedule with only 12 minutes at each station, you can make many different adjustments to fit the needs of your classroom. This time also varies depending on the school schedule. The only station students "must" be at during a specific time is Station 1 with the teacher. Thus, you could give students who are not at Station 1 a choice to complete any other station they would like when it is not their group's turn to be called to Station 1. Another flexibility option is to get rid of the 12-minute rotation timer. You may need to spend more time with different groups at station 1 depending on ability level. For example, the lower group might need 15 minutes and the higher-level group might only need 9. In this case, you would just announce the next group to come up once you have completed Station 1 with the current group.

For Lesson 1 on Reflections, students may struggle the most with graphing the line of reflection. Students may need a refresher on how to graph lines in $y = mx + b$ form as well as $x = a$ and $y = a$ lines. For Lesson 2 on Centroids and Orthocenters, students may struggle the most with the new vocabulary. I suggest putting on the front board a "cheat sheet" of vocabulary terms and definitions the students may need for the lesson along with sketches. For lesson 3 on Trigonometry, students may struggle with understanding why cofunctions are equal. This may need to be reviewed

as a warm-up for the next day and re-explained to the students. One option is to take out angle of elevation and depression from the mini-lesson and change the mini-lesson to go over more cofunction properties in depth. Then, you would teach angle of elevation and depression the next day. For lesson 4 on three-dimensional shapes, try to find as many manipulatives as possible to pass around while you are teaching or while the students are working in their stations. It's sometimes difficult for students to visualize 3-dimensional objects on 2-dimensional paper. This blended learning model requires adjustments based on the needs of the learners in your classroom. Try it out as it is written in the lesson plan the first time around, and then adjust as you see fit.

Conclusion

The goal of this curriculum is to demonstrate how the blended learning station rotation model can be implemented into the geometry classroom during various units. This project provides hands on activities, technology use, and small group activities to keep students engaged and meet the needs of all learners. I hope mathematics teachers found something helpful to use or incorporate into their own classroom. The answer keys for all warm-ups, lessons, stations, and exit tickets can be found in the appendix.

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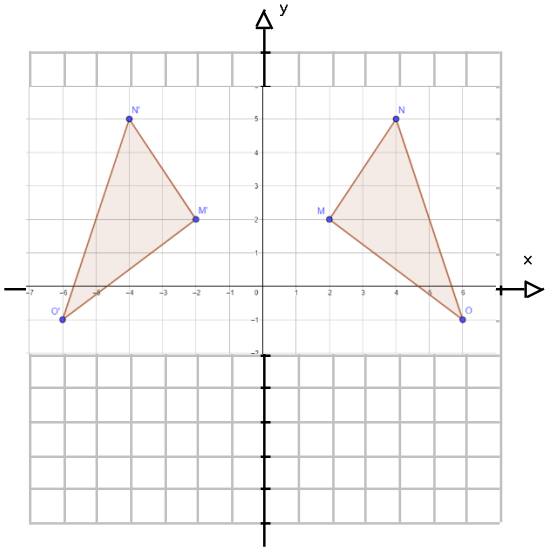
Appendix

Answer Key - Lesson 1 - Reflection Class Notes

Coordinate Geometry Given points *M* (2, 2), *N* (4, 5), and *O* (6, -1), graph △*MNO* and its reflection image as indicated. State the image point coordinates.

1. r_{y-axis}

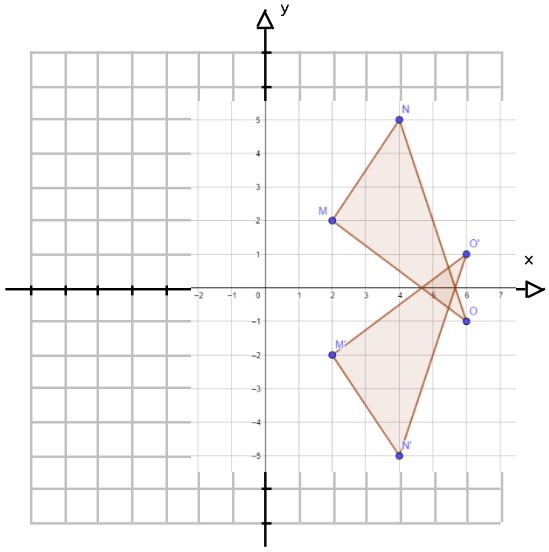


$M(2, 2) \rightarrow M'$: (-2, 2)

$N(4, 5) \rightarrow N'$: (-4, 5)

$O(6, -1) \rightarrow O'$: (-6, -1)

2. r_{x-axis}



$M(2, 2) \rightarrow M'$: (2, -2)

$N(4, 5) \rightarrow N'$: (4, -5)

$O(6, -1) \rightarrow O'$: (6, 1)

Is there a pattern?

Opposite coordinate changes sign

RULES:

To reflect over the y-axis:

y stays same, x changes sign

To reflect over the x-axis:

x stays same, y change sign

Is a line reflection considered a *rigid motion*? Why or why not?

Yes, distance and angle measure is preserved

Is orientation preserved in a line reflection? Why or why not?

No, pre-image and image have opposite letter orders

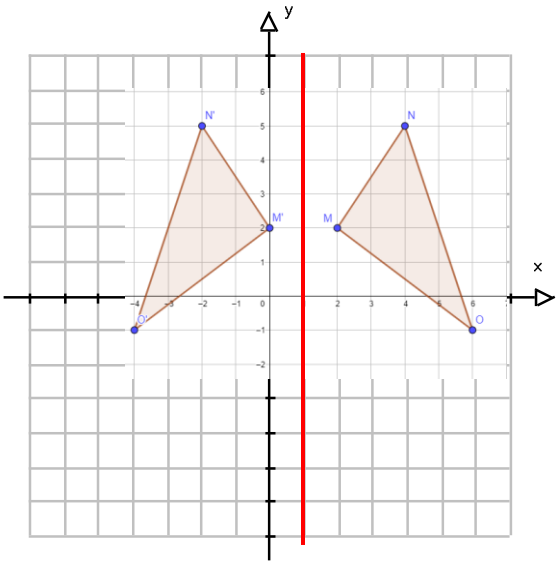
Reflecting over a Line that's Not an Axis

Steps:

1. Locate and plot the given point.
2. Locate and graph the line of reflection.
3. Move along the line PERPENDICULAR to the line of reflection until the given point and its image are equidistance from the given line. (Count the boxes away from the line of reflection, count the same number on the other side of the line.)

Coordinate Geometry Given points *M*(2, 2), *N*(4, 5), and *O*(6, -1), graph △*MNO* and its reflection image as indicated. State the image point coordinates.

3. $r_{x=1}$

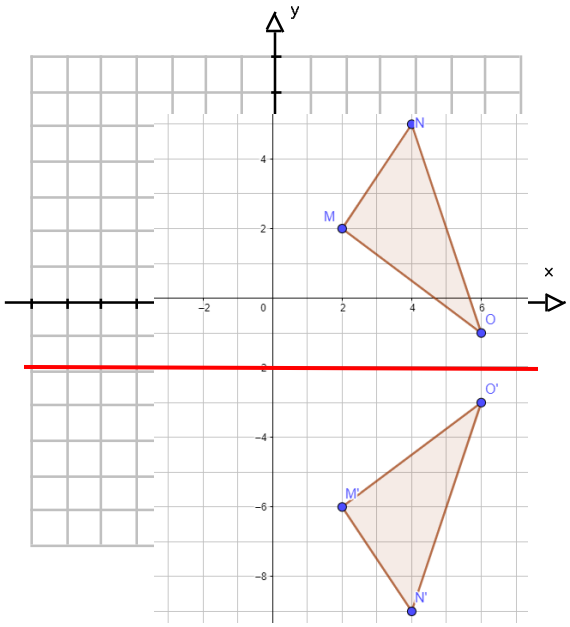


$M(2, 2) \rightarrow M'$: (0, 2)

$N(4, 5) \rightarrow N'$: (-2, 5)

$O(6, -1) \rightarrow O'$: (-4, -1)

4. $r_{y=-2}$



$M(2, 2) \rightarrow M'$: (2, -6)

$N(4, 5) \rightarrow N'$: (4, -9)

$O(6, -1) \rightarrow O'$: (6, -3)

In all the above examples, is… a) distance preserved?

yes

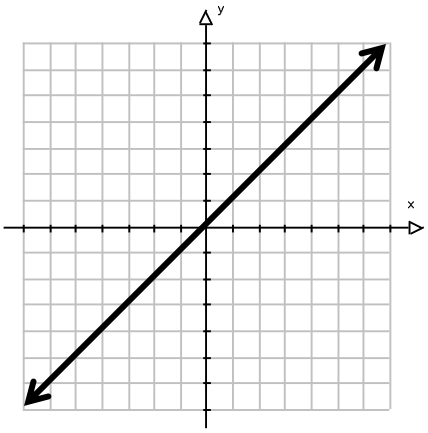
b) orientation preserved?

no

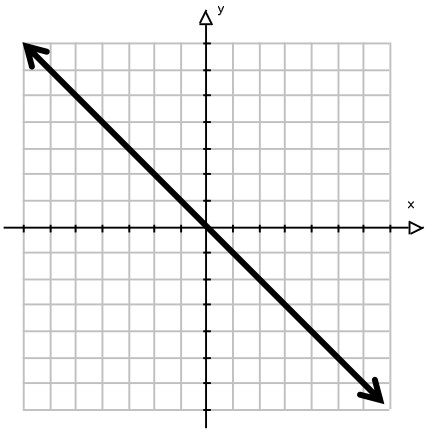
Reflecting Over Lines That Aren't Horizontal or Vertical

The two most common lines that are slanted and reflected over are:

$y = x$

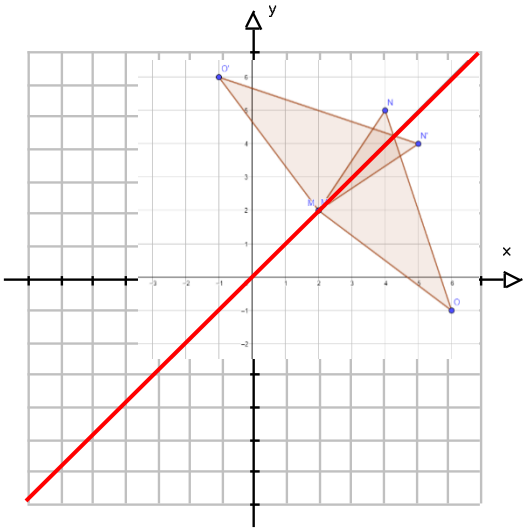


$y = -x$



Coordinate Geometry Given points *M*(2, 2), *N*(4, 5), and *O*(6, -1), graph △*MNO* and its reflection image as indicated. State the image point coordinates.

5. $r_{y=x}$



$M(2, 2) \rightarrow M'$: (2, 2)

$N(4, 5) \rightarrow N'$: (5, 4)

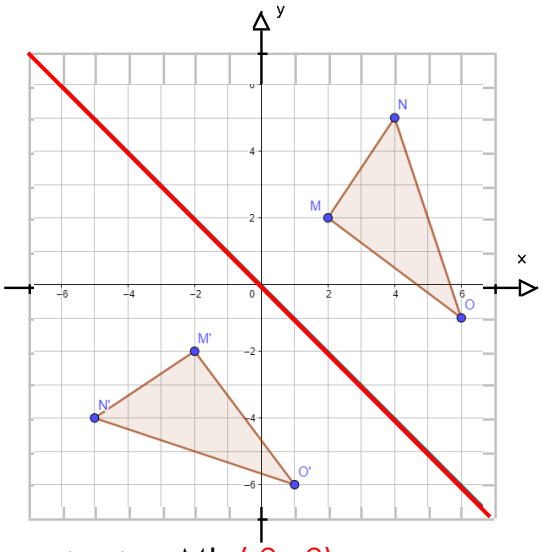
$O(6, -1) \rightarrow O'$: (-1, 6)

RULES:

To reflect over the line $y = x$,

Switch x and y

6. $r_{y=-x}$



$M(2, 2) \rightarrow M'$: (-2, -2)

$N(4, 5) \rightarrow N'$: (-5, -4)

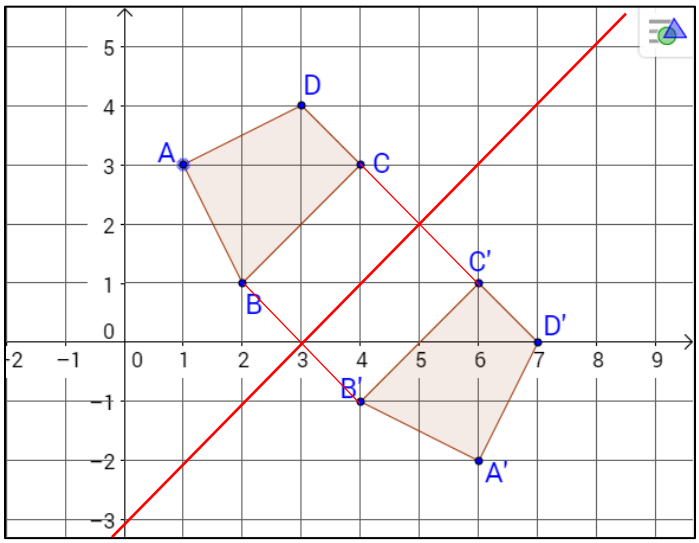
$O(6, -1) \rightarrow O'$: (1, -6)

To reflect over the line $y = -x$,

Switch and negate x and y

Writing Lines of Reflection

7. Write the equation of the line of reflection that maps *ABCD* onto A'B'C'D'.

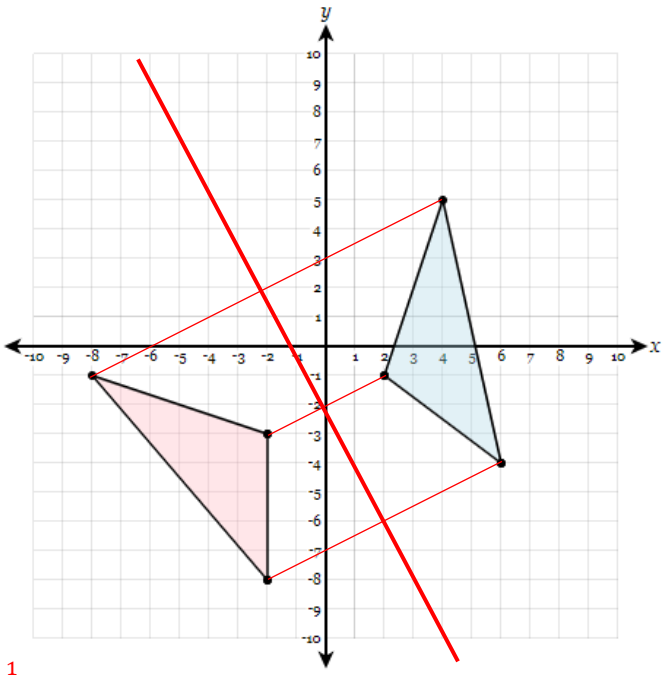


Slope: 1

Y-intercept: -3

Equation: $y = x - 3$

8.



Slope: $-\frac{1}{2}$

Y-intercept: -2

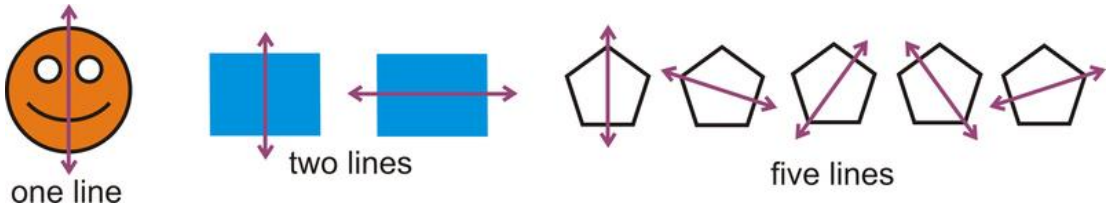
Equation: $y = -\frac{1}{2}x - 2$

Name _____

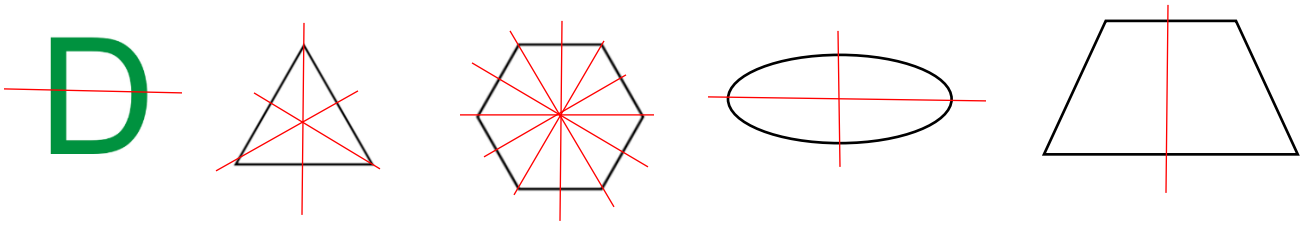
Station 1 – Mini Lesson

Symmetry: When a rigid motion maps a figure onto itself

Line Symmetry / Reflection Symmetry: If there is a reflection for which the figure is its own image.



Find all lines of symmetry for the shapes/letters below:

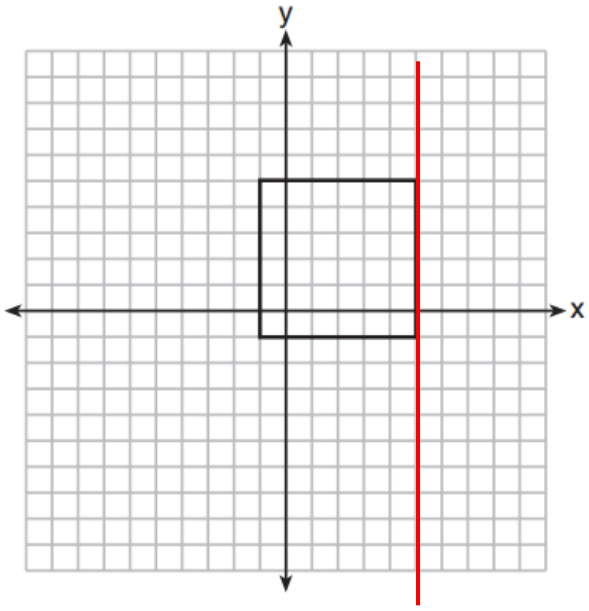


Draw your own shapes below! One with horizontal symmetry and the other with vertical symmetry:

Station 2 – At your desk

Practice:

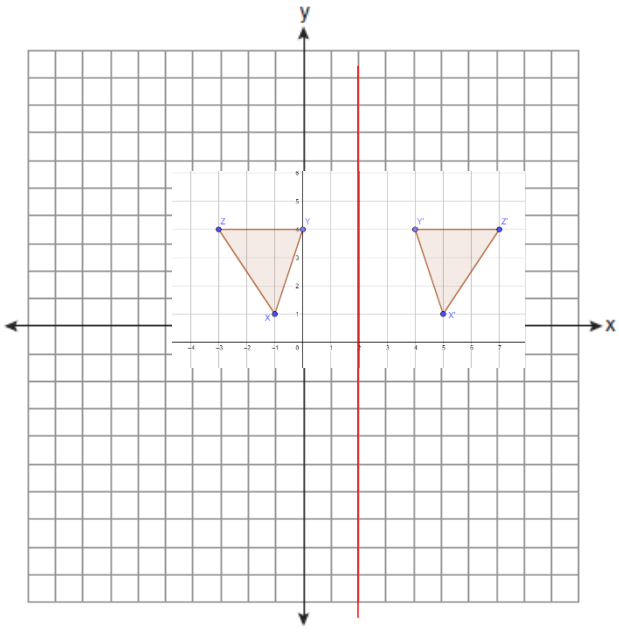
1. In the diagram below, a square is graphed in the coordinate plane.



A reflection over which line does *not* carry the square onto itself?

(1) $x = 5$
(2) $y = x$
(3) $y = 2$
(4) $y = -x + 4$

2. Triangle *XYZ*, shown in the diagram below, is reflected over the line $x = 2$. State the coordinates of $\triangle X'Y'Z'$, the image of $\triangle XYZ$.

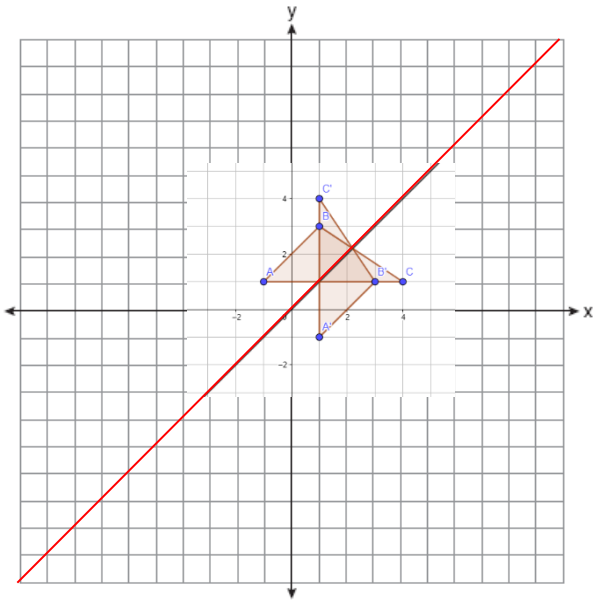


$X'(5, 1)$

Y' (4, 4)

Z' (7, 4)

3. Triangle *ABC* has vertices $A(-1,1)$, $B(1,3)$, and $C(4,1)$. The image of $\triangle ABC$ after a reflection over the line $y = x$ is $\triangle A'B'C'$. State and label the coordinates of $\triangle A'B'C'$.



4. Point $(-2,3)$ is reflected in the x-axis. In which quadrant does its image lie?

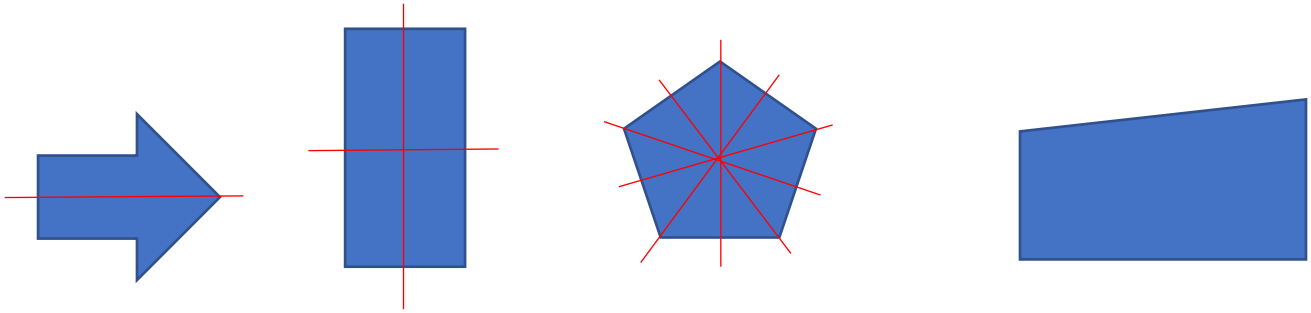
1) I
2) II
3) III
4) IV

5. Triangle *ABC* has coordinates $A(2,0), B(0,7)$ and $C(5,1)$. Under a reflection in the *y*-axis, which point remains invariant (does not move)?

5) A
6) B
7) C
8) None of the above

<u>If you already did the Station 1 mini-lesson, try the question below:</u>

6. Draw all lines of symmetry on the shapes below:



Station 3 – Technology

Directions:

1. Login to Delta Math

2. Complete the "Reflections Practice"

    - Find Line of Reflection Experimentally

    - Find Line of Reflection Methodically

3. You must answer 5 questions from each skill correct

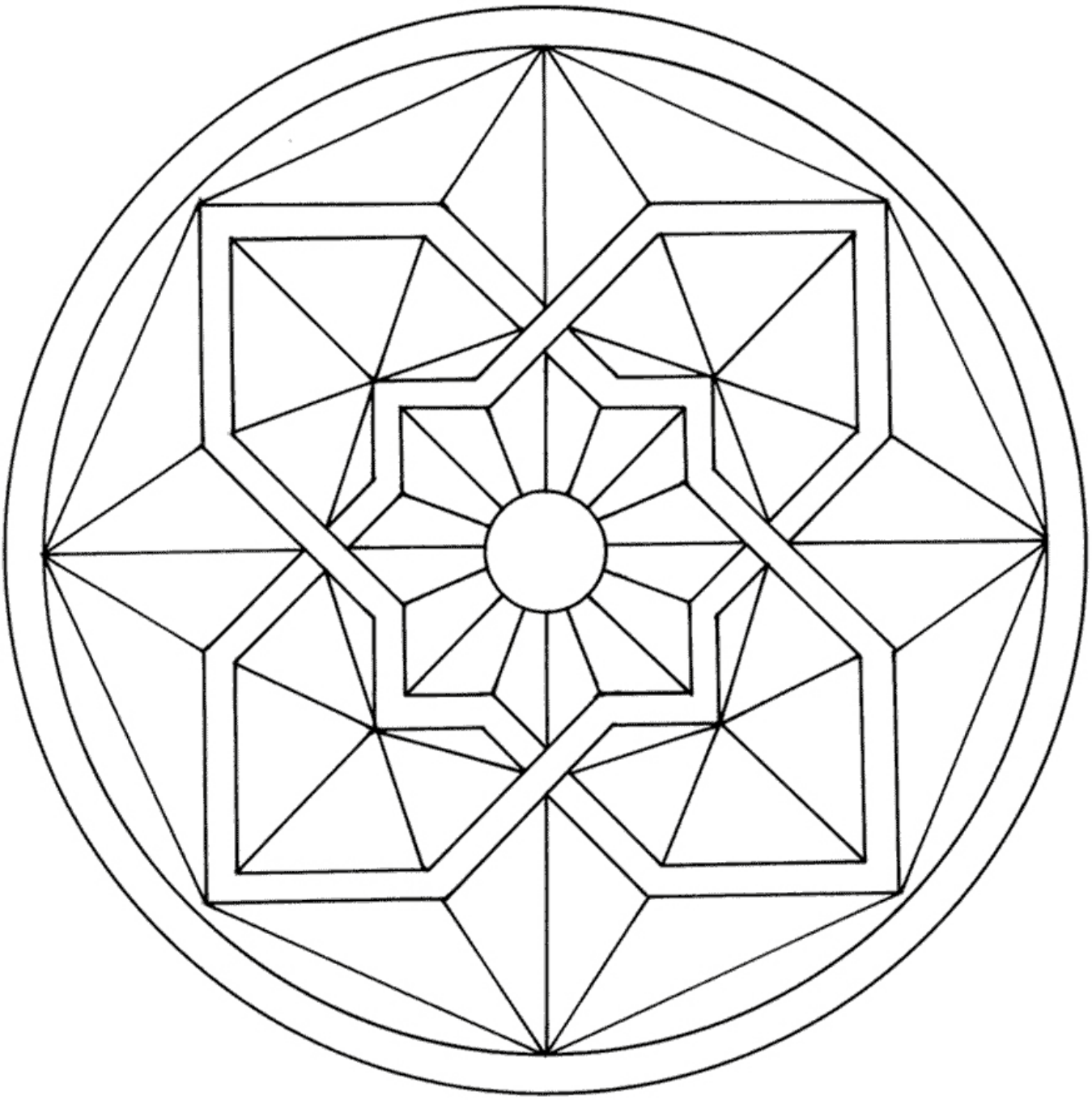
4. Finish Early? Keep trying more problems! Or go back and practice a skill from a previous unit

Checklist:

- ☑ Find Line of Reflection Experimentally- 5 questions correct
- ☑ Find Line of Reflections Methodically – 5 questions correct

Station 4 – Hands On

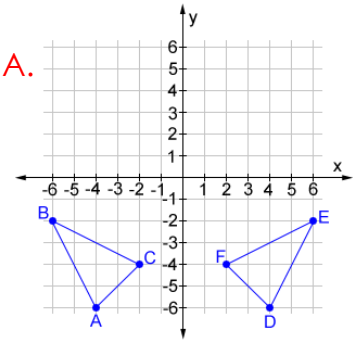
Directions: Color the mosaic below, making sure to create **reflexive symmetry.** Colored pencils are on the table! Answers will vary



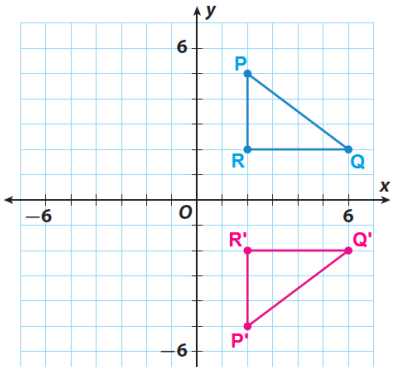
Teacher Materials

Anticipatory Set: Students will work in pairs with whiteboards. They will have 30 seconds to look at the images on the board and decide which transformation doesn't belong. Sample image sets are below:

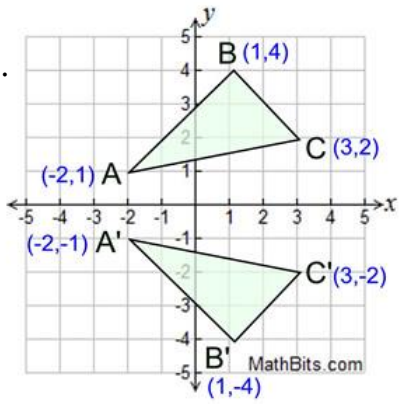
A.



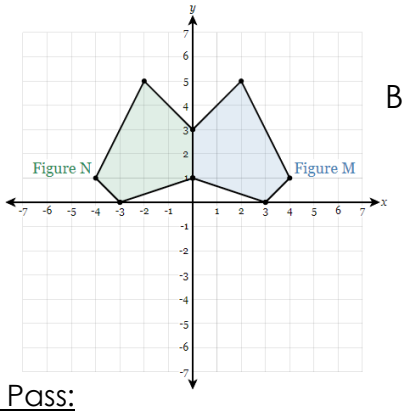
B.



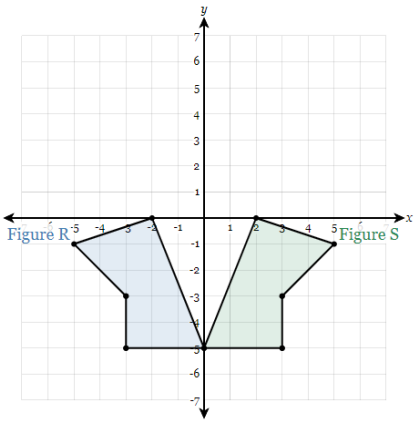
C.



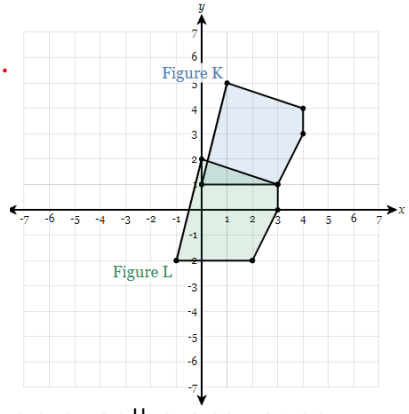
A.



B.



C.



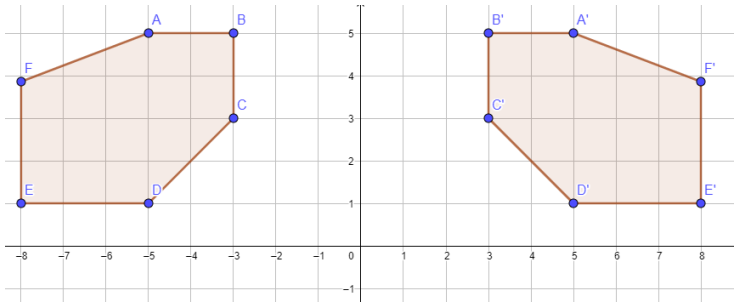
Exit Pass:

Students will complete this independently. Collect their papers, then go over the answer as a class.

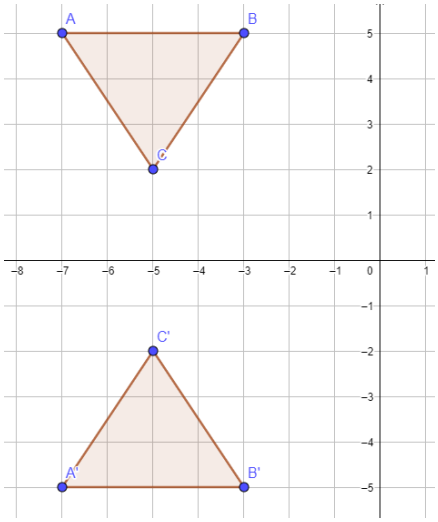
Name _____

Reflection Exit Ticket

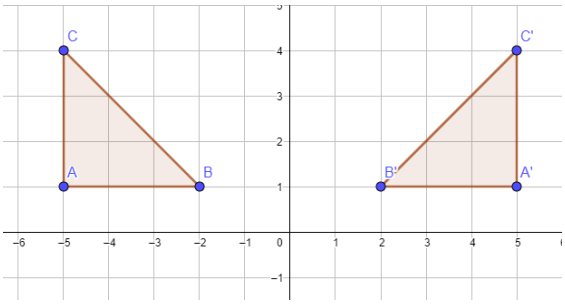
A.



B.



C.



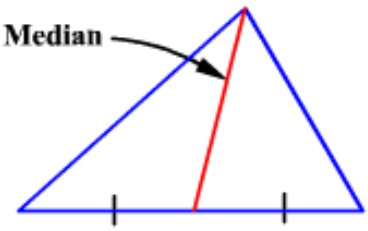
Which doesn't belong? (A, B, or C). Explain your answer.

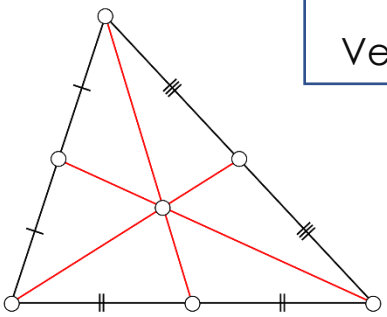
B, reflection over x-axis instead of y-axis

Lesson 2: Centroids and Orthocenters

Warm-up: Come up with your own definition of median and altitude using the pictures provided and the word bank

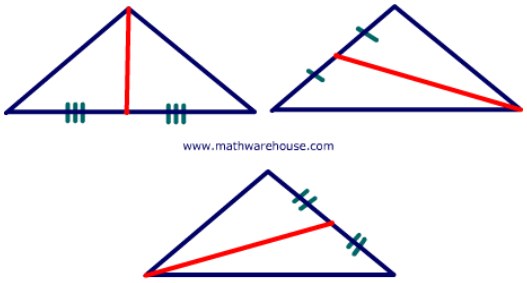
Median:





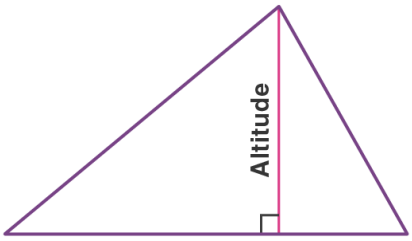
Word Bank:

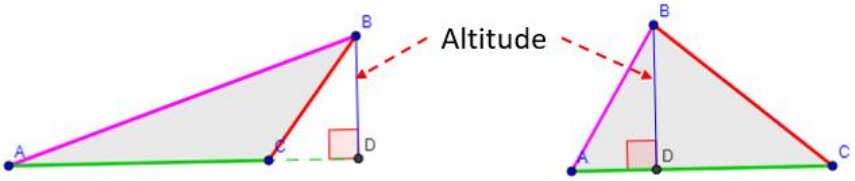
Vertex Midpoint Perpendicularly



Definition: The median connects the vertex of a triangle to the midpoint of the opposite side.

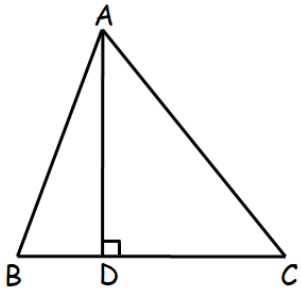
Altitude:

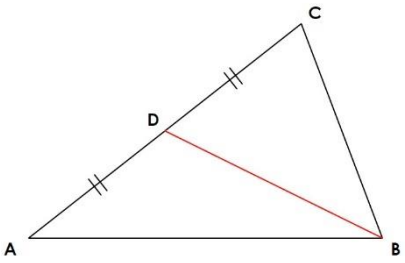


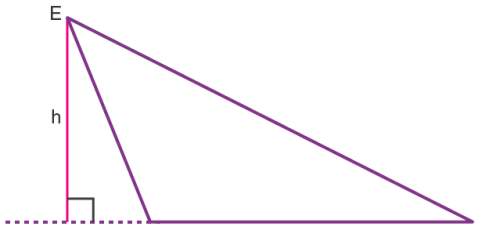


Definition: The altitude connects the vertex of a triangle to the opposite side perpendicularly.

Directions: Label the images below as altitudes or medians:



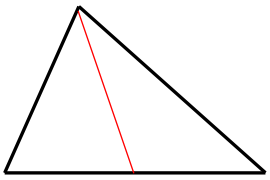


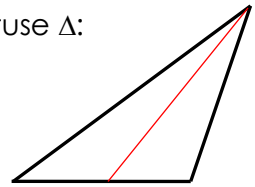


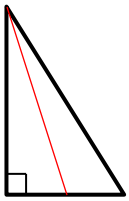
Altitude Median Altitude

Median: A line segment that connects the vertex of a triangle to the midpoint of the opposite side.

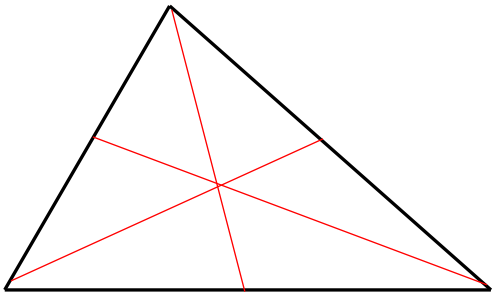
Draw a *median* for each type of triangle, starting at the upper vertex.

Acute Δ: 

Obtuse Δ: 

Right Δ: 

Every Triangle has 3 medians. Draw one from each vertex:



The concurrence of the three medians is called the centroid.

Concurrency of Medians Theorem: The medians of a triangle are concurrent at a point that is *two thirds* the distance from each vertex to the midpoint of the opposite side.

$DC = \frac{2}{3} DJ$ $EC = \frac{2}{3} EG$ $FC = \frac{2}{3} FH$

The ratio of the parts of each median is 2:1. The section of the median nearest the vertex is twice as long as the section near the midpoint of the triangle's side.

$DC : CJ = 2:1$ $CE : GC = 2:1$ $FC : CH = 2:1$

(Ex1) If DC = 8, what is the length of $\overline{DJ}$?

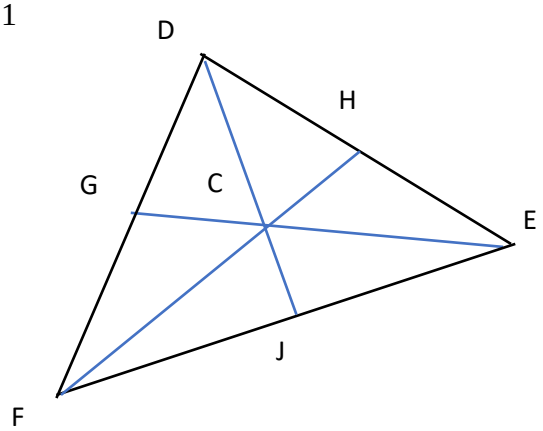
12

(Ex2) If EG = 9, what is the length of $\overline{GC}$?

3

(Ex3) If FH = 27, what is the length of $\overline{FC}$?

18

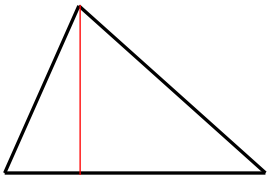


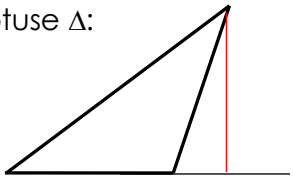
Remember, the median hits the side of the triangle at its **midpoint**

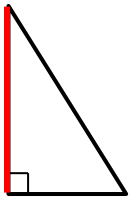
If $\overline{DH} = 3$, then $\overline{HE} = 3$ and $\overline{DE} = 6$

Altitude: A line segment that connects the vertex of a triangle to the opposite side <u>perpendicularly</u>.

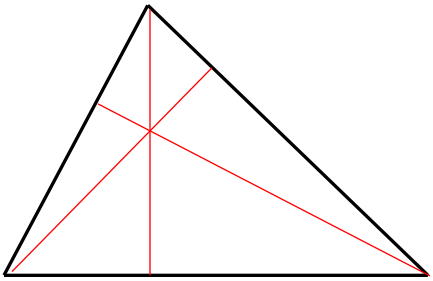
Draw an *altitude* for each type of triangle, starting at the upper vertex:

Acute Δ: 

Obtuse Δ: 

Right Δ: 

Every Triangle has 3 altitudes. Draw one from each vertex:



The concurrence of the three altitudes is called the Orthocenter.

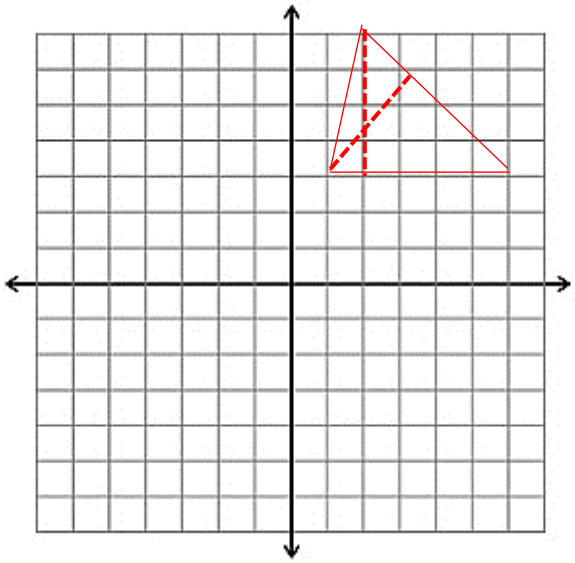
<u>Finding the orthocenter on the coordinate plane:</u>

Triangle ABC has vertices A(1,3) B(2,7) and C(6,3). What are the coordinates of the orthocenter?

Slope of BC = $-\frac{1}{1}$

Negative reciprocal: $\frac{1}{1}$

Orthocenter: (2, 4)



Hint:

Perpendicular lines have negative reciprocal slopes

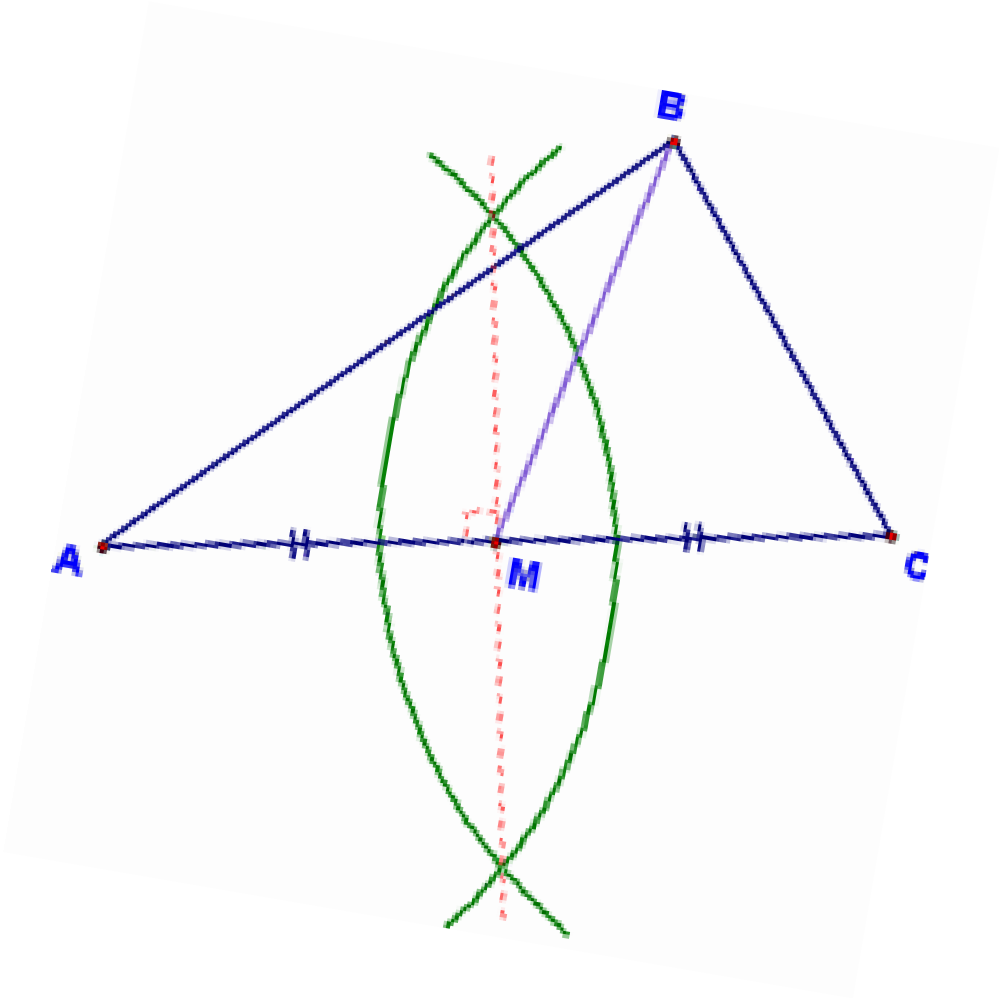
Name _____

Station 1 – Mini Lesson (Constructions)

Directions: Construct a median of ΔABC to side $\overline{AC}$ below.

Steps:

1. Construct the perpendicular bisector to find the midpoint, label it M.
2. Draw a line from the vertex B to the midpoint, M.



What do we know about $\overline{AM}$ and $\overline{MC}$?

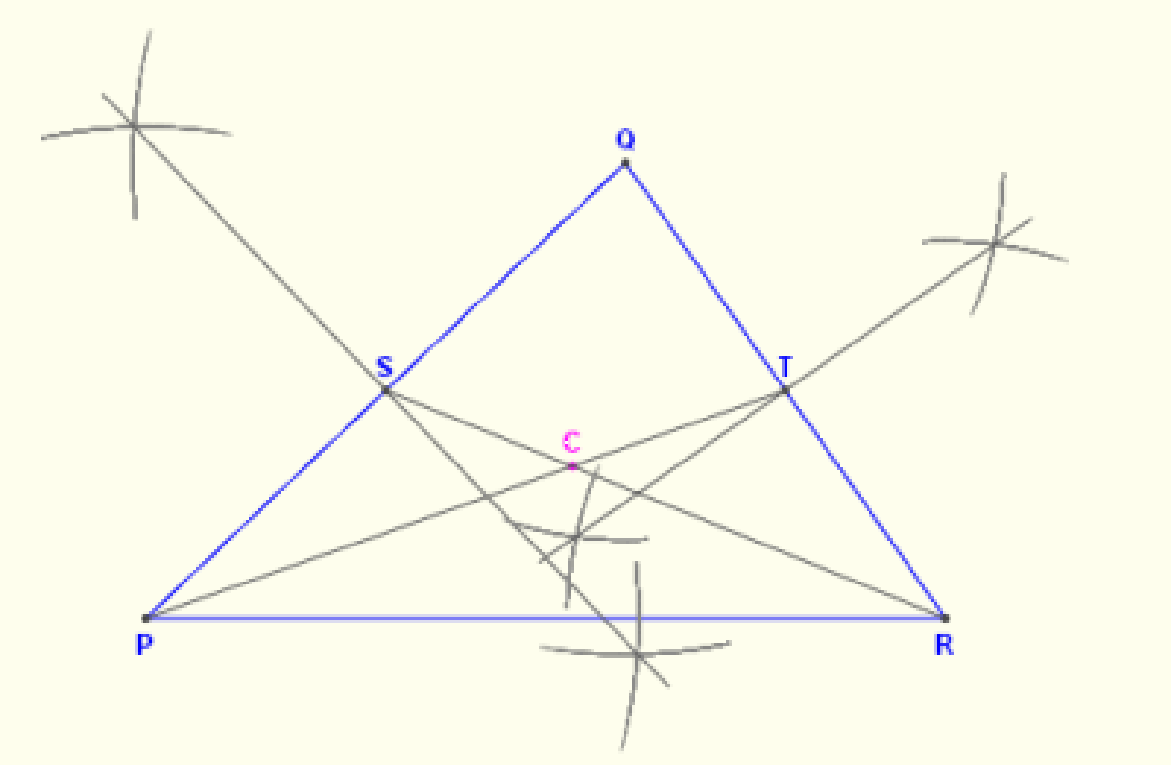
They are congruent

Directions: Construct the centroid of the triangle below with your compass and a straightedge. The centroid is where all 3 medians meet.

Steps:

1. Construct the medians

2. The point of intersection is the centroid, label it with a C.



Do we need to construct all 3 medians to find the centroid?

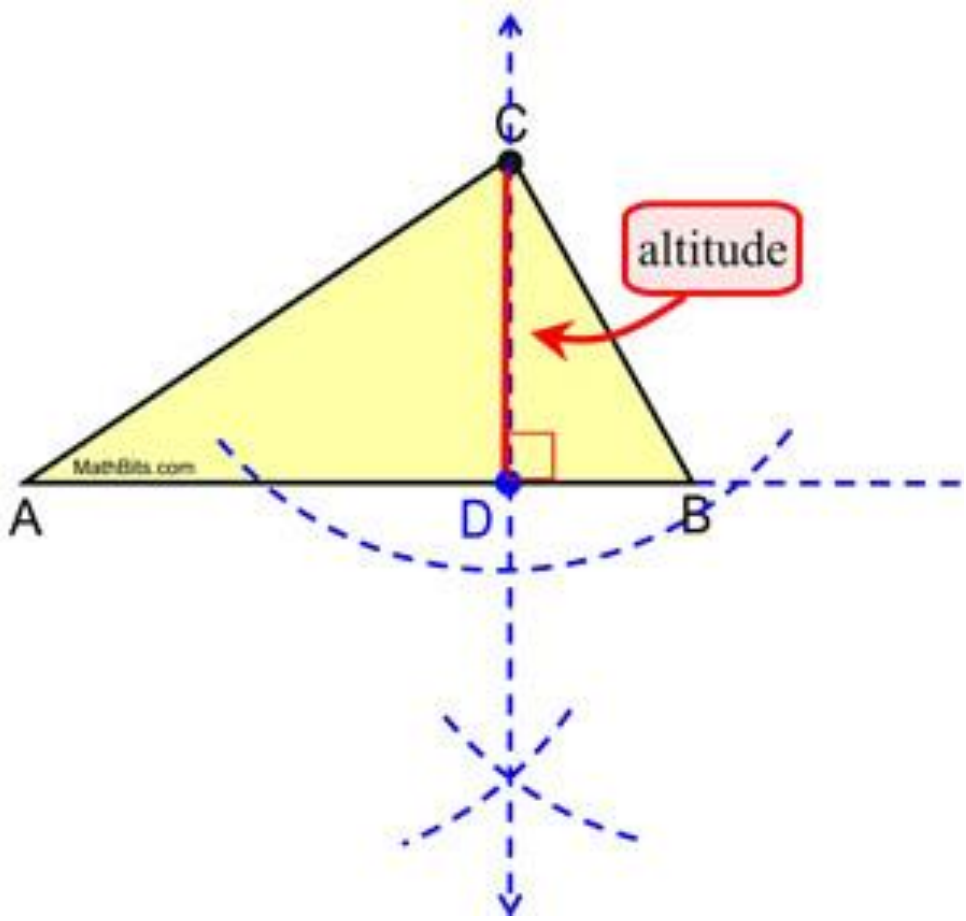
Directions: Construct an altitude of ΔABC to side $\overline{AB}$ below.

Steps:

1. Put your pointer on C, and draw an arc through side $\overline{AB}$

2. Put your pointer on the intersection points of $\overline{AB}$, then make a curvy arc below the triangle.

3. Connect C with your curvy arc below the triangle to create the altitude.



What do we know about the angle of intersection of the altitude with $\overline{AB}$?

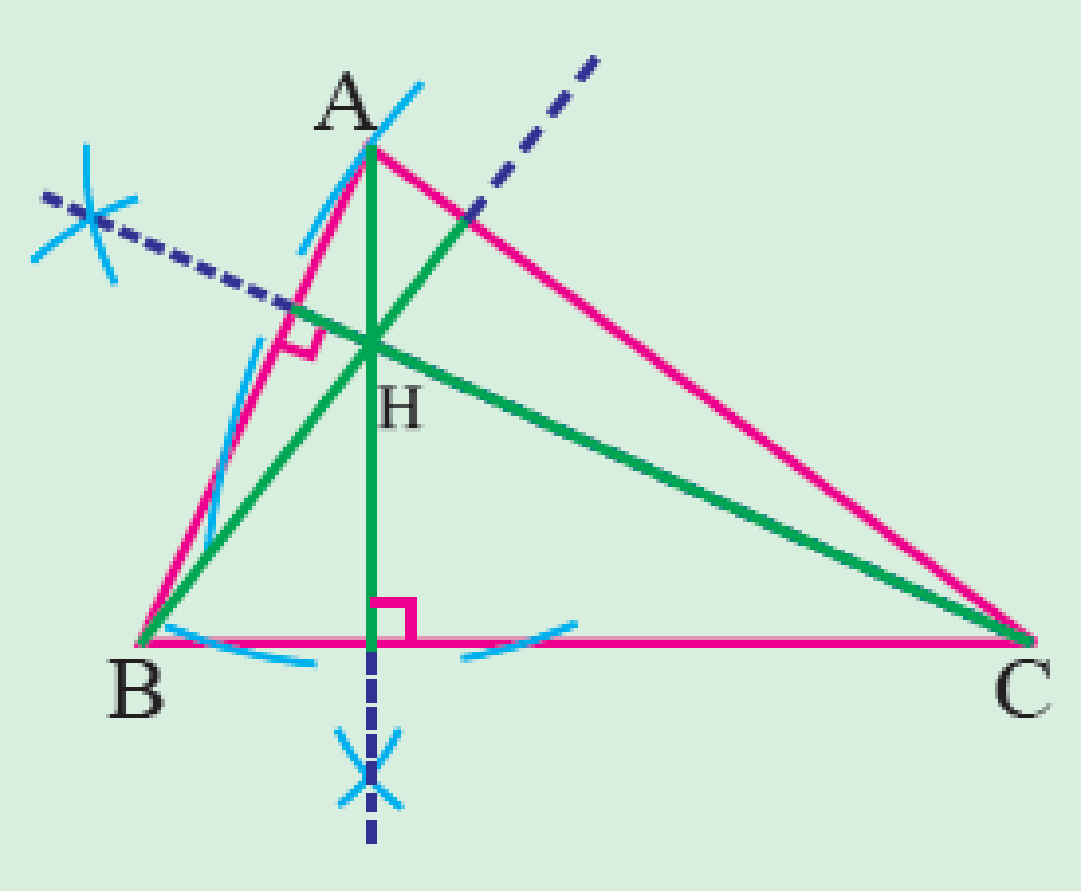
90 degrees (right angle)

Directions: Construct the orthocenter of the triangle below with your compass and a straightedge. The orthocenter is where all 3 altitudes meet.

Steps:

1. Construct the altitudes

2. The point of intersection is the orthocenter, label it with an H



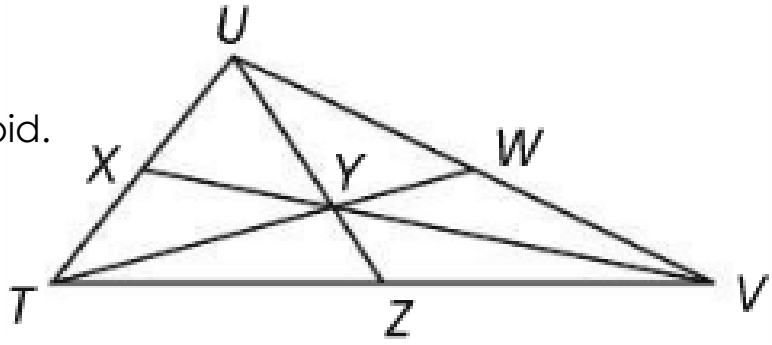
Do we need to construct all 3 altitudes to find the orthocenter?

No, only 2

Station 2 – At your Desk (Independent Practice)

Directions: Answer the following questions using your notes from the class lesson

1. In ΔTUV, Y is the centroid.



a. If $YW = 9$, find TY and TW.

$TY = 18$
$TW = 27$

b. If $YU = 8$, find ZU and ZY.

$ZU = 12$
$ZY = 4$

c. If $VX = 9$, find VY and XY.

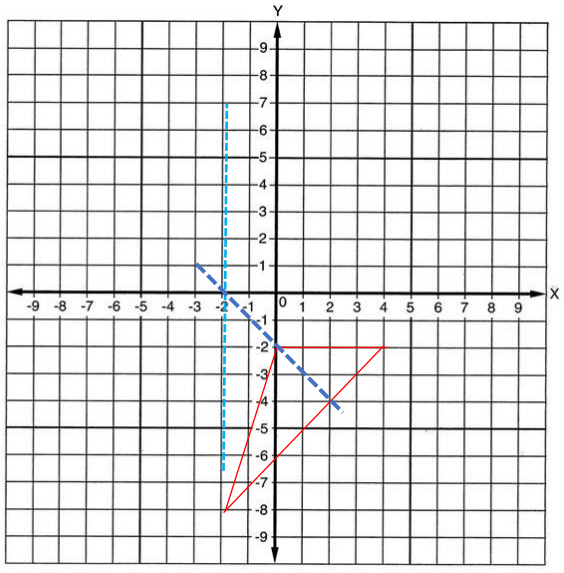
$VY = 6$
$XY = 3$

d. A centroid separates a median into two segments. What is the ratio of the length of the shorter segment to the length of the longer segment?

$1:2$

2. Find the coordinates of the orthocenter of ΔABC with the coordinates $A(0,-2)$, $B(4,-2)$, and $C(-2,-8)$.

$Orthocenter: (-2, 0)$



Station 3 – Technology

Directions: Open the GeoGebra Files and answer the questions below.

Altitude Link

1. The intersection of the altitudes in a triangle is called the orthocenter.

2. Move vertex B left or right until the Orthocenter appears outside of the triangle. What type of triangle did you create?

(Acute, Right, or Obtuse)

3. Move vertex B left or right until the Orthocenter lines up directly over vertex A or C. What type of triangle did you create?

(Acute, Right, or Obtuse)

Median Link

1. The intersection of the medians in a triangle is called the centroid.

2. What is true about the diagram?

a) $XC = CE$ b) $DC = CZ$

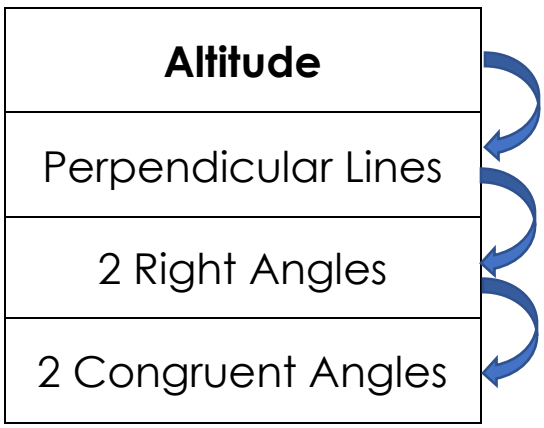
c) $XF = FZ$ d) $YE = \frac{2}{3}EZ$

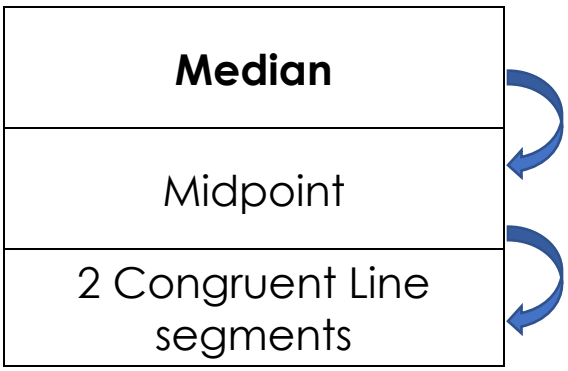
3. Move around the vertex of the triangle (X, Y, and Z). Can you make the centroid fall outside of the triangle? Why or why not?

No, the centroid must hit the midpoint of the other triangle side, so it cannot fall outside the triangle.

Station 4 – Hands on

Directions: Use the sequences provided on this sheet to arrange the proofs in the correct order. You may work with a partner.





Once you believe you have put your proof in the correct order, have the teacher check. You must correctly assemble at least one proof.

Easy: Blue Proof Teacher signature: _____

Easy: Green Proof Teacher signature: _____

Hard: Red Proof Teacher signature: _____

If you do not have a Co-Teacher, have students take pictures of how they arranged their statements and reasons and submit online. Then, go back and check their work later.

Red Proof Key

S- $\overline{CD}$ is a median of ΔABC	R- Given
S- D is the midpoint of $\overline{BA}$	R- Definition of Median
S- $\overline{BD} \cong \overline{DA}$	R- Definition of Midpoint
S- $\overline{CD}$ is an altitude of ΔABC	R- Given
S- $\overline{CD} \perp \overline{BA}$	R- Definition of Altitude
S- $\angle CDB \text{ and } \angle CDA$ are right angles	R- Definition of ⊥ lines
S- $\angle CDB \cong \angle CDA$	R- All right angles are ≅
S- $\overline{CD} \cong \overline{CD}$	R- Reflexive Postulate
S- $\Delta ADC \cong \Delta BDC$	R- SAS Theorem

Blue Proof Key

S- $\overline{GH}$ is an altitude of ΔGJK	R- Given
S- $\overline{GH} \perp \overline{JK}$	R- Definition of Altitude
S- $\angle GHJ \text{ and } \angle GHK$ are right angles	R- Definition of ⊥ lines
S- $\angle GHJ \cong \angle GHK$	R- All right angles are ≅
S- $\overline{JG} \cong \overline{KG}$	R- Given
S- $\overline{GH} \cong \overline{GH}$	R- Reflexive Postulate
S- $\Delta JGH \text{ and } \Delta KGH$ are right triangles	R- Right triangles have a right angle
S- $\Delta JGH \cong \Delta KGH$	R- HL Theorem

Green Proof Key

S- $\overline{YM}$ is a median of ΔXYW	R- Given
S- M is the midpoint of $\overline{XW}$	R- Definition of Median
S- $\overline{XM} \cong \overline{WM}$	R- Definition of Midpoint
S- $\overline{YW} \cong \overline{XY}$	R- Given
S- $\overline{MY} \cong \overline{MY}$	R- Reflexive Postulate
S- $\Delta YMW \cong \Delta YMX$	R- SSS Theorem
S- $\angle W \cong \angle X$	R- CPCTC

Exit Pass:

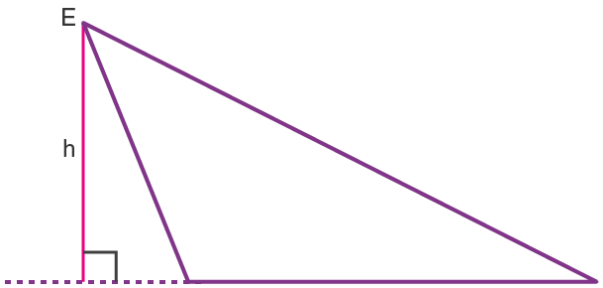
Students can discuss in pairs, then the class will complete the sorting activity together. This can be done on a SmartBoard, asking volunteers to drag and place an item into the correct box.

Answers Below:

Median

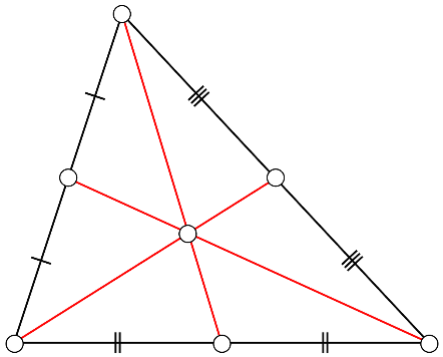
_____
Perpendicular Lines
2 Right Angles
2 Congruent Angles

A line segment that connects the vertex of a triangle to the opposite side <u>perpendicularly</u>.



Point of concurrency is the Orthocenter

Altitude



A line segment that connects the vertex of a triangle to the <u>midpoint</u> of the opposite side.

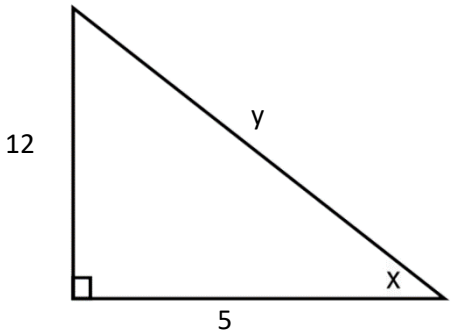
_____
Midpoint
2 Congruent Line segments

Point of concurrency is the Centroid

Lesson 3: Trigonometry

Warm-up: Find the missing side of the right triangle (y) using the Pythagorean Theorem.

$a^2 + b^2 = c^2$



$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 = c^2$$
$$144 + 25 = c^2$$
$$169 = c^2$$
$$\sqrt{169} = \sqrt{c^2}$$
$$13 = c$$

How do you know when to use trigonometry in a problem? (Hint: Can we use Pythagorean Theorem to find the value of x?)

Need to use trigonometry when an angle measure is involved

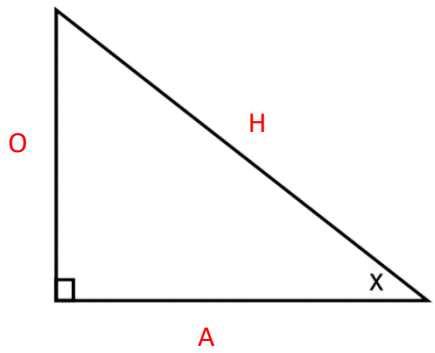
3 Sides of Right Triangles

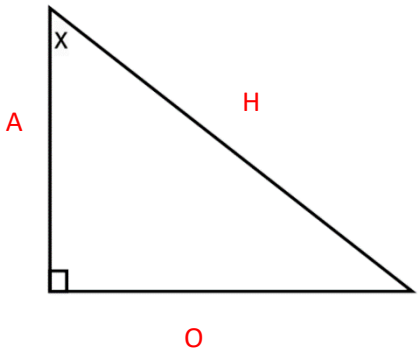
Hypotenuse: This is the longest side that is always opposite the right angle.

Opposite: This side is opposite the angle involved in the problem.

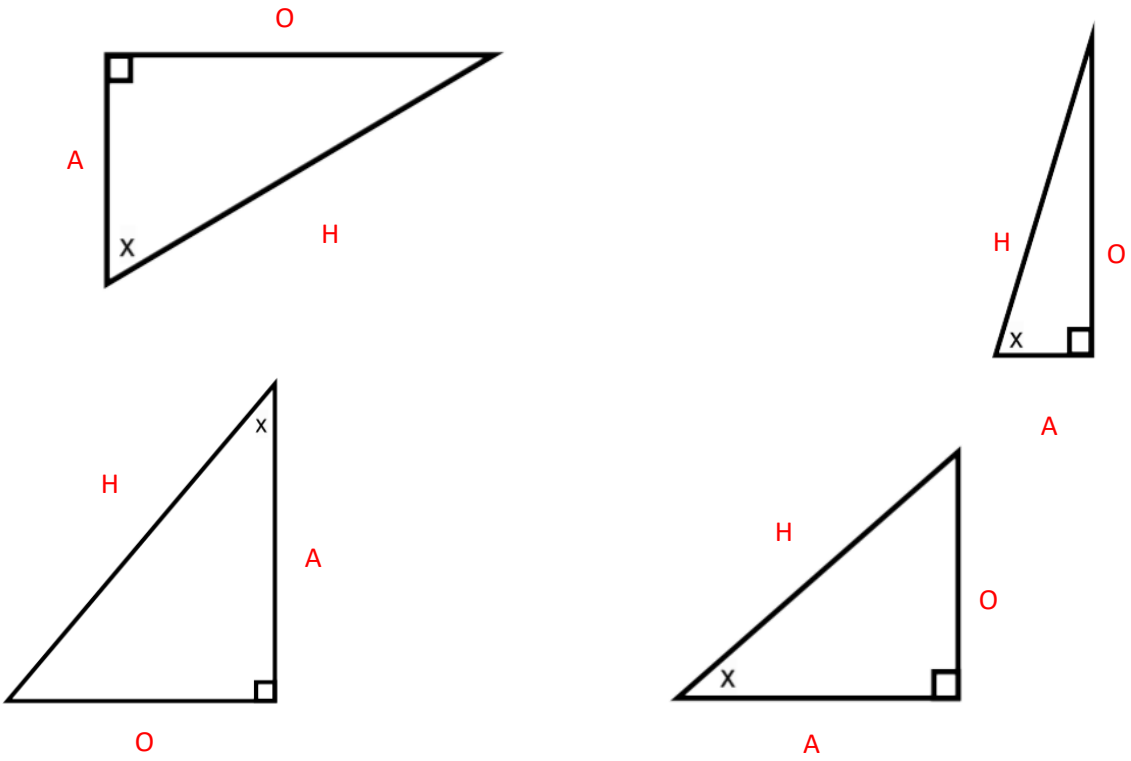
Adjacent: This is the last side, next to the angle.

Label the sides of the right triangle using the words opposite, adjacent, and hypotenuse based off angle x.





You try! Label the three sides of each triangle using H, O and A.



3 Trig Functions:

SOH-CAH-TOA

$$S = \frac{O}{H} \qquad C = \frac{A}{H} \qquad T = \frac{O}{A}$$

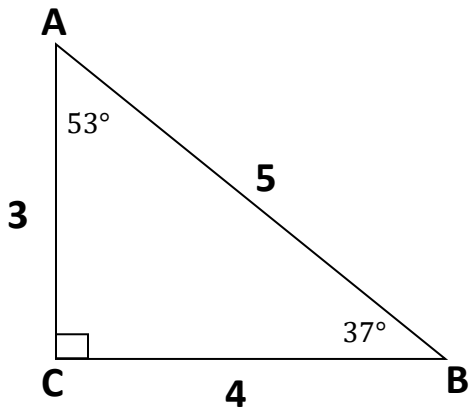
$$\text{Sin } x = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{Cos } x = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{Tan } x = \frac{\text{opposite}}{\text{adjacent}}$$

7. Label H (Hypotenuse is always across from the right angle)
8. Label A
9. Label O
10. Cross out unused side
11. Choose correct formula with the remaining two letters.
12. Plug in values

Find the following trigonometric ratios:



sin ∠ A: $\frac{4}{5}$ sin ∠ B: $\frac{3}{5}$

cos ∠ A: $\frac{3}{5}$ cos ∠ B: $\frac{4}{5}$

tan ∠ A: $\frac{4}{3}$ tan ∠ B: $\frac{3}{4}$

What do you notice about sin ∠ A and cos ∠ B?

$\sin\angle A = \cos\angle B$

Angles are Complementary: $53 + 37 = 90$

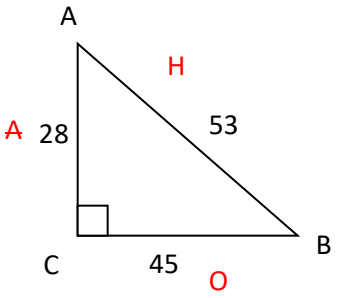
$\boldsymbol{\sin\angle A = \cos\angle B}$

$\boldsymbol{\angle A + \angle B = 90°}$

Finding Trigonometric Ratios:

Find the appropriate trig ratio for each example. Keep each ratio as a fraction.

1) Find the sin of $\angle A$

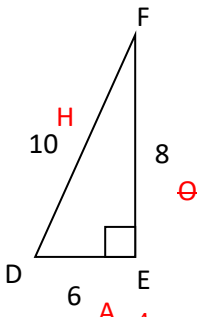


$S = \frac{O}{H}$

$\sin\angle A = \frac{45}{53}$

2) Find the cos of $\angle D$

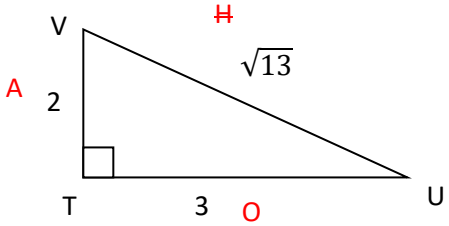
Label H, A, O



$C = \frac{A}{H}$

$\cos\angle D = \frac{6}{10} = \frac{3}{5}$

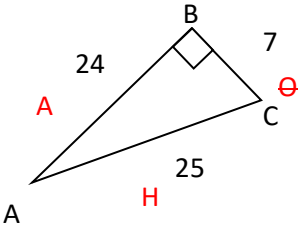
3) What is the tan of $\angle V$?



$T = \frac{O}{A}$

$\tan\angle V = \frac{3}{2}$

4) Find the cos of $\angle A$



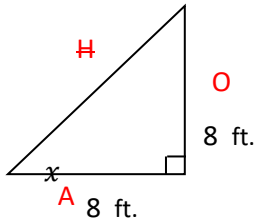
$C = \frac{A}{H}$

$\cos\angle A = \frac{24}{25}$

Finding Angle Measures:

Find the value of the indicated angle. Round each angle to the nearest degree.

1)

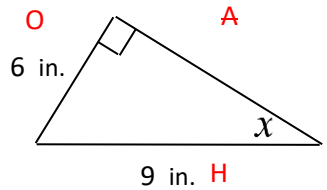


$\tan x = \frac{8}{8}$

$\tan^{-1}(\tan x) = \tan^{-1}\left(\frac{8}{8}\right)$

$x = 45°$

2)



$\sin x = \frac{6}{9}$

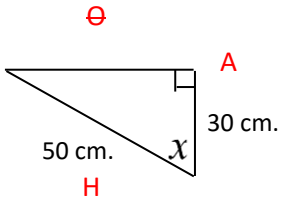
$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{6}{9}\right)$

$x = 42°$

-Put calculator in **"Degree Mode"**

-Use **inverse** trig functions!

3)

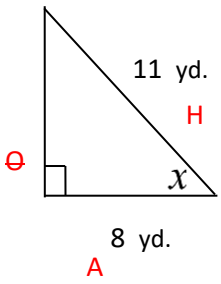


$\cos x = \frac{30}{50}$

$\cos^{-1}(\cos x) = \cos^{-1}\left(\frac{30}{50}\right)$

$x = 53°$

4)



$\cos x = \frac{8}{11}$

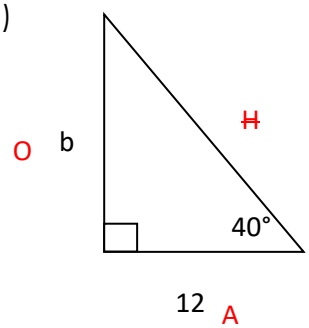
$\cos^{-1}(\cos x) = \cos^{-1}\left(\frac{8}{11}\right)$

$x = 43°$

Finding Side Lengths:

Find the length of each missing side to the **nearest hundredth**:

1)

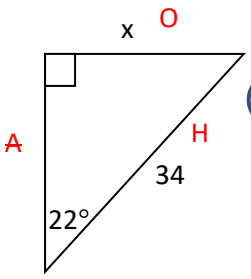


$\frac{\tan 40}{1} = \frac{b}{12}$

$12(\tan 40) = b$

$10.07 = b$

2)



$\frac{\sin 22}{1} = \frac{x}{34}$

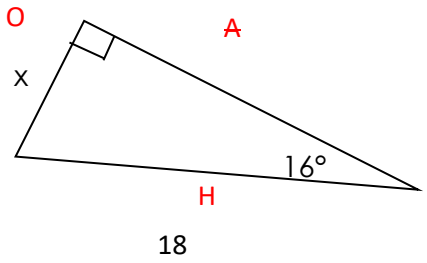
$34(\sin 22) = x$

$x = 12.74$

-Put trig function over 1

-Cross Multiply

3)

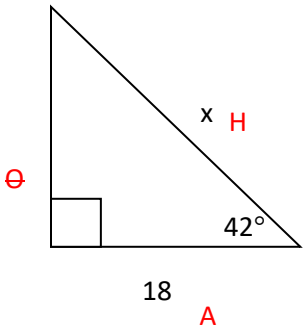


$$\frac{\sin 16}{1} = \frac{x}{18}$$

$$18(\sin 16) = x$$

$$4.96 = b$$

4)



$$\frac{\cos 42}{1} = \frac{18}{x}$$

$$x(\cos 42) = 18$$

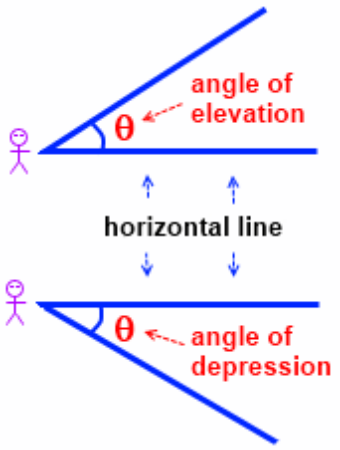
$$x = \frac{18}{\cos 42}$$

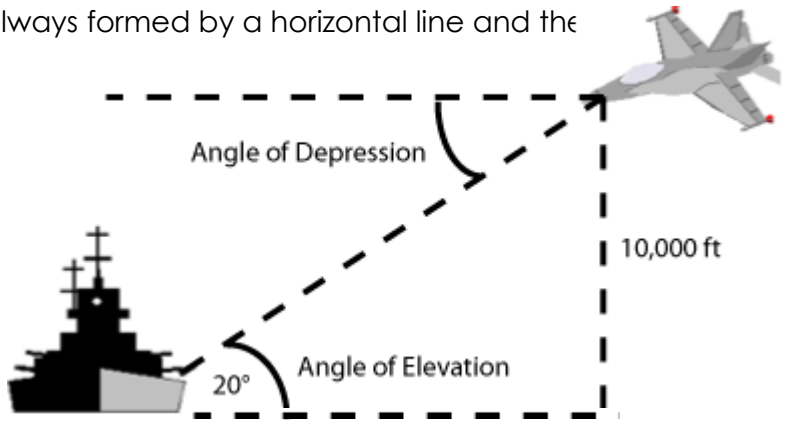
$$x = 24.22$$

Station 1 – Mini Lesson

Angle of Elevation vs. Angle of Depression

Angles of Elevation and Depression are always formed by a horizontal line and the sight.





Angle of Elevation: Measured from the bottom up. Always inside the triangle

Angle of Depression: Measured from the top down. Always outside the triangle

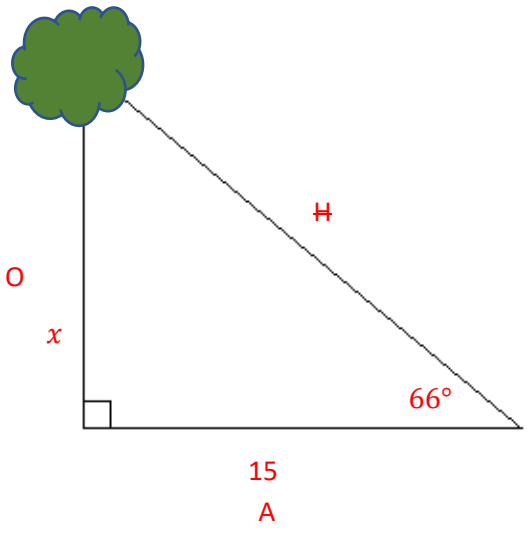
Angle of Elevation = Angle of Depression

Why?

When parallel lines are cut by a transversal, alternate interior angles are congruent

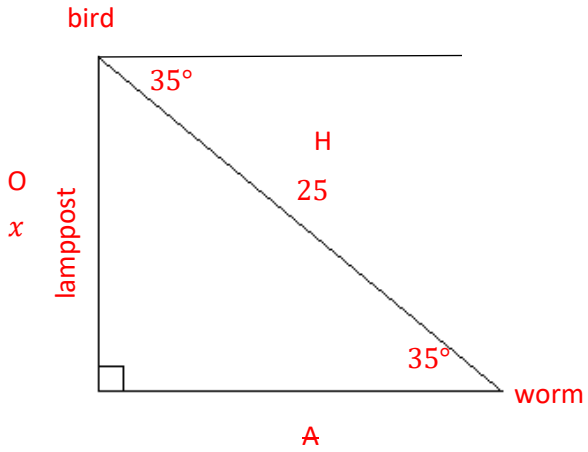
Practice Problems: Draw a picture!

1) A tree casts a shadow that is 15 feet long. The angle of elevation from the end of the shadow to the top of the tree is 66°. Determine the height of the tree, to the *nearest foot*.



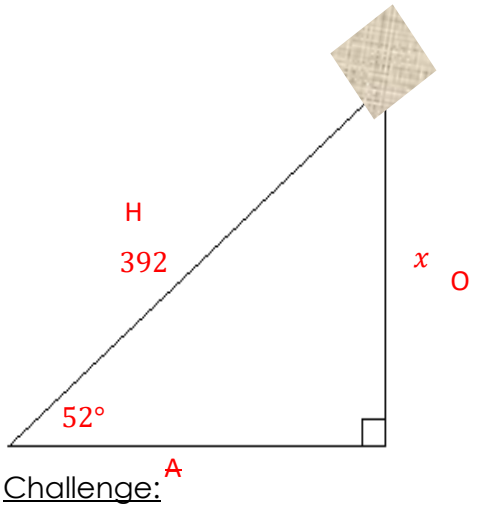
$$\frac{\tan 66}{1} = \frac{x}{15}$$
$$15\,(\tan 66) = x$$
$$34\ feet = x$$

2) A bird sits on top of a lamppost and spots a worm on the ground. The angle of depression from the bird to the worm is 35°. The distance from the bird to the worm is 25 meters. How tall is the lamppost to the nearest meter?



$$\frac{\sin 35}{1} = \frac{x}{25}$$
$$25\,(\sin 35) = x$$
$$14\ meters = x$$

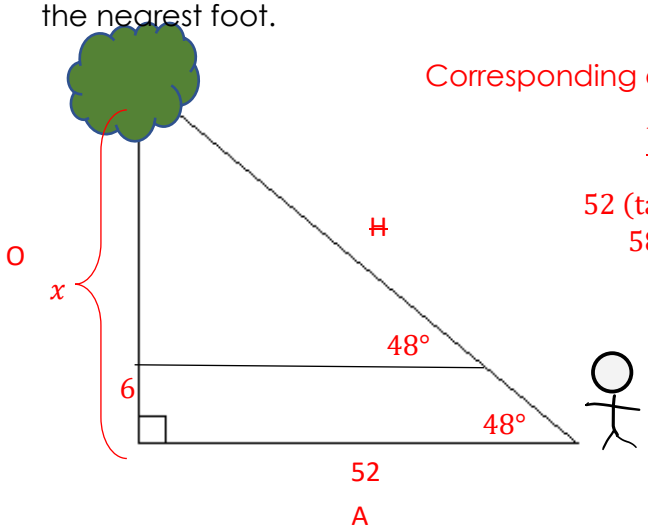
3) A boy flying a kite lets out 392 feet of string, which makes an angle of elevation of 52° with the ground. Assuming the string is tied to the ground, find to the nearest foot, how high the kite is above the ground.



$$\frac{\sin 52}{1} = \frac{x}{392}$$
$$392(\sin 52) = x$$
$$309\ feet = x$$

Challenge:

4) A person stands 52 feet from the base of a tree. The angle of elevation from his eye, which is 6 feet above the ground, to the top of the tree is 48°. Find the height of the tree to the nearest foot.

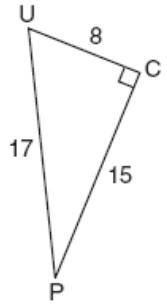


Corresponding angles are congruent

$$\frac{\tan 48}{1} = \frac{x}{52}$$
$$52\,(\tan 48) = x$$
$$58\ feet = x$$

Station 2 – At your Desk (Independent Practice)

1) The diagram at the right shows right triangle *UPC*.

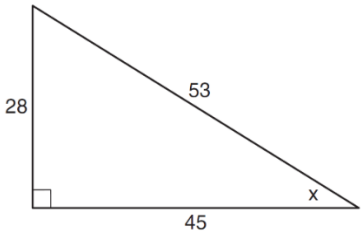


Which ratio represents the sine of $\angle U$?

1) $\frac{15}{8}$ 2) $\frac{15}{17}$ 3) $\frac{8}{15}$ 4) $\frac{8}{17}$

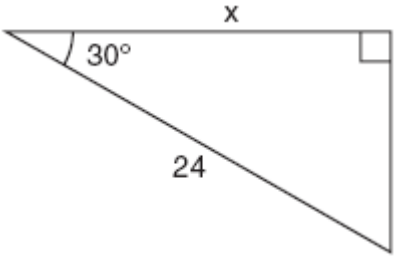
Label H, A, O

2) Which ratio represents $\cos x$ in the right triangle shown below?



1) $\frac{28}{53}$ 2) $\frac{28}{45}$ 3) $\frac{45}{53}$ 4) $\frac{53}{28}$

3) Find the value of x to the *nearest tenth of a foot.*

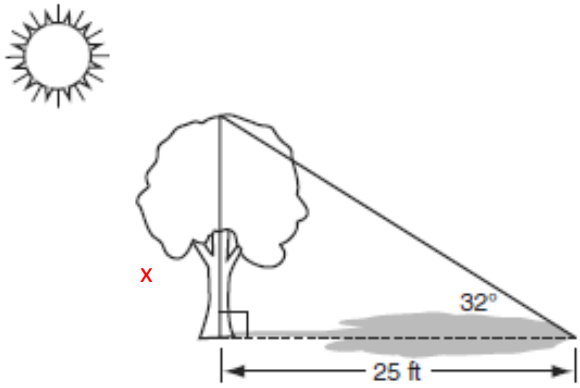


$$\frac{\cos 30}{1} = \frac{x}{24}$$
$$24(\cos 30) = x$$
$$20.8 = x$$

-Put trig function over 1

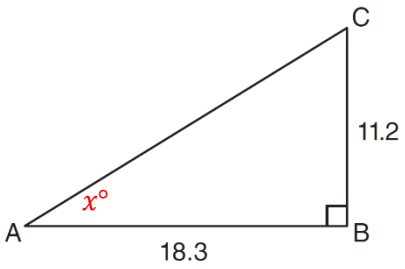
-Cross Multiply

4) Find the height of the tree to the *nearest hundredth of a foot.*



$$\frac{\tan 32}{1} = \frac{x}{25}$$
$$25\ (\tan 32) = x$$
$$15.62\ feet = x$$

5) What is the measure of $\angle A$, to the *nearest tenth of a degree*?



$$\tan x = \frac{11.2}{18.3}$$

$$tan^{-1}(tan\, x) = tan^{-1}\left(\frac{11.2}{18.3}\right)$$

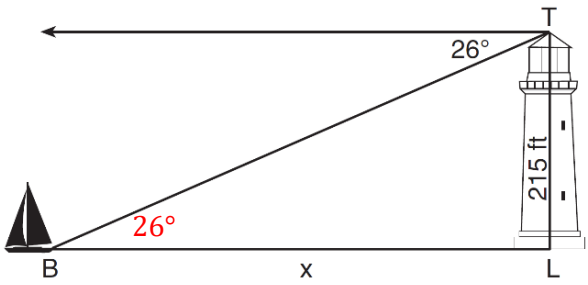
$$x = 31.5°$$

-Put calculator in **"Degree Mode"**

-Use **inverse** trig functions!

<u>If you already did the mini-lesson, try the question below:</u>

6) The top of a lighthouse, T, is 215 feet above sea level, L, as shown in the diagram below. The angle of depression from the top of the lighthouse to a boat, B, at sea is 26°. Determine, to the nearest foot, the horizontal distance, x, from the boat to the base of the lighthouse.



$$\frac{\tan 26}{1} = \frac{215}{x}$$

$$\frac{x(\tan 26)}{\tan 26} = \frac{215}{\tan 26}$$

$$x = 441\ feet$$

Station 3 – Technology

Directions:

1. Login to Delta Math

2. Complete the "Cofunctions Practice"

-Cofunction Equations

3. You must answer 5 questions from each skill correct

4. Finish Early? Keep trying more problems! Or go back and practice a skill from a previous unit

Checklist:

☑ Cofunction Equations – 5 questions

$\sin \angle A = \cos \angle B$

$\angle A + \angle B = 90°$

Stuck? Look at an example with the answer below:

If the angles are represented in degrees, find both angles:

$$\sin(x + 39) = \cos(6x - 12)$$

Angle 1: ☐° Angle 2: ☐° Submit Answer

When Sine and Cosine are equal → Angles add to 90

$$(x + 39) + (6x - 12) = 90$$
$$7x + 27 = 90$$
$$7x = 63$$
$$x = 9$$

Angle 1: $x + 39 = 9 + 39 = 48°$ Angle 2: $6x - 12 = 6(9) - 12 = 42°$

Station 4 – Hands On

Let's Explore Co-Functions without the use of a calculator! (Crazy!)

The chart to the right shows the approximate values of trig functions for different angle measures.

Use the chart to the right to find the following values. Highlight as you go!

1. sin (20°) = .3420 cos (70°) = .3420

2. sin (80°) = .9848 cos (10°) = .9848

3. cos (15°) = .9659 sin (75°) = .9659

4. cos (37°) = .7986 sin (53°) = .7986

What pattern did you notice? (Hint: add up the angle measures in each problem…)

Sine and Cosine of complementary angles are equal!

If two angles are **complementary**, then:

sin (angle) = cos (90° – angle)

$$\sin \angle A = \cos \angle B$$

$$\angle A + \angle B = 90°$$

Practice: *find the acute angle that makes the equation true.*

1. sin(15°) = cos (75°)

$90 - 15 = 75°$

2. sin(40°) = cos (50°)

$90 - 40 = 50°$

3. cos(83°) = sin(7°)

$90 - 83 = 7°$

4. cos(21°) = sin (69°)

$90 - 21 = 69°$

5. sin(90°) = cos(0°)

$90 - 90 = 0°$

6. cos(45°) = sin (45°)

$90 - 45 = 45°$

Deg	Sine (sin)	Cosine (cos)	Tangent (tan)	Deg	Sine (sin)	Cosine (cos)	Tangent (tan)
0	0.0000	1.0000	0.0000	45	0.7071	0.7071	1.0000
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.5000	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1.0000	90	1.0000	0.0000	undefined

Teacher Materials

Exit Pass: Project the statements on a screen and have the class show thumbs up or thumbs down for true or false. After each question, discuss the correct answer as a class. If false, correct the statement to make it true!

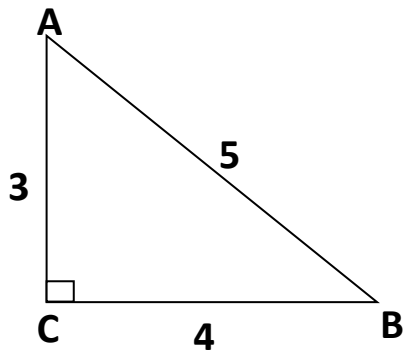
1. The Angle of Depression equals the Angle of Elevation

   True

2. If $\sin \angle A = \cos \angle B$, then $\angle A = \angle B$

   False, then $\angle A + \angle B = 90°$

Use the triangle to the right for questions 3 and 4.



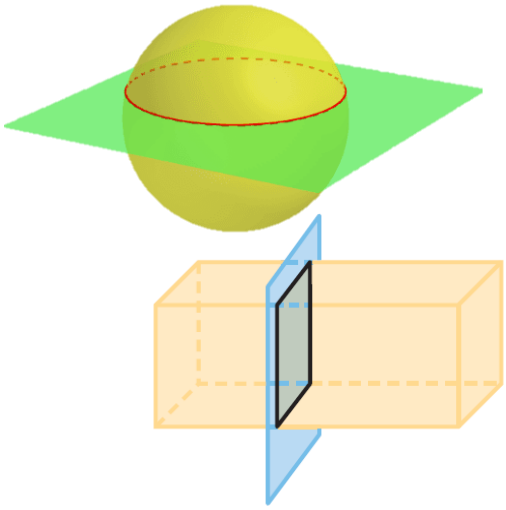
3. $\sin \angle A = \frac{4}{5}$

   True

4. $\cos \angle B = \frac{5}{4}$

   False, $\cos \angle B = \frac{4}{5}$

Lesson 4: 3D Geometry



Warm-up: Think about it!

1. What shape would you see if you sliced an orange?

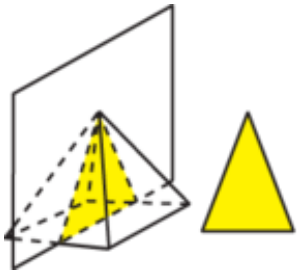
   circle

2. What shape would you see if you sliced a loaf of bread?

   Rectangle/square

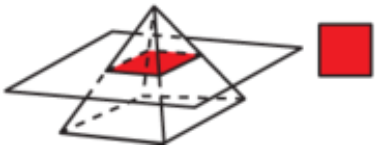
Cross Section: The intersection of a solid and a plane

Different Cross Sections of a Rectangular Pyramid:



Triangle

Perpendicular to the base



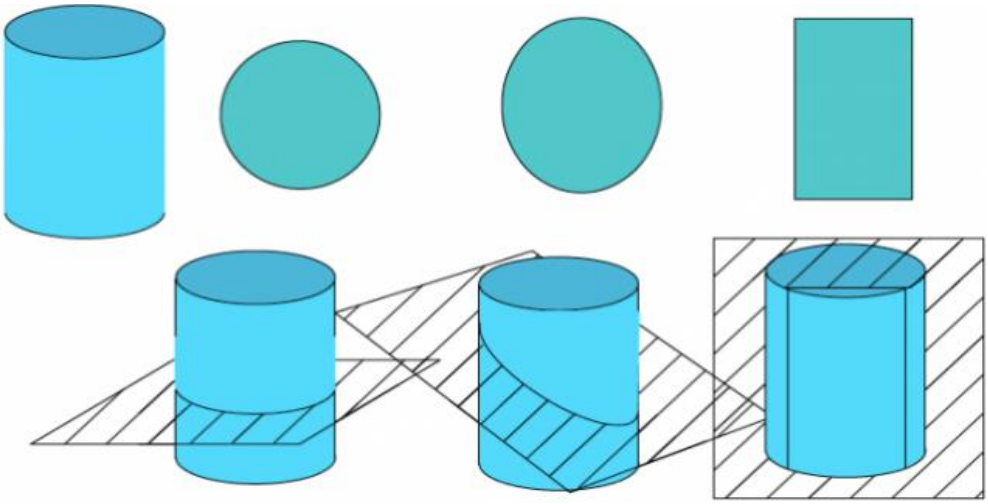
Rectangle

Parallel to the base

Trapezoid

Random Cross Section

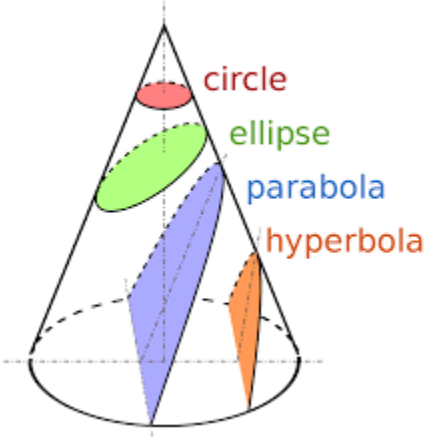
Different Cross Sections of a Cylinder:

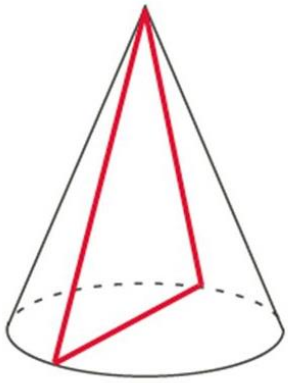


Parallel to the base

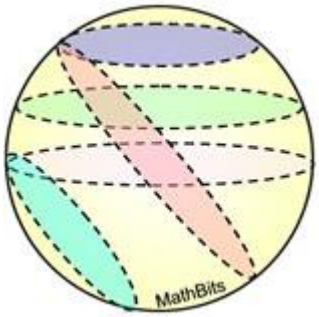
Perpendicular to the base

Different Cross Sections of a Cone:





Cross Sections of a Sphere
(all circles)



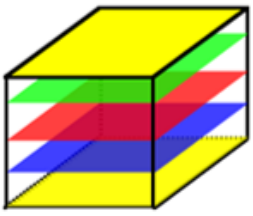
1) A plane intersects an octagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

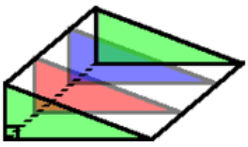
1) triangle
2) trapezoid
3) octagon
4) rectangle

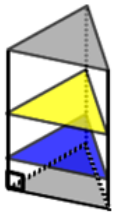
2) A two-dimensional cross section is taken of a rectangular prism. Which of the following cannot be the two-dimensional cross section?

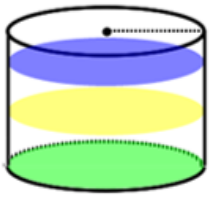
1) rectangle
2) triangle
3) square
4) circle

Cross Sections and Volume









Base area times ***height*** is a direct result of STACKING congruent cross sections on top of each other to "fill" in the shape. Above are a few visual examples of how cross sections are used to fill the shape and create volume. Try to think about the volume formulas for each.

General Prism Volume Formula:

$$V = Bh$$

$$B = area\ of\ Base, \qquad h = height\ of\ prism$$

Volumes of Prisms and Cylinders

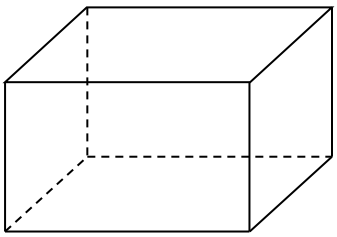
Volume: The space a figure occupies. Always measured in cubic units.

Rectangular Prism:

Volume Formula

V = Bh

where *B = AREA of the base*



The base of a rectangular prism is a rectangle

so the formula is $V = (lw)h$.

Practice: Find the volume of each rectangular prism below.

1) A rectangular prism with the dimensions 4 in., 6 in., 8 in.

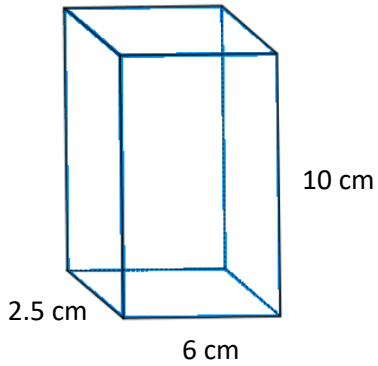
$V = Bh$

$V = (lw)h$

$V = (4 \cdot 6) \cdot 8$

$V = 192\ in^3$

2)



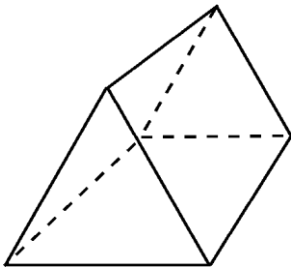
$V = Bh$

$V = (lw)h$

$V = (2.5 \cdot 6) \cdot 10$

$V = 150\ cm^3$

Triangular Prism:



Volume Formula

V = Bh

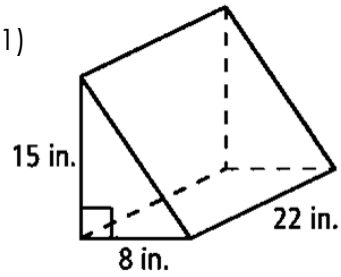
where *B = AREA of the base*

The base of a triangular prism is a triangle

so the formula is $V = \left(\frac{1}{2}bh\right)h$.

Practice: Find the Volume of each triangular prism below:

1)



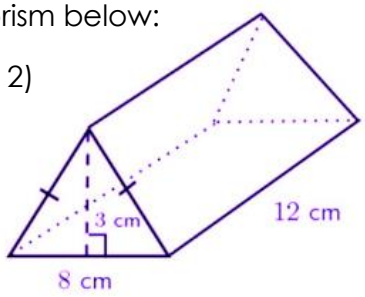
$V = Bh$

$V = \left(\frac{1}{2}bh\right)h$

$V = \left(\frac{1}{2}(8)(15)\right)(22)$

$V = 720\ in^3$

2)



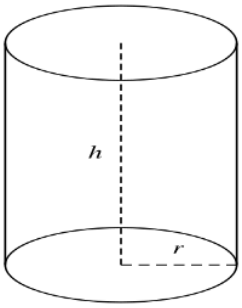
$V = Bh$

$V = \left(\frac{1}{2}bh\right)h$

$V = \left(\frac{1}{2}(8)(3)\right)(12)$

$V = 144\ cm^3$

Cylinder:



Volume Formula

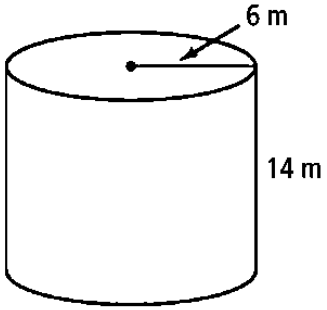
V = Bh

where *B = AREA of the base*

The base of a cylinder is a circle

so the formula is $V = (\pi r^2)h$.

Practice: Find the volume of the cylinder in terms of π and to the nearest hundredth



$V = Bh$

$V = (\pi r^2)h$

$V = (\pi(6)^2)(14)$

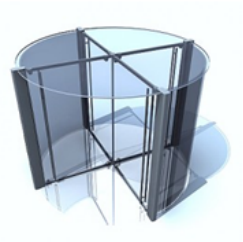
$V = 504\pi\ m^3$

$V = 1583.36\ m^3$

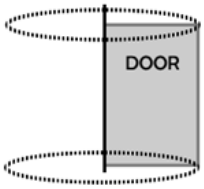
Revolving 2D Shapes to get volume

Instead of stacking congruent cross sections to form the volume of a solid, another technique is to rotate it about an axis. Thinking about a revolving door and how it swings (rotates) about the center creating a cylindrical shape.

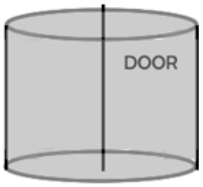
Revolving Door



Cross Section of Door



Imagine as the door revolves it filling the entire space of the cylinder

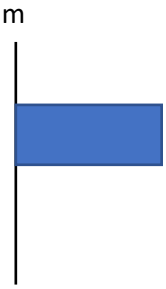


Describe the solid that is formed by rotating each of these figures about line m!

1.

Name: Cone

2.

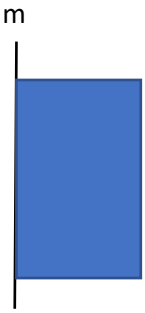


Name: Cylinder

3.

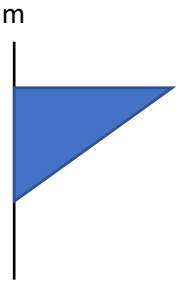
Name: Sphere

4.



Name: Cylinder

5.



Name: Cone

6.

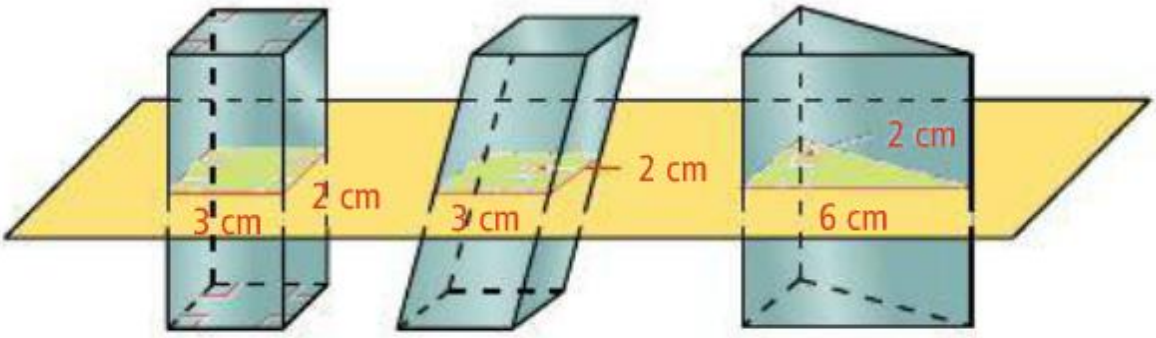
Name: Hemisphere

Station 1 – Mini Lesson

take note

Theorem 11-5 Cavalieri's Principle

If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.



A = bh

A = (3)(2)

A = bh

A = (3)(2)

A = bh/2

A = ((6)(2))/2

A = 12/2

1) Explain how Cavalieri's Principle applies to this stack of nickels.

The nickels has the same base area, and the same height.

Therefore, the stacks of nickels will have the same volume!

Regents Questions:

1. A rectangular prism has a base with a length of 25, a width of 9, and a height of 12. A second prism has a square base with a side of 15. If the volumes of the two prisms are equal, what is the height of the second prism?

In order for them to have the same volume, they must have the same base area and height. Therefore, the height of the second prism must also be 12.

2. Two prisms have equal heights and equal volumes. The base of one is a pentagon and the base of the other is a square. If the area of the pentagonal base is 36 square inches, how many inches are in the length of each side of the square base?

The area of the square base must also be 36 square inches.

$$\begin{aligned} A &= s^2 \\ 36 &= s^2 \\ 6\ inches &= s \end{aligned}$$

3. Tim has a rectangular prism with a length of 10 centimeters, a width of 2 centimeters, and an unknown height. He needs to build another rectangular prism with a length of 5 centimeters and the same height as the original prism. The volume of the two prisms will be the same. Find the width, in centimeters, of the new prism.

$$\begin{aligned} Area\ of\ first\ rectangular\ base &= Area\ of\ second\ rectangular\ base \\ lw &= lw \\ 10 \cdot 2 &= 5w \\ 20 &= 5w \\ 4\ centimeters &= w \end{aligned}$$

4. Two prisms with equal altitudes have equal volumes. The base of one prism is a square with a side length of 5 inches. The base of the second prism is a rectangle with a side length of 10 inches. Determine and state, in inches, the measure of the width of the rectangle.

$$\begin{aligned} Area\ of\ square\ base &= Area\ of\ rectangular\ base \\ s^2 &= lw \\ 5^2 &= 10w \\ 25 &= 10w \\ 2.5\ inches &= w \end{aligned}$$

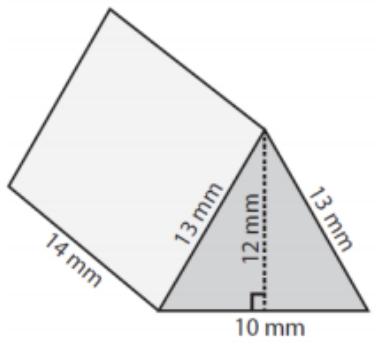
Station 2 – At your desk

1. A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of 6 feet and a height of 1 foot. The pool is going to be filled to the top with water. Determine the volume of the water in the pool, to the nearest cubic foot.

Diameter is 6, so radius is 3

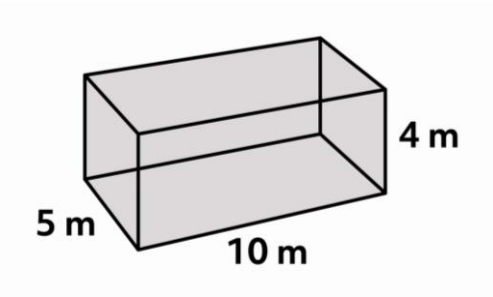
$$V = \pi r^2 h$$
$$V = \pi(3)^2(1)$$
$$V = 28\ ft^3$$

2. What is the volume, in cubic millimeters of the prism, rounded to the nearest cubic millimeter?



$$V = \left(\frac{1}{2}bh\right)h$$
$$V = \left(\frac{1}{2}(10)(12)\right)(14)$$
$$V = 840\ mm^3$$

3. What is the volume of the rectangular prism?



$$V = (lw)h$$
$$V = (5 \cdot 10)(4)$$
$$V = 200\ m^3$$

<u>If you already did the mini-lesson, try the question below:</u>

4. Two prisms with equal altitudes have equal volumes. The base of one prism is a square with a side length of 6 inches. The base of the second prism is a rectangle with a side length of 9 inches. Determine and state, in inches, the measure of the width of the rectangle.

$$Area\ of\ square\ base = Area\ of\ rectangular\ base$$
$$s^2 = lw$$
$$6^2 = 9w$$
$$36 = 9w$$
$$4\ inches = w$$

Station 3 – Technology

Directions:

1. Login to Delta Math

2. Complete the "Cross Sections Practice"

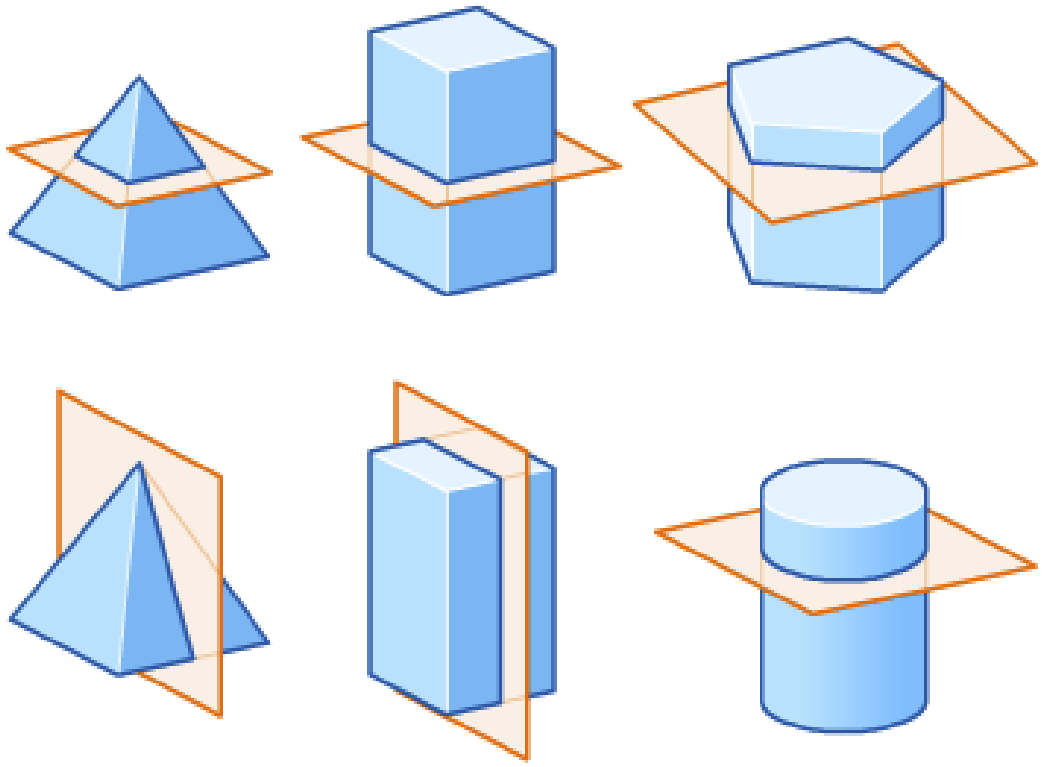
    -Identify Cross Sections of Solids

3. You must answer 10 questions correct

4. Finish Early? Keep trying more problems! Or go back and practice a skill from a previous unit

Checklist:

☑ Identify Cross Sections of Solids – 10 questions



Station 4 – Hands On

We are going to revolve 2D shapes to create 3D objects!

Directions:

4. Pick one shape to revolve (half circle, rectangle, or right triangle)
➔ Try to have each person in your group pick a different shape!

5. Cut out your shape on cardboard (the smaller the better)

6. Tape your shape to the side of a pencil toward the top

Answer the following questions:

4. The 2-dimensional shape I picked was half circle or rectangle or right triangle

5. When I spin the pencil in my hands very fast, I see a sphere or cylinder or cone (3-dimensional shape)

6. Upload a picture of your pencil to the drop box online!