The Effects of Heuristic Problem-Solving Strategies on Seventh Grade Students' Self-Efficacy and Level of Achievement in Mathematics

by

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Abstract

The ability to solve problems is an important and integral reason for learning mathematics. Teaching students to use heuristic problem-solving reasoning and strategies can help them become expert problem-solvers and assist them in transferring and applying their contextual knowledge to novel problems and situations. The purpose of this study was to examine the effects of teaching heuristic problem-solving reasoning and strategies on seventh-grade students' perceptions and level of achievement in mathematics. To do this, the researcher examined two aspects of problem-solving and student learning; the students' self-efficacy and their ability to solve non-routine problems in novel contexts. Two seventh-grade math classes participated in the study. One of the classes acted as a control group and received their standard problem-solving instruction. The other class acted as the intervention group which received explicit instruction on heuristic problem-solving reasoning and strategies.

The results of this study showed that students that were taught the heuristic reasoning and problem-solving strategies significantly improved in their level achievement compared to those that were not. The results also showed that for the group of students that received the intervention, there was a significant improvement in their positive perception of their problem-solving abilities, but not in their degree of self-efficacy. In fact, there was a significant decrease in their degree of self-efficacy after the intervention. However, this change in self-efficacy resulted in a significant increase in the correlation between the students' perception of their problem-solving ability and their actual ability to solve problems. This indicates that even when teaching students to use heuristic problem-solving reasoning and strategies does not improve their degree of self-efficacy, it does provide them with a more realistic perception of their problem-solving abilities.

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Chapter One: Introduction

In recent years there has been a growing determination for schools and teachers to develop students' problem-solving skills. The New York State (NYS) core curriculum standards in mathematics specifies that problem-solving skills are an essential part of a student's mathematical understanding and has incorporated a problem-solving process strand into the requirements of what a student should know and be able to do. According to the NYS Learning Standard for Mathematics, integrating problem-solving skills into the curriculum is necessary to produce students who will have mathematical knowledge, an understanding of mathematical concepts, and be able to apply mathematics in the solution of problems (NYS Department of Education, 2005).

Significance of the Problem

Learning mathematics is a vital part of preparing students for higher education and future success in an increasingly complex global society. Students who took math in high school were more likely to attend college, over 83 percent, as opposed to only 36 percent that did not take math classes (US Department of Education, NELS). This strong correlation between mathematics and future success has led to an increasing awareness of the need for higher quality mathematics education at earlier grade levels. Because math is a subject that builds on itself, it is crucial that students develop a solid foundation and high level of self-efficacy in mathematics at an early age. As a result, students are being tested by the state at the elementary and middle school grade levels, while teachers and schools are being held accountable for providing students with high quality mathematics education (US Department of Education, NCLB).

In an effort to provide this high quality education, schools have emphasized the integration of problem-solving skills into the mathematics curriculum. Recent approaches to instruction stress problem-solving that requires understanding, pattern seeking, experimentation, hypothesis testing, and an active seeking of solutions (Adiguzel & Akpinar, 2004). These types of problems require students to actively use their metacognitive abilities and cultivate lifelong problem-solving skills. Students who have mastered these problem-solving skills and gained proficiency in solving different types of problems are referred to expert problem-solvers. Expert problem-solvers are typically more successful at solving non-routine problems and are able to apply what they have learned to novel situations.

Expert problem-solvers can combine previously learned knowledge, concepts, and techniques to arrive at solutions where the path to the solution is not immediately known. They have a high degree of self-efficacy which has been shown to have a significantly positive relationship to increased levels of achievement (Hackett & Betz, 1989). As students achieve greater levels of success in the classroom, they will be better equipped to apply their knowledge and skills to other areas of life. This carryover into real world situations adds importance to the mathematics students study and will ultimately improve their everyday performances. However, despite progress in mathematics education reform, there are still significant differences in the way students learn and use mathematics in school, as well as, how workers apply mathematics in job settings (Bottge, Heinrichs, Mehta, Rueda, & Hung, 2004). Therefore, it is important to examine the effects of teaching problem-solving skills and strategies on students' self-efficacy and level of achievement.

Purpose

The purpose of this study was to examine the effects of teaching problem-solving strategies on seventh-grade students' perceptions and level of achievement in mathematics. To do this, the researcher examined two aspects of problem-solving and student learning. First, a student's self-efficacy towards the subject affects their attitude, level of engagement in the learning process, and their motivation to succeed. Teaching students strategies for successfully solving non-routine problems can greatly influence their self-efficacy and how they perceive themselves as problem-solvers. Second, learning problem-solving strategies can help students transfer and apply their contextual knowledge to other situations. This transfer and application of knowledge into other areas of learning can increase a student's overall level of achievement in mathematics.

Based on these aspects of learning, the researcher designed the following research questions:

- Does teaching problem-solving strategies improve seventh-grade students' perception of their self-efficacy in mathematical problem-solving?
- Does teaching problem-solving strategies improve seventh-grade students' level of achievement in mathematical problem-solving?

Rationale

Research has shown that problem-solving not only helps students to create connections in their learning, but it also helps students to generate a rich and highly developed schemata. (Owen & Sweller, 1989). George Polya was one of the greatest advocates in encouraging the use of problem-solving techniques in the learning of mathematics. Many of the educational recommendations and learning standards in mathematics education created the National Council of Teacher of Mathematics (NCTM) are based on the heuristic problem-solving theories Polya developed (Schoenfeld, 1987).

Polya's most significant contributions to the process of problem-solving were his theories of heuristic reasoning and heuristic problem-solving strategies. Heuristic reasoning should not be regarded as final and strict, but should be considered provisional and plausible (Polya, 2004). The goal of heuristic reasoning is to create a set of practical reasoning rules derived from the empirical, experimental method that directs thinking along the paths most likely to lead to success. Based on this, Polya identified four basic steps for solving problems which included understanding the problem, devising a plan, carrying out the plan, and looking back to examine the solution obtained (Marzano, Pickering, & Pollock, 2001). These heuristic reasoning steps were designed to act as a guideline in the process of problem-solving and work well on a variety of problems.

During the process of solving problems, students need to implement heuristic strategies. Heuristic strategies are devices which drastically limits the search for a solution in a large problem space (Polya, 2004). They provide a general suggestion or technique for solving different types of problems and can be used independently or in combination. The ten most common strategies include working backwards, finding a pattern, adopting a different point of view, solving a simpler analogous problem, considering extreme cases, making a drawing, intelligent guessing and testing, accounting for all possibilities, organizing data, and logical reasoning (Posamentier, Smith, & Stepelman, 2006). Although using these strategies does not guarantee finding an optimal solution, they do consistently lead students to a plausible solution, and are especially helpful when investigating a totally unfamiliar problem.

Teaching students when and how to implement heuristic reasoning and strategies can greatly improve their self-efficacy and level of achievement when they are engaged in problem-solving activities. This is especially helpful for novice problem-solvers who typically have difficulty understanding the underlying principles of the problems they encounter and are usually unsuccessful at solving them. These types of students experience a great deal of difficulty when they encounter non-routine problems or problems situated in an authentic context (Lee & Chen, 2009). Since heuristic problem-solving reasoning and strategies are designed to work on all types of problems, novice problem solvers can use them to develop their skills and abilities over time. As their skill at solving problems increases, their performance, level of achievement, and perceptions become more like those of expert problem-solvers (Schoenfeld & Hermann, 1982). Expert problem-solvers also benefit from learning in a problem-based curriculum, since continual exposure and practice has been shown to significantly increase a student's level of achievement (Schoenfeld, 1980).

Definition of Terms

The researcher defined a *problem* as a situation where the path to the solution is not immediately known. In mathematics, a *non-routine* problem is one that has multiple solutions, or a single solution that can be reached using multiple paths to achieve it. These types of problems require students to actively and consciously use their metacognitive abilities. On the other hand, *routine* problems are problems in which the path to the solution is readily known, and/or has only one solution and one method of acquiring it.

To solve non-routine problems, students can use a combination of heuristic reasoning and heuristic strategies. *Heuristic reasoning* is provisional and plausible instead of strict and procedural (Schoenfeld, 1987). This type of reasoning encourages students to use their metacognitive abilities instead of following step-by-step instructions. *Heuristic strategies* are derived from this type of reasoning and provide a general suggestion or technique for solving different types of problems (Schoenfeld, 1980). These types of strategies can help student solve problems in multiple formats and can be used in combination or separately.

The two types of problem-solvers are novice and expert. *Novice* problem solvers typically classify problems on the basis of surface features and have difficulty understanding the difference between a problem's setting and its structure. *Expert* problem solvers, on the other hand, tend to use heuristic reasoning and a logical thought process to work through problems. They are better able to correctly classify the underlying concepts within the problems and use effective strategies for solving it.

Two additional terms that the researcher defined for the study were self-efficacy and level of achievement. *Self-efficacy* was defined as the belief that one is capable of performing in a certain manner to attain certain goals (Zheng, McAlack, Wilmes, Kohler-Evans, & Williamson, 2009). The researcher defined level of *achievement* as the degree to which students are successful at completing a specified task. (Hackett & Betz, 1989).

Summary

Mathematics is a vital and necessary skill that middle school students must master in order to obtain future success in their educational and career goals. In an effort to provide students with high-quality mathematical instruction, the National Council of Teachers of Mathematics (NCTM) and New York State Board of Education have encouraged the integration of problem-solving skills into the mathematics curriculum. This approach to teaching mathematics helps students to actively use their metacognitive abilities and apply their knowledge and skills in real world contexts. However, in order to become better problem-solvers, students need to learn and understand the heuristic reasoning and strategies developed by Polya and recommended by the NCTM. This will assist them in becoming expert problem-solvers with a higher degree of self-efficacy, which research has shown results in significantly higher levels of achievement.

Chapter Two: Review of Literature

Traditionally, in modern society mathematics has been taught as an isolated subject with little or no connection to real world applications. However, learning to solve problems is the primary reason for studying mathematics and an important tool for helping students to develop their thinking abilities. To guide students through the process of solving problems, heuristic reasoning and strategies need to be explained and taught with the same importance as any other mathematical concept. Students should be exposed to non-routine problems that encourage them to use their metacognitive abilities and provided with opportunities to develop their problem-solving skills. This will result in expert problem solvers who are able to draw upon their prior knowledge and use effective techniques to find solutions to novel situations. This acquired proficiency in problem-solving will positively influence their self-efficacy and produce higher levels of achievement.

Importance of Problem-Solving

The origins of what is currently considered mathematics dates back to Ancient Egypt and Babylonian times, and was nothing more than a means of solving everyday problems (Burton, 2007). Over time, mathematics has grown and evolved into such a large and extensive body of knowledge that it is usually taught without any connection to the solving of real-world problems. Learning the principles, laws, and theories are no longer a means to an end, but an end in and of itself. This sentiment regarding mathematics as an isolated subject of study, owes more to tradition than to current knowledge of cognitive processes (Owen & Sweller, 1989). Unfortunately, until recently it had become the predominant method of teaching in modern societies.

However, recently there has been a shift back in thinking which views mathematics as a tool for solving problems. This shift is partly due to the fact that many jobs in modern societies increasingly require employees to use technology and demonstrate the ability to solve problems. It has also been further advanced by the integration of the Problem Solving Strand in Department of Education curriculum standards as well as legislation such as NCLB. This new approach to mathematical instruction is based on the belief that the principle reason for studying mathematics is to learn to solve problems (Posamentier, Smith, & Stepelman, 2006). Because the process of solving problems is such an integral part of mathematics instruction must stress problem-solving that requires understanding, pattern seeking, experimentation, hypothesis testing, and an active seeking of solutions (Adiguzel & Akpinar, 2004).

Furthermore, the process of problem-solving is required not only for solving problems, but also for building new mathematical knowledge. As such, the National Council of Teachers of Mathematics (NCTM) created four learning standards for problem-solving. The NCTM standards recommend that students be able to build new mathematical knowledge through problem-solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem-solving (NCTM, 2004). These standards can help teachers to focus on the most important aspects of the problem-solving process and how it can be applied. This in turn, will help to provide a foundation that teachers can use to scaffold their students to higher levels of mathematical thinking and learning

Heuristic Problem-Solving Reasoning and Strategies

Solving problems is a mental process that involves the use of metacognition, prior knowledge, reasoning, and strategies. Because of this, the majority of math content should be taught using a problem-solving approach. Research has shown that the process of problem-solving not only helps students to create connections in their learning, but also generate a rich and highly developed schemata (Owen & Sweller, 1989).

George Polya focused on the parts of the problem-solving process that were transferable to other areas of life. He believed that learning mathematics was about the discipline of discovery and not a purely deductive process (Polya, 2004). This led Polya to develop theories of heuristic reasoning and strategies which have become the basis of many of the learning standards and recommendations in mathematics education (Schoenfeld, 1987).

The aim of heuristic reasoning is to study the methods and rules of discovery and invention (Polya, 2004). It must be flexible enough work well on a variety of problems and help students to direct their thinking along the paths that are most likely to lead to a successful solution. It should also encourage problem-solving behavior that focuses on plausible and useful mental operations. Polya created a set of practical reasoning rules derived from the empirical, experimental method that directs thinking along the paths most likely to lead to success. His four basic stages for solving problems are to understand the problem, devise a plan, carry out the plan, and look back to examine the solution obtained (Marzano, Pickering, & Pollock, 2001). Although these reasoning stages appear simple and obvious, they are more complex than they seem to be and must be described and taught in detail to students.

The first stage in Polya's heuristic reasoning process is to understand the problem. Many students are not taught this crucial first stage in converting contextual information into conceptual understanding. The result is that students are often stymied in their efforts to solve problems simply because they do not fully understand what they are being asked to do (Polya, 2004). For students to be successful in this stage of the reasoning process, teachers need to teach students to ask themselves comprehension and conceptual understanding questions. Research has shown that students trained to use comprehension questioning in this stage of the process were significantly more successful at solving novel problems than students who were not (King, 1991).

The second stage of the problem-solving process is to devise a plan to solve the problem. This plan is not meant to be the step-by-step procedural instructions most students are taught in the classroom. It is actually the process of choosing an effective heuristic strategy to investigate an unfamiliar situation. Since most students are never explicitly taught problem-solving strategies, they are unable to perform this stage of the reasoning process. Unfortunately this can have a seriously negative impact on a student's ability to solve novel problems in authentic or real world contexts since that is the purpose of a heuristic strategy. Heuristic strategies provide students with a means of approaching an unfamiliar problem and have been shown to significantly improve a student's problem-solving abilities (Schoenfeld, 1987).

The third stage of Polya's reasoning process is to carry out the plan by implementing domain specific knowledge such as mathematical concepts, theorems, and formulas. This is the stage of the process that is the focus of most of the mathematical learning in the classroom. It is an important stage of the process since students need prior knowledge and a solid mathematical foundation to arrive at a solution. Research has shown that there is a significantly positive correlation between a student's level of prior knowledge and their ability to solve non-routine mathematical problems (Lee & Chen, 2009). However, teaching students mathematical concepts, theorems, formulas, and procedures for solving specific types of exercises and problems is not enough. Students not only need to have mathematical knowledge and an understanding of mathematical concepts, they must also be able to apply that knowledge and understanding.

Unfortunately, most of the classroom time and learning is spent on practicing these skills in isolated contexts with very little connection to each other or real world situations.

The final stage in Polya's heuristic reasoning process is to look back and examine the solution that was obtained. This is another important stage that is often partially overlooked when teaching students to solve problems. Although most students are taught to check their answer, they are rarely asked to reflect on how they arrived at the solution or if they could have used a more efficient method or strategy. It is this reflective thinking that eventually leads students to develop higher order problem-solving skills. Research has shown that students need these higher order problem-solving skills to recognize that there are multiple solutions and strategies for solving problems and to become expert problem-solvers (Adiguzel & Akpinar, 2004).

Polya also developed heuristic strategies which are used during the second stage of the problem-solving process. They are devices which aide students in their attempts to limit their search for a solution in a large problem space (Polya, 2004). They are especially helpful when students encounter problems that they do not recognize or have never seen before. In addition, research has shown that students had a significantly higher level of achievement when explicitly taught problem-solving strategies (Schoenfeld, 1980). However, in the decades since Polya began his work on developing heuristic problem-solving strategies, they have been refined and expanded upon. (Schoenfeld, 1987). Nevertheless, there are ten commonly accepted problem-solving strategies that all students should learn. These strategies include working backwards, finding a pattern, adopting a different point of view, solving a simpler analogous problem, considering extreme cases, making a drawing, intelligent guessing and testing, accounting for all possibilities, organizing data, and logical reasoning (Posamentier, Smith, & Stepelman, 2006).

Heuristic problem-solving strategies provide general guidelines for solving different types of problems and can be used independently or in combination. Students need to be taught to select the most appropriate strategies for the type of problem that they are given since it is impractical for them to try to work through all of the strategies. One method for doing this is to teach students to recognize similar problems or problems with similar characteristics and use the strategies that were most effective for solving them in the past. However, this necessitates that problem-solving strategies be taught with the same importance as any other mathematical concept (Schoenfeld, 1980). Furthermore, students should be given plenty of time and opportunities to practice using the different strategies, since problem-solving ability develops slowly over a prolonged period of time (Randall & Lester, 1984). Also, it is not feasible to thoroughly explore and teach all ten strategies at once. Instead, students should be taught one or two strategies and be allowed to practice them before introducing them to more. Since problem-solving strategies can be taught, and ability develops over time and with experience, all students would benefit from a problem-solving based curriculum.

Types of Problems

In mathematics, students are expected to be able to complete mathematical exercises and solve problems. These are two fundamentally different concepts which entail very different mental processes and skills. Mathematical exercises only required students to practice specific step-by-step procedures or to substitute numerical values into generalized formulas. While providing students with opportunities to practice domain specific knowledge is an important part of mathematical learning, it is not the same as having a student solve a mathematical problem.

A mathematical problem is a situation where the path to the solution is not immediately known. Problems require students to use their analytical and thinking skills in combination with their conceptual and background knowledge. The method of solving a problem involves inductive reasoning in an exploratory process of discovery. It also creates an environment where students build their own new mathematical knowledge to arrive at a solution to a novel situation. Problems are typically formatted as word problems and set in an authentic context from which students have to develop representations. These representations require students to translate written words into mathematical concepts which are then linked to appropriate mathematical knowledge. Students then use the procedural skills they practice in mathematical exercises to produce a solution.

There are two types of problems predominantly used in mathematics which are called routine or non-routine. A non-routine problem is one that has multiple solutions, or a single solution that can be reached using multiple paths to achieve it. These types of problems require students to actively and consciously use their metacognitive abilities. They tend to model real world situations and unlike routine problems, they are not contrived so that students can readily identify the process for solving it. Non-routine problems often include extraneous information, latent underlying principles, or multiple mathematical concepts. Solving non-routine problems helps students to see the meaning and relevance of what they learned and to facilitate the transfer of contextual knowledge to authentic situations (Lee & Chen, 2009).

On the other hand, a routine problem is one in which the path to the solution is readily known, and/or has only one solution and one method of acquiring it. These types of problems are more concrete and directly related to specific concepts or mathematical procedures. There is usually a prescribed or correct algorithm for solving them which is easy for teachers to assess. Most problems taught in the classroom are routine problems that can be solved by substituting specific data or following step-by-step examples. Very little time is spent teaching students the process of solving problems. As a result, students have difficulty transferring and applying domain knowledge to novel situations and problems. (Lee & Chen, 2009).

Types of Problem Solvers

Most students tend to fall into one of two categories where their problem-solving skills and abilities are concerned. The first category is that of the expert problem solver. These students typically have a solid mathematical foundation of prior knowledge and conceptual understanding. This is very important since research has shown that prior knowledge is significantly positively correlated with success in solving non-routine problems (Lee & Chen, 2009). However, prior knowledge alone is not enough to ensure that a student is an expert problem-solver. Research has also shown that their conceptual understanding needs to be well connected within the subject of mathematics, as well as in other subject areas, and that their knowledge is composed of a rich schemata (Lester, 1994). They are able to classify problems according to the underlying principles and tend to use problem-solving strategies more efficiently. They also spend more of their time during the problem-solving process engaged in logical reasoning and are able to follow through on paths of hypothetical thinking (Lester, 1994). All of this combines to produce students that are not only typically more successful at finding a solution, but they are also inclined to arrive at the most elegant solution for the problem.

The second type of problem-solvers is that of the novice, sometimes also referred to as concrete operational thinkers. Typically, these students have less background knowledge, and they experience difficulty applying the knowledge that they do have because it is an isolated understanding of specific mathematical concepts. In addition, research has shown that novice problem-solvers have trouble thinking hypothetically and often get lost in their reasoning efforts (Lester, 1994). As a result, these students tend to classify problems on the basis of surface features rather than seeking the underlying concept of the problem. They can order and organize information that is given to them, but have difficulty understanding the difference between a problem's setting and its structure. Therefore, these types of students experience a great deal of difficulty when they encounter non-routine problems or problems situated in an authentic context (Lee & Chen, 2009).

However, just because some students tend to be better problem-solvers than others, does not mean that all students can't improve their problem-solving abilities. In fact, based on prior research, the three most significant factors influencing a student's ability to solve problems is their prior knowledge, ability to use heuristic problem-solving reasoning and strategies, and their skill at self-monitoring and reflection. Prior knowledge can be taught, but for it to truly be effective in problem situations, teachers need to emphasize conceptual connections both within a mathematical context and in other areas of study. The heuristic stages of reasoning and heuristic problem-solving strategies can also be taught, and all students benefit greatly from systematically planned problem-solving instruction (Randall & Lester, 1984). However, these stages of reasoning and strategies need to be explained in detail and practiced regularly before students can become efficient as using them. Lastly, students need to learn the importance of monitoring their progress during the problem-solving process. By monitoring their progress, students can determine if what they are doing is leading towards a successful solution or not. Afterwards, they should be taught to reflect on what they have learned and discovered and how that knowledge can be applied in other situations.

Self-Efficacy and Achievement

As students acquire problem-solving skills their self-efficacy, level of achievement, and ability to apply their knowledge in novel situations becomes more like that of expert problem-solvers (Schoenfeld & Hermann, 1982). A student's self-efficacy can strongly affect how successful they are at solving non-routine problems since they require students to use their metacognitive skills. Research has shown that self-efficacy plays an important role in thinking ability, which in turn, is the base of motivation (Pimta, Tayruakham, & Nuangchalerm, 2009). Because the path to the solution of a problem should encourage discovery and not be readily evident, students need to be motivated and confident that they can arrive at a successful solution despite initially unsuccessful attempts. Students who lack confidence in their abilities are typically less motivated and tend to give up after only a few unsuccessful attempts. Therefore it is important that teachers encourage their students' self-efficacy in problem-solving situations since research has also shown that a student's self-efficacy is positively correlated to motivation and achievement in solving non-routine problems (Zheng et al., 2009).

Students that are expert problem-solvers demonstrate higher levels of mathematical thinking and achievement, both inside the classroom and in real world situations. Inside the classroom, the skills students develop during the problem-solving process helps them to build new mathematical knowledge and understanding. Students trained in the problem-solving process demonstrated higher levels of achievement in mathematics (Randall & Lester, 1984). They are able to create connections in their learning and generate a rich and highly developed schema. In addition, they are more skilled at seeking patterns, generating hypotheses, and actively using their inductive reasoning. Research has shown that these skills are positively correlated with the ability to apply knowledge to novel situations and in authentic contexts (King, 1991). It is this application of knowledge that is the focus of the attention to the integration of problem-solving in the mathematics curriculum recommended by the NCTM and legislation such as NCLB.

With this increased attention to achievement in problem-solving skills, mathematical thinking, which requires students to develop their abilities to recognize problems, patterns, and solutions, has been brought back to the real-world. It is clear that problem-solving skills are not only essential for solving everyday problems in real life, but are required for future success in the workplace environment. Students who are taught these skills and strategies are more capable problem-solvers in real-life contexts. This carryover into life-outside-of-the-schools adds importance to the mathematics our students study and will ultimately improve their everyday performances (Posamentier, Smith, & Stepelman, 2006). Therefore, it is imperative that teachers receive the support and training they need to integrate this process into their regular curriculum.

Summary

Learning to solve problems is the primary reason for studying mathematics, although until recently, very little effort has been made to ensure that students are trained in this essential skill. The NCTM standards and legislation such as NCLB has compelled teachers and schools to rethink how mathematics is taught and to use problem-solving as a process to scaffold their students to higher levels of mathematical thinking and learning. Heuristic reasoning and strategies can help guide students through the problem-solving process and build a solid foundation that can be applied in contexts both inside and outside of the classroom. Students need to be given ample opportunities to practice their skills so that they can become proficient and expert problem-solvers. This proficiency will in turn positively influence their self-efficacy and produce higher levels of achievement.

Chapter Three: Methodology

Introduction

This study was an initial investigation of the effects of teaching heuristic problem-solving reasoning and strategies on seventh grade students' self-efficacy and level of achievement. It was designed so that the students were divided into two groups based on the math class they were attending. One group acted as a control group and received their regular classroom instruction which included practice solving non-routine and authentic situational problems. The other group received an intervention which included explicit instruction on heuristic reasoning and three common heuristic problem-solving strategies.

Both groups of students were asked to complete a pre-survey to determine their degree of self-efficacy in problem-solving and a pre-test to measure their level of problem-solving achievement. Afterwards, one group of students was given the instructional intervention and both groups had time to practice solving non-routine problems. All of the students were then given a post-survey and post-test. The post-survey had the same questions as the pre-survey and the information was compared to determine if there was an increased degree of self-efficacy in either or both of the groups. The post-test asked the students to use the same conceptual knowledge required for the pre-test, however they needed to apply it in different contexts. This information was also analyzed and compared to determine if there was a higher level of achievement in either or both of the groups afterwards. The analyzed data was then used to answer the two questions posed by the researcher

Participants

A total of forty-one students from two seventh-grade math classes participated in the study. Of the forty-one students, twenty-five were males and sixteen were females. All of the participants were white, non-Hispanic students attending a rural middle school in Western New York. This was a good representation of the student population since the middle school had less than a one percent non-white populace. The math classes were general education classes that met every school day for forty minutes.

The participating teacher had over thirty years of experience teaching mathematics in the school. He was certified to teach adolescent mathematics for grades seven through twelve. He taught at least one seventh grade math class every year that he was employed at the school. According to the NYS Department of Education, the teacher was considered to be highly-qualified and over 90% of his students passed the NYS test for seventh grade mathematics each year.

The researcher did not actively participate in the classroom instruction prior to, or immediately following, the study. The role of the researcher was to administer the pre-survey and pre-test to both the control class and the class that received the problem-solving intervention. Furthermore, the researcher designed and taught the heuristic problem-solving intervention which took place over three consecutive forty minute classes. To reduce bias between the control group and the intervention group caused by the differences in instructional methods of the regular classroom teacher and the researcher, the researcher also taught the teacher-designed standard lessons to the control group for the same three day period. Afterwards, the researcher administered the post-survey and post-test to both groups.

Procedures for the Study

The regular classroom teacher had well-established schedules, classroom procedures, and assessments, which were not altered for the study. The mathematical units and concepts that the teacher taught were based on the NYS learning standards and educational goals for seventh grade mathematics. He organized the units so that each new concept only required the background knowledge and skills that the students had previously acquired. The teacher created his own schedule and based the length of time spent on each topic on his many years of experience teaching seventh grade mathematics. He formally assessed the students' learning using a combination of homework assignment, quizzes, and tests that he had developed, and which had been shown to be reliable measures of achievement. His classroom rules and procedures were unchanging from year to year, and his lessons followed a consistent pattern.

For this study, the researcher created a three-day heuristic problem-solving lesson plan that was integrated into the unit designed to teach the students about the mathematical concepts related to percents. At the end of each unit, the regular classroom teacher typically focused the last few lessons on applications of the unit's concepts. Because the researcher was interested in determining whether or not explicitly taught heuristic problem-solving strategies would improve a student's self-efficacy and ability to apply mathematical concepts to novel situations, the intervention occurred at the end of the unit. This was done to reduce a possible source of bias caused by significantly altering the typical organizational pattern established by the regular classroom teacher and to be consistent with the format of the units taught prior to the study.

The regular classroom teacher's lessons had a standard format and the students knew what was expected of them each day. He began each class with one or two activities designed to build the students basic math skills. Afterwards, the students were given an opportunity to ask questions regarding the homework assignment that had been assigned the previous class. Once the students had finished their questions, the homework was collected and previously graded assignments were returned to them. All of this was accomplished in the first five to ten minutes of class and the remainder of the time was spent learning new material.

This well-established lesson format was not altered for the study. Both the control class and the intervention class began each of the three lessons with a basic math skills activity. Afterwards, both classes had an opportunity to ask questions regarding the previously assigned homework assignments before they were collected. Furthermore, both classes were given the same homework assignments created by the regular classroom teacher for the three days during which the study took place. The only difference between the two classes occurred during the portion of the lessons in which new material was covered for the three days that the study was conducted.

Control Group

There were twenty-three students, fourteen males and nine females, attending the class that acted as a control for the study. All of the students in the class participated in the study. The three-day lesson plan for the control group consisted of notes and a real-world application activity created by the regular classroom teacher.

The first day of instruction, the students in the control group were taught how to find the total cost of an item after a percent discount and sales tax was applied to it. They were given teacher-provided examples and notes that they copied into their notebooks. This teacher-directed instruction was followed by student-centered instruction in which the students were given similar problems to practice solving independently. Afterwards they were given a homework assignment that was also created by the regular classroom teacher.

For the second day of instruction, the students in the control group were given an activity where they created and solved their own real-world problems. For Part 1 of the activity, the students owned a store and chose six out of ten items listed on the activity sheet to place on sale. They also chose the percentage that they wanted to discount each item and the sales tax. After they calculated the total cost of the items, they completed Part 2 of the activity. For Part 2 of the activity, they listed their six chosen items, discounts, and sales tax, then gave the worksheet to another student to calculate the total cost. The activity concluded with both students comparing their answers and discussing their solutions. This activity not only gave the students an opportunity to practice creating and solving non-routine percent application problems, but also provided them with an opportunity to reflect on the problem-solving process even though they did not receive any explicit problem-solving instruction. The lesson concluded with the students being given a homework assignment that required them to practice the skills they learned in class.

The last day of instruction, the students in the control group were taught the simple interest formula and how to apply it. They were given notes and a teacher-provided real-world example where they received a loan with a simple rate of interest for a fixed amount of time. The students then had to calculate the amount of interest, total amount of money to be paid back, and what their monthly payments would be. This instruction was followed by similar problems that the students practiced independently and a related homework assignment.

Intervention Group

There were eighteen students, eleven males and seven females, attending the class that acted as the intervention group for the study. All of the students in the class participated in the study. The three-day lesson plan for the intervention group followed a similar pattern of instruction which included the notes created by the regular classroom teacher and a real-world application activity. However, in addition to the regular classroom notes, the students also received instruction on specific heuristic problem-solving strategies. Furthermore, the real-world activity was modified by the researcher to focus the activity on the process of problem-solving rather than on finding the solution and repetition.

The first day of instruction, the students in the intervention group were also taught how to find the total cost of an item after a percent discount and sales tax was applied to it. They were given the same teacher-provided example and notes that the control group received, however their independent practice was to complete a modified version of the real-world problem activity sheet. For the modified version of the activity, the students were only asked to select three of the ten items to apply a discount and sales tax to, instead of six items. They also were not asked to complete the second part of the activity where they would have traded papers with another student and solved the problems that their classmate created. Instead, the students were asked to answer questions designed by the researcher that required them to reflect on what they had learned and how it could be applied to other situations. By decreasing the number of items that the students had to solve for, the researcher was able to still provide the students with the opportunity to practice creating and solving their own real-world problems. However, by eliminating the second part of the activity, the focus of the activity was shifted from repetition to reflection on the process of solving problems. Afterwards the students were given the same homework assignment created by the regular classroom teacher that the control group was given.

The second day of instruction, the students in the intervention group were taught the heuristic problem-solving strategies of Working Backwards and Guess and Check. The students were given a description of each of the problem-solving strategies. The researcher then demonstrated how the strategies could be applied to different types of non-routine and real-world problems. Afterwards, the students were given an opportunity to practice selecting the appropriate strategy and applying it to different types of percent application problems. The class concluded with the students being given the same homework assignment that the control group was given.

The last day of instruction, the students in the intervention group were taught the heuristic problem-solving strategy of Organizing Data and how to apply it to real-world situations involving the simple interest formula. Once again, the researcher gave a description of the strategy and demonstrated how it could be applied to different types of non-routine and real-world problems. One of the application problems included the same notes and example of simple interest that was given to the control group. This ensured that both the control group and the intervention group received identical notes and examples created by the regular classroom teacher, with only the exception of the problem-solving strategies that were explicitly taught to the intervention group. The last day of instruction for the intervention group ended with the students once again being given the same homework assignment that the control group received.

Prior To and Immediately Following the Intervention

All of the students in both the control class and the intervention class were asked to complete pre- and post- surveys and tests. The pre-survey and pre-test were administered to the students one week prior to the three days of lessons taught by the researcher. The post-survey and post-test were administered one week after the three days of instruction. . Because it was necessary to compare the information from pre- and post- surveys and tests to answer the research questions, the students were required to write their names on each. This presented a possible source of bias since the students may have been concerned that the teacher would know how they answered the questions on the surveys or the scores they received on the tests.

In an effort to have the students answer the surveys as honestly as possible and minimize this possible source of bias, the researcher administered the surveys to the students instead of the regular classroom teacher. The researcher also explained to the students that their answers were strictly confidential and that the teacher would not have access to their individual information. Furthermore, the teacher was either not present, or was otherwise occupied in another area of the classroom, when the surveys were administered and collected by the researcher.

To reduce the bias associated with pre-testing and post-testing, the researcher again administered the tests and modified the questions. At the time the researcher administered the tests, the students were informed that their scores would not have any effect on their regular classroom grades. The researcher also explained that the regular classroom teacher would not have access to their tests or their individualized scores. In addition, the questions on the two tests were specifically designed to assess the students' ability to apply their knowledge to novel situations. Therefore, the questions on the post-test were not exactly the same as the questions on the pre-test. This reduced the bias associated with improvement due to students recognizing or remembering problems and answers from the pre-test. However, to ensure that the pre-test and post-test scores provided a valid comparison of student achievement, the researcher used the same number and types of questions which required the application of the same mathematical concepts on both the pre- and post-tests, although the concepts themselves were applied in different contexts.

The data obtained from the pre- and post- surveys and tests was then analyzed by the researcher. The results from the analysis of the pre-survey and post-survey were used to answer the first research question regarding whether or not explicitly teaching students heuristic problem-solving strategies improved their self-efficacy. The results from the analysis of the pre-test and post-test were used to answer the second research question regarding the students' level of achievement in solving novel, non-routine problems.

Instruments for the Study

The instruments for this study were a self-efficacy survey, pre-test, and post-test. The self-efficacy survey was used to examine the students' perceptions of their ability to solve mathematical problems. The survey was administered one week prior to the three-day lesson plan. The same survey was administered again one week after the three-day lesson to determine if there was a change in the students' perceptions. The pre-test was used to provide a baseline measurement of the students' ability to apply mathematical concepts to novel, non-routine problems, and was also administered one week prior to the three-day lesson. The post-test contained the same number and types of problems as the pre-test, but it required the students to apply their knowledge in contexts that were different from those used in the pre-test. The results were used to measure the students' problem-solving skills after the intervention, and to determine if there was an improvement in the students' ability to apply their mathematical knowledge.

The mathematical problem-solving self-efficacy survey was created by the researcher and used in the study to measure students' perceptions of their problem-solving abilities. It was composed of eleven statements based on a four point Likert-scale, and one open-ended question. For the eleven Likert-scale statements, the students chose to strongly disagree, disagree, agree, or strongly agree with each of them. The first statement in the survey asked the students what their perception was of the applicability of mathematics to their daily lives. This was used to determine if there was a significant correlation between a student's ability to apply mathematics to novel problem situations and their perception of the usefulness of the concepts they were learning. The other ten statements related to the students' perceptions of their problem-solving attitudes, skills, and abilities. These statements were used to determine the students' degree of self-efficacy.

To provide some indication of the internal reliability of the mathematical problem-solving self-efficacy survey, the ten perception of problem-solving attitude, skills, and ability statements required students to examine five aspects of their problem-solving self-efficacy. These five aspects were ascribed into both a positive and a negative perceptual context, which resulted in a total of ten statements. If a student gave a positive perception statement a high score, then they should have given the related negative statement a low score. Likewise, if a negative perception statement was given a high score, then the related positive perception statement should have received a low score. Furthermore, when the survey was created all ten statements were assigned a number, then randomized using a computer program, and written in the order of the randomized numbers. This was done to make it more difficult for the students to detect a pattern in the questions and modify their answers accordingly.

In addition to the eleven Likert-scale statements, there was also an open ended question that asked the students if there was anything they wanted to tell the researcher regarding their thoughts of problem-solving in mathematics. This provided the students with an opportunity to express any of their concerns, questions, or comments regarding mathematical problem-solving that were not covered by the Likert-scale statements. This information was used by the researcher to help understand why the students assigned the scores they did to the eleven Likert-scale statements. In addition, the comments supplied by the students were used to provide supporting evidence for the conclusions in this study and may provide possible directions for future research and studies.

Another instrument used in the study was the pre-test created by the researcher to measure the students' ability to apply conceptual knowledge in novel contexts. The pre-test consisted of one routine and four non-routine mathematical problems. The purpose of the routine problem was to verify that the students had the knowledge and skills necessary to solve the non-routine problems. It was a straightforward problem based on the mathematical concepts that the students had just learned and was consistent with other routine problems the students had practiced.

The four non-routine problems on the pre-test required the students to apply their conceptual knowledge in contexts that they were new to them. Two of the four problems were based on the application of the concept of percents. They did not require any additional mathematical knowledge than the routine problem to solve, however, because they were non-routine problems, the path to the solution was not as readily apparent. The other two non-routine problems on the pre-test required only the mathematical knowledge of basic addition, subtraction, multiplication, and division. In addition, the students were provided with calculators to facilitate the calculations.

The last instrument used in the study was the post-test, which was also created by the researcher to measure the students' ability to apply their conceptual knowledge to novel problems. To make the post-test consistent with the pre-test, it was designed with one routine and four non-routine mathematical problems. All of the problems were based on the same mathematical concepts as the ones in the pre-test, however, the contexts for the problems were different. This was done to reduce the bias associated with learning effect, as well as, to determine whether or not the students were actually able to apply their knowledge to novel situations. The routine problem on the post-test was consistent with the routine problem on the pre-test and verified that the students had the necessary background knowledge and skills. Of the four non-routine problems, two of them were based on the application of percents and the other two required only basic addition, subtraction, multiplication, and division skills. Furthermore, the students were once again provided with calculators to facilitate their calculations.

Both the pre-test and the post-test were assigned scores between zero and one-hundred according to a grading rubric created by the researcher. The same grading rubric was used to assign scores to both tests in an effort to ensure that the scores were consistent and would provide a valid comparison of the students' level of achievement. The analysis of the scores was then used to answer the second research question posed by the researcher.

Data Analysis

All of the data obtained from the pre- and post- surveys and tests created by the researcher were analyzed using the Microsoft Excel data analysis software. On the mathematical problem-solving self-efficacy surveys, the researcher examined and compared the percentage of students that chose to strongly disagree, disagree, agree, or strongly agree with each statement. On the pre-test and post-test, the students' answers were assigned scores between zero and one-hundred based on a grading rubric. All of this data was then analyzed and compared by the researcher in an effort to answer the two research questions.

To answer the first research question and determine if teaching heuristic problem-solving strategies to seventh-grade math students improved their perception of their self-efficacy in mathematical problem-solving, the researcher compared the results of the mathematical problem-solving self-efficacy pre-survey and post-survey. As noted previously, the five aspects of the students' perception of their problem-solving skills and attitudes that were examined in the survey were formatted using both a positive and negative statement. The percentages of students' responses for the positive statements on the pre-survey were compared to the percentages of responses on the post-survey using a one-tailed z-test for two proportions at a ninety-five percent confidence level. Likewise, the percentages of students' responses for the negative statements on the pre-survey were compared to the percentages of responses on the post-survey using the same z-test.

In addition to determining if there was an overall improvement in the students' perception of their self-efficacy, the results from two of the survey statements were compared to the students' pre-test and post-test scores to determine if there was a correlation. The two statements examined were Statement 1 and Statement 5. Statement 1 asked the students for their perception of the applicability of mathematics in their daily lives. This comparison was performed to determine if there was a correlation between a student's perception of the usefulness of mathematics and their ability to apply mathematical concepts to solve novel problems. Statement 5 asked the students to evaluate their self-efficacy regarding their ability to solve math problems. For the purposes of the comparison, each of the student's responses to the two statements were assigned a numerical value from one to four with a value of one indicating that the students strongly disagreed with the statement and a four indicating that they strongly agreed with the statement. The correlation coefficient was then calculated using regression with a ninety-five percent confidence level.

Lastly, the open-ended question on the mathematical problem-solving self-efficacy surveys was also examined. The students' comments were compared to their responses on the Likert-scale statements to provide insights for why the students selected the responses that they did. In addition, the comments were used to provide qualitative supporting evidence regarding the students' perception of their self-efficacy in mathematical problem-solving situations.

The second research question posed by the researcher examined whether or not teaching heuristic problem-solving strategies to seventh-grade math students improved their level of achievement in mathematical problem-solving. To answer this question, the researcher assigned the students' answers to the pre-test and post-test a numerical score between zero and one-hundred based on a grading rubric. The numerical values were entered into a Microsoft Excel spreadsheet and the mean scores and standard deviations were calculated using the standard formulas incorporated in the data analysis software. The mean scores were then compared using paired t-tests and two-sample t-tests assuming unequal variances at a ninety-five percent confidence level. The results for the control group and the intervention group were then analyzed and compared to determine if there was a significant improvement in either, or both, of the groups' level of achievement.

Summary

This study was designed to investigate the effects of teaching heuristic problem-solving reasoning and strategies on seventh grade students' self-efficacy and level of achievement. Two seventh-grade math classes participated in the study with one of the classes acting as a control group and the other class acting as the intervention group. The researcher used a pre-survey, pre-test, post-survey, and post-test to gather data on students' perceptions of their self-efficacy and their ability to solve novel, non-routine mathematical problems. To minimize possible sources of bias, the researcher administered the pre- and post- surveys and test, and taught the three-day lesson plans to both classes. To ensure that the only difference between the two groups was that the intervention group received heuristic problem-solving instruction, the lessons for both classes followed a format that was consistent with prior units and lessons established by the regular classroom teacher. Furthermore, both groups were given the notes and examples created by the regular classroom teacher and participated in similar learning activities. Afterwards, the data obtained in the study was analyzed by the researcher in an effort to answer the two research questions.

Chapter Four: Results

Introduction

A total of forty-one students from two seventh-grade math classes participated in the study. Twenty-three students in one math class acted as a control group and received three days of standard classroom instruction. The eighteen students in the other math class received three days of intervention which included learning heuristic problem-solving strategies in addition to their standard instruction. A self-efficacy survey and pre-test was administered to both groups one week prior to the study. During the study, both groups were given opportunities to practice applying the mathematical concepts from the unit to real-world situations and to develop their problem-solving skills. One week after the study, the students in both groups were given the same self-efficacy survey and a post-test with problems similar to those on the pre-test. The students' responses to both surveys and their scores on the pre-test and post-test were analyzed in an effort to answer the two research questions posed by the researcher.

Results for the First Research Question

The first research question posed by the researcher was: *Does teaching problem-solving strategies improve seventh-grade students' perception of their self-efficacy in mathematical problem-solving?* To answer this question, the students' responses to the eleven Likert-scale statements from the researcher designed mathematical problem-solving self-efficacy surveys were used. The eleven statements were divided into three categories which consisted of the one statement regarding the students' perception of the applicability of mathematics, five statements that reflected a positive perception of their problem-solving self-efficacy, and five related statements that reflected a negative perception of their problem-solving self-efficacy.

To analyze the results, the researcher determined the percentage of students that chose to strongly disagree, disagree, agree, or strongly agree with each of the eleven statements. Then the percentage of students that selected either the strongly disagree or disagree choices from the four point Likert-scale options were then combined into one category as having disagreed with the statement. Likewise, the percentage of students that selected either to strongly agree or agree were then combined into one category as having agreed with the statement. These percentages from the pre-survey and post-survey for the control group and the intervention group were then compared using a one-tailed z-test for two proportions at a ninety-five percent confidence level to determine if there was a significant difference.

For the control group, the results showed that there was no significant change in the students' perception of their self-efficacy, problem-solving skills, or attitude between the pre-survey and post-survey. Furthermore, for the subcategory of the applicability of mathematics, over 80% of the students agreed that the mathematical concepts that they learned in school were useful in their daily lives. For the positive perception Statement 5 over 69% of the students agreed, indicating that they had a positive perception of their self-efficacy in problem-solving. The corresponding negative perception Statement 11 showed that over 65% of the students disagreed, indicating that the survey was a reliable measure of the students' perception of their self-efficacy. Lastly, the percentages of students that agreed with the positive perception statements for attitude and skills in problem-solving inversely correlated to the percentages of students that disagreed with the negative perceptions statements. This indicated that the survey was a reliable measure of these attributes as well. Table 1 shows the results for the students' responses to each of the perception subcategories for the control group.

Table 1

Control Group Student Perceptions of Problem-Solving Self-Efficacy

	Pre-Survey Disagreed	Post-Survey Disagreed	Pre-Survey Agreed	Post-Survey Agreed	z-score
Applicability					
Statement 1	17.4%	13.0%	82.6%	87.0%	0.033
Positive Statements					
Statement 5	26.1%	30.4%	73.9%	69.6%	0.026
Statement 6	60.9%	60.9%	39.1%	39.1%	0.003
Statement 8	60.9%	56.5%	39.1%	43.5%	0.024
Statement 9	47.8%	47.8%	52.2%	52.2%	0.003
Statement 10	13.0%	26.1%	87.0%	73.9%	0.740
Negative Statements					
Statement 2	43.5%	43.5%	56.5%	56.5%	0.003
Statement 3	56.5%	65.2%	43.5%	34.8%	0.254
Statement 4	13.0%	13.0%	87.0%	87.0%	0.003
Statement 7	73.9%	65.2%	26.1%	34.8%	0.343
Statement 11	65.2%	65.2%	34.8%	34.8%	0.003

Note. The z-score was calculated using a one-tailed z-test for two proportions at a ninety-five percent confidence level with $\alpha = 0.05$

For the intervention group, the results showed that there was a significant decrease in the students' perception of their self-efficacy. Over 94% of the students in the intervention group agreed with Statement 5 indicating that they had a positive perception of their self-efficacy on the pre-survey. However, only 61% of the students agreed with the same statement on the post-survey, which was a significant decrease. Furthermore, Statement 9 also showed a significant change from pre-survey to post-survey. Statement 9 was a positive perception statement that read: "I think, 'I can do it', even when a math problem seems hard". Only 44% of the students agreed with this statement on the pre-survey compared to over 77% that agreed on the post-survey. This indicated a significant increase in their positive perception of their problem-solving attitudes and abilities. Table 2 shows the results for the students' responses to each of the perception subcategories for the intervention group.

Table 2

Intervention Group Student Perceptions of Problem-Solving Self-Efficacy

	Pre-Survey Disagreed	Post-Survey Disagreed	Pre-Survey Agreed	Post-Survey Agreed	z-score
Applicability					
Statement 1	16.7%	16.7%	83.3%	83.3%	0.004
Positive Statements					
Statement 5	5.6%	38.9%	94.4%	61.1%	1.980
Statement 6	44.4%	33.3%	55.6%	66.7%	0.335
Statement 8	77.8%	61.1%	22.2%	38.9%	0.745
Statement 9	55.6%	22.2%	44.4%	77.8%	1.812
Statement 10	5.6%	5.6%	94.4%	94.4%	0.004
Negative Statements					
Statement 2	27.8%	22.2%	72.2%	77.8%	0.031
Statement 3	77.8%	83.3%	22.2%	16.7%	0.042
Statement 4	16.7%	16.7%	83.3%	83.3%	0.004
Statement 7	83.3%	88.9%	16.7%	11.1%	0.039
Statement 11	94.4%	94.4%	5.6%	5.6%	0.004

Note. The z-score was calculated using a one-tailed z-test for two proportions at a ninety-five percent confidence level with $\alpha = 0.05$

Lastly, the students' responses to Statement 1 and Statement 5 on the pre-survey and post-survey were compared to their scores on the pre-test and post-test. The student's responses were assigned a numerical value from one to four with a value of one indicating that the students strongly disagreed with the statement and a four indicating that they strongly agreed with the statement. The correlation coefficient was then calculated using regression with a ninety-five percent confidence level. The results from this comparison are shown in Table 3.

Table 3

Correlation between Survey Responses and Test Scores

	Control Group		Intervention Group	
	Pre-Survey	Post-Survey	Pre-Survey	Post-Survey
Statement 1	0.25	0.35	0.26	0.31
Statement 5	0.25	0.28	0.01	0.69

Note. The correlation coefficient was calculated using regression with a ninety-five percent confidence level

For Statement 1 regarding the applicability of mathematics to their daily lives, there was no significant correlation between the students' perceptions and their ability to apply mathematical concepts to novel situations with either the control group or the intervention group. The correlation coefficient for the control group between their pre-survey and pre-test scores was 0.25 and the correlation coefficient between their post-survey and post-test scores was 0.34. The correlation coefficient for the intervention group between their pre-survey and pre-test scores was 0.26 and the correlation coefficient between their post-survey and post-test scores was 0.31. Therefore, the students ability to solve problems did not affect their perception that math was useful in the real-world.

For Statement 5 regarding the students' perception of their self-efficacy in mathematical problem-solving, the results were mixed. The results for the control group showed no significant correlation between the students' perceptions and their ability to solve non-routine problems on either the pre-survey and pre-test or post-survey and post-test. The correlation coefficient for the control group between their pre-survey and pre-test scores was 0.25 and the correlation coefficient between their post-survey and post-test scores was 0.28. This indicates that for the students in the control group, their perception of their self-efficacy in problem-solving was not related to their actual ability to solve problems.

The results for the intervention group showed no correlation for Statement 5 on the pre-survey and pre-test, however there was a significant correlation for their post-survey and post-test. The correlation coefficient for the intervention group between their pre-survey and pre-test scores was 0.01 and the correlation coefficient between their post-survey and post-test scores was 0.69. This indicated the students' perception of their self-efficacy in problem-solving was not related to their actual ability to solve problems prior to the study. However, after the study, the students in the intervention group did demonstrate that their perception of their self-efficacy was directly related to their problem-solving ability.

Results for the Second Research Question

In an effort to answer the second research question, *Does teaching problem-solving strategies improve seventh-grade students' level of achievement in mathematical problem-solving?,* the results from the pre-tests and post-tests were compared. Both the pre-test and the post-test consisted of five problems based on the same mathematical concepts. Of the five problems, there was one routine problem and four non-routine problems. The routine problem was used to determine whether or not the students had the skills necessary to solve the non-routine problems. In addition, although the non-routine problems were based on the same mathematical concepts, they required the students to apply them in novel contexts. For both tests, the students were assigned a score ranging from zero to one-hundred based on the same grading rubric.

After the pre-test was administered to both the control group and intervention group, the researcher performed an analysis of the mean scores to determine if there was a significant difference between the two groups. A two-tailed t-test for two samples assuming unequal variance with a ninety-five percent confidence level was used and Table 4 shows the results. The difference between the means of the pre-tests showed a t-score of -1.12 with a p value of < 05. Based on these results, there was no statistically significant difference between the two groups prior to the study.

The same analysis was performed on the mean scores from the post-tests to determine if there was a significant difference between the two groups after completing the study. The difference between the means of the post-tests showed a t-score of -4.05 with a p value of <05.

These results show that there was a statistically significant difference between the mean scores for the two groups after the intervention. Table 4 also shows the results of this comparison.

Table 4

Comparison of Pre-Test and Post-Test Scores

	d.f.	Control Mean	Variance	Intervention Mean	Variance	t-score	Critical t-value
Pre-Test	39	44.35	616.60	52.50	465.44	-1.12	2.02
Post-Test	39	46.96	501.68	73.33	370.59	-4.05	2.02

Note. The t-score was calculated using a two-tailed t-test for two samples assuming unequal variance at a ninety-five percent confidence level with $p < 0.05$

To determine if there was a significant improvement in the students' level of achievement, the mean scores for the pre-test scores were compared to the mean scores for the post-test for both the control group and the intervention group. The researcher used a one-tailed t-test for paired sample means with a ninety-five percent confidence level. The results for the comparison between the pre-test and post-test for the control group showed a t-score of -1.37 with a p value of < 05. This indicates that there was no significant improvement in the level of achievement for the control group. The results for the comparison of mean scores for the intervention group showed a t-score of -10.26 with a p value of < 05. This indicates that there was a statistically significant level of improvement in achievement for the group of students that received the heuristic problem-solving intervention. The results are shown in Table 5.

Table 5

Difference in Test Scores for the Control and Intervention Groups

	N	d.f.	Pre-Test Mean	Variance	Post-Test Mean	Variance	t-score	Critical t-value
Control	23	22	44.35	616.60	46.96	501.68	-1.37	1.72
Intervention	18	17	52.50	465.44	73.33	370.59	-10.26	1.74

Note. The t-score was calculated using a one-tailed t-test for paired sample means at a ninety-five percent confidence level with $p < 0.05$.

Summary

The results from the study answered the second research question definitively, but showed mixed results for the first one. For the second research question, the results showed that teaching heuristic problem-solving strategies to seventh-grade students significantly improved their level of achievement in solving novel problems. On the other hand, the results for the first research question regarding whether or not teaching heuristic problem-solving strategies improved a student's degree of self-efficacy in problem-solving situations was mixed. There was a significant improvement in the positive perception of the students' problem-solving attitudes and abilities for the group that received the intervention, but not in their self-efficacy. In fact, there was a significant decrease in their degree of self-efficacy after the intervention. However, this change in self-efficacy resulted in a significant increase in the correlation between the students' perception of their problem-solving ability and their actual ability to solve problems. Therefore, although the answer to the second research question was clear, the answer to the first research question was more ambiguous.

Chapter Five: Discussion and Conclusions

Introduction

This study was designed to answer two research questions regarding the effectiveness of teaching seventh-grade students heuristic problem-solving reasoning and strategies. The first research question asked if heuristic problem-solving reasoning and strategies improved students' perception of their self-efficacy in mathematical problem-solving. Based on the analysis of the data obtained during the study, the results for this question were mixed. The second research question asked if teaching problem-solving strategies improved seventh-grade students' level of achievement in mathematical problem-solving. The results for this question clearly indicated that there is a significant level of improvement in achievement when students are taught heuristic reasoning and strategies for solving problems.

Conclusions

The first research question was based on the belief that as students learned to become better problem-solvers, they would demonstrate a higher degree of self-efficacy (Pimta, Tayruakham, & Nuangchalerm, 2009). Teaching students heuristic problem-solving reasoning and strategies is one way of helping students to become better at solving all types of problems. This is because heuristic reasoning and strategies works well on a variety of problems since it is not regarded as final and strict, but is actually a set of practical rules derived from the experimental method that directs thinking along the paths most likely to lead to the goal (Polya, 2004). Since self-efficacy is the belief that one is capable of performing in a certain manner to attain certain goals, the more success students experience in applying their contextual knowledge in novel situations, the more confident they become in their ability to solve problems. This is an important aspect of mathematical learning since previous research has shown that self-efficacy is positively correlated with motivation and achievement (Zheng et al., 2009).

However, based on the results from this study, the students that were taught heuristic problem-solving reasoning and strategies showed a significant decrease in their perception of their self-efficacy. On the pre-survey, over 94% of the students in the intervention group agreed with Statement 5 indicating that they had a positive perception of their ability to solve problems compared to only 61% on the post-survey. These results seem to imply that teaching seventh-grade students problem-solving strategies is detrimental to their self-efficacy, which contradicts previous research and studies. However, further analysis clearly shows that the results from this study actually support the findings and conclusions made by other researchers.

First, prior research has shown that one of the most significant differences between an expert and a novice problem-solver is that expert problem-solvers demonstrate the ability to monitor and reflect on their problem-solving efforts (Lester, 1994). Although the students that received the intervention showed a decrease in their level of self-efficacy, there was a significant increase in the correlation between their perception of their problem-solving abilities and their actual ability to solve problems. As shown in Table 3, the correlation coefficient for the students in the intervention group was 0.69 on the post-survey and post-test. This indicates that the students had a more realistic perception of their abilities after being taught the heuristic reasoning and strategies. Based on the comments to the open-ended question on the surveys, the students' more realistic perception is the result of an increase in their self-monitoring and self-reflection.

The open-ended question on the mathematical problem-solving self-efficacy survey asked the students to comment on anything they wanted to say about problem-solving in math.

Unlike the control group, there was a significant difference in the types of comments that the students in the intervention group made between the pre-survey and post-survey. On the pre-survey, most of the students' comments to the open-ended question were very general and related to the level of difficulty of the math problems or to external issues with them. Over 40% of the students made a comment that solving math problems was easy for them, which suggests that the routine problems they were being given did not challenge them. The other comments indicated that the students believed the math problems they were learning to solve had no real-world applications and that they did not like word problems because they were too hard to read.

In contrast, most of the comments on the post-survey for the students in the intervention group were very specific and related to their own abilities or difficulties in solving problems. For example, one student made the comment that "the hardest part [of solving problems] is deciding where to start". Other comments suggested that the students needed more practice identifying the important information in word problems. One of the students wrote, "I can organize [the information] but I don't always get everything I need to solve the problem". Lastly, the comments on the post-survey also showed that the students were learning to look for the underlying mathematical concepts and not focusing on the contextual details. One of the students wrote, "I knew right away that I had to find the [smallest] percent so I just used trial and error starting with the lowest". All of these comments on the post-survey demonstrated a drastic change in the students' thinking. Instead of being focused on external issues, the students were monitoring and reflecting on their own problem-solving skills.

Thus, the results of this study support previous research that shows that as students acquire problem-solving skills their perceptions become more like those of expert problem-solvers (Schoenfeld, & Hermann, 1982). The change in the types of comments that the students in the intervention group made on the surveys shows that they were more reflective in their thinking after the intervention. The significant increase in the correlation between the students' perception of their problem-solving self-efficacy and their ability to solve problems shows that the students were monitoring and tracking their success in solving problems. This supports prior research that showed teaching heuristic problem-solving reasoning and strategies helps students to monitor and reflect on their problem-solving abilities, which in turn makes them better problem-solvers (Lester, 1994).

The second reason that the results of the first research question in this study actually support prior research is that a student's problem-solving ability develops slowly over a prolonged period of time (Randall & Lester, 1984). It is not feasible to thoroughly teach and explore all ten of the common heuristic problem-strategies at one time. Students need to be taught one or two strategies, and then be given plenty of time and opportunities to practice them before introducing them to more. For this study, the students were only taught three of the ten strategies and they had only a few days in which to practice them. This unintended lack of sufficient time more than likely created the most significant and greatest limitation for this study. As a result, although the students developed a more realistic perception of their abilities, they did not have the time necessary for improving their skills, which would have in turn improved their self-efficacy.

Therefore, although the results for the first research question may at first appear to contradict previous research, upon further analysis they actually confirm what other studies have concluded. This is further supported by the fact that based on the results of the post-survey there was an increase in the students' problem-solving attitude after the intervention. As shown in Table 2, there was a significant increase in the number of students that agreed with the positive perception Statement 9 for the group that received the heuristic problem-solving instruction. Statement 9 asked students to agree or disagree with the proclamation that "I think, 'I can do it', even when a math problem seems hard". This indicates that although some of the students had a lower degree of self-efficacy after the intervention, most of them believed that they would eventually be successful in solving a problem. This showed that the students had a more positive attitude towards their ability to solve problems after the intervention. Therefore, based on the results from this study, although it is not possible to draw a concise conclusion regarding the first research question, the evidence does support the prior research. Furthermore, it is clear that teaching students heuristic problem-solving strategies can improve their attitude and may eventually lead to higher degrees of self-efficacy.

On the other hand, the students in the control group showed no significant change in their perception of their self-efficacy, problem-solving skills, or attitudes after the intervention. This group of students did not receive any explicit heuristic problem-solving instruction, but they were given opportunities to practice and develop their problem-solving skills. Prior to the study, 73.9% percent of the students had a positive perception of their self-efficacy, but there was only a 0.25 correlation between their perception and their ability to actually solve problems. After three days of student-centered problem-solving instruction and learning activities, there was still no significant correlation between their perception of their abilities and their actual ability to solve problems. On the post-survey, 69.6% of the students had a positive perception of their self-efficacy with only a 0.35 correlation to their level of achievement. This shows that having students solve problems without explicit problem-solving instruction is not enough to encourage them to reflect or monitor their problem-solving skills.

This conclusion is further supported by the fact that there was no significant change in the types of comments that the students in the control group made to the open-ended question on the pre-survey and post-survey. All of the comments for both surveys were very general and related to the level of difficulty of the math problems or to external issues with them. The most common comment on the pre-survey, which was made by over 65% of the students, was that they felt solving math problems was easy. This was also the most common comment on the post-survey with over 52% of the students indicating that the math problems they were solving were not challenging for them. The rest of the comments for both surveys included what the students liked, what they disliked, and their preference for different types of problems. None of the comments on either of the surveys indicated that before or after three days of practice solving problems they were monitoring and reflecting on their own problem-solving skills the way expert problem-solvers do.

The second research question was based on the belief that as students acquired problem-solving skills, they would demonstrate higher levels of achievement in mathematics and be able to apply their knowledge in novel situations (Schoenfeld & Hermann, 1982). The results from this study showed that there was a significant improvement in the level of achievement for the students that received the problem-solving instruction. As shown in Table 5, there was an increase of 20.83 points in the mean score for the intervention group between their pre-test and post-test. This supports the prior research that claimed students had a significantly higher level of achievement when explicitly taught problem-solving strategies (Schoenfeld, 1980). Furthermore, over 83.3% of the students in the intervention group that had the necessary skills to solve the routine problem were able to apply that knowledge to successfully solve the non-routine problems on the post-test.

On the other hand, there was no significant improvement in the level of achievement for the students in the control group after three days of practice solving real-world problems. The mean score on the pre-test for the control was 44.35 compared to 46.96 on the post-test. This clearly shows that practice solving problems alone is not sufficient to increase a student's level of achievement. Even after three days of solving real-world problems, the students in the control group still demonstrated significant difficulty in applying their conceptual knowledge in novel situations. These results are supported by the fact that over 78.3% of the students in the control group had the skills necessary to solve the routine problem on the post-test, but only 30.4% of them were able to apply that knowledge to solve the related non-routine problems.

The last aspect of a students' level of achievement that was examined in this study was the correlation between a student's ability to solve problems and their perception of the usefulness of mathematics in their everyday lives. The expectation was that a student's ability to apply mathematics to novel situations would be positively correlated to their perception of the usefulness of the mathematical concepts they were learning. To determine if this was the case, the researcher compared the students' responses to Statement 1 regarding the applicability of mathematics to their pre-test and post-test scores. The results, which were shown in Table 3, indicated that neither the control nor the intervention group demonstrated a significant correlation between their test scores and their survey responses for Statement 1. However, on the pre-test 82.6% of the students in the control group and 83.3% of the students in the intervention group agreed with the statement that mathematics was useful in their everyday lives. On the post-test, 87.0% of the students in the control group and 83.3% of the students in the intervention group agreed with the statement. Based on these results, the majority of seventh-grade students regard the subject of mathematics as being useful to them in the real-world for solving everyday problems, regardless of their own personal ability to apply the concepts they have learned.

Implications for Teaching

Based on the results of this study, as well as previous studies, it is clear that teaching students heuristic reasoning and problem-solving strategies can improve their level of achievement. However, teaching the students problem-solving reasoning and strategies alone is not enough. Students also need to be given sufficient time and opportunities to practice using what they have learned in order to improve their problem-solving self-efficacy. This is because problem-solving is a mental process that involves the use of metacognition, prior knowledge, and strategies which needs to be developed slowly over time. In addition, students should be taught how to select the most appropriate strategies for solving different types of problems since they cannot feasibly work through all of the strategies that they know for each problem they try to solve. As a result, teachers need to make a substantial commitment in time and effort to developing their students' problem-solving skills. They also need to teach problem-solving with the same importance that they teach other concepts in the mathematics curriculum. Research has shown that students will only use problem-solving strategies if they believe that are important, practical, and useful tools for solving problems (Schoenfeld, 1980).

Furthermore, all students would benefit greatly from systematically planned problem-solving instruction since solving problems helps students to see the meaning and relevance of what they learn and to facilitate the transfer of contextual knowledge to authentic situations (Lee & Chen, 2009). Because the process of solving problems requires students to create their own understanding, seek patterns, test hypotheses, and actively search for solutions it helps provide students with a solid foundation that teachers can use to scaffold their students to higher levels of mathematical thinking and learning (Adiguzel & Akpinar, 2004). Whether a student is a novice or expert problem-solver, this process is a lifelong learning skill that students can continue to improve upon with time and practice (Randall & Lester, 1984). In addition, problem-solving skills are not only vital to success in the subject of mathematics, but they also help students to solve problems in their everyday lives. As students become better problem-solvers, they will be able to combine previously learned knowledge, skills, concepts, and techniques to arrive at solutions to novel situations. Therefore, based on the results of this study and prior research, it is clear that teachers need to incorporate heuristic reasoning and problem-solving strategies into their curriculums.

However, teachers should also be cautioned that problem-solving instruction has limitations and issues just as any other method of instruction does. In addition to the considerable commitment in teaching time, integrating non-routine problem-solving into the curriculum may require additional planning and for many teachers to alter their current methods of assessment. Furthermore, some teachers have difficulty teaching problem-solving techniques to their students because they are novice problem-solvers themselves due to inadequate training in the process of solving problems. Other teachers may simply lack the knowledge of how problem-solving instruction can be smoothly incorporated into their regular curriculum. Lastly, some teaching professionals feel that problem-solving depends heavily on domain-specific knowledge, and therefore it should be emphasized in all subject areas, not just mathematics (McLeod, 1988).

Students may also have issues with this method of learning since prior knowledge and experience play an important role in problem-solving, although their effect can be either positive or negative (Shuell, 1990). Students bring preconceptions with them to the problem-solving process, as they do with any learning situation. Some students erroneously believe that a problem can only be solved one way, and that solving it any other way is either incorrect or cheating. If these types of misconceptions persist, then the student is likely to have difficulty finding an appropriate solution to the problem and will experience a high degree of frustration. Furthermore, when students are actively engaged in trying to solve non-routine mathematical problem, they often express a lot of emotion (McLeod, 1988). If they work on the problem over an extended period of time, their emotional responses may become quite intense. Students that are inexperienced or novice problem-solvers may quickly become frustrated and even feel a sense of panic. On the other hand, when students experience success in solving a challenging problem, it can make them feel a deep sense of satisfaction and even joy. Since the outcome to the problem-solving situation is something that teachers can control, this student issue actually becomes a pedagogical one.

Limitations and Improvements

This study was designed to gather information and analyze the results of students' perceptions of their self-efficacy and level of achievement in problem-solving after explicitly being taught how to use heuristic problem-solving reasoning and strategist. To accomplish this with the minimal amount of bias between the control group and intervention group, the researcher taught both groups their lessons during the three-day intervention. Furthermore, the lessons for both groups followed the same format and included the same notes, examples, and homework assignments created by the regular classroom teacher. Both groups of students also had opportunities to practice their problem-solving skills using real-world situations and participated in a student-centered learning activity. All of this was done in an attempt to control for differences between the two groups that were not related to the study of heuristic problem-solving. However there were still limitations and sources of bias that could not be controlled for.

In addition to the issues created by the time constraints of the study, some of the other limitations include sample bias, sample size, and reliability of the measurement instruments. Because the study used a sample of convenience, and the students were required to put their names on the survey, pre-test, and post-test so that their information could be compared, there was the possibility for sample bias. Furthermore, the study was conducted in a small rural school with a small number of participants. This creates issues with the researcher's ability to generalize the results to other student populations such as suburban and urban school students. Also, the results may have been vastly different if the study had been conducted on a larger sample size or more diverse student population. Finally, there was no proven reliability for the measurement instruments used in the study. The researcher did make attempts to verify that the instruments provided accurate measurements of what they were intended to measure by using both positively and negatively framed related statements on the surveys, as well as, using both routine and non-routine problems on the test. The results of the study suggested that the instruments did provide appropriate measurements, however these results are not proof that the instruments were reliable or valid. All of these issues create some limitations and possible sources of bias which may have affected the study. On the other hand, the majority of the results are consistent with those from other research. This suggests that despite these issues, the study did measure what it was intended to measure.

Some of the ways to improve this study would be to conduct the study again over a longer period of time, with students from other types of schools, and using a more diverse student population. As noted previously, problem-solving skills develop slowly and require time for students to practice them. By extending the time frame for the study, students would have plenty of time to learn all ten of the problem-solving strategies and improve their skill in using them. Conducting the study in multiple school settings that included suburban and urban schools would allow the results to be generalized with more accuracy. It would also eliminate the issues caused by a small sample size and the use of a sample of convenience. Although very little can be done to improve the measurement instruments themselves, if the same results were obtained with a more diverse population, it would go a long way towards demonstrating the reliability and validity of the survey, pre-test, and post-test.

Future Considerations

Based on the results of this study, further research should be conducted to determine quantitatively exactly how beneficial teaching heuristic problem-solving strategies are to students' self-efficacy and level of achievement. This study only taught the students the basic reasoning process and three strategies. It took place over a three-day period, which was not enough time for the students to truly explore and practice what they had learned. Future studies need to cover all of the strategies and provide students with ample opportunities to practice their skills. However, teaching all ten strategies in depth can be a time consuming process. Being able to provide teachers with concrete statistical proof of the value of teaching heuristic problem-solving reasoning and strategies would help to convince them that the benefits warrant the time involved.

Another area of interest that future studies should explore is how heuristic problem-solving reasoning and strategies can be used to enhance inquiry-based learning. Inquiry-based instruction is a method of teaching in which teachers create situations where students need to solve a problem in order to construct their own conceptual understanding. The lessons are typically designed so that as a student solves the problem, they are able to make connections to their previous knowledge, raise their own questions about their learning, and investigate ways to explore their ideas. This method of instruction has become an area of significant interest in recent years and since it requires students to solve problems, it is directly related to the study of heuristic problem-solving reasoning and strategies. Therefore, future studies that investigate this connection may lead to improvements in inquiry-based learning as well as problem-solving instruction.

Lastly, since teaching students heuristic problem-solving strategies clearly improves their level of achievement, future studies should focus on the effect it has on their self-efficacy. One future consideration should involve the length of time that it takes for students to develop their problem-solving skills enough to demonstrate a significant increase in their belief that they are capable problem-solvers. Another aspect is the amount of practice students require to feel confident in their problem-solving abilities. Furthermore, the factors that influence a student's self-efficacy in the process of solving problems such as attitude, interest, motivation, and level of engagement should also be explored. Learning more about these attributes of self-efficacy would not only help teachers to improve problem-solving instruction, it would also allow them to create a more student-centered learning environment that encourages and supports risk-taking while creating diverse opportunities for individual discovery.

Summary

In conclusion, although the results for the first research question were mixed and may at first appear to contradict previous research, upon further analysis it is clear that teaching students heuristic problem-solving strategies can improve their attitude and may lead to a higher degree of self-efficacy. To do this however, teachers need to make a substantial commitment in time and effort to developing their students' problem-solving skills. They need to teach problem-solving techniques with the same importance that they teach any other aspect of the mathematics curriculum. This is because students will only use the problem-solving strategies if they believe in them and have had time to practice and explore how to use them effectively. The results for the second research question clearly show that this investment is worthwhile since there was a significant improvement in the level of achievement for the students that received the problem-solving intervention. The fact that the students' achievement improved despite the meager time frame and other limitations of the study demonstrates just how beneficial teaching students heuristic problem-solving reasoning and strategies can be. Therefore, although this is not an easy task, it is a critically important one that will benefit students not only in the subject of mathematics, but ultimately it will also improve their ability to apply their knowledge and skills to other areas of their lives.

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Appendix A

Name: ____

Solving Problems in Math

	Strongly Disagree	Dis-agree	Agree	Strongly Agree
1. Mathematics is useful for the problems of everyday life.	☹	☹	☺	☺
2. I don't like doing word problems in math.	☹	☹	☺	☺
3. When a math problem seems hard, I think "I ***can't*** do it".	☹	☹	☺	☺
4. I like the easy math problems best.	☹	☹	☺	☺
5. I am good at doing math problems.	☹	☹	☺	☺
6. I like math problems that are challenging.	☹	☹	☺	☺
7. If I don't see how to do a math problem right away, I never get it.	☹	☹	☺	☺
8. I enjoy doing word problems in math.	☹	☹	☺	☺
9. I think, "I ***can*** do it", even when a math problem seems hard.	☹	☹	☺	☺
10. I can get the answer to a math problem if I work at it long enough.	☹	☹	☺	☺
11. Doing math problems is hard for me.	☹	☹	☺	☺

Comments: Is there anything ***you*** want to say about solving problems in math?

____

____

____

____

____

Appendix B

Name: <u>ANSWER KEY</u>

Real World Problems Pre-Test – SHOW ALL WORK

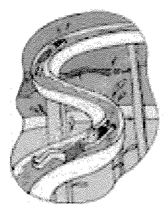
1. Noah wants to buy 6 items at Walmart that cost a total of $74.89. There is a sales tax of 7% on the items and Noah only has $80 with him. Does Noah have enough money to buy all six items? How much extra does he have ***or*** how much more would he need?

<u>Percent Skills</u>

$74.89 * .07 = 5.2423 = $5.24
$74.89 + $5.24 = $80.13

He does *<u>not</u>* have enough money.
He needs $0.13

2. Amber went to a water park and spent half of her money on the admission ticket. She forgot to bring a towel, so she spent half of the money that was left to buy a new one. Amber had $4.00 left, which she spent on lunch. How much money did she take to the water park?



<u>Working Backwards</u>

$4.00 is half of what she had before she bought the towel
$4.00 * 2 = $8.00

$8.00 is half of what she brought to the park
$8.00 * 2 = $16.00

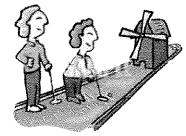
She brought $16.00 to the water park

3. Your team bought T-shirts for $10.50 each and sold them at a fundraiser $20.00 each. Your team has some T-shirts left over and wants to sell them for at least what you paid for them. What is the ***biggest*** discount in a 5% increment (meaning 5%, 10%, 15%, 20%, etc) that you can give?

<u>Guess and Check</u>

50% of $20.00 = $10.00, which is too big, but close
Try 45%: $20.00 * .45 = $9.00
$20.00 - $9.00 = $11.00
45% works, so it is the biggest discount we could give

4. Brittany played nine holes of miniature golf and had a final score of 22. She wanted to show her parents her score card, but she couldn't read her scores for the first two holes. She remembered that her score for the first hole was 1 stroke less than the score for the second hole. Using the score card below, what were her scores for the first two holes?



Brittany's Score Card									
Hole	1	2	3	4	5	6	7	8	9
Score			2	3	4	2	3	1	2

Organizing Data/Logical Thinking

We Know: Hole 1 + Hole 2 + 2 + 3 + 4 + 2 + 3 + 1 + 2 = 22

Hole 1 + Hole 2 + 17 = 22

− 17 −17

Hole 1 + Hole 2 = 5

Therefore, Logically

Hole 1 = 2 and Hole 2 = 3

5. Bryan wants to buy a mp4 player and found two stores that sold it for $127.00. Both stores put the player on sale last week, then lowered the price again this week with another sale. Bryan cannot remember the exact percentages of the two sales, but knows they were the same at both stores – except in reverse order. Store #1 had the bigger sale first, then the smaller sale. Store #2 had the smaller sale first then the bigger sale. Which store should Bryan buy the player at? Why?

Solve a simpler problem

Suppose it costs $100 and the sales were 10% and 25%

Store #1:	**Store #2**
$100 * .10 = $10.00	**$100 * .25 = $25.00**
$100 - $10.00 = $90	**$100 - $25.00 = $75**
$90 * .25 = $22.50	**$75 * .10 = $7.50**
$90 - $22.50 = $67.50	**$75 - $7.50 = $67.50**

OR

Trial and Error

Suppose the sales were 10% and 25%

Store #1:	**Store #2**
$127.00 * .10 = $12.70	**$127.00 * .25 = $31.75**
$127.00 - $12.79 = $114.30	**$127.00 - $31.75 = $95.25**
$114.30 * .25 = $28.58	**$95.25 * .10 = $9.53**
$114.30 - $28.58 = $85.72	**$95.25 - $9.53 = $85.72**

He could buy the player at either store, it does not matter what order the discounts are given in.

Appendix C

Name: ANSWER KEY

Real World Problems Post-Test – SHOW ALL WORK

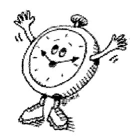
1. Tanya bought a PT Cruiser for $18,275 with an interest rate of 9% for 5 years. How much total will Tanya pay for the car?

Percent/Interest Skills

I = p * r * t
I = 18275 * .09 * 5
I = $8,333.75

Total = $18,275 + $8,333.75
= $26498.75

2. Brady was trying to decide what time to get up in the morning. He needs 45 minutes to get ready for school. It takes him 25 minutes to get to school. He wanted to get to school 20 mintes early to use the library. If school starts at 7:30, what time should he get up?

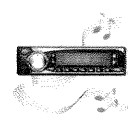


Working Backwards

7:30 – 20 minutes = 7:10
7:10 – 25 minutes = 6:45
6:45 – 45 minutes = 6:00

He should get up at 6:00

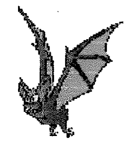
3. You work at a store that is selling a particular car stereo for $100.00. Another store is selling the same car stereo for less. Your manager wants you to put the stereo on sale so that it is less than the $89.00 that your competitor is selling it for. What is the ***smallest*** discount in 5% increments ((meaning 5%, 10%, 15%, 20%, etc) that you can give and still be less than your competitor?



Guess and Check

10% of $100.00 = $10.00, which would sell the stereo for $90
Close, but still too much
Try 15%: $100.00 * .15 = $15.00
$100.00 - $15.00 = $85.00
15% works, so it is the smallest discount you could give and still be less than $89.00

4. Sarah was studying how many bugs a bat eats each night. She watched one bat for four nights in a row and found that it ate a total of 1050 bugs. For her project, Sarah needs to show how many bugs the bat ate each night, but she lost the paper with her nightly score. She remembers that each night the bat ate 25 more bugs than it ate the night before. How many bugs did the bat eat each of the nights?



Sarah's Bat Project					
Night	1	2	3	4	Total
Number of bugs the bat ate	225	250	275	300	1050

Organizing Data/Logical Thinking

We Know: Bugs + Bugs + 25 + Bugs + 50 + Bugs + 75 = 1050

4 Bugs + 150 = 1050

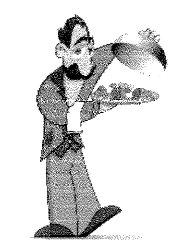
− 150 −150

4 Bugs = 900

Bugs = 225

Logically: Night 1 = 225, Night 2 = 250, Night 3 = 275, Night 4 = 300

5. John and his family went to a restaurant for dinner. The bill for the food was a total of $89.50. John wants to make certain that he gives the waiter enough money for the sales tax and the tip, but he can't remember which percentage to do first. Does it matter? Why? *(show your work)*



Solve a simpler problem

Suppose the bill was $100, the sales tax was 10% and the tip was 15%

Tax then Tip	Tip then Tax
$100 * .10 = $10.00	$100 * .15 = $15.00
$100 + $10.00 = $110	$100 + $15.00 = $115.00
$110 * .15 = $16.50	$115.00 * .10 = $11.50
$110 + $16.50 = $126.50	$115.00 + $11.50 = $126.50

OR

Trial and Error

Suppose sales tax was 10% and the tip was 15%

Tax then Tip	Tip then Tax
$89.50 * .10 = $8.95	$89.50 * .15 = $13.42
$89.50 + $8.95 = $98.45	$89.50 + $13.42 = $102.93
$98.45 * .15 = $14.77	$102.93 * .10 = $10.29
$98.45 + $14.77 = $113.22	$102.93 + $10.29= $$113.22

No: It will still cost the same amount of money

Yes: he will pay more (or less) in tax, and he will give the waiter a smaller (or bigger) tip

Appendix D

Pre-Survey and Post-Survey Data

	Pre-Survey				Post-Survey			
Applicability of Math	SD	D	A	SA	SD	D	A	SA
Control Group								
Statement 1	4.3%	13.0%	43.5%	39.1%	4.3%	8.7%	47.8%	39.1%
Intervention Group								
Statement 1	5.6%	11.1%	55.6%	27.8%	5.6%	11.1%	50.0%	33.3%

	Pre-Survey				Post-Survey			
Positive Statements	SD	D	A	SA	SD	D	A	SA
Control Group								
Statement 5	4.3%	21.7%	52.2%	21.7%	4.3%	26.1%	47.8%	21.7%
Statement 6	34.8%	26.1%	26.1%	13.0%	30.4%	30.4%	26.1%	13.0%
Statement 8	13.0%	47.8%	34.8%	4.3%	8.7%	47.8%	39.1%	4.3%
Statement 9	13.0%	34.8%	26.1%	26.1%	13.0%	34.8%	30.4%	21.7%
Statement 10	4.3%	8.7%	69.6%	17.4%	8.7%	17.4%	56.5%	17.4%
Intervention Group								
Statement 5	5.6%	0.0%	66.7%	27.8%	11.1%	27.8%	44.4%	16.7%
Statement 6	16.7%	27.8%	33.3%	22.2%	11.1%	22.2%	33.3%	33.3%
Statement 8	22.2%	55.6%	22.2%	0.0%	11.1%	50.0%	27.8%	11.1%
Statement 9	5.6%	50.0%	27.8%	16.7%	5.6%	16.7%	50.0%	27.8%
Statement 10	0.0%	5.6%	61.1%	33.3%	0.0%	5.6%	44.4%	50.0%

	Pre-Survey				Post-Survey			
Negative Statements	SD	D	A	SA	SD	D	A	SA
Control Group								
Statement 2	8.7%	34.8%	21.7%	34.8%	13.0%	30.4%	26.1%	30.4%
Statement 3	13.0%	43.5%	26.1%	17.4%	17.4%	47.8%	13.0%	21.7%
Statement 4	0.0%	13.0%	39.1%	47.8%	0.0%	13.0%	34.8%	52.2%
Statement 7	30.4%	43.5%	8.7%	17.4%	26.1%	39.1%	17.4%	17.4%
Statement 11	26.1%	39.1%	17.4%	17.4%	30.4%	34.8%	17.4%	17.4%
Intervention Group								
Statement 2	11.1%	16.7%	66.7%	5.6%	11.1%	11.1%	66.7%	11.1%
Statement 3	22.2%	55.6%	11.1%	11.1%	27.8%	55.6%	11.1%	5.6%
Statement 4	0.0%	16.7%	33.3%	50.0%	0.0%	16.7%	38.9%	44.4%
Statement 7	22.2%	61.1%	16.7%	0.0%	33.3%	55.6%	11.1%	0.0%
Statement 11	38.9%	55.6%	0.0%	5.6%	50.0%	44.4%	5.6%	0.0%

Note: SD = Strongly Disagree, D = Disagree, A = Agree, and SA = Strongly Agree

Appendix E

Pre-Test and Post-Test Data

Control Test Scores	
Pre-Test	Post-Test
65	70
35	30
45	55
85	80
45	50
20	30
85	70
30	40
60	55
10	20
20	30
85	80
85	80
30	30
50	45
5	15
25	20
30	20
45	40
30	40
20	30
50	65
65	85

Intervention Test Scores	
Pre-Test	Post-Test
65	85
50	80
60	85
15	40
30	70
55	65
10	35
70	80
85	100
35	65
50	80
25	35
70	80
50	75
65	85
65	80
80	100
65	80

Difference Between Pre-tests

t-Test: Two-Sample Assuming Unequal Variances

	Control Pre-Test	*Intervention Pre Test*
Mean	44.3478261	52.5
Variance	616.600791	465.4411765
Observations	23	18
Hypothesized Mean Difference	0	
df	39	
t Stat	-1.1233262	
P(T<=t) two-tail	0.26816649	
t Critical two-tail	2.0226909	

Difference Between Post-tests

t-Test: Two-Sample Assuming Unequal Variances

	Control Post Test	*Intervention Post Test*
Mean	46.95652174	73.33333333
Variance	501.6798419	370.5882353
Observations	23	18
Hypothesized Mean Difference	0	
df	39	
t Stat	-4.050767421	
P(T<=t) two-tail	0.000235216	
t Critical two-tail	2.022690901	

Difference Between Control Group Pre and Post Tests

t-Test: Paired Two Sample for Means

	Control Pre-Test	*Control Post Test*
Mean	44.34782609	46.95652174
Variance	616.6007905	501.6798419
Observations	23	23
Pearson Correlation	0.929992246	
Hypothesized Mean Difference	0	
df	22	
t Stat	-1.36672035	
P(T<=t) one-tail	0.092759435	
t Critical one-tail	1.717144335	

Difference Between Intervention Group Pre and Post Tests

t-Test: Paired Two Sample for Means

	Intervention Pre Test	*Intervention Post Test*
Mean	52.5	73.33333333
Variance	465.4411765	370.5882353
Observations	18	18
Pearson Correlation	0.917091433	
Hypothesized Mean Difference	0	
df	17	
t Stat	-10.25660859	
P(T<=t) one-tail	5.33304E-09	
t Critical one-tail	1.739606716	