How will teaching mathematics with inquiry based lessons using manipulatives impact 3^{rd} grade children's understanding of fractions?

By

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Abstract

During a five week unit on fractions I collected data using observations, interviews, and student work to discover how teaching mathematics with inquiry-based lessons using manipulatives impacts 3rd grade children's understanding of fractions. The main goal for student learning was to increase students' understanding of fractions and to get students working together to solve problems while practicing social skills. I wanted to learn if hands-on maniupulatives did have an impact on students' understanding of fractions and use any new information to improve my teaching of fractions. I have constructed a case study of each student and looked across many different kinds of data.

I have used my analysis to determine the impact of the easily accessible hands-on manipulatives, and study the methods students use to construct their own knowledge of fractions. These analyses have shown that inquiry-based lessons along with the use of manipulatives does have an impact on the students' understanding of fractions. Throughout the study I observed many different types of learning experiences and saw students using a combination of prior knowledge, hands-on tools, and collaboration to solve problems. Many students were excited about this new learning experience and the level of motivation increased as each lesson progressed to the next. This study has also helped me to learn about children's thinking and this has helped me to make decisions about how to best scaffold students' learning. By learning how students are thinking and using tools to learn and discover, I can

develop new strategies to introduce content and adjust the lessons to best fit the needs of each individual student.

While teaching mathematics in this manner it was difficult to change my teaching style from a traditional style to an inquiry approach where lessons are selected and designed through problems. These lessons enable the learner to acquire and use problem solving skills and self-directed learning strategies. Reviewing literature and studying theories designed around inquiry based education and constructivism helped me to change my style to help students learn, discover, and solve problems in a new and exciting way.

Chapter 1

Introduction

Background

Throughout my five years experience teaching third grade I have seen children struggle each time I begin my mathematics unit on fractions. The mathematics curriculum used by my school district is a text book based curriculum and students are required to learn the information through reading and lecture. Fractions seem to be the place where students get distracted and the traditional text book way of learning is not effective. There are no opportunities to develop problem-solving skills and students are not getting the opportunity to use hands-on objects to discover new concepts their own way.

Alsina (2002) states that "we can find in our materials great opportunities to bring "real" objects into the class and to provoke an experimental research approach by modeling by means of specific materials (p.246)". By centering our mathematics instruction on problem-solving and using manipulatives as experimental objects, my students have had the opportunity to learn mathematics in meaningful ways and connect math to real life materials. Introducing mathematics in this experimental way and letting children discover through hands-on experience is the approach I have used to foster the inquiry-based environment.

There are many reasons why teachers should begin centering their mathematics instruction on problem solving. Clouston and Whitcombe (2005) speak of the present drive for change in the government's agenda for higher education. The

National Council of Teachers of Mathematics (NCTM, 2000) developed a set of content standards introduced to support teachers in ensuring mathematics learning and teaching of the highest quality for all students. The National standards developed by NCTM describe the planning, implementation, and evaluation of high-quality mathematics programs and address the areas of mathematical understanding, knowledge, and skills that students should acquire from prekindergarten through grade twelve. Following these standards for improved mathematics education and increased expectations for the teaching of mathematics requires an increase in the quality of mathematics instruction. Clouston and Whitcombe (2005) believe in order to meet these standards as well as the needs of a diverse student group, educators must begin creating a creative learning environment that fosters the development of critical thinking and problem solving skills. By centering our mathematics instruction on problem solving we can enable students to learn through self-directed study and evoke student empowerment.

Research Question

How will teaching mathematics with inquiry based lessons using manipulatives impact 3rd grade children's understanding of fractions?

Rationale

The main purpose of this study was to examine and evaluate the impact that mathematics instruction grounded in constructivist principles has on children's understanding of fractions and use any new information learned to improve my teaching. Fractions are a concept with which I have seen many students struggle in the past. Fractions are also an area which many teachers in my district find difficult to teach effectively. The curriculum used in my school district does not enable students to discover new concepts using hands-on objects. Marjanna and Tapola (2004) believe through inquiry-based lessons and hands-on problem solving, teachers can improve student learning and the teaching of mathematics by providing meaningful and authentic learning tasks.

For many years I have been trying to convince my colleagues that teaching mathematics with hands-on manipulatives is the best possible way. It is my belief that children learn and construct understanding when they have the opportunity to learn with manipulatives and learn new concepts through discovery. Our current mathematics program at my school district is traditional. Our curriculum involves a text book and students are expected to read about mathematics and learn from it. There are very few hands-on mathematics lessons. I believe that when children actually do something, they learn and remember. Through my previous teaching experience through all subjects I have learned that incorporating hands-on learning has always resulted in an increase of student motivation and student learning. Heuser (2000) believes students will develop a better understanding of math concepts when encouraged to construct their own knowledge by physically interacting with their environment and exploring math in a research-based constructivist manner.

With an inquiry-based approach, teachers can design their materials and questioning to fit the needs of all students. Students have different abilities in our present classrooms. When we are able to meet the needs of these students and bring information and experiences to them on their own level and within their Zone of Proximal Development, they become successful. The Zone of Proximal Development is a zone where learners need to be moved for cognitive change to occur (Ross, 2005).

Teaching mathematics with maniuplatives is an important topic to study because manipulatives can be used to not only teach mathematics, but get children interested in the process of learning as well. In an inquiry-based approach, students construct their own knowledge and understanding through interacting in information-rich environments which promotes their natural inquisitiveness. According to Robins (2005), through exploration according to their personal interests, students are building knowledge and developing problem-solving strategies. Through the combination of manipulatives and inquiry-based learning, I hope to see an improvement in the children's understanding of fractions and gain an insight to how students construct their own mathematical meaning.

Definitions

Inquiry Based Learning: An inquiry-based classroom is based on students' natural inquisitiveness. In inquiry-based mathematics, students construct meaning by engaging in inquiry. The student inquiry takes a variety of forms through collaboration, participation, question formulation, and self assessment (Evans, 2001).

Constructivism: According to Ross (2005) "the principles of constructivism, as tied to theorists such as Vygotsky and Dewey, hold that students construct their mathematical knowledge through their social interactions and experiences in the word in which they live (p.235)".

Zone of Proximal Development: The space for learning where an individual can move with support of an expert. Learners can also create their own zone of proximal development when working and collaborating with each other. This construct, coined by Vygotsky, is a zone where learners need to be moved for cognitive change to occur (Ross, 2005).

Problems-Based Learning: Problem-based learning (PBL) is an approach to education where teachers carefully select and design problems that enable the learner to acquire and use problem solving skills, self-directed learning strategies, and communication and team participation skills. PBL creates a creative environment that enables students to be active and participative (Clouston & Whitcombe 2005).

Manipulatives: According to DeMestre & Richards (1991) "manipulatives are any hands-on materials used to assist students in the understanding and application of mathematical concepts (p.14)".

Study Approach

During a five week unit on fractions involving approximately 15-18 lessons, I collected data using observations, interviews, and student work. I also gave a short response survey to the students on the use of manipulatives before and after the study. This particular methodology is appropriate for this qualitative study because it has allowed me to construct a case study of each student. This triangulation has helped me to look across many different kinds of data and use the analysis to determine the impact of manipulatives and study the methods students are using to construct their own knowledge of fractions. The analysis of student work coupled with my observations and interviews has helped me to learn how students are using the manipulatives to help them learn and if these manipulatives introduced through the constructivist theory result in an impact on their learning of fractions. These analyses have shown if the inquiry-based environment along with the use of manipulatives has an impact on the students' understanding of fractions.

Chapter 2

Literature Review

Introduction

Many current studies have been done in the area of inquiry-based learning. These studies have revealed the significant impact inquiry-based learning has on the teaching and learning of mathematics. Reviewing these studies has helped me to gain a better understanding of how manipulatives can be used in a classroom setting and the effects that inquiry based learning and manipulatives have on students' understanding of mathematical concepts. Many of these studies have helped me to build ideas and acquire knowledge to conduct a similar study involving an inquiry-based teaching approach. I have also included a review of articles pertaining to the successful use of manipulatives and have become familiar with constructivist theory and use of constructivism in the mathematics classroom. The use of constructivist theory offers a student-centered learning environment where children construct their own knowledge through inquiry, problem solving, and the use of prior knowledge. This method of learning is linked to problems-based learning. Problems-based learning adheres to constructivist theory and supports the constructivist position by the student and facilitators exploration of multiple problem solving methods while building on their own unique experiences. The pieces of literature in this section have helped me to direct my present study and gain knowledge on constructivist theory and the impact it has on learning.

Implementing Inquiry to Teach Mathematics

An inquiry-based classroom is based on students' natural inquisitiveness. In inquiry-based mathematics, students construct meaning by engaging in inquiry. The student inquiry takes a variety of forms through collaboration, participation, question formulation, and self assessment (Evans, 2001). In these setting students are encouraged to make observations and find answers to problems through working and communicating with others. Students are offered hands-on experiences that are given to stimulate curiosity and solve problems. Evans is a professor who specializes in teaching teachers the use of inquiry. Evans shares what an inquiry-based classroom should look like and how it is beneficial to the learning of mathematics.

Evans (2001) states how the National Council of Teachers of Mathematics (NCTM) has identified the inquiry-based mathematics class as one of the most beneficial ways students can learn math. By beginning each lesson with a problem, teachers begin building children's natural curiosity. Students are given tools and encouraged to collaborate, communicate, and find answers and solutions through discovery. When the students have found answers and solutions, students present their findings by using the tools to show how they solved the problem. Students use critical, logical, and creative thinking to work on a solution. Evans believes inquiry to be the basis for creating a genuine and evolving educational culture where true learning occurs.

Fuller (2001) writes about a study in an urban area of southeastern Massachusetts where researchers examined 62 K-12 teachers' perceptions of changes in student learning after using the Partnerships Advancing the Learning of Math and Science (PALMS) program in their school district. The study involved 22 K-6 classrooms, and 40 7-12 classrooms. PALMS involved lessons that were constructed and taught with the constructivist framework and inquiry-based learning. PALMS was a state wide program funded by the Massachusetts Department of Education. Fuller writes that the main idea of PALMS is to create a hands-on, minds-on problem solving environment. Teachers were trained and supported for a period of 6 years to implement the PALMS approach and create a hands-on, inquiry-based environment in their classrooms. After the program was practiced in their classrooms, teachers from the area of southeastern Massachusetts were given a survey that questioned their perception of the approach and its effects on student learning and growth.

Fuller (2001) concluded that many teachers thought PALMS had a positive impact on classroom and school culture. The lessons were more enjoyable, and this resulted in significant teaching and learning outcomes. The data analysis indicated that there were educational benefits such as time for teachers to observe and help students with their individual learning needs and an increased students' understanding of concepts. The extra time with students allowed the teachers to maximize the students' learning potential. The data indicted significant increases in student participation, motivation, communication, and problem solving ability. The teachers involved in the study concluded that PALMS had an impact on student learning because teachers were able to take their students to a higher level of understanding and learning. The student-centered lessons allowed students to visualize the concepts and work, this resulted in a large increase in student involvement. The PALMS study helped teachers to create rich, active learning environment in which learning was built on discovery and reflection.

In a recent study in Finland, Marjanna and Tapola (2004) investigated the impact of inquiry learning situations on student achievement and goal orientation. Twenty-one primary schools participated in inquiry learning projects for a period of over 4 school years. The inquiry units were taught in 4-6 week periods and included 24-30 science lessons each to students in grades 3 and 4. The units were research projects and involved students generating a problem and gathering information on the topic. The student research was a collaborative effort where students worked in groups of two and three. In these groups they combined their knowledge while working together toward a solution through investigation and social interaction. The researchers collected data from student self-report questionnaires and video data on the students' learning and social interaction process.

Marjanna and Tapola (2004) believe there are three respects in which the traditional classroom learning situation differs from the new open learning or inquiry learning environment. These are the learning situations on which they based their study. In the inquiry setting the learning goals took shape during the process of learning. In the traditional setting the teacher sets the learning goals. In their study, the interaction process between student and teacher was consistent with the steps and processes they were involved in, unlike the traditional interaction process where the teacher directs all interaction. In this open inquiry study, the process mainly emphasized learning and the grades were not the only point of evaluation. In a traditional setting, learning is easily measured with quizzes and tests.

Marjanna and Tapola (2004) evaluated the student self report questionnaires and the video data on student learning and social interaction in the inquiry process that were collected throughout the learning projects. They concluded that some students showed a progressive tendency as the study continued to involve themselves more and more in the process of inquiry learning. The hands-on lessons encouraged more on task behavior for the students. When students were encouraged to design their own experiments and collaborate, they became more interested in the topic. There was also an increase in students' motivation to learn and participate. The findings of this study support that in a problems based, inquiry based learning environment teachers can solve some motivational problems that were present in the traditional classroom setting and can provide a more meaningful and authentic learning tasks.

Heuser (2000) shares how math workshops in the past have been used to differentiate instruction and increase student understanding. The workshops involve hands-on math and are usually center based and begin with a question or problem. Student directed workshops involve children developing their own questions and exploring with their own selection of materials to learn. Heuser believes math workshops can be used to create flexible environments where students actively learn, share, and explore concepts at their own pace. The philosophy and format of math and science workshops are based on techniques grounded in constructivist theory.

These workshops are designed around the philosophy that students need a lot of time to write, reflect, share, discover, and develop. Heuser shares the results from a three year study on the use of math workshops to increase student achievement in Glenview, Illinois. A group of teachers from Glenview used workshops in their 2nd grade classrooms during a three year study. The study was called the Mathematics Workshop Project. Heuser (2000) shares how the students explored math concepts in a research-based constructivist manner through workshops. The researchers collected data through observations, student performance data, and student reflections, to explore the impact of workshops on student achievement. The workshops focused on the learning of matter, measurement, and symmetry.

After analyzing the observations, student performance data, and student reflections Heuser (2000) concluded that the students developed a better understand of math concepts and the 2nd grade students outperformed comparable non-workshop classrooms in five out of seven measures of student achievement. The researchers also found that students developed the readiness to better understand concepts by exploring math and science in a research-based constructivist manner. Heuser also concluded that children learned best when they were actively involved in math and science. They must physically interact with their environment to learn mathematical concepts. Children in this study developed a deeper understanding of math when they were encouraged to construct their own knowledge.

The National Council of Teachers of Mathematics (NCTM) standards specify students in grades K-3 should build new mathematical knowledge through problem solving and apply and adapt a variety of appropriate strategies to solve problems. While constructing their own knowledge, students are also developing problem solving strategies through interactions and experiences. The impact of constructivism to solve mathematics problems and the many different uses of constructivism are discussed in the next section.

The Impact of Manipulatives on Children's Learning of Mathematics in the Classroom

Guha (2000) conducted a study on using manipulatives to improve children's mathematics skills. Guha studied a number of pre-school students during a series of visits in an inquiry-based mathematics setting and observed the effects manipulatives had on student achievement. The series of lessons used cookies and pizza to teach how parts of an object come from a whole. As part of Guha's study, the objective was to determine children's mathematical ability and improve the children's skills by using meaningful teaching methods through pictoral demonstrations, manipulatives, and problem solving situations. Guha collected data through observations and field notes. She observed students discovering objects, using them to solve problems, and finally making statements indicating their knowledge of mathematical concepts.

Guha's (2000) findings indicated that 90% of students understood the whole and piece concept after using pizza as a manipulative. Students were able to build the concept of part and whole and relate it to other objects such as cookies and blocks. Through the hands-on discovery with cookies and pizza, Guha observed students showing an increase in understanding. Students were able to make a connection between a piece of pizza and a part of pizza from the whole. The author also discussed how using actual hands on shapes in a classroom helped children learn the shapes. After holding the shapes, they compared them to real life objects around the room. Being able to use objects found in their everyday lives allowed the students to gain a true understanding of the concepts.

In another study using manipulatives, two teachers from an urban school district in Kentucky worked hard to modify their mathematics instruction. Borko and Elliott (1999) explained how their goal in the revamping was to model all problem solving approaches, introduce manipulatives, engage students in the learning process, and to connect their mathematics curriculum to real world situations. These two teachers taught in a 4th grade classroom and spent two years revamping their mathematics program to be specifically hands-on. The authors explained how they were looking for thought processes, reasoning, and a true understanding regarding manipulatives and their impact on the learning of mathematics. During daily oral mathematics, the two teachers used open-response tasks to help students learn to use appropriate problem-solving strategies to write mathematical explanations to accompany their problem solutions. Through interviews with students and observations of student interactions during lessons the teachers collected data to study the impact of their revamping. The teachers also used tests to measure student learning.

According to Borko and Elliott (1999), the results of their revamping were strong and successful. They had an extremely hard time revamping the program because of the push for high grades on the high stakes testing mandated by the district. After the first year, their high stake testing results met and exceeded the district's goals and high expectations the district had. After the first year, the researchers reported that the student test scores increased approximately 10 points. Ten percent of the students also scored on the "distinguished" level and this had never happened before. The students also scored higher on the open-response and multiple-choice questions than ever before.

One of the goals of their hands-on curriculum was to look for a lot more than just a student's answer on paper. Borko and Elliott (1999) analyzed their observations and found that students shared an increased interest in mathematics when using manipulatives. Students felt the manipulatives helped them to learn the concepts and construct understanding. The researchers concluded that by bringing real objects in to the room, they have provoked the students to experiment and as a result construct a better understanding of the concepts.

Rust (1999) also attempted to determine if manipulatives had an impact on 21 first grade students' understanding of mathematical concepts. The eight week study involved four mathematical concepts for research. The concepts were addition, subtraction, measurement, and fractions. During the eight week study the students used unifix cubes, chalkboards, and work mats along with other manipulatives. The teacher's method of data collection consisted of checklists used to document the manipulative used and notes taken during each student interview. For the first method of instruction, the teacher taught a lesson using textbooks. The other method of instruction involved the teacher instructing with manipulatives. The methods of instruction were both used simultaneously by splitting the class into two groups. One teacher in the room took a group and used manipulatives, and the other teacher took a group and used the workbook issued by the school district. After teaching each concept using the manipulatives and textbooks, students were tested with the same Knox County Skills Test.

Rust's analysis showed that each method used to teach the concepts worked equally well in terms of students' performance on the Knox County Skills Test. The researcher determined that lessons taught using manipulatives did not result in a significant difference in the understanding of the information. The results of the Knox County Math Skills Test also showed students learned equally well in each of the manipulative and workbook lessons. Although enthusiasm and interest was not evaluated, the data collected from teacher observations showed an increase in enthusiasm and motivation during the lessons taught with manipulatives. Rust concluded that although there was no significant increase in learning when using manipulatives, students did gain knowledge and practice solving problems. The data analyzed by Rust indicated that through the use of manipulatives to construct knowledge, her students gained problem solving techniques they can use throughout life. Although the researcher did not see a statistical impact on student learning, students were constructing knowledge by engaging in inquiry. Through inquiry learning, students were given the opportunity to use their natural curiosity and collaborate with others to construct knowledge.

Constructivist Theory

In an overview of educational psychology, Ross (2005) and McInerney (2005) each review the growth and importance of constructivism in the mathematics classroom. Ross discusses how the National Council of Teachers of Mathematics has attempted to promote the use of constructivist philosophy of learning over the past 20 years. Ross (2005) states that the principles of constructivism, as tied to theorists such as Vygotsky and Dewey, hold that students construct their mathematical knowledge through their social interactions and experiences in the word in which they live. The pedagogical strategies grounded in constructivism are learner centered and this allows students to develop strategies to solve real life situations and problems. Ross believes by introducing the theories of constructivism into daily mathematical teaching, students and teachers are not only engaged in the process of learning and discovery but also construct their mathematical knowledge as recommended by NCTM standards.

In McInerney's (2005) review of the developments of educational psychology over the past 25 years, he examines Piagetian and Vygotskian theories as well as their growth and benefits in the mathematics education. McInerney is a strong supporter of Piaget's theory of children developing their own ways of handling knowledge within their cognitive stages and Vygotsky's concentration on the social dimensions of learning. He believes through the combination and practice of these theories, learners are able to construct their own meanings within their social environments. What was considered a best practice 20 years ago has changed in today's time. McInerney believes constructivism has stood this test of time and is a crucial theory in the learning and teaching of mathematics. With the access of so many great manipulatives and teaching tools found in today's schools, constructivism is able to meet the learning needs and complexity of today's classrooms.

McInerney (2005) focuses on the study of Piaget's theories on human knowledge. McInerney believes Piaget's theory has formed a foundation for much of the constructivist theorizing that is reflected in today's research and theoretical practices. Piaget's notion of cognitive constructivism encompasses the theory that learners construct their own schemas through personal interaction with the world of experiences. Through this personal or cognitive constructivism, learners develop sophisticated ways of handling new knowledge.

According to Allen (2005) Vygotsky's concepts of mediational means and mental tools are elements developed by humans for the mastery of a person's mental process. Through Vygotskian studies, we learn that there are two types of tools that humans must distinguish between. The two tools, as coined by Vygotsky, are known as technical and psychological. The psychological tool is what we use to direct the mind and behavior. These psychological tools are used to mediate the relationship between the world and subject. The technical tool, as explained by Allen, is a tool which we use as a mediator between external objects or manipulatives and human activity. Allen believes educators extend children the opportunity to apply learning and new knowledge to the real world as they become familiar with the facilitation of mediational means and incorporate constructivist views such as a collaborative learning environment, social interaction, and genuine learning tasks into our daily routines.

The use of the Constructivism as a theoretical basis for the teaching of mathematics promotes higher-level critical thinking skills and allows students to learn within their zone of proximal development. Greene (2005) refers to this zone as the space for learning where an individual can move with support of an expert. Learners can also create their own zone of proximal development when working and collaborating with each other. This construct, coined by Vygotsky, is a zone where learners need to be moved for cognitive change to occur. When this learner-centered theory is in play, the student and teacher each enact curriculum in ways that grow out of their experiences (Ross, 2005). While students are in the zone of proximal development, shared problem solving occurs. Greene states how this zone encourages students and experts to jointly construct meaning while students find a balance between autonomy and dependence. The student has the opportunity to take learning spaces beyond those in which they may not traverse alone. Greene calls this journey the unknown future. These learning experiences will enable the student to develop an understanding of current knowledge and acquire appropriate skills for future problem solving situations.

Fowler and Poetter (2004) explored the French approach to the teaching of elementary mathematics. The researchers looked at the teaching of mathematics in these schools and analyzed what actually happens during a French mathematics lesson when taught using constructivism as a theoretical basis. The researchers investigated the type of pedagogy, the curriculum structure, and the instruments used to implement mathematics instruction in France. The researchers compared the impact of constructivisim in the French classroom to the American back-to-basics system where there is strict accountability and a high value on test scores. Through observations in 13 French primary schools for a total of 60 hours, the researchers observed constructivist-based mathematics lessons. The researcher also interviewed 20 French educators. These interviews focused on the French national curriculum for mathematics and questions pertaining to the curriculum and its clear guide that mandates teachers to connect math to the world, subjects, and the lives of the learners.

While observing the French teachers in their mathematical settings, the researchers observed an evident problems-based pedagogy and every mathematics lesson was embedded in a "real life context" (Fowler & Poetter, 2004). The children interact with their classmates and teachers in many activities related to what Fowler and Poetter call the "problem situation" while engaging in mathematical discussion. The researches learned through the interviews that that the French curriculum does not specify what process should be used to teach and there is no mandate of a text books or tests. There was also no evidence of a skill and drill approach and high stakes testing was not used.

Fowler and Poetter (2004) found the reason for the students' success in the learning of mathematics to be their teachers' skillful use of formative assessment, the use of knowledgeable professionals in the field of mathematics, and the use of the constructivist approaches to teach mathematics. Through the engagement of mathematical discussion coupled with the real life context of each lesson, Fowler and Poetter observed students constructing their own knowledge and connecting each new concept to their own personal lives. This method of learning resulted in what Fowler and Poetter call a personalized learning environment. Fowler and Poetter found that through a combination of student notebooks, a detailed student record, and a 3rd and 8th grade criterion reference tests, teachers and administrators were able to enhance the learning of each individual student. The researchers found that these three formative assessments did demonstrate the impact of the pedagogy and curriculum on student learning and achievement achievement. Fowler and Poetter concluded that through this study others can learn that the curriculum of real life context is what makes learning take hold and last.

Siegal's (2005) conducted a qualitative study of an 8th grade teacher's personal definition and teaching of cooperative learning while drawing on psychological constructivisim as the framework for teaching. Siegal's aim was to show that cooperative learning coupled with a constructivist framework can increase student achievement. Through interviews and observations the author collected data at the Iroquois School District in Albany. During 40 minute lessons for a period of ten weeks the author observed and interviewed a group of 8th grade students. The students worked on mathematics in cooperative groups and were given tasks that allowed them to construct their own knowledge. The students were introduced to new skills in a whole group setting. After a short introduction, students were put into groups of 2 or 4. In their groups they collaborated and used real-life applications to practice their new concept and solve problems pertaining to percentages and decimals.

Siegal's (2005) analysis indicates that cooperative learning coupled with a constructivist framework can increase student achievement and social skill development. For example, during the lessons, students were observed using receipts from restaurants. The analysis of these observations and interviews suggested an understanding of percentages and decimals when using the real-life objects. The data collected from the student interviews showed that students felt the partnership of cooperative learning and the way the lessons were constructed resulted in an exciting learning experience.

In a comparable study, Sharp (2002) examined children's thinking when they had the opportunity to construct personal knowledge about the division of fractions. The students were from a public school district in the mid-western United States. The study involved 23 5^{th} grade mathematics students. Through the use of pictures, symbols, and words students collaboratively solved problems and communicated their solutions to the given fraction problems. The students approached the division of fractions through problem solving by constructing their own knowledge. The students selected real world objects such as gum, candy bars, and cups of orange juice to learn and formulate problem solutions to equivalent fraction and mixed fraction problems. The researcher collected data through pre and post-tests, daily field notes, and daily work completed by students.

By analyzing the scores of the pre-test, Sharp (2002) learned that no student knew an algorithm for the division of fractions before the study began. After the study, the post-test determined that all of the children were able to resolve the situations using division concepts. The notes and observations taken by the researcher were analyzed and showed the students' solutions to problems were built from a combination of the use of real objects and students' existing knowledge of fractions and division. By reviewing the student work collected during the study, the researchers were able to review which manipulative was used and learn how the student created a solution to solving the problem. Students were given the opportunity to develop their own strategies to divide fractions, and through the constructivist framework, the data proves that students were able to construct their own solutions. Sharp (2002) states "through the window of constructivism, this study allowed the authors to glimpse children's constructions of knowledge and provide alternatives to the traditional view of the expected procedure (invert and multiply) that children should learn for the division of fractions" (p.333).

Problems-Based Learning

Problem-based learning (PBL) is an approach to education where teachers carefully select and design problems that enable the learner to acquire and use problem solving skills, self-directed learning strategies, and communication and team participation skills. Clouston and Whitcombe (2005) believe PBL creates a creative environment that enables students to be active and participative. The process of PBL helps to replicate the approach to solving problems and meeting our every day challenges that are encountered in life.

In a study in the Southern California area, Moseley (2005) examined students' early conceptions of rational number representations. The study consisted of 26 fourth grade students from 5 fourth grade classes. The students were broken into two groups. One group of 12 students received a curriculum that had an emphasis on constructing knowledge in mathematics through problems-based learning. The curriculum was based on the teaching and learning of part-whole relations. The second group received the same part-whole curriculum but was introduced to the information in an alternative traditional style. All students involved in this study were given a pre-diagnostic test of rational number knowledge. This was given to determine if the students in both groups had similar rational number knowledge at the beginning of the study. The two week study involved an eight lesson curriculum emphasizing part-whole relations in different perspectives. Students were given a task or problem involving part and whole relationships. The materials were also provided to the students and they were responsible for completing the task using manipulatives such as playing cards and pictures in any way the chose. Through video taping, individual pre and post assessments, and an analysis of student work the researchers collected data.

Moseley (2005) shared how each group scored equally on the pre-test. The analysis of the data collected from the post-test concluded that students in the problems-based group produced higher test scores then students from the traditional group. An analysis of student work also proved that students in the problems based learning group had a stronger understanding of part and whole concept then students from the traditional group. Through his video analysis, Moseley concluded that meaningful learning did occur when students were encouraged to relate new concepts to familiar ones in their everyday lives by meaningful activity. The analysis supports that there is a better understanding of the part-whole relation in fractions when students were involved in the problems-based learning.

Clouston and Whitcombe (2005) consider a number of factors that enable the consideration, introduction, and evaluation of problems based learning. The introduction of PBL was first developed in the 1960s to facilitate medical students' application and synthesis of knowledge through "real life" case studies. The use of problems based learning has carried over into education and provides a creative environment that enables students to be active and participative. Students have the empowerment to develop their own problem-solving skills and techniques in a self-directed study approach. In this approach the students' learning experiences are enhanced while building communication, collaboration, and self-reflective and evaluative skills. Clouston and Whitcombe share through their past experiences with PBL that for this method to be successful the teacher must create an open learning environment where traditional methods of lecture are set aside. Through their past learning experiences students formulate their own lens and use this lens to view and respond to other learning environments in different ways. By keeping this in mind teachers can support each learner and accept that each will have a unique relationship with the problem. While focusing on the solution to a problem, the teacher needs to be aware that an answer to a problem in this method might not be wrong, it is just perceived in a way that is best understood by the student. By using real life problems as triggers, Clouston and Whitcombe believe critical thinking and new learning can be applied and repeated until there is a true understanding.

Summary

The review of the literature in this section has helped me to formulate ideas and acquire knowledge to conduct a similar study involving an inquiry-based teaching approach involving manipulatives. While reviewing these studies I have begun to gain a better understanding of how to carry out a successful research study focused on inquiry- based teaching. The inquiry-based environment is what Fuller (2001) calls a rich, active learning environment in which learning is built on discovery and reflection.

I have also acquired numerous ideas and methods pertaining to the incorporation of manipulatives in a mathematics classroom. When manipulatives or hands-on objects are used to construct knowledge, Rust (1999) believes students are engaged in inquiry, gain problem solving techniques, and use their natural curiosity and to construct knowledge while collaborating with others. Along with using manipulatives as hands-on tool for learning, I will also teach my mathematics lessons using constructivism as a theoretical basis. By constructing their own knowledge and connecting each new concept to their own personal lives, students are a part of what Fowler and Poetter (2004) call a personalized learning environment.

While conducting my study, I will continue to re-visit these pieces of literature to gain more knowledge and ideas. I will also reflect back on these pieces and use them to familiarize myself with the effects that inquiry-based learning, constructivism as a pedagogical framework for the teaching of mathematics, and the use of manipulatives can have on student's understanding of fractions.

Chapter 3

Methods and Procedures

Introduction

While creating the unit on fractions for this study and preparing to carry out the project, many different methods and procedures were developed. I carefully selected four tools for collecting data and designed data analysis methods to investigate the impact manipulatives had on the students' understanding of fractions. Each of these methods and procedures were created with regard to the benefit of the three case study students involved. After much planning, preparation, and review of literature, I created a 5 week unit on fractions involving a total of 18 lessons that investigated how teaching mathematics with inquiry-based lessons using manipulatives impacts 3^{rd} grade children's understanding of fractions. This study was created to gain a better understanding on the impact of manipulatives and use this information to improve my teaching and therefore better the education of fractions for third grade students.

Assumptions

The students selected for this project each have different ability levels. I believe the different ability levels had an impact on the outcome of this study. Among the three students chosen each have a mathematical ability level of either high, medium, or low. The student ability levels were determined according to the students' previous report card scores in mathematics and assessment of their mathematic abilities according to previous observations and each student's unit examination score. Each student shared how this is the first study they have ever been a part of. I believe this has an impact on their class participation. Each was eager to learn and be a part of such an interesting grown-up event.

The materials chosen for this study were readily available through the school district. Since I used manipulatives seen in students' everyday lives such as money and food, I found these objects to be readily available. My classroom was stocked with play money and the candy used was a gift from the principal. The wooden shape blocks, along with other mathematics manipulatives used, were from my classroom supply cabinet.

As the classroom teacher and researcher I felt confident in my ability and opportunity to carry out all aspects of this research project. Much pre-planning and research went into this project and this helped me carry out each student's case study smoothly. Being the researcher as well as the teacher I assumed there would be times when my expectations were far different than the actual outcome. As the classroom teacher I had already worked with these children on a daily basis. As a researcher knowing each student's abilities and work habits, I found it difficult not to expect a particular outcome for each lesson.

Research Question

The objective of my research was to discover how teaching mathematics with inquiry-based lessons using manipulatives impacts 3rd grade children's understanding of fractions.

Participants

This research took place at L Elementary School. (A psuedonym was given to the school.) L Elementary School is an elementary school in a rural town located in western New York State. The U.S. Census Bureau indicated in their 2000 report that this predominately white community (96%) is populated by about 8,000 people. The average household income is around $40,000 and 3.8% of the families fall below the poverty line (U.S. Census Bureau).

L Elementary School currently enrolls 670 students. A total of $12,200 is spent on each child within the district. Out of those 670 students, 93.7% are white, 0.6% have limited English proficiency, and 25.2% receive free or reduced lunch making the school eligible for Title I funding. The school employs a total of 77 teachers, all of whom are considered to be highly qualified. An average class size at L. Elementary is 19 students, and each grade has no more than six sections.

The 3rd grade class in which this study took place consisted of 18 students. The classroom offered a comfortable and inviting atmosphere with student work displayed on each wall. There were class rules and procedures displayed on other bulletin boards in which were each designed and written by students. The desks were arranged in a comfortable fashion with small groups ranging from three to four students in each. The ceiling was covered with sticky hanging animal figures that glow when the lights were turned off. Among the comfortable atmosphere were a collection of twenty small posters that each relay a different motivational message along with famous quotes from popular authors.

From the 18 students in my 3rd grade classroom, the study involved 1 high, 1 medium, and 1 low achieving student in mathematics. Although the mathematics lessons were taught to the whole class, I collected data from the smaller number of regular education students. After the case students were randomly chosen, each of the 3 students was given pseudonyms or secret names to protect their identity. Concentrating on a smaller sample of students, I had the ability to spend more time evaluating and observing. This allowed me to collect more detailed data and get a true analysis of the students' understanding of fractions.

To assure parental permission I sent a letter and consent form home to the parents. This letter was used to share information about the purpose of the study along with a consent form to grant permission for their child to participate. Parents and students were notified of their rights and knew that at any time they could decline participation from the study. Parents were also notified that all personal information and data collected during the study would be securely stored and used for research purposes only. Oral assent from the students was also obtained and an explanation of data collection methods was given, as well. I also asked permission to use the data in this study. A letter of consent to the principal with an attached copy of the research proposal was also submitted. Student rights and confidentiality were assured by providing pseudonyms before data collection began. All student names were deleted from work that was collected, and identified only by pseudonyms. All collected data was locked in a filing cabinet in my classroom and student data was shredded after completion of the study.

Procedures

After reviewing literature on manipulatives and inquiry-based mathematics lessons, I began to formulate an inquiry-based unit on fractions. The unit was designed to foster children's natural curiosity and encourage children to discover learning and problem solving through collaboration and hands-on activities. Each activity chosen for the fractions unit enabled children to learn fractions through the use of hands-on manipulatives. The manipulatives used for each lesson were objects that each child is familiar with and use in their everyday lives. After collecting many easily accessible manipulatives found around my classroom and school, I began to design inquiry-based lessons around these manipulatives. By looking at the current mathematics curriculum I knew what needed to be covered and used this as an outline for my inquiry-based lessons. I focused on basic fraction introduction including parts and whole, equivalent fractions, and mixed fractions. While reviewing the curriculum I began to choose manipulatives that fit best with each lesson.

The main goal for student learning was to increase students' understanding of fractions. I wanted to learn if maniupulatives did have an impact on students' understanding of fractions and use any new information to improve my teaching of fractions. The lessons in this study were also designed to get students collaborating and working in small social settings. My goal was also to get students working together to solve problems and give them the opportunity to develop and practice social skills.

Throughout the unit I began to see the many different types of learning experiences generated from my inquiry-based lessons. Students were using a combination of prior knowledge and collaboration to solve problems. Many students were excited about this new learning experience and the level of motivation increased as each lesson built upon the prior lesson. Conversations were happening around the room during mathematics on a daily basis and it was great to see the level of social interaction increase in such a short period of time. The most interesting experience taking place during the unit was watching students learn through the use of maniulatives. Each student used the every day objects in their own personal way to solve problems and gain new knowledge. Buy discovering their own methods and using exciting objects to learn students began to move into their Zone of Proximal Development. As students used manipulatives to solve problems, I began to acquire a new role as a teacher. Instead of instructor I became a guide. I used my creativity and my personal knowledge of the maniuplatives to guide the students and introduce ways to use the manipulatives to learn fractions. Each child then found their own way in which they felt comfortable with and used the manipulative not only as a familiar object, but as a learning tool.

Through a series of inquiry-based mathematics lessons I used surveys, interviews, observations, and student work as instruments to learn if manipulatives do have an impact on childrens' understanding of fractions. Each lesson taught in this study has been grounded using constructivist theory. Ross (2005) believes that by introducing the theories of constructivism into mathematical teaching, students and teachers become engaged in the process of learning and discovery. This pedagogy of the construction of mathematical knowledge is also recommended by NCTM (2000) standards. Throughout this study I have transformed my classroom into a student-centered learning environment. Students have begun to construct their own knowledge through inquiry, problem solving, and the use of prior knowledge.

I collected data for this study using observations, interviews, and student work. I have also given a short response survey to the students on the use of manipulatives before and after the completion of the study. The pre and post survey was an instrument used to discover if students felt comfortable using manipulatives and if they felt as though manipulatives were beneficial in learning. Students answered three questions pertaining to their feelings towards the use of manipulatives. The pre and post surveys contained the same questions and were given to all students in the class. The only surveys analyzed for this study were the ones taken from the case study students. The interviews were also an important tool to investigate the impact of manipulatives on student motivation. The post surveys yielded data that helped me know if student feelings towards the use of maniuplatives changed after the study was complete. This information was also used to study the impact manipulatives had on students understanding of fractions (see Appendix A.).

While observing I used a journal as an instrument to record my observations on each of the student's interactions and progress. These notes were used to record the students' use of manipulatives to solve problems and communicate their mathematical thinking. The data yielded through my observations consisted of documented conversations heard between students, and notes taken on the student's use of manipulatives to solve problems. I wrote what I heard and saw in the journal during lessons while observing students working and interacting. I also used the journal to reflect at the end of the day on the lesson and document my summaries of the lesson as well as behaviors and student interactions. I used these observation notes to determine if the students were struggling with a lesson or had a good understanding of the lesson.

The interviews throughout the unit were conducted during the lessons and included short response questions about student ideas and questions about their methods of problem solving. Most questions asked were informal and pertained to the student's thinking and actions during each lesson. The interviews were open-ended and yielded data including each student's choice of learning methods, specific procedures used in problem solving, and how they used the manipulatives to help them understand and learn. Student interviews were documented on a student interview sheet and each question asked and student response was written. I also interviewed students after lessons to ask questions pertaining to the students understanding of information in the previous lesson as well as how they felt the use of manupulatives worked during the lesson (see Appendix B.).

I also collected student work produced during the lessons and this data was used to assess their understanding of fractions. Most of the student work was done on blank pieces of paper which students used to show how they solved a problem or used a manipulative. After students used a manipulative, they drew a picture to show their work. A brief explanation was usually added to the bottom of each piece of student work explaining how he got the answer and the methods he used to solve a problem.

Data Analysis

I analyzed the use of the manipulatives through my observation notes to learn if the manipulatives were a beneficial factor in each student's efforts to solve a problem. These notes also helped me to collect data on the effects the manipulative had on the student's understanding of fractions. The analysis of my student notes was done after each lesson was complete. As I sat down each afternoon I compared my notes on each student's activities from that day's lesson to the lesson objective and learning goals. By comparing the notes to the lesson objective and goals I learned if students were using the manipulative to help them understand fractions and I could determine the method they used to solve the problems. Analyzing what I saw during each lesson and reflecting back on these notes helped me to determine if students were making progress and which manipulatives had the biggest impact on the students' understanding of fractions.

I used constant comparison methods (Stringer, 2004) to look at each student's interview responses over time to uncover patterns and themes in each student's responses and secondly to look for patterns and themes across the interviews for all three students. The interviews were analyzed by reading the student responses on the interview response sheet and determining if students were using the maniuplatives to solve problems and how they were using the manipulatives. Throughout this study I realized students used manipulatives in completely different ways. By analyzing the response sheets and comparing the response sheet to their student work, I learned which way they used the manipulatives and determined if the manipulative did have an impact on their understanding of fractions. These response sheets became a crucial tool to not only analyze student understanding of fractions but also to assure students could explain how they used the manipulative and why they chose the method they chose. Many students showed the answers to the questions by using the manipulatives, yet the interviews helped me to dig deeper and encourage them to actually explain their thinking. This extension of student thought gave me a true answer to whether or not there was an impact on the students' understanding of fractions.

The analysis of student work has been a crucial part of this study. Student work was been used to show how students understand the content. By reviewing student work I have seen what different methods of problem solving were used to construct knowledge and I have searched for clues that show understanding of the content. The analysis of student work helped me to focus on the nature of each student's thinking and how he used manipulatives. When a student's work showed a correct problem solution, I could examine the tools they used to solve the problem and could conclude that manipulatives did have an impact on the students understanding of fractions. Throughout this study students used manipulatives in different ways to solve problems. Each student had his own way of using the manipulative to lean and discover. Analysis of data collected from student work also showed how students were using the manipulatives to solve problems. I analyzed the work to determine the method in which they used to solve problems. After examining the methods used through student work, I then compared each case study student's work to another. I was then able to determine how students used manipulatives differently and could then conclude the impact manipulatives had on each student's understanding of fractions.

The student surveys were collected before the study began and after the study was complete. I compared the surveys given before the unit to the ones given after the unit. This was done to learn if there was any change in student feeling toward the use of manipulatives. I also compared the answers given on the surveys to their student work and informal interview responses. The analysis of this triangulation of data helped me to compare feedback on the interviews to student work and informal interviews. This helped me to discover if a student's feelings towards maniuplatives had an impact on his understanding of fractions.

Through the data analysis of the surveys, student work, observations, and interviews I have constructed a case study of each student. This triangulation has helped me to look across many different kinds of data to determine the impact of manipulatives and study the methods students are using to construct their own knowledge of fractions. The analysis of student work coupled with my observations and interviews has helped me to learn how students are using the manipulatives to help them learn and if these manipulatives introduced through the constructivist theory have had in an impact on their learning of fractions.

Limitations

There were some predictable factors which affected the investigation. There was always potential for student absenteeism during this study. When absenteeism occurred, there were missing data on certain students during that time. Since I am the researcher and classroom teacher there were times in which my prior knowledge of the students did limit the study. By knowing each student on a personal basis as well as their academic ability before the study began, there were some circumstances where I did not push the student beyond their comfort zone. There were times in the beginning of the study when I did not push them past their comfort zone because I knew there was a possibility of them shutting down or loosing interest like they had in the past.

Time Schedule

Data collection began on April 30th, 2007 upon approval by the SUNY Brockport IRB. Data collection continued from April 30th to June 8th. Data were collected through a series of 18 inquiry-based math lessons each 40-60 minutes in length. Data analysis continued through the months of June, July, August, and September of 2007.

Summary

By creating my own inquiry-based unit on fractions and selecting the tools for collecting data and data analysis methods, I created a study that investigated how teaching mathematics with inquiry-based lessons using manipulatives impacts 3rd grade children's understanding of fractions. I have constructed a case study of each student by looking across many different kinds of data to determine the impact of manipulatives and study the methods students are using to construct their own knowledge of fractions. By concentrating on only three case study students, I had the ability to spend more time evaluating and observing the impact of manipulatives introduced through the constructivist theory. This allowed me to collect more detailed data and get a true analysis of the students' understanding of fractions. By carrying out this study I was able to gain a better understanding on the impact of manipulatives and use any new information to improve my teaching of fractions.

Chapter 4

Findings

The main purpose of this study is to examine and evaluate the impact that mathematics instruction grounded in constructivist principles has on children's understanding of fractions and use any new information learned to improve my teaching. The location of this study is in upstate New York. The entire study took place in my 3rd grade classroom. As the classroom teacher and researcher I randomly selected three students who each fall into a high, medium, and low mathematical achievement category. By using the students' prior Terranova testing scores, report card scores, and the review of student classroom work I was able to select the students according to their mathematic ability.

Participants

The first student selected for the study is R.J. (Pseudonyms are used for all case study students.) R.J. is an eight year old boy who falls into the average mathematical achievement range. R.J. loves math and has had a solid B average in all subjects throughout the year. His strengths are in the area of place value and counting money. R.J is interested in hot rods and loves Gym class, math, and snack time. Throughout the year R.J. has struggled with division of single digit numbers, and has had trouble remembering his multiplication facts and counting money. R.J does have some difficulty working in groups and is extremely selective when it comes to friends and work buddies.

The second student selected for the study was Flames. Flames is an eight year old boy and is extremely active. He loves animals and jumping his bike over ramps. He is a kind and respectful boy and loves school. Flames has his addition and subtraction facts memorized and having these memorized is one of his strengths. His greatest challenges were in the areas of multiplication, division, word problems, and elapsed time. Flames has difficulty with computation of multi-step problems and is also a student with ADHD.

The third student selected for this study is Tony. Tony is quite the mathematician and has had a 100% average in math throughout the year. Mathematics is not Tony's favorite subject and he has shared this feeling with the class many times. Although he does not enjoy math at times, he has never been a behavior problem in class. Tony loves to be challenged and I have made it a priority to challenge him in all subject areas through out the year. Tony struggles with motivation. This is likely due to the lack of a challenge in math throughout the past years in school. Tony likes hiking and camping. Reading *Harry Potter* books late at night is his favorite thing to do. I have encouraged him to teach mathematics mini-lessons to the class in the past and he believes this to be the best thing he has ever done in school.

I have constructed a case study of each student through analyzing data collected through surveys, student work, observations, and interviews. This triangulation has helped me to look across many different kinds of data to determine the impact of manipulatives and study the methods students used to construct their own knowledge of fractions. Throughout the entire study I collected data on Tony, R.J. and Flames. My analyses of the data yielded different results for each of the three students on not only how teaching mathematics with inquiry-based lessons using manipulatives impacted their understanding of fractions, but also in the areas of motivation and group work skills.

R.J's Case Study

The student for whom manipulatives had the greatest impact on the understanding of fractions was R.J. As the study progressed, R.J. began to participate more and share answers. He was so excited when each answer he shared was correct. Seeing R.J. use the maniuplatives to solve problems his own way and understand concepts was a rewarding experience.

The first sign of R.J's increase in understanding of fractions was seen on May 1st. The lesson was titled *What is a Fraction?* As the students were defining the term *fraction* in their own words, I noticed R.J.'s response on his lesson guide. He stated that a fraction was two numbers that are made to show a part of something out of the whole. On his paper he had the word *hole* next to the word *whole* in quotations. While watching R.J. write this on his paper, I also observed his facial expression. He flashed a large smile from ear to ear when I pointed out his perfect response. He explained how he remembered what we talked about yesterday and asked if he could help other students who might not understand. R.J. insisted on sharing how a fraction is part something taken from a whole. R.J. used the example of a cell phone. He explained how a fraction of the battery can have energy left in it. The cell phone was an example I had used the day before. This showed me he was listening to the previous day's lesson and made a connection between the two lessons. By connecting the two lessons R.J was constructing meaning. He used information he had already learned and connected it to new knowledge. R.J. had clearly understood the concept of *whole* and *parts.*

The previous day's lesson consisted of students taking objects from a hole (a black five gallon plastic bucket) and putting them into two boxes labeled "whole" and "parts". Since we were studying homophones for spelling that week, I thought it would be a good idea to incorporate spelling into our mathematics activity. This lesson was designed to introduce the terms *whole* and *part* while enabling students to discover the difference between the two. After depositing whole pieces and broken pieces of pencils, erasers, paper, and pipe cleaners into the correct boxes, students made fractions with the manipulatives as they pulled them back from the boxes. R.J. had remembered this lesson and was using information he had retained to help him understand the new information. When I realized R.J. was making connections to the *Whole and Parts* lesson from April 30th, I was pleased with his answer and he happily shared his definition with the class. While standing up, he stated that a fraction was two numbers that are made to show a part of something out of the whole. He held up his paper he displayed the word *hole* next to the word *whole* in quotations. Through the many lessons following, I realized R.J. was beginning to understand the new concepts and in turn I noticed an increase in his confidence.

The real life manipulatives which R.J. used to construct knowledge are what Clouston and Whitcombe (2005) call initial triggers. These triggers, or everyday objects, are used to solve real life problems and enable the student to reflect on any prior knowledge to construct new knowledge. When using the manipulatives to construct new knowledge, students are also building critical thinking skills. Clouston and Whitcombe believe these critical thinking skills to be the skills used for life long learning in not only mathematics but in everyday situations.

R.J. continued to show progress with concern to the understanding of fractions during the duration of the unit. I was especially pleased with his work on our lesson titled *A Sweet Lesson!* on May 3rd. While working with candy bars, the class was creating fractions using chunks of chocolate. Each chocolate candy bar had eight equal squares. While breaking squares off from the whole candy bar, students were discussing with each other what part was taken from the whole. They were creating fractions and sharing the fractions with each other. Students were also discussing how they made the fraction and discussing all of the possible fractions that could be made with eight as the denominator. As the students broke off a section of chocolate from a candy bar they were creating fractions and writing them on the lesson guide paper as well. They had free range and were not told how much to break off or what fraction to write. We did have a discussion before the activity involving where to put the whole number and how there were eight pieces of chocolate in each candy bar.

As the students were creating fractions I observed R.J. leaning over talking to the student next to him. Just as I was about to ask him to stay on task I observed the following conversation:

R.J.: You need to put the bigger number on the bottom.

Student: Ohh. What if I break 4 pieces off? Can I still put four on the bottom?

R.J. No. The whole number of pieces is eight. You gotta put eight on the bottom and four on the top. It's like one half now. See?

Student: So now I have four over eight. Oh.

I was glad I stopped myself because this was a fantastic conversation. R.J. was teaching another student what he had learned and this was the first time I had witnessed R.J. doing this. As the students were deeply involved in this inquiry-based lesson and engaged in using manipulatives, I observed first hand the power of discovery. R.J. was so excited with his newly found mathematical understanding and was equally excited to share his new knowledge with others.

I then asked R.J. an informal interview question pertaining to his thinking. The question was: How did you know to put eight on the bottom and four on the top? What was your reason for this? He explained how he remembered the first time we talked about fractions and explained to me that the bigger number goes below the line on the bottom. I was happy to see R.J. make these connections and share them with other classmates. He used his prior knowledge about the denominator and the maniuplative to not only learn new concepts and represent his answer, but to also help others. The chocolate had become a learning tool and had brought the two boys together in a meaningful discussion filled with discovery and connections. The cognitive change that had occurred at this moment proved the manipulative to be a tool which had supported the students at the students' new Zone of Proximal Development.

During our post-lesson conversation I asked R.J. to tell me how he knew four eighths was equal to one half. R.J. explained his thinking in terms of the manipulative (chocolate) and said it looked like the candy bar was cut perfectly in half. Since four was half of eight, and R.J. had been manipulating the chocolate for some time, he made the connection using numbers as well as holding the two equal pieces next to each other. Since the chocolate bar was divided into eight pieces, R.J knew that eight was the whole amount and this number belonged on the bottom of the fraction. When the candy bar was cut in half, R.J. noticed there were four pieces in each half. By using the manipulative, R.J discovered four eighths is equal to one half. According to Allen (2005), Vygotsky's concept of mediational means and mental tools are the tools which learners can use as a mediator between external objects and human activity. When introducing these tools to the learner, the learner has the opportunity to apply learning and new knowledge to the real world as they become familiar with the facilitation of mediational means.

On May 17th I started a lesson titled *Quarters and Halves.* The students were discovering the concepts of quarters and halves in fractions with relation to money. I cut play dollar bills in half and students had quarters on each desk. They were making connections with a half dollar and a quarter of a dollar and then writing the fraction on their paper. As I was interviewing the students, my interview with R.J. was where I noticed the impact manipulatives had on his understanding that day. R.J. had used the cut dollars and put them back together on his paper. He drew a line over the dollar pieces and put two quarters on the top. He wrote the fraction two fourths next to the figure he had made. I asked R.J. why he put the dollar back together and put it below the line on his fraction paper. R.J. explained how the dollar bill represented four quarters and instead of using four quarters for the bottom number in his picture he used the dollar for the denominator. He said, "I put the dollar back together because it's now a whole dollar and the whole dollar goes below the line. It called the denominator Mr.C… hello"! I was so happy he made this excellent connection through his own discovery and I responded by telling him he not only had a great representation of a whole but he also remembered a big vocabulary word from a previous lesson. I was also pleased when R.J. did not use four quarters for the denominator as many other students did. He has had trouble counting money all year and he had done a great job with it during this lesson by using the dollar bill in place of the four quarters.

While observing R.J throughout the rest of the lesson, I noticed he was using the quarters for the top of the fraction and dollar bills for the bottom. Each time he made a fraction on his desk he drew a picture and wrote the fraction next to the picture perfectly (see figure 4.1).

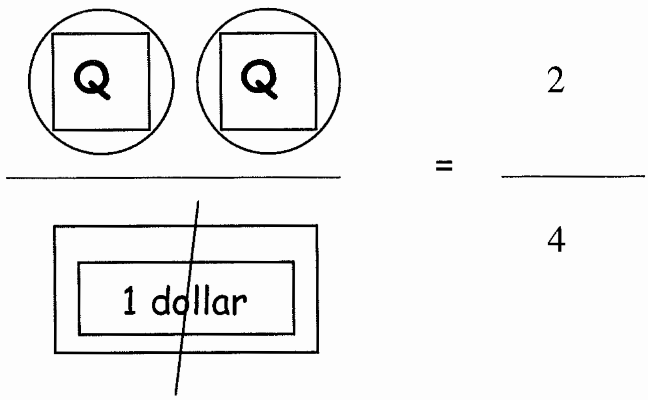

Figure 4.1

Moseley (2005) believes meaningful learning can occur when students are encouraged to relate new concepts, such as fractions, to familiar ones in their everyday lives by meaningful activity. In this lesson R.J. used a familiar object and related the object to a new concept. With this inquiry-based activity coupled with the use of everyday manipulatives, Clouston and Whitcombe (2005) believe students acquire what they call a true understanding. R.J. used the manipulatives as tools or mediational means. The technical tool, as explained by Allen (2005), is a tool which we use as a mediator between external objects or manipulatives and human activity. R.J. used the money as a means to develop his own representation of fractions. He used the maniuplatives to mediate the relationship between the money and the concept of quarters and halves.

While comparing R.J.'s post-survey to his pre-survey, I noticed his responses coincided with his progress during the study. I was drawn to the question asking if manipulatives or hands-on objects help to learn math. R.J.'s response to this question on his pre-survey was "yes". He said, "it's was fun to use maniuplatives to learn math but I never really get the chance".

After the unit was complete, I gave R.J. the same survey. His response to the same questions was: "I think manipulatives are cool because they help me answer hard math questions and I'm better at math than before." His answer to this question was exciting for me and I realized what an impact manipulatives had. R.J. believed he was better at math then before the unit began. The answer R.J. shared on his post-survey showed he was viewing manipulatives and mathematics in a whole new way. Instead of manipulatives being fun, they were now a tool for learning and in R.J.s case, had helped boost his confidence. During a quick meeting at my desk I told him how proud I was of his accomplishments and I explained to him how manipulatives gave him the opportunity to solve problems differently and we both agreed that the new way of learning really seemed to work for him.

While analyzing the data I was impressed by how manipulatives had such an effect on R.J. in terms of an impact on his understanding of fractions. R.J. was already doing well and had always scored within the 50th percentile for mathematics throughout his schooling. Looking back at R.J.'s previous report cards he has continuously done the same with the exception of one ten week term in 1st grade. I believe a reason for R.J.'s success is due to his place in the Zone of Proximal Development. With my inquiry-based lessons, I strategically designed my materials and questioning to fit the needs of all students. When I became able to meet the needs of R.J. and introduce content and experiences to him on his own level and within his

Zone of Proximal Development, he become successful. The Zone of Proximal Development is a zone where learners need to be moved for cognitive change to occur (Ross, 2005). This was a perfect example of the impact of manipulatives used to create the Zone of Proximal Development. R.J. has finally been challenged with support from his teacher and he has risen from the average achievement level in which he was almost stuck in the past.

With each correct answer and solved problem, I could see R.J.'s confidence building. He answered more questions and participated more than ever before. I am hoping the increase in understanding and gain in confidence can help R.J in other areas of mathematics as well as other subjects and daily routines.

Flames' Case Study

Flames is a boy who is easily distracted and has issues with self-control. Flames had some difficulty paying attention during this study. Although he thoroughly enjoyed himself through out this case study, I noticed only a slight increase in his understanding of fractions. Flames' ADHD coupled with no medication, the introduction of what he calls "toys", and a brand new approach of inquiry and discovery to learning were a recipe for distraction. As the study progressed I was able to adjust lessons and guide Flames to a point where there was some increase in his understanding of fractions.

While comparing Flames' pre-survey to his post survey I became aware of his response to the third question. The question asked if the student enjoys using manipulatives to learn, and if so, why? Flames response to this question on his pre-survey was "yes". He stated how manipulatives are like toys. He went on to share how he likes toys and he likes to play. He also stated how he really, really likes sling shots too. After the study was complete, I compared his post-survey answer to the same question. Flames responded, "I like using manipulatives to learn a real lot because they are not boring". As I interviewed Flames and we discussed his answers to this question he shared how he sometimes forgets what he is learning and just likes to play. This told me Flames was distracted by the manipulatives and sometimes saw them as a toy rather than a learning tool. As the case study progressed I adjusted the lessons and gave the students time before the lesson began to explore and play with the maniulatives. This time was beneficial to Flames and he was eventually able to use the manipulatives to his benefit rather than a distraction.

During the lesson titled *A Sweet Lesson!* on May 3rd students were working with candy bars. The class was creating fractions using chunks of chocolate. Students were breaking squares off from the whole candy bar and were discussing with each other what part was taken from the whole. As I observed Flames I noticed his chocolate was half eaten before the lesson began. He had nibbled a small bite from each corner of the chocolate bar and students around him had their hands raised trying to tell me. I approached him and asked why he had already eaten the candy. He told me he ate one fourth. He then reminded me that no one told him he couldn't. Even though Flames had eaten his candy, I did notice he had written some fractions on his white board. When I asked him to explain the fractions to me, he was able to show me how he ate 1 corner of his candy bar which was one square out of eight. When I asked him to explain what the one eighth on his board was, he told me how the one square was one out of eight. Since he had a correct fraction on his board which matched the candy he had eaten, I determined the manipulative was used to help Flames understand the concept.

During our one-on-one meeting after the lesson I asked Flames if he could tell me one way we used candy to learn about fractions. He said, "you take some, then you eat it." I asked him to tell me about his thinking in terms of part and whole from the candy bar. He explained to me how the candy bar was whole and he ate part when he was not supposed to. This told me Flames was distracted from the main idea of the lesson and used the candy to entertain himself in another way. I concluded that the manipulative I chose for this lesson could be distracting for some children, yet the manipulative did have an impact on Flames' understanding of the concept. During a free period that same day I reviewed what we had learned with Flames and I asked him to try extra hard to use self control next time we use manipulatives. I promised to remind him before each lesson some ways he can use the manipulatives to learn. He then promised me he would try his best to use self-control. I learned candy is exciting for children to work with and did increase interest and motivation, but it was still difficult for some children to stay on task and focus on the concepts being introduced.

Choosing the right manipulative for the lesson can sometimes be challenging. No particular manipulative will be effective as a tool for learning for every child. According to Sharp (2002), the selection of real-world contexts and manipulatives for the purpose of mathematics education must relate to the student's ethnomathematics. Sharp believes ethnomathematics is the student's unique personal knowledge of mathematics. The ethnomathematics or unique personal knowledge is what students find familiar in their own lives and consider to be real-world. What might be real-world to one child might not be real-world to another. This idea of ethnomathematics should be considered when choosing manipulatives to help students discover concepts and learn mathematics.

I began to see an improvement in Flames' behavior and understanding of fractions during our lesson titled *Fare Fractions* on May 15th. This lesson was designed to introduce students to the concept of equal fractions. Fare in the title *Fare Fractions* means equal or the same. We began with a brief reminder on the correct use of beans. I reminded students how important it was to respect the bean. "Respect the bean" means keep them out of your ears, nose, and mouth. We then continued with a mini-lesson on equal fractions and discussed how fractions can be equal even though there are different numbers in them. As we began our discovery of equal fractions using beans the students were making equal fractions with beans. They could use the beans any way they pleased to show equal fractions with either 2, 4, 6, 8, or 10 as the denominator. On each student's desk was a white board and each white board had 2 horizontal lines drawn across it. The white board was used as the fraction template and the beans were placed on the white board to show two equal fractions. While observing Flames I noticed he had represented two-fourths on one side of his board with beans and displayed four-eighths with beans on the other side of his board.

Between the two he had drawn an equal sign with his white board marker and wrote "fare" under the equal sign. (see figure 4.2)

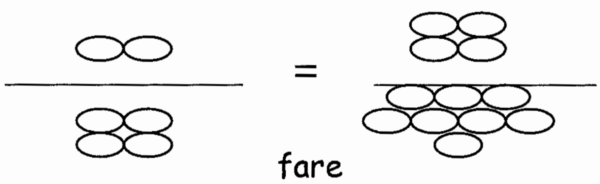

Figure 4.2

I asked how he knew two fourths was equal to four eighths. He told me they were both the same as one half because two is half of four and four is half of eight. Our discussion continued as follows:

Flames: I'm good at division Mr.C and I know about halves.

Teacher: Tell me why you put the equal sign there in the middle and you wrote fare too.

Flames: If you have a candy bar that has 4 pieces and eat 2 of them, you eat half of it. And if you have another one (candy bar) that has 8 small squares and eat 4 of them you eat half of it.

Teacher: What would you rather have four eighths or two fourths?

Flames: They're both fare Mr.C. That's why I wrote fare here.

I then asked Flames to make a fraction with two as the denominator as I did many students while walking around. He told me it was impossible because one couldn't go on top. I asked him to explain why and this is how the collaboration continued:

Flames: One over two means you just have one out of two.

Teacher: You are 100% correct! And what is half of two?

Flames: One...so that's one half?

Teacher: Yes, It's like breaking your candy bar directly in half. You then have one piece in your hand out of the two...or one half.

Flames: One half is still a "fare" fraction like four eighths and two fourths.

I was glad I had pushed Flames to think deeper. With a simple question and reassurance from the teacher Flames understood a concept that is difficult for many 3rd grade children to grasp. I told Flames I was pleased with his behavior during this lesson and I was happy he had used the beans correctly, too. I believe Flames got the most from this lesson because I stressed how important it was to use the manipulatives as learning tools and not toys before the lesson even began. I used proximity during the lesson and made sure I was near or even next to him periodically to remind him I was there and to help keep him on track. The beans were an excellent tool to help mediate his mathematical thinking. He was able to use the beans to show two equal fractions and I was pleased with Flames' on task behavior. The beans were more effective as a learning tool than chocolate, at least for Flames. The class was given 3 minutes to examine the beans and count them before the lesson began as we usually do with manipulatives for all lessons. I did not give the class time to do this before the candy lesson. It is possible that the time spent for exploration before the lesson began enabled the Flames and other students to become familiar with the manipulatives and think of ways the beans could be used as learning tools.

The remainder of the unit went well for Flames and I did see some progress in the lessons following. Green (2005) explains this as Vygotsky's shared problem-solving technique. At Flames' Zone of Proximal Development, meaning was jointly negotiated between the teacher or expert and learner. With my support and use of proximity during this lesson, Flames was able to get more from this lesson and carry out a task that was expected of him and reach a goal that would usually be beyond what green calls the learner's unassisted efforts. In the Zone of Proximal Development the student and teacher each enact curriculum in ways that grow out of their experiences. Greene (2005) states how this zone invites students and teachers to jointly construct meaning. During this collaboration students find a balance between autonomy and dependence. The student then has the opportunity to move in to learning spaces beyond those in which they would not have ventured into on their own.

On May 25th we began to talk about mixed fractions. During a lesson titled *Tangram Fractions, All Mixed Up*, students were discovering mixed fractions using wooden shape blocks from the previous geometry unit. The students were creating wooden trapezoid shapes that are made up of three triangles. When the triangles are put together correctly, they make a trapezoid. Students were showing mixed fractions using the wooden trapezoid shapes. When one whole trapezoid was completed from three small triangles it was considered a whole shape or a whole number. A collection of triangles in groups of two or smaller represented the mixed number. The mixed number always had a denominator of three. The students were drawing pictures of their completed fraction, and labeling the fractions they were creating. I was observing Flames as he created his first mixed fraction. He had 1 whole trapezoid made on his desk and to the right of the trapezoid he had 2 small triangles. He wrote the fraction one and two thirds next to the figure. (See figure 4.3)

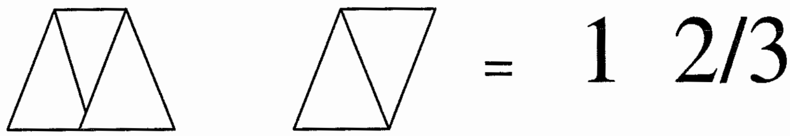

FIGURE 4.3

As I observed Flames put his final triangle in place, I asked him why he had only two triangles to the right and how he knew this was a mixed fraction. The following conversation took place after the interview question was asked:

Flames: I made a whole trapezoid to the left. It was one whole trapezoid so I put the number one in the fraction next to it…and two small triangles out of three next to the whole trapezoid…so two thirds.

Teacher: Tell me why this is a mixed fraction.

Flames: A mixed fraction has big and small numbers.

Teacher: Okay, I see what you mean but we don't call them big and small numbers like we call big and small letters. We call them what?

Flames: The big ones are whole numbers Mr.C., and the rest of the shapes left over make up a fraction.

Teacher: I am proud of your answer buddy, this looks fabulous!

The answer to my interview question told me Flames was on the right track. He had understood what whole numbers were and he was able to represent them with a manipulative. Flames' representation showed me he used the manipulatives to also construct meaning and solve a problem. The shapes were put together correctly and his answer was exactly what I had hoped to see. According to Borko and Elliott (1999) maniuplatives are used to show more than just an answer on paper, they are clues to the learner's thought process. Flames was able to represent the solution to the problem with the use of manipulatives. He also used them to solve the problem and show his mathematical thought process by arranging the shapes from left to right in the correct formation to show a mixed fraction.

After the lesson was complete and I was reviewing my notes I realized Flames' answer regarding big and small numbers was actually not wrong. It was his way of explaining what he saw. In a mixed fraction the whole number is usually written bigger than the fraction. He had used the term "big" in place of "whole" and this was his way of understanding the concept. Flames knew big and small letters or capitals and lower cases. He compared them to whole numbers and fractions and this was how he chose to represent the new information. This taught me a valuable lesson about discovery learning. When children learn through hands on experiences, they are using what they already know to help them solve a problem. Clouston and Whitcombe (2005) believe educators can't assume how a student might perceive a problem or how information might be understood. Each student develops a unique relationship to the new problem. Students bring personal views and perspectives to each new problem and the outcome is always seen differently through what Clouston and Whitcombe call each student's unique lens.

Flames had a rough beginning with this unit. Through my evaluation of his pre- and post-surveys, I determined that Flames still has a tendency to view manipulatives as toys and he admits they can be a distraction. After working with Flames on self-control, and giving some time to use the manipulative before each lesson, I finally began to see an improvement on the many concepts we had studied and learned. Although Flames saw the answers through his own special lens, as all learners do, he still made progress. Flames made his biggest improvement during the most difficult lesson taught on May 25^{h}. Mixed fractions are always extremely difficult for 3rd graders to understand. Flames proved that even some of the most difficult concepts for 8 year olds to grasp can be easily understood when introduced with the correct materials and approach. Through the construction of Flames' own learning of fractions he developed his own ways to use the manipulatives and developed his unique ways to represent the new concepts learned.

Sharp (2002) believes constructivist theory is a window thorough which educators can get a glimpse of the way children construct their own knowledge. An alternative to the expected procedure is provided and lessons are planned to strategically tap into the students' everyday worlds. Sharp believes we can then relate new information to what students already know. Flames used his prior knowledge of the alphabet, and although late in the unit, was able to construct his own understanding in his own way.

Tony's Case Study

The student for whom manipulatives had the least impact on the understanding of fractions was Tony. Although Tony did extremely well and his class work and participation were 100%, I believe all of his success was not directly related to the use of manipulatives. There were some lessons that interested Tony and during these few lessons I did see how manipulatives had an impact on his understanding of fractions. Tony is a visual learner and the traditional teaching approach of listening and lecture seem to benefit him best. Through his case study I was able to determine which lessons benefited Tony the most and which ones resulted in a lack of interest. Tony is a student who likes to know the correct answer and figure each problem out immediately with little time to waste. Tony was uncomfortable with the many acceptable answers to problems and the fact that the problems took time to solve. Borko and Elliott (1999) believe lessons based on the constructivist theoretical perspective involve open-ended problem solving tasks. These open-ended tasks do not usually end up having one right answer. Some tasks even have many acceptable answers. Having many acceptable answers bothered Tony and he began to lose his drive to use the manipulatives to solve problems and understand concepts.

On May 10th the students were involved in a lesson titled *Shoe Fractions.* This lesson was designed for students to discover different fractions using shoes. Each student took off his or her shoes and put them into a large pile in the middle of the room. Students came to the center of the pile one at a time and made fractions with the collection of shoes. Students made up their own fraction such as show the fraction of sneakers from the whole, and show the fraction of boots from the whole. They then represented their fractions by using the pile of foot wear in the center of the room. After they used the shoes to represent their fraction, they used the white board in the front of the room to write the fraction numerically and explain how they used the footwear to show their answer.

Tony came to the center of the room and the problem he chose to solve was finding the fraction of running sneakers in the pile. This was a question Flames skipped when he came to the front of the room. I believe this is the reason Tony chose this problem. He wanted to try a problem that others would not attempt. Instead of sorting the running sneakers into a separate pile and taking the time to count them and find the correct answer, Tony walked by the sneakers and came directly to the front board. When arriving there he quickly wrote 22/36 on the white board and said, “simple”. He said, “twenty-two thirty-sixths are sneakers”. His class members were impressed but not shocked. Tony is a student who always gets the correct answer and usually gets it quite quickly. To challenge him I asked what the fraction was for the remainder of shoes. Within five seconds he said, “oh, fourteen thirty sixths are other types of foot wear”. He sat back down and folded his arms with a large grin on his face. I asked him how he knew both correct answers so fast and this is what he revealed:

Tony: It was cake. I didn’t need to put them in a pile because I knew eleven people had sneakers on so I multiplied it by two. That’s twenty two.

Teacher: How did you know the denominator was thirty six?

Tony: There is eighteen people in our room and then I doubled it.

Teacher: Wow, I never though of doing it that way!

Tony: Like I said.. cake!

After sitting down that day and reviewing my notes I realized Tony was interested in finding the answer as quickly as possible. Using the manipulatives did not interest him. Finding the correct answer was what he wanted and he wanted to find it fast. Tony even answered the challenge question quickly and correctly. He answered the question so fast, I had to stop and think to figure out if it was correct. Since I was looking for the impact of manipulatives, it was evident that Tony looked at the shoes and didn't need to physically manipulate them. Although Tony counted the shoes correctly and made a correct fraction with them, he never had contact with the manipulative. Tony was the only student who doubled the number of shoes in his head that day. He was also capable of finding the difference using mental math and doubling that as well for the remainder of footwear. His lack of interest in using the maniuplative proved that the question was simple for Tony and he didn't need the manipulative. I believe he wanted to find the fraction quickly and handling the manipulatives would have taken too long.

Sharp (2002) believes children build knowledge through many different experiences in personal, realistic worlds. Each child will attack new problems using her or his own personal unique collection of knowledge in a realistic situation. In this particular problem Tony used his own style. Even though he chose to not use the tool offered, he still solved the problem in his own unique way using his own unique collection of knowledge. I used this lesson in the past and students usually love to handle their classmate's sneakers and sort them. Tony felt as though he did not need to handle them to learn the content and represent the answer to the problem. According to Allen (2005), Tony used the manipulatives as what Vygotsky calls psychological tools. Psychological tools are used by people to internally manipulate objects by directing the mind and behavior. This is done without having to physically manipulate the sneakers. Tony used the manipulatives as psychological tools and internalized the manipulatives as psychological tools for reasoning about fractional parts of whole collections.

On May 17th we began our lesson titled *Quarters and Halves.* The students were discovering the concepts of quarters and halves in fractions with relation to money. I cut play dollar bills in half and students had a collection of play quarters on each desk. They were making representations of half of a dollar and a quarter of a dollar with coins and dollar bills. Students were drawing pictures of their representation, then writing the fractions that went with their money on a piece of paper. After the students were done making fractions, they were then explaining answers to their classmates around them. Ten minutes into the lesson I walked by Tony to find his paper completely full of pictures and fractions. Each picture of money drawn had the correct fraction next to it. I then looked at his desk and noticed he had not touched the money. Tony was working from his bonus packet as my students do when they are finished with a task and have extra time. I asked Tony how he used the money to find the fractions on his paper. The conversation took place as follows:

Tony: I didn't touch or use the money because I found the answers in my head.

Teacher: Why didn't you use the money to solve any problems?

Tony: I can close my eyes and picture the money easier. I knew the dollar was the denominator and ¼, is one quarter over a dollar, 2/4 is .50 cents over a dollar, and ¾ is .75 cents over a dollar. They are all easy fractions to make. It's not like there is even a wrong answer because there is like five right answers.

After collecting Tony's paper and checking his work I realized that not only had he created correct fractions, but the only way he used the money was as a tracing stencil to draw perfect circles. After reflecting on our discussion during class and reviewing his work I concluded that Tony did benefit from the manipulatives that day. He used what he had already known about numbers and money along with using the manipulatives as a mental tool. This shows that Tony internalized the concrete manipulative, and then used that mental tool to think. Tony was comfortable solving the problems without actually touching the tools or support from peers or instructors. Since there were many ways to make fractions from money, Tony lost interest from physically using the manipulatives yet had internalized the manipulatives as a mental tool. He likes to be the first one done and loves finding the correct answer before anyone else. Since the whole class was not working to find one particular answer to the same question, Tony felt as though there was not as much competition and his drive to be the first done or the first to find the correct answer was lost. Tony had all of the correct answers on his paper and it was clear Tony used the manipulatives as not only as a tracing tool, but as a mental tool to learn concepts as well.

Clouston and Whitcombe (2005) believe when manipulatives are used as mental tools or a trigger to learn new concepts, they are called initial triggers. The manipulative was initially a trigger which Tony used to solve real life problem. The use of the tool as a trigger enabled Tony to reflect on any prior knowledge to construct new knowledge. Although he did not touch the tool, it was used mentally to learn.

In the months prior to this study, I developed many ways to challenge Tony and give him an extra push to learn material for all subject areas at his own level. This was a challenge for me as a teacher because there was such a wide range of mathematical ability levels in the classroom. Finding extra time to work with advanced students combined with finding advanced material for an advanced student can be difficult. I also found this challenging as a researcher because I had anticipated Tony to have a jump on other students, but did not realize it would be so accelerated. In the lessons that followed, I continued to adjust the material to challenge Tony by giving him extra problems to solve and enabling him to develop his own challenging problems. This helped to encourage him to use the manipulatives as tools to learn and discover new concepts.

Clouston and Whitcombe (2005) have spent many years studying the power of an inquiry-based learning environment. They believe educators have a challenge to teach such a diverse group of individuals while offering an opportunity for self-directed study and student empowerment. In Tony's case, and like many other students, he knows which style of learning he likes best. When confronted with a task, Tony chose to solve the problem his own way, under his own terms, and at his own speed. I believe this is what makes constructivism such a unique framework. In a traditional setting students are expected to learn under a set of conditions using one technique or skill chosen by the teacher to solve the problem. Tony has proven that when students are given different opportunities to solve problems in different ways, they will use the means that fit them best to construct their own meaning. The new knowledge acquired from this lesson then becomes much more personal.

While comparing Tony's pre and post-survey, I found the analysis to be interesting regarding his answers to the first question. He answered the same question extremely differently the second time. The survey question was: Do you think manipulatives or hands-on objects help you to learn math? Please explain why.

Tony answered this question the first time as follows:

I think they are cool and I like them because sometimes you get to eat them.

The same question was answered differently at the end of the study. Tony answered this question on the post-survey as follows:

I like learning fractions without manipulatives because it's faster.

After comparing these two answers I concluded that Tony had changed his mind about manipulatives. At first he thought they were a good tool and he was also excited about eating them. After the study was complete, Tony had proved that he could be just as successful without them and that he chose the fastest way to learn. I believe the data I collected from Tony proved that he did enjoy each lesson. He was always eager to share his answers and teach the class how he learned the concepts his own way. He was proud of himself for solving problems faster and in a different manner then the rest of the class. Tony did have a slight discomfort with the fact that there was not always one correct answer. He was so used to getting the answer and getting immediate satisfaction from teacher approval that he began to disregard the manipulatives and work problems out his own way.

I have struggled challenging Tony in mathematics throughout the whole year. As I went back into his school records I noticed that challenging Tony has been an issue for many of his teachers. As I began this study I was hoping to challenge Tony and possibly increase his level of motivation. Although I do not believe this happened, I do believe a door was opened to Tony that has introduced him to a selection of ways and tools he can use to solve problems. Tony chose ways to solve problems that were within his comfort zone. When offered manipulatives as a tool to solve problems, Tony chose to use what was familiar and comfortable.

Evans (2001) believes when students are engaged in inquiry, they are encouraged to collaborate, communicate and present their findings. Learning becomes a group effort and is shared with the class. Tony has always struggled with group work skills and getting along with others in small groups. His competitive attitude has held him back from allowing others to join him in the learning process. After beginning this study I predicted Tony to be a student whom manipulatives would have an impact on his learning of fractions. Although this did not happen, I know Tony gained a better understanding of what works well for him. He has developed his own learning style while being introduced to others that he may some day accept.

Summary

The use of manipulatives affected each case study student's understanding of fractions in different ways. After analyzing the data collected through my triangulation of surveys, student work, observations, and interviews, I was able to look across many different kinds of data to determine the impact of manipulatives. I also used this data to study the methods students used to construct their own knowledge of fractions. Some students showed an increase in the understanding of fractions and others did not. Each student was introduced to a new and unfamiliar way of learning. Many students are accustomed to the use of text books, lectures, and lessons in which they are told exactly what to do to learn mathematics. This study has helped to introduce a different way to learn that engages all and encourages collaboration, discovery, and social skills. These are all skills students need to be successful learners. I believe if more teachers adopt inquiry and constructivism as a pedagogical framework for their teaching, more students will be acquainted with these skills and this can have an impact on their success not only in the classroom but as they venture out into the real world.

Chapter 5

Discussion

I was extremely pleased with the results of this study. Using manipulatives introduced through the constructivist theory definitely had an impact on each case study student's understanding of fractions. Each student was introduced to a new and unfamiliar way of learning where they were encouraged to relate new concepts to familiar ones in their everyday lives through meaningful hands-on activity. This new way of learning and relating new concepts to familiar ones is what Ross (2005) calls active learning. Students used familiar objects in their unique ways and related the objects to new concepts. With inquiry-based activities and everyday manipulatives each student acquired a true understanding of the new concepts.

According to Evans (2001), inquiry-based instruction is the basis for the creation of this genuine and evolving educational culture. Evans believes true learning occurs when lessons are based on students' natural inquisitiveness. Students' natural inquisitiveness will stimulate curiosity and encourage students to collaborate, communicate, and solve problems through the use of hands-on manipulatives.

While the use of manipulatives impacted each case study student differently, each of the students used the manipulatives as tools, or mediational means. According to Allen (2005), Vygotsky's concepts of mediational means are elements humans use for the mental process to direct the mind and behavior or tools which we use as mediators between external objects and human activity. Each case study student used the manipulative as a means to develop his own representation of fractions. The maniuplatives were used to mediate the relationship between the object and the new concepts. For some students the manipulatives were tools that helped students solve problems that they couldn't solve without them. Working with the manipulatives enabled the student to think deeper and learn a new effective way to solve problems. For others they were used as a mental tool along with the student's prior knowledge of the concept. In this case the student used the manipulative as a mental tool to think about the problems.

According to Green (2005), by using Constructivism as a theoretical basis for the teaching of mathematics, teachers promote higher-level critical thinking skills. By promoting higher-level thinking and with support of an expert, students are encouraged to learn within their zones of proximal development. The zone of proximal development, coined by Vygotsky, is where learners need to be for cognitive change to occur. Ross, (2005) calls the zone of proximal development a learner-centered theory where student and teacher jointly construct meaning. In the ZPD students find a balance between autonomy and dependence. Through inquiry-based lessons and the use of manipulatives my case study students have been encouraged to discover this balance between autonomy and dependence. By encouraging independence, collaboration, and autonomy along with my support, my students have been challenged to try learning on a level which they might have been uncomfortable with in the past. By discovering ways to learn fractions and solve problems they have risen to a new zone of learning and built confidence to try difficult problems. As a result of this study, students have experienced cognitive change.

By adjusting lessons in different ways for different students I was able to see the impact manipulatives had on the students' understanding of fractions. Whether it was giving students time before the lesson began to explore and play with the maniulatives, or adding extra work for a challenge, each student used the manipulative to represent his findings, construct meaning, and solve problems. I was able to use the representations as clues as to how the student solved the problem and analyze the learner's thought process.

This study has introduced a different way to learn that has successfully engaged all students and encouraged collaboration, discovery, and social skills. I believe if more teachers adopt inquiry and constructivism as a pedagogical framework for their teaching, more students will be acquainted with these skills and this can have an impact on their academic success as well as real world problem solving.

Recommendations

My inquiry into learning about children's thinking has helped me to make decisions about how to best scaffold students' learning. I have created an assessment window by learning how students are thinking and using tools to learn. I will use this window to develop new strategies to introduce content and adjust the lessons to best fit the needs of each individual student. Sharp (2002) believes constructivism can be viewed as a window. Educators can use this window to view children's construction of knowledge. By observing the way students are thinking and constructing knowledge, we can provide alternatives to the traditional view of education. Having this new notion of an assessment window will now become part of my mathematics teaching practice. By constant assessment of student thinking I can teach mathematics in a way in which each individual child can understand and relate to.

Since each case study student benefited from the use of manipulatives, I will now begin to adjust mathematics lessons in other content areas to incorporate maniuplatives. Changing from the traditional text book way of educating will help to enable students to learn and discover. Manipulatives have helped me to connect mathematics curriculum to real world situations. Borko and Elliott (1999) believe by connecting mathematics curriculum to real world situations, we can engage students in the learning process, learn the concepts, and construct understanding.

This study has enabled me to gain a different understanding of how students use manipulatives as mediational means. Before I began this study I thought manipulatives were specifically used to demonstrate concepts. As this study has progressed I have seen first hand how students actually use manipulatives to mediate their thinking, and in return I have changed my thinking. By using the manipulative as what Clouston and Whitcombe (2005) call triggers, students use manipulatives not only as representations for their answers or solutions to problems, but as tools for thinking. The manipulatives are mental tools or triggers students use to learn new concepts and solve real life problems. These manipulatives enable students to connect new concepts to the tool at hand and reflect on any prior knowledge to construct new understanding.

Since fractions are challenging to teach, I will share new ideas, procedures, and theories learned through my research to help other teachers better their teaching of factions. By sharing the results of my study as well as newly developed ideas, teachers will begin to see the impact that hands-on learning can have on the teaching of fractions. It took some time and practice to adjust my teaching style to coincide with the inquiry-based lessons. I can use my skills and knowledge to help introduce ways which other teachers can successfully begin to use this method.

I would like to see this research continue with an extension for students with special needs. Since many classrooms are inclusion classrooms, I believe it will be helpful to study if teaching mathematics with inquiry based lessons using manipulatives impacts students with special needs. Teaching mathematics with inquiry-based lessons gave me the opportunity to spend time looking deeper into the students thought process. With this new opportunity I believe students with special needs can also benefit from teaching with manipulatives in the area of motivation, social skills, as well as academic.

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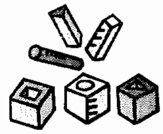
Appendix A

Student Survey



Pre__ Post__

Using Manipulatives in the Classroom

Name_________________

Date_________________

Please answer the questions below to the best of your ability. Try to remember any previous experience you have had using manipulatives and you can use those memories to answer the questions.

1. Do you think manipulatives or hands-on objects help you to learn math? Please explain why.

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2. How do manipulatives help you to learn math? If you believe manipulatives do not help, please explain why.

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3. Do you enjoy using manipulatives to learn? Please explain Why.

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Appendix B

Student Interview Response Sheet

Name______________________ **Date**________________

Context____

Questions asked by teacher: **Student Responses:**