Movement in the Mathematics Classroom

With a Focus on the Real Number System Unit in Algebra 1

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Abstract

This curriculum project was designed to increase student engagement and achievement by including movement in mathematics classrooms. This curriculum project presents six sequential lessons for the Real Number System unit in Algebra 1. Each of the six lessons includes at least one movement activity that can also be used for other topics outside the Real Number System. All these lessons are aligned with the Next Generation Standards for Algebra 1 used in New York State.

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Introduction

The traditional way of learning in a teacher centered classroom with some student interactions is often not the most effective for students to reach their full potential. Sitting for long periods of times can cause students to become bored and distracted in class, especially when the content is one that they struggle in, which is typical for mathematics, or if it is content that they simply do not like. One way mathematics teachers can support learning and increase interest is to integrate student movement into instruction. Movement in the classroom includes activities that allows students to practice mathematical skills while moving around the classroom. This often occurs in groupwork which also supports students sharing their own ideas, listening to others thinking, and can strengthen reasoning skills and mathematical understandings (Kilic et al., 2010). While this is beneficial for student learning, it can be intimidating for some mathematics teachers. Thus, this curriculum project provides a unit of material that includes movement in each lesson for the Real Number System unit in Algebra 1. The lessons are aligned with the Next Generation Standards used in New York State (link). Each lesson is designed to increase student engagement and maximize student learning. After using the lessons included in this curriculum project, teachers will have a small collection of various movement activities. The goal is to increase learning, and based on the authors' experiences, movement in the classroom will most likely become a regular part of classroom pedagogical strategies because of how they can engage students and support learning.

Literature Review

Movement and its Benefit

The World Health Organization (WHO) has recommended that children should receive 60 minutes of moderate to vigorous physical activity daily (WHO, 2010; McMullen, et al., 2019). Hannaford (2005) suggested that movement is important for children of all ages because it engages the mind and helps establish "new information and experience into our neural networks." (p.82). Engagement is a buzzword heard daily in schools. Teachers are constantly looking for ways to keep students focused and to include students as active learners in each lesson. Clearly, movement is a great way to do that.

There are multiple ways to include movement into lessons. Beaudoin and Johnston (2011) split types of movement in the classroom into two broad categories: unrelated movement and purposeful movement. Unrelated movement can keep students active through different fidgets, like a stress ball, while purposeful movement is physical activity that pertains to the instruction of the lesson (Beaudoin and Johnston, 2011). Purposeful movement was also found to increase student outcomes and attitudes (Beaudoin and Johnston, 2011). Benes et al. (2016) state that teachers have anecdotal evidence for using movement in the classroom that may not be connected to research in the field (Benes et al., 2016). They also state that movement in the classroom can be associated with supporting vocabulary and being engaged in and gaining a greater understanding of the content (Benes et al., 2016). For example, a teacher may have students spin for the word "rotation" in a Geometry class, which can in turn, aid in retention. Teachers also seek provide enjoyable lessons for their students." (McMullen, et al., 2019).

Informal Assessments

Integrating movement into the classroom can be beneficial to most lessons and it can also serve as a form of informal assessment for teachers. It is important that throughout instruction, teachers establish breaks to check in with student learning (Scammel, 2016). From there, teachers can adjust their instruction based on the needs of their students. Assessing students while they complete a movement activity allows teachers to observe how students are doing with the content without students even knowing they are being informally assessed (Eckhoff and Linder, 2020). Through such assessments teachers can appropriately plan future activities based on students' learning needs (Eckhoff and Linder, 2020).

One of the biggest takeaways of informal assessments is the feedback that teachers give to students. Beesley (2008) explained that "Feedback is an active part of the process and can address the task, the student's processing of the task, suggestions for what to work on next, and scaffolds for the individual student" (p.5). The feedback tells the teacher where to go from there, but it also can give students an idea of what they need to work on and what they have mastered. An example of a combination of movement and an informal

assessment is stations created where students move to and participate in learning tasks while teachers provide one-on-one feedback (Beesley, et al., 2016).

Dialogue in the Classroom

A study was done with preservice math teachers about teaching future teachers how to use movement in the classroom. One teacher shared that when the students were given the opportunity to move around the classroom that there was more content focused discussion among them (Romar et al., 2020). Likewise, Romar et al. (2020) discussed how movement improved the social climate of a classroom by students feeling safer in the classroom to share their ideas. Helgesson (2011) shared that when teachers included movement into their lessons, they were able to diminish anxieties and stress many students feel on a daily basis. This also allowed teachers to maintain a learning focus by avoiding disruptive behaviors. Kamppi et al. (2013) shared those teachers who increased physical activity throughout the school day found that it made the school environment more peaceful and enjoyable. Thus, adding movement to lessons has the potential to help teachers create a positive learning environment with fewer distractions and more academically successful students (Helgesson, 2011, p.84). As students continue in their education in the building, movement will hopefully continue to be a foundation of their learning and success. If students expect movement each day, then they will be more open to it and be an active learner in each lesson.

Curriculum

The following lessons are intended to be used in the Real Number System Unit in Algebra 1. The focus of the lessons is to incorporate more movement into the classroom. As more movement is introduced and implemented in the classroom, students will hopefully feel more comfortable in the classroom and will be more likely to actively participate in each lesson. Throughout this series of lessons, many movement activities are used that are not specific to the Real Number System, which means a teacher could use the same type of activity for a different content topic. Each lesson is outlined first, then the necessary materials and documents are attached. These lessons are aligned with the New York State Next Generation Standards for Algebra 1. The answer keys for each lesson are included in the appendix section at the end of the document.

Lesson Outline:

- Lesson 1: Classifying Real Numbers
- Lesson 2: Simplifying Radicals
- Lesson 3: Adding and Subtracting Radicals
- Lesson 4: Multiplying Radicals
- Lesson 5: Rationalizing the Denominator
- Lesson 6: Review

Real Number System Lesson 1 Topic: Classifying Real Numbers

The focus of this lesson to introduce students to the different types of real numbers. In addition, this lesson will be the first introduction into movement (if movement has not been a priority in other units prior to this). It is important for students to feel comfortable and for them to be able to see how movement can help them be more successful. The Do It Now (DIN) activity will help assess prior knowledge of where numbers fall on the number line.

If hybrid learning is a part of a school's current plan, this lesson could be done remotely, but the movement pieces would be more challenging to included. Some remote instructional ideas would be to create a Jamboard with numbers and assign students a "post it" on Jamboard with a number. Then have students place their numbers on the number line on Jamboard. The notes for this lesson could either be a video that students could watch, or it can be done live with screen sharing. The last activity can also be done on Jamboard. An idea for this would be to have a Jamboard slide for each type of number. Then have students write a number on a post it for each slide of the Jamboard. A class discussion can then be had about the numbers posted on each slide.

For flipped learning, students could watch a video of the notes, then this would allow for more time for the Do It Now (DIN) activities.

Standard(s):
AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers.
a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots
b.) Categorize the sum or product of rational or irrational numbers.
• The sum and product of two rational numbers is rational.
• The sum of a rational number and an irrational number is irrational.
• The product of a nonzero rational number and an irrational number is irrational.
• The sum and product of two irrational numbers could be either rational or irrational.

Objective: I can…
- Classify numbers as rational, irrational, whole, natural, or integer.

Materials:
- Index cards with various types of numbers or you can print out the numbers provided on paper or card stock
- Number line taped out on the floor with 0 marked in the middle. The number line will need to be fairly large. If you can go the length of the room, that would be best.
- Note sheet with real number system graphic organizer
- Post it notes
- Signs with the words "Rational, Irrational, Whole, Natural, and Integer"

DIN: (Do It Now)
1. On the blank number line on your index card, mark the general location of the number on your index card
2. Have all students line up on the number line on the floor

Class Activity #1: Notes

Students will take notes and fill out the real number system graphic organizer. Definitions and examples will be provided

Class Activity #2: Post it Notes Activity

Post the words "Rational, Irrational, Whole, Natural, and Integer" around the room. Have students write examples of each and go stick their post it notes under the word. Then once everyone has put their examples up, assign a few students to each poster to check their work. Have each group report their findings. Allow students to debate why a number does or does not belong if there is a disagreement.

Closure: True or False

Have true or false statements on paper or posted online and have students answer them.

Example: 45 is a rational number.
All integers are whole numbers

Detailed Materials:

- DIN: Flashcards with the numbers below listed (one number on each flashcard) or use the cards provided on the next page
    - 1, 3.5, 10/3, -2, -4, -7.6, -7.5, -9/2, $\sqrt{64}$, 100, 7^2, $-\sqrt{96}$, 1/2, -3/4, 8.6, -9, $-2\frac{2}{3}$, -10, -11, -1.5
    - **ANSWER KEY for the above numbers:**
        - -11, -10, $-\sqrt{96}$, -9, -7.6, -7.5, -9/2, -4, $-2\frac{2}{3}$, -2, -1.5, -3/4, 1/2, 1, 10/3, 3.5, $\sqrt{64}$, 8.6, 7^2, 100
    - More numbers can be added of the teacher's choosing if there are more than 20 students in the class.
    - If there are less than 20 students, teachers can choose which of the above numbers to use, or they can give multiple flashcards to each student/group of students.

- Class Activity #1: Note Sheet
    - This is a guided note sheet with many examples. Students are to follow along with the teacher and fill in the missing information. Then they will try some examples with the teacher and then on their own. These will all be reviewed before moving on to Class Activity #2
    - This notes sheet is included. The key is also attached.

- Class Activity #2: Posters and 5 post-its for each student.
    - There needs to be 5 posters labeled with Rational, Irrational, Whole, Natural, and Integer (one on each poster). There are half sheet posters provided in this document.

- Closure: True or False Statements to put into a Google Form (or similar program) and have students respond to. They need to state whether it is true or false. If they select false, they need to explain why it is false. This also could be done by displaying the statement on the board and having students record their answers on paper.
    1. -34 is a rational number
    2. 2.5 is an integer and a rational number
    3. 0 is a natural number

    KEY:

    1. True
    2. False, 2.5 is not an integer because an integer cannot have a decimal
    3. False, 0 is not a natural number because natural numbers are all whole numbers greater than 0

Cards for DIN:

1	3.5	10/3	-2
-4	-7.6	-7.5	-9/2
$\sqrt{64}$	100	7^2	$-\sqrt{96}$
1/2	-3/4	8.6	-9
$-2\frac{2}{3}$	-10	-11	-1.5

Posters for Class Activity #2:

RATIONAL

IRRATIONAL

INTEGER

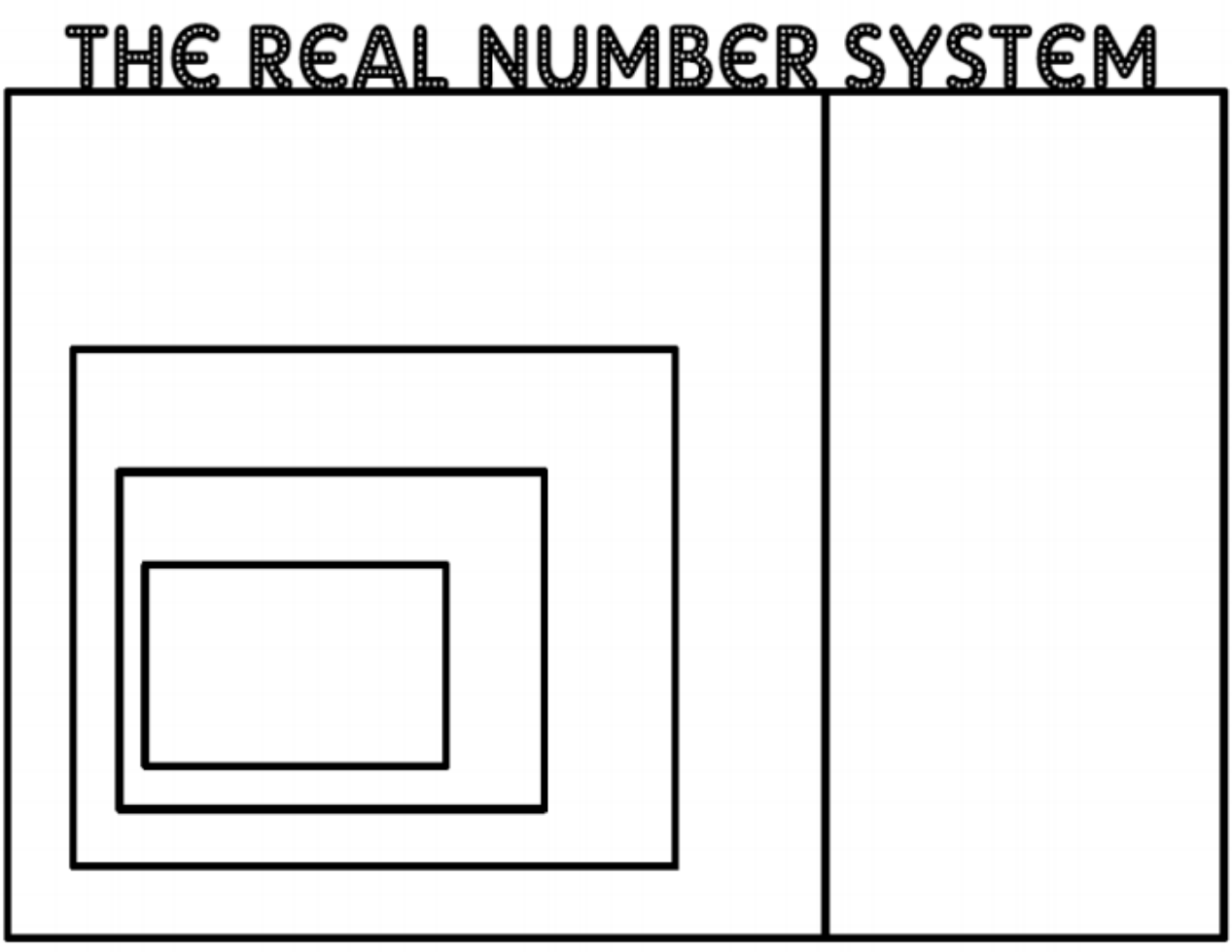
WHOLE

NATURAL

Lesson 1 Notes: Classifying Real Numbers

Vocabulary:

Natural Numbers	
Whole Numbers	
Integers	
Rational Numbers	
Irrational Numbers	
Real Numbers	



Carter, S. (2012). *Real number system graphic organizer*. [Image]. https://mathequalslove.net/real-number-system-graphic-organizer/

Put a check in all boxes that apply for each number listed in the table below:

	Natural	Whole	Integer	Rational	Irrational
0					
$\sqrt{21}$					
4/5					
-3					
-6.5					
100					

List all the classifications for each of the following numbers:

1. $\sqrt{84}$
2. -30
3. -7.3
4. 3/8
5. 17

True or False?

a. An integer is always a whole number.

b. A natural number is always a rational number

c. An irrational number can be an integer.

d. A whole number is always a natural number.

Brainstorm your own examples for each of the categories of real numbers. You will be posting 5 of your examples around the room.

Natural	
Whole	
Integer	
Rational	
Irrational	

Real Number System Lesson 2 Topic: Simplifying Radicals

This lesson is an introduction to radicals and how to simplify radicals. Students may be unfamiliar with the term "radical" and the idea of simplifying a radical is most likely new to them. There will also be some movement in this lesson to keep students actively engaged. The DIN activity will check to see if students understand what a perfect square number is. If hybrid learning is a part of a school's current plan, this lesson could be done remotely, but the movement pieces would not be included. Since integrating movement into the learning of mathematics is the focus of this lesson, I will not fully outline how to do this lesson remotely. Some ideas would be to use the card sort feature on Desmos to allow students to pair up the perfect square numbers similar to the original DIN activity. This could also be done on Jamboard, Google Slides, or Nearpod. The notes for this lesson could either be a video that students could watch, or it can be done live with screen sharing. The last activity can also be done in breakout groups in zoom or Google Meet. The rules of balance points would have to be adjusted, but it could still work. The teacher would have to jump between the breakout groups. Or, you could make the balance points activity a homework assignment and have students get their family involved by submitting pictures of their answers. This would be a great way to involve the family with their child's learning. For flipped learning, have students watch a video of the notes, then this would allow for more time for the DIN and Activity #2, which would also allow for more class discussion.

Standard(s):
AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers.
a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots
b.) Categorize the sum or product of rational or irrational numbers.
• The sum and product of two rational numbers is rational.
• The sum of a rational number and an irrational number is irrational.
• The product of a nonzero rational number and an irrational number is irrational.
• The sum and product of two irrational numbers could be either rational or irrational.

Objective: I can…
- Simplify radicals into simplest radical form

Materials:
- Index cards with perfect squares and other cards with numbers squared
- Note sheet
- Practice Problems

DIN: (Do It Now) Students need to find the match to their index card. For example, 2^2 and 4 would be paired together. Once students find their partner, they should make a list of perfect square numbers with their partner. Then introduce the radical symbol and talk about square roots.
Class Activity #1: Notes Students will take notes on how to simplify radicals. There will be many opportunities for students to practice. Show students how to use a graphing calculator to generate a list of factors for a number.
Class Activity #2: Balance Points - see blog Have students simplify the radical that is displayed on the board with a partner or small group. Then have them use balance points to express part of the answer (choose either the radicand or coefficient). It needs to be a fairly small number (for example, 21 would be too big of a number for balance points).
Closure: Have students write out the steps to simplifying radicals in their own words on a flashcard, post-it, or online (set up a Google Classroom Assignment or a Google Classroom Question).

Detailed Materials:

- DIN: Flashcards with the numbers/expressions listed below (one number/expression on each flashcard) or use the cards provided on the next page.
  - $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100$
  - **ANSWER KEY for the above numbers:**
    - $(1^2, 1)$ $(2^2, 4)$ $(3^2, 9)$ $(4^2, 16)$ $(5^2, 25)$ $(6^2, 36)$ $(7^2, 49)$ $(8^2, 64)$ $(9^2, 81)$ $(10^2, 100)$
  - More numbers can be added of the teacher's choosing if there are more than 20 students in the class.
  - If there are less than 20 students, teachers can choose which of the above numbers to use, or they can give multiple flashcards to each student/group of students. Make sure to remove pairs.
  - If there is an odd number of students in the class, then the teacher should participate.
- Class Activity #1: Note Sheet
  - This is a guided note sheet with many examples. Students are to follow along with the teacher and fill in the missing information. Then they will try some examples with the teacher and then on their own. These will all be reviewed before moving on to Class Activity #2
  - This notes sheet is included. The key is also attached.
- Class Activity #2: Balance Points
  - Need space for students to be working together in pairs. You can use the same pairs that were created in the DIN activity. Or you can choose to create new pairs, or even groups of three students. You may also want whiteboards or scrap paper for students to use to show their work.
- Closure: Flashcards, post-its, or online assignment for students to write out the steps to simplifying radicals in their own words.

Cards for DIN:

1^2	2^2	3^2	4^2
5^2	6^2	7^2	8^2
8^2	9^2	10^2	1
4	9	16	25
36	49	64	81
100			

Lesson 2 Notes: Simplifying Radicals

List of Perfect Square Numbers:

What is a perfect square number?

Calculator trick to get a list of perfect square numbers:

Estimate the following (without calculator):

$\sqrt{5}$ $\sqrt{50}$ $\sqrt{77}$

How to write radicals in **simplest radical form:**

Steps:	**Example: $\sqrt{48}$**

Practice Problems:

1. $\sqrt{24}$ 2. $\sqrt{54}$ 3. $\sqrt{98}$

Practice problems with a number in front of the radical:

4. $5\sqrt{80}$ 5. $-3\sqrt{75}$ 6. $-\sqrt{36}$

What happened in practice problem #6?

Think back to the vocabulary in Lesson 1.

a. What classification(s) could you give for your answer to practice problem #5?

b. What classification(s) could you give for your answer to practice problem #6?

Class Activity #2: Balance Points

If this is your first time using balance points in your classroom, I suggest following the tips on Sara Vanderwerf's blog about introducing Balance Points to your students. If you have already used this activity, then you can skip the intro or just remind students of the rules of Balance Points. The figure below demonstrates what it means to model an answer:

Figure 1: Balance Points (Shared from https://www.saravanderwerf.com/balance-points-a-mathmovement-activity)

On the next page there are problems to use for this activity, as well as how students should use movement to model their answer. They should work out their full answers on paper or on a personal whiteboard. Note, students will not be using movement to model their full answer, but instead, just part of it. Once you see that what they are modeling is correct, then double check their full answer. You could write these on the board, display them in a slideshow presentation, or print them out and hand out to students. I would suggest making sure students are all working on the same problem at the same time. If you have students who finish earlier than their classmates and their answer is correct, ask them to come up with a different way to model their answers.

Problem Set:

Problem	Answer	Directions to Model answer
#1: $\sqrt{8}$	$2\sqrt{2}$	Model the number in front of the radical
#2: $\sqrt{45}$	$3\sqrt{5}$	Model the number under the radical
#3: $2\sqrt{16}$	8	Model the answer
#4: $-5\sqrt{108}$	$-30\sqrt{3}$	Model the number under the radical
#5: $2\sqrt{40}$	$4\sqrt{10}$	Model the number in front of the radical
#6:$\sqrt{200}$	$10\sqrt{2}$	Model the number in front of the radical
#7: $\sqrt{72}$	$6\sqrt{2}$	Show the number in front of the radical
#8: $-3\sqrt{125}$	$-15\sqrt{5}$	Show the number under the radical

For problem #3, talk about the classification of the answer compared to the previous answers. Students should be able to recognize that #3 is a rational number while the previous answers were irrational.

Real Number System Lesson 3 Topic: Adding and Subtracting Radicals

This lesson focuses on adding and subtracting radicals. Students will use what they learned in Lesson 2 about simplifying radicals as they add and subtract radicals. A common misconception among students is that they can just add or subtract all numbers together, so it is important for students to understand what it means to have "like radicals." There will also be some movement in this lesson to keep students actively engaged. The DIN (Do It Now) activity will check to see how students are doing with simplifying radicals since it is an important skill to understand before moving on to adding and subtracting radicals. If hybrid learning is a part of a school's current plan, this lesson could be done remotely, but the movement pieces would not be included. Since integrating movement into the learning of mathematics is the focus of this lesson, I will not fully outline how to do this lesson remotely. Some ideas would be to use breakout groups to pair students for the speed dating activity. The only problem with this is that breakout rooms take some time to load and it will take a little extra time for students to switch partners. The notes for this lesson could either be a video that students could watch, or it can be done live with screen sharing. For flipped learning, have students watch a video of the notes, then this would allow for more time for the DIN and practice problems, which would also allow for more class discussion. In addition, you could have more of a focus on the Closure activity and have students solve each other's problems.

Standard(s):
AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers.
a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots
b.) Categorize the sum or product of rational or irrational numbers.
• The sum and product of two rational numbers is rational.
• The sum of a rational number and an irrational number is irrational.
• The product of a nonzero rational number and an irrational number is irrational.
• The sum and product of two irrational numbers could be either rational or irrational.

Objective: I can…
- Add and subtract radicals and write the answer in simplest radical form

Materials:
- Speed dating cards
- Class Activity 1 half sheet
- Note sheet

DIN: (Do It Now) Speed dating Students will be given a card that has a radical they need to simplify. They should place their solution on the back of the card. Students then will first share their problem with their partner and work on that problem. Once they think they have solved it, they check their work with their partner. Then students will switch to a new partner, by rotating one partner and having the other partner stay put. Make sure students always take their own card.
Class Activity #1: Exploring adding and subtracting radicals with a calculator Have a list of expressions for students to test in their calculator (on half sheet). See if students can come to any conclusions about adding and subtracting radicals. They will be using their calculators to test their ideas and see if they can come up with the process of adding/subtracting radicals on their own.
Class Activity #2: Notes Students will take notes on how to add and subtract radicals. There will be many opportunities for students to practice within the notes.
Closure: Have students create their own problem and put the solution on the back of it. Make sure to check these problems and solutions before the next class.

Detailed Materials:

- DIN: Speed Dating Cards
    - More problems can be added of the teacher's choosing if there are more than 20 students in the class.
    - If there are less than 20 students, teachers can choose which of the problems to use, or they can give multiple flashcards to each student/group of students. Make sure to remove pairs.
    - If there is an odd number of students in the class, then the teacher should participate.
- Class Activity #1: Exploration Half Sheet
    - This half sheet has questions for students to think about which will hopefully help them to discover how to add and subtract radicals.
- Class Activity #2: Notes
    - This is a guided note sheet with many examples. Students are to follow along with the teacher and fill in the missing information. Then they will try some examples with the teacher and then on their own.
    - This notes sheet is included. The key is also attached.

- Closure: Flashcards
    - Each student needs one flashcard to create their own addition or subtraction problem. The solution to the problem should be written on the back of the flashcard.

Speed Dating Cards for DIN:

$\sqrt{90}$	$\sqrt{200}$	$\sqrt{80}$
$\sqrt{175}$	$\sqrt{27}$	$\sqrt{147}$
$\sqrt{56}$	$\sqrt{48}$	$\sqrt{108}$
$\sqrt{72}$	$\sqrt{162}$	$\sqrt{192}$
$\sqrt{12}$	$\sqrt{8}$	$\sqrt{45}$
$\sqrt{112}$	$\sqrt{125}$	$\sqrt{128}$
$\sqrt{180}$	$\sqrt{150}$	

Class Activity #1 Half Sheet:

We are going to be adding and subtracting radicals today. I want you to try to come up with the process for adding radicals.

a. First, how would you add $3x + 5x$?

b. Given the expression $3\sqrt{7} + 5\sqrt{7}$, how do you think you would add those together? What would the sum be? (Note: there should be a radical in your answer). To check your work, first type in the expression $3\sqrt{7} + 5\sqrt{7}$ into your calculator to see the decimal value. Then type in what you think the sum is to see its decimal value. If they are the same decimal values, then you have the correct sum, if not then you need to try a different process for adding radicals.

c. Given the expression, $2\sqrt{3} + 4\sqrt{75}$, how do you think you would add these together? Repeat the same process from above to check your work.

Class Activity #1 Half Sheet:

We are going to be adding and subtracting radicals today. I want you to try to come up with the process for adding radicals.

a. First, how would you add $3x + 5x$?

b. Given the expression $3\sqrt{7} + 5\sqrt{7}$, how do you think you would add those together? What would the sum be? (Note: there should be a radical in your answer). To check your work, first type in the expression $3\sqrt{7} + 5\sqrt{7}$ into your calculator to see the decimal value. Then type in what you think the sum is to see its decimal value. If they are the same decimal values, then you have the correct sum, if not then you need to try a different process for adding radicals.

c. Given the expression, $2\sqrt{3} + 4\sqrt{75}$, how do you think you would add these together? Repeat the same process from above to check your work.

Lesson 3 Notes: Adding and Subtracting Radicals

- When adding and subtracting radicals, we want to treat the radical as a _____.

- We only add/subtract the numbers _____ the like radicals.

- If we do not have like radicals, then we need to _____ first.

Examples:

1. $4\sqrt{5} - 8\sqrt{5}$	2. $3\sqrt{3} + 3\sqrt{48}$
3. $6\sqrt{7} + \sqrt{7}$	4. $6\sqrt{2} - 3\sqrt{8} + 2\sqrt{32}$
5. $5\sqrt{2} + 6\sqrt{5} - 3\sqrt{2} + \sqrt{5} + \sqrt{7}$	6. $-5\sqrt{32} + 7\sqrt{80}$

Practice Problems

1. $4\sqrt{12} + 3\sqrt{8}$

2. $-2\sqrt{3} + 3\sqrt{27}$

3. $6\sqrt{54} - 3\sqrt{24} - 8\sqrt{96}$

4. $\sqrt{50} + \sqrt{32}$

5. $-4\sqrt{18} + 5\sqrt{8} - \sqrt{24}$

6. $-\sqrt{27} - 2\sqrt{45} - \sqrt{20}$

7. $-9\sqrt{56} + \sqrt{126}$

8. $-3\sqrt{12} + 5\sqrt{3} + 7\sqrt{20}$

Real Number System Lesson 4 Topic: Multiplying Radicals

This lesson focuses on multiplying radicals. Students will use what they learned in Lesson 2 about simplifying radicals as they multiply radicals. There will be some movement in this lesson to keep students actively engaged. The DIN (Do It Now) activity will check to see how students are doing with adding and subtracting radicals. If hybrid learning is a part of a school's current plan, this lesson could be done remotely, but the movement pieces would not be included. Since integrating movement into the learning of mathematics is the focus of this lesson, I will not fully outline how to do this lesson remotely. Some ideas would be to use breakout groups to pair students for the DIN activity. Students could share their screen with their problem on it and have the other partner solve the problem. I would suggest using a shared Jamboard between the students, so both students can edit and draw on it. The notes for this lesson could either be a video that students could watch, or it can be done live with screen sharing. For flipped learning, have students watch a video of the notes, then this would allow for more time for the DIN and practice problems, as well as the stations activity.

Standard(s): AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers. a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots b.) Categorize the sum or product of rational or irrational numbers. • The sum and product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. • The sum and product of two irrational numbers could be either rational or irrational.
Objective: I can… • Multiply and divide radicals and write the answer in simplest radical form
Materials: • Student created problems from the previous lesson • Note sheet • Cycle stations • Exit ticket sheet
DIN: (Do It Now) Pass out student-created problems from the previous lesson, and have students solve them.
Class Activity #1: Notes Students will take notes on how to multiply radicals. There will be many opportunities for students to practice within the notes.

Class Activity #2: Cycle Stations Hang up the station posters around the room. Students start at any station and solve the problem. They need to look for the station with their answer to know what their next problem is. This continues until students loop back to their first problem. This will have mostly multiplying problems with a few adding and subtracting problems.
Closure: Have students reflect on how they are feeling about operations with radicals by answering the questions on the exit ticket.

Detailed Materials:

- DIN: Student created problems from the last lesson.
    - Have students work on a problem from yesterday, and then check their answer with the solution on the back.
- Class Activity #1: Notes
    - This is a guided note sheet with many examples. Students are to follow along with the teacher and fill in the missing information. Then they will try some examples with the teacher and then on their own.
- Class Activity #2: Cycle Stations
- Closure: Exit Ticket Sheet
    - This has questions for students to answer about operations with radicals.

Lesson 4 Notes: Multiplying Radicals

Steps for Multiplying Radicals:

1.

2.

3.

Practice:

1. $\sqrt{8} \cdot \sqrt{3} =$

2. $5\sqrt{5} \cdot 3\sqrt{10} =$

3. $3\sqrt{20}(\sqrt{5})$

4. $(3\sqrt{2})(-6\sqrt{5}) =$

5. $5\sqrt{12} \cdot 2\sqrt{7} =$

Distributing with Radicals:

a. $5\sqrt{3}(6+2\sqrt{5})$

b. $-2\sqrt{15}(-3\sqrt{3}+3\sqrt{5})$

Binomials with Radicals:

- **Use box method or distribution**

a. $\left(2\sqrt{2}+6\right)\left(\sqrt{5}-7\right)$

b. $\left(8+3\sqrt{12}\right)\left(-4+2\sqrt{6}\right)$

c. $\left(5-4\sqrt{5}\right)\left(-2+\sqrt{5}\right)$

d. $\left(3-4\sqrt{6}\right)^2$

Radicals Cycle Stations
Student Work Page

Write the station letter of the first station in the start box. Then solve that problem and find the answer on another station card. Record that letter in the next box and solve the problem. Keep repeating this process until you have filled the boxes. The answer to the last station you write in the box should be from the first station. You can start at any station.

Start:

Show your work for each station below:

Station _____	**Station _____**
Station _____	**Station _____**
Station _____	**Station _____**

Station _____	**Station _____**
Station _____	**Station _____**

STATION A

ANSWER:

$$20 + 2\sqrt{2}$$

QUESTION:

$$7\sqrt{5}(9\sqrt{15})$$

– –

STATION B

ANSWER:

$$-3\sqrt{6}$$

QUESTION:

$$6\sqrt{2} \cdot 8\sqrt{14}$$

STATION C

ANSWER:

$$315\sqrt{3}$$

QUESTION:

$$-3\sqrt{3}\left(2+\sqrt{6}\right)$$

– –

STATION D

ANSWER:

$$-8+26\sqrt{2}$$

QUESTION:

$$8\sqrt{108}+\sqrt{12}$$

STATION E

ANSWER:

$$-96\sqrt{7}$$

QUESTION:

$$\left(1-2\sqrt{2}\right)\left(-4-3\sqrt{8}\right)$$

– –

STATION F

ANSWER:

$$31+10\sqrt{6}$$

QUESTION:

$$5\sqrt{160}-2\sqrt{10}$$

STATION G

ANSWER:

$$3$$

QUESTION:

$$\sqrt{3}\left(-5\sqrt{2}+\sqrt{8}\right)$$

– –

STATION H

ANSWER:

$$18\sqrt{10}$$

QUESTION:

$$\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$$

STATION I

ANSWER:

$$50\sqrt{3}$$

QUESTION:

$$(\sqrt{6}+5)^2$$

– –

STATION J

ANSWER:

$$-6\sqrt{3}-9\sqrt{2}$$

QUESTION:

$$\left(-5-\sqrt{2}\right)\left(4-2\sqrt{18}\right)$$

Exit Ticket

1. How do you feel about simplifying radicals, adding/subtracting radicals, and multiplying radicals?

2. What is your strongest skill out of the lessons up to this point?

3. What do you need more practice with?

Exit Ticket

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Exit Ticket

1. How do you feel about simplifying radicals, adding/subtracting radicals, and multiplying radicals?

2. What is your strongest skill out of the lessons up to this point?

3. What do you need more practice with?

Real Number System Lesson 5 Topic: Rationalizing the Denominator

This lesson focuses on rationalizing the denominator. Students will use what they learned in Lesson 2 about simplifying radicals as well as what they learned in Lesson 4 about multiplying radicals. There will be some movement in this lesson to keep students actively engaged. The DIN (Do It Now) activity will have students explore what happens when you multiply a radical by itself. The hope is that students figure out that the radical "disappears" and can come up with an explanation for why that happens. If hybrid learning is a part of a school's current plan, this lesson could be done remotely, but the movement pieces would not be included. Since integrating movement into the learning of mathematics is the focus of this lesson, I will not fully outline how to do this lesson remotely. Some ideas would be to use breakout rooms to group students for the DIN activity. This will allow students to share their ideas on what happens when you square a radical. Students could use a digital whiteboard like Jamboard to record their ideas as a group. The notes for this lesson could either be a video that students could watch, or it can be done live with screen sharing. For flipped learning, I would suggest using having students still do the DIN. This could be done in Desmos, Edpuzzle, or any other program that allows students to share their thoughts. Then, have students watch a video of the notes. This would allow for more time for the practice problems, as well as the Quizlet Live activity.

Standard(s):
AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers.
a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots
b.) Categorize the sum or product of rational or irrational numbers.
• The sum and product of two rational numbers is rational.
• The sum of a rational number and an irrational number is irrational.
• The product of a nonzero rational number and an irrational number is irrational.
• The sum and product of two irrational numbers could be either rational or irrational.

Objective: I can…
- Rationalize the denominator and write the answer in simplest radical form

Materials:
- DIN problem
- Note sheet with practice problems
- Quizlet Live activity (online) - LINK
- Whiteboards, dry erase markers, and erasers or scrap paper and pen/pencil for the Quizlet Live activity
- Closure problem

DIN: (Do It Now)
Have students explore what happens when you multiply a radical by itself. Display this expression ($\sqrt{3} \cdot \sqrt{3}$) on the board, and have students work in pairs or small groups to discuss their ideas. Then have a whole class discussion to allow students to share what they noticed.

Class Activity #1: Notes
First, students will learn when it is necessary to rationalize the denominator. Then, students will take notes on how to rationalize the denominator. There will be many opportunities for students to practice within the notes.

Class Activity #2: Quizlet Live - LINK
Quizlet Live is a game created by Quizlet and uses a flashcard set that is already created. Students must work with their group members to match the current problem with its corresponding answer. The catch is that each member of the group has different answers on their device screen, so they must figure out who has the correct answer and then select it to move onto the next problem. On the teacher's projected screen, it will show the status of each team, and allows everyone to see who is in the lead.

When you are ready to start the activity, have students get up and go to their Quizlet live groups. This are automatically created by the Quizlet Live Program. Students will need a whiteboard, dry erase marker, and eraser or scrap paper and a pen or pencil to complete the problems in the activity. Change groups each time you play to get students up and moving more.

Closure: Put the following problem on the board and have students solve it on their whiteboards or scrap paper. Go around the room to check how students did.

Problem: $\frac{6}{\sqrt{7}}$ ANSWER: $\frac{6\sqrt{7}}{7}$

Detailed Materials:

- Class Activity #1: Notes Worksheet
    - This is a guided note sheet with many examples. Students are to follow along with the teacher and fill in the missing information. Then they will try some examples with the teacher and then on their own. These will all be reviewed before moving on to Class Activity #2.
    - This notes sheet is included. The key is also attached.
- Class Activity #2: Quizlet Live Link
    - Here is the LINK to the deck that will be used for Quizlet Live.
    - Teachers can create a free account to use this activity. Students do not need an account.
    - Make sure students have either whiteboards, dry erase markers, and erasers, or scrap paper and a pen/pencil to work on the problems during the game.

Lesson 5 Notes: Rationalize the Denominator

For an expression to be in simplest radical form, there cannot be any radicals in the denominator of a fraction.

Here's an example:

We need a way to "remove" the radical from the denominator, and we will do this by rationalizing the denominator.

Steps:	Example: $\frac{14}{\sqrt{7}}$
1. Multiply the numerator and denominator by the radical in the denominator 2. Simplify the radicals (if necessary) 3. Simplify the fraction (if necessary)	

Practice:

1. $\frac{5}{\sqrt{6}}$

2. $\frac{7}{\sqrt{3}}$

3. $\frac{13}{\sqrt{5}}$

4. $\frac{-10}{\sqrt{2}}$

5. $\frac{1}{\sqrt{8}}$

6. $\frac{11}{\sqrt{9}}$

Real Number System Lesson 6 Topic: Review

This lesson focuses on reviewing all the material from this unit. Students will use what they learned in each lesson to complete the activities in this lesson. There will be some movement in this lesson to keep students actively engaged. The DIN (Do It Now) activity will have students working in pairs or small groups to create a brief outline/study guide for the entire unit. The teacher should be walking around to answer any questions. If there is time, have each pair or group share their outline, and allow the other groups to add to their outlines. If hybrid learning is a part of a school's current plan, this lesson could be done remotely, but the movement pieces would not be included. Since integrating movement into the learning of mathematics is the focus of this lesson, I will not fully outline how to do this lesson remotely. Some ideas would be to use breakout rooms to group students for the DIN activity. This will allow students to work together to create an outline. Students could use a digital whiteboard like Jamboard to record their ideas as a group. For class activity #1, this could be done in many ways. One example would be having a Kahoot (or similar program) to classify numbers. For class activity #2, this could be done using google slides or google forms.

For flipped learning, have students create a more-detailed outline before class. Students could choose to do this on paper or digitally. Then, the class time can be used to focus on the activities.

Standard(s): AI-N.RN.3 Use properties and operations to understand the different forms of rational and irrational numbers. a.) Perform all four arithmetic operations and apply properties to generate equivalent forms of rational numbers and square roots b.) Categorize the sum or product of rational or irrational numbers. • The sum and product of two rational numbers is rational. • The sum of a rational number and an irrational number is irrational. • The product of a nonzero rational number and an irrational number is irrational. • The sum and product of two irrational numbers could be either rational or irrational.
Objective: I can… • Classify numbers • Rationalize the denominator and write the answer in simplest radical form • Perform addition, subtraction, multiplication, and division with radicals and write the answer in simplest radical form
Materials:

• Scrap paper for outlines • Numbers for classification activity in a slideshow (LINK) • Mathlib activity
DIN: (Do It Now) Have students create a quick outline of the topics in this unit in pairs or small groups. Then have students share their outlines and allow time for the other groups to add to their own outlines.
Class Activity #1: Classifying Numbers Label areas of the room with the following titles using the posters from Lesson 1: rational, irrational, integer, whole, and natural. Show a number on the board from the slideshow (LINK) and have students walk to the correct classification (you can choose either most specific classification or let students choose one classification)
Class Activity #2: Mathlib Similar to stations, but students solve problems to get words or phrases that go into their "Mathlib" (like a madlib). These problems will be a mix of all the operations with radicals.
Closure: Allow students to look through their notes and review materials. If they have questions, they should ask them at this time to prepare for the assessment.

Detailed Materials:

- DIN:
    - Scrap Paper
    - Have students work in groups to create a brief outline for the unit. Each student should have their own copy to study from.
- Class Activity #1: Classifying Numbers
    - Use the posters from Lesson 1 to create areas for each type of number around the room.
    - Slideshow (LINK) of numbers for students to classify. You can choose if you want most specific classification or any classification. If the students do not agree, have them explain why they think they are correct.
    - Answers are provided within the slideshow.
- Class Activity #2: Mathlib
    - Posters with problems to hang around the room. They do not need to be in number order.
    - Story sheets/work sheets for students to show their work and record their answers.

Radicals Mathlib

Directions: Solve each problem and find the answer. Then write the name, item, or phrase that corresponds to the answer in the correctly numbered space in the story below. Show all work in the correctly numbered box on this worksheet.

(1)_____ decided to go for a walk after his (2)_____ today in (3)_____. While on his walk, he saw a (4)_____ without an owner that he decided to take with him. Since his new friend did not have a name, he decided to name it (5)_____. It was well past lunchtime, so he stopped and got a (6)_____ and then continued on his walk. (7) _____ joined on the walk and they started looking for the owner of their new friend. After (8) _____ they saw someone calling out "(9)_____," and ran over to see if this is the owner. The owner was so excited to have their pet back that they gave (10)_____ for a reward!

Question 1: _____	Question 2: _____
Question 3: _____	Question 4: _____

Question 5: _____	Question 6: _____
Question 7: _____	Question 8: _____
Question 9: _____	Question 10: _____

Question 1

Write in simplest radical form:

$$\sqrt{64}$$

a. 8	Buzz Lightyear
b. $2\sqrt{16}$	Mickey Mouse
c. 32	Mr. Incredible
d. $8\sqrt{8}$	Gaston

Question 2

Write in simplest radical form:

$$2\sqrt{3} + 4\sqrt{75}$$

a. $6\sqrt{78}$	Performance
b. $22\sqrt{3}$	Interview
c. $2\sqrt{3} + 4\sqrt{75}$	Doctor's Appointment
d. $102\sqrt{3}$	Dentist Appointment

Question 3

Write in simplest radical form:

$$-3\sqrt{2}+3\sqrt{20}-3\sqrt{8}$$

a. $-3\sqrt{7}$	Orlando
b. $-9\sqrt{2}+6\sqrt{5}$	New York City
c. $-6\sqrt{2}+6\sqrt{5}$	San Francisco
d. $6\sqrt{5}$	Chicago

Question 4

Write in simplest radical form:

$$2\sqrt{3} \cdot 7\sqrt{2}$$

a. $9\sqrt{5}$	Cat
b. $14\sqrt{5}$	Bunny
c. $9\sqrt{6}$	Snake
d. $14\sqrt{6}$	Dog

Question 5

Write in simplest radical form:

$$\frac{1}{\sqrt{3}}$$

a. $\frac{\sqrt{3}}{9}$	Pluto
b. $\frac{\sqrt{3}}{\sqrt{9}}$	Goofy
c. $\frac{\sqrt{3}}{3}$	Doug
d. 9	Slinky

Question 6

Write in simplest radical form:

$$\sqrt{6}(\sqrt{6}+2\sqrt{3})$$

a. $12\sqrt{2}$	Hamburger
b. $6+6\sqrt{2}$	Pizza
c. $6+2\sqrt{18}$	Hot Dog
d. $36+6\sqrt{2}$	Sub

Question 7

Write in simplest radical form:

$$\sqrt{98} - \sqrt{128}$$

a. $-\sqrt{2}$	Frozone
b. $-\sqrt{30}$	Bo Peep
c. $15\sqrt{2}$	Olaf
d. $-15\sqrt{2}$	Minnie Mouse

Question 8

Write in simplest radical form:

$$\sqrt{72}$$

a. $4\sqrt{18}$	10 minutes
b. $8\sqrt{9}$	An hour
c. $6\sqrt{2}$	30 minutes
d. 36	Two hours

Question 9

Write in simplest radical form:

$$\frac{4}{\sqrt{6}}$$

a. $\frac{\sqrt{6}}{9}$	Pepper
b. $\frac{4\sqrt{6}}{6}$	Bruno
c. $\frac{4\sqrt{6}}{36}$	Spots
d. $\frac{2\sqrt{6}}{3}$	Lucky

Question 10

Write in simplest radical form:

$$4\sqrt{5} \cdot 10\sqrt{10}$$

a. $200\sqrt{2}$	$500 cash
b. $40\sqrt{50}$	A diamond ring
c. $80\sqrt{5}$	AirPods
d. $20\sqrt{2}$	A TV

My Turn

Movement in the mathematics classroom is something I truly believe in because not only am I a mathematics teacher, but I am also a dance teacher in the private sector. I have experienced how important movement is to growing children and how it can have many educational benefits. This became even more evident to me after teaching through the pandemic where I had to limit student contact, and therefore, limit student movement in my classroom.

I have been including movement in my classroom since my first year of teaching. Although the movement may not directly relate to the content, getting students up and moving helps increase engagement and focus if done correctly. If you are introducing movement into your classroom for the first time, you may receive some push back from students. High schoolers tend to complain at first when I introduce various movement activities, however as they become acclimated to my classroom and teaching style, they learn to appreciate and enjoy these movement activities when you provide specific expectations for the movement activity. It is common to have to work with various ways of grouping students to assure students are staying on task and working to their full potential.

The curriculum provided includes movement activities that I have had great success within my own classroom. Although these activities are directed towards the real number system, I have used most of these activities for other topics. Having some consistent movement activities from unit-to-unit help make it more effective in keeping students engaged while helping them be more successful in my class. One of the best feelings is when I announce to my students that we will be doing a Mathlib. They are always so excited to solve the mystery that they forget about all the mathematics they are going to have to do. Just by simply putting practice problems into an activity, teachers can change students' attitudes towards learning and doing mathematics. At the end of each school year, I have my students fill out an evaluation form and students regularly mention that the movement activities were their favorite part of my class. Not only do they mention that they are fun, but they say they truly feel like it positively impacted their education. As I continue with my

teaching career, I will also continue to include movement into the classroom as I have found it to increase student success and change the negative outlook that many students have towards learning mathematics.

Conclusion

Movement in mathematics classrooms can help increase student engagement and achievement. It is the author's hope that other teachers can use this curriculum to increase movement in their own classroom, and likewise increase student participation and success.

References

Beaudoin, C. R., & Johnston, P. (2011). The impact of purposeful movement in algebra instruction. *Education*, *132*(1), 82–96.

Beesley, A. D., Clark, T. F., Dempsey, K., & Tweed, A. (2018). Enhancing formative assessment practice and encouraging middle school mathematics engagement and persistence. *School Science & Mathematics*, *118*(1/2), 4–16. https://doi.org/10.1111/ssm.12255

Benes, S., Finn, K. E., Sullivan, E. C., & Yon, Z. (2016). Teachers' perceptions of using movement in the classroom. *Physical Educator*, *73*(1), 110–135. https://doi.org/10.18666/TPE-2016-V73-I1-5316

Eckhoff, A., & Linder, S. M. (2020). Practical Approaches to Informal Assessment. *Teaching Young Children*, *14*(1), 28–31.

Helgeson, J. (2011). 4 simple ways to add movement in daily lessons. *Kappa Delta Pi Record*, *47*(2), 80–84.

Kilic, H., Cross, D.I., Ersoz, F.A., Mewborn, D.S., Swanagan, D., & Kim, J., (2010). Different types of instructional facilitation influence students' thinking and reasoning; reflecting on your own practices can help you determine your role as an instructor and increase your competence. Teaching children Mathematics, NCTM, 350-357.

McMullen, J. M., MacPhail, A., & Dillon, M. (2019). "I want to do it all day!"—Students' experiences of classroom movement integration. *International Journal of Educational Research*, *94*, 52–65. https://doi.org/10.1016/j.ijer.2018.11.014

Romar, J.-E., Bjorkgren, M., Snellman, J. E., Ruostekoski, A., Harjunpaa, P., & Juslenius, V. (2020). *Preservice secondary subject teachers incorporating movement integration into classroom practice*. https://brockport.illiad.oclc.org/illiad/pdf/365908.pdf

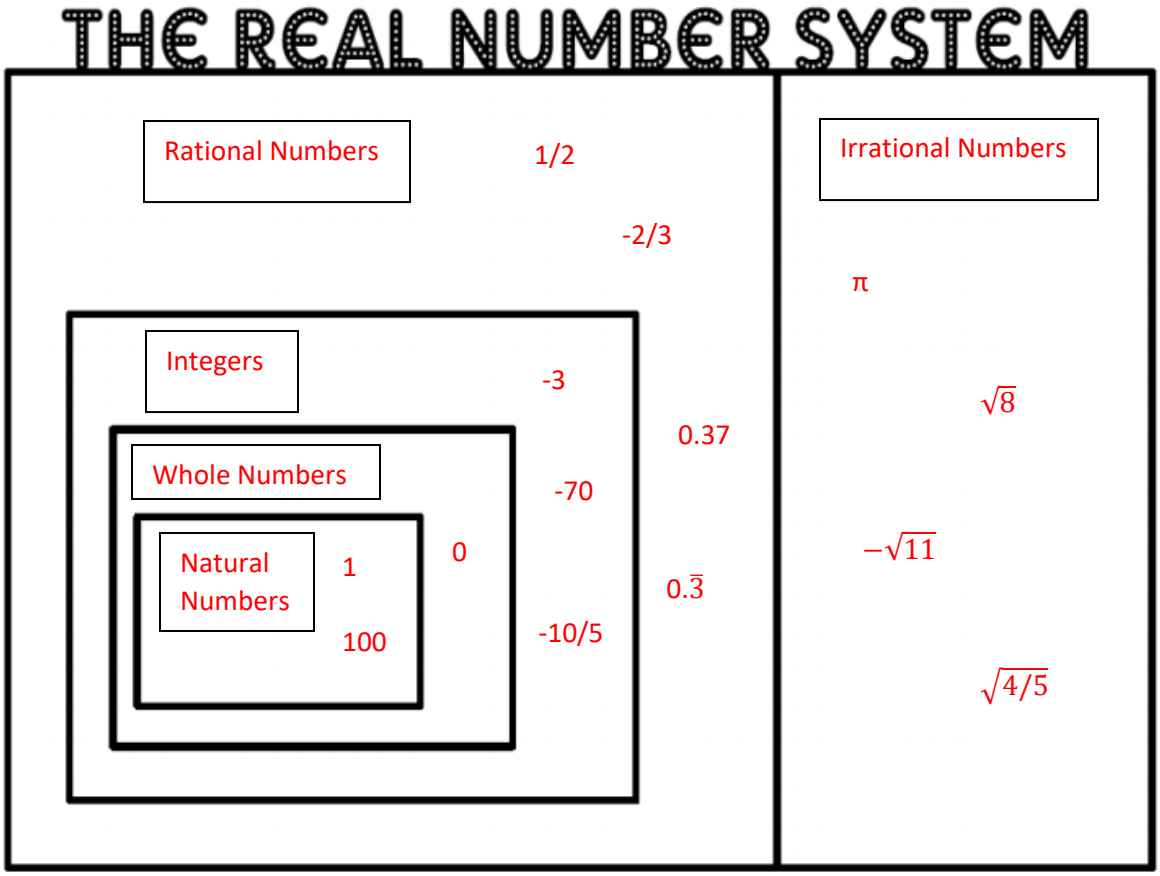
Scammell, J. (2016). Improving student achievement in mathematics through formative assessment in instruction. *Delta-k*, *53*(1), 10–14.

Appendix

KEY Lesson 1 Notes: Classifying Real Numbers

Vocabulary:

Natural Numbers	Start with 1 and then count up; "Counting Numbers" Examples: 1, 2, 3, …
Whole Numbers	Start with 0 and count up; 0 and all natural numbers Examples: 0, 1, 2, …
Integers	Positive and negative numbers – no decimals or fractions Examples: …, -3, -2, -1, 0, 1, 2, 3, …
Rational Numbers	Any number that can be written as a fraction; terminating decimals or repeating decimals only Examples: 1/2, 1/3, -2/5, $6.\overline{3}$, 300
Irrational Numbers	Any number that cannot be written as a fraction; any nonrepeating infinite decimal Examples: π, $\sqrt{24}$, $-\sqrt{10}$
Real Numbers	All rational and irrational numbers



Carter, S. (2012). *Real number system graphic organizer*. [Image]. https://mathequalslove.net/real-number-system-graphic-organizer/

Put a check in all boxes that apply for each number listed in the table below:

	Natural	Whole	Integer	Rational	Irrational
0		✓	✓	✓	
$\sqrt{21}$					✓
4/5				✓	
-3			✓	✓	
-6.5				✓	
100	✓	✓	✓	✓	

List all the classifications for each of the following numbers:

1. $\sqrt{84}$ - irrational
2. -30 – integer, rational
3. -7.3 - rational
4. 3/8 - rational
5. 17 – natural, whole, integer, rational

True or False?

a. An integer is always a whole number. False – negative numbers cannot be whole numbers

b. A natural number is always a rational number. True

c. An irrational number can be an integer. False

d. A whole number is always a natural number. False – 0 is not a natural number but is a whole number

Brainstorm your own examples for each of the categories of real numbers. You will be posting 5 of your examples around the room. Answers will vary by student

Natural	
Whole	
Integer	
Rational	
Irrational	

KEY Lesson 2 Notes: Simplifying Radicals

List of Perfect Square Numbers:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, …

What is a perfect square number?

A number that results in a whole number when taking the square root of it; the result of squaring a whole number

Calculator trick to get a list of perfect square numbers:

Go to y=, type in x^2 into y_1, then click 2nd – graph to look at the table. The y column is all perfect square numbers. The x column is the square root of each perfect square number.

Estimate the following (without calculator):

$\sqrt{5}$	$\sqrt{50}$	$\sqrt{77}$
About 2.2	About 7.1	About 8.8

How to write radicals in **simplest radical form:**

Steps:	**Example: $\sqrt{48}$**
1.Figure out where the number under the radical lies in the list of perfect squares. 2.Divide the number under the radical by the perfect squares less than it until you get a whole number. 3.Split the radical into 2 radicals: one with the perfect square and the other with the quotient from step 2 4.Simplify the radical with the perfect square.	1.The number 48 falls between 36 and 49 in the list of perfect square numbers. 2.Divide 48 by the perfect squares that are less than it, until you get a perfect square number: 48/16=3 3. $\sqrt{48} = \sqrt{16} \cdot \sqrt{3}$ 4. $4\sqrt{3}$

Practice Problems:

1. $\sqrt{24}$

   $\sqrt{4} \cdot \sqrt{6}$

   $2\sqrt{6}$

2. $\sqrt{54}$

   $\sqrt{9} \cdot \sqrt{6}$

   $3\sqrt{6}$

3. $\sqrt{98}$

   $\sqrt{49} \cdot \sqrt{2}$

   $7\sqrt{2}$

Practice problems with a number in front of the radical:

4. $5\sqrt{80}$

$5 \cdot \sqrt{16} \cdot \sqrt{5}$

$5 \cdot 2\sqrt{5}$

$10\sqrt{5}$

5. $-3\sqrt{75}$

$-3 \cdot \sqrt{25} \cdot \sqrt{3}$

$-3 \cdot 5 \cdot \sqrt{3}$

$-15\sqrt{3}$

6. $-\sqrt{36}$

-6

What happened in practice problem #6?

The radical "disappeared" because 36 is a perfect square number.

Think back to the vocabulary in Lesson 1.

c. What classification(s) could you give for your answer to practice problem #5?
irrational

d. What classification(s) could you give for your answer to practice problem #6?

Natural, whole, integer, rational

KEY Lesson 3 Speed Dating DIN activity

$\sqrt{90}$ $3\sqrt{10}$	$\sqrt{200}$ $10\sqrt{2}$	$\sqrt{80}$ $4\sqrt{5}$
$\sqrt{175}$ $5\sqrt{7}$	$\sqrt{27}$ $3\sqrt{3}$	$\sqrt{147}$ $7\sqrt{3}$
$\sqrt{56}$ $4\sqrt{14}$	$\sqrt{48}$ $4\sqrt{3}$	$\sqrt{108}$ $6\sqrt{3}$
$\sqrt{72}$ $6\sqrt{2}$	$\sqrt{162}$ $9\sqrt{2}$	$\sqrt{192}$ $8\sqrt{3}$
$\sqrt{12}$ $2\sqrt{3}$	$\sqrt{8}$ $2\sqrt{2}$	$\sqrt{45}$ $3\sqrt{5}$
$\sqrt{112}$ $4\sqrt{7}$	$\sqrt{125}$ $5\sqrt{5}$	$\sqrt{128}$ $8\sqrt{2}$
$\sqrt{180}$ $6\sqrt{5}$	$\sqrt{150}$ $5\sqrt{6}$	

KEY Lesson 3 Class Activity #1

We are going to be adding and subtracting radicals today. I want you to try to come up with the process for adding radicals.

a. First, how would you add $3x + 5x$? Combine the like terms by adding the coefficients, so the sum would be 8x.

b. Given the expression $3\sqrt{7} + 5\sqrt{7}$, how do you think you would add those together? What would the sum be? (Note: there should be a radical in your answer). To check your work, first type in the expression $3\sqrt{7} + 5\sqrt{7}$ into your calculator to see the decimal value. Then type in what you think the sum is to see its decimal value. If they are the same decimal values, then you have the correct sum, if not then you need to try a different process for adding radicals.
Allow students the opportunity to make mistakes. Observe to see if there are any common misconceptions. The decimal value of the sum is 21.1660104885 and the sum in simplest radical form is $8\sqrt{7}$. Make sure all students eventually come to the conclusion that you keep the like radical the same and just add the coefficients (similar to part a).

c. Given the expression, $2\sqrt{3} + 4\sqrt{75}$, how do you think you would add these together? Repeat the same process from above to check your work.
Again, allow students to try a few of their own ideas. Then, make sure all students come to the conclusion that they need to simplify the radical first in order to get like radicals. Then add. The decimal value of the sum is 38.1051177665, and the sum in simplest radical form is $22\sqrt{3}$.

KEY Lesson 3 Notes: Adding and Subtracting Radicals

- When adding and subtracting radicals, we want to treat the radical as a variable.
- We only add/subtract the numbers in front of the like radicals.
- If we do not have like radicals, then we need to simplify the radical(s) first.

Examples:

1. $4\sqrt{5} - 8\sqrt{5}$ $-4\sqrt{5}$	2. $3\sqrt{3} + 3\sqrt{48}$ $3\sqrt{3} + 3\sqrt{16}\sqrt{3}$ $3\sqrt{3} + 3(4)\sqrt{3}$ $3\sqrt{3} + 12\sqrt{3}$ $15\sqrt{3}$
3. $6\sqrt{7} + \sqrt{7}$ $7\sqrt{7}$	4. $6\sqrt{2} - 3\sqrt{8} + 2\sqrt{32}$ $6\sqrt{2} - 3\sqrt{4}\sqrt{2} + 2\sqrt{16}\sqrt{2}$ $6\sqrt{2} - 3(2)\sqrt{2} + 2(4)\sqrt{2}$ $6\sqrt{2} - 6\sqrt{2} + 8\sqrt{2}$ $8\sqrt{2}$
5. $5\sqrt{2} + 6\sqrt{5} - 3\sqrt{2} + \sqrt{5} + \sqrt{7}$ $2\sqrt{2} + 7\sqrt{5} + \sqrt{7}$	6. $-5\sqrt{32} + 7\sqrt{80}$ $-5\sqrt{16}\sqrt{2} + 7\sqrt{16}\sqrt{5}$ $-5(4)\sqrt{2} + 7(4)\sqrt{5}$ $-20\sqrt{2} + 28\sqrt{5}$ No like terms so cannot simplify anymore

Practice Problems:

1. $4\sqrt{12}+3\sqrt{8}$

$4\sqrt{4}\sqrt{3}+3\sqrt{4}\sqrt{2}$

$4(2)\sqrt{3}+3(2)\sqrt{2}$

$8\sqrt{3}+8\sqrt{2}$

2. $-2\sqrt{3}+3\sqrt{27}$

$-2\sqrt{3}+3\sqrt{9}\sqrt{3}$

$-2\sqrt{3}+3(3)\sqrt{3}$

$-2\sqrt{3}+9\sqrt{3}$

$7\sqrt{3}$

3. $6\sqrt{54}-3\sqrt{24}-8\sqrt{96}$

$6\sqrt{9}\sqrt{6}-3\sqrt{4}\sqrt{6}-8\sqrt{16}\sqrt{6}$

$6(3)\sqrt{6}-3(2)\sqrt{6}-8(4)\sqrt{6}$

$18\sqrt{6}-6\sqrt{6}-32\sqrt{6}$

$-20\sqrt{6}$

4. $\sqrt{50}+\sqrt{32}$

$\sqrt{25}\sqrt{2}+\sqrt{16}\sqrt{2}$

$(5)\sqrt{2}+(4)\sqrt{2}$

$9\sqrt{2}$

5. $-4\sqrt{18}+5\sqrt{8}-\sqrt{24}$

$-4\sqrt{9}\sqrt{2}+5\sqrt{4}\sqrt{2}-\sqrt{4}\sqrt{6}$

$-4(3)\sqrt{2}+5(2)\sqrt{2}-(2)\sqrt{6}$

$-12\sqrt{2}+10\sqrt{2}-2\sqrt{6}$

$-2\sqrt{2}-2\sqrt{6}$

6. $-\sqrt{27}-2\sqrt{45}-\sqrt{20}$

$-\sqrt{9}\sqrt{3}-2\sqrt{9}\sqrt{5}-\sqrt{4}\sqrt{5}$

$-(3)\sqrt{3}-2(3)\sqrt{5}-(2)\sqrt{5}$

$-3\sqrt{3}-6\sqrt{5}-2\sqrt{5}$

$-3\sqrt{3}-8\sqrt{5}$

7. $-9\sqrt{56}+\sqrt{126}$

$-9\sqrt{4}\sqrt{14}+\sqrt{9}\sqrt{14}$

$-9(2)\sqrt{14}+(3)\sqrt{14}$

$-18\sqrt{14}+3\sqrt{14}$

$-15\sqrt{14}$

8. $-3\sqrt{12}+5\sqrt{3}+7\sqrt{20}$

$-3\sqrt{4}\sqrt{3}+5\sqrt{3}+7\sqrt{4}\sqrt{5}$

$-3(2)\sqrt{3}+5\sqrt{3}+7(2)\sqrt{5}$

$-6\sqrt{3}+5\sqrt{3}+14\sqrt{5}$

$-\sqrt{3}+14\sqrt{5}$

KEY: Lesson 4 Notes: Multiplying Radicals

Steps for Multiplying Radicals: $\boldsymbol{outside\sqrt{inside}}$

1. Multiply outside numbers

2. Multiply inside numbers

3. Simplify the radical

Practice:

1. $\sqrt{8}\cdot\sqrt{3}=$

$\sqrt{24}$

$\sqrt{4}\sqrt{6}$

$2\sqrt{6}$

2. $5\sqrt{5}\cdot 3\sqrt{10}=$

$15\sqrt{50}$

$15\sqrt{25}\sqrt{2}$

$15\cdot 5\sqrt{2}$

$75\sqrt{2}$

3. $3\sqrt{20}(\sqrt{5})$

$3\sqrt{100}$

$3\cdot 10$

30

4. $(3\sqrt{2})(-6\sqrt{5})=$

$-18\sqrt{10}$

5. $5\sqrt{12}\cdot 2\sqrt{7}=$

$10\sqrt{84}$

$10\sqrt{4}\sqrt{21}$

$10\cdot 2\sqrt{21}$

$20\sqrt{21}$

Distributing with Radicals:

a. $5\sqrt{3}(6+2\sqrt{5})$

$30\sqrt{3}+10\sqrt{15}$

b. $-2\sqrt{15}(-3\sqrt{3}+3\sqrt{5})$

$6\sqrt{45}-6\sqrt{75}$

$6\sqrt{9}\sqrt{5}-6\sqrt{25}\sqrt{3}$

$6\cdot 3\sqrt{5}-6\cdot 5\sqrt{3}$

$18\sqrt{15}-30\sqrt{3}$

Binomials with Radicals:

- **Use box method or distribution**

a. $(2\sqrt{2}+6)(\sqrt{5}-7)$

$2\sqrt{10}-14\sqrt{2}+6\sqrt{5}-42$

b. $(8+3\sqrt{12})(-4+2\sqrt{6})$

$-32+16\sqrt{6}-12\sqrt{12}+6\sqrt{72}$

$-32+16\sqrt{6}-12\sqrt{4}\sqrt{3}+6\sqrt{36}\sqrt{2}$

$-32+16\sqrt{6}-12\cdot 2\sqrt{3}+6\cdot 6\sqrt{2}$

$-32+16\sqrt{6}-24\sqrt{3}+36\sqrt{2}$

c. $(5-4\sqrt{5})(-2+\sqrt{5})$

$-10+5\sqrt{5}+8\sqrt{5}-4\sqrt{25}$

$-10+13\sqrt{5}-4\cdot 5$

$-10+13\sqrt{5}-20$

$-30+13\sqrt{5}$

d. $(3-4\sqrt{6})^2$

$(3-4\sqrt{6})(3-4\sqrt{6})$

$9-12\sqrt{6}-12\sqrt{6}+16\sqrt{36}$

$9-24\sqrt{6}+16\cdot 6$

$9-24\sqrt{6}+96$

$105-24\sqrt{6}$

KEY Lesson 4

Radicals Cycle Stations

Student Work Page

Write the station letter of the first station in the start box. Then solve that problem and find the answer on another station card. Record that letter in the next box and solve the problem. Keep repeating this process until you have filled the boxes. The answer to the last station you write in the box should be from the first station. You can start at any station.

Start: This should cycle through no matter where students start.

A	**C**	**J**	**D**	**I**	**F**	**H**	**G**	**B**	**E**

Show your work for each station below:

Station ____A____

$$63\sqrt{75}$$
$$63\sqrt{25}\sqrt{3}$$
$$63 \cdot 5\sqrt{3}$$
$$315\sqrt{3}$$

Station ____C____

$$-6\sqrt{3} - 3\sqrt{18}$$
$$-6\sqrt{3} - 3\sqrt{9}\sqrt{2}$$
$$-6\sqrt{3} - 3 \cdot 3\sqrt{2}$$
$$-6\sqrt{3} - 9\sqrt{2}$$

Station ___J_____

$$-20 + 10\sqrt{18} - 4\sqrt{2} + 2\sqrt{36}$$
$$-20 + 10\sqrt{9}\sqrt{2} - 4\sqrt{2} + 2 \cdot 6$$
$$-20 + 10 \cdot 3\sqrt{2} - 4\sqrt{2} + 12$$
$$-20 + 30\sqrt{2} - 4\sqrt{2} + 12$$
$$-8 + 26\sqrt{2}$$

Station ___D_____

$$8\sqrt{36}\sqrt{3} + \sqrt{4}\sqrt{3}$$
$$8 \cdot 6\sqrt{3} + 2\sqrt{3}$$
$$48\sqrt{3} + 2\sqrt{3}$$
$$50\sqrt{3}$$

Station ____I____

$$(\sqrt{6} + 5)(\sqrt{6} + 5)$$
$$6 + 5\sqrt{6} + 5\sqrt{6} + 25$$
$$31 + 10\sqrt{6}$$

Station ___F_____

$$5\sqrt{16}\sqrt{10} - 2\sqrt{10}$$
$$5 \cdot 4\sqrt{10} - 2\sqrt{10}$$
$$20\sqrt{10} - 2\sqrt{10}$$
$$18\sqrt{10}$$

Station ___H____	**Station ___G____**
$5+\sqrt{10}-\sqrt{10}-2$ $5-2$ 3	$-5\sqrt{6}+\sqrt{24}$ $-5\sqrt{6}+\sqrt{4}\sqrt{6}$ $-5\sqrt{6}+2\sqrt{6}$ $-3\sqrt{6}$
Station ____B____	**Station ___E____**
$-48\sqrt{28}$ $-48\sqrt{4}\sqrt{7}$ $-48\cdot 2\sqrt{7}$ $-96\sqrt{7}$	$-4-3\sqrt{8}+8\sqrt{2}+6\sqrt{16}$ $-4-3\sqrt{4}\sqrt{2}+8\sqrt{2}+6\cdot 4$ $-4-3\cdot 2\sqrt{2}+8\sqrt{2}+24$ $-4-6\sqrt{2}+8\sqrt{2}+24$ $20+2\sqrt{2}$

KEY Lesson 5 Notes: Rationalize the Denominator

For an expression to be in simplest radical form, there cannot be any radicals in the denominator of a fraction.

Here's an example: $\frac{1}{\sqrt{6}}$

We need a way to "remove" the radical from the denominator, and we will do this by rationalizing the denominator.

Steps:	Example: $\frac{14}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$
1. Multiply the numerator and denominator by the radical in the denominator	$\frac{14\sqrt{7}}{\sqrt{49}}$
2. Simplify the radicals (if necessary)	$\frac{14\sqrt{7}}{7}$
3. Simplify the fraction (if necessary)	$2\sqrt{7}$

Practice:

1. $\frac{5}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$\frac{5\sqrt{6}}{6}$

2. $\frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{7\sqrt{3}}{3}$

3. $\frac{13}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\frac{13\sqrt{5}}{5}$

4. $\frac{-10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{-10\sqrt{2}}{2}$

$-5\sqrt{2}$

5. $\frac{1}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$

$\frac{\sqrt{8}}{8} = \frac{2\sqrt{2}}{8}$

$\frac{\sqrt{2}}{4}$

6. $\frac{11}{\sqrt{9}}$

9 is a perfect square number!

$\frac{11}{3}$

Mathlib KEY

(1) Buzz Lightyear decided to go for a walk after his (2) interview today in (3) New York City. While on his walk, he saw a (4) dog without an owner that he decided to take with him. Since his new friend did not have a name, he decided to name it (5) Doug. It was well past lunchtime, so he stopped and got a (6) pizza and then continued his walk. (7) Frozone joined on the walk, and they started looking for the owner of their new friend. After (8) 30 minutes they saw someone calling out "(9) Lucky" and ran over to see if this is the owner. The owner was so excited to have their pet back that they gave (10) $500 cash for a reward!

Question 1

Write in simplest radical form:

$$\sqrt{64}$$

a. 8	Buzz Lightyear
b. $2\sqrt{16}$	Mickey Mouse
c. 32	Mr. Incredible
d. $8\sqrt{8}$	Gaston

Question 2

Write in simplest radical form:

$$2\sqrt{3} + 4\sqrt{75}$$

a. $6\sqrt{78}$	Performance
b. $22\sqrt{3}$	Interview
c. $2\sqrt{3} + 4\sqrt{75}$	Doctor's Appointment
d. $102\sqrt{3}$	Dentist Appointment

Question 3

Write in simplest radical form:

$$-3\sqrt{2}+3\sqrt{20}-3\sqrt{8}$$

a. $-3\sqrt{7}$	Orlando
b. $-9\sqrt{2}+6\sqrt{5}$	New York City
c. $-6\sqrt{2}+6\sqrt{5}$	San Francisco
d. $6\sqrt{5}$	Chicago

Question 4

Write in simplest radical form:

$$2\sqrt{3} \cdot 7\sqrt{2}$$

a. $9\sqrt{5}$	Cat
b. $14\sqrt{5}$	Bunny
c. $9\sqrt{6}$	Snake
d. $14\sqrt{6}$	Dog

Question 5

Write in simplest radical form:

$$\frac{1}{\sqrt{3}}$$

a. $\frac{\sqrt{3}}{9}$	Pluto
b. $\frac{\sqrt{3}}{\sqrt{9}}$	Goofy
c. $\frac{\sqrt{3}}{3}$	Doug
d. 9	Slinky

Question 6

Write in simplest radical form:

$$\sqrt{6}(\sqrt{6}+2\sqrt{3})$$

a. $12\sqrt{2}$	Hamburger
b. $6+6\sqrt{2}$	Pizza
c. $6+2\sqrt{18}$	Hot Dog
d. $36+6\sqrt{2}$	Sub

Question 7

Write in simplest radical form:

$$\sqrt{98} - \sqrt{128}$$

a. $-\sqrt{2}$	Frozone
b. $-\sqrt{30}$	Bo Peep
c. $15\sqrt{2}$	Olaf
d. $-15\sqrt{2}$	Minnie Mouse

Question 8

Write in simplest radical form:

$$\sqrt{72}$$

a. $4\sqrt{18}$	10 minutes
b. $8\sqrt{9}$	An hour
c. $6\sqrt{2}$	30 minutes
d. 36	Two hours

Question 9

Write in simplest radical form:

$$\frac{4}{\sqrt{6}}$$

a. $\frac{\sqrt{6}}{9}$	Pepper
b. $\frac{4\sqrt{6}}{6}$	Bruno
c. $\frac{4\sqrt{6}}{36}$	Spots
d. $\frac{2\sqrt{6}}{3}$	Lucky

Question 10

Write in simplest radical form:

$$4\sqrt{5} \cdot 10\sqrt{10}$$

a. $200\sqrt{2}$	$500 cash
b. $40\sqrt{50}$	A diamond ring
c. $80\sqrt{5}$	AirPods
d. $20\sqrt{2}$	A TV