Metacognition and Student Achievement in Mathematics

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Abstract

Students often struggle when solving with mathematical problems on tests. While students may have content knowledge to solve problems, often they do not have the ability to identify when specific content knowledge should be applied. Also, students make simple computational errors or give up on problem solving because they do not feel that they were on the right path. Students need to be equipped with metacognitive strategies that may help them overcome such issues. This thesis discusses metacognition and its' impact on student achievement in mathematics. In this study, students were given three New York State (NYS) Regents exam problems before and after being taught two metacognitive strategies: a) think aloud; and b) planning, monitoring and evaluating. The result found that there was a significant difference in the pre-test and post-test scores, overall and in particular with female participants.

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Chapter One: Introduction

Problem Statement

Mathematical achievement is based on the knowledge a student has and the ability to utilize this knowledge. Students' ability to utilize their knowledge often is compromised when encountering problems which they have not previously seen, such as on final exams. The students must determine what the question is asking and what procedures to use in order to arrive at the mathematical answer. One reason for this result could be that students are not taught how to examine their own thought processes when problem solving. This leads students to second guess themselves and give up easily.

Significance of Problem

As teachers, it is our job to facilitate student learning. Students need to be equipped with the knowledge to solve problems successfully. Knowledge of content is a necessity, especially when the problem has never been seen before. Many times students begin class stating that they did not understand how to complete a mathematical homework problem. In this case, they have looked at it and decided that they didn't know what to do or maybe attempted it only to realize that they are unsure about what they were doing and quit half way through the problem. Students also tend to decide on a way to solve problems and continue no matter what. What leads students to just quit during problem solving? How do students decide how to begin a problem? What happens when students get stuck on a problem? These questions are important to consider in order to improve student achievement. Thus, students should be taught how to use their knowledge to solve problems that they have never seen before. Schoenfeld defines problem solving as working problems that required more thought process that allow students to decide how to solve the problem based on their knowledge and experiences

(1992, pg. 14). Administrators have determined that certain classes, like Algebra 2, are the gateway to higher level learning and have the potential to increase the number of students taking Advanced Placement courses. As teachers, we need to help our students become successful by teaching them how to help themselves and use their knowledge to be successful.

Purpose

The purpose of this research is to investigate if teaching students to become more aware of their thought processes, during mathematical problem solving will improve test performance. This research was done by introducing students to metacognitive strategies which should allow them to be more aware of their thought processes during problem solving. Students were taught two techniques: (a) think aloud and (b) planning, monitoring and evaluating. These techniques may help them determine what the best course of action would be when mathematical problem solving. As a result of this investigation, it is hopeful that there will be an increase in student achievement on tests.

Rationale

This research examines metacognitive strategies and the connection to test performance on mathematical problem solving tasks. Research previously conducted on metacognition and mathematical problem solving has shown that when students were taught interventions or strategies that students were able to plan, monitor and continue with problem solving tasks and not just give up or stop part way through a problem (Schoenfeld, 1992; Garofalo & Lester,1985). When using metacognitive strategies, in addition to their knowledge of mathematics, students' ability to solve problems that they have not seen before can increase. This research will be useful in determining how metacognitive strategies impact student performance with problem solving tasks.

Chapter Two: Literature Review

Many studies have been conducted on problem solving and the processes used in problem solving such as metacognition, and cognition (Schoenfeld, 1992; Garofalo & Lester, 1985; Gray, 1991; Montague & Applegate, 1993; Mayer, 1993). Research on how metacognitive processes impact student learning of secondary students is ongoing. Schoenfeld (1992) indicated that cognition and metacognition greatly impact the successfulness of students when problem solving. Mayer (1993) indicated that differences in student processes play a role in how successful students are when using the processes of metacognition and cognition. This paper presents the definitions of problem solving, the processes involved in problem solving, in particular metacognition, and cognition, and the impact on learning for general education students and students with disabilities.

Definition of Problem Solving

In order to discuss the processing involved with problem solving, the notion of problem solving itself needs further clarification. Schoenfeld (1992) presented the difference between solving problems and problem solving. The idea of solving problems is that students are completing "routine exercises organized to provide practice on a particular mathematical technique that, typically, has just been demonstrated to the student" (pg. 11). In contrast, problem solving requires students to think about and solve problems with no set algorithm. Schoenfeld refers to Stanic and Kilpatrick's three historical themes of problem solving and identifies that two of the themes identify problem solving as traditionally used as a tool to achieve other goals. The third theme identified by Schoenfeld refers to problem solving as "the heart of mathematics" (1992, pg. 14). In this theme, problem solving is viewed as working

problems that required more thought process that allow students to decide how to solve the

problem based on their knowledge and experiences.

Wilson, Fernandez and Hadaway's (1993) study also distinguished the difference

between solving problems and problem solving, noting that when speaking about mathematical

problem solving, many different notions come to mind. According to Wilson et al., (1993),

problem solving should involve "exploration, pattern finding, and mathematical thinking" with

consideration about teaching "How to think" as opposed to "What to think" or "What to do" (pg.

60). Wilson et al. (1993) noted that many American textbooks contain linear problem solving

models which focus on a set of procedures to solve problems as opposed to genuine problem

solving which involve teaching students to think.

Lester (1994) studied mathematical problem solving research from the past and compared

it to more recent research. In this study, Lester concluded that "problem solving has been most

written about, but possibly the least understood" (pg. 661). His findings show that in the past,

research focused more on traditional word problems and that more recent research has focused

on the aspects of problem solving.

Hiebert, et. al. (1996) study on problem solving discusses problem solving as presenting

problems so that the students are allowed to think and search for different ways to solve them.

They distinguish between acquiring knowledge and applying knowledge. In their study,

participants were not taught what to do, but allowed to determine what methods they would use

to solve a problem. By doing so, the participants came up with many different methods to solve

the problem, yet all arrived at the same answer.

Problem Solving Processes

Schoenfeld (1992) reflected on the history of thinking mathematically and the problem solving processes involved in doing so. Schoenfeld discussed the concept of cognitive processes, which refers to how information is stored and organized. Schoenfeld (1992) acknowledged that a student's knowledge base greatly impacts their ability to problem solve. If students do not have the knowledge base or the knowledge base is incorrect, then a student's ability to problem solve is diminished. Within the realm of cognitive processes is the process of metacognition. Schoenfeld's research (1992) on problem solving with new problems indicated that students will often quickly decide on an idea to solve a problem and then continue to work with that problem solving path no matter what. However, with interventions that teach students how to think about the processes used in problem solving, the students become more successful. The results showed that the more aware students were to their thought processes, the more successful they were when problem solving. These findings did not differentiate between general education students and students with disabilities.

Kantowski's (1977) research delved into the processes involved when participants are given non-routine problems. Participants consisted of higher ability ninth grade students. The participants were first observed for behaviors of thinking aloud and coded for correctness and processes used. In the final stage of the research, participants were given a post-test. At all stages participants were asked to think aloud when problem solving. The process of thinking aloud allowed Kantowski to code the participants on the use of heuristics and formation of solutions. The results showed that scores were higher for participants who used goal-oriented heuristics. Additionally, participants who used goal-oriented heuristics had less misdirections when problem solving. Kantowski also made the observations that the heuristic strategy of "looking back" did not improve a participant's ability to problem solve successfully and that

students success with problem solving also depended upon their accuracy not only on the information a participant has but also on their ability to recall it (as cited in Kantowski, 1977).

Mayer (1993) discussed the problem solving processes of cognition, metacognition and motivation and the impact on learning. Mayer determined that each of these alone is not enough to be successful at problem solving. Mayer discussed how a student who knows how to subtract single digits and borrow successfully may still fail at subtracting double digit problems because they cannot apply their skill to a more complex problem. Similarly, Mayer described how students could compute operations successfully; yet fail at finding the correct solution because they did not use their metacognition to think about what they were doing and why they were doing it. Mayer also discussed student motivation during problem solving, namely in regards to interest, self-efficacy and attributions. His findings indicated that each played a role in a student's ability to successfully problem solve. In particular, Mayer described how students with disabilities can increase their problem solving ability when they were taught not only cognitive strategies, but also motivational strategies about attributing success or failure to effort rather than ability.

Garofalo and Lester's (1985) paper also discussed the role of metacognition in mathematical problem solving. Garofalo and Lester's research demonstrated that cognition alone is inadequate when problem solving. Garofalo and Lester discussed a cognitive-metacognitive framework that is used when problem solving. The framework involves four categories used in mathematical task performance: orientation, organization, execution and verification. There are distinctive metacognitive behaviors associated with each of the categories. Orientation involves behaviors to understand and figure out what a problem is asking. Some of the metacognitive behaviors this category includes analysis of information and assessment of familiarity of task.

Organization or planning of behavior and action is the second category. This category uses

metacognitive behavior such as identification of goals and planning. Execution, meaning

regulation of behavior, is the third category which uses the metacognitive behaviors of

monitoring of progress and trade-off decisions. The final category is Verification which involves

evaluation of decisions and outcomes. It uses the metacognitive strategies of evaluation and

checking the adequacy of representation and performance (p.171)

Goos, Galbraith and Renshaw (2002) discussed metacognitive processes used when

interacting with their peers and its impact on problem solving. Their research was built upon

Vygotsky's notion of the zone of proximal development (ZPD), which proposed that when in

peer groups, students were able to act above their normal level and perform at a higher level.

The study conducted by Goo et al. included observations of participants through video and audio

tape, interviews with participants and questionnaires. The goal of the study was to determine

how the participants created a culture of mathematical inquiry and explored collaborative

mathematical activity (2002). The participants were given mathematical problems to solve and

their conversations were recorded and coded. The results of the study supported findings that

poor metacognition in small groups led to poor collaboration and problem solving whereas more

metacognition in small groups lead to higher success in problem solving. That is, when students

in small groups listened, questioned, engaged and justified their reasoning with others, they were

more likely to establish a collaborative ZDP. This indicated the students were more successful

with problem solving.

Montague and Applegate (1993) studied cognitive and metacognitive strategies used by

students with disabilities, average ability students and gifted middle school aged students. The

study involved having the participants solve six word problems, three of which were solved by

thinking aloud. The results showed that students with disabilities and average ability students lacked cognitive strategies that were used in problem solving when compared to gifted students. The results also showed that students with disabilities lacked the knowledge about metacognitive strategies thus resulting in incorrect answers. From this study, Montague and Applegate (1993) determined that their results supported previous research, which concluded that students with disabilities attempted problem solving differently than general education students. Students with disabilities therefore need interventions and specific instruction in using metacognitive processes to help them become more successful problem solvers.

Mayer (1998) also discussed mathematical problem solving with regards to individual differences. He acknowledged the need for metacognitive processes in problem solving. Mayer (1998) reported that students may know how the meaning of every word in the problem and know how to complete the mathematical operations, but still arrive at the incorrect answer. The reason behind this is that students do not understand the meaning of the problem. Students need to understand what a problem is calling for and how to use their content knowledge in order to correctly solve the problems. Mayer refers to Schoenfeld's finding on control meaning that students need to be taught how cognitive strategies when problem solving. As an implication towards instruction, Mayer discussed that programs need to be developed that explicitly teach strategies to use the different processes of cognition and metacognition when problem solving.

Similarly, Gray (1991) discussed the relationship between metacognition and mathematical problem solving. Gray discussed teaching students the process of metacognition in order to strengthen the process of problem solving. Gray's instruction of teaching included the plan, monitor and evaluate model, thus providing students with a way to begin problem solving and continue until the solution is found.

Chapter Three: Method

In this chapter, the population of the study and procedure of the study will be discussed. The goal of the study would prove the hypothesis that metacognitive strategies lead to higher student achievement.

Participants

The population of the study consisted of 19 secondary students who were enrolled in Algebra 2/Trigonometry. From this population, seven participants were in their sophomore year, eleven were in their junior year and one was in their senior year in a suburban school in upstate New York. There were seven male and twelve female participants. The ethnicities of the participants were white (15), African American (1) and Multiracial (3). All participants were on the honor roll, which consisted of maintaining a grade point average of 80 or higher during the most recent marking period and having no referrals. The school district operated six schools- four elementary buildings housing grades K- 5, a middle school for grades 6-8 and a high school for grades 9-12. There were 4320 students and 354 teachers in the district (2005).

Procedure of Study

The procedure of the study involved a pre-test, treatment, and post-test. The study took place within a normal classroom and involved all students who volunteered to participate in the study. First, the students completed a pre-test which consisted of three part III questions from past New York State Regents exams. Students had learned the content of the questions throughout the year, yet had never seen the specific questions before. After the students took the pre-test, metacognitive strategies were modeled and taught. In particular, the metacognitive strategies of (a) think aloud, and (b) planning, monitoring and evaluating were modeled and taught using direct instruction and guided practice in the classroom. The treatment for teaching

metacognitive strategies can be found in appendix i. The method of instruction of the

metacognitive strategies was modeled after Yoong's (2002) "Helping Your Students to Become

Metacognitive in Mathematics: A Decade Later". Students practiced using both metacognitive

strategies and were guided in the correct use of the strategies. The length of practice of treatment

was 320 minutes over two weeks. When the students were familiar with how to use the

strategies, the students were then given the post-test. The post-test consisted of three part III

New York State Regents exam questions. The post-test questions were different from the pre-test

questions. Again, students had learned the content throughout the year, but had not seen the

specific questions before. Students were scored on both exams using the New York State

Regents exam rubrics on a scale from one to four points per question to see if the use of the

metacognitive strategies had an impact on achievement. The pre-test questions, post-test

questions, pre-test scoring rubric and post-test scoring rubric can be found in appendix ii,

appendix iii, appendix iv, and appendix v respectively.

Chapter 4: Results

The data gained from this study included participants' individual scores on each question

for both the pre-test and post-test. The data was then categorized into participant scores, average

scores, and average scores by gender.

Participant Scores

Each participant was scored on the pre-test questions and post-test questions. When

comparing the pre-test scores to the post-test scores, of the nineteen participants, scores

increased for fifteen participants, decreased for three participants and remained the same for one

participant. When this data was analyzed using the Test of Within-Subjects Contrasts, there was

a significant difference between the scores, p-value = 0.18, $(p < 0.05)$.

Average scores

Data from the study was compared using the average of all participants' scores on the pre-test and the average of all participants' scores on the post-test. Data was also compared based on gender. Table 1 details the average of the participant's scores on the pre-test compared to the average of participant's scores on the post-test for all participants and for male and female participants separately.

Table 1

Comparison of Pre-test Scores to Post-Test Scores (Maximum score: 100)

Category	Pre-Test Score	Post-Test Score
Average of All Participants	29.37	45.61
Average of Male Participants	35.70	44.05
Average of Female Participants	25.68	46.52

The results show an increase of student achievement across all three categories. This data was then analyzed using a two variable T-test and the results are shown on Table 2. The results of the two variable t-test show that for the Pre-Post-test Average of all participants, there was a significant difference, $p = 0.007$, ($p < 0.05$). The results also show that for the Pre-Post-test Average of Female Participants, there was a significant difference, $p = 0.0073$, ($p < 0.05$). There was not a significant difference for the Post-Pre-test Average for the male participants.

Table 2

T-test Results

Category	t-test values	p-values
Post-Pre Average All Participants df = 36, n = 19	2.58	0.007***
Post-Pre Average Male Participants, df = 12, n = 7	0.77	0.227
Post-Pre Average of Female Participants, df = 22, n = 12	2.646	0.0073***

df = $n_1 + n_2 - 2$, where n_1 = pre-test participants and n_2 = post-test participants
*p <.05; **p <.01; ***p <.001

Chapter 5: Summary and Reflection

The goal of this study was to determine if teaching specific metacognitive strategies could lead to higher mathematical achievement on state assessments, specifically, the NYS Algebra 2/Trigonometry Regents exam. The results of this study have many implications.

Participants' scores and average participants scores

The first implication occurs when comparing the participants' pre-test scores to the post-test scores. When comparing this data, fifteen out of nineteen students had an increase in scores, three had a decrease in scores and one participant had no change in scores. When analyzed, the results showed a significant difference, p = 0.18. Additionally, when the averages of the pre-test scores and averages of the post-test scores were analyzed, there was a significant difference (p = 0.007). This result supports the hypothesis that metacognitive strategies lead to higher mathematical achievement. This implies that teaching students metacognitive strategies is useful in mathematics.

Average Scores by Gender

The second implication occurs when comparing the pre-test average scores with the post-test average scores for both the males and females. When taking into account the gender of the students who participated in the study and the results of increased scores, fifteen participants' scores increased. Of the fifteen participants, there were ten females and five males. Of the three participants that had a decrease in scores, two were female and one was male. The gender of the participant whose score was unchanged was male. While the gender of the participants who scores decreased or remained the same were equal, two male and two female, the remaining fifteen participants were represented by a ratio of 2:1, female to male.

The results showed that there was not a significant difference in the pre-test average scores and the post-test average scores for the males, yet there was a significant difference in the pre-test average scores and the post-test average scores for the females. One possible explanation for this result could be that males feel that their success in mathematics in based upon their ability whereas females feel that their success in mathematics is based upon their effort (Pedro, Wolleat, Fennema, and Becker, 1981). In this study, there were more female participants, twelve females compared to seven males. If the female participants did believe that the metacognitive strategies would help them to be more successful, then they would have put forth more effort to use the strategies. Males on the other hand, who believe that their success is based on their ability and not effort, may not have been as serious in learning or using the metacognitive strategies.

Reflection

The results of this study show that there is a benefit to teaching students metacognitive strategies in order to increase mathematical achievement. Think aloud and planning, monitoring and evaluating are just two types of metacognitive strategies that can be beneficial to students

and are easily taught within the classroom environment. This study was conducted over a short

period in time and resulted in positive results. Metacognitive strategies that are included in the

curriculum and taught over the course of a year, along with the content of mathematics could

have an even greater impact on student achievement.

This study also provides support for using metacognitive strategies to increase the

mathematical achievement among females. Females who believe that their success is due to

effort will use metacognitive strategies in order to attain higher grades. This awareness of their

thought processes may enable these female to recognize that their success is due to not only

effort, but also their ability. This study shows that further research into this metacognitive

strategies and mathematical achievement is warranted.

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Appendix i

Teaching Metacognitive Strategies

<u>Think aloud</u>

Direct Instruction and Modeling: The instructor will explain to the students that they will be learning the metacognitive strategy of Think aloud. The Think aloud strategy helps students to become more aware of their thought processes while problem solving. The teacher will explain to the students that they will think about their thought processes and verbalize their thoughts while problem solving.

The teacher will model how to use the Think aloud strategy while solving the problem: $x^3 + 5x^2 = 4x + 20$. The teacher will also have the students give a problem to the teacher to solve and the teacher will verbalize her thought processes while solving the problem.

Students will then be given the following two problems to solve and be asked to use the Think aloud strategy.

Problem 1: Solve the equation $2 \tan C - 3 = 3 \tan C - 4$ algebraically for all values of C in the interval $0° \leq C < 360°$.

Problem 2: Express $(\frac{2}{3} x - 1)^2$ as a trinomial.

Student will then be instructed to use the Think aloud strategy while completing their homework assignment and all future problems. Students will be monitored by the teacher and assisted in using the Think aloud strategy so that it is learned and used consistently.

<u>Planning, Monitoring and Evaluating</u>

The teacher will explain to students the importance of planning, evaluating and monitoring when problem solving. The teacher will discuss with the students how the following questions can help guide them through the problem solving process and help them to determine if they need to continue problem solving or change to a different method in order to determine the correct answer.

What are you doing?

Why are you doing it this way?

How does this help?

The teacher will model planning, monitoring and evaluating when solving the problem: Simplify the expression $\left(\dfrac{w^{-5}}{w^{-9}}\right)^{\frac{1}{2}}$ by answering the above questions as the problem is solved. The teacher will then model a second problem: Find the roots of the equation $2x^2 + 7x - 3 = 0$ and again answer the questions above while solving the problem.

Students will be instructed to answer the same questions when solving the following two questions.

Problem 1: Factor completely: $10ax^2 - 23ax - 5a$

Problem 2: Express the sum of $7 + 14 + 21 + 28 + \ldots + 105$ using sigma notation.

Students will be instructed to use the planning, monitoring and evaluating questions while completing their homework and all future problems. Students will be monitored by the teacher and assisted in using the Planning, Monitoring, and Evaluating strategy so that it is learned and used consistently.

Students will be encouraged to use both strategies when problem solving. The teacher will continue to model and reinforce the strategies when teaching. Students will continue to use the strategies so that both strategies become a regular part of problem solving.

Strategies are from *Helping Your Students to Become Metacognitive in Mathematics: A Decade Later* by Dr. Wong Khoon Yoong which was adapted from Schoenfeld's *Cognitive Science and Mathematics Education,* Chapter 8, What's All the Fuss About Metacognition?

Problems from New York State Regents exams

Appendix ii: Pre-test Questions

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Write the binomial expansion of $(2x - 1)^5$ as a polynomial in simplest form.

37 In △ABC, m∠A = 32, a = 12, and b = 10. Find the measures of the missing angles and side of △ABC. Round each measure to the nearest tenth.

38 The probability that the Stormville Sluggers will win a baseball game is $\frac{2}{3}$. Determine the probability, to the nearest thousandth that the Stormville Sluggers will win at least 6 of their next 8 games.

Appendix iii – Post-test Questions

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Solve algebraically for x: $\dfrac{1}{x+3} - \dfrac{2}{3-x} = \dfrac{4}{x^2-9}$

37 Find all values of θ in the interval $0° \leq \theta \leq 360°$ that satisfy the equation $\sin 2\theta = \sin \theta$.

38 The letters of any word can be rearranged. Carol believes that the number of different 9-letter arrangements of the word "TENNESSEE" is greater than the number of different 7- letter arrangements of the word "VERMONT." Is she correct? Justify your answer.

Appendix iv – Pre-test Rubric

Part III

For each question, use the specific criteria to award a maximum of four credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(36)   [4] $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$, and appropriate work is shown.

[3] Appropriate work is shown, but one computational or simplification error is made.

[2] Appropriate work is shown, but two or more computational or simplification errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

[1] Appropriate work is shown, but one conceptual error and one computational or simplification error are made.

or

[1] A correct substitution is made into the binomial expansion formula, but no further correct work is shown.

or

[1] $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

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(37)   [4] $m\angle B = 26.2$, and $m\angle C = 121.8$, $c - 19.2$, and appropriate work is shown.

[3] Appropriate work is shown, but one computational, rounding, or labeling error is made.

or

[3] Appropriate work is shown, but only the measures of one angle and one side are found and labeled correctly.

[2] Appropriate work is shown, but two or more computational, rounding, or labeling errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown, but only the measures of $\angle B$ and $\angle C$ are found and labeled correctly.

or

[2] Appropriate work is shown, but only the measure of side c is found.

[1] Appropriate work is shown, but one conceptual error and one computational, rounding, or labeling error are made.

or

[1] Appropriate work is shown to find $m\angle B = 26.2$, but no further correct work is shown.

or

[1] $m\angle B = 26.2$, $m\angle C = 121.8$, and $c - 19.2$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(38)   [4] 0.468, and appropriate work is shown.

         [3] Appropriate work is shown, but one computational or rounding error is made.

or

         [3] Appropriate work is shown to find $\dfrac{3072}{6561}$ or an equivalent fraction, but no further correct work is shown.

         [2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

         [2] Appropriate work is shown, but one conceptual error is made, such as finding P(at most 6 games).

or

         [2] $_8C_6\left(\dfrac{2}{3}\right)^6\left(\dfrac{1}{3}\right)^2 + {}_8C_7\left(\dfrac{2}{3}\right)^7\left(\dfrac{1}{3}\right)^1 + {}_8C_8\left(\dfrac{2}{3}\right)^8\left(\dfrac{1}{3}\right)^0$, but no further correct work is shown.

         [1] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or

         [1] The probability of winning *exactly* 6 baseball games is calculated correctly.

or

         [1] 0.468, but no work is shown.

         [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Appendix v – Post-test Rubric

Part III

For each question, use the specific criteria to award a maximum of four credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(36)   **[4]** $\frac{1}{3}$, and appropriate algebraic work is shown.

[3] Appropriate work is shown, but one computational error is made.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $x - 3 + 2(x + 3) = 4$ or an equivalent equation is written, but no further correct work is shown.

or

[2] $\frac{1}{3}$, but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or

[1] $\frac{1}{3}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[8]

(37)   **[4]** 0, 60, 180, and 300, and appropriate work is shown.

    [3] Appropriate work is shown, but one computational, factoring, or substitution error is made.

<div align="center">*or*</div>

    [3] Appropriate work is shown, but only three of the four correct solutions are found.

    [2] Appropriate work is shown, but two or more computational, factoring, or substitution errors are made.

<div align="center">*or*</div>

    [2] Appropriate work is shown, but one conceptual error is made.

<div align="center">*or*</div>

    [2] Appropriate work is shown, but only two of the four correct solutions are found.

<div align="center">*or*</div>

    [2] Appropriate work is shown, and $\sin \theta = 0$ and $2 \cos \theta - 1 = 0$ is written, but no further correct work is shown.

    [1] Appropriate work is shown, but one conceptual error and one computational, factoring, or substitution error are made.

<div align="center">*or*</div>

    [1] Appropriate work is shown, and $\sin \theta (2 \cos \theta - 1) = 0$ is written, but no further correct work is shown.

<div align="center">*or*</div>

    [1] 0, 60, 180, and 300, but no work is shown.

    [0] A correct substitution is made for $\sin 2\theta$, but no further correct work is shown.

<div align="center">*or*</div>

    [0] 0 or 60 or 180 or 300, but no work is shown.

<div align="center">*or*</div>

    [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

<div align="center">[9]   [OVER]</div>

(38)   [4] No, and an appropriate justification is given, such as comparing 7! for Vermont
        to $\dfrac{9!}{4!2!2!}$ for Tennessee.

        [3] Appropriate work is shown, but one computational error is made, but an
        appropriate justification is given.

                                            or

        [3] Appropriate work is shown to find 5,040 and 3,780, but No is not stated.

        [2] Appropriate work is shown, but two or more computational errors are made.

                                            or

        [2] Appropriate work is shown, but one conceptual error is made.

                                            or

        [2] Appropriate work is shown to find 3,780, but no further correct work is shown.

        [1] Appropriate work is shown, but one conceptual error and one computational
        error are made.

                                            or

        [1] Appropriate work is shown to find 5,040, but no further correct work is shown.

                                            or

        [1] 5,040 and 3,780 and No, but no work is shown.

        [0] 5,040 and 3,780, but no work is shown.

                                            or

        [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct
        response that was obtained by an obviously incorrect procedure.

The College at
BROCKPORT
STATE UNIVERSITY OF NEW YORK

Institutional Review Board

Date: 5/29/2015

To: Nancy Laistner

From: Julie Wilkens
              IRB Coordinator

Re: IRB Project # 2014-198

Project Title: Metacognition and its impact on student achievement in mathematics

Your proposal has been approved effective 5/22/15.

You must use only the approved consent form or informational letter and any applicable surveys or interview questions that have been approved by the IRB in conducting your project. If you desire to make any changes in these documents or the procedures that were approved by the IRB, you must obtain approval from the IRB prior to implementing any changes.

If you wish to continue this project beyond one year, federal guidelines require IRB approval before the project can be approved for an additional year. A reminder continuation letter will be send to you in eleven months with the specific information that you will need to submit for continued approval of your project. Please note also that if the project initially required a full meeting of the IRB (Category III proposal) for the first review, then continuation of the project after one year will again require full IRB review.

Please contact Julie Wilkens, IRB Coordinator, Grants Development Office, at (585) 395-2779 or jwilkens@brockport.edu **immediately** if:

- the project changes substantially,
- a subject is injured,
- the level of risk increases
- changes are needed in your consent document, survey or interview questions or other related materials.

Best wishes in conducting your research.

PARENT/GUARDIAN STATEMENT OF INFORMED CONSENT

Dear Parent or Guardian,

In the month of May and June, I will be conducting research in my classroom for completion of my Master's degree at College at Brockport SUNY. This research is being done with the College at Brockport SUNY and the department for Human Education and Development.

The purpose of my research is to see how metacognitive strategies increase student achievement in mathematics. Metacognitive strategies are those which allow students to become more aware of their thought processes when solving mathematical problems. As part of our regular classwork for the Regents review, I will be asking the students to first complete three Part III Regents exam questions. I will then teach the class the metacognitive strategies of 1) Think aloud and 2) Planning, Monitoring and Evaluating. When the students understand how to use these strategies, they will then as part of regular classwork for Regents review complete three different Part III Regents exam questions.

In order for your child to participate in the study, your informed consent is required. You are being asked to make a decision whether or not your child may participate in the project. If you want your child to participate in the project, please read the statements below and sign your name in the space provided at the end. <u>You may change your mind at any time ant your child may leave the study without penalty, even after the study has begun.</u>

Sincerely,

Mrs. Nancy Laistner

I understand that:

1. My child's participation is voluntary and they have the right to refuse to answer any questions.

2. My child's confidentiality is protected. A pseudonym will be used and there will be no way to connect him to the work. If any publication results from this research, his name and school will not be identified.

3. There will be no anticipated personal risks in the participation of this project except for the time it takes to complete the six Regents questions and instruction on metacognitive strategies. An anticipated benefit of my child's participation in the research could be a better score on the final exam.

4. My child's participation involves completing a total of six Part III Regents exams questions, and learning about the metacognitive strategies of 1) Think aloud and 2) Planning, Monitoring and Evaluating. Three questions will be completed before instruction about metacognition strategies and three questions will be completed after instruction.

5. The participants of this study are the students in Mrs. Laistner's Algebra 2/Trigonometry classes who volunteer to participate, a maximum of 65 total students. The results will be used for the completion of a master's thesis by the primary researcher.

6. Participation in this research project will not affect your child's grades or class standings.

7. Data will be kept in a locked filling cabinet by the investigator. When research has been accepted and approved, data and consent forms will be shredded and destroyed.

I am 18 years of age or older. I have read and understand the above statements. All of my questions about my child's participation have been answered to my satisfaction. I agree to let my child participate in the study realizing I may withdraw without penalty at any time during the research process.

If you have any questions you may contact:

Primary Researcher	Faculty Advisor
Nancy Laistner	Dr. Carol Wade
(585) xxx-xxxx	Department of Education and Human Development, Brockport, SUNY (585) 395-5569
Email address: nlaistner@xxxxxxxxxxxxx.org	Email address: cwade@brockport.edu

Signature of Parent/Date: _____

Child's Name: _____

STATEMENT OF MINOR ASSENT

Dear Student,

In the month of May and June, I will be conducting research in my classroom for completion of my Master's degree at College at Brockport SUNY. This research is being done with the College at Brockport SUNY and the department for Human Education and Development.

The purpose of my research is to see how metacognitive strategies increase student achievement in mathematics. Metacognitive strategies are those which allow students to become more aware of their thought processes when solving mathematical problems. As part of our regular classwork for the Regents review, I will be asking you to first complete three Part III Regents exam questions. I will then teach the class the metacognitive strategies of 1) Think aloud and 2) Planning, Monitoring and Evaluating. When you understand how to use these strategies, you will then as part of regular classwork for Regents review complete three different Part III Regents exam questions.

I understand that:

1. My participation is voluntary and I have the right to refuse to answer any questions.

2. My confidentiality is protected. A pseudonym will be used and there will be no way to connect me to the work. If any publication results from this research, my name and school will not be identified.

3. There will be no anticipated personal risks in the participation of this project except for the time it takes to complete the six Regents questions and instruction on metacognitive strategies. An anticipated benefit of my participation in the research could be a better score on the final exam.

4. My participation involves completing a total of six Part III Regents exams questions, and learning about the metacognitive strategies of 1) Think aloud and 2) Planning, Monitoring and Evaluating. Three questions will be completed before instruction about metacognition strategies and three questions will be completed after instruction.

5. The participants of this study are the students in Mrs. Laistner's Algebra 2/Trigonometry classes who volunteer to participate, a maximum of 65 total students. The results will be used for the completion of a master's thesis by the primary researcher.

6. Participation in this research project will not affect my grades or class standings.

7. Data will be kept in a locked filling cabinet by the investigator. When research has been accepted and approved, data and consent forms will be shredded and destroyed.

Please sign below to indicate that you have read and understand the above statements and that you agree to participate in the research study. You may change your mind and withdraw from

the study at any time. If you have any questions, please do not hesitate to contact me at 585-xxx-xxxx or Dr. Carol Wade, my faculty advisor at (585) 395-5569.

Sincerely,

Mrs. Laistner

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

To be completed by the student:
Please print your name: _____

Signature: _____ Date: _____

Grade and Birthdate of Participant: _____

Signature of witness _____Date: _____
(18 years of age or older)