

AN ATTEMPT TO DETERMINE A HIERARCHY IN  
THE ACQUISITION OF ELEMENTARY  
MATHEMATICS CONCEPTS

THESIS

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## Abstract

The purpose of this thesis was to attempt to discover a workable hierarchy in the acquisition of basic mathematics concepts. Research indicated that, to date, no definite hierarchy had been established.

Ninety-nine subjects, ranging in age from three years to nine years and eleven months, were asked to perform fourteen tasks. These tasks represented seven mathematics concepts: cardinality-counting, discrimination, one-to-one correspondence, ordination, seriation, classification, and conservation. Responses to the items were recorded by the examiner as either correct or incorrect.

Green's (1956) Scalogram Analysis was used with the data to determine whether or not a hierarchy did exist in the represented mathematics concepts. Findings indicated that although the items were independent of one another, a definite hierarchy did exist. This hierarchy was maintained in each of two subgroups, one consisting of easier items and the other of more difficult items.

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## Chapter I

### Statement of Problem

#### Purpose

The purpose of this thesis was to attempt to discover a workable hierarchy in the acquisition of basic mathematics concepts.

Often a child's development of mathematics concepts is found to be behind that of his classmates. Frequently the problem rests with his inability to grasp a particular concept or it may lie with his lack of understanding of some prerequisite concept. Due to the hierarchical structure of mathematics (Gagne 1968), this can pose more severe problems than in other subjects. Any single concept could be vital to the acquisition of future concepts.

Every teacher is interested in teaching mathematics concepts correctly and therefore tries to present these concepts in a manner which is most meaningful to the students. This objective would be best accomplished if a definite order of acquisition of these concepts was known. The concepts referred to in this study were limited to cardinality-counting, classification, conservation, discrimination, one-to-one correspondence, ordination, and seriation.

### Definition of terms

A hierarchy is an arrangement in a graduated series, based on the degree of skill and responsibility each element entails. In defining specifically the hierarchy of learning sets relevant to any given learning task, Gagné' (1961) suggests beginning with the following question: "What would the individual have to know how to do in order to be able to achieve this (new) task, when given only instructions?"

The following are other definitions used in this study. The use of number to indicate quantity or number in a set is known as cardination. Classification refers to the arrangement of values according to one or more factors. Conservation is the recognition that the number, area, or volume of objects does not change when rearranged. The ability to pick out a particular object(s) from a group of others that are different is known as discrimination. One-to-one correspondence is the establishment of a relationship between two distinct sets of elements such that every member of the first set can be paired with a unique element in the second set. The use of numbers to indicate the position of an element in a group or set is known as ordination. Finally, the formation of an arrangement in a series of orderly sequence according to some characteristic is referred to as seriation.

### Limitations

A study of this type has several limitations. Concepts can be broken into subcategories, for example, classification can be one-dimensional, two-dimensional, three-dimensional, etc. Each concept can also be represented by an endless number of tasks for testing purposes. Often the tasks are found to overlap two or more concepts. It would be virtually impossible for this study to have attempted to place every aspect of each concept in the hierarchy. Therefore, only certain tasks were chosen to see if the concepts they represent could be placed in a hierarchy. In addition, concept areas were chosen so as to reduce the possibility of tasks representing more than one concept.

## Chapter II

### Review of Related Literature

The related literature was reviewed for the purposes of improving on past errors, finding good procedures for experimentation and determining if any part of a workable hierarchy had been discovered.

The literature was found to be most consistent in that, to date, no firm hierarchy comprised of these concepts had been established.

Rea and Reys (1970) studied 727 entering kindergarteners. Nearly forty percent of them correctly counted to or beyond twenty. Over fifty percent of them correctly counted beyond fourteen. Prewitt (1975) studied the twenty-one to forty-eight month old child showing a relationship between age and the development of the mathematical concept of seriation. These two studies showed the existence of mathematical ability of preschool children.

Godfrey (1974) found significant differences between rote and conceptual learning. He stated that the method of specifying a hierarchy of mathematical concepts, as well as proceeding to study the relations between them is promising.

Brainerd (1973, 1974) did several studies dealing strictly with the concepts of ordination and cardination. He stated that ordination must occur prior to arithmetic

proficiency, while cardination is not essential. The ordinal property was found to be much easier to train than the cardinal property. Also, ordinal training appeared to transfer better than cardinal training.

Lamb (1975) felt that with a well developed conception of number, children will be aided in their reasoning ability. Rathmell (1974) followed along the same path indicating the belief that many children develop the ability to read and write numerals without understanding their basic ideas. They can rename numbers in a skill situation but generally don't understand any process behind it.

Brashear (1970) seemed to be leading in the right direction. In her study she converted tasks to picture materials and administered the tasks to a large number of subjects in order to establish a hierarchy. In Beers (1974) preliminary study into certain mathematics concepts it was indicated that a hierarchy would show up if there had been less overlap in response and fewer, more distinct items tested.

All of these studies are bits and pieces of a possible answer. They all indicate a concern for the classroom problems that can arise due to poor concept acquisition. The identification of a hierarchy would assist the classroom teacher in dealing with these problems in a more efficient manner.

## Chapter III

### Design of the Study

#### Hypotheses

The purpose for this study was to discover through responses to certain items (basic mathematical tasks), a hierarchy in the acquisition of the concepts represented by the items. If discovered, this would indicate the order in which the items had developed and the optimum order for classroom presentation.

The hypotheses upon which this study was based were:

1. There exists a reproducible hierarchy in the acquisition of the mathematics concepts represented by the total fourteen items.
2. There exists a reproducible hierarchy in the acquisition of the mathematics concepts represented by the seven easier items.
3. There exists a reproducible hierarchy in the acquisition of the mathematics concepts represented by the seven more difficult items.

The mathematics concepts represented in these hypotheses are cardination-counting, classification, conservation, discrimination, one-to-one correspondence, ordination, and seriation.

## Methodology

### Subjects

There were ninety-nine subjects used for this study. Seventy-five children were from the Brockport Central Schools, and twenty-four were from the Brockport Child Care Center. The subjects ranged in age from three years through nine years and eleven months. There were fifteen subjects from each of the age groups four through nine, and eight subjects that were in the three year old age group. According to Prewitt (1975) there are definite mathematical concepts present in the twenty-one to forty-eight month old child. The sample of preschoolers, therefore, as well as school-age children was necessary to establish the concepts that are present before a child actually enters formal schooling.

Subjects were randomly selected by age group from the participating classes. The subjects were tested individually for approximately fifteen minutes. Each was asked to perform all fourteen tasks in a single session.

### Instruments

The instrument used in this study was comprised of a series of fourteen questions (see Appendix A) or tasks. Concrete materials (see Appendix A) were used to perform the tasks which were representative of seven particular mathematics concepts. Each concept had two tasks representing it, one of which was relatively easy and one which

was more difficult. The tasks were designed carefully to make sure that no overlap of concepts existed within a task.

Table 1 contains a list of the specific concepts used and a general idea of the type of question asked. Although usually thought of as two separate concepts, it was difficult to distinguish between cardinality and counting. It was difficult to determine a distinct task that would keep them from overlapping. They were therefore combined into a single concept.

Paper and pencil were used by the examiner to record only correct or incorrect responses to each individual item. The subject answered questions orally.

#### Procedure

The procedure consisted of examining the subject in a one-to-one situation. Each subject was asked to perform each task or answer the related question after observing the examiner. Responses were recorded by the examiner as either correct (as indicated by a check mark) or incorrect (as indicated by an X). A tape-recorder was used to avoid misinterpretations of responses made by the subjects. Prior to being erased, each tape was reviewed at the end of each session, and the scoring validated.

Table 1

## Attributes and Characteristics of Concepts

Concept Area	Attribute	Characteristic
1. discrimination	size	biggest one
2. discrimination	mass	heaviest one
3. seriation	size	small to large
4. seriation	length	short to long
5. cardination- counting	counting	number in set
6. cardination- counting	subtraction	number in set if some are removed
7. classification	one- dimensional	shape
8. classification	two- dimensional	shape and color
9. conservation	number	equal rows
10. conservation	area	repositioning blocks on a sheet of paper
11. one-to-one correspondence	provoked	make an equivalent set
12. one-to-one correspondence	spontaneous	make the two sets equivalent
13. ordination	one- dimensional	first
14. ordination	two- dimensional	second in third row

Each subject performed all fourteen tasks and the tasks were presented in the same order for all subjects. Actual wording of directions for each task, as well as the physical arrangements were kept the same for all subjects. Physical arrangements consisted of an isolated room to avoid interruption or distraction, and materials not being used placed out of the way or covered.

Positive responses to all items were collected and ranked in popularity order. Green's (1956) Scalogram Analysis was used to assign an Index of Reproducibility.

## Chapter IV

### Analysis of Data

After all data was collected and recorded, items were ranked in their order of popularity. The item receiving the most correct responses was ranked first and the item receiving the least correct responses was ranked last. Green's (1956) Scalogram Analysis was applied to the data to establish an Index of Reproducibility. This Index of Reproducibility was used to determine the existence of a hierarchy.

### Findings

There were several findings in this study. Table 2 contains the total number of correct and incorrect responses to each of the items in the instrument, in the order that the items were presented. Table 3 shows the popularity order of items. The most popular item is the one receiving the most correct responses. The least popular item received the fewest correct responses.

The entire group of items was then divided into the two subgroups. Each subgroup contained representative items of one of the seven concepts used in the study. One group contained the seven easier items, one from each concept area. The second group contained the seven more difficult items, one from each concept area. This division into two subgroups was based on responses from the subjects.

Table 2  
Correct and Incorrect Responses

Item Number	Correct Responses	Incorrect Responses
1 (discrimination)	98	1
2 (discrimination)	97	2
3 (seriation)	66	33
4 (seriation)	65	34
5 (cardination- counting)	85	14
6 (cardination- counting)	80	19
7 (classification)	91	8
8 (classification)	67	32
9 (conservation)	64	35
10 (conservation)	48	51
11 (one-to-one correspondence)	86	13
12 (one-to-one correspondence)	67	32
13 (ordination)	91	8
14 (ordination)	27	72

Table 3  
Popularity Order of Items

Item Number	Popularity Rank	Number of Correct Responses
1 (discrimination)	1	98
2 (discrimination)	2	97
7 (classification)	3	91
13 (ordination)	4	91
11 (one-to-one correspondence)	5	86
5 (cardination- counting)	6	85
6 (cardination- counting)	7	80
12 (one-to-one correspondence)	8	67
8 (classification)	9	67
3 (seriation)	10	66
4 (seriation)	11	65
9 (conservation)	12	64
10 (conservation)	13	48
14 (ordination)	14	27

Table 4 contains the easier items ranked in order of popularity and Table 5 contains the more difficult items ranked in order of popularity. Table 6 shows the

most popular and least popular items by particular age group. This was calculated by finding the highest and lowest numbers of correct responses to the particular items by age.

Table 4  
Popularity Order of Items  
in Subgroup A

Item Number	Popularity Rank	Number of Correct Responses
1 (discrimination)	1	98
7 (classification)	2	91
13 (ordination)	3	91
11 (one-to-one correspondence)	4	86
5 (cardination- counting)	5	85
3 (seriation)	6	66
9 (conservation)	7	64

Table 5  
Popularity Order of Items  
in Subgroup B

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Item Number	Popularity Rank	Number of Correct Responses
2 (discrimination)	1	97
6 (cardination- counting)	2	80
8 (classification)	3	67
12 (one-to-one correspondence)	4	67
4 (seriation)	5	65
10 (conservation)	6	48
14 (ordination)	7	27

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Table 6  
 Most and Least Popular Items  
 by Age Group

Age Group	Most Popular Items	Number of Correct Responses	Least Popular Items	Number of Correct Responses
3-0 to 3-11	1	8	4,8,9, 10,12,14	0
4-0 to 4-11	1,2	15	14	0
5-0 to 5-11	1,2,13	15	14	2
6-0 to 6-11	1,2,5,7,11	15	14	0
7-0 to 7-11	1,2,3,4,5, 6,7,12,13	15	14	7
8-0 to 8-11	1,2,3,4,5, 6,7,11,13	15	14	7
9-0 to 9-11	1,2,3,4,5,6, 7,8,9,10,11, 12,13	15	14	11

### Interpretations

Scalogram Analysis by Green (1956) was used to determine if the order of popularity displayed, did in fact show a distinct hierarchy. The degree of reproducibility of the items ranked in order of popularity is determined by the Index of Reproducibility calculated through Green's process. For the total set of fourteen items the Index of Reproducibility was 0.97 indicating scalability.

The same result was found when the fourteen items were divided into the two subgroups. For the seven easier items, Subgroup A, the same formula was used, and the Index of Reproducibility displayed with these items, was 0.97. The Index of Reproducibility found with the group of more difficult items, Subgroup B, was 0.97.

An optional part of Green's Scalogram Analysis (1956) is to estimate the reproducibility that would occur by chance if the items had their observed popularities but were mutually independent. This Index of Reproducibility by Chance for the total fourteen items was 0.98. For Subgroup A, the easier items, the Index of Reproducibility by Chance was 1.00. For the more difficult items, Subgroup B, the Index of Reproducibility by Chance was 0.99.

A final procedure was used to determine the Index of Consistency between the Index of Reproducibility and

the Index of Reproducibility by Chance. For the entire set of fourteen items the Index of Consistency was -0.50. The Index of Consistency for Subgroup A, the easier items, was -1.0 and for Subgroup B, the more difficult items, was also -1.0. Negative numbers indicate some negative correlation in the sample. This means that there were pairings of correct and incorrect responses to particular items by an individual respondent that were not consistent with general or overall tendencies.

#### Summary

In summation, the Scalogram Analysis indicates an extremely high degree of reproducibility in the group of total items as well as the two subgroups of seven items. According to Green (1956), this indicates a high degree of scalability, or a definite hierarchy.

Values in the Index of Reproducibility by Chance in all three groups indicate that the items tend to be independent of one another. The order represented by the rank of popularity seems to be the hierarchy in which children acquire concepts used in this study. The Index of Reproducibility by Chance indicates that the actual items are independent of one another, or that the order in which children learn these concepts could be changed. Mastery of one concept does not rely on mastery of the previous concept.

## Chapter V

### Conclusions and Implications

The purpose of this study was to discover if a hierarchy existed in the seven concept areas, and if a hierarchy did exist, to identify it. This could assist classroom teachers in presenting material in these areas to their students.

#### Conclusions

Due to the high Index of Reproducibility found, a definite hierarchy is present in all the total fourteen items, and the two subgroups. The hierarchy found in the total fourteen items, beginning with the most popular item was: discrimination, size; discrimination, mass; classification, one-dimensional; ordination, one-dimensional; one-to-one correspondence, provoked; cardination-counting, counting; cardination-counting, subtraction; one-to-one correspondence, spontaneous; classification, two-dimensional; seriation, size; seriation, length; conservation, number; conservation, area; and ordination, two-dimensional.

The hierarchy found for Subgroup A, the easier items, was: discrimination, size; classification, one-dimensional; ordination, one-dimensional; one-to-one correspondence, provoked; cardination-counting, counting; seriation, size; conservation, number. The hierarchy for Subgroup B, the

more difficult items, was: discrimination, mass; cardination-counting, subtraction; classification, two-dimensional; one-to-one correspondence, spontaneous; seriation, length; conservation, area; ordination, two-dimensional.

The hierarchies for the two subgroups are identical to that for all fourteen items. Subgroup B has two items of equal popularity. For the purposes of this study they were kept in numerical order but according to Green (1956) either order is acceptable. The strength of the hierarchy found in the total group of items is emphasized by the preservation of that hierarchy in each subgroup.

When the items had been selected, extreme caution was taken to prevent any overlap among the concepts. This is seen in the high Index of Reproducibility by Chance figures. High chance figures indicate independence. Although a hierarchy was shown, because of the careful selection of the concept items, the items used are completely independent of one another. In other words, you would not need the information from one item in order to do the next.

According to Guttman (1944), perfect scales are not to be expected in practice. The deviation from perfection has been measured by the Index of Reproducibility. Since it is 0.85 or better, it can be used as an efficient approximation of a perfect scale.

An order of development has been established among the concept items represented. Even though this is true, the items are not dependent on one another. The way Guttman (1944) describes something like this is, if someone came from another planet, he could probably develop a particular concept within the hierarchy without prior knowledge of other concepts. However, for our intents and purposes, the order of development of these concepts in children has been established.

#### Implications for Research

A good implication for further research would be to determine the cause for extremely high Index of Reproducibility by Chance figures. If it were possible, it would be better to select items that would produce a low Index of Reproducibility by Chance value. With a high Index of Reproducibility, a low Index of Reproducibility by Chance would make an even stronger hierarchy than the one established in this study. The reason for this is that the items would be quite dependent on one another stressing the importance for the hierarchy.

#### Implications for Classroom Practice

The hierarchy discovered should be useful in the classroom. Teachers can assume that if a student has acquired a concept, he also has acquired those preceding it. It may also be assumed that none of the concept areas are prerequisites for any of those following. A teacher

may wish to present concept skills and ideas in the order found in this study. To be more specific, a teacher may wish to look at the popular items found for the age group that they teach. This could assist them in presenting them in an order most suitable to their particular age group of children.

In addition, the teacher could use the items within her classroom, when the need is there for clear-cut and distinct items of a particular concept.

The classroom teacher may even wish to give the items to particular children to see if they are in agreement with the popularities for their age group. If not, the teacher may use the hierarchy as an example to alter teaching strategies within her own classroom, to assist children with their special learning needs.

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APPENDIX A

List of Items

1. discrimination, size

Examiner places three colored rods of different lengths in front of child.

Examiner says, "Look at the rods in front of you and hand me the biggest one."

2. discrimination, mass

Examiner holds three red balls (weighted ping-pong balls) in a cut-off egg carton in front of the child.

Examiner says, "Pick up each one of these balls, one at a time, and tell me which one is the heaviest."

3. seriation, size

Examiner places five cardboard triangles of different sizes in front of child.

Examiner says, "Please put these shapes in order from smallest to largest."

4. seriation, length

Examiner asks child to watch as he puts three colored rods side-by-side in order of length. Examiner then places ten colored rods in front of child and asks him to do the same.

Examiner says, "Watch me while I make a staircase with these rods. Now, what I'd like you to do is make your own staircase using all of these rods."

5. cardination-counting, counting

Examiner places six blocks in front of child.

Examiner says, "Look at the blocks and tell me how many there are."

6. cardination-counting, subtraction

Examiner uses same six blocks from item number 5.

Examiner says, "If I were to take two of these blocks away, how many would be left?"

7. classification, one-dimensional

Examiner places several shapes in front of child(wooden).

Examiner says, "Please pick out all the round ones."

8. classification, two-dimensional

Examiner places several shapes in front of child (card-board). The shapes are different colors with all circles having the color red. Red is also represented, however, by two other shapes.

Examiner says, "Look at the shapes in front of you. Are all the circles red? Are all the red ones circles?"

9. conservation, number

Examiner places six blocks in front of child in a row. Examiner then places six blocks in front of himself in a row that is equal in length to the row in front of the child.

Examiner says, "Look at the rows of blocks. Do you have more blocks, do I have more blocks, or do we have the same number of blocks?"

Examiner then spreads his rows of blocks out so that his row, although still containing the same number of blocks is longer in length than the child's row.

Examiner says, "Now, do you have more blocks, do I have more blocks, or do we have the same number of blocks?"

10. conservation, area

Examiner places two pieces of green paper flat on a table between him and the child. Four blocks are placed on each paper in the exact same position, near center. Pieces of paper are the exact same size as well as the blocks.

Examiner says, "Here are two farms. (The paper is green). On these farms are some barns. The cows on these farms just love to eat the grass when they're hungry. If the green is the grass, is there more grass on this farm for the cows to eat (examiner points to one farm), is there more grass on this farm for the cows to eat (examiner points to other farm), or is there the same amount of grass on both farms?"

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Examiner repositions the barns on one of the farms by spreading them out. The same procedure is followed.

Examiner says, "Now, is there more grass on this farm for the cows to eat, is there more grass on this farm for the cows to eat, or is there the same amount of grass on both farms?"

11. one-to-one correspondence, provoked

Examiner places a piece of cardboard in front of himself. On the cardboard are six blocks that are glued on, evenly spaced apart. The child is given a pile of ten blocks.

Examiner says, "Look at my row of blocks. What I'd like you to do is, from your pile of blocks, pick out enough blocks so that you have one block for each one of my blocks."

12. one-to-one correspondence, spontaneous

Examiner puts out a row of five blocks in front of himself. Examiner places seven blocks in front of the child in a row that is equal in length to the examiner's five blocks.

Examiner says, "Look at the two rows of blocks. What I'd like you to do is make your row of blocks have the same number of blocks as my row."

13. ordination, one-dimensional

Examiner places four plastic three-dimensional toy bears in a row, front to back. A long brown envelope is placed in front of the row, or line, with the opening facing the bears.

Examiner says, "Let's pretend that this is a cave (points to envelope). These little bears are on their way into the cave. If the bears stay in line, which bear is going to be the first one to get into the cave?"

14. ordination, two-dimensional

Examiner places a model of a classroom made with a sheet of paper using blocks in rows to represent the students in their desks. A window, door, and teacher's desk were drawn on the paper.

Examiner says, "Let's pretend that this is a classroom. This is the door to get in, this is the teacher's desk, and this is a window on the side of the room. What I'd like you to do is pick out the second child from the window in the third row back from the teacher's desk."