Productive Struggle in an Algebra 1 Classroom

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Abstract

This curriculum project aims to transform students' struggles with Algebra 1 into opportunities for growth and learning. Recognizing that almost all students encounter challenges with mathematical concepts, the curriculum seeks to shift students' mindset from self-identification as a student who is not good at mathematics to one that embraces productive struggle. By actively engaging students with various algebraic topics and providing comprehensive support for teachers, this curriculum can support teachers who seek to help students overcome struggles in learning mathematics. Students will see opportunities for productive struggle as steppingstones rather than stumbling blocks. This curriculum includes four Algebra 1 lessons with teacher support materials, student materials and answer keys. These materials are located in the appendix.

Introduction

Struggle is an inherent aspect of life. Regardless of the situation or position, everyone experiences struggle. These challenging times often become our greatest teachers, imparting some of life's most valuable lessons. Students, especially in the classroom, are no exception. Every student will encounter difficulties with mathematics at some point, whether in first grade or calculus. Unfortunately, many students engage in what is termed as "destructive struggle," (Permatasari, p. 96) where their confidence wanes, leading to disengagement and a negative self-image regarding their academic abilities.

As educators, we have the unique opportunity to guide students towards productive struggle (Permatasari, p. 95), where they use challenges as powerful learning experiences. This curriculum project is designed to highlight various methods for fostering productive struggle and to provide teachers with strategies that support their students through this process. Covering a range of nonsequential Algebra 1 topics, the lessons aim to engage students in productive struggle through active problem-solving and critical thinking. The lessons begin with literal equations, where students use their notes and independent thinking to solve problems without teacher support, pushing them into a space of productive struggle. They then move into exponential equations, which encourages students to expand their understanding of using calculators as problem-solving tools. The final lessons focus on factoring, in which students break down polynomials into simpler factors, and quadratic equations, whereby students solve and analyze quadratic functions.

The key objective of this curriculum is to promote productive struggle by helping students perceive challenges as essential to the learning process and fostering resilience and a growth mindset in mathematical thinking. Additionally, it supports teacher guidance by

providing educators with strategies to assist students through their challenges and supplying materials and detailed notes to facilitate effective instruction and intervention. The resources available for teachers include customizable blank slides for instructional use, practice problems and activities aligned with lesson objectives, comprehensive solutions for all practice problems to provide quick feedback, and detailed explanations, tips, and strategies for effectively teaching each concept. By engaging in these thoughtfully designed lessons, students will be better equipped to understand the material deeply and form lasting connections with the content. This approach can be used to show that struggles become opportunities for profound learning and personal growth.

Literature Review

Why do students struggle with mathematics?

Across the United States (US), students struggle with learning mathematics (Pew Research Center, 2017) . Some struggle as early as kindergarten or first grade, and others struggle later in life with more advanced mathematical concepts. But when looking into the why behind this seemingly universal struggle, it's important to consider this question *why is the struggle unique to learning mathematics?* Students struggle in other subjects like reading and writing, but mathematics shines as an outlier in the following statistics. According to the 2015 P.I.S.A. (Program for International Student Assessment), 15 year old students in the US scored on average, 27 points lower in mathematics than reading or science. The US falls below the international average in mathematics, and above it in reading and science (PEW Research Center, 2017). Regarding standardized assessments. why is it that students perform lower in mathematics? The answer is simple, learning mathematics is complex. As defined by van Merriënboer et al. (2006), "complex tasks have many different solutions, are ecologically valid [e.g., real world connections], cannot be mastered in a single session and pose a very high load on the learner's cognitive system" (p.1). Most mathematics teachers will agree that learning mathematics requires students to study concepts over time for them to be mastered. Mathematics is a multifaceted subject, which requires students to constantly break down their understandings and reconstruct them, using new information to further their learning. Sometimes what appears to be the best way to attack a problem is later revealed to be the longest, most laborious way. For example, we require students to learn the formal definition of a derivative (required for understanding the concept of the derivative) before teaching them tools like the product rule, quotient rule, and chain rule, etc. Much of what is learned in mathematics can be taken outside

the classroom, making it ecologically valid, or applicable to the real world, such as in the science fields. Anyone who has become proficient in mathematics understands the high cognitive load that is associated with learning mathematics. For example, the operation addition is a transferable skill required to learn multiple concepts such as combining like terms, factorization, and the definition of the derivative, however students are not able to complete such complex tasks when they first learn to take the sum. Learning mathematics takes time, practice, repetition, and struggle. Tall (2004) described three distinct mathematical worlds that students encounter throughout their education that grows in sophistication . As they travel different paths along their mathematical growth, obstacles occur that require earlier ideas to be reconsidered and reconstructed making the journey different for every traveler. In summary, students struggle learning mathematics because it is complex, so it is important for educators to see the value in the struggle and utilize students' struggle as part of their learning.

What is productive struggle? How do we avoid destructive struggle?

When students struggle with learning mathematics the struggle can either help to further their understanding or it can lead them to developing a negative self-image toward their ability to learn mathematics. The goal of educators is to encourage and engage students in productive struggle and to help students avoid destructive struggle. Students who are engaging in productive struggle do not shut down and refuse to engage but acknowledge that they are struggling and continue working towards arriving at an acceptable solution. When students disengage from a task, or develop learned helplessness, they are less likely to engage in productive struggle and are more likely to engage in destructive struggle. They might develop a negative self-image in relation to their abilities or determine that mathematics is not something that they can do. This can lead to students stopping their pursuit of understanding mathematics altogether, and instead

rely on short term memorization of procedures. Destructive struggle replaces effort to learn with complacency. As educators, we cannot force students to engage in productive struggle, however we can provide the optimal conditions for it to occur. Schoenfeld (2017) referred to the "happy medium between spoon-feeding mathematics in bite sized pieces and having the challenges so large that students are lost at sea" (p. 9). Maintaining this balance is crucial to helping students engage in productive struggle, as well as helping them avoid destructive struggle. There are many factors that contribute to this balance, and as always, each student's unique needs are at the center of it all. According to Permatasari (2016), there are three areas of focus for educators to consider when attempting to engage students in productive struggle, mathematical self-image, the student's disposition, and teacher support and feedback.

Mathematical self-image is related to a student's perception about their mathematical ability. They often think that "mathematical ability is something that some students are skilled at rather than a behavior that everyone can develop" (Permatasari, p. 3). Students at any age come to school with some sort of preconceived notion of their ability. They gather this from their peers, families, and surroundings at home and school. Students need to believe that they are capable of solving a problem before they will truly engage with it.

Student's disposition with challenging mathematical tasks depends on whether they find the task interesting, if they believe that can solve the problem, and if they believe that solving the task is worth the effort (Permatasari, 2016). Regarding the last point, students need to believe that their efforts are warranted. When students ask *when am I ever going to need this,* they are revealing their doubt about the need-to-know. Educators can do more than just provide an uninspiring answer to move forward with the lesson. They can generate curiosity, provide students with encouragement, and allow them to see the value in what they are being asked to do.

Effective feedback communicates the learning goal, progress made towards that goal, and how to make progress from where the student currently is (Permatasari, 2016). Students need to feel supported and know that teachers have an invested interest in their success in learning mathematics. By providing feedback on students' progress students can become aware of their teacher's invested interest in their success.

How do we know productive struggle has an impact on student learning?

As teachers, it is difficult to watch students struggle through a problem, especially when they get so close to an acceptable answer. But this struggle is important. When students engage in productive struggle, they are working towards understanding, and truly learning. How do we know? That depends on two things, first how do you define learning? Second, how do you measure the impact of different classroom learning strategies? Learning has many definitions, but one definition has always stood out to me; "learning can be characterized as a permanent change in long term memory" (Wade, page 3). This means that what was learned can be recalled and used to support more learning of mathematics. Hattie (2015) researched the impact that different factors have on student learning and categorized over 250 influences and placed them into five zones. The zones range from negative impact to highly positive impact. Each influence has a rating on a scale of -.2 to 1.6 (with 1.6 being the most impactful). When looking at productive struggle in light of Hattie's work, it is easy to align some of the influences with the attributes of productive struggle. For example, Hattie has "self-efficacy" as having an impact of 0.71, which makes it is one of the highest impact influences. When students engage in productive struggle, they are also engaging in self-efficacy. After struggling through a problem, and arriving at a solution, students can see their own ability, and begin to believe in themselves. When students increase their mathematical self-image, they increase their ability to engage in

productive struggle. It's a repeating cycle that leads to an increase in understanding. Another example from Hattie with an impact on learning of 0.93, during productive struggle, students are forced to dig into their mathematical toolboxes, and determine what learned processes they can use to help them tackle a new task, whether it be solving a problem, or analyzing a graph. When students engage in productive struggle, they are accelerating the process of learning by making the use of highly impactful learning strategies. By setting the conditions for productive struggle in our classrooms, we can enable students to further their own learning.

Material Creation Statement

All lesson materials contained in the following lessons were created by Frederick Finter as part of the Victor Central School District 15:1+1 Algebra 1 Curriculum, aligning to New York State's Next Gen Standards in Algebra 1. References made to either specific units or lesson numbers can be ignored as they align to that curriculum.

The lessons focus on different ways that you can emphasize productive struggle in your classroom, looking at lessons across different topics in algebra 1. These lessons are not consecutive, but are linked by the central purpose of culminating productive struggle.

Some problems were pulled from old New York State Regent's Exams, which can be found on JMAP (https://www.jmap.org/JMAP_REGENTS_EXAMS.htm).

Lesson 1 - Literal Equations from Solving Linear Equations and Inequalities

Standards - NYS Next Generation Mathematics Standards:
AI-A.CED
4. Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

AI-A.REI
3. Solve linear equations and ~~inequalities~~ in one variable, including equations with coefficients represented by letters.

Learning Objective - "I will be able to solve literal equations for a specific variable by using inverse operations".

Assessment of Learning - Students will turn in their practice activity from the lesson, this will serve as a formative assessment.

Planned Learning Experiences

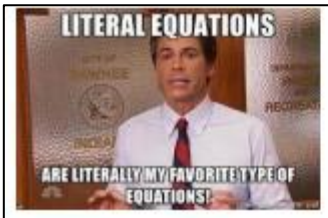
1. Warm Up - The warmup problem for this lesson is a regent's multiple choice where students need to correctly identify the expression that has been correctly converted from syntax to symbols. (*see lesson materials*)
2. White Board Practice Problems - Each student will require a white board (or blank piece of paper) to work through two examples of solving literal equations by isolating the desired variable.
3. Formal Notes and Student Led examples – Students will take notes (either in a spiral or blank paper) on Literal equations, and then complete several problems to help solidify their understanding.
4. The Hamster Heist - Each student will receive a hamster heist packet, which contains 5 literal equations problems. The students will form groups and attempt to solve the 5 problems. Each correctly solved problem will grant that group a clue to help them solve the case of the hamster heist. A timer will be set at the beginning of the activity for 10-20 minutes, depending on time constraints. Until this timer goes off, students will not be able to seek out adult support on those questions. They will need to work through the problems with their group. Once the timer goes off, groups can ask questions regarding the problems.

Productive Struggle - Throughout the lesson, students will be engaging in productive struggle through mathematical investigation. Students will be completing the hamster heist packet independently, using their group members / notes to help them work through the problems. It is important to allow students to engage in this struggle, and to not step in too early in their problem-solving process.

Teacher Notes:

- If students all complete the activity early, go through the problems together to clear up any misconceptions.
- Be sure to collect all packets at the end of class to use as an assessment of learning.

Lesson 1 Blank Slide Deck



Section 2.2:

Literal Equations

- Warm Up
- Literal Equations Intro & Notes
- Practice in Packet
- The Hamster Heist Activity...

Warm Up: Activity Packet

3. Three times the sum of a number and four is equal to five times the number, decreased by two. If x represents the number, which equation is a correct translation of the statement?

1) $3(x+4) = 5x - 2$

2) $3(x+4) = 5(x-2)$

3) $3x + 4 = 5x - 2$

4) $3x + 4 = 5(x-2)$

Learning Statement

I can solve literal equations for a specific variable by using inverse operations.

To "solve" a literal equation is to solve for a different variable.

Ex) Solve for m in terms of T and v:

$$T = \frac{1}{2}mv^2$$

*Kinetic Energy formula

Ex) Solve for v in terms of T and m:

$$T = \frac{1}{2}mv^2$$

Page 18 in Activity Packet...

NYS Regents

12) The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$. The height, h, of the trapezoid may be expressed as

1) $2A - b_1 - b_2$

2) $\frac{2A - b_1}{b_2}$

3) $\frac{1}{2}A - b_1 - b_2$

4) $\frac{2A}{b_1 + b_2}$

Practice Worksheet

"The Hamster Heist"

Lesson 1 Blank Student Problems

2. When 3 times a number is decreased by 3, the result is the same as 13 subtracted from five times the number. Write an equation and find the number.

3. Three times the sum of a number and four is equal to five times the number, decreased by two. If x represents the number, which equation is a correct translation of the statement?

1) $3(x + 4) = 5x - 2$

2) $3(x + 4) = 5(x - 2)$

3) $3x + 4 = 5x - 2$

4) $3x + 4 = 5(x - 2)$

4. The second side of a triangle is one inch less than the first side, and the third side is twice as long as the first side. If the perimeter of the triangle is 59 inches, find the lengths of all three sides.

9) Solve for **b**: $a^2 + b^2 = c^2$

10) Solve for **a**: $r = p(220 - a)$

11) Boyle's Law involves the pressure and volume of gas in a container. It can be represented by the formula $P_1V_1 = P_2V_2$. When the formula is solved for P_2, the result is

1) $P_1V_1V_2$
2) $\frac{V_2}{P_1V_1}$
3) $\frac{P_1V_1}{V_2}$
4) $\frac{P_1V_2}{V_1}$

12) The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$. The height, h, of the trapezoid may be expressed as

1) $2A - b_1 - b_2$
2) $\frac{2A - b_1}{b_2}$
3) $\frac{1}{2}A - b_1 - b_2$
4) $\frac{2A}{b_1 + b_2}$

Lesson 1 Blank Hamster Heist Student Handout

Name _____ Literal Equations Practice

The Hamster Heist

When students returned to school after a holiday break, they discovered their pet hamster, Ham, had disappeared. His cage was not in its usual place on the table near the window.

They thought maybe Miss Anita, the custodian, had moved Ham, but when they asked her about it, she reported that the cage had been in the classroom on Monday. She said she had unlocked the classroom door, cleaned the room, then locked the door and left. On Thursday morning, when she unlocked the room to clean it again, Ham's cage was gone. She had no idea where it was.

Solve the following literal equations to get clues to help you solve the mystery of who took Ham the Hamster. Check your answer for each problem and for each question you get right, you get a clue to the mystery. Good luck!

1. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Rewrite the formula solving for the radius, r.

2. The formula $Ax + By = C$ represents the equation of a line in standard form. Which expression represents y in terms of A, B, C, and x?

(1) $\frac{C - Ax}{B}$

(2) $\frac{C - A}{Bx}$

(3) $\frac{C - A}{x + B}$

(4) $\frac{C - B}{Ax}$

3. The amount of energy, Q, in joules, needed to raise the temperature of m grams of a substance is given by the formula $Q = mC(T_f - T_i)$, where C is the specific heat capacity of the substance. If its initial temperature is T_i, an equation to find its final temperature, T_f, is

(1) $T_f = \frac{Q}{mC} - T_i$

(2) $T_f = \frac{Q}{mC} + T_i$

(3) $T_f = \frac{T_i + Q}{mC}$

(4) $T_f = \frac{Q - mC}{T_i}$

4. The temperature inside a cooling unit is measured in degrees Celsius, C. Josh wants to find out how cold it is in degrees Fahrenheit, F.

Solve the formula $C = \frac{5}{9}(F - 32)$ for F so that Josh can convert Celsius to Fahrenheit.

5. A formula for determining the finite sum, S, of an arithmetic sequence of numbers is $S = \frac{n}{2}(a + b)$, where n is the number of terms, a is the first term, and b is the last term. Express b in terms of a, S, and n.

Cross out terms that do not apply to each person, and circle those that do. Remember that if you circle something, it eliminates many other possibilities.

NURSE	PRINCIPAL	LIBRARIAN	COACH
white truck	white truck	white truck	white truck
red van	red van	red van	red van
black truck	black truck	black truck	black truck
silver van	silver van	silver van	silver van
key #25	key #25	key #25	key #25
key #30	key #30	key #30	key #30
key #35	key #35	key #35	key #35
key #40	key #40	key #40	key #40
Monday	Monday	Monday	Monday
Tuesday	Tuesday	Tuesday	Tuesday
Wednesday	Wednesday	Wednesday	Wednesday
Thursday	Thursday	Thursday	Thursday

CLUES (TO CUT OUT AND HAND TO KIDS AS THEY GET QUESTIONS RIGHT!)

The coach was issued key #35 and drives a white truck which was spotted at school early in the week.

Both the librarian and the owner of the red van were out of town on Monday and were also not at school on Thursday.

The one who was issued key #25 drives a silver van, which was only spotted in the school parking lot on Wednesday.

The principal drives a black truck and carries the key with the highest number.

FINAL CLUE

Only keys with even numbers open all classrooms.

Lesson 2 - Exponential vs. Linear Functions

Standards - NYS Next Generation Mathematics Standards:

AI-F.LE 1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Justify that a function is linear because it grows by equal differences over equal intervals, and that a function is exponential because it grows by equal factors over equal intervals.

Learning Objective - I can distinguish between an exponential and linear model given a mathematical and/or contextual situation.

Assessment of Learning - There is an exit ticket at the end of the lesson, that can be used as an assessment of learning for this lesson. Students should be able to answer both of the multiple-choice questions correctly.

Planned Learning Experiences

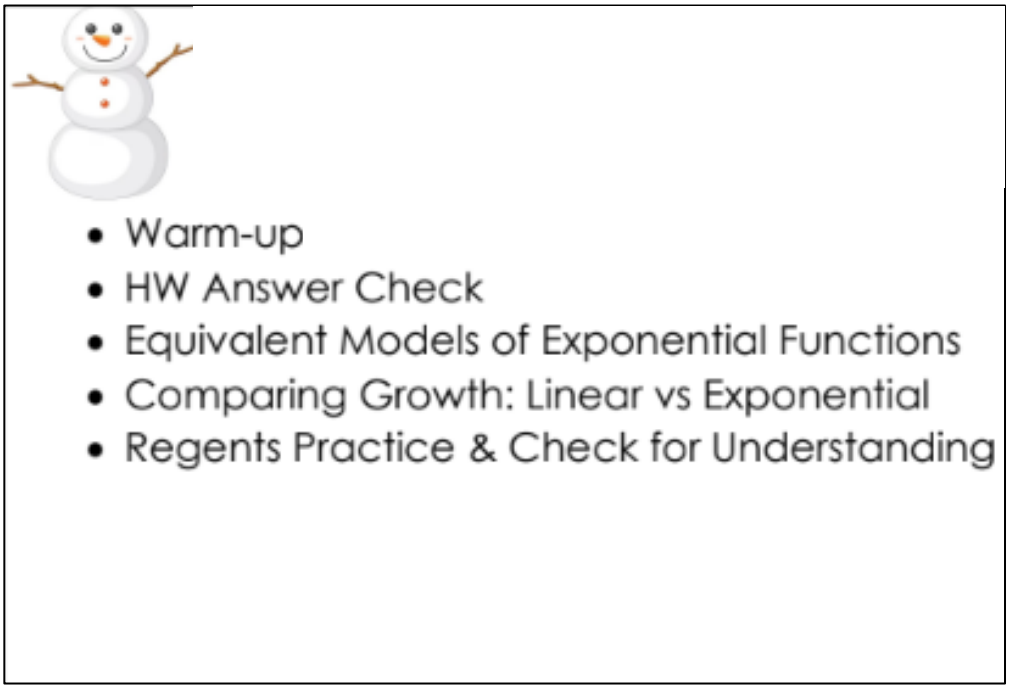
1. Students will complete a warmup, which includes 3 multiple choice questions focused on interpreting the context of 3 different exponential equations. Questions are in the packet.
2. Students will be asked to think through 2 questions (old regents questions) where they need to utilize their calculators to check for equivalence. Adult support should not be given during this time, students should work through problems on their own, utilizing their resources to help them push through. After a given time, go through the problems with students, allowing them to share their thinking before you give them the answers.
3. Students will then attempt 3 more equivalent equation problems where they need to use their calculator. Again, this should be independent.
4. Students will have to work through a problem asking them to compare the population of 2 towns, with 1 town being represented by a linear equation, and the other by an exponential one.
5. Then, students will be walked through the process of using the calc feature on their calculators, to determine the exact point of intersection.
6. Students will record notes comparing linear vs. exponential functions and their features.
7. If time allows present students with extra practice with determining whether each situation is linear or exponential, and then have them complete the check for understanding.

Productive Struggle – Throughout this lesson, students should be invited to engage in productive struggle. During the lesson, ensure that students are engaging in mathematical discourse with their classmates, and investigating problems and solutions. Push students to engage in higher level thinking by asking open ended questions as you float to support students in this process.

Teacher Notes:

- Timing and number of problems for each learning experience can be altered depending on time constraints.
- For the second learning experience, students should be pushed to persevere through those problems, but should not be given direct adult support.
- For #7, the problems outlined in the slides are present in the activity packet.
- A majority of the work in this lesson can be done on whiteboards, or scrap paper. For the notes sections, consider printing off that slide so that students can fill it in with you.

Lesson 2 Blank Slide Deck



2. Mike uses the equation $b = 1300(2.65)^x$ to determine the growth of bacteria in a laboratory setting. The exponent represents
   1) the total number of bacteria currently present
   2) the percent at which the bacteria are growing
   3) the initial amount of bacteria
   4) the number of time periods

3. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, y, of one account that was left inactive for a period of x years. What is the y-intercept of this equation and what does it represent?

   1) 0.98, the percent of money in the account initially
   2) 0.98, the percent of money in the account after x years
   3) 5000, the amount of money in the account initially
   4) 5000, the amount of money in the account after x years

Equivalent Models:

1. A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$, where n is the number of the note in the sequence and f(n) is the note frequency in hertz. Which function will generate the same note sequence as f(n)?
   1) $g(n) = 12(2)^{4n}$
   2) $h(n) = 6(2)^{4n}$
   3) $p(n) = 12(4)^{2n}$
   4) $k(n) = 6(8)^{2n}$

2. Nora inherited a savings account that was started by her grandmother 25 years ago. This scenario is modeled by the function $A(t) = 5000(1.013)^{t+25}$, where A(t) represents the value of the account, in dollars, t years after the inheritance. Which function below is equivalent to A(t)?
   1) $A(t) = 5000[(1.013^t)]^{25}$
   2) $A(t) = 5000[(1.013)^t + (1.013)^{25}]$
   3) $A(t) = (5000)^t (1.013)^{25}$
   4) $A(t) = 5000(1.013)^t (1.013)^{25}$

3. Mario's $15,000 car depreciates in value at a rate of 19% per year. The value, V, after t years can be modeled by the function $V = 15{,}000(0.81)^t$. Which function is equivalent to the original function?
   1) $V = 15{,}000(0.9)^{9t}$
   2) $V = 15{,}000(0.9)^{2t}$
   3) $V = 15{,}000(0.9)^{t/9}$
   4) $V = 15{,}000(0.9)^{t/2}$

4. The number of bacteria grown in a lab can be modeled by $P(t) = 300 \cdot 24^t$, where t is the number of hours. Which expression is equivalent to P(t)?
   1) $300 \cdot 8^t$
   2) $300 \cdot 16^t$
   3) $300^t \cdot 2^4$
   4) $300^{2t} \cdot 2^{2t}$

5. The expression $300(4)^{x+3}$ is equivalent to
   1) $300(4)^x (4)^3$
   2) $300(4^x)^3$
   3) $300(4)^x + 300(4)^3$
   4) $300^x(4)^3$

Warm Up:

In 2000, Littleton had a population of 11,510 people. Littleton's population increased by 600 people each year. In 2000, Tinyville had a population of 10,000 people. Tinyville's population increased by 10% each year.

In what year were the population's equal?

Using the graph feature on our calculators:

- May need to adjust window
- 2nd TRACE, 5:Intersect

Linear vs Exponential

Graphically:

Equations:

Tables:

Rates of Change:

Scenarios in context:
(Real world examples)

What type of function is described?

"equal factors over equal intervals"

"equal differences over equal intervals"

Practice - p. 24 - 26, #1-10

- Complete #1-3 and check answers with key on board
- Complete #4-7 and check answers with key on board
- Complete #8-10 and check answers with key on board

Check for Understanding

Which situation could be modeled as a linear equation?

1- **The value of a motorcycle decreases by 15% every year**
2- **The number of dolphins in the ocean doubles every 6 years**
3- **Two liters of water evaporate from a pool every day**
4- **The amount of caffeine in a person's body decreases by ¼ every 3 hours**

Lesson 2 Blank Student Practice Questions

1. A laboratory technician studied the population growth of a colony of computations. bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

Which function would accurately model the technician's data?

(1) $f(t) = 25^t$
(2) $f(t) = 25^{t+1}$
(3) $f(t) = 25t$
(4) $f(t) = 25(t + 1)$

2. The table below represents the function F.

x	3	4	6	7	8
F(x)	9	17	65	129	257

The equation that represents this function is

(1) $F(x) = 3^x$
(2) $F(x) = 3x$
(3) $F(x) = 2^x + 1$
(4) $F(x) = 2x + 3$

3. The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposit or withdrawn after the initial amount is deposited.

Which type of function best models the given data?

(1) linear function with a negative rate of change
(2) linear function with a positive rate of change
(3) exponential decay function
(4) exponential growth function

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

4. Tables of values for four functions are shown below

Which table best represents an exponential function?

1) f(x)
2) g(x)
3) h(x)
4) j(x)

x	f(x)
0	6
1	7
2	10
3	15
4	22

x	h(x)
0	1
1	2
2	4
3	8
4	16

x	g(x)
0	0
1	−2
2	−2
3	0
4	4

x	j(x)
0	2
1	5
2	8
3	11
4	14

5. Which of the three situations given below is best modeled by an exponential function?

I. A bacteria culture doubles in size every day.
II. A plant grows by 1 inch every 4 days.
III. The population of a town declines by 5% every 3 years.

1) I, only
2) II, only
3) I and II
4) I and III

6. The highest possible grade for a book report is 100. The teacher deducts 10 points for each day the report is late. Which kind of function describes this situation?

1) linear
2) quadratic
3) exponential growth
4) exponential decay

7. Which situation could be modeled as a linear equation?

1) The value of a car decreases by 10% every year.
2) Two liters of water evaporate from a pool every day.
3) The number of fish in a lake doubles every 5 years.
4) The amount of caffeine in a person's body decreases by 1 3 every 2 hours.

8. One Saturday afternoon, three friends decided to keep track of the number of text messages they received each hour from 8 a.m. to noon. The results are shown below.

- Emily said that the number of messages she received increased by 8 each hour.
- Jessica said that the number of messages she received doubled every hour.
- Chris said that he received 3 messages the first hour, 10 the second hour, none the third hour, and 15 the last hour.

Which of the friends' responses best classifies the number of messages they received each hour as a linear function?

1) Emily, only
2) Jessica, only
3) Emily and Chris
4) Jessica and Chris

9. If $f(x) = 3^x$ and $g(x) = 2x + 5$, at which value of x is $f(x) < g(x)$?

1) −1
2) 2
3) −3
4) 4

10. Alicia has invented a new app for smart phones that two companies are interested in purchasing for a 2-year contract. Company A is offering her $10,000 for the first month and will increase the amount each month by $5000. Company B is offering $500 for the first month and will double their payment each month from the previous month. Monthly payments are made at the end of each month. For which monthly payment will company B's payment first exceed company A's payment?

1) 6
2) 7
3) 8
4) 9

11. Suppose you have a choice of two different jobs at graduation. Which offer would you choose? Why?

- Start at $30,000 with a 6% per year increase
- Start at $40,000 with $1200 per year raise

Lesson 3 - Review of Factoring Polynomials

Standards - NYS Next Generation Mathematics Standards:

AI-A.SSE 1: Interpret expressions that represent a quantity in terms of its context.
a. Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.
b. Interpret expressions by viewing one or more of their parts as a single entity.

AI-A.SSE 2: Recognize and use the structure of an expression to identify ways to rewrite it.

AI-A.SSE 3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
c. Use the properties of exponents to rewrite exponential expressions.

AI-A.APR 1: Add, subtract, and multiply polynomials and recognize that the result of the operation is also a polynomial. This forms a system analogous to the integers.

Learning Objective - This is a review day, so the learning objectives apply to the unit

- I can completely factor a polynomial using GCF, DOPS, or Trinomial Factoring.
- I can rewrite a polynomial in standard form or factored form.
- I can add/subtract and multiply polynomial expressions, including from a context.

Assessment of Learning - The Unit 6 review HW will be collected in the following class, and will serve as a quick check for understanding. The next class day is the unit test, so there is a summative assessment in the near future.

Planned Learning Experiences

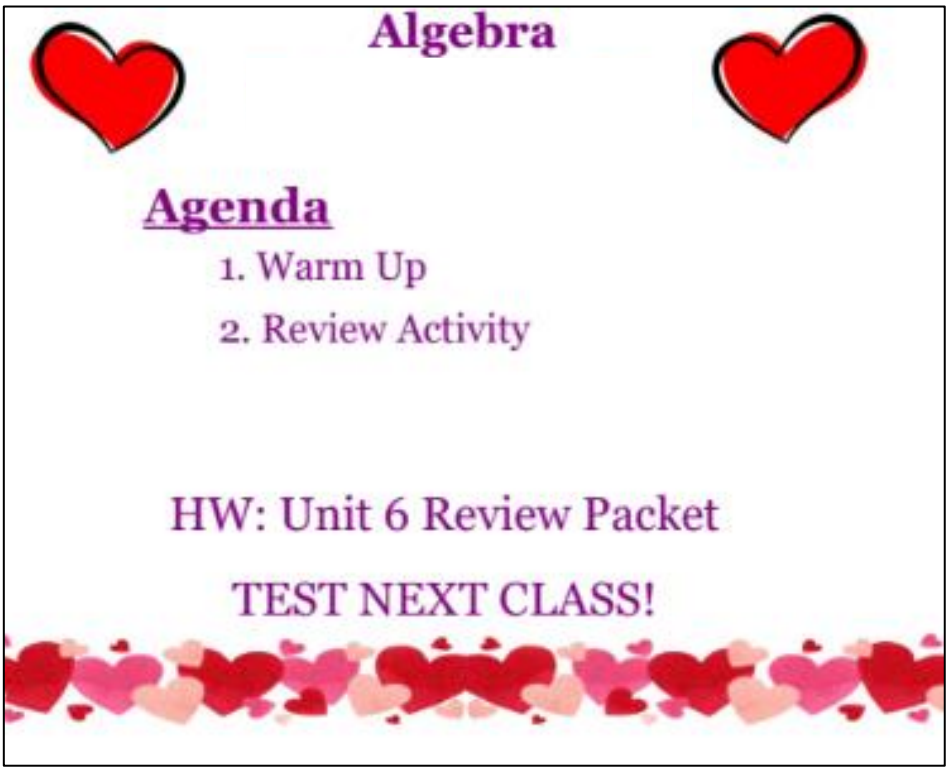
1. There is a warm up that asks students to look at a solution that was "incorrectly" produced, and identify where the error was made as well as the correct solution.
2. Students will be given the Unit 6 Review Activity, which contains 4 stations, each covering standards and objectives from above. Students will have to work, either in a group or pairs to complete each station. Students need to complete the whole station before they can have it checked, or ask questions.
3. If groups finish the stations, they can begin working on the review homework. Both the stations packet and review homework are similar to the unit test, so if students are able to complete both the packet and homework problems, they should be in good shape for the test.
4. If time allows, you can go over the station packet problems, correcting any common misconceptions that you see during work time.

Productive Struggle – This lesson is a review of topics already covered in class. Students should be able to use what they've learned to help them solve each problem in the review materials. During the warmup, students should be conducting a mathematical investigation to discover the error shown in the example work. During their practice, students need to engage in mathematical discourse within their groups in order to solve each problem on the page, before they can have it checked / ask questions. Encourage students to participate in these discussions and praise them when you see it.

Teacher Notes:

- During the station practice, students should complete the whole station before they have it checked. You can float and support if you want, but students should be pushed to work through their problems with their groups and come up with the solutions on their own.
- If your groups are evenly mixed, you can create a game out of the station packet, rewarding the group that gets every station completed first, or each station individually. Keep in mind your students, and how they react to competition in the classroom.
- Post the key to the review homework before the test, so students can check their work and attempt to clear any of their own misconceptions before their test.

Lesson 3 Blank Slide Deck



Warm Up

1) Mrs. Wagner factored the following expression, but did it incorrectly. Explain the error she made AND correctly factor the expression.

Original	Mrs. Wagner's Answer
$10x^3 + 5x^2 + 5x$	$5x(2x^2 + x)$

2) Which expression is equivalent to $16x^2 - 36$?

1) $4(2x-3)(2x-3)$
2) $4(2x+3)(2x-3)$
3) $(4x-6)(4x-6)$
4) $(4x+6)(4x+6)$

Practice Time!

- 4 sets of questions
- Finish a section & get it checked

Extra Practice if needed:

Homework Packet

You guys got this!

Lesson 3 Blank Student Review Activity

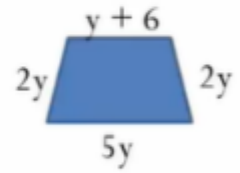
Station 1: Adding/Subtracting

1) State a polynomial that is three times the sum of $5x^2 - 5x - 3$ and $-7x^2 - 2x + 6$

2) The revenue, $R(x)$, from selling x units of a product is represented by the equation $R(x) = 35x$, while the total cost, $C(x)$, of making x units of the product is represented by the equation $C(x) = 20x + 500$. The total profit, $P(x)$, is represented by the equation $P(x) = R(x) - C(x)$. For the values of $R(x)$ and $C(x)$ given above, what is $P(x)$?

1) $15x$
2) $15x + 500$
3) $15x - 500$
4) $10x + 100$

3) Find the perimeter of the following quadrilateral in standard form.



4) Subtract $2y(x - y)$ **from** $4(x^2 + 3xy)$

Station 2: Multiplying

1) Express $(2p + 6)^2$ in standard form.

2) What is the product of $5x^2y^3$ and $-2xy^7$?

3) The length of a rectangular garden is $3x + 4$ and the width is $x - 1$. State the area.

4) Perform the operation $(5x - 2)(x^2 + 3x - 8)$. Write your answer in standard form.

Station 3: Factor

1) Write the expression in factored form:

$$4g^3 - 16g^{10}$$

2) Factor the following expression:

$$x^4 - 36$$

3) Factor the following expression:

$$x^2 + x - 20$$

Station 4: Factor Completely

1) Factor completely: $3x^2 + 18x + 15$

2) Factor completely: $2x^2 - 50$

3) Factor completely: $j^4 - 16$

Lesson 3 Blank Student Review Homework

Name _____ Date _____

Algebra 1 Review HW

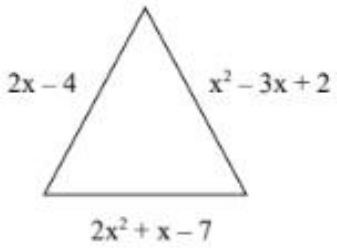
1. Find the sum of the following expressions and write your answers in standard form:

   a) $4c^2 - 2c + 7$ and $-2c^2 + 3c - 1$ is

   b) $4c^3 + 5c$ and $9c^2 - 12c$

2. Subtract $4x^3 - 2x^2 + 3x$ **from** $6x^3 + 4x^2 + 6$

3. Find the perimeter of the following triangle.



4. Which of the following polynomials is a degree of 4, has 3 terms, and 2 variables?

(A) $3x^4 - 4x^3 + 3x^2 - 3xy - 5$

(B) $3x^4 - 4x^3 - 3xy$

(C) $3x^4 - 4x^3 - 3x$

(D) $3x^2 - 3xy - 5$

Factor each of the following:

5. $x^2 - 5x - 6$

6. $25 - 16x^2$

7. $6x^2 - 3x$

Factor completely:

8. $3x^3 + 12x^2 + 9x$

9. $16p^4 - 1$

Find the product of the following expressions. Write your answer in standard form:

10. $(3xy)$ and $(4x^3y^4)$

11. $(3x - 1)(4x + 1)$

12. $(x - 3)(2x^2 + x + 1)$

13. $(k - 4)^2$

14. A student did the following factoring: $m^2 - 10m + 24 = (m - 12)(m + 2)$. Is the student correct? How do you know? **Justify** and **Explain** your answer.

15. Simplify each expression and write your final answers in standard form.

a. Subtract $2a(a^2 - 4b)$ **from** $3(a - ab)$.

b. Find three times the sum of $3x^2 - 8x + 7$ and $-5x^2 + 2x - 9$

Lesson 4 - Solving Quadratics by Factoring

Standards - NYS Next Generation Mathematics Standards:

AI-A.APR 3: Identify zeros of polynomial functions when suitable factorizations are available.

Learning Objective - 1. I can identify a quadratic equation in various forms. 2. I can solve a quadratic equation using factoring and the zero-product property.

Assessment of Learning - Students will be completing a page of their activity packet and having it checked at the end of the lesson. This serves as the formative assessment for the lesson.

Planned Learning Experiences

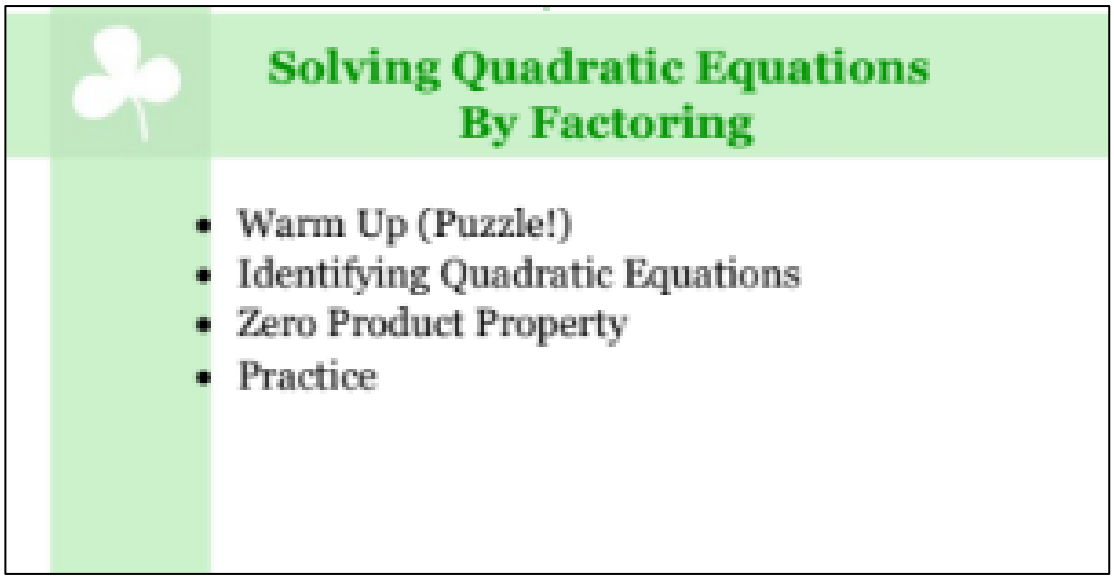
1. Students will attempt to solve the "nine-dot-puzzle". Have students attempt the puzzle on whiteboards, or blank paper. Read students the instructions and give them 5 minutes to attempt and solve the puzzle. Make sure to encourage students to keep trying for the entire 5 minutes, and to not give up! At the end of the time, show the solution (in the appendix), and talk with students about how they struggled for those 5 minutes, and how they took their failures and learned from them during the puzzle. Encourage them to do this when solving mathematical problems.
2. Have students examine the three functions on the slide. See if they can name either the red or blue function and describe what the equation looks like. Once a student either gets it right, or gets close, reveal the equation from behind the rectangle. See if students can guess what's different about the green function, or what it's called. Reveal to students both the name, and the form of the equation for a quadratic function.
3. Have students go through the list of equations and determine whether each one is quadratic or not. Once you go over any misconceptions on the examples, have students turn their packets to page 1, and have them practice identifying a quadratic equation.
4. Discuss with students the zero-product property and fill in the notes section on page 2 of their packets. Go through the examples with the students, peeling back support as you get towards the bottom of the page.
5. Have students turn to page 3 of their packets, and graph the quadratic equation given. Once students have it graphed, ask them to look at the table for the function, and see if they can identify the pattern.
6. Have students work through page 4, either in pairs or individually. Go over problems 1-4 with students after you've had a chance to check in on each group. Ask students to complete the problems on page 5 of their packets as their exit ticket for the class.

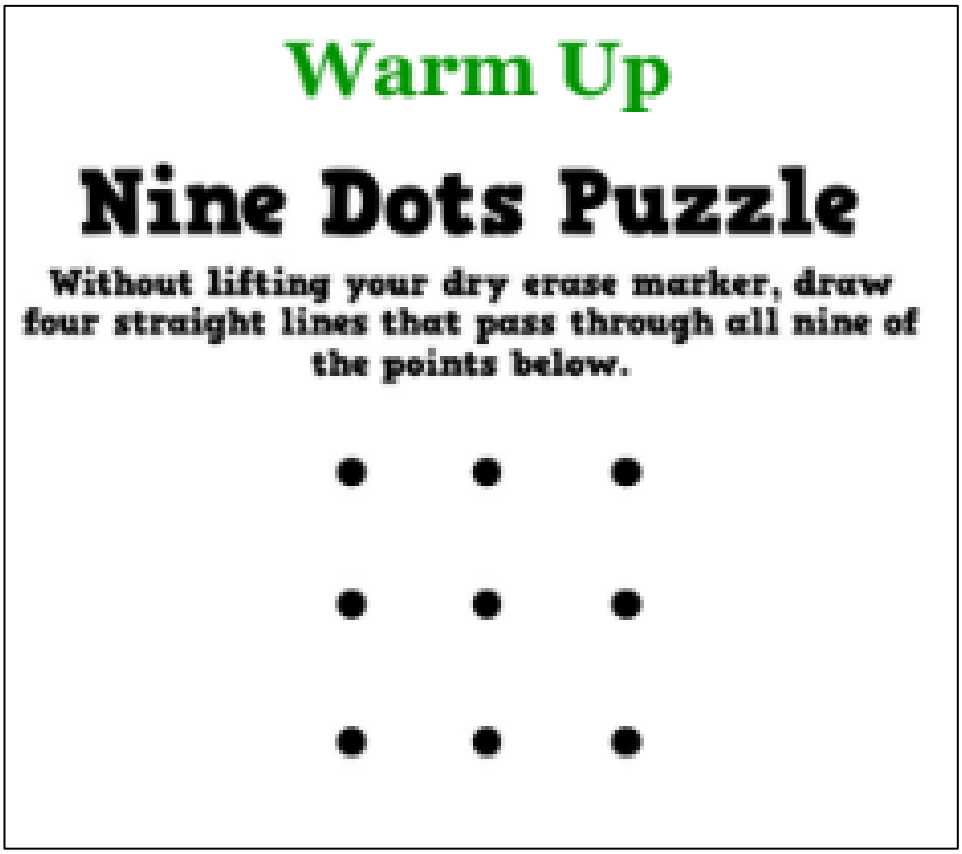
Productive Struggle – The warmup of this lesson allows students to directly engage in productive struggle, using a non-curricular. Encourage them to work through their problems in a similar fashion during instruction. Being honest with students about the purpose of productive struggle can help them to further engage with it.

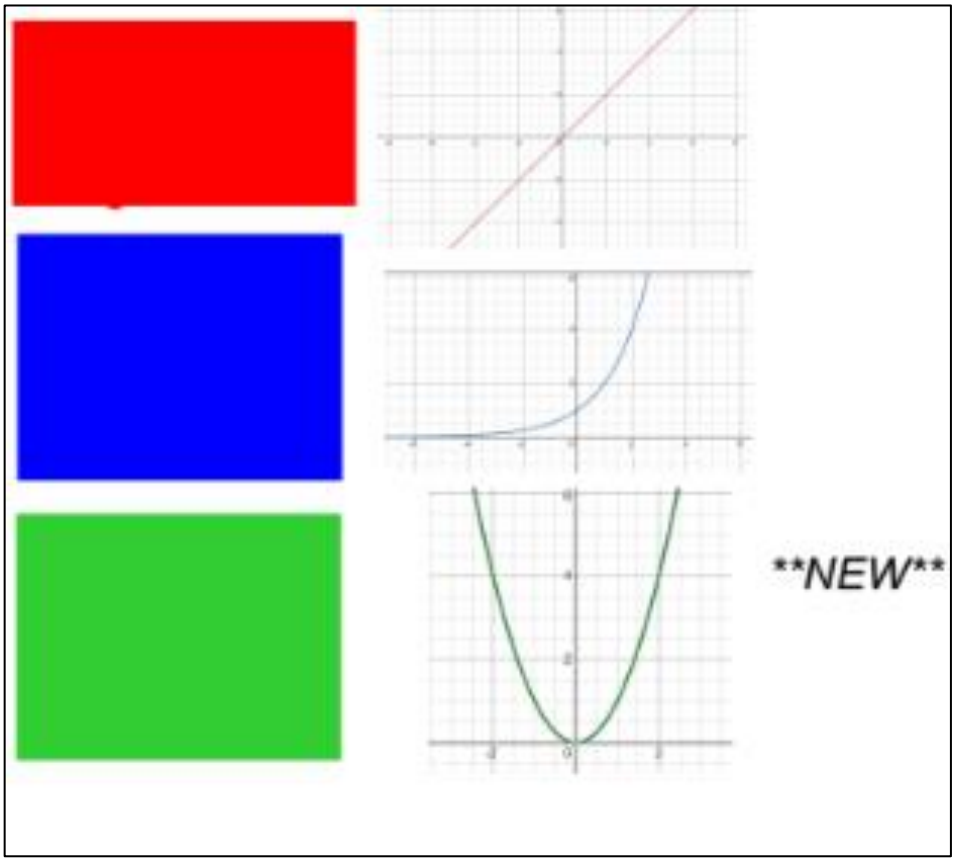
Teacher Notes:

- For learning experience 1, you can alter the length of the conversation around productive struggle to your needs.
- For learning experience 3, you could have students work in groups here as well or push them to do it individually.
- For learning experience 5, you can just talk through part c, depending on time.

Lesson 4 Blank Slide Deck







1) I can identify a quadratic equation in various forms.

2) I can solve a quadratic equation by using factoring and the zero product property.

A quadratic equation is an equation of degree 2

Which of the following are quadratic equations??

A) $(x-7)(x+2)=0$

B) $x^3-x^2+3x-1=0$

C) $x^2=x+4$

D) $x-2x^2=1+x^5$

E) $x^2+7x+12=0$

F) $x^2=49$

G) x^2-x-6

H) $3x-4=x+1$

The _____ _____ property states that if $ab = 0$, then either $a = 0$ or $b = 0$, or both a and b are 0. When the product of factors equals **zero**, one or more of the factors must also equal **zero**.

EX #1: Use this property from above to answer the following questions:

(a) If $(6)(a) = 0$, then what is the value of a?

(b) If $3(x + 4) = 0$, then what is the possible value of x?

(c) If $x(x - 4) = 0$, then what are the possible values of x?

(d) If $(x - 7)(x + 7) = 0$, then what are the possible values of x?

(e) If $(x - 8)(x - 2) = 0$, then what are the possible values of x?

Section 1: Solving Quadratic Equations by Factoring

INTRO

A quadratic equation is an equation that has a degree of 2.

Identify which equations below are quadratic or not.

1.) $x^2 + 4x - 12 = 0$ a. quadratic equation b. not quadratic equation

2.) $y^3 + y^2 - 5y = 24$ a. quadratic equation b. not quadratic equation

3.) $z^2 + 8 = -12x$ a. quadratic equation b. not quadratic equation

4.) $x + 7 = 0$ a. quadratic equation b. not quadratic equation

5.) $y^2 - 25 = 0$ a. quadratic equation b. not quadratic equation

6.) $(3x - 1)(x - 1) = 0$ a. quadratic equation b. not quadratic equation

7.) $4m^2 = 0$ a. quadratic equation b. not quadratic equation

8.) $x^2 + 6x = 0$ a. quadratic equation b. not quadratic equation

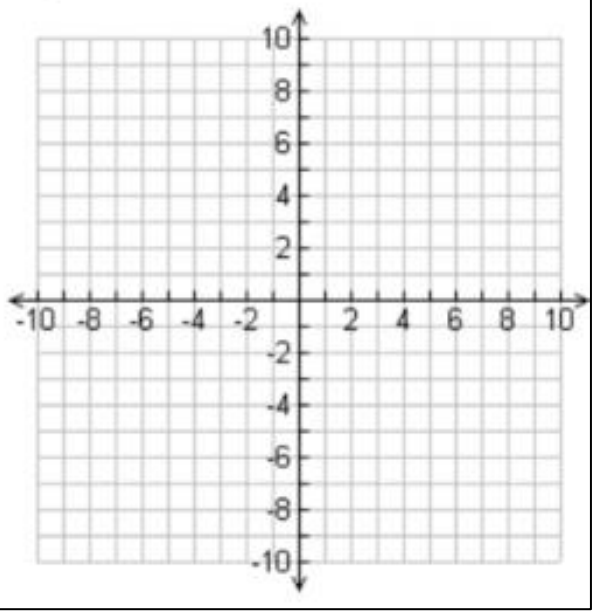
9.) $y^2 - 5y = 24$ a. quadratic equation b. not quadratic equation

10.) $(7y - 2)(7y - 2)$ a. quadratic equation b. not quadratic equation

EX #2:

a) Graph the quadratic equation $y = x^2 + 2x - 8$ on the grid below.

x	y



b) Find all possible x-values that satisfy the equation $x^2 + 2x - 8 = 0$.

c) What do you notice from parts a and b?

PRACTICE

Directions: For #1-8, solve the following equations by factoring. Express your answer(s) as a solution set.

1) $(x - 10)(x + 2) = 0$

2) $x^2 - 12x + 31 = -5$

3) $7x^2 + 7x = 0$

4) $2x^2 - 18 = 0$

5) $2x^2 + 10x = -12$

6) $(x - 3)(x + 3) = 6x - 14$

7) $(x - 2)(x^2 - 16) = 0$

8) $(x + 3)(x^2 - 5x - 6) = 0$

Validity

When determining the focus for my curriculum project, I reflected on various professional development sessions I had attended as a new teacher. One session that particularly stood out to me was titled "The Power of Productive Struggle," offered during a conference day last year. Initially, the title piqued my interest, and during the session, we delved into the profound impact that struggle can have on a student's learning process. We explored how engaging in struggle activates different parts of the brain and teaches students to use their problem-solving skills independently. The concept was both obvious and eye-opening, resonating deeply with me as I realized its potential impact on my students.

Inspired by this session, I decided to integrate elements of productive struggle into my lessons and discuss its importance with my students. I frequently reminded them of Thomas Edison's perseverance, noting how he failed 999 times before inventing the lightbulb. Implementing this approach required patience, but over time, I observed my students beginning to engage in productive struggle. Although it can be challenging for a teacher to watch students struggle, understanding that it enhances their learning makes it worthwhile.

The results of my formative and summative assessments following these lessons underscored the effectiveness of productive struggle. Students demonstrated improved retention of information and were able to apply their knowledge more effectively over time. This positive outcome reinforced my commitment to using productive struggle across all my classes. I believe this approach not only benefits students academically but also equips them with valuable life skills that extend beyond the classroom.

Conclusion

Productive struggle is a powerful classroom tool that both students and teachers can leverage to enhance the understanding of complex concepts. The lessons in this project provide a glimpse into the various methods of integrating productive struggle into a curriculum, not only in mathematics but across different subjects. My hope is that you can incorporate these lessons into your curriculum and gain a deeper understanding of what productive struggle entails and how to effectively engage students in it.

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Appendix

What follows are all the filled in teacher materials, as well as the keys for any student materials included in the curriculum project.

Lesson 1 Answer Keys

Warm Up: Activity Packet

Page 12:

3. Three times the sum of a number and four is equal to five times the number, decreased by two. If x represents the number, which equation is a correct translation of the statement?

1) $3(x + 4) = 5x - 2$

2) $3(x + 4) = 5(x - 2)$

3) $3x + 4 = 5x - 2$

4) $3x + 4 = 5(x - 2)$

Piece by Piece!

$3(x+4)=5x-2$

Section 2.2: Literal Equations

Literal Equation - equations with more than one variable

To "solve" a literal equation is to solve for a different variable.

Ex) Solve for m in terms of T and v:

**Kinetic Energy formula*

$$\frac{T}{v^2} = \frac{1}{2}\frac{mv^2}{v^2} \quad \cdot 2$$

$$m = \frac{2 \cdot T}{v^2}$$

Ex) Solve for v in terms of T and m:

$$\frac{T}{m} = \frac{1}{2}\frac{mv^2}{m} \quad \cdot 2$$

$$\sqrt{\frac{2T}{m}} = \sqrt{v^2}$$

$$v = \sqrt{\frac{2T}{m}}$$

Page 18 in Activity Packet...

NYS Regents

12) The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$. The height, h, of the trapezoid may be expressed as

1) $2A - b_1 - b_2$

2) $\frac{2A - b_1}{b_2}$

3) $\frac{1}{2}A - b_1 - b_2$

4) $\frac{2A}{b_1 + b_2}$

$2 \cdot A = \frac{1}{2}(b_1 + b_2)h \cdot 2$

$\frac{2A}{(b_1 + b_2)} = \frac{(b_1 + b_2)h}{(b_1 + b_2)}$

Name _____ Literal Equations Practice

The Hamster Heist

When students returned to school after a holiday break, they discovered their pet hamster, Ham, had disappeared. His cage was not in its usual place on the table near the window.

They thought maybe Miss Anita, the custodian, had moved Ham, but when they asked her about it, she reported that the cage had been in the classroom on Monday. She said she had unlocked the classroom door, cleaned the room, then locked the door and left. On Thursday morning, when she unlocked the room to clean it again, Ham's cage was gone. She had no idea where it was.

Solve the following literal equations to get clues to help you solve the mystery of who took Ham the Hamster. Check your answer for each problem and for each question you get right, you get a clue to the mystery. Good luck!

1. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Rewrite the formula solving for the radius, r.

$3V = \frac{1}{3}\pi r^2 h$

$\frac{3V}{\pi h} = \frac{\pi r^2 h}{\pi h}$

$\sqrt{\frac{3V}{\pi h}} = \sqrt{r^2}$

$r = \sqrt{\frac{3V}{\pi h}}$

2. The formula $Ax + By = C$ represents the equation of a line in standard form. Which expression represents y in terms of A, B, C, and x?

(1) $\frac{C - Ax}{B}$

(2) $\frac{C - A}{Bx}$

(3) $\frac{C - A}{x + B}$

(4) $\frac{C - B}{Ax}$

$\frac{By}{B} = \frac{C - Ax}{B}$

$y = \frac{C - Ax}{B}$

3. The amount of energy, Q, in joules, needed to raise the temperature of m grams of a substance is given by the formula $Q = mC(T_f - T_i)$, where C is the specific heat capacity of the substance. If its initial temperature is T_i, an equation to find its final temperature, T_f, is

(1) $T_f = \frac{Q}{mC} - T_i$

(3) $T_f = \frac{T_i + Q}{mC}$

(2) $T_f = \frac{Q}{mC} + T_i$

(4) $T_f = \frac{Q - mC}{T_i}$

4. The temperature inside a cooling unit is measured in degrees Celsius, C. Josh wants to find out how cold it is in degrees Fahrenheit, F.

Solve the formula $C = \frac{5}{9}(F - 32)$ for F so that Josh can convert Celsius to Fahrenheit.

$32 + \frac{9}{5}C = F - 32 + 32$

$F = \frac{9}{5}C + 32$

5. A formula for determining the finite sum, S, of an arithmetic sequence of numbers is $S = \frac{n}{2}(a + b)$, where n is the number of terms, a is the first term, and b is the last term. Express b in terms of a, S, and n.

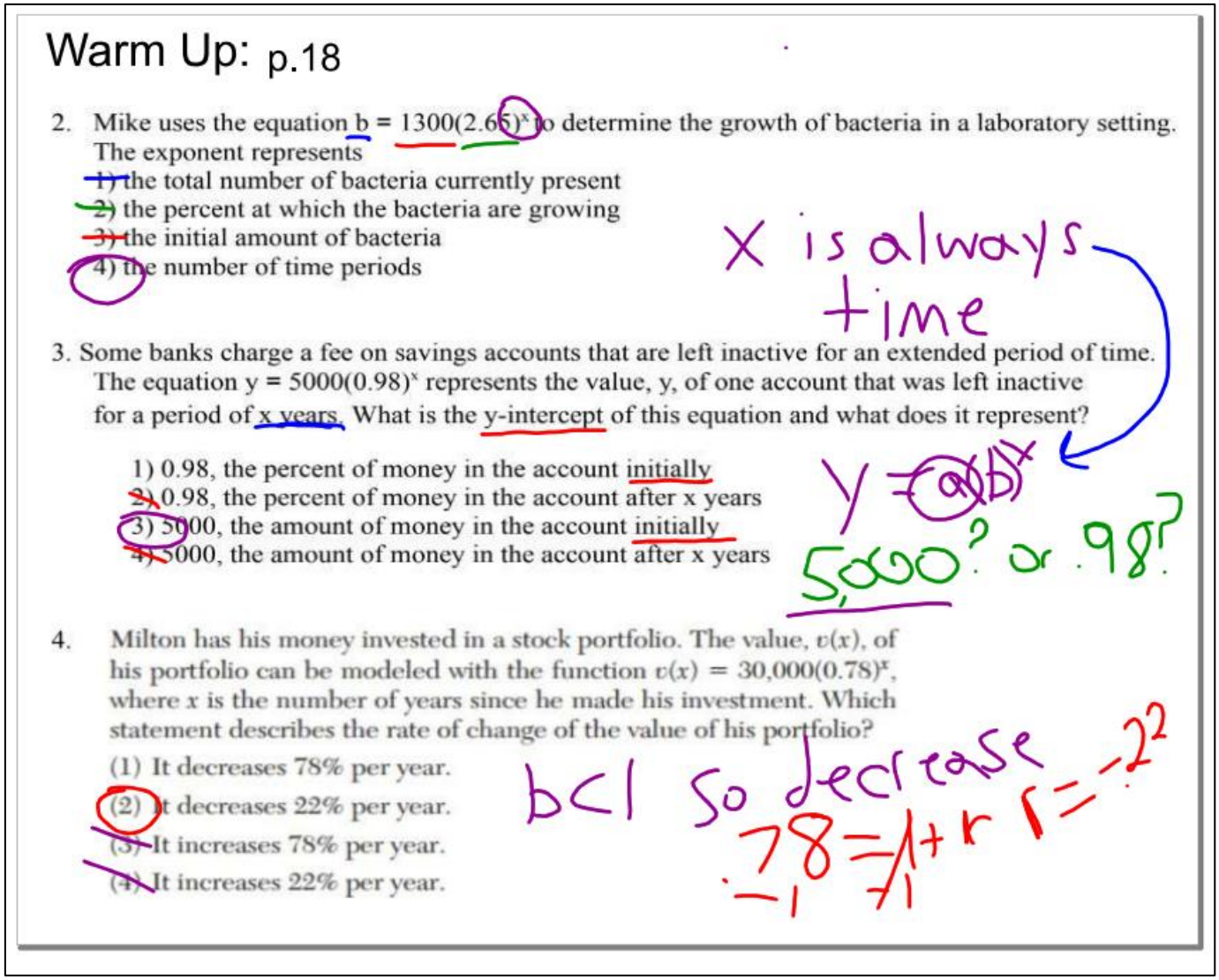
$-a + \frac{2S}{n} = a + b - a$

$b = \frac{2S}{n} - a$

Cross out terms that do not apply to each person, and circle those that do. Remember that if you circle something, it eliminates many other possibilities.

NURSE	PRINCIPAL	LIBRARIAN	COACH
white truck	white truck	white truck	white truck
red van	red van	red van	red van
black truck	black truck	black truck	black truck
silver van	silver van	silver van	silver van
key #25	key #25	key #25	key #25
key #30	key #30	key #30	key #30
key #35	key #35	key #35	key #35
key #40	key #40	key #40	key #40
Monday	Monday	Monday	Monday
Tuesday	Tuesday	Tuesday	Tuesday
Wednesday	Wednesday	Wednesday	Wednesday
Thursday	Thursday	Thursday	Thursday

Lesson 2 Answer Keys



Warm Up: p.18

2. Mike uses the equation $b = 1300(2.65)^x$ to determine the growth of bacteria in a laboratory setting. The exponent represents
   1) the total number of bacteria currently present
   2) the percent at which the bacteria are growing
   3) the initial amount of bacteria
   4) the number of time periods

x is always time

3. Some banks charge a fee on savings accounts that are left inactive for an extended period of time. The equation $y = 5000(0.98)^x$ represents the value, y, of one account that was left inactive for a period of x years. What is the y-intercept of this equation and what does it represent?

   1) 0.98, the percent of money in the account initially
   2) 0.98, the percent of money in the account after x years
   3) 5000, the amount of money in the account initially
   4) 5000, the amount of money in the account after x years

$y = ab^x$

5,000? or .98?

4. Milton has his money invested in a stock portfolio. The value, $v(x)$, of his portfolio can be modeled with the function $v(x) = 30{,}000(0.78)^x$, where x is the number of years since he made his investment. Which statement describes the rate of change of the value of his portfolio?
   (1) It decreases 78% per year.
   (2) It decreases 22% per year.
   (3) It increases 78% per year.
   (4) It increases 22% per year.

b<1 so decrease

$.78 = 1 + r$

$r = -.22$

Other Equivalent Models

1. A computer application generates a sequence of musical notes using the function $f(n) = 6(16)^n$, where n is the number of the note in the sequence and f(n) is the note frequency in hertz. Which function will generate the same note sequence as f(n)?

   1) $g(n) = 12(2)^{4n}$
   2) $h(n) = 6(2)^{4n}$
   3) $p(n) = 12(4)^{2n}$
   4) $k(n) = 6(8)^{2n}$

a stays the same.

Use y=

2. Nora inherited a savings account that was started by her grandmother 25 years ago. This scenario is modeled by the function $A(t) = 5000(1.013)^{t+25}$, where A(t) represents the value of the account, in dollars, t years after the inheritance. Which function below is equivalent to A(t)?

   1) $A(t) = 5000[(1.013^t)]^{25}$
   2) $A(t) = 5000[(1.013)^t + (1.013)^{25}]$
   3) $A(t) = (5000)^t (1.013)^{25}$
   4) $A(t) = 5000(1.013)^t (1.013)^{25}$

Stuck? Use y=

3. Mario's $15,000 car depreciates in value at a rate of 19% per year. The value, V, after t years can be modeled by the function $V = 15{,}000(0.81)^t$. Which function is equivalent to the original function?
   1) $V = 15{,}000(0.9)^{9t}$
   2) $V = 15{,}000(0.9)^{2t}$
   3) $V = 15{,}000(0.9)^{t/9}$
   4) $V = 15{,}000(0.9)^{t/2}$

4. The number of bacteria grown in a lab can be modeled by $P(t) = 300 \cdot 2^{4t}$, where t is the number of hours. Which expression is equivalent to P(t)?
   1) $300 \cdot 8^t$
   2) $300 \cdot 16^t$
   3) $300^t \cdot 2^4$
   4) $300^{2t} \cdot 2^{2t}$

5. The expression $300(4)^{x+3}$ is equivalent to
   1) $300(4)^x (4)^3$
   2) $300(4^x)^3$
   3) $300(4)^x + 300(4)^3$
   4) $300^x(4)^3$

Linear

In 2000, Littleton had a population of 11,510 people. Littleton's population increased by 600 people each year. In 2000, Tinyville had a population of 10,000 people. Tinyville's population increased by 10% each year.

In what year were the populations equal?

Are they both exp?

No

$y = 600x + 11,510$

$10\% \rightarrow .10$

$y = 10,000(1.10)^x$

Use $y=$ to compare

After 3 years $y = 13,310$

Time to chat...

Linear vs Exponential

	Linear	Exponential
Graphically:		
Equations:	$y = mx + b$	$y = a\,b^{x}$
Tables:	$+\ -$	$\times\ \div$
Rates of Change:	constant	non-constant

What type of function is described?

"equal factors over equal intervals"

exp

"equal differences over equal intervals"

Lin

Check for Understanding

Which situation could be modeled as a linear equation?

1) The value of a motorcycle decreases by 15% every year. %

2) The number of dolphins in the ocean doubles every 6 years. Doubles

3) Two liters of water evaporate from a pool every day

4) The amount of caffeine in a person's body decreases by 1/4 every 3 hours

Which of the contexts below represents exponential decay?

1) A certain population of 17 aggressive zombies quadruples every minute. Inc

2) A town's population grows at a rate of 5.9% every year.

3) A car depreciates at a rate of 4.7% per year.

4) An elevator descends at a rate of 19 feet per second. Linear

Lesson 3 Answer Keys

Warm Up

1) Mrs. Wagner factored the following expression, but did it incorrectly. Explain the error she made AND correctly factor the expression.

5
1 5
10
1 0
1 · 5
2 · 5

Original

$10x^3 + 5x^2 + 5x$

$5x(2x^2+x+1)$

Mrs. Wagner's Answer

$5x(2x^2 + x) + 1$

Where'd the +1 go?

2) Which expression is equivalent to $16x^2 - 36$?

1) $4(2x-3)(2x-3)$
2) $4(2x+3)(2x-3)$
3) $(4x-6)(4x-6)$
4) $(4x+6)(4x+6)$

$4(4x^2-9)$

$4(2x+3)(2x-3)$

Key

Station 1: Adding/Subtracting

1) State a polynomial that is three times the sum of $5x^2 - 5x - 3$ and $-7x^2 - 2x + 6$

$3(5x^2-5x-3-7x^2-2x+6)$

$3(-2x^2-7x+3)$

$-6x^2-21x+9$

2) The revenue, $R(x)$, from selling x units of a product is represented by the equation $R(x) = 35x$, while the total cost, $C(x)$, of making x units of the product is represented by the equation $C(x) = 20x + 500$. The total profit, $P(x)$, is represented by the equation $P(x) = R(x) - C(x)$. For the values of $R(x)$ and $C(x)$ given above, what is $P(x)$?

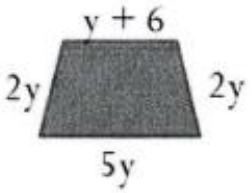
1) $15x$
2) $15x + 500$
3) $15x - 500$
4) $10x + 100$

$35x-(20x+500)$

$35x-20x-500$

$15x-500$

3) Find the perimeter of the following quadrilateral in standard form.



$2y+2y+y+6+5y$

$10y+6$

4) Subtract $2y(x - y)$ **from** $4(x^2 + 3xy)$

$4(x^2+3xy)-2y(x-y)$

$4x^2+12xy-2xy+2y^2$

$4x^2+10xy+2y^2$

Key

Station 2: Multiplying

1) Express $(2p + 6)^2$ in standard form.

$(2p+6)(2p+6)$

$4p^2+12p+12p+36$

$4p^2+24p+36$

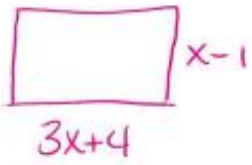
2) What is the product of $5x^2y^3$ and $-2xy^7$?

$5x^2y^3(-2xy^7)$

$-10x^3y^{10}$

3) The length of a rectangular garden is $3x + 4$ and the width is $x - 1$. State the area.

$(3x+4)(x-1)$



$3x^2-3x+4x-4$

$3x^2+x-4$

4) Perform the operation $(5x - 2)(x^2 + 3x - 8)$. Write your answer in standard form.

	x^2	$3x$	-8
$5x$	$5x^3$	$+15x^2$	$-40x$
-2	$-2x^2$	$-6x$	$+16$

$5x^3+13x^2-46x+16$

Key

Station 3: Factor

1) Write the expression in factored form:

$4g^3 - 16g^{10}$

$4g^3(1-4g^7)$

2) Factor the following expression:

$x^4 - 36$

$(x^2+6)(x^2-6)$

3) Factor the following expression:

$x^2 + x - 20$

$(x-4)(x+5)$

Key

Station 4: Factor Completely

1) Factor completely: $3x^2 + 18x + 15$

$3(x^2 + 6x + 5)$

$3(x+1)(x+5)$

2) Factor completely: $2x^2 - 50$

$2(x^2 - 25)$

$2(x+5)(x-5)$

3) Factor completely: $j^4 - 16$

$(j^2 + 4)(j^2 - 4)$

$(j^2+4)(j+2)(j-2)$

Name Key

Date _____

Algebra 1

Review HW

1. Find the sum of the following expressions and write your answers in standard form:

   a) $4c^2 - 2c + 7$ and $-2c^2 + 3c - 1$ is

   $2c^2 + c + 6$

   b) $4c^3 + 5c$ and $9c^2 - 12c$

   $4c^3 + 9c^2 - 7c$

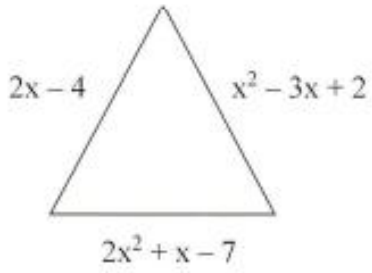
2. Subtract $4x^3 - 2x^2 + 3x$ **from** $6x^3 + 4x^2 + 6$

   $(6x^3 + 4x^2 + 6) - (4x^3 - 2x^2 + 3x)$

   $6x^3 + 4x^2 + 6 - 4x^3 + 2x^2 - 3x$

   $2x^3 + 6x^2 - 3x + 6$

3. Find the perimeter of the following triangle.



   $2x - 4 + x^2 - 3x + 2 + 2x^2 + x - 7$

   $3x^2 - 9$

4. Which of the following polynomials is a degree of 4, has 3 terms, and 2 variables?

   (A) $3x^4 - 4x^3 + 3x^2 - 3xy - 5$

   (B) $3x^4 - 4x^3 - 3xy$

   (C) $3x^4 - 4x^3 - 3x$

   (D) $3x^2 - 3xy - 5$

Factor each of the following:

5. $x^2 - 5x - 6$

m: −6
a: −5

$(x+1)(x-6)$

6. $25 - 16x^2$

$(5+4x)(5-4x)$

7. $6x^2 - 3x$

$3x(2x-1)$

Factor completely:

8. $3x^3 + 12x^2 + 9x$

$3x(x^2+4x+3)$

m: 3
a: 4

$3x(x+3)(x+1)$

9. $16p^4 - 1$

$(4p^2+1)(4p^2-1)$

$(4p^2+1)(2p+1)(2p-1)$

Find the product of the following expressions. Write your answer in standard form:

10. $(3xy)$ and $(4x^3y^4)$

$12x^4y^5$

11. $(3x - 1)(4x + 1)$

$12x^2+3x-4x-1$

$12x^2 - x - 1$

12. $(x - 3)(2x^2 + x + 1)$

	$2x^2$	$+x$	$+1$
x	$2x^3$	$+x^2$	$+x$
-3	$-6x^2$	$-3x$	-3

$2x^3 - 5x^2 - 2x - 3$

13. $(k - 4)^2$

$(k-4)(k-4)$

$k^2-4k-4k+16$

$k^2 - 8k + 16$

14. A student did the following factoring: $m^2 - 10m + 24 = (m - 12)(m + 2)$. Is the student correct? How do you know? **Justify** and **Explain** your answer.

$m^2 - 10m + 24$ $\quad m: 24$ $\quad a: -10$

$(m - 4)(m - 6)$

$(m-12)(m+2)$

$m^2 + 2m - 12m - 24$

$m^2 - 10m - 24$

The student picked the wrong factors so the constant is negative instead of positive.

15. Simplify each expression and write your final answers in standard form.

a. Subtract $2a(a^2 - 4b)$ from $3(a - ab)$.

$3(a - ab) - 2a(a^2 - 4b)$

$3a - 3ab - 2a^3 + 8ab$

$-2a^3 + 5ab + 3a$

b. Find three times the sum of $3x^2 - 8x + 7$ and $-5x^2 + 2x - 9$

$3(3x^2 - 8x + 7 - 5x^2 + 2x - 9)$

$3(-2x^2 - 6x - 2)$

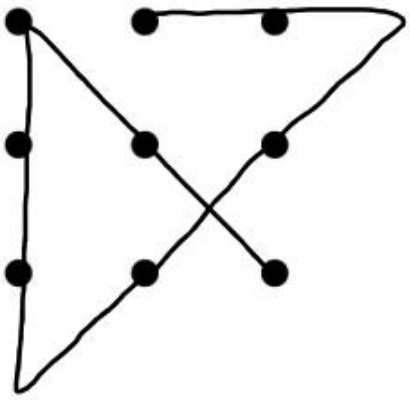
$-6x^2 - 18x - 6$

Lesson 4 Answer Keys

Warm Up

Nine Dots Puzzle

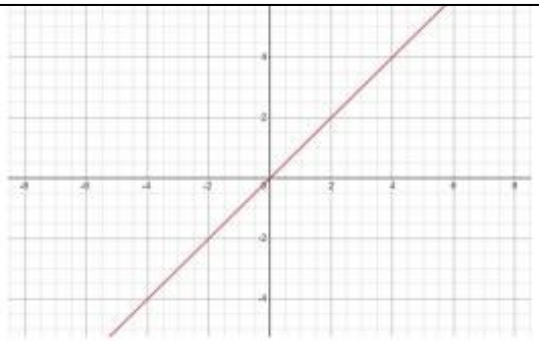
Without lifting your dry erase marker, draw four straight lines that pass through all nine of the points below.



Linear Function

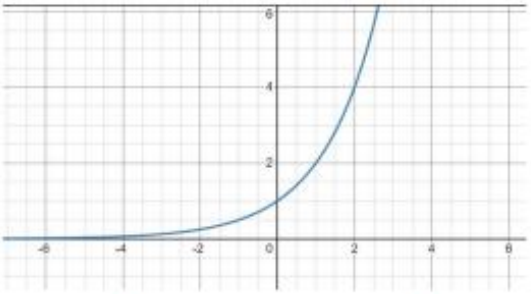
$y = x$

degree of 1



Exponential Function

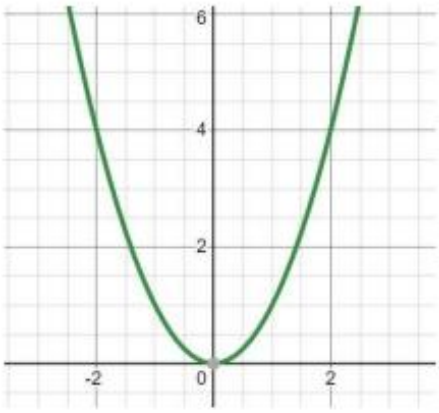
$y = 2^x$



Quadratic Function

$y = x^2$

degree of 2



NEW

A quadratic equation is an equation of degree 2

Which of the following are quadratic equations??

A) $(x-7)(x+2)=0$

B) $x^3-x^2+3x-1=0$

C) $x^2=x+4$

D) $x-2x^2=1+x^5$

E) $x^2+7x+12=0$

F) $x^2=49$

G) x^2-x-6

H) $3x-4=x+1$

$(x-7)(x+2)$

$x^2+7x+2x-14$ ✓

Activity Packet-page 1

Section 1: Solving Quadratic Equations by Factoring

INTRO

A quadratic equation is an equation that has a degree of 2.

Identify which equations below are quadratic or not.

1.) $x^2 + 4x - 12 = 0$ a. quadratic equation b. not quadratic equation

2.) $y^3 + y^2 - 5y = 24$ a. quadratic equation b. not quadratic equation

3.) $z^2 + 8 = -12x$ a. quadratic equation b. not quadratic equation

4.) $x + 7 = 0$ a. quadratic equation b. not quadratic equation

5.) $y^2 - 25 = 0$ a. quadratic equation b. not quadratic equation

6.) $(3x - 1)(x - 1) = 0$ a. quadratic equation b. not quadratic equation

7.) $4m^2 = 0$ a. quadratic equation b. not quadratic equation

8.) $x^2 + 6x = 0$ a. quadratic equation b. not quadratic equation

9.) $y^2 - 5y = 24$ a. quadratic equation b. not quadratic equation

10.) $(7y - 2)(7y - 2)$ a. quadratic equation b. not quadratic equation

Activity Packet-page 2

The 0 Product property states that if ab = 0, then either a = 0 or b = 0, or both a and b are 0. When the product of factors equals **zero**, one or more of the factors must also equal **zero**.

EX #1: Use this property from above to answer the following questions:

(a) If (6)(a) = 0, then what is the value of a?

Does 6 = 0? No!

a = 0

(b) If 3(x + 4) = 0, then what is the possible value of x?

3 ≠ 0 so... x + 4 = 0

−4 −4

x = −4

(c) If x(x – 4) = 0, then what are the possible values of x?

Hm... Test both

x = 0, x − 4 = 0

+4 +4

x = 4

(d) If (x – 7)(x + 7) = 0, then what are the possible values of x?

x − 7 = 0

+7 +7

x = 7

x + 7 = 0

−7 −7

x = −7

(e) If (x – 8)(x – 2) = 0, then what are the possible values of x?

1. Split

2. Set = 0

3. Solve

(x − 8)

x − 8 = 0

+8 +8

x = 8

(x − 2)

x − 2 = 0

+2 +2

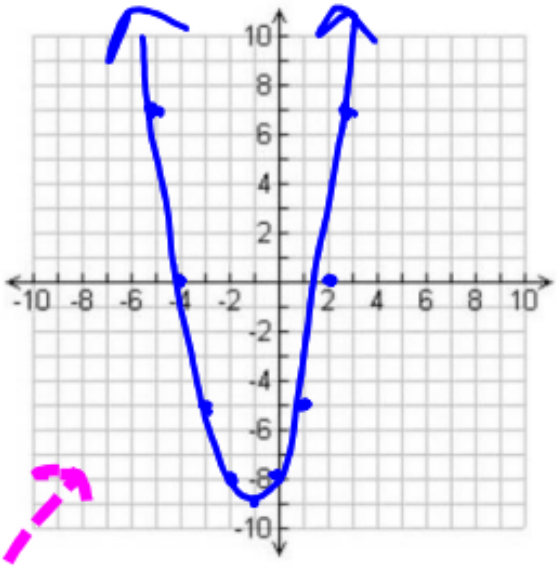
x = 2

Activity Packet-page 3

EX #2:

a) Graph the quadratic equation $y = x^2 + 2x - 8$ on the grid below.

x	y
-5	7
-4	0
-3	-5
-2	-8
-1	-9
0	-8
1	-5
2	0
3	7



b) Find all possible x-values that satisfy the equation $x^2 + 2x - 8 = 0$.

$(x-2)(x+4) = 0$

$x - 2 = 0$ $\quad$ $x + 4 = 0$

$+2 \quad +2$ $\quad$ $-4 \quad -4$

$x = 2$ $\quad$ $x = -4$

$+ \quad \times$

-8

$1 \cdot -8$

$-1 \cdot 8$

$-2 \cdot 4$

$2 \cdot -4$

c) What do you notice from parts a and b?

Activity Packet-page 4

PRACTICE

Directions: For #1-8, solve the following equations by factoring. Express your answer(s) as a solution set.

1) $(x - 10)(x + 2) = 0$

$x - 10 = 0 \quad x + 2 = 0$

$+10 \quad +10 \quad -2 \quad -2$

$x = 10 \quad x = -2$

2) $x^2 - 12x + 31 = -5$

Have to +5 first

$(x-6)(x-6) = 0$

$x - 6 = 0$

$+6 \quad +6$

$x = 6$

3) $7x^2 + 7x = 0$

$7x(x+1) = 0$

$\frac{7x}{7} = \frac{0}{7} \qquad x + 1 = 0$

$-1 \quad -1$

$x = 0 \qquad x = -1$

4) $2x^2 - 18 = 0$

$2(x^2 - 9) = 0$ ... Dyr!

$(x+3)(x-3) = 0$

$x = -3, 3$

Activity Packet-page 5

5) $2x^2 + 10x = -12$

$2(x^2+5x+6)=0$

$(x+2)(x+3)=0$

$x=-2,-3$

6) $(x-3)(x+3) = 6x - 14$

$x^2-3x+3x-9-6x+14=0$

$x^2-6x+5=0$

$x-2=0 \quad x-3=0$

$x=2,3$

7) $(x-2)(x^2-16) = 0$

$x-2=0$

$+2 \quad +2$

$x=2$

$(x+4)(x-4)$

$x=-4,4$

8) $(x+3)(x^2-5x-6) = 0$

$x=-3$

$(x-6)(x+1)$

$x=-1,6$