The Influence of Learning Theories on the Teaching and Learning of Algebra

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This paper reviews the influence of learning theories from cognitive science and constructivism on the teaching and learning of algebra. Through an artifact analysis, we document the changing nature of algebraic instruction. Four articles were randomly selected from each of the last three decades of the twentieth century, along with three from 2000-2005 to total fifteen articles analyzed All the articles analyzed had classroom teachers as an intended audience. The analysis showed that as the dominant learning theory shifted from cognitive science to constructivism, the use of authentic learning activities increased and reflected the influence of both rational and social constructivist learning theories.

Keywords: Authentic Learning, Algebra, Middle School, High School

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The teaching of algebra has changed in the past thirty years or so because of the influence of various learning theories, primarily cognitive science in the earlier years, and constructivism more recently. Many are familiar with the nature of the constructivist learning theories that inspired the NCTM *Standards* (1989, 2000). These documents as well as the related NCTM publications for classroom teachers, such as *Mathematics Teacher*, connect theory to practice. Students today need to "deal with quantitative situations in their lives outside of school… such as the duration of rechargeable batteries, or the cost, size, and gas mileage of automobiles." (NCTM, 2005). As such, authentic learning experiences are an integral part of these *Standards.* Many call for the increased use of authentic learning techniques including connections to real world situations, problem solving, and critical thinking (NCTM, 2000; Shepard, 2000; Stein, Grover, & Henningsen, 1996). However, we could not locate a historical review of the nature of classroom activities published in journals of professional practice for mathematics teachers. Therefore, this paper addresses the following research question: Did the transition from cognitive science to constructivism as the predominant learning theory change classroom practice? We begin by briefly reviewing the influences of cognitive science and constructivism on mathematics education. Then, we provide a historical synopsis of recent developments in algebraic instruction. Finally, we document through artifact analysis how algebraic instruction in practice has responded to the influences of constructivism.

Influences of Learning Theories on Algebraic Instruction

Cognitive science studies grounded in structural orientation and gestalt theory suggested meaningful learning involves an understanding of mathematical structures. In a gestalt view of learning, students use global organizing processes to help solve mathematical problems. Gestalt theorist Katona (Resnick & Ford, 1981, p. 153) advocated that when underlying principles are understood, solutions can be reconstructed, extended and remembered. Information processing theorists argued that children could be taught to efficiently perform algorithms and believed "learning hierarchies result[ed] from analyzing tasks into target behaviors with both stimuli and responses specified" (Resnick & Ford, 1981, p. 205). Cognitive scientists sought to build initial problem representation, and offered the students appropriate procedures for solving problems. The primary goal of mathematics instruction under this theory was "well-structured knowledge" (Resnick & Ford, 1981, p. 235). Authentic tasks were not necessarily an integral component of mathematics instruction because the instructional emphasis was on the cognitive capabilities of the students to perceive mathematical structure and to transfer that knowledge to subsequent problems.

In contrast, constructivism grew out of Piaget's teaching experiments with students. In a basic or weak constructivist framework, theorists recognized that "knowing is active, that it is individual and personal, and that it is based on previously constructed knowledge" (Ernest, p. 336). This belief emanated from von Glasersfeld's first principle of constructivism: "Knowledge is not passively received but actively built up by the cognizing subject" (von Glasersfeld, 1989, p. 182). Constructivism advanced the belief that children learn by participating in the building of their own mathematical knowledge rather than by memorizing facts isolated from their own experiences. Students should be challenged to understand the richness and complexities inherent in their mathematical world (Brooks, 1993; NCTM, 2000). Two variations of constructivist theory extend it in slightly different manners. Socially oriented theories of constructivism include the influence of group behavior and group dynamics on the learning of individuals (Ernest, 1996). Rational constructivist theories include the concept of multiple representations in their definition of constructivism. In the rational constructivist paradigm, Goldin and Kaput (1996) argued this work "characterized the building of powerful systems of representation as an overarching goal of mathematics learning and development" (p. 398). These "systems of representation" include graphs, tables, narratives, and algebraic equations, and are often connected to real-world, authentic tasks for school children.

Historical Overview

As Kieran (1992) noted in her seminal piece on the teaching and learning of algebra, instruction in algebra remained quite static through most of the 20th century. In the 1960s, "discovery learning" became popular. Ernest (1996) aptly describes why this type of learning was *not* based upon constructivist learning theories:

> "Discovery learning" from the 1960s onward was often bound up with a romanticism that in the end was not wholly productive for learners, and we must guard against constructivism becoming identified with this position. There is an undoubted need to interact with learners to negotiate a passage toward socially accepted knowledge. However, forms of discovery learning in which teachers always "funnel" learners toward predetermined solutions presuppose that the teacher is in possession of "the truth," rather than someone aware of the conventional nature of knowledge. (p. 336)

In 1989, Thorpe affirmed that:

> the teaching of algebra in the schools is not significantly different today from what it was fifty years ago

> …[and] the "new math" movement of the 1960s attempted (and briefly, succeeded) in introducing some new ideas and new approaches into algebra instruction. But the changes… have been more cosmetic than substantial" (p. 11)

Elementary algebra classes typically symbolized relationships and performed manipulations (Usiskin, 1988; Kieran, 1992; Brenner et al., 1997). Algebraic instruction usually began with adjustments from arithmetic to algebra patterning a historical learning of algebra in a procedural to structural progression. Here, the end goal was to see algebraic expressions as mathematical objects and for students to be able to carry out operations such as simplifying, factoring, and evaluating algebraic expressions. Most adults today still associate symbolic manipulation such as solving equations and simplifying algebraic expressions with the study of algebra (NCTM, 2000).

In 1980, the NCTM published the *Agenda for Action* promoting problem solving in the mathematics curriculum. By the end of the 1980s, advocates for mathematics education called for instruction in algebra to reflect changes in technology and the way mathematics is used (Kaput, 1989). Algebra, they claimed, should be seen as a tool, rather than simply a "bag of tricks" (Thorpe, 1989, p. 12). A vision emerged in the first NCTM *Standards* (1989) in which algebraic instruction would promote structural understanding in the context of *why* the skill was important (Thorpe, 1989). The corresponding proliferation of technology resulted in the call for using algebra to solve problems rather than doing algebra for the sake of symbolic manipulation (Booth, 1989). Many called for a change in the curriculum to promote different forms of representation including graphical and tabular forms, and the use of manipulatives and other learning aids (Kaput, 1989; NCTM, 1989; Booth, 1989). The NCTM *Standards* (1989) were followed by the *Professional Standards for Teaching Mathematics* (1991). This publication furthered the vision of this implementation by teachers of both rational and social constructivist learning paradigms in the classrooms when they are

> modeling mathematical ideas through the use of representations… as vehicles for examining mathematical ideas. Not only do teachers need to be familiar with a variety of representations, they must be comfortable with helping students construct their own representations… [and] teachers need to focus on creating learning environments that encourage students' questions and deliberations—environments in which the students and teacher are engaged with one another's thinking and function as members of a mathematical community (p. 152).

The NCTM elaborated their vision of elementary algebra in a separate content strand in the most recent version of the *Principles and Standards* (2000). Four major themes permeate the study of algebra in the *Principles and Standards*: all students should (1) understand patterns, relations, and functions; (2) represent and analyze mathematical situations and structures using algebraic symbols; (3) use mathematical models to represent and understand quantitative relationships; and (4) analyze change in various contexts. Understanding change is a foundational concept in algebra and one effective way is through the use of technology to model and interpret data in contexts directly related to a real-world experiences.

Methodology

We documented the influence of cognitive science and constructivism on the teaching and learning in algebra through the analysis of a random selection of articles since 1970 in the NCTM publications for classroom teachers: *Teaching Mathematics in the Middle School*, a publication for grades 5-8; and *Mathematics Teacher,* a publication generally for grades 8-12. The artifact analysis follows a chronological order to document the influences of changing learning theories from cognitive science to constructivism.

Our first step was to identify the articles from 1970 to 2005 that appeared to address the elementary algebra topics typically taught in grades eight or nine. Using the ERIC educational research database for the two publications *Mathematics Teacher* and *Teaching Mathematics in the Middle School*, we first identified the number of articles available.

Table 1. Number of articles on algebra in NCTM journals

Journal	**Years**	**Number of Articles**
Mathematics Teacher	1970-1979	275
	1980-1989	187
	1990-1999	196
	2000-2005	59
Teaching Mathematics in the Middle School	1994-1999	34
	2000-2005	40

Note that in 1994, the NCTM divided the elementary and middle school publication *Arithmetic Teacher* into two subsequent publications, *Teaching Children Mathematics* and *Teaching Mathematics in the Middle School.*

Then, because of space limitations, we decided to choose four of these articles from each of our first three arbitrary time periods (the 1970s, 1980s, and 1990s), and three articles from 2000-2005. Next, we assigned a number to each of the identified articles within each time period and picked articles randomly using a random number generator. We intend this small sample of randomly selected examples to represent the kind of articles that were published during each time period.

Results

Articles from the 1970s

The first example is entitled *Numerical Solutions of Linear Equations* (Niebaum, 1973). Niebaum presented a detailed procedural explanation to verify the accuracy of solutions obtained by solving linear equations. The article also described a second procedural manipulation that students could use to verify the accuracy of their results. The article highlighted the importance of accuracy. The second article, *A Discovery in Linear Algebra,* Nicolai (1974) described a "discovery" made by the author's junior high school class that was "stumbled" upon during class. They found that when given a pair of linear equations like $ax + by = c$ and $dx + ey = f$, the solution to the system is always (-1, 2) if the coefficients (a, b, c, d, e, and f) are consecutive integers or any arithmetic sequence of real numbers.

The third example, *A Strategy for Using LSD!* Hancock (1976) addressed the difficulty that students commonly experience when faced with algebraic word problems, specifically, problems involving a difference. As a result, the author proposed the use of 'LSD', where 'L' stands for larger, 'S' stands for smaller and 'D' stands for difference. Using these, students are taught to remember three formulas: L – D = S, L – S = D, and S + D = L. Thus, as the students read problems that fit the necessary criteria, they then input the given information from the problem into an appropriate relationship using one of the three formulas. Finally, in *Errors in First-Year Algebra*, Laursen (1978) outlined common mistakes related to reducing algebraic fractions. Laursen lamented the introduction of shortcuts, which students often remember how to do while losing the context of when it is proper to do so. This article emphasized the importance of strict step-by-step manipulations using terms that are carefully defined to the students.

Articles from the 1980s

The first example, entitled *Families of Lines* (Hirsch, 1983), targeted grades 8-11. The objectives in the activity called for students to (1) graph pairs of linear equations and use the results to discover an algebraic technique for solving a system of equations, and (2) complete, run, and modify a BASIC program for solving a system of linear equations. The activity directed students to *discover* an algebraic technique with the careful guidance of a set of worksheets and the teacher. The problems are all numerical in nature and they are not set in a context.

The second example, *Microcomputer Unit: Graphing Parabolas* (Hastings and Peterman, 1986), involved graphing quadratic functions using Apple computers. Here, students used the computer to help investigate how the value of the coefficients in the equation $y = Ax^2 + Bx + C$ influenced the shape of a parabola. Hastings and Peterman emphasized that the "activity is sequential, and thus it is important that each sheet be completed [in order]" (p. 713). Students would summarize their discoveries at the end of class.

In our third example, Wallace (1988) described an activity in which students were exposed to concepts of linear functions in his article, *Activities: Using Linear Functions*. The problem in the activity made reference to a real life experience about electricians and their work. Wallace hoped that by introducing the linear function concepts concretely using graphs and geometric aspects, the students would be better prepared to learn and understand the abstract manipulations. Attached to the article were worksheets the students would use to complete the activity. Many aspects of the worksheets were based on procedural manipulations. There were a few questions that required the students to explain their reasoning, but for the most part there was an emphasis on skill development and procedures.

In our final example, *Using Diagrams to Solve* problems, Stimpson (1989) described an activity where students were divided into groups and given a series of worksheets related to using diagrams to represent algebraic expressions, such as, "How can you represent that Mutt and Jeff ate thirty-four cans?" or "How can we show that Mutt is eating half as much?" Problems became increasingly more difficult. Stimpson advised classroom teachers who might implement the ideas in the article to stick with the pictorial representations until students are ready to move to the more formal notations of algebra.

Activities from the 1990s

Our first example from the 1990s, *Relating Graphs in Introductory Algebra* (Van Dyke, 1994), referenced the use of graphing calculators, demonstrating a shift from programming computers to hand-held technology characteristic of the early 1990s. The activity for grades 6-12 showed students "that a graph and the corresponding algebraic statements represent the same set of points" (p. 427). The activity called for students to match an appropriate graph with a verbal statement, draw an appropriate graph to match a verbal statement, and interpret information from a graph. Activities related to students' lives such as "we climbed up a hill, paused to rest, and then sledded down it" would be matched with an appropriate graph. Answers could vary for the graphs. Students critiqued answers in small groups. Van Dyke included open-ended questions to stimulate critical thinking and reflection on the part of the students.

The Write Way: A look at Journal Writing in First Year Algebra (Dougherty, 1996) is the second example chosen for the 1990s. Dougherty explained that carefully selected writing prompts can enhance student understanding in algebra because writing forces students to think critically about the mathematics they are using and helps students make connections to other topics within mathematics, as well as topics outside of the area of mathematics. Dougherty further explained that in his classroom, he found writing prompts helped develop diverse thinking and helped his students to appreciate multiple solution strategies.

In the next article *Telephones and Algebra* (Appelbaum, 1997), the author encouraged teachers to interest students in algebra by implementing more ideas from their culture and everyday lives into the problems that might be used in class. The author summarized her attempt to use the booming interest in cell phones as an opportunity to encourage student interest in algebra. Students researched rates offered by cell phone companies and learned to represent those rates as algebraic expressions and graphs. Students were also challenged to interpret those graphs to find the best deal for a given consumer usage.

Finally, Thomas and Thomas (1999) also implemented technology to scaffold students learning strategies of linear equations. In *Discovery Algebra: Graphing Linear Equations*, students used a computer program, Lesson Graph, to learn how to graph linear equations and develop their own mathematical reasoning. Here, students worked in pairs first developing conjectures on the concepts of slope and y-intercepts, and then testing those conjectures as they entered different equations into the Lesson Graph simulator. The teacher noted that "the strategies of controlling variables, making and testing conjectures, and building on the knowledge of others were foreign to these students" and that the teacher "was challenged to anticipate, recognize, and capitalize on all the valuable opportunities that arose" (p. 572).

Articles from 2000-2005

Algebra in the Middle Grades (Lambdin, Lynch & McDaniel, 2000), our first example from this time period, encouraged students to think about rate of change and the shapes of graphs in a specific context. The students engaged in a data collecting activity taken from the C*onnected Mathematics Program* (Lappan et. al., 1998). To simulate fatigue in sporting events, students worked in groups taking turns collecting data on the number of jumping jacks completed in two minutes. After students collected the data, the teacher provided graph paper but the students decided how big to make their graphs and what scale to use. The activity helped students develop insights into how changes in data and related rates of change affected the shapes of the graphs. The authors reported that "some heated discussion took place about whether graphs should consist of discrete points or of points connected with smooth lines" (p. 198).

In our second example, *Building Students' Sense of Linear relationships by Stacking Cubes,* Gregg (2002) helped students learn to understand linear relationships and connect the graphical and symbolic representations of these relationships. In these activities, students were given a pattern, asked to identify the pattern, and draw sequential towers in the pattern. Students were directed to focus on the change in height from one tower to another. The mathematical pattern in the first activity was 2*p* + 1 and students were asked questions that would help them focus on building a formula on the basis of the change from one tower to the next. Students could draw or build the towers. The highlights of these activities were the questions the teacher asked to promote student thinking, such as "how many cubes would be in the zeroth tower?" [the tower preceding the given first tower] or "if you start with the zeroth tower, how many more cubes would you need to build the third tower?" (p. 331). This questioning technique was used throughout the lesson. The series of activities was intended to help students think of the concept of slope as a rate of change.

The final example was entitled *Promoting Problem Solving across Geometry and Algebra by Using Technology* (Erbas et al, 2005). This article explained why the use of technology is not only useful, but needed in every mathematics classroom that claims to promote problem-solving. The authors used an example of the Pythagorean Theorem to show how technology challenges students to see multiple representations, find multiple solutions, and interact with problems. The activities attempt to make the problem attractive to students by using Geometer's Sketchpad, spreadsheets, and graphing calculators to provide multiple technological representations of the Pythagorean theorem.

Discussion

In this section we briefly analyze the example articles in each of the time periods, relating the articles to cognitive science and constructivist learning theories and the teaching styles associated with those learning theories. Three of the articles from the 1970s emphasized teaching procedural processes to avoid common mistakes and solve traditional word problems in a teacher-centered environment. The other article related a discovery made by students in the author's teacher-centered classroom that could be demonstrated by other teachers in their own classrooms. All of these activities were context-free. The influence of cognitive science learning theories, the predominant learning theory at the time, was evident in the emphasis upon well structured knowledge presented in a sequential order. The nature of these articles also corroborates Thorpe's (1989) observations that the teaching of algebra had not changed much over the twentieth century.

During the 1980s, two of the activities discussed in the articles were context-free computer-based activities that sequentially investigated algebraic relationships through graphs with the purpose of leading students to procedures and rules. The other two activities used teacher-provided limited contexts to develop procedure, skills, and representations. This "discovery" reflected the point made by Ernest (1996) earlier in this paper. Deviations from that pattern generally would not be accepted. The computer programs directed students to find a solution devoid of any connection or application to a real life problem. The objectives reflected the influence of cognitive science and information processing. Following the lesson, students learned correspondence, integration, and connectedness from practicing the skills on the worksheets. The use of computers began to emerge during this decade and there is some evidence of weak constructivism, possibly emanating from the work of Piaget. However, in an overall sense, the predominant learning theory was still based upon cognitive science. The lack of articles in this sample illustrating other principles of teaching and learning that would be promoted in the 1989 Principles leads one to the possible conclusion that such teaching strategies were rare.

The articles sampled from the 1990s illustrated major changes in the types of activities they described. These activities taken as a whole included group work, pairing of students, graphing calculators, comparing representations, open-ended questions, different solutions to verbal problems, different solution strategies, student conjectures and investigation, a real-world context researched by the students, writing in the math classroom, and making connections. These activities were more student-centered and required the teacher to appreciate and use student thinking. Evidence of social constructivist learning theories is apparent in that students developed their conjectures together and shared subsequent findings. These activities prompted students to have frequent opportunities to use (and discuss) multiple representations such as words, tables, graphs, and equations to solve a contextualized authentic problem involving change, thus reflecting the importance of multiple representations from rational constructivism. This dramatic change in the nature of the activities published in the NCTM journals for teachers illustrated that some teachers were using at least some of the principles in their classrooms and were willing to write articles about their practice. Teachers who read these articles were encouraged to develop more authentic learning environments by using real-world data and constructivist processes that reflect the genuine practice of professionals who use mathematics.

Finally, between 2000-2005, the activities described in the literature continued to emphasized student-centered learning and extended that philosophy by promoting the use of multiple technologies used concurrently, student discussions of mathematical ideas, and the changing role of the teacher in the mathematical classroom. The mathematics teacher is portrayed as the facilitator of learning, not the source of mathematical knowledge. One essential aspect of the teacher as facilitator illustrated by one of the examples is the importance of teacher questions in the learning process. These articles were again replete with authentic learning connections and driven by both social and rational constructivism. Not only was the use of technology promoted, but the use of multiple technologies further supported the evidence of rational constructivism. Perhaps the best characterization of these activities is that they were in fact process-driven rather than being driven by learning procedures and drilling skills.

Conclusion

Over the years covered in this investigation, the roles of the teachers and students in the published activities have changed dramatically. The teacher's role in the earliest articles, based upon learning theories from cognitive science, was primarily concerned with teaching specific procedural manipulations to the students. Then, as constructivist learning theories proliferated, the nature of algebraic instruction shifted as well as the role of the teacher. While instructors still placed an emphasis on procedures, they also attempted to make mathematical connections to real world applications. The changing emphasis in the articles from a teacher-centered, cognitive science-based focus on procedural knowledge and ways to package that knowledge for students in the earlier articles to student-centered conjecture and discussion-based activities using constructivist ideas, technology, and authentic contexts is obvious and dramatic.

Although we acknowledge that the representation of these articles reflects the optimal vision of practice and perhaps the views of the NCTM editors and reviewers, they do help support the claim that many educators are making the shift from lecturing to facilitating, with a repertoire of authentic learning experiences. Students still need to be proficient at algebraic manipulation, but they now have the opportunity to connect mathematics to real world examples, engage in problem solving, make and test conjectures, use various technologies, and integrate their own strategies into their work. The learning behaviors and teaching styles that are evidenced in the latest articles display the implementation of the principles of constructivist learning theories that incorporate authentic learning.

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