Vocabulary Emphasis to support English Language Learners Perseverance in Solving Exponential Function Problems

Christina Bernacett

State University of New York (SUNY)

College at Brockport

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Abstract

This curriculum project showcases a collection of lessons designed to promote mathematical literacy by unpacking vocabulary terms that have multiple meanings. The project includes four sequential lesson plans that follow the Exponential Functions unit. The motivation for each lesson is to encourage student discourse by creating a space for exploration and discussion. Each lesson aligns with the New York State Next Generation Learning Standards for Algebra 1.

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Introduction

One major challenge English Language Learners (ELLs) face is vocabulary comprehension. Within the Algebra 1 course, the Exponential Functions unit has mathematical terminology that can be both confusing and unclear. The gaps between the English language and mathematical terminology can be defeating for ELLs. One issue is that many of the vocabulary words in mathematics have specific meanings that are different than their colloquial meanings. This causes confusion and frustration for ELLs. The first Standard of Mathematical Practice in the New York State Next Generation Standards is *make sense of problems and persevere in solving them*, which clearly indicates that a space should be created within lessons for ELLs to explore unfamiliar mathematical terminology. This space supports ELLs in using prior knowledge, technology, and peer discourse to dissect mathematical meanings from vocabulary, definitions, and terms. The implementation of positive reinforcement initiatives that unpack vocabulary not only promotes academic language and development, but it can also support ELLs in developing the ability to make sense of problems and persevere in solving them.

The purpose of this curriculum project is to provide support for teachers as they teach ELLs the algebra 1 functions. This unit is designed for students to make comparisons between algebraic and exponential functions so they can correctly identify them based on the representation presented. Providing opportunities where students can communicate with each other on their responses can promote language development. “A discourse approach to learning mathematics means considering the different ways of talking about mathematical objects and points of view of mathematical situations that students bring to classroom discussions.” (Moschkovich, 1999, p. 11) Within the Functions unit, there are different vocabulary terms students learn about which include linear, quadratic and exponential. Roberts and Truxaw (year) discuss the ambiguity ELL students face when learning about math vocabulary. In order for

students to master different mathematical skills and the standard for mathematical practice, teachers should take the time to go through vocabulary development. There are many challenges with math vocabulary such as the many different names for specific term and multiple meanings.

Literature Review

Academic Language Challenges of English Language Learners

Between unfamiliar mathematical language and challenges in understanding content, ELLs face many learning obstacles. Learning mathematics is particularly problematic for English Language Learners (ELLs) as they are learning English while they are learning mathematical language (Adoniou & Qing, 2014). Building academic language thus presents instructional and learning challenges. Teachers often face the challenge of being equipped to bridge the learning gaps. According to Lager (2004), the three language obstacles are misinterpreting vocabulary, incomplete understanding of syntax and contradictory visual cues. "The overwhelming majority of vocabulary (coefficient, function, factor, etc.) as well as many of the language and discourse structures in mathematical English are unique to English"(Lager, 2004, p. 2). Highlighting vocabulary terms can help ELLs develop both academic language and mathematical ideas. Demonstrating how to identify root words, prefix, suffixes from vocabulary terms are a ways to develop academic language skills. A framework created by Mary Brenner to bridge the learning gap would be "communicating about mathematics, communicating in mathematics, and communicating with mathematics." (Winsor, 2007, p. 373) The results from Winsor's (2007) his experiment on using the approach *Mathematics is the Second Language* improved student communication and the use of mathematical vocabulary. Winsor incorporated group work, journals and projects within his instruction which addressed the student learning gaps to develop understanding of the mathematics making connections by emphasizing academic language. From

what he stated about his findings "…ELL students, in order to learn mathematics as well as English, must have the chance to communicate mathematically." (Winsor, 2007, p. 378)

Sociocultural theory also provides a framework to support teachers in this work. This theory expands on the idea that children learn how to use language from other people. Roy-Campbell (2013) stated that "students who come to the classroom with languages other than English bring with them cultural understandings that can impact how they receive instruction from teachers (p. #). Providing such support for teachers is especially important as the ELL student population continues to increase. "Between 1998 and 2009 there was a 51% increase in the number of ELL students in United States (US) schools—from 3.5 to 5.3 million—representing about 10% of the student population."(Roy-Campbell, 2013, p. 3)

Connecting Standards of Mathematical Practice with the Function's Unit

Making sense of problems and persevere in solving them is a mathematical practice that aims to promote conceptual understanding before engaging in problem solving. For ELL students, having discussions on the content before encouraging independent work is one example that can develop this practice. "Two important functions of productive classroom discussions are uncovering the mathematical content in student contributions and bringing different ways of talking and points of view into contact." (Moschkovich, 1999, p. 11) Moschkovich organized mathematical discussion into frameworks where vocabulary and comprehension skills are highlighted. ELL students need to be able to not only understand the meanings of mathematical terms but also be able to communicate through verbal/written form. To promote mathematical discourse where students can make sense of problems and solve them, teachers should provide opportunities for mathematical discussions.

The standard for mathematical practice on making sense of problems relies heavily on student conceptual understanding. The strategies Roberts and Truxaw recommend are word walls and graphic organizers. For ELL students, creating a Frayer model where they can identify the differences between each function can develop their academic language. “The graphic organizers used throughout a unit can and should be revisited to support long-term retention.”(Roberts & Truxaw, 2013, p. 33) Promoting mathematical understanding for ELLs needs to have an emphasis on building vocabulary through different representations which will help achieve the mathematical practice of making sense of problems and solving them.

Curriculum Overview

The following lessons within this curriculum project were designed to promote vocabulary comprehension while introducing mathematical concepts within the New York State (NYS) Algebra 1 exponential functions unit. The vocabulary highlighted will be a combination of mathematical terms and Blooms taxonomy higher order levels. A vocabulary chart is thus provided in each lesson for students to complete and refer back to throughout the lesson. In the second column of the chart, students are encouraged to either jot down familiar descriptions and specific mathematical definitions, as they are often different. Teachers are encouraged to review the mathematical definitions and use the sentence starter writing prompts to support students’ development of mathematical literacy. These practices can be especially beneficial for English Language Learners (ELLs) who often need additional support with the English language.

Purpose: The purpose of this lesson is to go through the properties of exponents, via remote instruction, with a focus on vocabulary comprehension. Students will use the Google Suite platform to respond to question prompts. The Teacher will assess student understanding based on student responses through Desmos, the Collaborative Google Doc and Exit Ticket.

Lesson 1: Properties of Exponents (Zero Exponent, Negative Exponent and Power of a Power)

Goals

Focus Question: How can we simplify the expression using the Zero Exponent, Negative Exponent and Power of a Power properties?

Next Generation Learning Standards

MP1 Make sense of problems and persevere in solving them.

AI-A.SSE Write expressions in equivalent forms to reveal their characteristics.
- c. Use the properties of exponents to rewrite exponential expressions

Learning Target
By the end of today's lesson, I will be able identify the Zero Exponent, Negative Exponent and Product of Powers properties in order to evaluate and simplify expressions.

Materials	**Skill**
• Google Slides • Google Forms (Exit Ticket) • Google Doc (Collaborative Document) • Desmos	-Identify the Zero Exponent, Negative Exponent and Power of a Power properties -Evaluating expressions using the Zero Exponent, Negative Exponent and Power of a Power properties

MOTIVATION AND INTRODUCTION OF LESSON
(Student Discussion) Five students will be selected to lead the discussion using their prior knowledge of exponents. A conversation prompt box will be visible on Google Slides to assist students in the discussion. After the five students discuss, the rest of the class will be encouraged to also share their thoughts. Teacher will select one student to share the class consensus answers.

Do Now: Simplify the following below.

i. $(2^2)(2^3) =$

ii. $\frac{2^5}{2^2} =$

iii. $(y^3)^3 =$

$\left(\frac{4}{3}\right)^3 =$

Lesson Introduction:

Prompt Question: What are the properties of exponents?

The teacher will go over the definitions and properties of exponents.

- Exponent
- Base
- Zero Exponent Property
- Negative Exponent Property
- Power of a Power Property

Building Academic Language-"All About Words" slide:

The Teacher will display the following slide titled "All About Words" which will include the 'Math Terms Support' Chart and the following image of a base and exponent. The Teacher will then ask students to use the image and prior knowledge to then write down the definitions for the following terms: Base, Exponent, Identify and Evaluate.

Math Terms Support Chart: The Teacher will have students complete the Vocabulary Support Chart to include the words discussed. In the second column, give students 2 minutes to look online for other definitions of the word.

For example:
Base-

- The bottom of something considered as its support
- (Biology) that part of a bodily organ by which it is attached to another more central structure of the organism
- (Architecture) the lower part of a wall, pier, or column considered as a separate architectural feature

Exponent-

- a person who believes in and promotes the truth or benefits of an idea or theory.

Identify-

- associate (someone) closely with; regard (someone) as having strong links with.

Evaluate-

- form an idea of the amount, number, or value of; assess.

The following definitions will be discussed with the class. Then students will fill in the last column to write down an example for each.
Base: the number that gets multiplied by an exponent.
Exponent: for the symbol placed above and to the right of another to indicate by what power the base number is to be raised.
Identify: establish or indicate who or what (someone or something) is.
Evaluate: to determine or calculate the numerical value of (a formula, function, relation, etc.).

Math Term	Where have you heard or seen this word? (Discuss with your peer. Then browse online for some definitions.)	Math Definition	Example

Learning Activities:

[Zero Exponent/Negative Exponent Property] The Teacher will have the class go over the following examples using the I Do/We Do/You Do protocol. Students will put their answers on Desmos for the Teacher to monitor.

I Do/We Do
Directions: Evaluate each expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

a. 6.7^0 **b.** $(-2)^{-4}$

Written Explanation: The steps I took to evaluate each expression were….

You Do (Checking For Understanding)
Directions: Evaluate each expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

1. $(-9)^0$ **2.** 3^{-3}

Written Explanation: The steps I took to evaluate each expression were….

Answer key will be displayed for students to check their work.

Scaffold: A helpful hint box will be displayed on each slide for students who need to recall the properties.
Helpful Hint:
Algebra $a^0 = 1$, where $a \neq 0$

Algebra $a^{-n} = \frac{1}{a^n}$, where $a \neq 0$

[Power of a Power] The Teacher will have the class go over the following examples using the I Do/We Do/You Do protocol. Students will put their answers on Desmos for the Teacher to monitor.

I Do/We Do
Directions: Evaluate each expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

a. $(4^2)^3$ b. $(12^7)^x$

Written Explanation: The steps I took to evaluate each expression were….

You Do (Checking For Understanding)
Directions: Simplify the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$(8^5)^2$

Written Explanation: The steps I took to evaluate each expression were….

Answer key will be displayed for students to check their work.

Scaffold: A helpful hint box will be displayed on each slide for students who need to recall the property.
Helpful Hint:
Algebra $\mathbf{(a^m)^n}$

Asynchronous Work:
Students will be organized in their breakout teams to complete a Google Doc. Each task sheet will be differentiated based on student needs.

A conversation prompt box will be on the document for students to use when discussing with their team members.
How to talk to your group members:

"What do you think the answer to question ___ is?"

"I think the answer is ___ because…"

"I disagree, I think the answer is___ because.."

Closure:
Students will complete the following exit ticket via Google Forms.

Which expression is equivalent to

$(5^{2x})^{3y}$

- Option 1: 5^{5xy}
- Option 2: 25^{5xy}
- Option 3: 25^{6xy}
- Option 4: 125^{6xy}

Breakout Team Google Document

Properties of Exponents Collaborative Document

Breakout Group 1

Group Member Names:

Directions: Evaluate each expression and identify the property used. Show all work. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$(-8)^0$	$(-4)^{-5}$	$(6^3)^5$
Property Used:_____	Property Used:_____	Property Used:_____

Written Explanation: The steps I took to evaluate each expression were….

Breakout Group 2

Group Member Names:

Directions: Simplify the expression and identify the property used. Write your answer using only positive exponents. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$9x^0y^{-3}$	$3x^{-4}y^0z^0$	$(4^{6m})^{5n}$
Property Used:_____	Property Used:_____	Property Used:_____

Written Explanation: The steps I took to evaluate each expression were….

Breakout Group 3

Group Member Names:

Directions: Describe and correct the error in simplifying the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$4x^0y^{-3}=\frac{4x^0}{y^3}$	$7j^0k^4m^{-2}=\frac{7j^0}{k^4m^2}=\frac{7}{k^4m^2}$
My Explanation:	My Explanation:

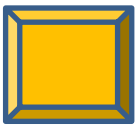
Written Explanation: The steps I took to evaluate each expression were….

Breakout Group 4

Group Member Names:

Directions: Complete the problem below by answering each section. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

Sophia wants to know the area of the square picture frame. A side length is $6x^2y$.



i) What is the formula for the area of a square? _____

ii) State the property of exponent to use in finding the area of the picture frame. _____

iii) Show all work to find the area of the picture frame.

Written Explanation: The steps I took were….

Purpose: The purpose of this lesson is to go through the properties of exponents via remote instruction with a focus on vocabulary comprehension. Students will use the Google Suite platform to respond to question prompts. The Teacher will assess student understanding based on student responses through Desmos, the Collaborative Google Doc and Exit Ticket.

Lesson 2: Properties of Exponents (Product of Powers and Quotient of Powers)

Goals

Focus Question: How can we simplify the expression using the Product of Powers and Quotient of Powers properties?

Next Generation Learning Standards

MP1 Make sense of problems and persevere in solving them.

AI-A.SSE Write expressions in equivalent forms to reveal their characteristics.
- c. Use the properties of exponents to rewrite exponential expressions

Learning Target
By the end of today's lesson, I will be able identify the Product of Powers and Quotient of Powers properties in order to evaluate and simplify the given expression.

Materials	**Skill**
• Google Slides • Google Forms (Exit Ticket) • Google Doc (Collaborative Document) • Desmos	-Identify the Product of Powers and Quotient of Powers properties -Evaluating expressions using the Product of Powers and Quotient of Powers properties

MOTIVATION AND INTRODUCTION OF LESSON

(Student Discussion) Five students will be selected to lead the discussion. A conversation prompt box will be visible on Google Slides to assist students in the discussion. After the five students discuss, the rest of the class will be encouraged to also share their thoughts. Teacher will select one student to share the class consensus answers.

Do Now: Find the value of x and y. Explain how you found your answer.

$12m^{x}n^{2y}=\frac{12}{n^{6}}$

Prompt Hint Box: Identify the exponent properties discussed in yesterday's lesson.

Lesson Introduction:

Prompt Question: What are the remaining properties of exponents?

The teacher will go over the following properties of exponents.
- Product of Powers Property
- Quotient of Powers Property

Building Academic Language-"All About Words" slide:

The Teacher will display the following slide titled "All About Words" which will include the 'Math Terms Support' Chart and the following image of a base and exponent. The Teacher will then ask students to recall the definitions from the previous lesson and complete each of the columns. Afterwards, the Teacher will select students to read out loud the definitions they came up with. The Teacher will then go over each vocabulary term.

Base: the number that gets multiplied by an exponent.
Exponent: for the symbol placed above and to the right of another to indicate by what power the base number is to be raised.
Identify: establish or indicate who or what (someone or something) is.
Evaluate: to determine or calculate the numerical value of (a formula, function, relation, etc.).

Math Term	Math Definition	Example

Learning Activities:

[Product of Powers/Quotient of Powers] The Teacher will have the class go over the following examples using the I Do/We Do/You Do protocol. Students will put their answers on Desmos for the Teacher to monitor.

I Do/We Do
Directions: Evaluate each expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

a. $3^2 \cdot 3^6$ **b.** $\frac{(-4)^2}{(-4)^7}$

Written Explanation: The steps I took to evaluate each expression were….

You Do (Checking For Understanding)
Directions: Simplify the expression. Write your answer using only positive exponents. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$10^4 \cdot 10^{-6}$

Written Explanation: The steps I took to evaluate each expression were….

Answer key will be displayed for students to check their work.

Scaffold: A helpful hint box will be displayed on each slide for students who need to recall the properties.
Helpful Hint:
Algebra $a^m \bullet a^n = a^{m+n}$

Algebra $\frac{a^m}{a^n} = a^{m-n}$, where a ≠ 0

Asynchronous Work:
Students will be organized in their breakout teams to complete a Google Doc. Each task sheet will be differentiated based on student needs and include all properties of exponents discussed between both days.

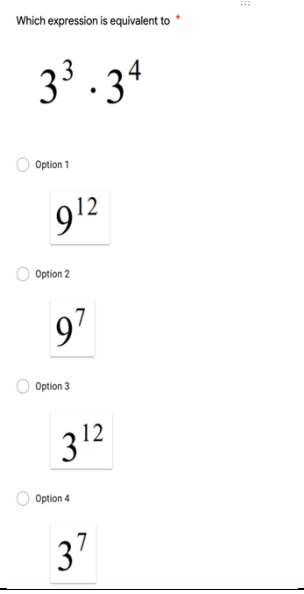
A conversation prompt box will be on the document for students to use when discussing with their team members.
How to talk to your group members:

"What do you think the answer to question ___ is?"

"I think the answer is ___ because…"

"I disagree, I think the answer is___ because.."

Closure:
Students will complete the following exit ticket via Google Forms.



Breakout Team Google Document

Properties of Exponents Collaborative Document

Breakout Group 1

Group Member Names:

Directions: Evaluate each expression and identify the property used. Show all work. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$6^2 \bullet 6^4$	$\frac{8^5}{8^3}$	$\frac{2^{-1}}{-9^0}$
Property Used:_____	Property Used:_____	Property Used:_____

Written Explanation: The steps I took to evaluate each expression were….

Breakout Group 2

Group Member Names:

Directions: Simplify the expression and identify the property used. Write your answer using only positive exponents. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$9x^0y^{-3}$	$3x^{-4}y^0z^0$	$(4^{6m})^{5n}$
Property Used:_____	Property Used:_____	Property Used:_____

Written Explanation: The steps I took to evaluate each expression were….

Breakout Group 3

Group Member Names:

Directions: Describe and correct the error in simplifying the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$3^4 \bullet 3^5 = (3 \bullet 3)^{4+5}$ $= 9^9$	$\frac{m^4 \bullet m^2}{m^3} = \frac{m^6}{m^3} = m^{6/3} = m^2$
My Explanation:	My Explanation:

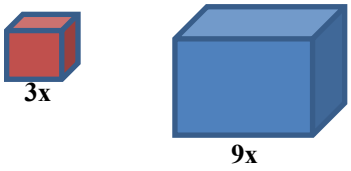
Written Explanation: The steps I took to evaluate each expression were….

Breakout Group 4

Group Member Names:

Directions: Complete the problem below by answering each section. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

Anthony has two different size cubes.



i) What is the volume of the orange cube?_____ Volume of the blue cube?_____

ii) State the property of exponent to use in simplifying the expression for the volume of each cube.

iii) Determine how many times greater is the blue cube from the orange cube using one of the properties of exponents.

Written Explanation: The steps I took were….

Purpose: The purpose of this lesson is to rewrite radical and rational exponents via remote instruction while promoting vocabulary comprehension. Students will use the Google Suite platform to respond to question prompts. The Teacher will assess student understanding based on student responses through Desmos, the Collaborative Google Doc and Exit Ticket.

Lesson 3: Radicals and Rational Exponents

Goals

Focus Question: How can you write and evaluate the nth root of a number?

Next Generation Learning Standards

MP1 Make sense of problems and persevere in solving them.

AI-A.REI Solve equations and inequalities in one variable

Learning Target

By the end of today's lesson, I will be able to rewrite terms with rational exponents in order to evaluate.

Materials

- Google Slides
- Google Forms (Exit Ticket)
- Google Doc (Collaborative Document)
- Desmos

Skill

-Rewrite expressions involving radicals

MOTIVATION AND INTRODUCTION OF LESSON

(Student Discussion) Students will have 3 minutes to complete the Do Now. Teacher will select 1 student to share answer. Then teacher will open discuss for class to agree or disagree with students answer and come to a class consensus.

Do Now: Evaluate the following below.

a) $\sqrt{36}$ **b)** $\sqrt[3]{8}$

Prompt Hint Box: Use the Desmos scientific calculator to verify your answers.

Lesson Introduction: The Teacher will go over the difference between square roots and cube roots.

Recall that you cube a number as follows.

3rd power

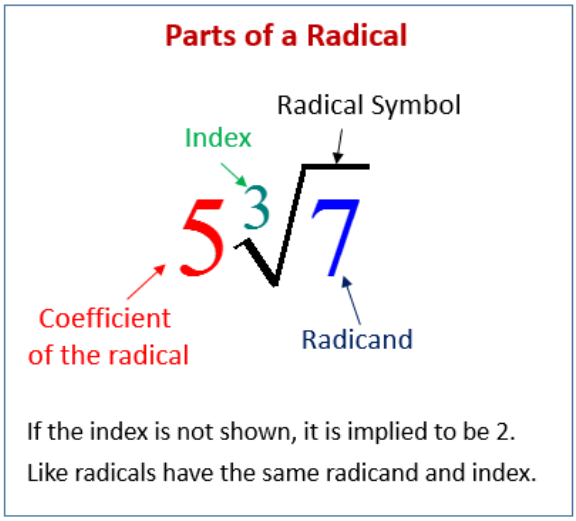
$2^3 = 2 \cdot 2 \cdot 2 = 8$ 2 cubed is 8.

To "undo" cubing a number, take the cube root of the number.

Symbol for cube root is $\sqrt[3]{\ }$.

$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$ The cube root of 8 is 2.

Building Academic Language- "All About Words" slide:



The Teacher will display the following slide titled "All About Words" which will include the 'Math Terms Support' Chart and the image titled 'Parts of a Radical'. The Teacher will then ask students to use the image and prior knowledge to then write down the definitions for the following terms: Radical, Radicand, Index, and Coefficient.

Math Terms Support Chart: The Teacher will have students complete the Vocabulary Support Chart to include the words discussed. In the second column, give students 2 minutes to look online for other definitions of the word.

For example:
Radical-

- (Politics) a person who advocates thorough or complete political or social reform
- (Science) a group of atoms behaving as a unit in a number of compounds

Index-

- (From a book) a more or less detailed alphabetical listing of names, places, and topics along with the numbers of the pages on which they are mentioned or discussed
- (English) something used or serving to point out; a sign, token, or indication

Coefficient-

- (Physics)a number that is constant for a given substance, body, or process under certain specified conditions, serving as a measure of one of its properties
- (from **co-** + **efficient**) that which unites in action with something else to produce a given effect,"

The following definitions will be discussed with the class. Then students will fill in the last column to write down an example for each.
Radical: forming the root/The √ symbol that is used to denote square root or nth roots.
Radicand: to take ***root/*** The value inside the ***radical*** symbol
Index: the power or exponent which is raised to a number or a variable
Coefficient: a number multiplied by a variable

Math Term	Where have you heard or seen this word? (Discuss with your peer. Then browse online for some definitions.)	Math Definition	Example

Rules for Rational Exponents: The Teacher will go over the rules for rational exponents when evaluating.
Let $a^{\frac{1}{n}}$ be an nth root of a, and let m be a positive integer.

Algebra $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$

Numbers $\mathbf{27^{\frac{2}{3}} = (27^{\frac{2}{3}})^2 = (\sqrt[3]{27})^2}$

Learning Activities:
The Teacher will have the class go over the following examples using the I Do/We Do/You Do protocol. Students will put their answers on Desmos for the Teacher to monitor.

I Do/We Do
Directions: Evaluate each expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

a) $(16)^{\frac{3}{4}}$ b) $(27)^{\frac{4}{3}}$

c) $\sqrt[4]{256}$ d) $\sqrt[3]{-343}$

Written Explanation: The steps I took to evaluate each expression were….

You Do (Checking For Understanding)

Directions: Evaluate the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

a) $(64)^{\frac{2}{3}}$ b) $\sqrt[3]{-125}$

Written Explanation: The steps I took to evaluate each expression were….

Answer key will be displayed for students to check their work.

Scaffold: A helpful hint box will be displayed on each slide for students who need to recall the rule for evaluating rational exponents.
Helpful Hint:
Algebra $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$

Asynchronous Work:
Students will be organized in their breakout teams to complete a Google Doc. Each task sheet will be differentiated based on student needs.

A conversation prompt box will be on the document for students to use when discussing with their team members.
How to talk to your group members:

"What do you think the answer to question ___ is?"

"I think the answer is ___ because…"

"I disagree, I think the answer is___ because.."

Closure:
Students will complete the following exit ticket via Google Forms.

Evaluate the expression.

$32^{3/5}$

- 8
- 9
- 10

Breakout Team Google Document

Radicals and Rational Exponents Collaborative Document

Breakout Group 1

Group Member Names:

Directions: Evaluate the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$9^{\frac{5}{2}}$	$\sqrt[3]{27}$
$-\sqrt[4]{16}$	$256^{\frac{3}{4}}$
Written Explanation: The steps I took to evaluate each expression were….	

Breakout Group 2

Group Member Names:

Directions: Evaluate the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$128^{\frac{1}{7}}$	$-\sqrt[5]{1024}$
$\sqrt[4]{256}$	$(-243)^{\frac{2}{5}}$
Written Explanation: The steps I took to evaluate each expression were….	

Breakout Group 3

Group Member Names:

Directions: Describe and correct the error in evaluating the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$$(-81)^{\frac{3}{4}} = [(-81)^{\frac{3}{4}}]^3$$
$$= (-3)^3$$
$$= -27$$

My Explanation:

Breakout Group 4

Group Member Names:

Directions: Complete the problem below. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

The Algebra Team was given a rectangular carpet to put in the tutoring room. The dimensions are in feet. Find the area.

$4^{1/2}$

$\sqrt[6]{729}$

My Explanation:

Purpose: The purpose of this lesson is to identify and evaluate exponential functions via remote instruction while promoting vocabulary comprehension. Students will use the Google Suite platform to respond to question prompts. The Teacher will assess student understanding based on student responses through Desmos, the Collaborative Google Doc and Exit Ticket.

Lesson 4: Exponential Functions

Goals

Focus Question: How can you write and evaluate the nth root of a number?

Next Generation Learning Standards

MP1 Make sense of problems and persevere in solving them.

AI-A.REI Solve equations and inequalities in one variable

Learning Target

By the end of today's lesson, I will be able to identify and evaluate the characteristics of an exponential function in order to determine different representations.

Materials	**Skill**
• Google Slides • Google Forms (Exit Ticket) • Google Doc (Collaborative Document) • Desmos	-Evaluate exponential functions

MOTIVATION AND INTRODUCTION OF LESSON

(Student Discussion) Students will have 3 minutes to complete the Do Now. Teacher will select 1 student to share answer. Then teacher will open discuss for class to agree or disagree with students answer and come to a class consensus.

Do Now: Which expression does not belong with the other three? Explain your response.

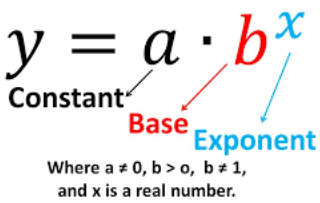
$(\sqrt[3]{27})^2$ $27^{\frac{2}{3}}$ 3^2 $(\sqrt[2]{27})^3$

Prompt Hint Box: What steps do you need to take to identify the expression that does not belong?

Lesson Introduction: The Teacher will go over the definition of exponential functions.

An **exponential function** is a nonlinear function of the form $y = ab^x$, where $a \neq 0$, $b \neq 1$, and $b > 0$. As the independent variable x changes by a constant amount, the dependent variable y is multiplied by a constant factor, which means consecutive y-values form a constant ratio.

Building Academic Language- "All About Words" slide:



The Teacher will display the following slide titled "All About Words" which will include the 'Math Terms Support' Chart, the exponential function definition and the image of the exponential function form. The Teacher will then ask students to use the image and prior knowledge to then write down the definitions for the following terms: Constant, Independent/ Dependent Variable and Consecutive.

Math Terms Support Chart: The Teacher will have students complete the Vocabulary Support Chart to include the words discussed. In the second column, give students 2 minutes to look online for other definitions of the word.

For example:
Constant-

- a situation or state of affairs that does not change.
- (Physics) a number expressing a relation or property which remains the same in all circumstances, or for the same substance under the same conditions.

Independent/ Dependent Variable-

- (Independent)free from outside control; not depending on another's authority.
- (Dependent)requiring someone or something for financial, emotional, or other support.

Consecutive-

- following continuously.

The following definitions will be discussed with the class. Then students will fill in the last column to write down an example for each.
Constant: a quantity or parameter that does not change its value whatever the value of the variables
Independent/ Dependent Variable: An independent variable is a variable that represents a quantity that is being manipulated in an experiment. A dependent variable represents a quantity whose value depends on those manipulations.
Consecutive: Numbers that follow each other continuously in the order from smallest to largest

Math Term	Where have you heard or seen this word? (Discuss with your peer. Then browse online for some definitions.)	Math Definition	Example

Learning Activities:
The Teacher will have the class go over the following examples using the I Do/We Do/You Do protocol. Students will put their answers on Desmos for the Teacher to monitor.

I Do/We Do
Directions: (Table) Does each table represent a linear or an exponential function? **(Equation)** Evaluate the function for the given value of x. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

1)

x	0	1	2	3
y	2	4	6	8

2)

x	0	1	2	3
y	4	8	16	32

3) $y=-2(5)^x$; x=3

4) $y=3(0.5)^x$; x= -2

Written Explanation: This table/equation represents a _____ function because…

You Do (Checking For Understanding)
Directions: (Table) Does each table represent a linear or an exponential function? **(Equation)** Evaluate the function for the given value of x. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

1)

x	0	1	2	3
y	8	4	2	1

2)

x	-4	0	4	8
y	1	0	-1	-2

3) $y=1.5(2)^x$; x= -2

Written Explanation: This table/equation represents a _____ function because…

Answer key will be displayed for students to check their work.

Scaffold: A helpful hint box will be displayed on each slide for students who need guidance on how to identify the function based on the table of values.
Helpful Hint:

Table	**Equation**
Check how the y-values increase by: -Add/Subtract (Linear) -Multiply (Exponential)	1)Substitute the 'x' value into the function 2) Then simplify

Asynchronous Work:
Students will be organized in their breakout teams to complete a Google Doc. Each task sheet will be differentiated based on student needs.

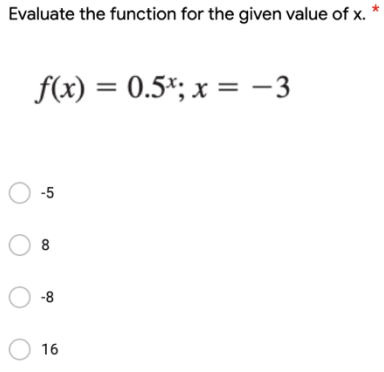
A conversation prompt box will be on the document for students to use when discussing with their team members.
How to talk to your group members:

"What do you think the answer to question ___ is?"

"I think the answer is ___ because…"

"I disagree, I think the answer is___ because.."

Closure:
Students will complete the following exit ticket via Google Forms.



Breakout Team Google Document

Exponential Functions Collaborative Document

Breakout Group 1

Group Member Names:

Directions: (Table) Does each table represent a linear or an exponential function? **(Equation)** Evaluate the function for the given value of x. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

<table><tr><td>**x**</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>**y**</td><td>0.25</td><td>1</td><td>4</td><td>16</td></tr></table>	<table><tr><td>**x**</td><td>-3</td><td>0</td><td>3</td><td>6</td></tr><tr><td>**y**</td><td>10</td><td>1</td><td>-8</td><td>-17</td></tr></table>
$y=(3)^x$; x= 2	$y=(0.5)^x$; x= -3
Written Explanation: This table/equation represents a _____ function because…	

Breakout Group 2

Group Member Names:

Directions: Evaluate the expression. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

x	-1	0	1	2
y	-2	0	2	4

x	1	2	3	4
y	6	12	24	48

$y=-4(5)^x$; x= 2

$y=\frac{1}{3}(6)^x$; x= 3

Written Explanation: This table/equation represents a _____ function because…

Breakout Group 3

Group Member Names:

Directions: Describe and correct the error in evaluating the function. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

$$g(x)= 6(0.5)^{x} \text{ ; } x=-2$$

$$g(-2)= 6(0.5)^{-2}$$

$$= 3^{-2}$$

$$= \frac{1}{9}$$

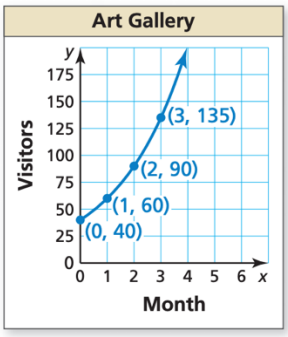
My Explanation:

Breakout Group 4

Group Member Names:

Directions: Complete the problem below. Then complete the written explanation. Be sure to include at least one math term from your 'Math Term Support Chart' in your explanation.

The graph represents the number y of visitors to a new art gallery after x months.



a. Write an exponential function that represents this situation.

b. Approximate the number of visitors after 5 months.

My Explanation:

How this Worked in The Classroom

A veteran bilingual teacher with experience working directly with ELLs reviewed the lessons and provided the following feedback for other teachers to consider as they seek to implement these materials in their own classrooms.

What were the strengths of the lessons?

After introducing each lesson, providing time for students to connect with unfamiliar vocabulary terms promoted student both student discourse and understanding. Students appeared to be more comfortable discussing the terms out loud. The Teacher noted that students were including the mathematical terms in their explanation responses. The Teacher also noted that providing a writing space in each task was also a great way to have students develop conceptual understanding and make connections to the topics presented.

What were the weaknesses of the lessons?

The time spent on having students explore the meanings of the mathematical vocabulary independently cut into the mini lesson and collaborative activity. After receiving this feedback, I changed the exploration of the mathematical terms support chart to have students discuss with their peers the word meanings to promote student peer discourse. Also, after building a routine with the students using the support chart, there was more time to have students practice completing the lesson topic tasks.

With your experience working with English Language Learners, do you believe the tasks promoted student understanding in unpacking the meanings of the mathematical vocabulary?

Overall, the Teacher mentioned each of the lessons were planned extremely well and believe that ELLs can benefit from the Math Support Chart because it increases student

confidence in making connections with the math vocabulary while exploring how words can have alternate meanings.

Conclusion

English Language Learners face many challenges especially learning the English language. Within the math classroom, there are many high leverage vocabulary terms than can be difficult for ELLs to decipher the meanings. It is critical to their development for teachers to design lessons with a focus on vocabulary comprehension. The exponential functions unit is one of many example units where a math support chart can be used for ELLs to learn about new word meanings and how they can also have multiple meanings. To increase student comprehension by promoting a growth mindset using the support chart can positively increase their confidence. It is my hope that algebra teachers find this curriculum useful and can be used in future lessons to bridge the learning gaps between mathematics and vocabulary comprehension.

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Wiest, Lynda R. "Problem-Solving Support for English Language Learners." *Teaching Children Mathematics*, vol. 14, no. 8, National Council of Teachers of Mathematics, 2008, pp. 479–84.

Appendix

Lesson 1 Answer Key

Zero Exponent/Negative Exponent Property Examples:

I Do/We Do:

a. $6.7^0 = 1$

*Zero Product Property

b. $(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$

* Negative Exponent Property

You Do (Checking For Understanding)

1. $(-9)^0 = 1$

*Zero Product Property

2. $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$

* Negative Exponent Property

Power of a Power Property Examples:

I Do/We Do:

a. $(4^2)^3 = 64^6$

b. $(12^7)^x = 12^{7x}$

You Do (Checking For Understanding)

$(8^5)^2 = 64^{10}$

Breakout Team Google Document KEY

Breakout Group 1:

$(-8)^0$	$(-4)^{-5}$	$(6^3)^5$
$(-8)^0 = 1$	$(-4)^{-5} = \frac{1}{(-4)^5} = \frac{1}{1024}$	$(6^3)^5 = 7776^{15}$
Property Used: Zero Exponent	Property Used: Negative Exponent	Property Used: Power of a Power

Breakout Group 2:

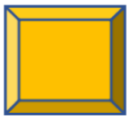
$9x^0y^{-3}$	$3x^{-4}y^0z^0$	$(4^{6m})^{5n}$
$9 \cdot 1 \cdot \frac{1}{y^3}$ $\frac{9}{y^3}$	$3 \cdot \frac{1}{x^4} \cdot 1 \cdot 1$ $\frac{3}{x^4}$	1024^{30mn}
Property Used: Zero Exponent/ Negative Exponent	Property Used: Negative Exponent/ Zero Exponent	Property Used: Power of a Power

Breakout Group 3:

$4x^0y^{-3}=\frac{4x^0}{y^3}$	$7j^0k^4m^{-2}=\frac{7j^0}{k^4m^2}=\frac{7}{k^4m^2}$
$4x^0y^{-3}=\frac{4\cdot 1}{y^3}=\boxed{\frac{4}{y^3}}$ My Explanation: The student did not use the Zero Exponent property.	$7j^0k^4m^{-2}=\frac{7j^0k^4}{m^2}=\frac{7\cdot 1\cdot k^4}{m^2}=\boxed{\frac{7k^4}{m^2}}$ My Explanation: The student incorrectly applied the Negative Exponent property to k^4

Breakout Group 4:

Sophia wants to know the area of the square picture frame. A side length is $6x^2y$.



i) What is the formula for the area of a square? $A=s^2$

ii) State the property of exponent to use in finding the area of the picture frame. Power of a Power Property

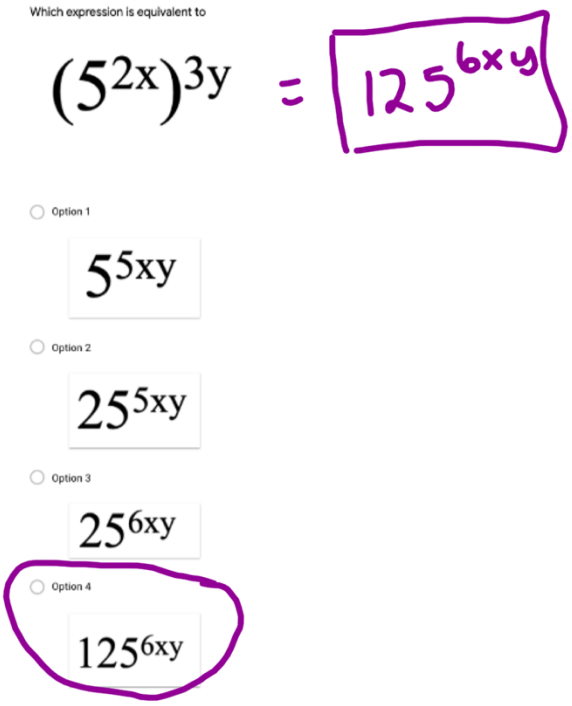
iii) Show all work to find the area of the picture frame.

$A=s^2$

$=(6x^2y)^2$

$=\boxed{36x^4y^2}$

Exit Ticket:


Which expression is equivalent to
$(5^{2x})^{3y}$ = 125^{6xy}
Option 1
5^{5xy}
Option 2
25^{5xy}
Option 3
25^{6xy}
Option 4
125^{6xy}

Lesson 2 Answer Key

Product of Powers/Quotient of Powers Examples:

I Do/We Do:

a. $3^2 \cdot 3^6 = 3^{2+6} = \boxed{3^8}$

*Product of Powers

b. $\frac{(-4)^2}{(-4)^7} = -4^{2-7} = -4^{-4} = \frac{1}{-4^4} = \boxed{\frac{1}{256}}$

* Quotient of Powers/ Negative Exponent

You Do (Checking For Understanding)

$10^4 \cdot 10^{-6} = 10^{4-6} = 10^{-2} = \frac{1}{10^2} = \boxed{\frac{1}{100}}$

* Product of Powers / Negative Exponent

Breakout Team Google Document KEY

Breakout Group 1:

$6^2 \bullet 6^4$	$\frac{8^5}{8^3}$	$\frac{2^{-1}}{-9^0}$
$6^2 \cdot 6^4 = 6^{2+4} = 6^6$	$\frac{8^5}{8^3} = 8^{5-3} = 8^2$	$\frac{2^{-1}}{-9^0} = \frac{2^{-1}}{1} = \frac{1}{2}$
Property Used: Product of Powers	Property Used: Quotient of Powers	Property Used: Zero Exponent / Negative Exponent

Breakout Group 2:

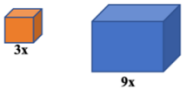
$3^{-4} \bullet 3^{-11}$	$\frac{9^{-7}}{9^4}$	$\frac{4^{-3}a^0}{b^{-7}}$
$3^{-4} \cdot 3^{-11} = 3^{-4+(-11)} = 3^{-15} = \frac{1}{3^{15}}$	$\frac{9^{-7}}{9^4} = 9^{-7-4} = 9^{-11} = \frac{1}{9^{11}}$	$\frac{1}{4^3} \cdot \frac{1}{1} \cdot \frac{b^7}{1}$ $\frac{b^7}{64}$
Property Used: Product of Powers/ Negative Exponent	Property Used: Quotient of Powers/Negative Exponent	Property Used: Negative Exponent/Zero Exponent

Breakout Group 3:

$3^4 \bullet 3^5 = (3 \bullet 3)^{4+5} = 9^9$	$\frac{m^4 \bullet m^2}{m^3} = \frac{m^6}{m^3} = m^{6/3} = m^2$
$3^4 \cdot 3^5 = 3^{4+5} = \boxed{3^9}$	$\frac{m^6}{m^3} = m^{6-3} = \boxed{m^3}$
My Explanation: The student incorrectly used the Product of Powers Property. To multiply powers with the same base, add their exponents.	My Explanation: To divide powers with the same base, add their exponents.

Breakout Group 4:

Anthony has two different size cubes.



i) What is the volume of the orange cube? $(3x)^3$ Volume of the blue cube? $(9x)^3$

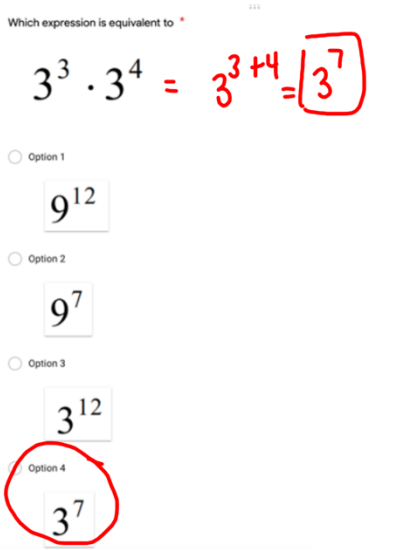
ii) State the property of exponent to use in simplifying the expression for the volume of each cube.
The property to use is Power of a Product Property.

iii) Determine how many times greater is the blue cube from the orange cube using one of the properties of exponents.

$$\frac{(9x)^3}{(3x)^3} = \frac{729x^3}{27x^3} = \boxed{27}$$

The volume is 27 times greater.

Exit Ticket:


Which expression is equivalent to *
$3^3 \cdot 3^4 = 3^{3+4} = 3^7$
Option 1
9^{12}
Option 2
9^7
Option 3
3^{12}
Option 4
3^7

Lesson 3 Answer Key

Do Now: Evaluate the following below.

a) $\sqrt{36} = 6$ b) $\sqrt[3]{8} = 2$

I Do/We Do:

a) $(16)^{\frac{3}{4}}$

$16^{3/4} = (16^{1/4})^3$

$= (2)^3$

$= 8$

b) $(27)^{\frac{4}{3}}$

$27^{4/3} = (27^{1/3})^4$

$= (3)^4$

$= 81$

c) $\sqrt[4]{256}$

$\sqrt[4]{256} = \sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4}$

$= 4$

d) $\sqrt[3]{-343}$

$\sqrt[3]{-343} = \sqrt[3]{(-7)\cdot(-7)\cdot(-7)}$

$= -7$

You Do (Checking For Understanding)

a) $(64)^{\frac{2}{3}}$

$64^{\frac{2}{3}} = (64^{\frac{1}{3}})^2$

$= (4)^2$

$= 16$

b) $\sqrt[3]{-125}$

$\sqrt[3]{-125} = \sqrt[3]{(-5)\cdot(-5)\cdot(-5)}$

$= -5$

Exit Ticket:

Evaluate the expression.

$32^{3/5}$

$32^{\frac{3}{5}} = (32^{1/5})^3$

$= (2)^3$

$= 8$

- (8)
- 9
- 10

Breakout Team Google Document KEY

Breakout Group 1:

$9^{\frac{5}{2}} = (9^{1/2})^5$ $= (3)^5$ $= 243$	$\sqrt[3]{27} = \sqrt[3]{3 \cdot 3 \cdot 3}$ $= 3$
$-\sqrt[4]{16} = -\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$ $= -2$	$256^{\frac{3}{4}} = (256^{1/4})^3$ $= (4)^3$ $= 64$

Breakout Group 2:

$128^{\frac{1}{7}} = 2$

$-\sqrt[5]{1024}$

$-\sqrt[5]{1024} = -\sqrt[5]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$

$= -4$

$\sqrt[4]{256} = \sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4}$

$= 4$

$(-243)^{\frac{2}{5}}$

$-243^{\frac{2}{5}} = (-243^{\frac{1}{5}})^2$

$= (-3)^2$

$= 9$

Breakout Group 3:

$$(-81)^{\frac{3}{4}} = [(-81)^{\frac{3}{4}}]^3$$
$$= (-3)^3$$
$$= -27$$

$(-81)^{3/4} = (-81^{\frac{1}{4}})^3$

-81 does not have a real fourth root

Breakout Group 4:

$A_{\square} = l \times w$

$= 4^{\frac{1}{2}} \cdot \sqrt[6]{729}$

$= 2 \cdot 3$

$= \boxed{6 ft^2}$

Lesson 4 Answer Key

Do Now: Which expression does not belong with the other three? Explain your response.

$(\sqrt[3]{27})^2$ = 3 $\quad 27^{\frac{2}{3}}$ = 3 $\quad 3^2$ = 3 $\quad (\sqrt[2]{27})^3$ = 140

It is the only expression not equivalent to 9.

I Do/We Do:

1)

x	0	1	2	3
y	2	4	6	8

+2 +2 +2

Linear

2)

x	0	1	2	3
y	4	8	16	32

×2 ×2 ×2

Exponential

3) $y=-2(5)^x$; x=3

$y = -2(5)^3$

$= -250$

4) $y=3(0.5)^x$; x= -2

$y = 3(0.5)^{-2}$

$= 12$

You Do (Checking For Understanding)

1)

x	0	1	2	3
y	8	4	2	1

×½ ×½ ×½

Exponential

2)

x	-4	0	4	8
y	1	0	-1	-2

-1 -1 -1

Linear

3) $y=1.5(2)^x$; x= -2

$y = 1.5(2)^{-2}$

$= 0.375$

Exit Ticket:

Evaluate the function for the given value of x. *

$f(x) = 0.5^x; x = -3$

$f(-3) = 0.5^{-3}$
$= 8$

- -5
- 8
- -8
- 16

Breakout Team Google Document KEY

Breakout Group 1:

x	-1	0	1	2
y	0.25	1	4	16

x4 x4 x4 Exponential

x	-3	0	3	6
y	10	1	-8	-17

-9 -9 -9 Linear

$y=(3)^x; x= 2$

$y = (3)^2$
$= 9$

$y=(0.5)^x; x= -3$

$y = (0.5)^{-3}$
$= 8$

Breakout Group 2:

x	-1	0	1	2
y	-2	0	2	4

+2 +2 +2

Linear

x	1	2	3	4
y	6	12	24	48

×2 ×2 ×2

Exponential

$y=-4(5)^x$; x= 2

$y= -4(5)^2$

$=-100$

$y=\frac{1}{3}(6)^x$; x= 3

$y= \frac{1}{3}(6)^3$

$=72$

Breakout Group 3:

$g(x)= 6(0.5)^x$; x=-2

$g(-2)= 6(0.5)^{-2}$

$= 8^{-2}$

$= \frac{1}{9}$

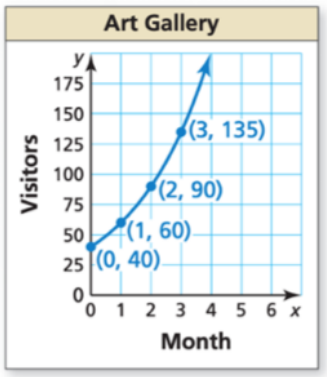
$g(-2)= 6(0.5)^{-2}$

$= 6(4)$

$= 24$

Breakout Group 4:

The graph represents the number y of visitors to a new art gallery after x months.



x	0	1	2	3
y	40	60	90	135

$\times\frac{3}{2}$ $\times\frac{3}{2}$ $\times\frac{3}{2}$

a. Write an exponential function that represents this situation.

$a = 40$

$b = \frac{3}{2}$

$y = ab^x$

$y = 40\left(\frac{3}{2}\right)^x$

b. Approximate the number of visitors after 5 months.

$y = 40\left(\frac{3}{2}\right)^x \; ; \; x = 5$

$= 40\left(\frac{3}{2}\right)^5$

$=$ 304 visitors