Increasing Growth Mindset through Collaborative Learning in Algebra 1

By

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Abstract

This curriculum project focuses on instructional practices that promote a growth mindset through peer collaboration. Teachers seeking materials that support student learning in small groups or partnerships will find these lessons beneficial. The lessons focus on the first eight days of the school year towards developing sociomathematical classroom norms of collaboration, allowing for students to be challenged and encouraged by their peers. This curriculum project includes the following topics; order of operations, evaluating expressions, properties of real numbers and translating English to algebra.

Keywords: Growth mindset, mathematical mindset, algebra 1, collaborative learning, high school transition

Introduction

The transition from middle school to high school can be difficult for students for several reasons, including peer relationships, decreased parental supervision, new teachers and often new school locations (Neild, 2009). Through this transition and other factors students can develop a fixed mindset about mathematics. A student with a fixed mindset would be "invested in looking smart never looking foolish; they avoid effort because it makes them feel dumb" (Dweck, 2014, p.10). Students often think that only some people can be good at mathematics. curriculum design focus "on the process of learning and maintaining a problem-solving approach which [may] counter a student's insecurities about what they perceive as their inability" (Howard & Whitaker, 2011, p. 14). The beginning of the school year is a vital time for teachers to create a positive classroom environment. This curriculum is designed to be focused on introducing students into an inclusive high school classroom, with activities to encourage student dialogue and peer collaboration.

Literature Review

Transition from Middle School to High School

The transition from middle school to high school can be difficult for students. A factor in this difficult transition is "that ninth grade coincides with life-course changes, such as reduced parental supervision and increased peer influence" (Neild, 2009, p. 53). According to Neild (2009), some students are simply not prepared for the rigor of high school. "Failing math or English in the middle grades was a better predictor than standardized test scores of academic difficulties in ninth grade" (Neild, 2009, p. 60). Neild expressed that the way in which high school is organized can be difficult for students to feel connected to their teachers or peers. "The traditional social organization of high school, in which teachers' primary allegiance is to subject-matter departments and students are hurried from one forty-five-minute class period to another, can leave students feeling anonymous and alienated" (Neild, 2009, p.61).

To support the transition for students, it is suggested that educators focus on the vertical alignment of content and teaching strategies (Brown & Seeley, 2010). As students move from younger grades to high school, there is often a decrease in "hands-on exploration and less individualized attention" in mathematics classrooms (Brown & Seeley, 2010, p. 355). Brown & Seeley (2010) suggested that middle school and high school mathematics teachers work together to ease the transition. The teachers need to "clarify what content each is responsible for teaching, how that content is taught, and how it could be assessed" (Brown & Seeley, 2010, p 356). Another suggestion is for teachers to create a growth mindset atmosphere where students are excited to learn mathematics. "Students' sense of belonging in the mathematics classroom can significantly affect whether -- and the degree to which -- they chose to engage in their learning" (Brown & Seeley, 2010, p. 356).

Growth Mindsets

The National Council of Teachers of Mathematics (NCTM) states that a positive, inclusive and nurturing classroom climate will support students learning of mathematics (Sole, 2019). A classroom environment with a focus on growth mindset will encourage students to persevere on harder problems and develop their critical thinking skills (Sole, 2019). In contrast to a fixed mindset where students are "invested in looking smart never looking foolish; they avoid effort because it makes them feel dumb" (Dweck, 2014, p .10). A growth mindset can be promoted through "challenge seeking, hard work, good strategies, focus and persistence instead of ability or intelligence" (Dweck, 2014, p. 10).

Strategies for students to be successful in mathematics classrooms include; "attendance, taking notes, and students' ability to ask questions effectively...[and] accurate homework completion" (Howard & Whitaker, 2011, p.3). Researcher Howard & Whitaker (2011) suggest, "focusing on the process of learning and maintaining a problem-solving approach may counter a student's insecurities about what they perceive as their inability" (Howard & Whitaker, 2011, p. 14). Focusing on the learning process can include lessons centered on student engagement, homework help time at the end of class, and test/assignment reflections (Howard & Whitaker, 2011).

Cooperative Learning

As expressed in *Principles to Action* (2014) by NCTM, a successful mathematics classroom must include meaningful dialogue. "Students must have the opportunities to talk with, respond to, and question one another as part of the discourse community, in ways that support the mathematics learning of all students in the class" (NCTM, 2014, p. 30). In a study conducted by Qaisar, Dilshad, and Butt (2015), students perceived mathematical abilities were influenced

positively by peer collaboration. The researchers believe that since students were given "opportunities to share ideas, challenge each other and justify hypothesis" (Qaisar, et al., 2015, p. 83). Yackel and Cobb (1996) introduced these ideas as sociomathematical classroom norms, which they defined as "normative aspects of mathematical discussion that are specific to students mathematical learning activities" (p. 458).

Curriculum Design

This curriculum focuses on the first few lessons of the first mathematics course that counts as a Carnegie unit for high school graduation in most US schools, algebra 1. These lessons were designed to give students opportunities to discuss and justify their mathematical reasoning to their peers. These lessons focus on algebraic basics while reviewing eighth grade material. Each lesson is organized for a 40-minute block. To establish sociomathematical classroom norms grounded in the growth mindset, classroom instruction should include students working together in partnerships or small groups.

Day 1: Mathematical Partnerships

Teacher Goals: Set the tone for the school year, create an inclusive and positive environment, create classroom rules for partner work.

Student's I can: I can work productively with a partner in math class.

Standards of Mathematical Practice:

- Construct Viable Arguments and critique the reasoning of others.

Common Core Standards:

- N/A

Outline: Students will begin with an individual activity, where each student will identify important aspects of partnerships in classrooms. This individual time will give students time to reflect personally before sharing their ideas with their seat partner. Students will then work with their seat partner to think more deeply about partnership in mathematics. The class will come together to discuss and make a poster identifying the classroom rules for partnerships. This lesson is adapted from Youcubed.org which is a resource created by Jo Boaler and her team of growth mindset specialists (Boaler et al., 2019).

Name: _____ Date: _____
Activity 1 Partnerships and Growth Mindset

Individual Activity:

In Algebra 1 you will be working with a partner almost every day. Write down 3 key aspects of a productive mathematical partnership.

1. _____
2. _____
3. _____

Partner Activity:

Do this in a mathematical partnership	Do NOT do this in a mathematical partnership

Day 2: Four Fours Activity using the Order of Operations

Teacher Goals: Provide a low-stress opportunity for students to use the classroom rules, review the order of operations

Student's I can: I can use the order of operations to simplify expressions

Standards of Mathematical Practice:

- Reason abstractly and quantitatively
- Attend to precision
- Make sense of problems and persevere in solving them

Common Core Standards:

- Work with radicals and integer exponents. (NY-8.EE)
- Interpret the structure of expressions (AI-A. SSE)
- Write expressions in equivalent forms to reveal their characteristics (AI-A.SSE)

Outline: Students will review the order of operations, then use them to work on the Four Fours activity with their partner. This activity is adapted from Youcubed.org (Boaler et al., 2019). In this activity, students can only use the numbers; 4, 4, 4, and 4 to create as many numbers as possible. Students must use all four of the fours. Students can use any operation to create new numbers. The goal is not for students to create the most numbers, but for students to work collaboratively while using the order of operations correctly. After giving students an appropriate amount of time to develop solutions, ask each set of partners to identify one example that they are most proud of. Have each group share out their proud moment!

Name:_____ Date: _____
Activity 2 Order of Operations

I can _____

Do now: Fill in the acronym below.

P	
E	
M	
D	
A	
S	

Examples:

1.) $\frac{3*4}{6} + (9-3)^2$

2.) $7(-2)^2 + 4(5)^2$

Four Fours Activity

Challenge: Use the order of operations and 4 fours to result in the numbers 1 - 16. You can use any combinations of operations including; multiplication, addition, subtraction, division, square root. You must use only the 4 fours, but you can put them together (44). Be sure to include parentheses when needed!!

1.
2.
3.
4.
5.
6.
7.
8.

9.
10.
11.
12.
13
14.
15.
16.

Day 3: Evaluating Expression

Teacher Goals: Assess students' current ability to evaluate expressions, review the importance of parentheses, provide classroom time for students to work collaboratively

Student's I can: I can evaluate expressions using the order of operations.

Standards of Mathematical Practice:

- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning.

Common Core Standards:

- Work with radicals and integer exponents. (NY-8.EE)
- Interpret the structure of expressions. (AI-A.SSE)
- Write expressions in equivalent forms to reveal their characteristics (AI -A. SSE)

Outline: Students will build on their understanding from yesterday to practice evaluating expressions using the order of operations. There will be a focus on evaluating expressions that include exponents and negative numbers. The lesson will begin with a review of the Order of Operations, and then students will work with their partner to practice evaluating expressions. Remind students to use the classroom rules while working with their partner.

Name:_____ Date: _____
Algebra 1 - Day 3 PEMDAS & Expressions

I can _____

Do Now: Use the order of operations to simplify the following expressions.

1.) $\frac{(4+4)^2}{(4+4)}$ 2.) $4 \cdot 4 + \frac{4}{4}$

Explain: What is the difference between an **equation** and an **expression**?

Ex 1: Evaluate the following expression when $x = 9$.

$$4x - 17$$

Ex 2: Evaluate $16 - 3x^2$ when $x = 2$.

Ex 3: Evaluate $16 - 3x^2$ when $x = -2$.

Ex 4: Evaluate $x^2 - 6x + 3$ when $x = -3$

Work on the following with your partner:

1.) Evaluate $x^2 - 3x + 1$ when $x = -1$	2.) Evaluate $x^2 + 3x + 6$ when $x = -1$
3.) Evaluate $(a + 4)(3 - a)$ when $a = -4$	4.) Evaluate $(a - 4)(3 + a)$ when $a = -6$
5.) Evaluate $(z - 2)^2$ when $z = -3$	6.) Evaluate $-3(1 - t)^2$ when $t = -4$
7.) Evaluate $w(-3 - w)$ when $w = -3$	8.) Evaluate $r^2 + 3$ when $r = -1$
9.) Evaluate $\frac{4(t)^2}{t}$ when $t = -9$	10.) Evaluate $\frac{(f-3)^2}{5}$ when $f = -7$

Day 4: Associative and Commutative Properties

Teacher Goals: Introduce students to the concepts of associative and commutative properties.

Student's I can: I can explain the commutative and associative properties.

Standards of Mathematical Practice:

- Construct Viable Arguments and critique the reasoning of others
- Attend to precision
- Look for and make use of structure

Common Core Standards:

- Understand solving equations as a process of reasoning and explain the reasoning (AI-A.REI)

Outline: Students will learn about the associative and commutative properties. As their warm-up, students will analyze the definitions of associate and commute. With their partner, students will write two sentences using each of the words. Students will share out their sentences.

The class will then play a game organized by the teacher. Instruct students to sit on top of their desks. Much like Simon Says, the students will be doing various tasks like jump on one foot, high five your partner etc. This game will include the instruction to use the associative property and the commutative property. Before playing the game, discuss as a class how to use those properties. Write the rules on the Smart Board, students can also write the instructions on the back of their worksheet. Guide students to switch seats with a partner for the commutative property and to find a new seat and a new partner for the associative property. Play the game for a couple of rounds. Bring the class together, and guide the students to closure using the worksheet.

Name: _____ Date: _____
Algebra 1 - Day 4 Commutative & Associative Properties

I can _____

Vocabulary:
According to Merriam-Webster Dictionary:

Associate (Verb)

- to join as a partner, friend or companion
- to join or connect together: COMBINE

Associate (Noun)

- One associated with another

With your partner write two (2) sentences using the word associate.

1. _____
2. _____

Vocabulary:
According to Merriam-Webster Dictionary:

Commute (Verb)

- To travel back and forth regularly
- To yield the same mathematical result regardless of order
- To give in exchange for another
- To convert into another form

With your partner write two (2) sentences using the word commute.

1. _____
2. _____

What happened when we used the commutative property?

_____

What does that look like as an Algebraic representation?

What happened when we used the associative property?

_____

What does that look like as an Algebraic representation?

Rearrange the following expressions using the **commutative** property but do not solve:

1.) $3 + 4 + 9 =$____ + ____ + ____ 2.) $8 \times 3 =$ ____ × _____

3.) $1 \cdot 4 \cdot 7 =$ _____ ·_____·_____ 4.) $-9 + 7 + -3 =$ _____ + _____ + _____

The examples above only use multiplication and addition. Can we use the rule for subtraction or division? With your partner make two numerical examples.

Subtraction	Division

Day 5: Associative and Commutative Properties

Teacher Goals: Reinforce the difference between the associative and commutative properties, represent a visual for students to remember for the commutative and associative properties

Student's I can: I can explain the associative and commutative properties.

Standards of Mathematical Practice:

- Construct Viable Arguments and critique the reasoning of others
- Attend to precision
- Look for and make use of structure

Common Core Standards:

- Interpret the structure of expressions (AI-A. SSE)
- Write expressions in equivalent forms to reveal their characteristics. (AI-A. SSE)
- Understand solving equations as a process of reasoning and explain the reasoning. (AI-A. REI)

Additional Materials: Large print out versions various numbers, variables, multiplication signs, addition symbols, and parentheses

Outline: The commutative and associative properties were introduced on day 4. Today the focus is to allow students to see a visual representation of the differences between the properties. After reviewing the Do Now, ask for three volunteers. Give one volunteer a plus sign, and the other two a variable or number. Ask them to set up an expression, have students write the expression in the first box on the table. Instruct the students to use the commutative property on the expression. Students should write down the new expression. Ask for two more volunteers, and repeat the same process. Introduce two parentheses, and complete the associative side of the table.

Name: _____ Date: _____
Algebra 1 - Day 5 Commutative & Associates Properties Day 2

I can _____

Do Now: Explain the commutative and associative properties:

The Commutative Associative Game -

Commutative		Associative	

Organize the following as the associative or commutative properties and justify your reasoning.

1.) $(3 + 4) + 9 = (4 + 3) + 9$ _____

_____

2.) $(3 \cdot 4) \cdot 5 = (3 \cdot 5) \cdot 4$ _____

_____

3.) $(1 + 2) + 3 = (3 + 1) + 2$ _____

_____

4.) $7(8 \cdot 9) = 7(9 \cdot 8)$_____

_____

5.) $m + (t + p) = t + (m + p)$ _____

_____

6.) $(r + y) + (w + s) = (y + r) + (s + w)$_____

_____

7.) $(q \cdot v) \cdot (h \cdot a) = (h \cdot a) \cdot (q \cdot v)$_____

_____

8.) $\frac{1}{2} \cdot (b \cdot c) = (\frac{1}{2} \cdot b) \cdot c$ _____

_____

Summarize your understanding here:

Commutative Property Definition _____

Examples:

Associative Property Definition _____

Examples:

Day 6: Distributive Property

Teacher Goals: Focus on creating equivalent expressions by using the distributive property and combining like terms

Student's I can: I can use the distributive property to simplify expressions.

Standards of Mathematical Practice:

- Attend to precision
- Look for and make use of structure
- Reason abstractly and quantitatively

Common Core Standards:

- Understand solving equations as a process of reasoning and explain the reasoning. (AI-A.REI)
- Perform arithmetic operations on polynomials. (AI-A.APR)
- Write expressions in equivalent forms to reveal their characteristics. (AI-A.SSE)

Outline: Students will write two sentences using the word distributive in a mathematical context. Guide students through the examples of the distributive property. Allow students to work with their partner and to show all of their work

Name: _____ Date: _____
Algebra 1 - Day 6 Distributive Property

I can _____

Vocabulary:
According to Merriam-Webster Dictionary:

Distribute (Verb)

- To divide among several or many
- To spread out as to cover something

With your partner write two (2) sentences using the word distribute.

1. _____
2. _____

Examples of the Distributive Property.

1.) $2(3+5)$

2.) 2(x+3)

3.) 2(x+y)

4.) z(x+y)

5.) $4(3x-9)$

6.) $-2(t+10)$

7.) $-8(-2p-5)$

8.) $-5(3x^2-4x+1)$

9.) $3(y^2+6y-2)$

10.) $2x(3-4x^2)$

Simplify the following expressions by combining like terms. Be sure to use the distributive property carefully.

1.) $-2(3x+1)+4x$

2.) $-(x-9)+10$

3.) $-7(x-7)+3(x\ -1)$

4.) $5(2x+6)-3(x+10)$

5.) $2(4-x)\ -8(x)$

6.) $-(x+16)\ +(5x-3)$

7.) $2x(x+3)\ -(x-2)$

8.) $-3x(5x-8)\ +4(10-3x)$

Day 7: Translating English to Algebra

Teacher Goals: Introduce students to reading and writing algebraic expressions

Student's I can: translate English to Algebra.

Standards of Mathematical Practice:

- Construct Viable Arguments and critique the reasoning of others
- Attend to precision
- Look for and make use of structure

Common Core Standards:

- Understand solving equations as a process of reasoning and explain the reasoning. (AI-A.REI)
- Write expressions in equivalent forms to reveal their characteristics. (AI-A.SSE)
- Create equations that describe numbers or relationships. (AI-A.CED)

Outline: Students will write synonyms for addition, subtraction, multiplication and division. Review with students examples of variables. Students will complete the remainder of the worksheet with their partner. Review at the end of the class period.

Name: _____ Date: _____
Algebra 1 - Day 7 Translating English to Algebra

I can _____

Do Now: Jot down as many synonyms for the following operations.

Addition	Subtraction	Multiplication	Division

Variables	
Example	Non Example
Visual	Definition

Translate each of the following from English to Algebra

1. Five less than 10 times a number, x. _____
2. The sum of ½ of a number, y, and 14. _____
3. The difference between 33 and four times x. _____
4. The product of 9 and a number z._____
5. The ratio of x to 12. _____

Translate each of the following from Algebra to English. Write each two different ways!

1. $1 - 4x$ _____

   _____

2. $\frac{2x}{3}$ _____

   _____

3. $\frac{1}{4}$x+9 _____

   _____

4. $\frac{1}{4}(x + 9)$ _____

   _____

Day 8: Translating English to Algebra Day 2

Teacher Goals: Students will practice reading and writing algebraic expressions, students will make a quiz for

Student's I can: translate English to Algebra.

Standards of Mathematical Practice:

- Construct Viable Arguments and critique the reasoning of others
- Attend to precision
- Look for and make use of structure

Common Core Standards:

- Understand solving equations as a process of reasoning and explain the reasoning. (AI-A.REI)
- Write expressions in equivalent forms to reveal their characteristics. (AI-A.SSE)
- Create equations that describe numbers or relationships. (AI-A.CED)

Outline: Students will complete the Do Now with their partner and the following exercise. Students should then create 10 written expressions with solutions to handed in.

Name: _____ Date: _____
Algebra 1 - Day 8 Translating English to Algebra 2

I can _____
Do Now: How many ways can you express the following in English?

x-8		z+12	
$2 \cdot y$	Two times y	$\frac{w}{2}$	

The following two examples are very similar. Explain how they are different from each other, then identify translate into an algebraic expression.

1.) Twice the sum of the variable, x and 10.

2.) The sum of twice the variable x and 10.

Validity

This curriculum was implemented in three sections of Common Core Algebra 1. This lesson was implemented on the first instruction school day of the academic year. The class rules are still posted in the classroom as a reminder of student expectations. Students seemed actively engaged in all aspects of the learning process through peer collaboration and student dialogue. Students still ask to play the commutative and associative game. Students did need more refreshers of 8th grade topics such as; exponents, square roots, and combining like terms. Notes from each day's activities are bulleted below:

Day 1 - Partnerships and Growth Mindset

- Students enjoyed this activity.
- Great way to start the school year.
- This activity only took about 20 minutes to complete.

Day 2 - Four Fours

- Students were definitely engaged! Great dialogue!
- Students needed a refresher of the square root.
- Students struggled to find a lot of the numbers, but could find multiple ways to find a few.

Day 3 - PEMDAS & Expressions

- Students struggle to use parentheses.
- Students wanted to use calculator, but struggled to use the correct techniques. Students were using graphing calculators for the first time. Students needed to be shown the difference between negative and subtraction on the Ti-84.
    - Proposed Edit: Include more calculator work.

Day 4 - Commutative & Associative Properties

- Students struggled to write creative sentences to describe associative and commutative.
- Students enjoyed the game, but struggled to translate the game into an algebraic representation.
- Students quickly caught on that subtraction and division would not work for the properties.
- Students were enjoying themselves.
- Some students even noted; "I didn't know that math could be fun."

Day 5 -Commutative & Associative Properties Day 2

- This activity could have been better organized.

- Students expressed that they loved the visual, but could have done as partner activities with cut outs.
- Some students struggled to keep up with this activity.
- Students struggled with #6.
- Students needed a more concrete way of seeing the examples.
    - Proposed Edit: Have the teacher keep track of the examples, as a class, and focus on the discussion.

Day 6 - Distributive Property

- Students needed a review of exponent rules and like terms.

Day 7 - Translate English to Algebra

- Students struggled to write their own sentences (especially the last two).

Day 8 - Translate English to Algebra Day 2

- Students struggled to write creative and challenging expressions but enjoyed the process.

Conclusion

As expressed by Cobb et al., (2001) students are expected to "act, and learn as they participate in and contribute to the development of a system that is larger than themselves, the classroom community" (p. 153). The beginning of the school year can be both intimidating and exciting for a first year teacher when it is necessary to creating a positive classroom environment. The introduction of algebraic basics with review of eight grade content through peer collaboration allowed for the idea of mathematics as a team activity. A team activity where students will continue to work hard on their mathematical content, dialogue and mindset throughout the school year. This curriculum was created with the intention for teachers to use and adapt in their algebra 1 classroom.

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Appendices

Name: Key Date: _____

Activity 1 Partnerships and Growth Mindset

Individual Activity:

In Algebra 1 you will be working with a partner almost everyday. Write down 3 key aspects of a productive mathematical partnership.

1. Team work
2. Equal work
3. Be kind

Partner Activity:

Do this in a mathematical partnership	Do NOT do this in a mathematical partnership
• helpful • Be patient • be mindful of partner's space • Both put in effort • encourage partner to work hard	• give away answers • make the other feel bad • ghost • Touch personal space.

Name: Ky

Date: _____

Activity 2

Order of Operations

I can use the order of operations to simplify expressions

Do now: Fill in the acronym below.

P	Parentheses ( ) [ ]
E	exponents a^2 x^3
M	multiplication
D	division
A	addition
S	subtraction

→R

L→R

Examples:

1.) $\frac{3*4}{6} + (9-3)^2$

$\frac{12}{6} + (6)^2$

$2 + 36$

38

2.) $7(-2)^3 + 4(5)^2$

$7(-8) + 4(25)$

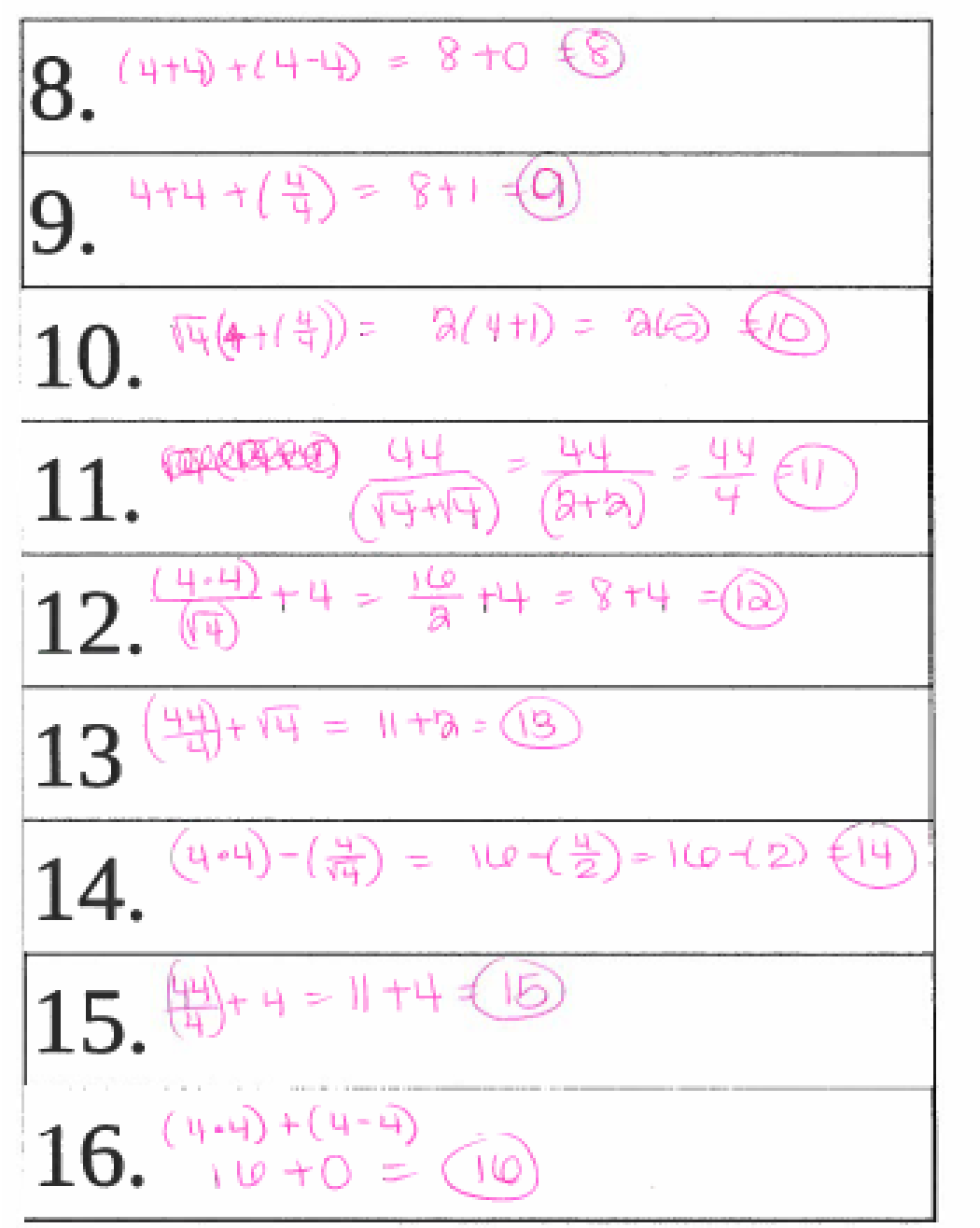
$-56 + 100$

-156

Four Fours Activity

Challenge: Use the order of operations and 4 fours to result in the numbers 1 - 20. You can use any combinations of operations including; addition, subtraction, division, square root. You must use only the 4 fours, but you can put them together (44). Be sure to include parentheses when needed!!

1. $\frac{(4+4)}{(4+4)} = \frac{8}{8} = 1$
2. $\frac{(4 \cdot 4)}{(4+4)} = \frac{16}{8} = 2$
3. $\sqrt{4} + \sqrt{4} - \left(\frac{4}{4}\right) = 2 + 2 - 1 = 4 - 1 = 3$
4. $(\sqrt{4} + \sqrt{4}) + (4 - 4) = (2 + 2) + (0) = 4$
5. $\sqrt{4} + \sqrt{4} + \left(\frac{4}{4}\right) = 2 + 2 + 1 = 5$
6. $\frac{(4 \cdot 4)}{4} + \sqrt{4} = \frac{16}{4} + 2 = 4 + 2 = 6$
7. $4 + 4 - \left(\frac{4}{4}\right) = 4 + 4 - 1 = 8 - 1 = 7$


8. (4+4)+(4-4) = 8+0 = 8
9. 4+4+(4/4) = 8+1 = 9
10. √4(4+1(4/4)) = 2(4+1) = 2(5) = 10
11. 44/(√4+√4) = 44/(2+2) = 44/4 = 11
12. (4·4)/(√4) + 4 = 16/2 + 4 = 8+4 = 12
13 (44/4)+√4 = 11+2 = 13
14. (4·4)-(4/√4) = 16-(4/2) = 16-(2) = 14
15. (44/4)+4 = 11+4 = 15
16. (4·4)+(4-4)
16+0 = 16

I can evaluate expressions using the order of operations

Do Now: Use the order of operations to simplify the following expressions.

1.) $\frac{(4+4)^2}{(4+4)}$

$\frac{(8)^2}{8} = \frac{64}{8} = 8$

2.) $4 \cdot 4 + \frac{4}{4}$

16 + 1

17

Explain: What is the difference between an **equation** and an **expression**?

An equation has an equal sign.

An expression does not have an equal sign.

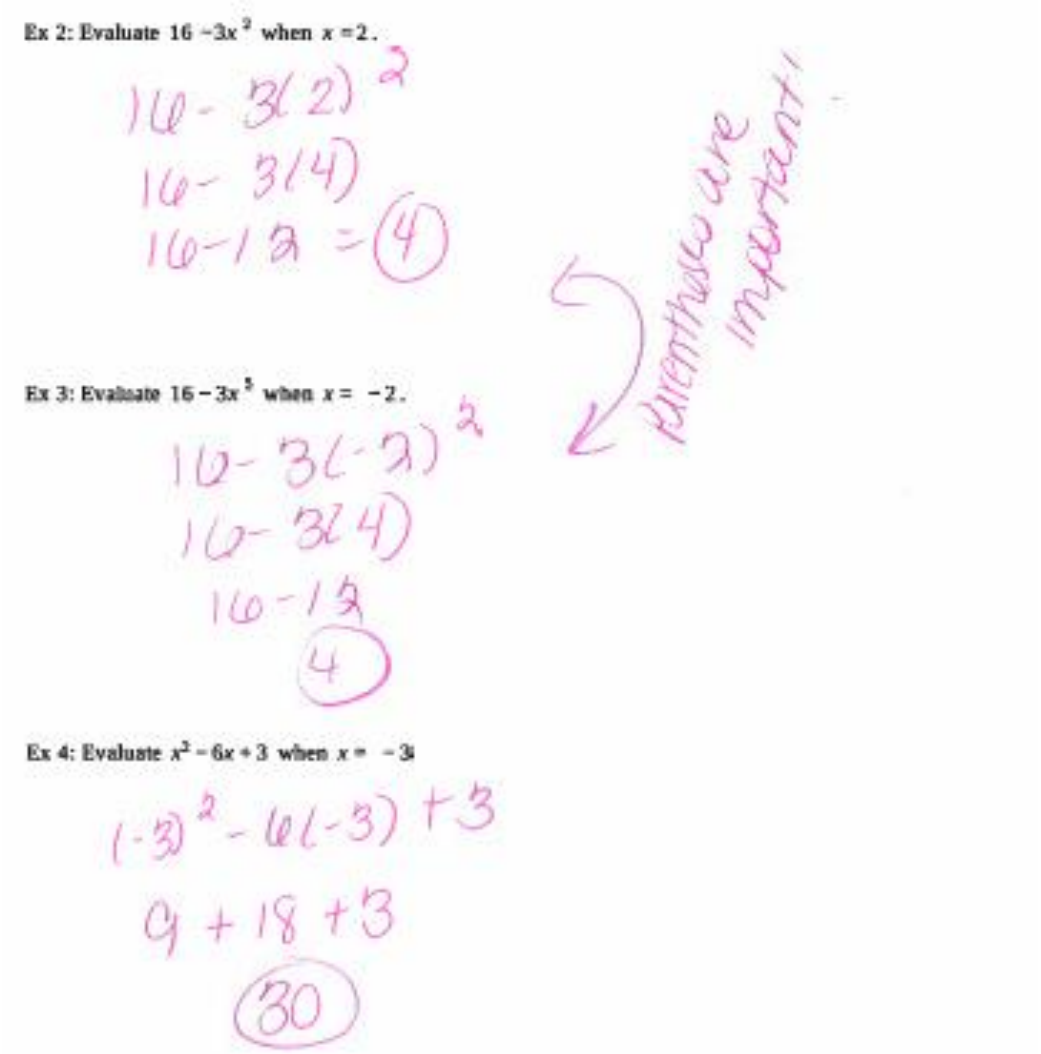
Ex 1: Evaluate the following expression when $x = 9$.

$4x - 17$

4(9) - 17

36 - 17

19


Ex 2: Evaluate $16 - 3x^2$ when $x = 2$.
$16 - 3(2)^2$
$16 - 3(4)$
$16 - 12 = 4$
parentheses are important!
Ex 3: Evaluate $16 - 3x^2$ when $x = -2$.
$16 - 3(-2)^2$
$16 - 3(4)$
$16 - 12$
4
Ex 4: Evaluate $x^2 - 6x + 3$ when $x = -3$
$(-3)^2 - 6(-3) + 3$
$9 + 18 + 3$
30

Work on the following with your partner:

1.) Evaluate $x^2 - 3x + 1$ when $x = -1$ $(-1)^2 - 3(-1) + 1$ $1 + 3 + 1 = 5$	2.) Evaluate $x^2 + 3x + 6$ when $x = -1$ $(-1)^2 + 3(-1) + 6$ $1 - 3 + 6 = 4$
3.) Evaluate $(a + 4)(3 - a)$ when $a = -4$ $((-4) + 4)(3 - (-4))$ $(0)(7) = 0$	4.) Evaluate $(a - 4)(3 + a)$ when $a = -6$ $((-6) - 4)(3 + (-6))$ $(-10)(-3) = 30$
5.) Evaluate $(z - 2)^2$ when $z = -3$ $((-3) - 2)^2 = (-5)^2$ 25	6.) Evaluate $-3(1 - t)^2$ when $t = -4$ $-3(1 - (-4))^2 = -3(5)^2$ $-3(25) = -75$
7.) Evaluate $w(-3 - w)$ when $w = -3$ $(-3)(-3 + (+3))$ $(-3)(0) = 0$	8.) Evaluate $r^2 + 3$ when $r = -1$ $(-1)^2 + 3 = 1 + 3 = 4$
9.) Evaluate $\frac{4t^2}{t}$ when $t = -9$ $\frac{4(-9)^2}{-9} = \frac{4(81)}{-9} = -36$	10.) Evaluate $\frac{(f-3)^2}{5}$ when $f = -7$ $\frac{(-7-3)^2}{5} = \frac{100}{5} = 20$

I can explain the commutative & associative properties

Vocabulary:

According to Merriam-Webster Dictionary:

Associate (Verb)

- to join as a partner, friend or companion
- to join or connect together: COMBINE

→ groupings

Associate (Noun)

- One associated with another

With your partner write two (2) sentences using the word associate.

1. Cows are associated with milk.
2. My math teacher's associate is the Science Teacher.

Vocabulary:

According to Merriam-Webster Dictionary:

Commute (Verb)

- To travel back and forth regularly
- To yield the same mathematical result regardless of order
- To give in exchange for another
- To convert into another form

With your partner write two (2) sentences using the word commute.

1. The students commute to school.
2. We must commute USD to CAD.

What happened when we used the commutative property?

Switch Sides

What does that look like as an Algebraic Representation ?

$3+4 = 4+3$ $\quad 9 \cdot 10 = 10 \cdot 9$

$x+y = y+x$

What happened when we used the associative property?

Change grouping

What does that look like as an Algebraic Representation?

$(3+4)+5 = (5+4)+3$ $\quad (9 \cdot 10) \cdot 1 = 9 \cdot (10 \cdot 1)$

$(a+b+c) \neq (a+b)+c$

Rearrange the following expressions using the commutative property but do not solve:

1.) $3+4+9 = 9 + 4 + 3$ $\quad$ 2.) $8 \times 3 = 3 \times 8$

3.) $1 \cdot 4 \cdot 7 = 7 \cdot 4 \cdot 1$ $\quad$ 4.) $-9+7+-3 = -3 + 7 + -9$

The examples above only use multiplication and addition. Can we use the rule for subtraction or division? With your partner make two numerical examples.

Subtraction	Division
$9-4 \neq 4-9$ $5 \neq -9$ $(3-1)-4 \neq 3-(1-4)$	$\frac{10}{2} \neq \frac{2}{10}$ $16 \div (4 \div 2) \neq (16 \div 4) \div 2$

I can explain the associative and commutative property

Do Now: Explain the commutative and associative properties:

switch seats ↙ ↘ new partners ( ) grouping

The Commutative Associative Game –

Commutative		Associative	
$3+x$	$x+3$	$(x+3)+y$	$=(x+y)+3$
$x+y+z$	$z+y+x$	$(z+x)+y$	$=(y+x)+z$
$y \cdot x \cdot z$	$=z \cdot x \cdot y$	$(y \cdot x) \cdot z$	$= y \cdot (x \cdot z)$
$3 \cdot x$	$x \cdot 3$	$3(x \cdot y)$	$=(3 \cdot x) \cdot y$

* include tricky examples $(3 \cdot x) \cdot y = y \cdot (3 \cdot x)$

$3 \cdot (x \cdot y) = 3 \cdot (y \cdot x)$

↑ commutative property

Organize the following as the associative or commutative properties and justify your reasoning.

1.) $(3+4)+9=(4+3)+9$ commutative
3 2 4 switched locations

2.) $(3\cdot4)\cdot5 = (3\cdot5)\cdot4$ Associative
5 went into the parentheses

3.) $(1+2)+3 = (3+1)+2$ Associative
2 left the group

4.) $7(8\cdot9)=7(9\cdot8)$ commutative
8 & 9 switched

5.) $m+(t+p)=t+(m+p)$ Associative
m moved into ( )

6.) $(r\cdot y)\cdot(w\cdot s)=(y\cdot r)\cdot(s\cdot w)$ commutative
r & y and s & w switched

7.) $(q\cdot v)\cdot(h\cdot a)=(h\cdot a)\cdot(q\cdot v)$ ~~Associative~~ commutative
(q·v) moved

8.) $\frac{1}{2}\cdot(b\cdot c)=(\frac{1}{2}\cdot b)\cdot c$ Associative
c moved out

Summarize your understanding here:

Commutative Property Definition switched parts

Examples:

Associative Property Definition same location / new grouping

Examples:

I can use the distributive property to simplify expressions

Vocabulary:
According to Merriam-Webster Dictionary:
Distribute (Verb)
- To divide among several or many
- To spread out as to cover something

With your partner write two (2) sentences using the word distribute.

1. We must distribute the cookies evenly.
2. Please distribute the papers to the students.

Examples of the Distributive Property. use the ~

1.) $2(3+5)$

$2(8) = 16+10$
$16 = 16$ ✓

2.) $2(x+3)$

$2x+6$

3.) $2(x+y)$

$2x+2y$

4.) $2(x+y)$

$2x+2y$

5.) $4(3x-9)$

$12x-36$

6.) $-2(t+10)$

$-2t-20$

7.) $-8(-2p-5)$

$16p+40$

8.) $-5(3x^2-4x+1)$

$-15x^2+20x-5$

9.) $3(y^2+8y-2)$

$3y^2+8y-6$

10.) $2x(3-4x^2)$

$6x-8x^3$

Simplify the following expressions by combining like terms. Be sure to use the distributive property carefully.

1.) $-2(3x+1)+4x$

$-6x-2+4x$

$-2x-2$

2.) $-(x-9)+10$

$-x+9+10$

$-x+19$

3.) $-7(x-7)+3(x-1)$

$-7x+49+3x-3$

$4x+46$

4.) $5(2x+6)-3(x+10)$

$10x+30-3x-30$

$7x$

5.) $2(4-x)-8(x)$

$8-2x-8x$

$8-10x$

6.) $-(x+15)+(5x-3)$

$-1x-16+5x-3$

$4x-19$

7.) $2x(x+3)-(x-2)$

$2x^2+6x-x+2$

$2x^2+5x+2$

8.) $-3x(5x-8)+4(10-3x)$

$-15x^2+24x+40-12x$

$-15x^2+12x+40$

I can translate English to Algebra

Do Now: Jot down as many synonyms for the following operations.

Addition	Subtraction	Multiplication	Division
plus increase more than sum	take away subtract less than decrease difference	times product (double)	split quotient ratio

talk about squared

Variables	
Example x, y, z	Non Example 7, 10, -4
Visual 3x + y	Definition Represents an unknown

Translate each of the following from English to Algebra

1. Five less than 10 times a number, x. $10x - 5$
2. The sum of ½ of a number, y and 14. $\frac{1}{2}y + 14$
3. The difference between 33 and four times x. $4x - 33$
4. The product of 9 and a number z. $9 \cdot z$
5. The ratio of x to 12. $\frac{x}{12}$

Translate each of the following from Algebra to English. Write each two different ways!

1. $1 - 4x$
   The difference between 1 and 4 times x
   Subtract four times x from 1
2. $\frac{2x}{3}$
   2 times x divided by 3
   The product of 2 & x split by 3
3. $\frac{1}{4}x + 9$
   ¼ times x plus 9
   The sum of the product of ¼ & x & 9
4. $\frac{1}{4}(x + 9)$
   x increased by 9 then multiplied by ¼
   ¼ times the sum of x & 9

I can translate English to Algebra

Do Now: How many ways can you express the following in English?

x-8	x minus 8 subtract 8 from x 8 less than x the difference of x & 8	z+12	z increased by 12 z plus 12 the sum of z & 12
2 · y	Two times y double y the product of 2 & y multiply 2 & y	$\frac{w}{2}$	divide w & 2 split w in 2 the ratio of w to 2 the quotient of w & 2

The following two examples are very similar. Explain how they are different from each other, then identify translate into an algebraic expression.

1.) Twice the sum of the variable, x and 10. 2(x+10)

2.) The sum of twice the variable x and 10. 2x + 10

In the 1st one the sum of x & 10 are being multiplied by 2. In 2nd only x is being multiplied by 2.