Using Relational and Instrumental Understandings to Prepare Students for Success With Solving Quadratic Equations on End of Course Exams

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Abstract

Solving quadratic equations is one of the most important units taught in Algebra, but also one of the most difficult for student understanding. This curriculum is to get students using their calculator and blending the meaning of solving equations with graphing functions, with the end goal of students performing better on end of course exams. The majority of students struggle to remember all the methods used to solve quadratic equations, so this curriculum contains 6 lessons focusing on three methods that can be used to solve any quadratic equation. The lesson methods in the curriculum are graphing, factoring, and using the quadratic formula to solve quadratic equations in equation and word problem forms. The assessment will require the application for solving quadratic equations with a focus on students' independently determining the best method to solve.

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Introduction

When creating lessons educators seek instructional practices that best supports student learning and preparation for end of year assessments. Some may simplify instruction and implement mathematical tricks, e.g., please excuse my dear Aunt Sally to teach the order of operations. Such tricks mostly focus on the use of procedures. Such instruction is believed to ease learning and help students get the right answer more quickly (Skemp, 2015). Other teachers may tend to focus more conceptual understanding. However, what most teachers really want is a balance between the two because students need an understanding of why a mathematical procedure or trick works. Otherwise they cannot retain the concept, leading van Merrienboer, Kester, and Paas (2006) to refer to this phenomenon as the transfer paradox. Teachers believe what they are doing is supporting learning but in reality, without combining conceptual understanding with procedures learning does not transfer (van Merrienboer et al., 2006). This curriculum provides lessons on solving quadratic equations that relates procedures supported with calculator strategies and conceptual understanding. Solving quadratic equations is an essential skill for many higher-level mathematics classes, therefore this curriculum project was designed to present a balance between procedures and concepts with the goal of students learning and being better prepared for end of year exams. Skemp (2006) referred to this as instruction that combines relational understanding (knowing what to do and why) and instrumental understanding (rules without reason).

Literature Review

As teachers of mathematics we all seek to teach for understanding but how we do that can vary. Some may teach rules, that to students may seem to exist without reasons, while others teach methods with a motive, which Skemp (2006) referred to as *knowing what to do and why*. These differences in instructions can be referred to as instrumental and relational understanding, respectively. According to Skemp (2006), instrumental understanding can be effective in many classrooms because it is easier to understand and apply, but it is harder to adapt to new tasks; such as when teachers use the trick of *two negatives make a positive*. This common phrase is used often in mathematics instruction, but may lead students to think that this trick applies to any operation, not just multiplication and division. Relational understanding is an effective instructional practice in classrooms because it allows teachers to support students in understanding the why in mathematics (Skemp, 2006). For example, when teachers have students link finding zeros graphically to the factors of quadratic equations, they are helping students understand the why. Obviously, the main reason many teachers use instrumental understanding instead of relational understanding is because the relational concepts may be too difficult or take too long to teach. In addition, teachers have end of year assessments that the students will need to pass, so it's easier to show students a trick for understanding rather than taking the time to teach the understanding of why something works.

The process of teaching for understanding and assessing learning is complex. Unquestionably, one challenge is knowing whether or not students fully understand the mathematical concepts. According to Wade, Sonnert, Sadler and Hazari (2017) pragmatic pressures toward drill and practice and teaching for understanding faces this challenge, "no one knows exactly what instructional experiences promote learning" (p.6). Additionally, the Algebra 1 curriculum is so broad regarding what students are to learn, students may simply move through the course learning procedures rather than concepts. This is critical because Algebra 1 is foundational for all higher-level mathematics, so if students fail to learn mathematical concepts they may have trouble assimilating their knowledge further. For example, the basic concept of area is developed in elementary mathematics and continues into integral calculus. Learning in the end is possible, but circumstances need to allow students to engage and spend time on academic tasks (Wade, et al. 2017).

Instrumental understanding can be connected well with the use of procedures. Star (2005) stated that the ability to work with problem solving efficiently is referred to as thinking with flexibility while using procedures. A more flexible solver with a deep knowledge of procedures will navigate a problem using techniques to produce a solution rather than use an over- practiced skill (Star, 2005). Merging knowing what to and why with procedural flexibility to support student understanding can be beneficial toward preparing students for end of course exams. The importance of this curriculum project is that it was designed for teachers based on these principles.

One topic in Algebra 1 that has limited research on teaching and learning is solving quadratic equations. Didis and Erbas (2015) conducted a study of 217 tenth graders from three public high schools in Turkey on solving quadratic equations comprising of 12 open- ended items on symbolic equations and word problems. This test was given after a unit of 15 lessons on quadratic equations emphasizing the symbolic approaches of factorization, quadratic formula, and completing the square. After the test students were interviewed to get their opinions on the questions and strategies they used to solve or attempt to solve the various quadratic equations. The results of students' responses revealed that a majority of the students attempted to use either factorization or the quadratic formula. Students that attempted to complete the square were not able to accurately utilize it to solve a quadratic equation (Didis, et al. 2015). Also, it was concluded that students focused primarily on the calculation of symbols without understanding the meaning of what solutions to quadratic equations are. Similarly, it was found that the comprehension of word problems was a central reason for student difficulties. Many students struggled to find factor pairs of larger constant values for factorization. As Didis and Erbas stated "It is important to design teaching and learning environments that can actively promote creative mathematically founded reasoning over rote learning and imitative reasoning." It could be that many of these students experienced the learning of factorization as rules without reason. Based on Skemp's (2006) research, if instruction had been supported with relational understanding their learning could have been more successfully demonstrated.

Curriculum

This unit plan is aligned with the Algebra 1 curriculum to teach quadratic equations. The lessons in the unit plan use a combination of "relational and instrumental understanding" to increase student success on end of course exams. A Unit Overview is provided, so teachers can better manage lesson planning. Based on a 40- minute class schedule the lessons require about 7- 8 days. For 90- minute block schedules these will need to be modified. Each lesson is written with fill-in guided notes, partner discussion questions and independent questions. This provides students the opportunity to master quadratic equation content as a class, and with a classmate, and by practicing the skill on their own. At the end of the unit there is an end of unit assessment that contains NYS past assessment questions to better gage student comprehension of the lessons and test the major skills that would be on an end of course exam. The unit plan includes two lessons on solving quadratic equations through area and projectile motion word problems. End of year exams often include questions that are computational and comprehensive, so it is important for students to have continued practice, but also know what quadratic equations can be used and solved for. There are end of year exams that may require students to solve using completing the square and these students may never be able to earn full points for these questions, but have a higher possibility of earning points on an exam.

All notes and worksheets for this curriculum are created to maximize information and time in the classroom, so it does not follow APA format. Answer keys and teacher notes to each lesson are found after the unit notes and assessment. Graphing with a graphing calculator is encouraged. By viewing the image of a graph crossing the x-axis students can find solutions and develop an understanding of factoring. This visualization aid can provide students with more confidence because they do not have to use the guess and check method. They can instead develop an understanding of the relationship between zeros and factors. This supports relational understanding, or knowing what to do and why.

Unit Plan Introduction:

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UNIT PLAN: SOLVING QUADRATIC EQUATIONS

<u>Subject Area</u>: • 9^{th} Grade Algebra 1
<u>Approximate Time Needed:</u> • 7 Days (including a testing day) • At Least 40-minute class periods
<u>Vocabulary:</u> • Quadratic Equations • Standard Form/ Factored Form • Zeros/ Solutions/ x- intercepts/ Roots • Vertex • Solve • Factors • Factoring • Quadratic Formula • Square root/ Radical • Area • Projectile Motion
<u>New York State Regents Standards</u>: **N-RN**: Use properties of rational and irrational numbers. 3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **A-SSE**: Interpret the structure of expressions. 1a: Interpret parts of an expression, such as terms, factors, and coefficients. 2: Use the structure of an expression to identify ways to rewrite it. **A-SSE**: Write expressions in equivalent forms to solve problems. 3a: Factor a quadratic expression to reveal the zeros of the function it defines. **A-APR**: Understand the relationship between zeros and factors of polynomials. 3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **A-CED**: Create equations that describe numbers or relationships. 1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* 2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **A-REI.4**: Solve equations and inequalities in one variable. 4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

F-IF: Understand the concept of a function and use function notation.

4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF: Analyze functions using different representations.

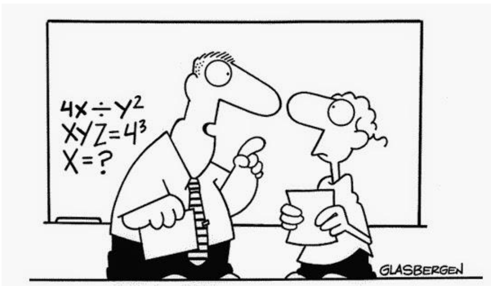
7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.

7c: Graph polynomial functions, identify zeros when suitable factorizations are available, and showing end behavior.

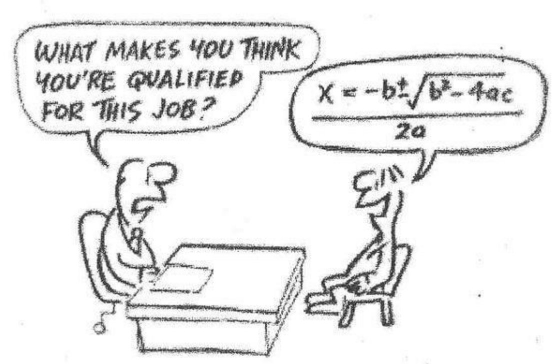
8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Name: ___________________________ CC Algebra I

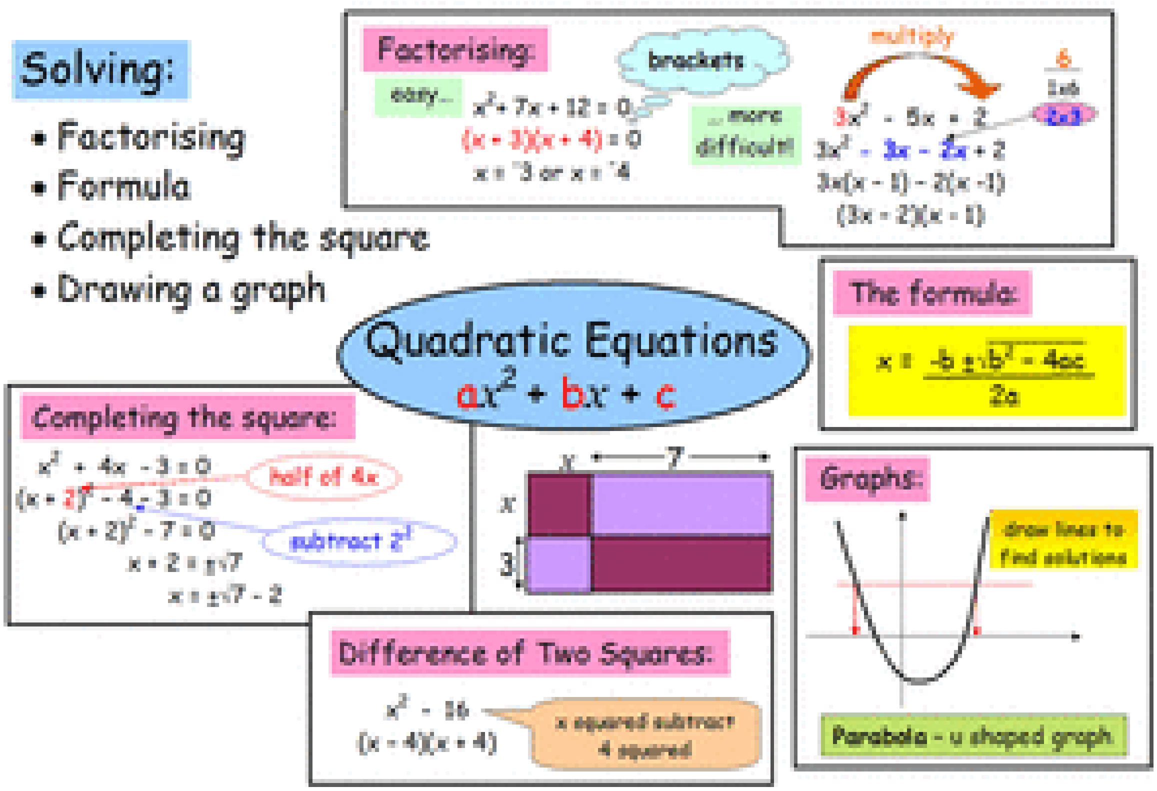
Period: ___________



"Algebra class will be important to you later in life because there's going to be a test six weeks from now."



Unit: Solving Quadratic Equations



Day 1: Solving Quadratic Equations by Graphing

Standards:

F-IF: Understand the concept of a function and use function notation.

4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF: Analyze functions using different representations.

7a: Graph linear and quadratic functions and show intercepts, maxima, and minima.

7c: Graph polynomial functions, identify zeros when suitable factorizations are available, and showing end behavior.

Objective(s):

Students will be able to determine the zeros of quadratic equations graphically (Post to board for student visual)

- Rewrite a quadratic equation in standard form and vertex form.
- Graph quadratic equations by hand and in the calculator.
- Identify key features of quadratic function graphs, such as x-intercepts, vertex, and maximum/ minimum.

Essential Questions:

1. How are zeros represented in the table of a quadratic function?
2. How are zeros represented on the graph of a quadratic function?
3. What synonym words can be used to determine the zeros of a quadratic equation?
4. How can we use the calculator to help graph quadratic functions?

Materials:

Doc Cam, Worksheets (see attached), Graphing Calculators

Instructional Plan:

1. ***Guided Notes***: Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Doc Cam. (2 questions)
2. ***Partner Practice***: Students will discuss multiple- choice questions in pairs, where they will have to compromise on an answer. (2 questions)
3. ***Independent Practice***: Before the end of the period, students will complete questions on their own in order to check the understanding of the objectives for each student. (5 questions)

Name: ______________________________ Date: ___________________

Objective:

Students will be able to determine the zeros of quadratic equations graphically.

What are Zeros of Quadratic Equations?

Zeros are:

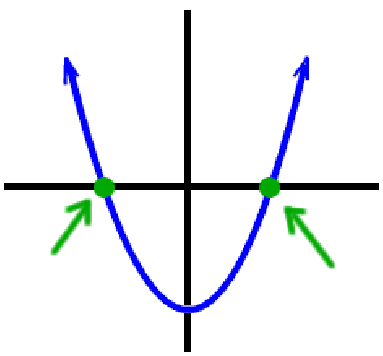
The _______ value of a function when $y = 0$ or the _______________________ of the function and the x- axis

Synonyms for Zeros Algebra:

___________________________, ________________________, and ___________________.

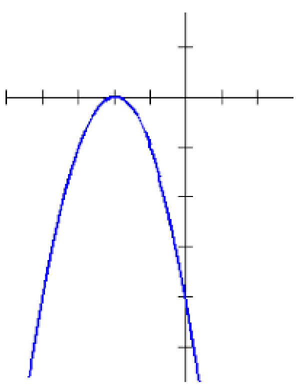
Identifying Zeros on Graphs and in Tables?

____________ zeros

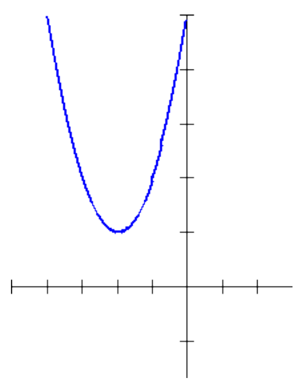


	x	$f(x)$
	−2	3
solution	−1	0
	0	−1
solution	1	0
	2	3
	3	8

____________ zero

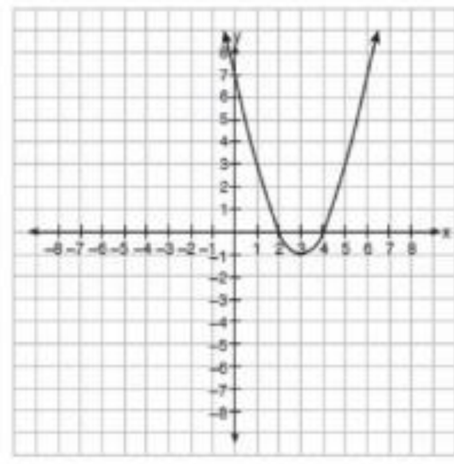


____________ zeros



Class Example #1:

Given the graph of the Quadratic Equation below:



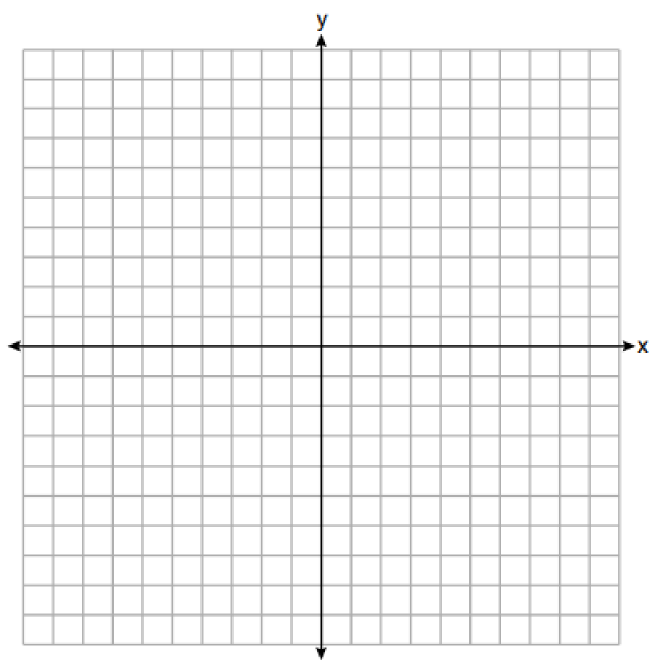
How many zeros does the function have?

What are the zeros of the function?

Class Example #2:

Determine the vertex and root(s) of the function $g(x) = -x^2 + 12x - 36$, graphically.

x	y



Root(s): ___________ ____________

Vertex: ____________

Partner Example #1:

How many real- number solutions does $4x^2 + 2x + 5 = 0$ have?

(1) One

(2) Two

(3) Zero

(4) Infinitely many

Partner Example #2:

What is the vertex and zeros of the parabola $y = x^2 - 16x + 63$?

(1) Vertex: $(8, -1)$; Zeros: $7, 9$

(2) Vertex: $(8, -1)$; Zeros: $-9, -7$

(3) Vertex: $(8, 1)$; Zeros: $7, 9$

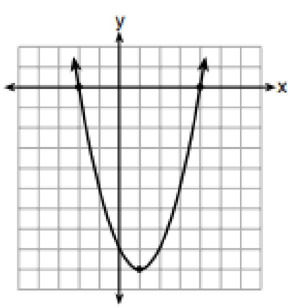
(4) Vertex: $(8, 1)$; Zeros: $-9, -7$

Independent Example #1:

Which function has zeros of -4 and 2?

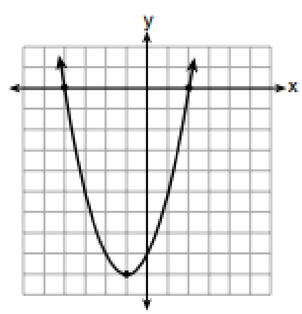
(1) $f(x) = x^2 + 7x - 8$

(2)



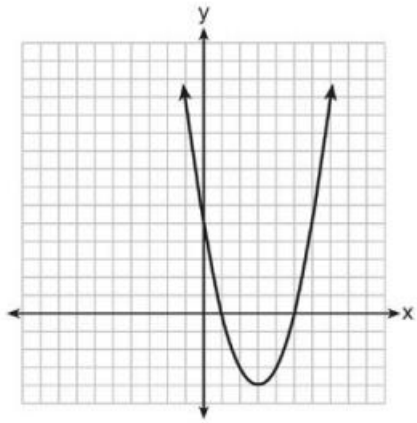
(3) $g(x) = x^2 - 7x - 8$

(4)



Independent Example #2:

The equation $y = (x-h)^2 + k$ is graphed on the set of axes below.



Based on the graph, what are the roots and vertex of the equation?

(1) 1 and 5; $(3, -4)$

(2) 1 and 0; $(3, -4)$

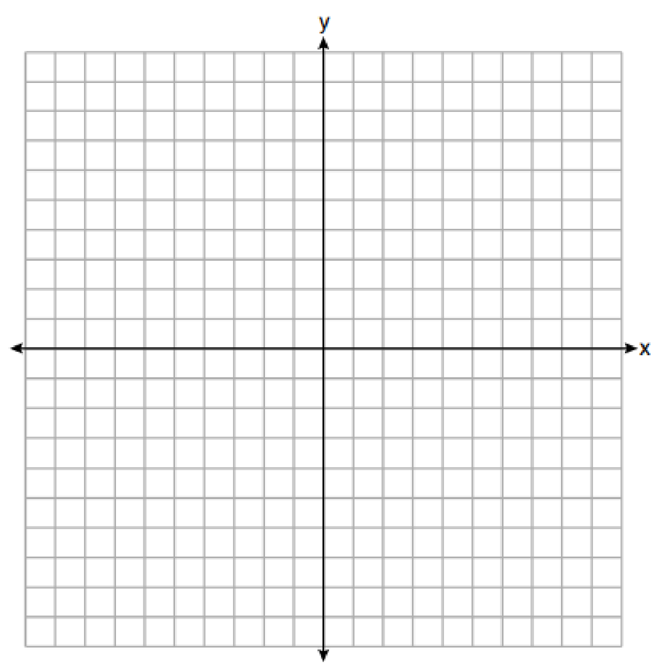
(3) 1 and 5; $(-4, 3)$

(4) 1 and 0; $(-4, 3)$

Independent Example #3:

On the set of axes below, graph $y = 2x^2 - 4x - 6$.

x							
y							



State the roots of the equation

$$0 = 2x^2 - 4x - 6.$$

Independent Example #4:

In your own words describe the difference between one, two and no zeros for a quadratic function.

____

____

____

____

____

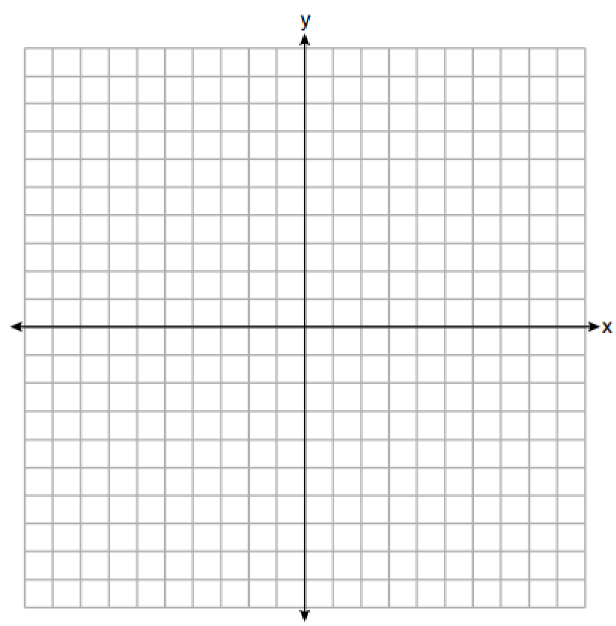
____

Independent Example #5:

Find the solutions graphically to quadratic equation below.

$$2x - 8 = 2x^2 + 6x - 3$$

x							
y							



Day 2: Solving Quadratic Equations by Graphing and Factoring

Standards:

A-SSE: Interpret the structure of expressions.
1a: Interpret parts of an expression, such as terms, factors, and coefficients.
2: Use the structure of an expression to identify ways to rewrite it.

A-SSE: Write expressions in equivalent forms to solve problems.
3a: Factor a quadratic expression to reveal the zeros of the function it defines.

A-APR: Understand the relationship between zeros and factors of polynomials.
3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED: Create equations that describe numbers or relationships.
1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI.4: Solve equations and inequalities in one variable.
4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Objective(s):

Today's Goal: Students will be able to determine the zeros of quadratic equations graphically and by factoring (Post to board for student visual)

- Graph quadratic equations in the calculator.
- Identify zeros of quadratic function graphs.
- Explain the connection between zeros and factored form of a quadratic equation (zero product property).
- Set quadratic equations equal to zero and solve the quadratic equations by factoring.

Essential Questions:

1. How are solutions represented in a table, on a graph, or in factors of a quadratic equation?
2. What synonym words can be used to determine the x- intercepts of a quadratic equation?
3. How can we use the calculator to help graph quadratic functions?
4. How can we use the calculator to help identify quadratic equations factors?
5. How can the zeros determine the factors of a quadratic equation?

Materials:

Doc Cam, Worksheets (see attached), Graphing Calculators

Instructional Plan:

1. *Guided Notes*: Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Doc Cam. (2 questions)
2. *Partner Practice*: Students will discuss multiple- choice questions in pairs, where they will have to compromise on an answer. (2 questions)
3. *Independent Practice*: Before the end of the period, students will complete questions on their own in order to check the understanding of the objectives for each student. (5 questions)

Name: ______________________________ Date: ____________________

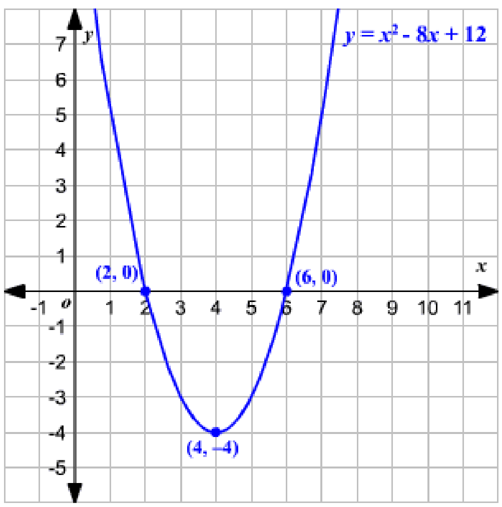
Objective:

Students will be able to determine the zeros of quadratic equations graphically and by factoring

Zeros Can Identify Factors:

Factored Form: $y = (x + \_\_\_\_)(x + \_\_\_\_)$

Zeros are the _____ intercepts of a quadratic equation, which means find the _____ value when $y = 0$.



So if $y = 0$, then…

$\_\_\_\_\_\_ = (x + r_1)(x + r_2)$

Also,

$\_\_\_\_\_ = x + r_1$ or $\_\_\_\_\_ = x + r_2$

Therefore,

r_1 and $r_2 =$ __________ of the zeros (x-intercepts)

Factoring to Find Zeros:

1.) Rewrite the quadratic equation in ______________________ form (______________________).

2.) Identify the zeros ________________________.

3.) ________________ the quadratic expression.

4.) Set each factor equal to ______________.

5.) ____________ each equation for the solutions or zeros.

Class Example #1:

Solve the quadratic equation. $0 = x^2 + 4x - 5$

<u>Step 1</u>: Identify the zeros from the table in the calculator and box them in as the answers.

<u>Step 2</u>: Write the factored form of a quadratic equation, but replace r_1 and r_2 with the inverses of the zeros.

<u>Step 3</u>: Set each factor equal to zero and solve both for x.

Class Example #2:

Determine the zeros to the polynomial equation below using factoring.

$$x^3 = 8x^2 + 20x$$

Partner Example #1:

Which equation has the same solutions as $x^2 + 16x + 64 = 0$?

(1) $(x + 4)(x - 4) = 0$

(2) $(x + 8)(x - 8) = 0$

(3) $(x - 8)(x - 8) = 0$

(4) $(x + 8)(x + 8) = 0$

Partner Example #2:

What are the solutions to the equation $(x - 2)(x - a) = 0$?

(1) -2 and a

(2) 2 and $-a$

(3) -2 and $-a$

(4) 2 and a

Independent Example #1:

Which polynomial function has zeros at $-3, 0$, and 4?

(1) $f(x) = (x + 3)(x^2 + 4)$

(2) $f(x) = (x^2 - 3)(x - 4)$

(3) $f(x) = x(x + 3)(x - 4)$

(4) $f(x) = x(x - 3)(x + 4)$

Independent Example #2:

Solve the quadratic equation using the factoring method.

$$x^2 - 12x + 35 = 0$$

Independent Example #3:

Determine the zeros to the quadratic equation using factoring.

$$x^2 - 3x + 2 = 0$$

Independent Example #4:

Determine the zeros to the quadratic equation using factoring.

$$x^2 + 3x - 4 = 50$$

Independent Example #5:

Determine the roots to the quadratic equation using factoring.

$$6x^2 - 7x - 60 = 4x^2 + 3x - 12$$

Day 3: Solving Quadratic Equations by Graphing and Factoring – Real World

Standards:

A-SSE: Interpret the structure of expressions.
1a: Interpret parts of an expression, such as terms, factors, and coefficients.
2: Use the structure of an expression to identify ways to rewrite it.

A-SSE: Write expressions in equivalent forms to solve problems.
3a: Factor a quadratic expression to reveal the zeros of the function it defines.

A-APR: Understand the relationship between zeros and factors of polynomials.
3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED: Create equations that describe numbers or relationships.
1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI.4: Solve equations and inequalities in one variable.
4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Objective(s):

Today's Goal: Students will be able to determine solutions of scenario based quadratic equations by graphing and factoring (Post to board for student visual)

- Graph quadratic equations in the calculator.
- Explain the connection between zeros and factored form of a quadratic equation.
- Set quadratic equations equal to zero and solve the quadratic equations by factoring.
- Translate area scenario diagrams into quadratic equations.
- State the positive solutions and reject negative solutions.

Essential Questions:

1. What is the question asking for?
2. What is the difference in solving for x and finding a length and/or width?
3. What is the formula to find the area of a rectangle?
4. How can we use the calculator to identify quadratic equation factors?
5. How can the zeros determine the factors of a quadratic equation?
6. Why are negative solutions rejected?

Materials:

Doc Cam, Worksheets (see attached), Graphing Calculators

Instructional Plan:

4. *Guided Notes*: Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Doc Cam. (2 questions)
5. *Partner Practice*: Students will discuss multiple- choice questions in pairs, where they will have to compromise on an answer. (2 questions)
6. *Independent Practice*: Before the end of the period, students will complete questions on their own in order to check the understanding of the objectives for each student. (4 questions)

Name: ______________________________ Date: ______________________

Objective:

Students will be able to determine solutions to scenario quadratic equations graphically and by factoring

Area:

The __________________ occupied by a flat shape or the surface of an object.

Equation: $A = (L)(W)$

A: __________ of the shape or object

L: _______________ of the shape or object $(x + r_1)$

W: _______________ of the shape or object $(x + r_2)$

Length and Width:

To find the length and width of any quadratic area problem make sure to know if the solved value of x needs to be _____________________ back into an expression.

Class Example #1:

A rec center basketball court measures 100 feet by 150 feet. The rec center's owner plans to increase the area of the court by 20%. He will do this by increasing the length and width by the same amount, x. Which equation represents the area of the new field?

(1) $(100 + 2x)(150 + 2x) = 18{,}000$

(2) $2(100 + x) + 2(150 + x) = 15{,}000$

(3) $(100 + x)(150 + x) = 18{,}000$

(4) $(100 + x)(150 + x) = 15{,}000$

Class Example #2:

The length of a rectangular sign is 6 inches more than half its width. The area of this sign is 432 square inches.

Label the diagram of the desired object.

Write an equation in one variable that could be used to find the number of inches in the dimensions of this sign.

Solve this equation algebraically to determine the dimensions of this sign, in inches.

Partner Example #1:

A 4 inch by 6 inch photo is put into a picture frame with a border of constant width. If the area of the frame, including the picture, is 80 square inches. Which equation can be used to find the width of the border, w?

(1) $2(4+w)+2(6+w)=80$

(2) $(4+2w)+(6+2w)=80$

(3) $(4+w)(6+w)=80$

(4) $(4+2w)(6+2w)=80$

Partner Example #2:

Maria is building a rectangular dog pen for her new puppy. The width of the pen is 1 yard less than the length. If the area of the dog pen is 20 square yards, how many yards of fencing will she need for each length and width of the pen?

(1) $l=15, w=5$

(2) $l=5, w=4$

(3) $l=10, w=2$

(4) $l=10, w=10$

Independent Example #1:

The length of a rectangular patio is 7 feet more than its width, w. The area of a patio, $A(w)$, can be represented by the function

(1) $A(w)=w+7$

(2) $A(w)=4w+14$

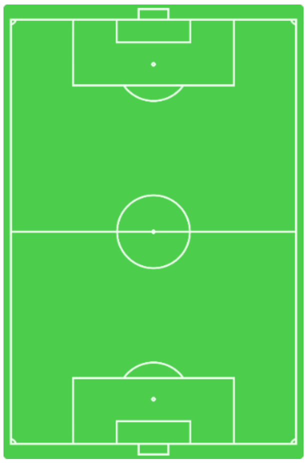
(3) $A(w)=w^2+7w$

(4) $A(w)=4w^2+28w$

Independent Example #2:

A landscaper is creating a rectangular soccer field at the local park with an area of 6,000 square yards. The length of the soccer field must be 40 yards longer than its width. Determine algebraically the dimensions of the new soccer field, in yards.

Diagram:



Equation:

Independent Example #3:

The area of a rectangular television screen is 1590 square inches. The width of the screen is 23 inches longer than the height. Therefore, the quadratic equation that can be used to represent the area of the television screen is $h(h + 23) = 1590$. Determine the dimensions of the television.

Diagram:

Equation:

Independent Example #4:

Tyrone wants to be the best goal tender in New York. He felt the best way to get started was to know the dimensions of the goal. He knows the area of the goal is 9,322 square inches and that the length of the goal post is 40 less than twice the height. What are the dimensions of the goal, so Tyrone can know how far he will need to jump and dive to save goals.

Diagram:

Equation:

Day 4: Solving Quadratic Equations by Quadratic Formula

Standards:

N-RN: Use properties of rational and irrational numbers.

3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A-SSE: Interpret the structure of expressions.

1a: Interpret parts of an expression, such as terms, factors, and coefficients.

2: Use the structure of an expression to identify ways to rewrite it.

A-REI.4: Solve equations and inequalities in one variable.

4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Objective(s):

Today's Goal: Students will be able to determine the zeros of quadratic equations using the quadratic formula (Post to board for student visual)

- Identify and substitute coefficients of quadratic equations into the quadratic formula.
- Rewrite quadratic formula as two solutions.
- Use the calculator to simplify arithmetic in the quadratic formula.
- Explain why the quadratic formula can be used to find solutions of any quadratic equation.
- Explain how many solutions a quadratic function has based on the quadratic formula

Essential Questions:

1. What is the formula for the quadratic formula?
2. When can a student use the quadratic formula?
3. What are the a, b, and c coefficients in a quadratic equation?
4. How can we use the calculator to simplify the quadratic formula?
5. How many possible solutions can you have for a quadratic equation?

Materials:

Doc Cam, Worksheets (see attached), Graphing Calculators

Instructional Plan:

1. *Guided Notes*: Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Doc Cam. (2 questions)
2. *Partner Practice*: Students will discuss multiple- choice questions in pairs, where they will have to compromise on an answer. (2 questions)
3. *Independent Practice*: Before the end of the period, students will complete questions on their own in order to check the understanding of the objectives for each student.

Name: ______________________________ Date: ______________________

Objective:

Students will be able to use the quadratic formula to find solutions to quadratic functions.

Quadratic Formula:

A formula that gives the ___________________ to any general quadratic equation.

Standard Equation of a Quadratic:

$$ax^2 + bx + c = 0$$

Formula used to find quadratic solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using the Quadratic Formula:

Step 1: Make sure the quadratic equation is in _______________________ form.

Step 2: Identify _______, ________, and ________.

Step 3: ____________________ a, b, and c into the quadratic formula.

Step 4: ________________ the formula.

$$\underset{a}{6}x^2 - \underset{b}{x} - \underset{c}{2} = 0$$

a = 6 b = -1 c = -2

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-2)}}{2(6)}$$

$$x = \frac{-(-1) \pm \sqrt{1 + 48}}{12}$$

$$x = \frac{1 \pm \sqrt{49}}{12}$$

Class Example #1:

Write down the values of a, b, can c in each case, substitute the values into the quadratic formula and simplify.

______ Solutions

$$2x^2 + 4x - 6 = 0$$

a = ________

b = ________

c = ________

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \times \square \times \square}}{2 \times \square}$$

______ Solutions

$$x^2 + 6x + 9 = 0$$

a = ________

b = ________

c = ________

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \times \square \times \square}}{2 \times \square}$$

______ Solutions

$$-2x^2 - 3 = 0$$

a = ________

b = ________

c = ________

$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \times \square \times \square}}{2 \times \square}$$

Class Example #2:

If the quadratic formula is used to find the zeros of the equation $-2x^2 - 3x = -4$, determine the zeros.

Partner Example #1:

The roots of the equation $2x^2 + 7x - 3 = 0$ are

(1) $-\frac{1}{2}$ and -3

(3) $\frac{-7\pm\sqrt{73}}{4}$

(2) $\frac{1}{2}$ and 3

(4) $\frac{7\pm\sqrt{73}}{4}$

Partner Example #2:

Solve the quadratic $2x^2 + 11x = 2$ using the quadratic formula.

Independent Example #1:

The roots of $x^2 - 5x - 4 = 0$ are

(1) 1 and 4

(3) -1 and -4

(2) $\frac{5\pm\sqrt{41}}{2}$

(4) $\frac{-5\pm\sqrt{41}}{2}$

Independent Example #2:

Find the roots of the quadratic equation $3x^2 - x + 2 = 0$.

Independent Example #3:

Determine the solutions to the quadratic equation $6x^2 = 3x + 5$.

Independent Example #4:

Solve $-7x + x^2 = 4$ using the quadratic formula.

Independent Example #5:

Using the quadratic formula, solve the equation $3 = 2x^2 - 9x$ for x.

Day 5: Solving Quadratic Equations by Quadratic Formula Continued

Standards:

N-RN: Use properties of rational and irrational numbers.

3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A-SSE: Interpret the structure of expressions.

1a: Interpret parts of an expression, such as terms, factors, and coefficients.

2: Use the structure of an expression to identify ways to rewrite it.

A-REI.4: Solve equations and inequalities in one variable.

4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Objective(s):

Today's Goal: Students will be able to determine the zeros of quadratic equations using the quadratic formula (Post to board for student visual)

- Identify and substitute coefficients of quadratic equations into the quadratic formula.
- Rewrite quadratic formula as two solutions.
- Use the calculator to simplify the quadratic formula in decimal form.
- Reduce radicals to simplest form.
- Explain why the quadratic formula can be used to find solutions of any quadratic equation.
- Explain how many solutions a quadratic function has based on the quadratic formula.

Essential Questions:

1. What is the formula for the quadratic formula?

2. When can a student use the quadratic formula?

3. What are the a, b, and c coefficients in a quadratic equation?

4. How can the calculator be used to simplify the quadratic formula and/or convert to a decimal?

5. How many possible solutions can you have for a quadratic equation?

6. How can a radical be simplified using perfect squares?

7. When simplifying the quadratic formula equation why can't you reduce a number under a radical and a denominator?

Materials:

Doc Cam, Worksheets (see attached), Graphing Calculators

Instructional Plan:

4. *Guided Notes*: Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Doc Cam. (2 questions)
5. *Partner Practice*: Students will discuss multiple- choice questions in pairs, where they will have to compromise on an answer. (2 questions)
6. *Independent Practice*: Before the end of the period, students will complete questions on their own in order to check the understanding of the objectives for each student.

Name: ______________________________ Date: ______________________

Objective:

Students will be able to simplify quadratic solutions using the quadratic formula.

Simplifying Square Roots:

The square root of the product ab is ____________ to the product of each square root. ($\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$)

Examples:

$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} =$ _________

$\sqrt{50} = \sqrt{25} \cdot \sqrt{2} =$ __________

$\sqrt{80} = \sqrt{8} \cdot \sqrt{10} = \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{5} =$ ________

$\sqrt{12} = \sqrt{6} \cdot \sqrt{2} = \sqrt{3} \cdot \sqrt{2} \cdot \sqrt{2} =$ ________

$\sqrt{45} =$ __________________=_________

$\sqrt{32} =$ _________=___________=________

Simplifying Quadratic Formula:

Step 1: _____________ the product of the square root.

$$x = \frac{4 \pm \sqrt{72}}{2} = \frac{4 \pm \sqrt{36 \cdot 2}}{2} = \frac{4 \pm 6\sqrt{2}}{2}$$

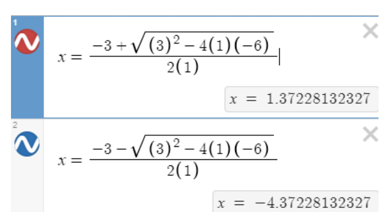
Step 2: ___________ the fractions.

$$x_1 = \frac{4 + 6\sqrt{2}}{2} = 2 + 3\sqrt{2} \qquad x_2 = \frac{4 - 6\sqrt{2}}{2} = 2 - 3\sqrt{2}$$

Quadratic Formula Solutions as a Decimal:

Write both the addition and subtraction solutions into the calculator as

___________________.



Class Example #1:

Solve the equation $6x^2 - 2x - 3 = 0$ and express the answer in simplest radical form.

Class Example #2:

Given: $g(x) = 2x^2 + 3x + 10$

$k(x) = 2x + 16$

Solve the quadratic equation $g(x) = 2k(x)$ algebraically for x, to the *nearest tenth*.

Partner Example #1:

If the quadratic formula is used to find the roots of the equation $x^2 - 6x - 19 = 0$, the correct roots are

(1) $3 \pm 2\sqrt{7}$

(2) $-3 \pm 2\sqrt{7}$

(3) $3 \pm 4\sqrt{14}$

(4) $-3 \pm 4\sqrt{14}$

Partner Example #2:

Lena was asked to go to the board and solve the quadratic equation $2x^2 + 8x - 3 = 0$ using the quadratic formula. Her work is shown below.

$2x^2 + 8x - 3 = 0$ $\quad a = 2 \quad b = 8 \quad c = -3$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-3)}}{2(2)} = \frac{-8 \pm \sqrt{64+24}}{4} = \frac{-8 \pm \sqrt{88}}{4} = -2 \pm \sqrt{22}$$

When she got back to her seat, the teacher stated that she made one error while simplifying. Identify the error Lena made and write the correct answer.

Independent Example #1:

Solve the quadratic equation $3x^2 - 4x - 6 = 0$ using the quadratic formula. Round solutions to the ***nearest hundredth***.

(1) $x = 2.46, -3.79$

(3) $x = 3.79, -2.46$

(2) $x = 2.23, -0.90$

(4) $x = 0.90, -2.23$

Independent Example #2:

Solve the equation $2x^2 - 5 = 4x$ and express the answer in simplest radical form.

Independent Example #3:

Solve for x to the *nearest tenth*: $x^2 + x - 5 = 0$

Independent Example #4:

Use the method of quadratic formula to determine the exact values of x for the equation $x^2 - 8x + 6 = 0$.

Independent Example #5:

Solve $6x^2 - 42 = 0$ for the exact values of x.

Day 6: Solving Projectile Motion Equation by Quadratic Formula – Real World

Standards:

N-RN: Use properties of rational and irrational numbers.

> 3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A-SSE: Interpret the structure of expressions.

> 1a: Interpret parts of an expression, such as terms, factors, and coefficients.
> 2: Use the structure of an expression to identify ways to rewrite it.

A-REI.4: Solve equations and inequalities in one variable.

> 4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

Objective(s):

Today's Goal: Students will be able to determine zeros of quadratic projectile motion scenarios (Post to board for student visual)

- Solve quadratic equations using the quadratic formula.
- Simplify the quadratic formula to decimal answers.
- Identify and label solutions with the correct units.
- Relating zeros to a projectile hitting the ground.

Essential Questions:

1. What is the quadratic formula?
2. Why would the quadratic formula be the best method to find zeros with projectile motion?
3. How are zeros represented in projectile motion scenarios?
4. How can the calculator be used to help simplify the quadratic formula equations?
5. What are the most logical units for zeros with projectile motion questions?

Materials:

Doc Cam, Worksheets (see attached), Graphing Calculators

Instructional Plan:

7. *Guided Notes*: Each student is given a copy of the skeleton notes, which they are expected to follow along as the teacher works on the Doc Cam. (2 questions)
8. *Partner Practice*: Students will discuss multiple- choice questions in pairs, where they will have to compromise on an answer. (2 questions)
9. *Independent Practice*: Before the end of the period, students will complete questions on their own in order to check the understanding of the objectives for each student.

Name: ______________________________ Date: ______________________

Objective:

Students will be able to determine zeros of quadratic projectile motion scenarios.

Projectile Motion:

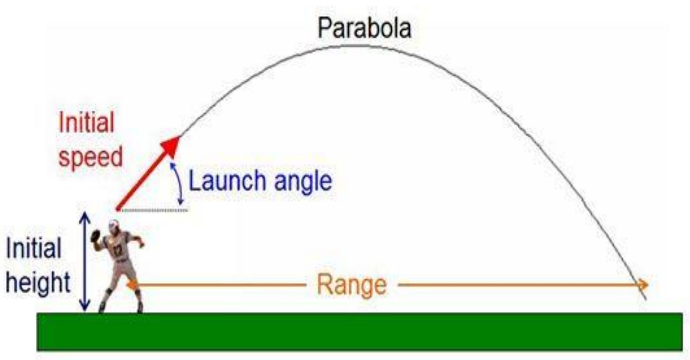
The path objects take when they are ________________, and ________________ either up or down.

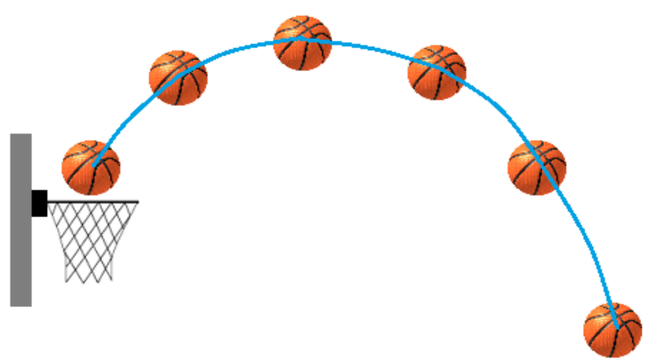
<u>Equation</u>: $y = ax^2 + bx + c$

a: Force of gravity on the object (represented by either _______ or _______)

b: Velocity at which the object travels

c: The initial height of the object (____________________)





Zeros in Projectile Motion:

The **<u>Zeros</u>** represent the ______________ it takes for an object to hit the ______________ (height of zero).

Class Example #1:

The height of a rocket launched from the ground is given by the function $h(x) = -16x^2 + 200x$, where $h(x)$ is the height, in feet, after x seconds.

What is the initial height of the rocket (c- value in the quadratic equation)?

When will the rocket reach the ground?

Class Example #2:

Justin Tucker needs to kick a 45-yard (135 feet) field goal for the Ravens to beat the Stealers. The height of the kicked football, $f(x)$, in feet, can be modeled by the equation $f(x) = -0.01x^2 + 1.28x$, where x is the ball's horizontal distance, in feet, from the ground.

How far will the football travel in feet? Does Justin Tucker win the Ravens the game?

Partner Example #1:

A ball is thrown into the air from the top of a building. The height, $h(t)$, of the ball above the ground t seconds after it is thrown can be modeled by $h(t) = -16t^2 + 64t + 80$. How many seconds after being thrown will the ball hit the ground?

(1) 5 seconds

(2) 2 seconds

(3) 80 seconds

(4) 144 seconds

Partner Example #2:

Omar shoots a basketball at basketball practice. The equation that models the path of the ball is $p(t) = -16t^2 + 34t + 5.25$, where $p(t)$ is the height of the ball in feet and t is the number of seconds after the ball was shot. If the basketball hoop is 8 feet tall, how long is the ball in the air before the ball goes in the hoop?

Independent Example #1:

Lebron threw a basketball from half court toward the hoop. The quadratic function that models the height, in feet, of the ball after t seconds is $h(t) = -16t^2 + 12t + 6$. If the hoop is 8 feet high, how long is the ball in the air before going into the hoop.

(1) $1\frac{1}{2}$ second (3) 2 seconds

(2) 1 second (4) $2\frac{1}{2}$ seconds

Independent Example #2:

A cliff diver on a Caribbean island jumps from a height of 105 feet, with an initial upward velocity of 5 feet per second. An equation that models the height, h(t), above the water, in feet, of the diver in time elapsed, t, in seconds, is $h(t) = -16t^2 + 5t + 105$. How many seconds, to the ***nearest hundredth***, does it take the diver to fall 45 feet below his starting point?

(1) 1.45 (3) 2.10

(2) 1.84 (4) 2.72

Independent Example #3:

Dasani pulled the plug in her bathtub and it started to drain. The amount of water in the bathtub as it drains is represented by the equation $L = -5t^2 - 8t + 120$, where L represents the number of liters of water in the bathtub and t represents the amount of time, in minutes, since the plug was pulled.

How many liters of water were in the bathtub when Dasani pulled the plug?

Determine, to the *nearest tenth* of a minute, the amount of time it takes for all the water in the bathtub to drain.

Independent Example #4:

Tyrone is a test pilot for the United States Air Force. He is cruising at an altitude of 9,000 feet, when he is forced to eject from the aircraft. The function $h(t) = -16t^2 + 128t + 9000$ models the height, in feet, of the pilot above the ground, where t is the time, in seconds, after he is ejected.

Determine and state the zero of the function *rounded to the nearest second*.

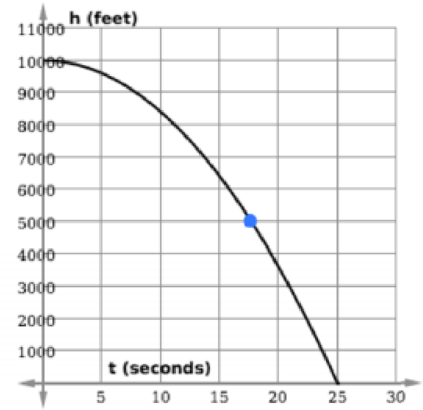
Explain what the zero represents in the context of the problem.

____

____

Independent Example #5:

You jump out of an airplane. The graph shows the function $h(t) = -16t^2 + 10{,}000$ that models the height $h(t)$ you are above ground after t seconds. If you release your parachute at 5,000 feet, after how many seconds *rounded to the nearest hundredth* do you do this?



Day 7: Solving Quadratic Equations Assessment – End of Year Test Questions

Standards:

N-RN: Use properties of rational and irrational numbers.

3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

A-SSE: Interpret the structure of expressions.

1a: Interpret parts of an expression, such as terms, factors, and coefficients.
2: Use the structure of an expression to identify ways to rewrite it.

A-SSE: Write expressions in equivalent forms to solve problems.

3a: Factor a quadratic expression to reveal the zeros of the function it defines.

A-APR: Understand the relationship between zeros and factors of polynomials.

3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-CED: Create equations that describe numbers or relationships.

1: Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A-REI.4: Solve equations and inequalities in one variable.

4b: Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

F-IF: Understand the concept of a function and use function notation.

4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF: Analyze functions using different representations.

7c: Graph polynomial functions, identify zeros when suitable factorizations are available, and showing end behavior.
8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Objective(s):

Today's Goal: Students will be able to determine the zeros of quadratic equations using the calculator graphically, factoring, and/or the quadratic formula. (Post to board for student visual)

- Determine the number of solutions for a given quadratic function.
- Identify key feature of x-intercepts for quadratic function graphs given a calculator.
- Solve for zeros based on quadratic projectile motion scenarios.
- Solve for the side lengths of a rectangular area scenario using quadratic equation zeros.
- Solve quadratic equations using factoring, graphing and the quadratic formula.

Materials:

Unit Test (see attached), Reference Sheet, scrap graph paper, and Graphing Calculators

Instructional Plan:

1. Students will get the entire class period to work on the Quadratic Equations Unit Test (7 multiple choice and 3 short response).

2. Students will be given the reference sheet that they will also receive on their end of year assessment (should include the quadratic formula)

3. Once students are finished have a vocabulary word search or matching activity students can do as preparation for the next unit.

Name: ______________________________________ Date: ______________

Quadratics Equations Unit Test

Answer each multiple-choice question. Each question is worth 2 points and partial credit will go to students that showed correct work.

1.) How many real- number solutions does $4x^2 + 2x + 5 = 0$ have?

(1) One (3) Zero

(2) Two (4) Infinitely Many

2.) Solve the equation $x^2 - 64 = 0$ for x.

(1) 8 and -8 (3) 8

(2) -8 (4) 16 and -4

3.) The zeros of the function $p(x) = x^2 - 2x - 24$ are

(1) -8 and 3 (3) -4 and 6

(2) -6 and 4 (4) -3 and 8

4.) Which function has zeros at -4 and 2?

(1) $f(x) = x^2 + 7x - 8$

(2) $h(x) = (x + 2)(x - 4)$

(3) $g(x) = x^2 - 7x - 8$

(4) $j(x) = (x + 4)(x - 2)$

5.) The solution to the quadratic equation $2x^2 + 5x - 1 = 0$ is

(1) $\frac{5 \pm \sqrt{17}}{4}$

(2) $\frac{-5 \pm \sqrt{17}}{4}$

(3) $\frac{5 \pm \sqrt{33}}{4}$

(4) $\frac{-5 \pm \sqrt{33}}{4}$

6.) What are the solutions to the equation $x^2 - 8x = 10$?

(1) $4 \pm \sqrt{10}$

(2) $4 \pm \sqrt{26}$

(3) $-4 \pm \sqrt{10}$

(4) $-4 \pm \sqrt{26}$

7.) A toy rocket is launched upward from the ground with an initial velocity of 128 feet per second and is modeled by the equation $h(t) = -16t^2 + 128t$, where h is the height in feet and t is the time in seconds. How long will it take for the rocket to hit the ground?

(1) 10 seconds

(2) 6 seconds

(3) 4 seconds

(4) 8 seconds

Answer each constructed response question. Each question is worth 2 points and partial credit will go to students that showed correct work.

8.) How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.

Answer the constructed response question. This question is worth 4 points and partial credit will go to students that showed correct work.

9.) Determine algebraically the zeros of $f(x) = 3x^3 + 21x^2 + 36x$.

10.) A school is building a rectangular football field that has an area of 6,360 square yards. The football field must be 67 yards longer than its width. Determine algebraically the dimensions of the football field, in yards.

My Turn

I have been teaching Algebra 1 for 5 years, working particularly in urban school districts. Many of my students come to my 9th and 10th grade classes with very limited algebraic and computational skills. Thus, the solving quadratic equations unit was designed for teachers to support students who struggle to remember and/or comprehend multiple methods and skills required to learn a topic. In my classroom, we also use a calculator, but I keep it simple because many of my students don't have calculators that they can take home. For the students that struggle with learning quadratic equations, I focus my attention to graphing, factoring, and the quadratic formula. In the past I had additionally taught completing the square but found that students were rushed to cover content and did not have time to develop an understanding of graphing, factoring, and the quadratic equation. When these problem-solving methods were supported and taught for understanding then I learned that my students can solve any quadratic equation.

Conclusion

Solving quadratic equations can be one of the most difficult topics for students to learn. These difficulties can both be procedural and relational, so it is important that teachers not only focus on the steps to solving the equations, but also explain the why. By shortening the number of methods used to solve quadratic equations I found that students gained a deeper understanding of the topics. This experience aligns with the famous saying of American Mathematics curriculum, *it is a mile wide and an inch deep*. Completing the square is a logical method for proving the quadratic formula, but requires advanced mathematics skills. Maybe this topic fits better in the Algebra II curriculum? I found that students choose to use the quadratic formula to solve non-factorable quadratic equations so they can get by without learning this skill in Algebra 1. The quadratic formula will be a formula they will either memorize or be given, but it will always work to solve a quadratic equation factorable or not. If students are going to be prepared for an end of year exam they need both the instrumental and relational understanding of any topic, by limiting the focus of the Algebra 1 solving quadratic equations curriculum students will succeed. Overall, I hope other teachers find success using this curriculum as I have.

References

Didis, M.G., & Erbas, A. K. (2015). Performance and Difficulties of Students in Formulating and Solving Quadratic Equations with One Unknown. *Educational Sciences: Theory & Practice, 15(4)*, 1137-1150.

JMAP BY TOPIC - JUMBLED. (n.d.). http://www.jmap.org/JMAP_RESOURCES_BY_TOPIC.htm#AI.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. *National Governors Association Center for Best Practices.*

Skemp, R. R. (2006). Relational understanding and instrumental Understanding. *Mathematics Teaching in the Middle School, 12(2),* 88-95.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for research in mathematics education*, *36*(5), 404-411.

Van Merriënboer, J. J., Kester, L., & Paas, F. (2006). Teaching complex rather than simple tasks: Balancing intrinsic and germane load to enhance transfer of learning. *Applied Cognitive Psychology: The Official Journal of the Society for Applied Research in Memory and Cognition*, *20*(3), 343-352.

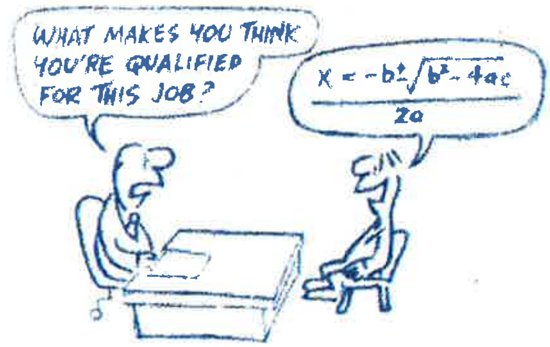
Wade, C. H., Sonnert, G., Sadler, P. M., & Hazari, Z. (2017). Instructional Experiences that Align with Conceptual Understanding in the Transition from High School Mathematics to College Calculus. *American Secondary Education, 45(2)*, 4–21.

Appendix

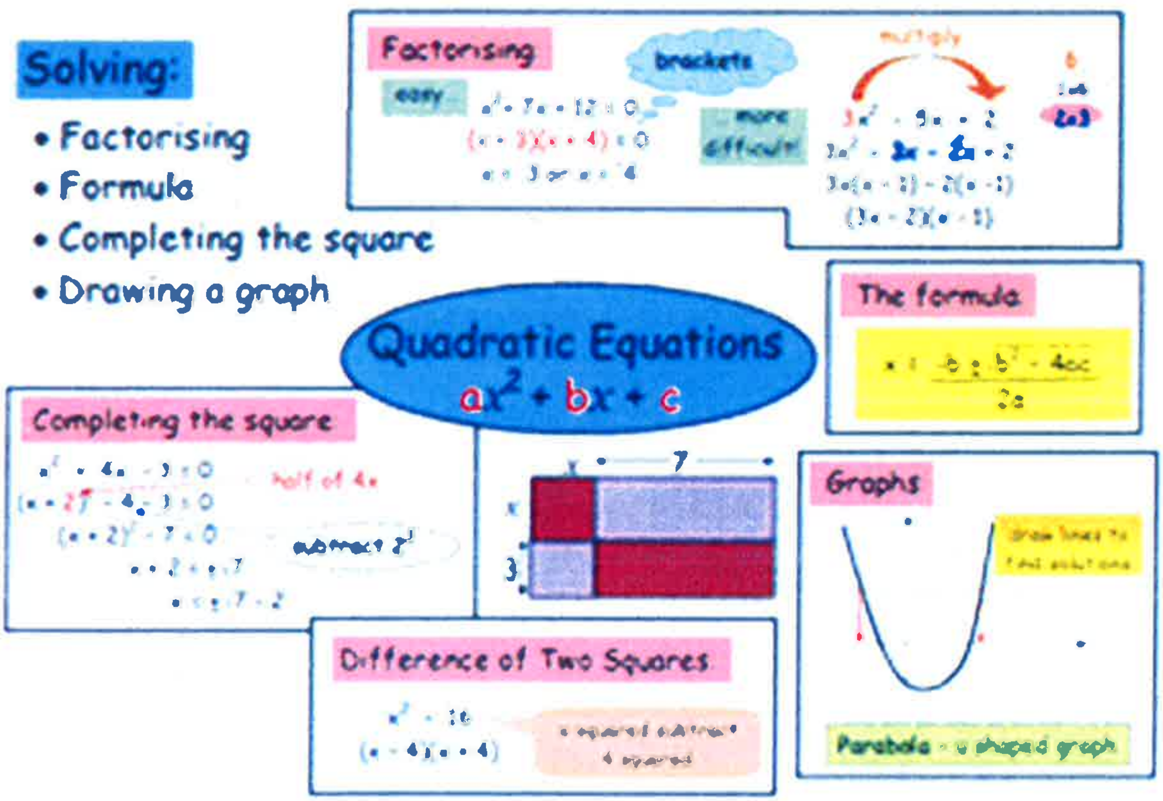
Name: Answer Key

Period: ____________

"Algebra class will be important to you later in life because there's going to be a test six weeks from now."



Unit: Solving Quadratic Equations



Name: ______________________ Date: ______________

Objective:

Students will be able to determine the zeros of quadratic equations graphically.

What are Zeros of Quadratic Equations? $\rightarrow x^2$

Zeros are:

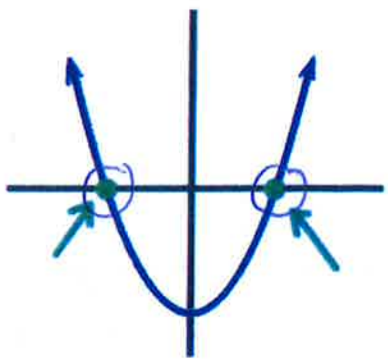
The __X__ value of a function when $y = 0$ or the __intersection__ of the function and the x- axis

Synonyms for Zeros Algebra:

__X-intercepts__, __solutions__, and __roots__.

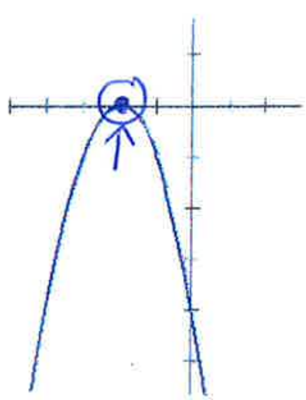
Identifying Zeros on Graphs and in Tables? *Will use calculator to compare Equations

__two__ zeros

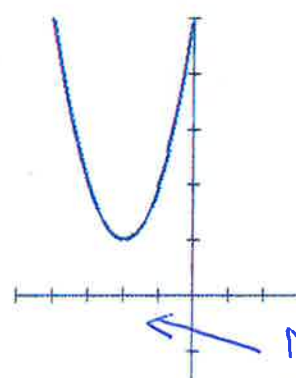


x	f(x) ← Y
-2	3
-1	0 ← y=0
0	-1
1	0 ← y=0
2	3
3	8

__One__ zero



__No__ zeros



Never intersects the x-axis

Class Example #1:

Given the graph of the Quadratic Equation below:

y=0

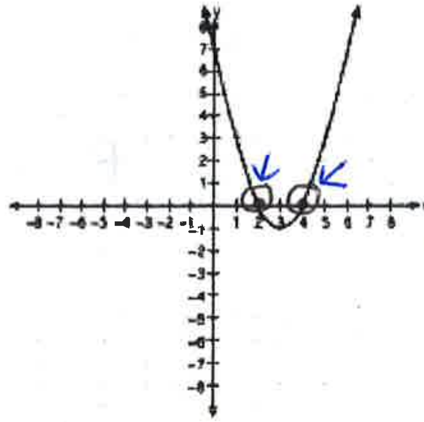
How many zeros does the function have?

2 zeros

y=0

What are the zeros of the function?

X=2 X=4



Class Example #2:

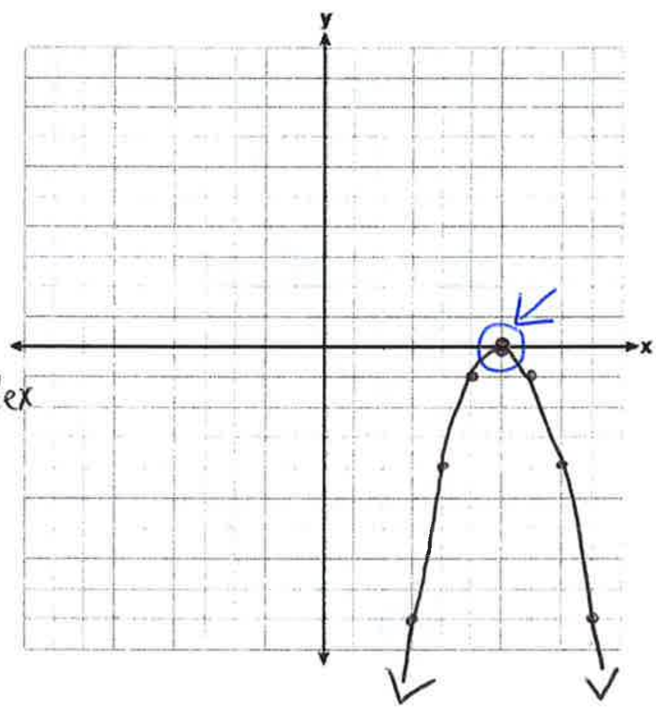
graph in calculator

Determine the vertex and root(s) of the function $g(x) = -x^2 + 12x\ 36$, graphically.

x	y
3	-9
4	-4
5	-1
6	0
7	-1
8	-4
9	-9

One Root

start table at Vertex

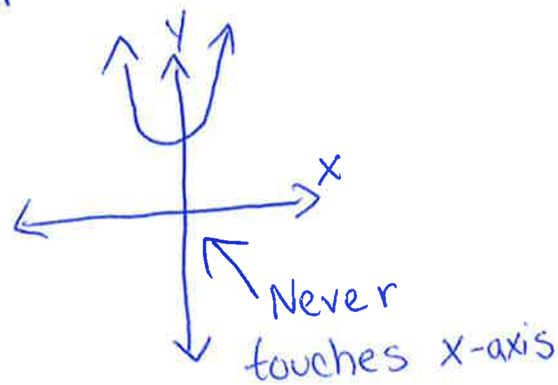


Root(s): X = 6 ______

Vertex: (6,0)

Partner Example #1:

How many real- number solutions does $4x^2 + 2x + 5 = 0$ have? ← graph in calculator



(1) One (3) Zero

(2) Two (4) Infinitely many

Partner Example #2:

What is the vertex and zeros of the parabola $y = x^2 - 16x + 63$? ← graph in calculator

(1) Vertex: $(8, -1)$; Zeros: $7, 9$

~~(3)~~ Vertex: $(8, 1)$; Zeros: $7, 9$

(2) Vertex: $(8, -1)$; Zeros: $-9, -7$

~~(4)~~ Vertex: $(8, 1)$; Zeros: $-9, -7$

x	y
6	3
7	0
8	-1
9	0
10	3

zeros

Vertex → (8, -1)

Independent Example #1:

Which function has zeros of -4 and 2? ← Graph in calculator

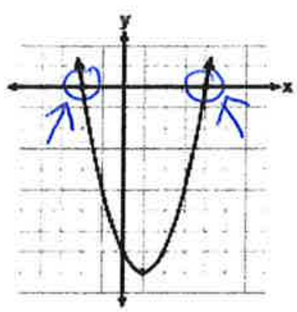
~~(1)~~ $f(x) = x^2 + 7x - 8$

$x = 1, -8$

(3) $g(x) = x^2 - 7x - 8$

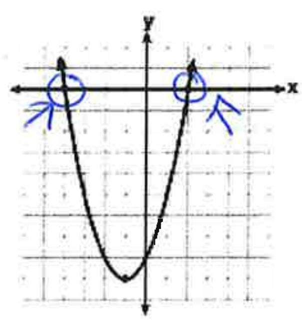
$x = -1, 8$

~~(2)~~



$x = -2, 4$

(4)

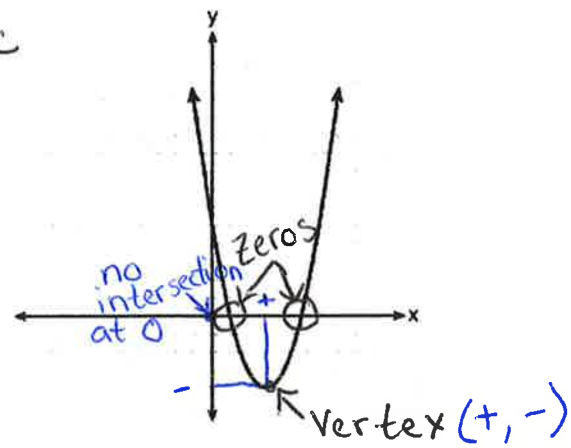


$x = -4, 2$

Independent Example #2:

The equation $y = (x - h)^2 + k$ is graphed on the set of axes below.

Quadratic



Based on the graph, what are the roots and vertex of the equation?

(1) 1 and 5; $(3, -4)$

(2) 1 and 0; $(3, -4)$

doesn't intersect at the origin

~~(3)~~ 1 and 5; $(-4, 3)$

~~(4)~~ 1 and 0; $(-4, 3)$

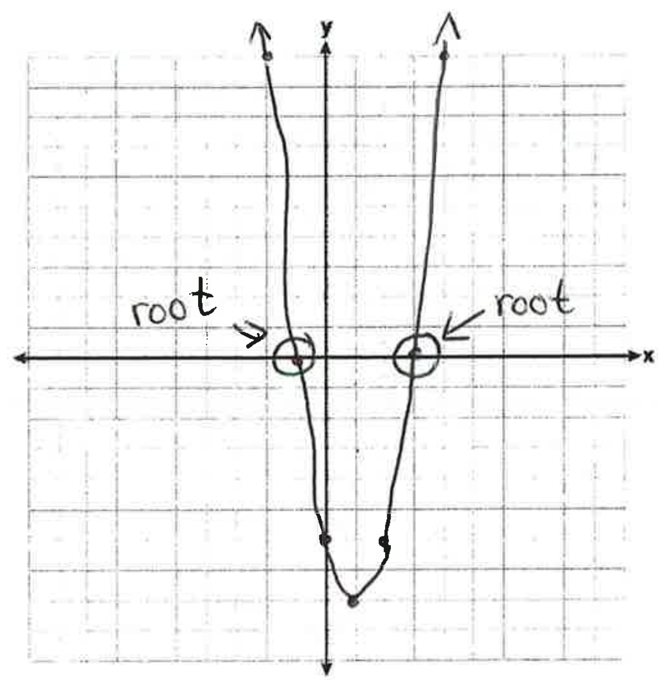
Independent Example #3:

graphed in calculator

On the set of axes below, graph $y = 2x^2 - 4x - 6$.

x	-2	-1	0	1	2	3	4
y	10	0	-6	-8	-6	0	10

zeros $(y=0)$



State the roots of the equation

$y=0 \rightarrow 0 = 2x^2 - 4x - 6.$

$x = -1, 3$

Independent Example #4:

In your own words describe the difference between one, two and no zeros for a quadratic function.

The difference in the number of zeros for a quadratic function depends on the number of intersection a function and the x-axis have. Therefore, one zero is one intersection, two zeros is two intersections and ~~two~~ no zeros are no intersections.

key point

Independent Example #5:

zeros (solve for zero)

Find the solutions graphically to quadratic equation below.

$$2x - 8 = 2x^2 + 6x - 3$$
$$-2x \qquad\qquad -2x$$
$$-8 = 2x^2 + 4x - 3$$
$$+8 \qquad\qquad +8$$
$$0 = 2x^2 + 4x + 5$$

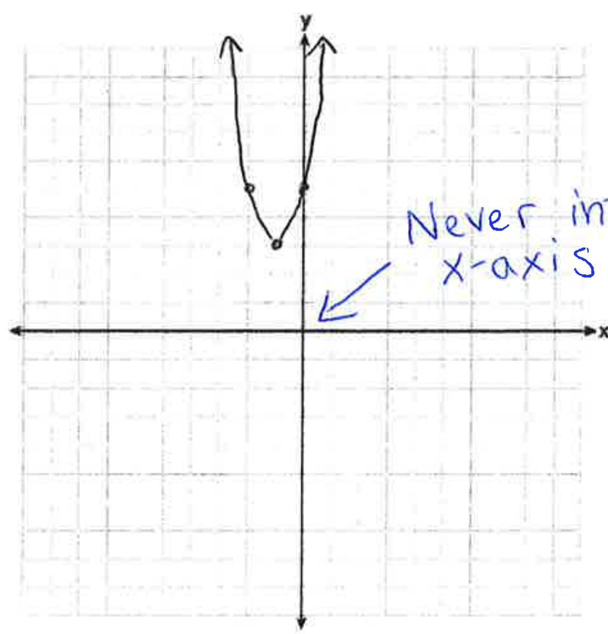
Struggle to Solve for zero:

$$\begin{cases} y_1 = 2x - 8 \\ y_2\ 2x^2 + 6x - 3 \end{cases}$$

no places the equations are equal

$\boxed{\text{no solutions}}$

x	-4	-3	-2	-1	0	1	2
y	21	11	5	3	5	11	21



Never intersects x-axis

Name: ______________________ Date: ______________

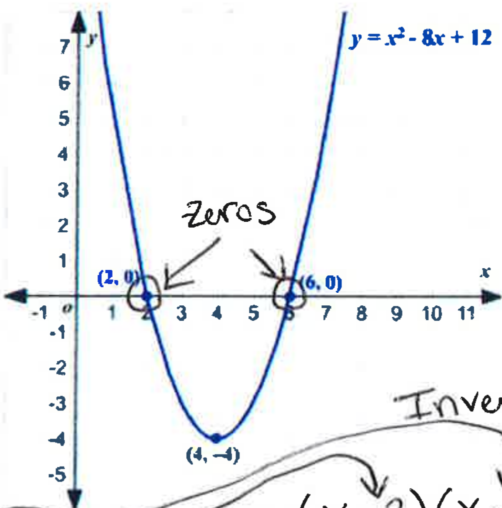
Objective:

Students will be able to determine the zeros of quadratic equations graphically and by factoring

Zeros Can Identify Factors:

Factored Form: $y = (x + \underline{r_1})(x + \underline{r_2})$

Zeros are the __X__ intercepts of a quadratic equation, which means find the __X__ value when $y = 0$.



So if $y = 0$, then...

$\underline{0} = (x + r_1)(x + r_2)$

↳ Only way to get 0 w/ multiplication is one of them equal to zero

Also,

$\underline{0} = x + \cancel{r_1}$ or $\underline{0} = x + \cancel{r_2}$

$-r_1 \quad -\cancel{r_1}$ $\qquad -r_2 \quad -\cancel{r_2}$

Therefore, $-r_1 = x$ $\qquad -r_2 = x$

r_1 and r_2 = __inverse__ of the zeros (x-intercepts)

Inverse

$(x-2)(x-6)=0$

$X = 2$

$X = 6$

$x - \cancel{2} = 0$ | $x - \cancel{6} = 0$
$+\cancel{2} \quad +2$ | $+\cancel{6} \quad +6$
$x = 2$ | $x = 6$

Factoring to Find Zeros:

1.) Rewrite the quadratic equation in __Standard__ form ($y = ax^2 + bx + c$).

2.) Identify the zeros __graphically__.

3.) __Factor__ the quadratic expression.

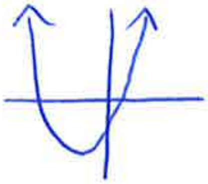
4.) Set each factor equal to __zero__.

5.) __Solve__ each equation for the solutions or zeros.

Class Example #1:

Solve the quadratic equation. $f(x) = x^2 + 4x - 5$

Step 1: Identify the zeros from the table in the calculator and box them in as the answers.



$X = 1$ $\quad$ $X = -5$

Step 2: Write the factored form of a quadratic equation, but replace r_1 and r_2 with the inverses of the zeros.

$$0 = (x-1)(x+5)$$

Step 3: Set each factor equal to zero and solve both for x.

$x - 1 = 0$
$+1 \quad +1$
$\boxed{x = 1}$

$x + 5 = 0$
$-5 \quad -5$
$\boxed{x = -5}$

Class Example #2:

Determine the zeros to the polynomial equation below using factoring.

$x^3 = 8x^2 + 20x$
$-8x^2 \quad -8x^2$
$x^3 - 8x^2 = 20x$
$-20x \quad -20x$
$x^3 - 8x^2 - 20x = 0$

$$\boxed{\begin{array}{l} x = -2 \\ x = 0 \\ x = 10 \end{array}}$$

$$(x+2)(x+0)(x-10) = 0$$

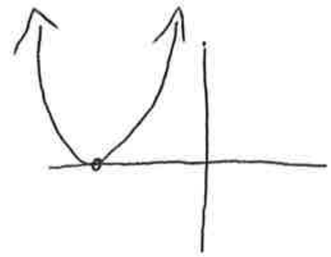
$x + 2 = 0$
$-2 \quad -2$
$x = -2$

$x + 0 = 0$
$-0 \quad -0$
$x = 0$

$x - 10 = 0$
$+10 \quad +10$
$x = 10$

Partner Example #1:

Which equation has the same solutions as $x^2 + 16x + 64 = 0$? ← Graph in calculator



(1)	$(x+4)(x-4)=0$	(3)	$(x-8)(x-8)=0$
(2)	$(x+8)(x-8)=0$	(4)	$(x+8)(x+8)=0$

$X=-8$

Inverse

Partner Example #2:

What are the solutions to the equation $(x-2)(x-a)=0$?

(1)	-2 and a	(3)	-2 and $-a$
(2)	2 and $-a$	(4)	2 and a

$$\begin{array}{rl} x - \cancel{2} & = 0 \\ + \cancel{2} & +2 \\ \hline x & = 2 \end{array} \qquad \begin{array}{rl} x - \cancel{a} & = 0 \\ + \cancel{a} & +a \\ \hline x & = a \end{array}$$

Independent Example #1:

Which polynomial function has zeros at $-3, 0,$ and 4?

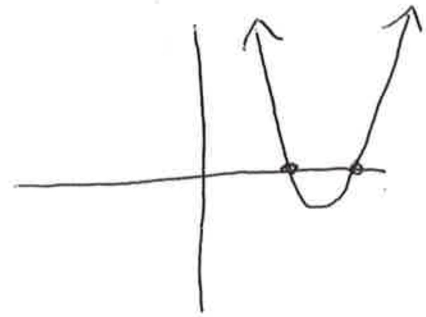
↳ $x=-3$ $x=0$ $x=4$) Inverse

(1)	$f(x)=(x+3)(x^2+4)$	(3)	$f(x)=x(x+3)(x-4)$
(2)	$f(x)=(x^2-3)(x-4)$	(4)	$f(x)=x(x-3)(x+4)$

Independent Example #2:

Solve the quadratic equation using the factoring method.

$$x^2 - 12x + 35 = 0$$



$$(x-5)(x-7)=0$$

Inverse

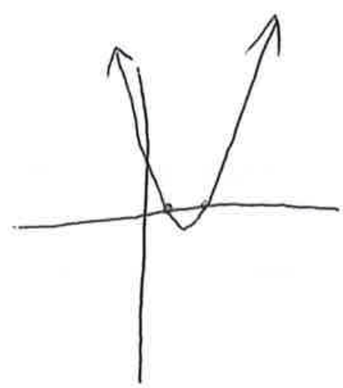
$$x-5=0 \qquad x-7=0$$
$$+5 \quad +5 \qquad +7 \quad +7$$
$$x=5 \qquad x=7$$

$$\boxed{x=5} \qquad \boxed{x=7}$$

Independent Example #3:

Determine the zeros to the quadratic equation using factoring.

$$x^2 - 3x + 2 = 0$$



$$(x-1)(x-2)=0$$

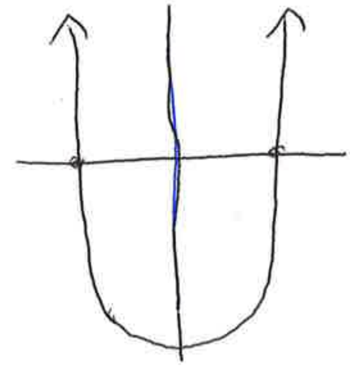
Inverse

$$x-1=0 \qquad x-2=0$$
$$+1 \quad +1 \qquad +2 \quad +2$$
$$x=1 \qquad x=2$$

$$\boxed{x=1} \qquad \boxed{x=2}$$

Independent Example #4:

Determine the zeros to the quadratic equation using factoring.



$$x^2 + 3x - 4 = 50$$

$$-50 \quad -50$$

$$x^2 + 3x - 54 = 0$$

Inverse

$$(x+9)(x-6) = 0$$

$$x + 9 = 0 \qquad x - 6 = 0$$

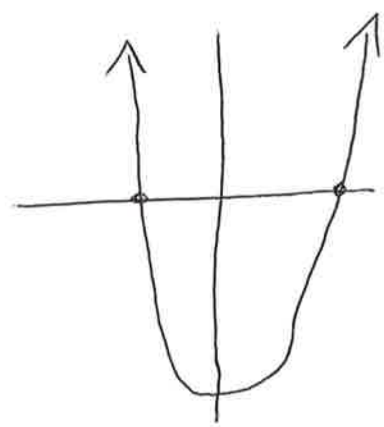
$$-9 \quad -9 \qquad +6 \quad +6$$

$$x = -9 \qquad x = 6$$

$$\boxed{x = -9} \qquad \boxed{x = 6}$$

Independent Example #5:

Determine the roots to the quadratic equation using factoring.



$$6x^2 - 7x - 60 = 4x^2 + 3x - 12$$

$$-4x^2 \quad -4x^2$$

$$2x^2 - 7x - 60 = 3x - 12$$

$$-3x \quad -3x$$

$$2x^2 - 10x - 60 = -12$$

$$+12 \quad +12$$

$$2x^2 - 10x - 48 = 0$$

$$(x+3)(x+8) = 0$$

$$x + 3 = 0 \qquad x + 8 = 0$$

$$-3 \quad -3 \qquad -8 \quad -8$$

$$x = -3 \qquad x = -8$$

$$\boxed{x = -3} \qquad \boxed{x = 8}$$

Name: ______________________ Date: ______________________

Objective:

Students will be able to determine solutions to scenario quadratic equations graphically and by factoring

Area:

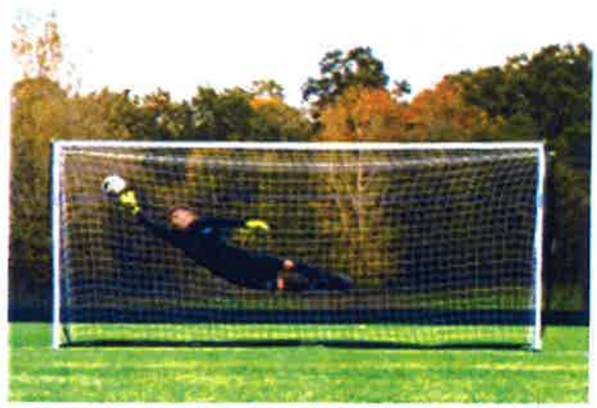
The Space occupied by a flat shape or the surface of an object.

Equation: $A = (L)(W)$

A: Area of the shape or object

L: Length of the shape or object $(x + r_1)$

W: Width of the shape or object $(x + r_2)$



Length and Width:

To find the length and width of any quadratic area problem make sure to know if the solved value of x needs to be Substituted back into an expression.

Class Example #1:

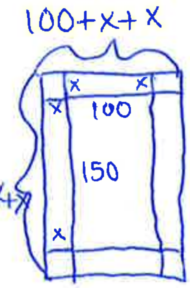
$A = (100)(150) = 15000$

A rec center basketball court measures 100 feet by 150 feet. The rec center's owner plans to increase the area of the court by 20%. He will do this by increasing the length and width by the same amount, x. Which equation represents the area of the new field?

$15000(20\%)$

$.20 + 1$

$\to A = (L)(W)$



(1) $(100 + 2x)(150 + 2x) = 18{,}000$

~~(3)~~ $(100 + x)(150 + x) = 18{,}000$

~~(2)~~ $2(100 + x) + 2(150 + x) = 15{,}000$

~~(4)~~ $(100 + x)(150 + x) = 15{,}000$

$15000(1.20) = 18000$ new area $\to A = (100+2x)(150+2x)$

Class Example #2:

The length of a rectangular sign is 6 inches more than half its width. The area of this sign is 432 square inches.

$+$ $\frac{1}{2}$

~~Draw and~~ label ~~a~~ the diagram of the desired ~~shape/~~ object.

W

$6 + \frac{1}{2}W$

Write an equation in one variable that could be used to find the number of inches in the dimensions of this sign.

$A = (L)(W)$

$\boxed{432 = (6 + \frac{1}{2}W)(W)}$

Solve this equation algebraically to determine the dimensions of this sign, in inches.

$432 = (6 + \frac{1}{2}W)(W)$

$432 = 6W + \frac{1}{2}W^2$

$-432 \qquad -432$

$0 = \frac{1}{2}W^2 + 6W - 432$

$(x - 24)(x + 36) = 0$

$x - 24 = 0$, $+24 \; +24$, $x = 24$

$x + 36 = 0$, $-36 \; -36$, $x = -36$

$\boxed{W = 24\text{in}}$

~~$W = -36$~~ Reject negative ~~length~~ width

$L = 6 + \frac{1}{2}W$

$L = 6 + \frac{1}{2}(24)$ $\boxed{L = 18\text{in}}$

Partner Example #1:

A=(L)(w)

A 4 inch by 6 inch photo is put into a picture frame with a border of constant width. If the area of the frame, including the picture, is 80 square inches. Which equation can be used to find the width of the border, w?

→ A=80

6+2x

4 in

4+2x

6 in

(1) ~~$2(4+w)+2(6+w)=80$~~

(2) ~~$(4+2w)+(6+2w)=80$~~

(3) $(4+w)(6+w)=80$

(4) $(4+2w)(6+2w)=80$

Partner Example #2:

Maria is building a rectangular dog pen for her new puppy. The width of the pen is 1 yard less than the length. If the area of the dog pen is 20 square yards, how many yards of fencing will she need for each length and width of the pen?

L - 1

→ A=20

(1) $l=15, w=5$

(2) $l=5, w=4$ ← logical choice

(3) $l=10, w=2$

(4) $l=10, w=10$

L

$\boxed{A=20}$ L-1

$A = L(w)$

$20 = L(L-1)$

$20 = L^2 - 1L$

$-20 \qquad -20$

$0 = L^2 - 1L - 20$

$0 = (L-5)(L+4)$

$0 = L-5$

$+5 \qquad +5$

$\boxed{5 = L}$

$L+4=0$

$-4 \qquad -4$

$\cancel{L=-4}$

Reject negative length

$W = L-1$

$W = 5-1$

$\boxed{W=4}$

Independent Example #1:

A=(L)(w)

The length of a rectangular patio is 7 feet more than its width, w. The area of a patio, $A(w)$, can be represented by the function

(1) $A(w)=w+7$

(2) $A(w)=4w+14$

(3) $A(w)=w^2+7w$

(4) $A(w)=4w^2+28w$

w

7+w

$A = (L)(w)$

$A = (7+w)(w)$

$A = \boxed{7w + w^2}$

Independent Example #2:

$\swarrow A = 6000$

A landscaper is creating a rectangular soccer field at the local park with an area of 6,000 square yards. The length of the soccer field must be 40 yards longer than its width. Determine algebraically the dimensions of the new soccer field, in yards.

$+$

Diagram:

$40+w$

w

Equation:

$A = (L)(w)$

$6000 = (40+w)(w)$

$6000 = 40w + w^2$

$-6000 \qquad -6000$

$0 = w^2 + 40w - 6000$

$0 = (w-60)(w+100)$

$0 = w - 60$

$+60 \qquad +60$

$\boxed{60\text{ yd} = w}$

$w + 100 = 0$

$-100 \qquad -100$

$w = \cancel{-100}$

Reject negative width

$L = 40 + w$

$L = 40 + 60$

$\boxed{L = 100\text{ yd}}$

Independent Example #3:

$\swarrow A = 1590$

The area of a rectangular television screen is 1590 square inches. The width of the screen is 23 inches longer than the height. Therefore, the quadratic equation that can be used to represent the area of the television screen is $h(h+23) = 1590$. Determine the dimensions of the television.

$+$

Diagram:

h

$23+h$

$W = 23 + h$

$W = 23 + 30$

$\boxed{W = 53\text{ in}}$

Equation:

$h(h+23) = 1590$

$h^2 + 23h = 1590$

$-1590 \qquad -1590$

$h^2 + 23h - 1590 = 0$

$(h-30)(h+53) = 0$

$h - 30 = 0$

$+30 \qquad +30$

$\boxed{h = 30\text{ in}}$

$h + 53 = 0$

$-53 \qquad -53$

$h = \cancel{-53}$

Reject negative height

Independent Example #4:

Tyrone wants to be the best goal tender in New York. He felt the best way to get started was to know the dimensions of the goal. He knows the area of the goal is 9,322 square inches and that the length of the goal post is 40 less than twice the height. What are the dimensions of the goal, so Tyrone can know how far he will need to jump and dive to save goals.

$-$ (than) $\times 2$

Diagram:

$2h-40$

h

Equation:

$A=(L)(W)$

$9322 = (2h-40)(h)$

$9322 = 2h^2-40h$

$-9322 \qquad -9322$

$0 = 2h^2-40h-9322$

$0 = (h-79)(h+59)$

$0 = h-79$

$+79 \qquad +79$

$79 = h$

$\boxed{h = 79\text{in}}$ $\cancel{h=-59}$

Reject negative height

$h+59 = 0$

$-59 \qquad -59$

$h = -59$

$L = 2h-40$

$L = 2(79)-40$

$\boxed{L = 118\text{in}}$

Name: ____________________ Date: ____________________

Objective:

Students will be able to use the quadratic formula to find solutions to quadratic functions.

Quadratic Formula:

A formula that gives the Solution to any general quadratic equation.

Standard Equation of a Quadratic:

$$ax^2 + bx + c = 0$$

Formula used to find quadratic solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using the Quadratic Formula:

Step 1: Make sure the quadratic equation is in standard form.

Step 2: Identify a, b, and c.

Step 3: Substitute a, b, and c into the quadratic formula.

Step 4: Simplify the formula.

$$\underset{a}{6}x^2 \underset{b}{-x} \underset{c}{-2} = 0$$ 1st

a = 6 b = -1 c = -2 2nd

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-2)}}{2(6)}$$ 3rd

$$x = \frac{-(-1) \pm \sqrt{1 + 48}}{12}$$

$$x = \frac{1 \pm \sqrt{49}}{12}$$ 4th

Class Example #1:

Write down the values of a, b, can c in each case, substitute the values into the quadratic formula and simplify.

2 Solutions

$2x^2 + 4x - 6 = 0$

a = 2

b = 4

c = -6

$$x = \frac{-\boxed{4} \pm \sqrt{\boxed{4}^2 - 4 \times \boxed{2} \times \boxed{-6}}}{2 \times \boxed{2}}$$

$$X = \frac{-4 \pm \sqrt{64}}{4}$$

$$X = \frac{-4+\sqrt{64}}{4} \qquad X = \frac{-4-\sqrt{64}}{4}$$

1 Solutions

$x^2 + 6x + 9 = 0$

a = 1

b = 6

c = 9

$$x = \frac{-\boxed{6} \pm \sqrt{\boxed{6}^2 - 4 \times \boxed{1} \times \boxed{9}}}{2 \times \boxed{1}}$$

$-6+0=-6-0$

$$X = \frac{-6 \pm \sqrt{0}}{2}$$

$$X = \frac{-6}{2}$$

No Solutions

$-2x^2 - 3 = 0$

a = -2

b = 0

c = -3

$$x = \frac{-\boxed{0} \pm \sqrt{\boxed{0}^2 - 4 \times \boxed{-2} \times \boxed{-3}}}{2 \times \boxed{-2}}$$

Reject $\sqrt{neg.}$

$$X = \frac{0 \pm \sqrt{-24}}{-4}$$

X = Error

Class Example #2:

If the quadratic formula is used to find the zeros of the equation $-2x^2 - 3x = -4$, determine the zeros.

$$-2x^2 - 3x = -4$$

$$+4 \qquad\qquad +4$$

$$-2x^2 - 3x + 4 = 0$$ standard form

a = -2 b = -3 c = 4

$$X = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(-2)(4)}}{2(-2)}$$

$$X = \frac{3 \pm \sqrt{41}}{-4}$$

$$\boxed{X = \frac{3+\sqrt{41}}{-4}} \qquad \boxed{X = \frac{3-\sqrt{41}}{-4}}$$

Partner Example #1:

The roots of the equation $2x^2 + 7x - 3 = 0$ are

(1) $-\frac{1}{2}$ and -3

(2) $\frac{1}{2}$ and 3

(3) $\frac{-7\pm\sqrt{73}}{4}$

(4) $\frac{7\pm\sqrt{73}}{4}$

$2x^2+7x-3=0$

$a=2 \quad b=7 \quad c=-3$

$x=\frac{-7\pm\sqrt{(7)^2-4(2)(-3)}}{2(2)}$

$x=\frac{-7\pm\sqrt{73}}{4}$

Partner Example #2:

Solve the quadratic $2x^2 + 11x = 2$ using the quadratic formula.

$2x^2+11x=2$

$-2 \qquad -2$

$2x^2+11x-2=0$

$a=2 \quad b=11 \quad c=-2$

$x=\frac{-11\pm\sqrt{(11)^2-4(2)(-2)}}{2(2)}$

$x=\frac{-11\pm\sqrt{137}}{4}$

$\boxed{\frac{-11+\sqrt{137}}{4}}$

$\boxed{\frac{-11-\sqrt{137}}{4}}$

Independent Example #1:

The roots of $x^2 - 5x - 4 = 0$ are

(1) 1 and 4

(2) $\frac{5\pm\sqrt{41}}{2}$

(3) -1 and -4

(4) $\frac{-5\pm\sqrt{41}}{2}$

$1x^2-5x-4=0$

$a=1 \quad b=-5 \quad c=-4$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-4)}}{2(1)}$

$x=\frac{5\pm\sqrt{41}}{2}$

Independent Example #2:

Find the roots of the quadratic equation $3x^2 - x + 2 = 0$.

$$3x^2 - 1x + 2 = 0$$

$a = 3 \quad b = -1 \quad c = 2$

$$\boxed{\text{no solutions}}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{-23}}{6}$$

← negative radical is an error

Independent Example #3:

Determine the solutions to the quadratic equation $6x^2 = 3x + 5$.

$$\begin{array}{r|l} 6x^2 = & 3x + 5 \\ -3x & -3x \\ \hline 6x^2 - 3x = & 5 \\ -5 & -5 \\ \hline 6x^2 - 3x - 5 = & 0 \end{array}$$

$a = 6 \quad b = -3 \quad c = -5$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(6)(-5)}}{2(6)}$$

$$x = \frac{3 \pm \sqrt{129}}{12}$$

$$\boxed{x = \frac{3 + \sqrt{129}}{12}} \qquad \boxed{x = \frac{3 - \sqrt{129}}{12}}$$

Independent Example #4:

Solve $-7x + x^2 = 4$ using the quadratic formula.

$$
\begin{array}{r|l}
-7x + 1x^2 = & 4 \\
-4 & -4 \\
\hline
1x^2 - 7x - 4 = & 0
\end{array}
$$

$a = 1 \quad b = -7 \quad c = -4$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{65}}{2}$$

$$\boxed{x = \frac{7 + \sqrt{65}}{2}} \qquad \boxed{x = \frac{7 - \sqrt{65}}{2}}$$

Independent Example #5:

Using the quadratic formula, solve the equation $3 = 2x^2 - 9x$ for x.

$$
\begin{array}{r|l}
3 = & 2x^2 - 9x \\
-3 & -3 \\
\hline
0 = & 2x^2 - 9x - 3
\end{array}
$$

$a = 2 \quad b = -9 \quad c = -3$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{9 \pm \sqrt{105}}{4}$$

$$\boxed{x = \frac{9 + \sqrt{105}}{4}} \qquad \boxed{x = \frac{9 - \sqrt{105}}{4}}$$

Name: ______________________________ Date: ______________________

Objective:

Students will be able to simplify quadratic solutions using the quadratic formula.

Simplifying Square Roots:

The square root of the product ab is equal to the product of each square root. ($\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$)

Examples:

$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

$\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$

$\sqrt{80} = \sqrt{8} \cdot \sqrt{10} = \sqrt{4}(\sqrt{2})(\sqrt{2})\sqrt{5} = 2 \cdot 2\sqrt{5}$ OR $4\sqrt{5}$

$\sqrt{12} = \sqrt{6} \cdot \sqrt{2} = \sqrt{3} \cdot (\sqrt{2})(\sqrt{2}) = 2\sqrt{3}$

$\sqrt{45} = \sqrt{5} \cdot \sqrt{9} = 3\sqrt{5}$

$\sqrt{32} = \sqrt{8} \cdot \sqrt{4} = (\sqrt{4})\sqrt{2}(\sqrt{4}) = 4\sqrt{2}$

Simplifying Quadratic Formula:

Step 1: Simplify the product of the square root.

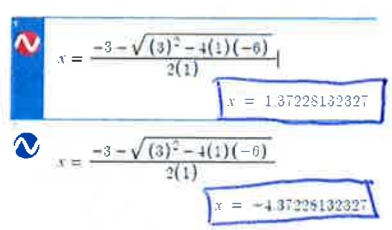
$$x = \frac{4 \pm \sqrt{72}}{2} = \frac{4 \pm \sqrt{36 \cdot 2}}{2} = \frac{4 \pm 6\sqrt{2}}{2}$$

Step 2: Reduce the fractions.

$$x_1 = \frac{4 + 6\sqrt{2}}{2} = 2 + 3\sqrt{2} \qquad x_2 = \frac{4 - 6\sqrt{2}}{2} = 2 - 3\sqrt{2}$$

Quadratic Formula Solutions as a Decimal:

Write both the addition and subtraction solutions into the calculator as fractions.



Class Example #1:

Solve the equation $6x^2 - 2x - 3 = 0$ and express the answer in simplest radical form.

$a=6 \quad b=-2 \quad c=-3$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{2 \pm \sqrt{76}}{12}$$

$$x = \frac{2 \pm \sqrt{19 \cdot 4}}{12}$$

$$x = \frac{2 \pm 2\sqrt{19}}{12}$$

$$x = \frac{2 + 2\sqrt{19}}{12} \qquad x = \frac{2 - 2\sqrt{19}}{12}$$

$$\boxed{x = \frac{1 + 1\sqrt{19}}{6}} \qquad \boxed{x = \frac{1 - 1\sqrt{19}}{6}}$$

Class Example #2:

Given: $g(x) = 2x^2 + 3x + 10$

$k(x) = 2x + 16$

$2(2x+16)$

Solve the quadratic equation $g(x) = 2k(x)$ algebraically for x, to the *nearest tenth*.

$$2x^2 + 3x + 10 = 2(2x+16)$$

$$2x^2 + 3x + 10 = 4x + 32$$

$$-4x \qquad -4x$$

$$2x^2 - 1x + 10 = 32$$

$$-32 \qquad -32$$

$$2x^2 - 1x - 22 = 0$$

$a=2 \quad b=-1 \quad c=-22$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-22)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{177}}{4}$$

$$x = \frac{1 + \sqrt{177}}{4} = 3.576\ldots \qquad x = \frac{1 - \sqrt{177}}{4} = -3.07\ldots$$

$$\boxed{x = 3.6} \qquad \boxed{x = -3.1}$$

Partner Example #1:

If the quadratic formula is used to find the roots of the equation $x^2 - 6x - 19 = 0$, the correct roots are

$a=1 \quad b=-6 \quad c=-19$

(1) $3 \pm 2\sqrt{7}$

(2) $-3 \pm 2\sqrt{7}$

(3) $3 \pm 4\sqrt{14}$

(4) $-3 \pm 4\sqrt{14}$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-19)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{112}}{2} = \frac{6 \pm \sqrt{16 \cdot 7}}{2}$$

$$x = \frac{6 \pm 4\sqrt{7}}{2} = 3 \pm 2\sqrt{7}$$

Partner Example #2:

Lena was asked to go to the board and solve the quadratic equation $2x^2 + 8x - 3 = 0$ using the quadratic formula. Her work is shown below.

$2x^2 + 8x - 3 = 0$ $\qquad a = 2 \qquad b = 8 \qquad c = -3$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-3)}}{2(2)} = \frac{-8 \pm \sqrt{64+24}}{4} = \frac{-8 \pm \sqrt{88}}{4} = -2 \pm \sqrt{22}$$

When she got back to her seat, the teacher stated that she made one error while simplifying. Identify the error Lena made and write the correct answer.

Lena divided √88 by 4 instead of Simplifying √88 first and only dividing by 4 after.

$$x = \frac{-8 \pm \sqrt{88}}{4} = \frac{-8 \pm \sqrt{8 \cdot 11}}{4} = \frac{-8 \pm \sqrt{2 \cdot 4 \cdot 11}}{4} = \frac{-8 \pm 2\sqrt{22}}{4}$$

$$= \boxed{-2 \pm \frac{1\sqrt{22}}{2}}$$

Independent Example #1:

Solve the quadratic equation $3x^2 - 4x - 6 = 0$ using the quadratic formula. Round solutions to the ***nearest hundredth***.

$a=3 \quad b=-4 \quad c=-6$

$$X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)}$$

(1) $x = 2.46, -3.79$ (3) $x = 3.79, -2.46$

(2) $x = 2.23, -0.90$ (4) $x = 0.90, -2.23$

$$X = \frac{4 \pm \sqrt{88}}{6}$$

$$X = \frac{4 \pm \sqrt{4 \cdot 22}}{6}$$

$$X = \frac{4 \pm 2\sqrt{22}}{6}$$

$$X = \frac{2}{3} \pm \frac{1}{3}\sqrt{22} \quad \text{OR} \quad \frac{2 + 1\sqrt{22}}{3} = 2.230\ldots$$

$$\frac{2 - 1\sqrt{22}}{3} = -0.896\ldots$$

Independent Example #2:

Solve the equation $2x^2 - 5 = 4x$ and express the answer in simplest radical form.

$$\begin{array}{rcl} 2x^2 - 5 & = & 4x \\ -4x & & -4x \\ \hline 2x^2 - 4x - 5 & = & 0 \end{array}$$

$a=2 \quad b=-4 \quad c=-5$

$$X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-5)}}{2(2)}$$

$$X = \frac{4 \pm \sqrt{56}}{4} = \frac{4 \pm \sqrt{8 \cdot 7}}{4} = \frac{4 \pm \sqrt{4 \cdot 2 \cdot 7}}{4}$$

$$X = \frac{4 \pm 2\sqrt{14}}{4}$$

$$\boxed{1 + \frac{1\sqrt{14}}{2}}$$

$$\boxed{1 - \frac{1\sqrt{14}}{2}}$$

Independent Example #3:

Solve for x to the *nearest tenth*: $x^2 + x - 5 = 0$

$1x^2 + 1x - 5 = 0$

$a=1 \quad b=1 \quad c=-5$

$$X = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-5)}}{2(1)}$$

$$X = \frac{-1 \pm \sqrt{21}}{2}$$

$$\frac{-1 + \sqrt{21}}{2} = 1.791\ldots \quad \boxed{X = 1.8}$$

$$\frac{-1 - \sqrt{21}}{2} = -2.791\ldots \quad \boxed{X = -2.8}$$

Independent Example #4:

Use the method of quadratic formula to determine the exact values of x for the equation $x^2 - 8x + 6 = 0$.

$1x^2 - 8x + 6 = 0$

$a = 1 \quad b = -8 \quad c = 6$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{40}}{2}$$

$$x = \frac{8 \pm \sqrt{10 \cdot 4}}{2}$$

$$x = \frac{8 \pm \sqrt{2 \cdot 5 \cdot 4}}{2}$$

$$x = \frac{8 \pm 2\sqrt{10}}{2} = 4 \pm 1\sqrt{10}$$

$\boxed{4 + \sqrt{10}}$

$\boxed{4 - \sqrt{10}}$

Independent Example #5:

Solve $6x^2 - 42 = 0$ for the exact values of x.

$6x^2 + 0x - 42 = 0$

$a = 6 \quad b = 0 \quad c = -42$

$$x = \frac{-0 \pm \sqrt{0^2 - 4(6)(-42)}}{2(6)}$$

$$x = \frac{0 \pm \sqrt{1008}}{12}$$

$$x = \frac{\pm\sqrt{16 \cdot 63}}{12}$$

$$x = \frac{+\sqrt{16 \cdot 9 \cdot 7}}{12} = \frac{4 \cdot 3\sqrt{7}}{12} = \frac{12\sqrt{7}}{12} = \boxed{\sqrt{7}}$$

$$x = \frac{-\sqrt{16 \cdot 9 \cdot 7}}{12} = \frac{-4 \cdot 3\sqrt{7}}{12} = \frac{-12\sqrt{7}}{12} = \boxed{-\sqrt{7}}$$

Name: ______________________ Date: ______________________

Objective:

Students will be able to determine zeros of quadratic projectile motion scenarios.

Projectile Motion:

The path objects take when they are launched, and travel either up or down.

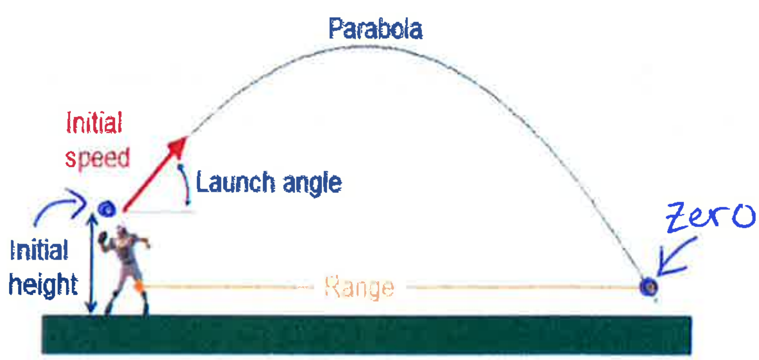
Equation: $y = ax^2 + bx + c$

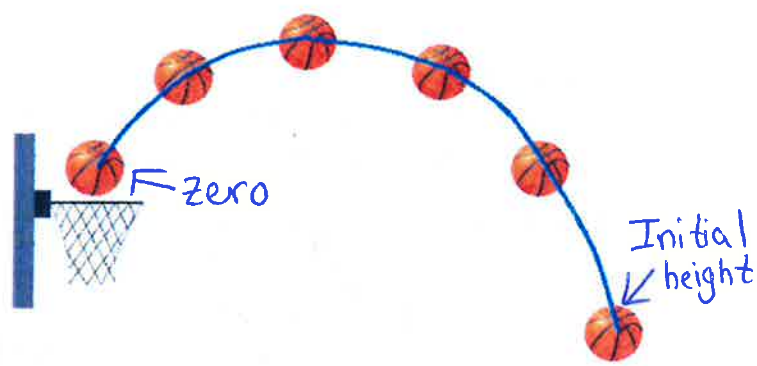
a: Force of gravity on the object (represented by either -16 or -4.9)

feet meters

b: Velocity at which the object travels

c: The initial height of the object (y-intercept)





Zeros in Projectile Motion:

The **Zeros** represent the time it takes for an object to hit the ground (height of zero).

Class Example #1:

The height of a rocket launched from the ground is given by the function $h(x) = -16x^2 + 200x$, where $h(x)$ is the height, in feet, after x seconds.

What is the initial height of the rocket (c- value in the quadratic equation)?

$\boxed{0 \text{ feet}}$

$h(x) = -16x^2 + 200x + 0$

$a = -16 \quad b = 200 \quad c = 0$

When will the rocket reach the ground?

$\hookrightarrow h(x) = 0$

$a = -16 \quad b = 200 \quad c = 0$

$$x = \frac{-200 \pm \sqrt{(200)^2 - 4(-16)(0)}}{2(-16)}$$

$$x = \frac{-200 \pm \sqrt{40000}}{-32}$$

$$x = \frac{-200 \pm 200}{-32}$$

$$\frac{-200+200}{-32} = 0 \quad \text{Reject starting time}$$

$$\frac{-200-200}{-32} = 12.5$$

OR

$0 = -16x^2 + 200x + 0$

$0 = (x+0)(x-12.5)$

$0 = x + \cancel{0} \qquad x - 12.5 = 0$

$-0 \quad \cancel{-0} \qquad +12.5 \quad +12.5$

$0 = x \qquad x = 12.5$

$\boxed{12.5 \text{ seconds}}$

Class Example #2:

Justin Tucker needs to kick a 45-yard (135 feet) field goal for the Ravens to beat the Stealers. The height of the kicked football, $f(x)$, in feet, can be modeled by the equation $f(x) = -0.01x^2 + 1.28x$, where x is the ball's horizontal distance, in feet, from the ground.

How far will the football travel in feet? Does Justin Tucker win the Ravens the game?

$f(x) = 0$

$0 = -0.01x^2 + 1.28x + 0$

$0 = (x+0)(x-128)$

$0 = x + \cancel{0} \qquad x - 128 = 0$

$-0 \quad \cancel{-0} \qquad +128 \quad +128$

$0 = x \qquad x = 128$

reject starting time

OR

$a = -0.01 \quad b = 1.28 \quad c = 0$

$$x = \frac{-1.28 \pm \sqrt{(1.28)^2 - 4(-0.01)(0)}}{2(-0.01)}$$

$$x = \frac{-1.28 \pm \sqrt{1.6384}}{-0.02}$$

$$= \frac{-1.28 + \sqrt{1.6384}}{-0.02} = 0 \quad \text{Reject starting time}$$

$$= \frac{-1.28 - \sqrt{1.6384}}{-0.02} = 128$$

128 feet and No, Justin will not make the field goal; short 7 feet.

Partner Example #1:

A ball is thrown into the air from the top of a building. The height, $h(t)$, of the ball above the ground t seconds after it is thrown can be modeled by $h(t) = -16t^2 + 64t + 80$. How many seconds after being thrown will the ball hit the ground?

↳ $h(t) = 0$

(1) 5 seconds (3) 80 seconds

(2) 2 seconds (4) 144 seconds

$0 = -16t^2 + 64t + 80$

$0 = (x-5)(x+1)$

$0 = x - 5$ $x + 1 = 0$

$+5 \quad +5$ $-1 \quad -1$

$5 = x$ $x = -1$

Reject negative time

$a = -16 \quad b = 64 \quad c = 80$

$$x = \frac{-64 \pm \sqrt{(64)^2 - 4(-16)(80)}}{2(-16)}$$

OR

$$x = \frac{-64 \pm \sqrt{9216}}{-32}$$

$$x = \frac{-64 + \sqrt{9216}}{-32} = -1 \quad \text{Reject}$$

$$x = \frac{-64 - \sqrt{9216}}{-32} = 5$$

Partner Example #2:

Omar shoots a basketball at basketball practice. The equation that models the path of the ball is $p(t) = -16t^2 + 20t + 6$, where $p(t)$ is the height of the ball in feet and t is the number of seconds after the ball was shot. If the basketball hoop is 10 feet tall, how long is the ball in the air before the ball goes in the hoop?

↳ $p(t) = 10$

$10 = -16t^2 + 34t + 6$

$-10 \qquad\qquad -10$

$0 = -16t^2 + 34t - 4$

$a = -16 \quad b = 34 \quad c = -4$

$$x = \frac{-34 \pm \sqrt{(34)^2 - 4(-16)(-4)}}{2(-16)}$$

$$x = \frac{-34 \pm \sqrt{900}}{-32}$$

$$\frac{-34 + \sqrt{900}}{-32} = .125 \quad \text{Reject initial time}$$

$$\frac{-34 - \sqrt{900}}{-32} = 2$$

$\boxed{\text{2 seconds}}$

Independent Example #1:

Lebron threw a basketball from half court toward the hoop. The quadratic function that models the height, in feet, of the ball after t seconds is $h(t) = -16t^2 + 12t + 6$. If the hoop is 8 feet high, how long is the ball in the air before going into the hoop.

$h(t) = 8$

(1) $\frac{1}{2}$ second

(2) 1 second

(3) 2 seconds

(4) $2\frac{1}{2}$ seconds

$$8 = -16t^2 + 12t + 6$$
$$-8 \qquad\qquad -8$$
$$0 = -16t^2 + 12t - 2$$

$a = -16 \quad b = 12 \quad c = -2$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(-16)(-2)}}{2(-16)}$$

$$x = \frac{-12 \pm \sqrt{16}}{-32}$$

$$x = \frac{-12 + \sqrt{16}}{-32} = \cancel{\frac{1}{4}}$$ Reject Initial time

$$x = \frac{-12 - \sqrt{16}}{-32} = \frac{1}{2}$$

Independent Example #2:

A cliff diver on a Caribbean island jumps from a height of 105 feet, with an initial upward velocity of 5 feet per second. An equation that models the height, h(t), above the water, in feet, of the diver in time elapsed, t, in seconds, is $h(t) = -16t^2 + 5t + 105$. How many seconds, to the ***nearest hundredth***, does it take the diver to fall 45 feet below his starting point?

$\hookrightarrow h(t) = 105 - 45 = 60$

(1) 1.45

(2) 1.84

(3) 2.10

(4) 2.72

$$60 = -16t^2 + 5t + 105$$
$$-60 \qquad\qquad -60$$
$$0 = -16t^2 + 5t + 45$$

$a = -16 \quad b = 5 \quad c = 45$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(-16)(45)}}{2(-16)}$$

$$x = \frac{-5 \pm \sqrt{2905}}{-32}$$

$$x = \frac{-5 + \sqrt{2905}}{-32} = \cancel{-1.5280\ldots}$$ Reject negative time

$$x = \frac{-5 - \sqrt{2905}}{-32} = 1.8405\ldots$$

Independent Example #3:

Dasani pulled the plug in her bathtub and it started to drain. The amount of water in the bathtub as it drains is represented by the equation $L = -5t^2 - 8t + 120$, where L represents the number of liters of water in the bathtub and ~~L represents the number of liters of water in the bathtub and~~ t represents the amount of time, in minutes, since the plug was pulled.

How many liters of water were in the bathtub when Dasani pulled the plug?

$\hookrightarrow X=0$

$\boxed{\text{120 Liters of Water}}$

Determine, to the ***nearest tenth*** of a minute, the amount of time it takes for all the water in the bathtub to drain.

$\hookrightarrow L=0$

$0 = -5t^2 - 8t + 120$

$a=-5 \quad b=-8 \quad c=120$

$X = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(-5)(120)}}{2(-5)}$

$X = \dfrac{8 \pm \sqrt{2464}}{-10}$

$X = \dfrac{8 + \sqrt{2464}}{-10} = \cancel{-5.8}$

Reject negative time

$X = \dfrac{8 - \sqrt{2464}}{-10} = 4.2$

$\boxed{t = 4.2 \text{ mins.}}$

Independent Example #4:

Tyrone is a test pilot for the United States Air Force. He is cruising at an altitude of 9,000 feet, when he is forced to eject from the aircraft. The function $h(t) = -16t^2 + 128t + 9000$ models the height, in feet, of the pilot above the ground, where t is the time, in seconds, after he is ejected.

Determine and state the zero of the function, rounded to the nearest second.

$\hookrightarrow h(t)=0$

$0 = -16t^2 + 128t + 9000$

$a=-16 \quad b=128 \quad c=9000$

$X = \dfrac{-128 \pm \sqrt{(128)^2 - 4(-16)(9000)}}{2(-16)}$

$X = \dfrac{-128 \pm \sqrt{592384}}{-32}$

$X = \dfrac{-128 + \sqrt{592384}}{-32} = \cancel{-20.05}$

Reject negative time

$X = \dfrac{-128 - \sqrt{592384}}{-32} = 28.05$

$\boxed{t = 28 \text{ secs.}}$

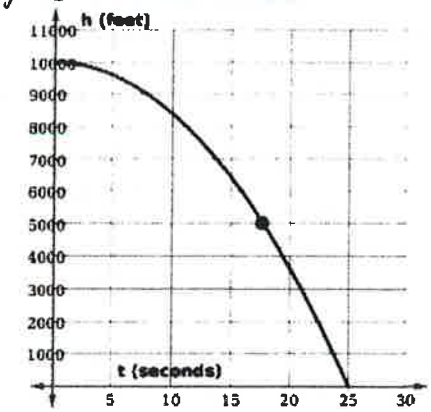
Explain what the zero represents in the context of the problem.

28 seconds represents the amount of time it took Tyrone to reach the ground.

Independent Example #5:

You jump out of an airplane. The graph shows the function $h(t) = -16t^2 + 10{,}000$ that models the height $h(t)$ you are above ground after t seconds. If you release your parachute at 5,000 feet, after how many seconds *rounded to the nearest hundredth* do you do this?

$h(t) = 5000$



$$\cancel{5000} = -16t^2 + 10{,}000$$
$$\cancel{-5000} \qquad -5000$$

$$0 = -16t^2 + 5000$$
$$0 = -16t^2 + 0t + 5000$$

$a = -16 \quad b = 0 \quad c = 5000$

$$t = \frac{-0 \pm \sqrt{(0)^2 - 4(-16)(5000)}}{2(-16)}$$

$$t = \frac{\pm\sqrt{320000}}{-32}$$

$$\frac{+\sqrt{320000}}{-32} = \cancel{-17.6776...}$$

Reject negative time

$$\frac{-\sqrt{320000}}{-32} = 17.6776...$$

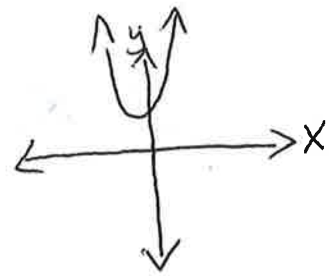
$$\boxed{t = 17.68 \text{ seconds}}$$

Name: ______________________________ Date: ______________

Quadratics Equations Unit Test

Answer each multiple-choice question. Each question is worth 2 points and partial credit will go to students that showed correct work.

1.) How many real- number solutions does $4x^2 + 2x + 5 = 0$ have?



(1) One

(2) Two

(3) Zero

(4) Infinitely Many

OR $a=4 \quad b=2 \quad c=5$

$$X = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(5)}}{2(4)} = \text{Error}$$

2.) Solve the equation $x^2 - 64 = 0$ for x.

table: $X = 8, -8$

(1) 8 and −8

(2) −8

(3) 8

(4) 16 and −4

OR $a=1 \quad b=0 \quad c=-64$

$$X = \frac{-0 \pm \sqrt{0^2 - 4(1)(-64)}}{2(1)}$$

$$X = \frac{\pm\sqrt{256}}{2}$$

$\frac{\sqrt{256}}{2} = 8$

$\frac{-\sqrt{256}}{2} = -8$

3.) The zeros of the function $p(x) = x^2 - 2x - 24$ are

$0 = x^2 - 2x - 24$ table: $X = -4, 6$

(1) −8 and 3

(2) −6 and 4

(3) −4 and 6

(4) −3 and 8

OR

$a=1 \quad b=-2 \quad c=-24$

$$X = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-24)}}{2(1)}$$

$$X = \frac{2 \pm \sqrt{100}}{2}$$

$\frac{2+\sqrt{100}}{2} = 6$

$\frac{2-\sqrt{100}}{2} = -4$

4.) Which function has zeros at -4 and 2?

~~(1)~~ $f(x) = x^2 + 7x - 8$ $X=1$ $X=-8$

~~(3)~~ $g(x) = x^2 - 7x - 8$ $X=-1$ $X=8$

~~(2)~~ $h(x) = (x+2)(x-4)$

$0 = (x+2)\ (x-4)$

$0 = x+2$ $x-4 = 0$

$-2 \quad -2$ $+4 \quad +4$

$-2 = x$ ✗ $x = 4$ ✗

(4) $j(x) = (x+4)(x-2)$

$0 = (x+4)\ (x-2)$

$0 = x+4$ $x-2 = 0$

$-4 \quad -4$ $+2 \quad +2$

$-4 = x$ ✓ $x = 2$ ✓

5.) The solution to the quadratic equation $2x^2 + 5x - 1 = 0$ is

(1) $\frac{5 \pm \sqrt{17}}{4}$

(3) $\frac{5 \pm \sqrt{33}}{4}$

(2) $\frac{-5 \pm \sqrt{17}}{4}$

(4) $\frac{-5 \pm \sqrt{33}}{4}$

$a=2 \quad b=5 \quad c=-1$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{33}}{4}$$

6.) What are the solutions to the equation $x^2 - 8x = 10$?

$x^2 - 8x = 10$

$-10 \qquad -10$

$x^2 - 8x - 10 = 0$

(1) $4 \pm \sqrt{10}$

(3) $-4 \pm \sqrt{10}$

(2) $4 \pm \sqrt{26}$

(4) $-4 \pm \sqrt{26}$

$x^2 - 8x - 10 = 0$

$a=1 \quad b=-8 \quad c=-10$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{104}}{2}$$

$\sqrt{104}$

$\sqrt{4}$ $\sqrt{26}$

$\sqrt{2}$ $\sqrt{13}$

$$x = \frac{8 \pm 2\sqrt{26}}{2}$$

$$x = \frac{8}{2} \pm \frac{2\sqrt{26}}{2} = 4 \pm \sqrt{26}$$

7.) A toy rocket is launched upward from the ground with an initial velocity of 128 feet per second and is modeled by the equation $h(t) = -16t^2 + 128t$, where h is the height in feet and t is the time in seconds. How long will it take for the rocket to hit the ground?

t $\quad$ $y=0$

(1) 10 seconds

(2) 6 seconds

(3) 4 seconds

(4) 8 seconds

$0 = -16t^2 + 128t$

$0 = (t+0)(t-8)$

$0 = t + \cancel{0}$ $\quad$ $t - 8 = 0$

$-0 \quad -\cancel{0}$ $\quad$ $+8 \quad +8$

$0 = t$ $\quad$ $t = 8$

$h(t) = -16t^2 + 128t + 0$

OR

$a = -16 \quad b = 128 \quad c = 0$

$$X = \frac{-128 \pm \sqrt{(128)^2 - 4(-16)(0)}}{2(-16)}$$

$$X = \frac{-128 \pm \sqrt{16384}}{-32}$$

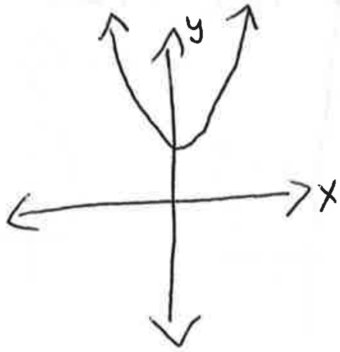
$$\frac{-128 + \sqrt{16384}}{-32} = 0$$

$$\frac{-128 - \sqrt{16384}}{-32} = 8$$

Answer each constructed response question. Each question is worth 2 points and partial credit will go to students that showed correct work.

looking for 2 or less zeros

8.) How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.



The equation $x^2 - 2x + 5 = 0$ has no real solutions because at no point is the graph of the function have $y = 0$.

OR No Real solutions because $x = \frac{-(2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \text{Error}$

Answer the constructed response question. This question is worth 4 points and partial credit will go to students that showed correct work.

looking for 3 or less zeros

9.) Determine algebraically the zeros of $f(x) = 3x^3 + 21x^2 + 36x$.

$0 = 3x^3 + 21x^2 + 36x$

$0 = (x+0)(x+4)(x+3)$

Factors found in table using calculator.

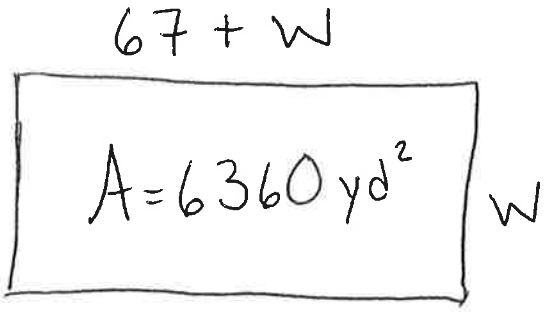
$0 = x + \cancel{0}$ $x + 4 = 0$ $x + \cancel{3} = 0$

$-0 \quad \cancel{-0}$ $-4 \quad -4$ $\cancel{-3} \quad -3$

$0 = x$ $x = -4$ $x = -3$

$\boxed{x = 0,\ x = -4,\ x = -3}$

10.) A school is building a rectangular football field that has an area of 6,360 square yards. The football field must be 67 yards longer than its width. Determine algebraically the dimensions of the football field, in yards.



$$A = L \cdot W$$

$$6360 = (67 + w) \cdot (w)$$

$$6360 = 67w + w^2$$

$$\cancel{6360} - \cancel{6360} \qquad -6360$$

$$0 = 67w + w^2 - 6360$$

$$0 = w^2 + 67w - 6360$$

$$0 = (w - 53)(w + 120)$$

Factors found in table using calculator

$$0 = w - \cancel{53} \qquad w + \cancel{120} = 0$$

$$+53 \quad +\cancel{53} \qquad -\cancel{120} \quad -120$$

$$\boxed{53 = w} \qquad \cancel{w = -120}$$

reject neg.

$$L = 67 + 53 = \boxed{120}$$