

Evidence-based practices for teaching mathematics to students with disabilities

By:

Daniel Figliole

August 2013

A thesis submitted to the Department of Education and Human Development of the State  
University of New York College at Brockport in partial  
Fulfillment of the requirements for the degree of  
Master of Science in Education

## TABLE OF CONTENTS

Title page.....	1
Table of contents.....	2
Abstract.....	3
Part I.....	4
Part II.....	24
Part III.....	50
Conclusion.....	75
Appendices.....	76

## **Abstract**

This research study examines the effectiveness of three evidence-based strategies on students with disabilities. The participants include five students in a self-contained special education classroom. Depending on ability levels, the participants were exposed to one or more of the evidence-based strategies which included; self-regulation, concrete-representational-abstract, and copy, cover, compare. The research study found that students need a considerable amount of instruction and assistance for using the self-regulation strategy and its corresponding checklist. However, the concrete-representational-abstract strategy as well as the copy, cover, compare strategy proved to be beneficial in helping students to efficiently count coin money to a designated amount using the fewest amount of coins possible.

## Part I

Ever since the passage of No Child Left Behind in 2001, then emphasized with the passing of the Individuals with Disabilities Education Act, evidence-based practices have been a central focus in the education arena. NCLB opened the flood gates for increasing the role research evidence plays in education. Then, in 2004, IDEA established that research evidence was no longer a suggestion, but that it now had become a law. The term evidence-based practice implies that a specific strategy is guaranteed to work with all students. However that is not the case. Before making a judgment about any evidence-based practices, it is important to first understand what it means to be evidence based. Then once that foundation has been set, one needs to tackle the assumption that an evidence-based practice will be effective for all students. When the phrase all students is used, does the average person know that that means general education as well as special education students? Furthermore, does that same person know that much like a snow flake, no two special education students are identical?

Prior to the topic of evidence-based practices in special education being discussed, it is essential that the term evidence-based practice be explained. As it is explained in an article by Spencer T., Detrich R., and Slocum T., the education world has struggled for quite some time to distinguish the difference between practices that have evidence from multiple researches behind them and practices that are universally used. Just because a particular teaching strategy is being used by a significant amount of teachers across the United States, does not mean that it is evidence based, even if it leads to positive results from a majority of those teachers. In order for a teaching strategy to be considered to be evidence based, it needs to go through a rigorous process. Moreover, this process does not happen overnight. Rather, in much the same way it takes years for a particular drug to become generic, it takes years for a given teaching strategy to

become evidence based. “A gap between research and practice means that consumers are not receiving services that are based on the best research evidence that exists and therefore may suffer from poorer outcomes and unnecessary costs associated with ineffective treatments.” (Spencer T., Detrich R., and Slocum T. p.128). By this statement, the authors of this article are suggesting that if the best practices are not being used, then the consumers, in this case the students, are suffering from inadequate teaching strategies. With the goal of every teacher being to help every student realize and achieve their fullest potential, the teachers should be using the best strategies available that have been supported by scientific research.

Detrich (2008) argued that the evidence-based movement was an attempt to protect the students by using strategies that research suggests are likely to be the most effective. He goes on to say that the effectiveness of a given strategy can only be determined according to the established goals. Therefore, Detrich is saying that a strategy can only be deemed effective if the student meets their targeted goal. For example, according to Detrich, if a student has a goal of being able to count a set of mixed coins up to \$5.00, the strategy used to teach this skill to the student can only be considered effective if the student is able to count out \$5.00 in mixed coins by the end of a specific period of time. This does not take into account any progress that the student might make under the \$5.00 mark. In other words, if the student is only able to count up to \$4.30, the strategy that the teacher used is not effective. In the world of intense teacher evaluations, there is no room for a teacher to try new strategies if this is how effectiveness is measured. Then you will get teachers all using the same strategies, most likely direct instruction lecture style, to teach their students in the hopes that enough students will learn to give that teacher a positive evaluation score.

The authors of this article propose that evidence-based practices are based on three factors: best available evidence, professional judgment, and client values and context. The term best available evidence infers that there is a wide range of research and teachers should select the best from what is accessible. Thus, when choosing what strategies to use when teaching their students, teachers need to take time to sort through all of the evidence and select the best strategies. The teacher will need to consider the research that is most pertinent to the decision and is most likely to be effective. However, Spencer T., Detrich R., and Slocum T explain that if educators only consider research of the highest quality, they will find very few evidence-based practices. They go on to say that educators are not able to wait until a significant amount of research has been carried out before they make their decisions on the best practices to choose to use in their classrooms.

The implication that there is minimal research regarding evidence-based practices may be plausible especially when referring to a specific strategy. While data is good, it does take a considerable amount of time to record enough data on a particular strategy to make an educated decision on its effectiveness. When trying to determine if a specific strategy will be effective for teaching a student to multiply double-digit equations, it may take weeks before the teacher can gather enough data to arrive at an accurate conclusion. Furthermore, it takes time to find a strategy that on paper will best meet the individual student's needs.

In much the same way, Cook, Tankersley, and Landrum evaluated what it means to be evidence based and came to the conclusion that there needs to be multiple studies regarding a specific teaching strategy, the strategy needs to be used as intended, and that the strategy must have a significant positive effect on student results. The authors laid out a proposed set of guidelines for evidence-based practices specifically for special education that includes quality

indicators aimed at four distinct research designs: group experiments studies, single-subject research (SSR), correlational research, and qualitative research. They go on to explain that in general education, What Works Clearinghouse only evaluates randomized trials and quasi-experiments studies when defining effective strategies. In addition, they require that one or two studies have positive, potentially positive, potentially negative, or negative results. “To consider a practice to be evidence-based in special education, Horner et al. (2005) specified a minimum of five SSR studies that involve a total of at least 20 total participants and that at least three different researchers conduct across at least three different geographical locations.” (Cook, Tankersley, and Landrum p. 372).

Just like with any research, there needs to be a protection plan against all biasness and tampering. Moreover, the research should be conducted using a variety of samples. The part that is mildly perplexing is the requirement for at least 20 total participants. In most special education settings there are twelve students to a classroom for a 12:1:1 ratio of students to teachers and can go to 7:1:4. Therefore, in order to get at least 20 students, a researcher would need to collect data from two or three classrooms. A critical analyst might say that this may be taking too much data from one demographic. However, another analyst might say that two or three classes from one school or even one region would give more solid credibility to the data that is collected.

In an effort by (Cook, Tankersley, and Landrum p. 372) to wrap up their guidelines, they said that for a strategy to be considered evidence-based, using SSR in special education, there needs to be proof that there is a correlation between the strategy and student behavior. To clarify this, the authors devised indicators for a visual inspection: immediacy of effects, overlap of data points, magnitude of change, and consistency in data patterns. Thus, meaning that a teacher or

researcher can use the art of observation to assess whether or not a strategy is effective by using the four visual inspection indicators. Not only will these criteria work for one student, but they will work on a larger scale as well. They contain measurable, sustainable data that can be quickly collected by a teacher or researcher and then organized in a graph. However, they do not protect against biasness if the data is collected by the teacher. In relation to the article by Spencer, Detrich, and Slocum, the criteria laid out by Cook, Tankersley, and Landrum alludes to the fact that it is to be expected that a substantial amount of time goes by for an efficient amount of data to be collected. If the data in question is regarding aggressive behavior by a student, the teacher might not have the necessary amount of time for an acceptable amount of data to be collected.

Once a teacher understands what an evidence-based practice is, the next step is to know how to implement the strategies that they chose to use. However, this can often times be the more challenging part. It is not as simple as just flipping through a book of teaching strategies. As was mentioned in the previous two articles, finding a high quality evidence-based practice takes some research on the teacher's part. Not only does this include determining if there is enough research behind the strategy to qualify it as evidence-based, but it also includes the teacher knowing the students' characteristics and figuring out what will work best for the students. In a general education classroom, this is much more stress-free. However, in a special education classroom where there can be anywhere from seven to twelve or even fifteen students who all have different characteristics, finding an evidence-based practice that works needs to be individualized.

Choosing and implementing an evidence-based practice can be a more difficult undertaking that one might imagine. However, authors such as Torres, Farley, and Cook have laid out

a guide for successfully implementing evidence-based practices. Reflective of the articles by Cook, Tankersley, and Landrum as well as Spencer, Detrich, and Slocum, the authors of this article explain that evidence-based practices are proven with high quality research suggesting meaningful improvement in student outcomes. Furthermore, it needs to have been verified to be effective through multiple research studies. This is a fairly universal understanding among researchers in this field. Even though a practice may be widely used, researchers will not consider any teaching strategy to be evidence-based until there has been a considerable amount of high quality research that substantiates the strategy's effectiveness. Going back to the medicine analogy, a consumer would not want to be prescribed a medication that has not been tested and verified as being effective. Much the same goes for education practices.

Communities expect that their children are being taught using the best teaching practices available.

To make the selection and implementation process more stress-free, Torres, Farley, and Cook have proposed a 10-step evidence-based implementation process. The steps are as follows: determine, search, select, identify, implement, check, monitor, adapt, make, and share. While this may seem objectively straight forward, this process will need to be followed until the teacher finds a strategy that is effective. In addition, the teacher needs to have the time and patients to let the process play itself out. To assist with the process, the authors of this article have also created a checklist for the teacher to use.

In step one; the teacher is required to determine the characteristics of the student, themselves, and the environment. Next, the teacher must do their own homework by scanning sources of evidence-based practices and then select one that best fits with the characteristics identified in step one. In order for this process to continue forward, the teacher needs to identify

the main components of the practice that they chose. Once all of these steps have been completed, the teacher is ready to implement the strategy that they chose. Contrary to popular belief, the teacher's work is not done there. They next need to monitor and collect data to determine if the strategy is showing to be effective. The remaining steps describe how the teacher should continue to monitor progress and adapt the strategy to meet the individual student's needs. The last step in implementing an evidence-based practice in special education is for the teacher to become an advocate for the strategy and share what they have discovered.

Torres, Farley, and Cook explain that in order for new evidence-based practices to be sustained, "communities of practice" need to be created by the teachers to share their findings and coach fellow teachers as they too try the new strategy. This part can be especially effective when it comes to the adaptation step in the implementation process. It will give the teachers an opportunity to offer insight and evidence from how they implemented the same strategy. However, the issue with this process is the part regarding teachers searching several sources for the best practices. Surely there will be backlash from several teachers who say that they do not have the time to be able to sort through a plethora of research articles looking for one or two strategies that are supposed to work in their classrooms. In support of this point, with the new common core standards that are going into effect, it is true that the teachers are expected to teach more material to their students. Furthermore, with the continuous cuts to education, they are ordered to do these additional things with fewer resources. On the other hand, this could be when evidence-based practices show to be most effective. After all, they don't cost anything to use.

Now that the definition of evidence-based practices has been laid out and the processes of verifying and implementing these processes has been explained, it is time to examine several

strategies that have been researched and found to be effective. The first article by Jitendra et al looks at increasing potential for students with Attention Deficit Hyperactivity Disorder (ADHD). The article mentions that a significant majority of the research regarding this topic has focused on the behavior rather than academics. The most common solutions to the problem of behavior are medication and management strategies. Although they have shown to help increase the student's attention span, they have proven to be ineffective in improving the student's academic achievement.

Two of the most common strategies for refining academic achievement for students with ADHD are peer mediated interventions and computer assisted instruction. The article explains that peer mediated interventions improved not only the student's academic performance, but also was responsible for decreasing their off task behavior. As for computer assisted instruction, "research on CAI for small samples of children with ADHD has shown clinically significant gains in oral reading fluency and basic mathematics computation skills." (Jitendra et al p. 326). While studies have shown peer mediated and computer assisted instruction to be effective. It is important to mention that the students first need quality instruction from their teachers. These interventions as well as others are meant to be an assistant to formal instruction as opposed to a substitute. They enhance the student's academic achievement by focusing their attention on either another student or a computer screen. Additional strategies specifically designed to help students with ADHD improve their math skills are cover, copy, compare, and schema-based instruction. Both of these strategies provide the student with a more immediate feedback and allow them to develop self-monitoring skills.

In an article written by Kim Paulson, she mentions that 5-8% of school-aged children are identified as having a mathematics disability. Although that number sounds low, it can be

assumed that the real percentage is much higher. Her article is directed towards educating those who will be teaching the students with disabilities. She goes on to say that “in school, skill in mathematics is important for success; after school, mathematics competence contributes to gainful employment, income, and work productivity...” (Paulson p. 21). Therefore, it is essential that the teacher have a solid foundation themselves so that they can correctly implement evidence-based practices in the process of helping their students to become successful in math.

In accordance with the Council for Exceptional Children’s Division of Learning Disabilities Knowledge and Skills Compliance in Math, Parmer and Cawly (1997) have discovered six standards that all teachers should know before they begin their teaching careers: modeling good mathematics teaching; knowledge of mathematics; knowing students as learners of math; knowing mathematics pedagogy; developing as a teacher of mathematics; and teacher’s role in professional development. Once a teacher candidate has a concrete understanding of those six standards, they are ready to incorporate evidence-based practices into their teaching. The ironic thing about teaching is that good teachers are to never stop learning. Although these standards are meant for pre-service teachers, it can only be beneficial for veteran teachers to familiarize themselves with the standards as well. After all, there are always new strategies being created and new ideas being passed around.

Much like the article by Jitendra et al, Paulson’s article offers examples of evidence-based practices for students with disabilities. She provides suggestions such as advance organizers, modeling, guided practice, scaffolding, and distributive review. Paulson states that these practices will allow more opportunities for student responses in addition to amplified student on-task time. Advance organizers are used to capture students’ attention and connect previous knowledge to current knowledge. Modeling and guided practice are connected in that

first a teacher models the correct way for a task to be carried out then the students are instructed to practice with the help of the teacher guiding them. Distributive review involves reviewing material over an extended period of time. With the exception of advance organizers, the other three strategies are understandably effective for students with disabilities. Most students with disabilities, especially ones with recall or processing delays are not always able to think back to previous knowledge. Those that can are not always able to make the connection between previous and current knowledge. As for modeling and guided practice, they demonstrate proper procedure and then assist the student in completion until the student builds the knowledge and confidence to do it on their own. Distributive review seems the most probable in improving the mathematics achievement of students with disabilities. The continuous review of old material will refresh the students' memory enough times until they are able to process and comprehend it.

As soon as any of these four practices are in place, Paulson states that curriculum based measurements are required to monitor student progress. The data collected from the curriculum based measurements is then shared between teachers and parents to exhibit the progress that the student is making. Students use these progress reports as motivation to continue to increase their achievement. It is a little known fact that students enjoy monitoring their own progress and Paulson picked up on that and shared that knowledge with teacher candidates. In general, the level of learning increases when students, even students with disabilities, are able to make their learning more meaningful. One of the ways to do that is through self-monitoring their progress. These are the useful tips that every pre-service teacher craves before they begin their careers and although they may seem simple, they have proven to be effective.

Similarly to the previous articles mentioned, the article by Farley et al, explains how evidence-based practices go through a rigorous evaluation process as they are required to meet

high standards. However, Farley et al. focuses on the effects of evidence-based practices on students with emotional behavioral disabilities (EBD). She adds that “the National Secondary Transition Technical Assistance Center (NSTTAC) separates out studies conducted specifically with students with EBD when identifying EBPs. This is important because practices that are identified as EBPs based on research involving students without disabilities, or students with disabilities other than EBD, may not be as effective for students with EBD.” (Farley et al p. 38). This idea correlates with the philosophy of differentiating instruction. The theory that a strategy that has proven to be successful for one student or a group of students may not be effective for a different student or group of students.

This article also gives examples of evidence-based practices that will work specifically for students with EBD. Conversely, Farley et al’s article shares identical ideas with Jitendra et al’s article. Both articles state that peer mediation (or class wide peer tutoring) will be effective for the population of students that they studied; Jitendra et al with students with ADHD and Farley et al with students with EBD. In a medical dictionary these two populations of students are not replicas of each other, but they do share similar academic struggles. Both articles mention that ADHD and EBD students struggle with academic achievement, on task time, and personal behavior. Furthermore, both articles agree that CWPT widely recognized models of peer tutoring. Yet, Farley et al goes one step further, this article states that CWPT is the most researched practice for peer tutoring.

The difference between both articles is that Farley et al provides a more thorough explanation of the process of peer tutoring. The main points expressed were to use CWPT for about 20 minutes each day, three times a week, and that the teacher needs to model and practice with the students; an idea expressed in the article by Paulson of modeling and guided practice.

In addition, students as well as teachers conveyed approval for the process due to academic and social gains made by the students. Another strategy described in Farley et al's article is self-management interventions. In this practice, students are responsible for monitoring their own behavior as well as academic achievement. The National Professional Development Center on Autism Spectrum Disorders and the National Autism Center identified self-management strategies as being evidence-based practices across grade levels.

There are five different types of self-management interventions: self-monitoring; self-evaluation; self-instruction; goal-setting; and strategy instruction. Most teachers are familiar with the first three interventions, but might not be as well informed about goal-setting and strategy instruction. Goal-setting is much like a student contract. The students choose a goal and create personal guidelines to help them reach that goal. Strategy is the process of teaching students techniques that will help them move towards their goal independently. To the average special educator who works with students who have been diagnosed as having EBD, these interventions are what a student of this population needs. These students want to be completely independent and want to be in absolute control. Self-management strategies allow for this control. With a mild amount of instruction to explain the technique to the student, the teacher can allow the student to be independent and according to the research, progress towards their goal.

Farley et al lays out an identical implementation process to the one that Torres, Farley, and Cook identified in their article. The most likely explanation for this commonality is that three of the authors are the same for both articles with the addition of Walehua in the article pertaining to EBD students. An additional difference is that the Farley et al article was published in *Beyond Behavior*, a property of Council for Children with Behavioral Disorders. And the

Torres, Farley, Cook article was published in *Teaching Exceptional Children*, a product of the Council for Exceptional Children. Both magazines are highly credible in the field of special education. Furthermore, the fact that both magazines published the same implementation process indicates that it is credible.

An article by Donaldson and Zager evaluates evidence-based practices for students with high functioning Autism (HFA), Asperger's syndrome (AS), and nonverbal learning disability (NLD). It describes how students with NLD and HFA have similar characteristics and thus, would benefit from similar implementation strategies. Some of these characteristics pertaining to math include: impaired visuo-spatial ability, motor skill deficits, difficulty remembering operations throughout an entire equation, comprehending language in instructions or word problems, and organizing information on a page.

Donaldson and Zager also recommend that teachers utilize a self-regulation strategy. The one described in this article involves students completing a checklist as they complete the equation. The article mentions that "Dunlap and Dunlap (1989) found that with self-regulation, student solution accuracy increased as their mood became more positive." (Donaldson and Zager p. 43). They also suggested the implementation of goal setting, or what they call goal structure, which is connected to a rewards system for the completion of math tasks. Lastly, this article describes a concrete-representational abstract (CRA) strategy. In this strategy, students are first shown a concrete example, next move to a representation such as a picture, and then an abstract depiction of the concept. All of these strategies slow down teaching to allow the students more time to process the information that is presented to them. This population of students needs to have a step-by-step process laid out for them because they absolutely need to know what is coming up and when; any deviation from the correct process would cause a great

deal of distress for them. Furthermore, the CRA strategy will be beneficial for students with Autism because thinking abstractly is a difficult task for them. By presenting a concrete object then working through a representation, to an abstract object, the student is allowed the time to make the connection between each object. The research presented in the article by Donaldson and Zager correlates well to the typical characteristics of students on the Autism spectrum.

Scruggs et. al. wrote an article evaluating the effects of mnemonic strategies for enhancing learning and memory in students with mild disabilities. The article begins by explaining that a mnemonic is designed to improve one's memory and that there are several different types of mnemonic strategies; keyword method, pegword method, and letter strategies. In the keyword method, students are shown a picture that represents the keyword as well as the associated information. The pegword method is based on rhyming. This strategy involves numbers that rhyme with specific words and is often used to help a learner remember numbered or ordered information. It is possible for the pegword and keyword methods to be connected when unfamiliar terms are related to numbers. Letter strategies are the most common mnemonic strategies. This is the one where each letter represents a word.

This article states that “in addressing the complex requirements of content area learning, it is necessary to combine a number of mnemonic strategies to address different memory needs. Information can be organized by its familiarity and concreteness, and appropriate reconstructions created.” (Scruggs et al. p. 81). Familiar information that is already concrete can be shown using representative pictures, while information that is concrete but not familiar can be displayed through symbolic pictures. As for information that is neither concrete or familiar, acoustic representations can be utilized through the keyword or pegword methods. Letter strategies are best suited for instances where lists of information are required to be memorized.

Scruggs et al cites the research of several other researchers when he lists the disabilities that can best be addressed through the use of mnemonics. These disabilities include: learning disabilities, mental retardation, social/ emotional conditions, and aggressiveness. The article goes on to say that “mnemonic strategies likely are effective at addressing these deficits because they build upon familiarity or meaningfulness and provide verbal elaboration to enhance learning.” (Scruggs et al. p. 84). The theory behind this is that the mnemonics minimize the learning weaknesses of the students while maximizing their strengths. This has been supported through three decades of research.

Research regarding mnemonics dates back to the early 1980s; since then over 40 experiments containing more than 2,000 participants who had mild disabilities. They found mnemonics strategies to be very effective leading the Council for Exceptional Children Division of Learning Disabilities and Division for Research to label Mnemonics instruction as a “Go for it!” strategy. However, the articles notes that mnemonic strategies do not meet all school objectives, therefore, they should not be declared a remedy for all facets of school learning. Regardless of this warning, mnemonics strategies remain a significant evidence-based practice for special education teachers.

In comparison to the article by Farley et al., Ryan, Pierce, and Mooney discovered that a majority of interventions that have been used for students with emotional and behavioral disabilities has addressed behavior modification and completely ignored the academic deficiencies. The researchers of this article divided the interventions into three separate categories: peer-mediated interventions, self-mediated interventions, and teacher mediated interventions. The practice of peer-mediated intervention has resurfaced. Ryan, Pierce, and Mooney stated that peer-mediated interventions have led to dramatically positive results related

to improved academic performance for students with EBD. They also noted that large gains were seen in mathematics. Furthermore, it was discovered that regardless of what role the students played in the intervention; tutor, tutee, or sharing both roles, there were clear benefits for the students. Utley and Mortweet (1997) found that “peer-mediated interventions provide both an effective means for offsetting high teacher-pupil ratios and an effective alternative to one-on-one instruction for students with severe academic deficiencies.”

Much like with peer-mediated interventions, self-mediated interventions require that the students are responsible for the academic instruction. The only difference is that they are the only ones involved, there are no other students assisting them. The teachers are responsible for initially teaching the students how to carry out the activities, but once the students demonstrate an understanding of the concept, the teacher reduces their role to an observer. Ryan, Pierce, and Mooney also discussed the self-management strategy of Cover, Copy, and Compare. Jitendra et al. also mentioned this strategy. The basis behind this practice is providing the students immediate feedback. Both articles explain that the cover, copy, and compare strategy increases student accuracy leading to greater academic gains. An additional commonality is that the authors of both articles said that CCC works best when a reinforcement plan accompanies it.

Unlike self-mediated interventions, teacher-mediated interventions focus on stopping problem behaviors before they occur. The goal is to intervene prior to the negative behavior affecting the student’s academic performance. Teacher-mediated interventions are designed to help teachers identify what reinforcers work best for students and use those to extract appropriate responses to instruction. Despite these interventions being implemented for only a short period of time, they were convincingly effective. However, other research suggests that interventions are not maintained long enough to see significant and sustainable improvements for students

with EBD. It is also noteworthy to mention that the studies discussed in this article did not include a random enough sample and contained too few amount of students. Therefore, it is problematic to assume these findings apply to all students.

For years researchers have disagreed on the relationship between a student's behavior and their academic performance. Only recently, have connections been made between behavior and academic performance. This is arguably a shocking conclusion. The most plausible explanation is that several years ago when little was known about special education, especially EBD students, it can be understood how teachers assumed that the students were not doing well in school because they were being defiant or noncompliant. However, this was often times not by the choice of the students. The same assumptions were seen for students with Tourette syndrome and ADHD. Teachers and people in general did not know much about these disabilities and thus, were unable to understand their relationship to academic performance. It is for this reason that when gains are seen for these students, people outside of the education world are astonished at academic improvements for EBD students. Only recently have scientists begun to understand Autism and its related disabilities. Unfortunately, it could be quite some time before they begin to understand EBD individuals.

Although there has been a significant amount of research for students with Autism, little has been done to integrate the results into school curriculums. Given that individuals with Autism spend an overwhelming majority of their intervention time in schools, one would assume that there would be more urgency to get research results into school programs. There has also been a significant lack of social validity for the interventions. The article by Callahan, Henson, and Cowen examines this deficiency. They look at whether the goals on the interventions are socially approved. The authors look particularly at the appropriateness as well as the

acceptability by practitioners and care givers of the practices used in clinics, homes, and classrooms. This is something that is frequently overlooked. When educators create strategies to be implemented into classrooms that are intended to help improve students' academic performance, they do not consider whether or not the practice will be acceptable to the parents and other professionals that work with those children. It could be one way to develop a more cohesive bond between the parents and the school if parents were notified and educated on the practices that will be used to increase their child's academic proficiency.

What Callahan, Henson, and Cowan found is that there was an incredible amount of support from parents, teachers and administrators to implement evidence-based interventions into public schools. They stated that "the research confirms there is widespread social validity for basic intervention strategies which have served as the traditional, legal, and ethical foundations of high-quality special education programming." (Callahan, Henson, and Cowan p. 689). Just like with general education parents, parents of students in special education programs want the best for their children. They also want to be informed on what is being done to help their children to reach their fullest academic potential. While this study did not test the effectiveness of evidence-based practices in special education, specifically Autism programs, it did address the lack of social validity that other research pertaining to evidence-based practices in special education have left out.

Research emerged 30 years ago regarding the effectiveness of explicit instruction in teaching mathematics. Explicit instruction involves advance organizers, teacher modeling and guided practice, independent practice, and then finally, maintenance checks. Paulson also mentioned these steps in her article pertaining to prospective teacher candidates. However, some educators have suspicions on whether the information discovered 30 years ago is still pertinent

today. Research conducted by Kroesbergen and Van Luit (2003) concluded “that explicit teaching was more effective for students with special needs than reform-based instructional practices for the learning of basic mathematic skills.” (Miller and Hudson p. 48). The authors of this article also mentioned concrete-representation-abstract.

Similarly to the articles by Donaldson and Zager as well as Paulson, Miller and Hudson clarified that CRA involves using manipulatives to represent a concept. They stated that three lessons in the concrete stage and three lessons in the representational stage with each lesson lasting containing about 20 problems is necessary for students with disabilities to understand the mathematics concept being taught. Mastery is achieved in the representational stage once the student reaches an 80% accuracy during independent practice. Then in the abstract stage, the use of manipulatives is prohibited. The researchers indicated that CRAs have helped students with learning disabilities master several mathematics concepts such as coin sums.

Miller and Hudson also mentioned a second type of knowledge that they say is needed for students to master in order to become proficient in mathematics. This second type of knowledge is procedural knowledge. In the article, procedural knowledge is defined as “the ability to follow a set of sequential steps to solve a mathematical task. (Bottage, 2001; Carnine, 1997; Goldman et al., 1997)” Procedural knowledge can be used to solve word problems, or other computational problems. It can also be used to solve real-world problems such as making change. Following a procedure is a difficult challenge for any student with a disability. Many diagnosed disabilities contain a processing delay that inhibits a learner from hearing, processing and storing information in a systematic way. This is why strategies such as concrete-representational-abstract have become so effective. As all three articles that relate to the topic

mention, breaking things down so that students with disabilities are able to relate the new content to something that they already have a solid understanding of makes the learning much easier.

One thing that Miller and Hudson have done differently is to clarify each step in teaching students with learning disabilities to develop procedural knowledge. First, the strategy should contain a consecutive set of steps that leads to the answer of the problem. Next, the steps must be generalized. Then, each step should direct the student to execute an overt action, use either a cognitive or metacognitive procedure, or implement a rule. Fourth, the language should be short and brief. Lastly, mnemonic devices can be useful in helping students remember the steps. Overall, it is essential that special education teachers arrange for a balanced instruction across all math standards.

Teachers are under a great deal of pressure to help their special education students meet their academic yearly progress goals. The strategies that have been proven to be evidence-based practices have all gone through an extensive evaluation process and have been deemed to be effective for students on a vast array of criteria and settings. While any given strategy may not work for all students with disabilities, the researchers mentioned have connected certain evidence-based practices to specific disabilities. Granted, it is rare that a teacher will have a class of all EBD students, but that does not mean that peer-mediated instruction will not work for students with learning disabilities. It simply means that even though special education teachers will continue to have to differentiate instruction, they now have a set of practices that have been comprehensively tested and proven to be effective.

After completing the literature review portion of the thesis, I have selected four evidence-based practices that were mentioned in several articles during the course of my research. In

summation of all the articles that were used in this literature review, the consensus is that there are several strategies that work for specific disabilities. On the other hand, while two or three articles discussed the same strategies, which were the extent of the overlapping. No articles stated that any given strategy would work for all disabilities across the board. Therefore, in my original research I have selected four evidence-based practices to test with students whom I work with. Those strategies are: concrete-representational-abstract, self-regulation, peer-tutoring, and cover, copy, compare. I may also test computer assisted instruction with my students as well.

Since I work with dually diagnosed students, I am looking to see what strategies will work best. All of the articles discussed specific disabilities when testing their evidence-based practice, however, students who are dually diagnosed will pose an additional challenge to testing each strategy's effectiveness. It is very possible that the strategies will work as described. It is also possible that the strategies fail to increase the student's academic performance. Something that I will be especially looking for is if any strategies will work for all of the students in the sample group. This would provide credibility for evidence-based practices working across the board for special education as well as general education.

## **Part II**

### **Self-monitoring/ Self-regulation**

The first strategy that I began testing was self-regulation. In the self-regulation strategy the students monitor their own work and progress. They are responsible for checking their work for errors and correcting those errors without being prompted by a teacher. In this strategy, the students were given a check list that they were required to reference when they completed a math

work sheet. The check list asked them to: 1) double check their answers, 2) erase their errors, 3) re-read the question/ directions, 4) re-answer the problem, and 5) raise their hand. The purpose of testing this strategy on students with disabilities is that often times; the students will rush through their work and not notice the details in the directions or questions. Thus, they will occur what can be called “careless” errors on answers that they should have gotten correct if they had slowed down. Therefore, with this strategy, they will learn to slow down and check their work. Also, the self-regulation strategy will teach the students to check their own work before they hand it in to a teacher.

At the onset of experimentation using this particular strategy, they students were given minimal instruction on how to check their own work. The reason for the lack of instruction is because the students had been given instruction on how to use a check list in other content areas; such as language arts. Furthermore, I wanted to also test their ability to fully self-regulate themselves. I wanted to see if they could not only read the directions on their worksheets, but also read the steps on the check list. For those students who have difficulties with reading text, the check lists also contained picture cues representing the words that they depicted. Since the check lists were laminated, the students were able to use a dry erase marker and mark a check in the box next to each step after they have completed that particular step.

Student “A” was the first to participate in the experiment of the self-regulation strategy. With little instruction of the elf-regulation strategy, the student raised his hand indicating that he was finished with the worksheet that he was working on. At that time I told the student to go back over his work to double check for errors. After I told him to double check his answers, I could see the student re-reading the directions as well as the questions. The worksheet that he was working on required him to practice telling time. On the top half of the worksheet, he had to

look at four different analog clocks and write their times underneath the specific clocks. Then on the bottom half of the worksheet, he had to look at all four digital clocks and number them in chronological order from one to four. After he re-read the directions and questions, he said that all of his answers were correct and they were. Next, the student began working on a second math worksheet. When he was finished, I again told him to double check his work. He immediately said that he had done that specific worksheet before so he was confident that his answers were correct. On this worksheet he had to look at a line of characters and answer what place they were in; first through seventh.

On day two when he was asked to double check his work, he looked at the worksheet and after about three seconds said that he was finished double checking. Knowing that this was not enough time to double check that particular worksheet, even if he had done it before, I told the student to double check it again. Upon a second round of double checking, he re-read the questions and changed a couple of his answers. When he was finished double checking his work for the second time, he raised his hand and turned in the worksheet. All of the answers were correct. This is one example in which a student rushes through his/ her work in order to be done sooner. If this individual student had not been told to double check his work a second time, he would have handed it in with two errors and been asked to fix them. For students with disabilities, being told that they are wrong and/ or have to do something again causes them a great deal of frustration. Had this specific student needed to fix a couple of his answers, he would have “shut down” and refused to fix his errors. However, since he was strongly encouraged to double check his answers again and reminded to go more slowly, he found his errors on his own and avoided the frustration that could have occurred.

Student “B” is a very good candidate for the self-regulation strategy. He has a tendency to rush through his work and make several errors that could have been easily avoided if he had slowed down, taken his time, and carefully read the directions. On one occasion, student “B” had a worksheet in which he was practicing ordinal numbers. He was required to read directions that instructed him as to what flower on the paper he had to color and what color he was supposed to use. Shortly after being given the worksheet, he got up from his seat and said that he was finished. Before I even looked at the worksheet, I told him to go back to his desk and double check his work. I reminded him to re-read the directions at the top of the page as well as each question to make sure that he colored the correct flower the correct color. Since this student has a history of going back to his desk and immediately turning around while saying that he fixed all his errors, I told the student that double checking that worksheet would take more than five seconds.

The student again got up from his desk and brought his worksheet to me. The directions asked him to color five specific flowers, five specific colors. The student returned with all ten flowers colored in. The first five flowers were not even colored the correct colors. When given the benefit of the doubt, it is understandable that the student could not fully follow all of the steps of the check list because step two says to erase the errors, but it is not possible to erase marker. Therefore, I gave the student a brand new copy of the same worksheet to re-do. The student came back moments later with only four out of the five questions answered and one of them is incorrect. The student was again told to take the paper back to his desk and double check his work. I reminded him to look at the check list before he said he was finished. He then asked for me to tell him which questions were incorrect. I told the student that part of self-regulation is that he checks his work on his own before a teacher checks it.

At the same time, student “B” also brought up a second worksheet in which he was required to answer single-digit addition questions, cut out the answers and glue them in the column according to the answers( either 5, 6, or 7); something that is well within this student’s ability range. Six out of the nine answers were glued in the wrong column. At this time, in an effort to help the student self-regulate himself and his work, he was asked to write the answer to each equation next to the problem before re-gluing them. The student went back to his desk, but repeatedly brought that same worksheet back to me without changing or even erasing any of the answers. This was an indication that the student either did not understand or was not following the check list. In an effort to ensure that the student understood what the directions said and also to demonstrate how to follow the self-regulation check list, I asked the student to verbally tell me the answers to the questions that he got wrong. Then I told him to go back to his desk and fix the ones that were in the wrong column. When he brought the worksheet again, he had all of the equations in the correct column.

As for the flower coloring worksheet, the student re-read the first two questions and placed a check mark above the flower that the question asked him to color. However, one of the two flowers that he placed a check over was still wrong. I then asked the student to read the directions out loud to me. At first he read “color” as “circle,” but then corrected himself. He read the remainder of the question with minimal assistance from me. I then asked him to point to the flower that the question asked him to color. This showed me that the student had some difficulty deciphering the words in the questions, but once he knew what the questions said, he knew exactly what they were asking him to do.

Student “B” completed a third worksheet on this day. Like the previous two worksheets, there were several errors on this worksheet. This worksheet contained four wheel-like objects in

which there was a number followed by an addition sign at the center then a ring around it sectioned off with a single number in each section, and a third ring that was sectioned off with blank sections in which the answer from the first two rings should go. When the student brought this worksheet to me, I asked him if he double checked his work before he brought the worksheet up to me, he insisted that he did. I then told him that he had many errors on the worksheet. He refused to re-check his work and maintained that all of his answers were correct. The student brought up the worksheet several more times without re-checking his work. It was clear that he was not following the check list because the worksheet did not have any eraser marks on it, thus he did not erase his answers and therefore, could not have re-read and re-wrote his answers.

Student “C” had a much different reaction to the self-regulating strategy. This student was working on a worksheet in which he was practicing skip counting. While he was working, he noticed that he had an error. Furthermore, he realized that the error was caused by a typing error on the worksheet and not a mistake on his part. The student raised his hand and said that he thinks there might be a mistake on the paper, but he wanted to be sure. On a second page in which he was asked to count hundreds, tens, and ones, he raised his hand, said he was done, and I told him to double check his work. As I walked over to him, I saw him erasing and fixing one of his answers. On a third worksheet, after he said he was finished and was told to double check, he went through each individual question as if he were starting with a blank worksheet. He determined that all of the questions were correct and in deed they were. It can be said that this student is at a higher ability level compared to the other two students and that is why he had much better results with the self-regulation strategy. However, all of the students have the ability to follow the check list either by reading the words or looking at the pictures.

When student “D” began working on the self-monitoring strategy, he was working on an addition worksheet. Although he was using a calculator to complete the worksheet, he did get one question wrong. As he went back to his desk, I went over the check list for making corrections with him. I went step-by-step through the check list with him so that I could be sure that he understood what to do at each step. Once he showed a solid understanding of what to do at each step of the check list, I walked away to let him begin working on his second worksheet for that math period. After he completed the second worksheet, he asked me if he could do the check list by himself. I gave him permission to proceed independently. While he did not get any questions wrong on this particular worksheet, he followed each of the steps of the check list appropriately.

Student “B” was working on a worksheet in which he was practicing single and double digit addition. Out of twenty questions, the student got one of the questions wrong. I explained the making corrections check list to him briefly and modeled it before demonstrating through guided practice. The student made the correction to the question that he got wrong, he moved onto the next worksheet. Although the student demonstrated that he understood the making corrections check list, he seemed to be rushing through the check list and not fully re-reading the directions and questions.

While the self-monitoring strategy does help the students to develop a greater sense of independence which is a desired trait for students with disabilities, it is a difficult concept for them to understand. Their processing delays hinder their ability to think through each of the steps of the check list. Furthermore, two of the students use calculators which dramatically lessen the amount of mathematical errors that might occur. As for the group as a whole, they respond well overall when I remind them to double check their work. Self-monitoring teaches

the students to slow down while working. When paired with “double checking” it teacher the students to take their time when completing their class work and to check for their own errors as well as to fix them. Lastly, it works on developing a greater sense of independence for them.

### **Concrete-Representational-Abstract**

I decided to test this strategy by using plastic, replica coins to teach the students to be able to count up to \$5.00 in mixed coins. I am using plastic, replica coins at the start of this strategy because they are a concrete object that the students will be able to manipulate while they are counting. The students will be able to touch the coins that they are counting and make more meaning out of the task. In front of each student are four separate piles of coins, each pile containing all the same coins (i.e. pennies in one pile, nickels in another, etc.). I began by showing student “B” a card with an amount of money written on it. I asked the student what amount the card was asking him to count to with the coins that were available to him. Once the student told me what the amount was on the card, I asked him to show me that amount in coins. The student got the first amount correct, but then got the next two incorrect. At this point I decided to have the student count out amounts that I figured would be simpler for him. I then asked the student if he knew how many cents were in one dollar. Since he was unsure of the correct answer, I told him that 100 cents equals one dollar. I then asked him if he could count out 100 cents with like coins (i.e. all quarters, all dimes, etc.). He kept getting confused with counting the quarters, but was able to count up to 100 cents with the other coins. Once he was completed, I showed him how to count out 100 cents in quarters. Next, I had the student count out the 100 cents in quarters with me. He continued to have trouble understanding that 100 cents equals one dollar, but was still able to count out 100 cents in pennies, nickels, and dimes. Although, when he got to the end, he would always say 100 instead of one dollar.

During the mixed coin counting, he was able to count sets of coins without using quarters after he continually counted quarters as five cents. However, he did self-correct himself on several occasions, most of the time it was with the quarters. When he counted sets of mixed coins, he would frequently start with the nickels, then dimes, and pennies last. I encouraged him to start counting with the coin(s) that are worth the most, but he repeatedly assumed that the largest coin in that set was worth the most. Other times, it seemed as though he preferred to start with the nickels because they were the largest since he was very hesitant to use quarters. However, even though he refused to use quarters when he counted out the amounts either written on the cards or the amounts that I verbally told him as well as wrote on a white board, he got most of the amounts correct. I pointed out to the student that just because the nickel was the largest coin that he selected into his set of coins doesn't mean that it is worth the most. I told the student that if he didn't want to use quarters, he should start with dimes, then nickels, then pennies. In a set of similar coins, the student was able to count out the correct amount almost every time, but when there were mixed coins, he struggled going from one type of coin to a different one.

I began with student "D" much the same as how I started with student "B." I began having him show me the coins that added up to the amounts on the cards. Since he got all of the cards correct, I had him show me one dollar using all of the same coins. The student was able to do this correctly as well. It is worth noting that the student did not always use the least amount of coins possible to add up amounts over one dollar. Next, I asked him to add up to two dollars. He used several coins to do this (not the least amount possible). Student "D" did well with amounts under one dollar, but showed signs of struggling with amounts over one dollar. Like student "B," student "D" insisted on counting the coins out of order. Meaning, he did not always

start with the coin that was worth the most out of the coins that he selected from the respective piles. He too preferred to start with the coins that were the largest and work towards the coins that were the smallest.

I continued working with student “A” on counting sets of mixed coins. I used the same set of 10 flash cards that I used in previous trials. Below is a chart of student “A’s” results.

43 cents: 4 dimes, 4 pennies	56 cents: 1 nickel, 6 pennies	65 cents: 1 nickel, 6 pennies
60 cents: 6 dimes	10 cents: 1 dime	12 cents: 1 dime, 2 pennies
70 cents: 7 dimes	25 cents: 1 quarter	63 cents: 6 dimes, 3 pennies
	15 cents: 1 dime, 1 nickel	

Figure 1

When I asked student “A” to count out fifty-six cents for me, he pulled out the nickel at first, followed by two pennies and then realized that he had the wrong coins after he re-counted the coins that he had pulled out in front of him. He then put all of those coins back into their respective piles and pulled out new coins. This time he pulled out a quarter and a penny. It is worth noting that the student is still using plastic, replica coins. I asked the student to re-count the coins that he had pulled out in front of him. This time he counted out twenty-six cents correctly. Seeing that the student was having trouble counting the fifty-six cents and knowing from the trial pictured in figure 1, I knew the student knew that one quarter is worth twenty-five cents. Therefore, I asked the student to show me fifty cents. He then counted out five dimes. Then to continue to fifty-six cents, he added six pennies. Although the student did technically get the amount correct, he did not use the least amount of coins possible.

Next, I asked the student to re-do the 65 cents. He counted out six dimes and five pennies. Once again, I asked the student to show me fifty cents, but this time, I asked him to show me the fifty cents without using any dimes. After thinking of how he could complete this task, he pulled out ten nickels. I then asked him if it was possible to use quarters to get to fifty cents, he thought about it for a moment and then proceeded to pull out two quarters. I then asked him if he could show me one dollar, however, before I allowed him to select his coins, I asked the student if he knew how many cents are in one dollar. He did not know the correct answer, so I told him that there is 100 cents in one dollar. Then he immediately picked out four quarters at a time until he reached eight dollars. It is assumed that the student would have kept going with his groups of four quarters, but he only had enough to make exactly eight dollars out of only quarters. After he was finished counting out the eight dollars in quarters, he proceeded to count out one dollar in dimes.

In the next phase of this work session, I wrote a random amount on a white board and asked the student to show me that amount using the plastic, replica coins. He pulled out a combination of mixed coins and started counting with the coin that was lined up farthest to his left, but he quickly became confused with the mixed set of coins that he created. So I asked him to separate his pile of mixed coins into different piles, each with the same coins (pile of pennies, pile of nickels, etc.). I then asked him to show me what coin is worth the most. He correctly pointed to the pile of quarters and then counted out the two quarters in his pile of coins. He said the dimes were next in the order, but I could tell that he was getting confused with all the thinking/ adding in his head. I helped him go from the quarters to the dimes, then skip count the nickels and lastly the pennies. With my help he correctly counted the amount of change he had

in the pile in front of him. He definitely knows which coins are which and how much they are worth. Similarly to the other students, student “A” enjoyed using the replica coins.

In a second trial, Student “A” was asked to count out the amounts indicated on the designated set of cards. His answers are recorded in the chart below:

56 cents: 1 nickel, 1 penny	65cents: 6 dimes, 5 pennies	15 cents: 3 nickels
63 cents: 6 dimes, 3 pennies	25 cents: 1 quarter	70 cents: 7 dimes
12 cents: 12 pennies	10 cents: 1 dime	60 cents: 6 dimes
	43 cents: 4 dimes, 3 pennies	

Figure 2

Since the student was showing an overall command of the amounts on the cards, I decided to ask the student to show me amounts above one dollar. Therefore, I wrote the amount of money that I wanted the student to count to on a white board. The student’s results are displayed in figure 3 below.

\$1.25: 5 quarters	\$2.25: 9 quarters	\$4.50: 18 quarters
\$1.36: 4 quarters, 3 dimes, 6 pennies	\$3.71: 12 quarters, 7 dimes, 1 penny	
\$4.94: 16 quarters, 9 dimes, 4 pennies	\$2.67: 8 quarters, 6 dimes, 7 pennies	
\$1.45: 4 quarters, 4 dimes, 5 pennies	\$3.16: 12 quarters, 16 pennies	
	\$2.35: 8 quarters, 3 dimes, 5 pennies	

Figure 3

Now that the data supports the fact that the student knows how to successfully count monetary amounts up to five dollars, I am going to turn my attention towards using the CRA strategy to help the student learn to use the least amount of coins to reach a designated amount of money. Student “A” as well as other students in this research study are able to correctly count

out any amount of money using a mixed set of coins, but they all favor using dimes and pennies to reach any amount.

For the first amount of 56 cents, I kept asking the student if there was a way to get to fifty-six cents by using less coins than what he used in the second trial. I wanted to see if he could realize on his own that it is possible to get to fifty cents without needing to use five dimes and six pennies. The student thought about it for a moment, then pulled out two quarters. I then asked him to count out six cents. He began to count six pennies, but then I asked him once again if there were fewer coins he could use to get to six cents without using six pennies. He took a few seconds to think about it, and then pulled out a nickel and a penny.

Next up was sixty-five cents. When I asked him how he could get this amount using fewer coins, he counted six dimes and one nickel. I then asked him if he could get sixty cents by using fewer coins than the six dimes that he used. I asked the student if it was possible to use quarters to get to sixty cents or at least close to it. He said it was possible but was unsure of how to do it. I suggested that he start with one quarter and add one at a time until he had more than sixty-five cents. At first he counted out four quarters and one nickel. Going from two quarters to the third quarter was very confusing for this student. He could count the first two quarters correctly, but then would count the third quarter as five cents. I then proceeded to show the student one quarter at a time and with each quarter added, I asked him how much he had. After two quarters, I told him that he needed ten more cents so I asked him how he could do that. He counted out a nickel and five pennies. I then told the student that he now had sixty cents but he needed to get to sixty-five cents. When I said this, the student became confused, particularly with how he would get from sixty to sixty-five cents. He confused himself regarding the value

of a nickel. I asked him to clarify how much a nickel is worth and he correctly told me that a nickel is worth five cents.

Student “D” had similar results in his trial. When asked to count out fifty-six cents, he started with a nickel and six pennies, then corrected himself when I asked him to count his pile for me one more time. He said “fifty” to himself as he pulled out two quarters followed by six pennies. Next was sixty-five cents, which he immediately pulled out two quarters, one dime and one nickel. When he got to the next amount of fifteen cents, he pulled out three dimes, then put one back. He looked at me indicating that he was finished. I told him that the coins that he had in front of him added up to the incorrect amount, so he pulled the third dime back out. In an effort to check the student’s understanding of what coin he was working with at the moment, I asked him what coin he had in front of him and he correctly told me “dime.” When I asked him how much a dime is worth, he said “ten cents.” Next, I asked him to count the three coins that he had in front of him. After realizing that he had too much, he put one dime back and counted the two remaining dimes. He counted one more time and put one more dime back and pulled out a nickel before counting to the correct amount.

The next amount that the student was asked to count was sixty-three cents. The first coin that the student decided to use was dimes. However, he was counting the dimes as five cents. He then pulled out three pennies. After realizing that he had an incorrect amount, he corrected himself with two quarters, one dime, one nickel, and three pennies. I then told the student that he had too much money there so he removed the dimes and I told him that now he did not have enough in his pile. He then returned the nickel and pulled the dime back out to give himself the correct amount. The next amount was twenty-five cents. The student pulled out two nickels. He

then flipped the flash card over to peak at the answer on the back and saw that according to the card, he needed two dimes and one nickel.

When the student was asked to count out seventy cents, he pulled out seven nickels. After I told him that he was incorrect, he put all of the nickels back and pulled out two quarters and one nickel. I told the student that he was getting closer, but was still a little off so he added one dime. Then, after some thinking, he pulled out two more dimes. At that point I told the student that he now had too much money in front of him so he put one nickel and one dime back. Once he made this correction, he was left with the correct amount. The student was then asked to count out twelve cents. He pulled out two nickels and two pennies. As soon as he was asked to count out ten cents, he immediately put the two pennies back into their proper pile. Unlike student “A,” when student “D” was asked to count out sixty cents, he selected two quarters and one dime. Although the student thought about using nickels and a second dime, he corrected himself before he said that he was finished. The last amount for the flash cards was forty-three cents, which the student quickly pulled out four dimes and three pennies.

As I did with student “A,” I wrote monetary amounts above one dollar onto a white board and asked student “D” to count out the amounts on the board which are the same amounts that student “A” counted out. First was \$1.25 which the student counted out five quarters. Next, for \$2.25 the student counted out nine quarters and placed them in front of himself. The student also answered \$4.50 correctly with all quarters. These first three amounts were chosen to test the students’ knowledge of quarters. When the student was asked to count out \$1.36, he pulled out four quarters and three dimes for his final answer. Then, he pulled out five pennies. I asked the student to double check the coins that he grabbed a sixth penny and said he now had the correct amount.

The next amount was \$3.71. First the student selected twelve quarters, two dimes, and one nickel. I told the student that he had too much money so he pulled out three more nickels. I then asked the student what coin he had and he told me that he had a nickel. Therefore, I asked him how much a nickel is worth and he told me “five cents.” At that point, I reminded him that he needed to count up to seventy-one cents. The student proceeded to put all the coins that he already had back into their respective piles and then selected seven dimes and one nickel. When the student was asked to count out \$4.94, he chose to use sixteen quarters, four dimes, and one nickel. I told him that he had an incorrect amount in front of him. I asked him how to get to ninety-four cents first, and then get the four dollars later. To get to ninety-four cents, student “D” pulled out three quarters and said that that added up to seventy-five cents. Next, I asked him how he could get from seventy-five cents to eighty cents. He selected a nickel and then four pennies. Seeing that he was having trouble getting from seventy-five to eighty cents, I counted with him. Once he got to eighty cents, he knew that a dime would get him to ninety, plus the four pennies and the sixteen quarters gave him a total of four dollars and ninety-four cents. Student “D” did the next question of \$2.67 with minimal difficulties. He chose eight quarters, six dimes, and seven pennies.

The next amount was \$1.45. The student selected four quarters, one nickel, and four pennies. I kept reminding the student to count out the forty-five cents first. He pulled out one quarter, two nickels, and four pennies. I asked him to count the coins that he had and he realized that he had the wrong amount. He then put all the coins back and pulled out four dimes and five pennies. The student was then asked to count out \$3.16 in mixed coins. While still using the plastic, replica coins, he counted out twelve quarters, two nickels, and six pennies. The student started to pull out a third nickel as he began to say “fifteen,” but he then put it back and pulled

out the six pennies. The last amount that the student was asked to count out was \$2.35. He chose to use eight quarters, one dime, and one nickel. I told him that he was close to the correct answer so he pulled out a second dime. After I told the student that he was incorrect, he put the nickels back and pulled out three dimes and five pennies.

Overall, student “D” was very proficient in using quarters to get to the dollar amounts. However, he was very hesitant to use the quarters to get the change/ cents amounts. He preferred to use dimes and pennies when counting the change portion of the amounts. He was also good at self-correcting himself after I told him that he was incorrect. Although, he did have some difficulty counting from one coin to a different coin (ex: dime to nickel), much like student “A” did as well.

I began working with student “E” on sorting coins according to the value of each coin. This particular task requires the student to know the value of each coin and place each coin into a container labeled with its corresponding value. The coins that were used for this task were the penny, nickel, dime, and quarter. The student began with a pile of mixed coins in front of him. He then needed to sort the coins into their proper containers according to the coin’s value. This task also used replica coins. On his first trial, he correctly sorted the coins into their respective containers with 100% accuracy. Next, I asked the student to complete the task with pictures of animated coins like in a standard clip art document. At one point he put a picture of a nickel into the twenty-five cent container, then immediately self-corrected himself by picking it out of the twenty-five cent container and correctly placing it into the five cent container without any prompting. Once again, he scored a 100% accuracy with the picture coins. Furthermore, he showed little hesitation and completed the task in a short amount of time.

Next, I asked student “E” to sort a pile of pieces of paper, each labeled with the name of a coin on it. He was asked to sort the pile of coin names into the same containers that he placed the plastic, replica coins and the pictures of coins into. In this task, Student “E” correctly sorted the papers labeled dime and penny. However, while he put all the papers labeled quarter into the same cup and all of the papers labeled nickel into the same cup, he got the two cups switched. He put the papers labeled quarter into the five cent container and all of the papers labeled nickel into the twenty-five cent container. When asked how much a nickel is worth, the student responded “ten cents.” I then asked him how much a quarter is worth; he responded “a dime.” Yet, when he had the replica coins and the animated coins, he had a 100% accuracy rate with matching the coins to their respective values.

Seeing as how student “E” did very well with the animated pictures of coins, I decided to test student “A’s” proficiency with them. His trial results are displayed in figure 3 below.

43 cents: 4 dimes, 3 pennies	60 cents: 6 dimes	10 cents: 1 dime
12 cents: 1 dime, 2 pennies	70 cents: 7 dimes	25 cents: 1 quarter
63 cents: 6 dimes, 3 pennies	15 cents: 1 dime, 5 pennies	
56 cents: thought about using quarters but changed his mind, 5 dimes, 6 pennies		
65 cents: 6 dimes, 5 pennies		

Figure 4

Since student “A,” as well as the other students in this classroom, this student has a strong without using dimes. For this trial, I only asked the student four questions because I wanted to conclude testing before he entered the frustration stage. His results for this trial are displayed below.

43 cents: 8 nickels, 3 pennies

60 cents: pulled out 6 nickels but put them back after I asked him to re-count. The then pulled out 11 nickels, and 5 pennies.

10 cents: first he started to pull out 10 pennies, but then switched to 1 nickel and 5 pennies

12 cents: 1 nickel, 7 pennies

Figure 5

In an attempt to narrow in on using quarters, I asked student “A” to show me seventy cents. However, in this solo trial I supplied the student with only seven quarters, four dimes, and four pennies. When the student was given permission to begin, he pulled out six quarters. He counted the first two quarters by twenty-five, but then counted the remaining quarters as five cents each. I provided the student with some assistance in so far as I asked him if he could use one quarter to get to seventy cents, then two quarters. At the third quarter, which I helped the student add to get seventy-five cents, I asked the student if we would be able to use that quarter. The student had a confused look on his face before hesitantly saying “no.” Next, I reminded the student that we were at fifty cents and needed to get to seventy, so we needed twenty more cents. The student then pulled out two dimes.

Although I knew that student “A” had not mastered the representational stage, I wanted to see where he was ability wise on the abstract stage. He did the first four with no problems; seventy cents: seven dimes, twenty-five cents: one quarter, sixty-three cents: six dimes and three pennies, and fifteen cents: one nickel and ten pennies. At this point, I was convinced that even on the abstract stage, student “A” was proficient with the use of dimes due to his preference to use them. Therefore, I removed all of the pieces of paper that were labeled dime. I continued the trial by asking the student to show me fifty-six cents. At first he pulled out six pennies, then

four quarters. As he did with the animated picture coins and the replica, plastic coins, he counted the first two quarters by twenty-five, but then counted by five for the next two quarters. I then asked him to show me fifty cents. He pulled out two pieces of paper labeled quarter and slid them along the table to me. Then, I asked him to show me six pennies. On the flash card, I covered up the “6” first, then the “5” as I tried to break down the number for him. The next flash card was sixty-five cents. Rather than asking him to count out the amount on the card, I broke down the number for him. I first asked him to show me sixty cents. He tried, but then said he did not know how to get sixty cents without using dimes. Therefore, I asked him to show me fifty cents instead. The student immediately pulled out two quarters. Then I told him that he now needed to get to sixty cents. I reminded him that he only needs ten more cents. He first started to pull out a nickel and five pennies, but he put the pennies back and decided to use a second nickel. Once he realized that he had sixty cents, I asked him to show me five cents and he quickly pulled out five pennies.

As with the other two students at this point in the research study, student “A” has a very difficult time counting more than two quarters. However, he and the other students know that four quarters equals one dollar. Nevertheless, he prefers to use dimes, but when dimes are not available, he resorts to doubling the nickels. When this student has assistance in breaking down the amounts into smaller portions, he has a much easier time reaching the correct amount.

### **Copy, Cover, Compare**

As I begin to wrap up the experimental phase of this research study, I want to see if using concrete objects has had any positive effects. I also want to implement the copy, cover, compare technique which research has stated to be evidence-based. The first student to begin testing this

strategy was student “E.” I began by asking him to point to the pile of pennies, then nickels, followed by the dimes, and quarters. Then, I asked him to show me the pile that is worth one cent, five cents, ten and lastly, twenty-five cents. Next, I asked the student if he could show me one cent. Since he was able to do this correctly, I asked him to show me two cents, which he did correctly as well. When I asked the student to show me three cents, he showed some difficulty at first, and then proceeded to pull the entire pile of about twenty pennies closer to him. He then pointed to random pennies in the pile as he counted “one, two, four.” Therefore, I reduced the pile to ten pennies and counted to ten with the student. Since he was having trouble counting to ten, I reduced the pile further to five pennies. I then worked with the student on counting the five pennies one at a time. We counted the pennies by cents (ex: one cent, two cents, three cents, etc.). However, the student had a hard time counting past two cents. The student would repeatedly count “one, two, four.” I would ask the student to show me different amounts from one to five cents. As before, he was able to show me one cent with no problem. Although there was some hesitation when I asked him to show me two cents, he was able to do this correctly. Even with hand-over-hand assistance, he had trouble counting more than two cents.

Next, I showed the student money flash cards with the amounts of one, four, and five cents on them. The flash cards had the number and word forms of the amount on one side and pictures of coins needed to get to that amount on the other side. I taught the student to match the replica coins to the pictures of the coins on the flash cards by placing a replica coin on top of the coin picture. I repeatedly showed the student each card with the amount and words side first. Then, I asked the student how much the card was asking him to count to. He was able to tell me the correct amount indicated on the card with no problems. Next, I would flip the card over and ask the student to match the replica coins to the pictures on the cards. After he matched the

coins, I asked the student to tell me how many cents he had on the card. He was able to tell me the correct amount. Then, I would remove the coins from the cards and ask him to tell me how much money he had in front of him. Even though I still had the picture side of the card facing up, the student was not always able to tell me how much he had in front of him. If it turned the card over to show him the number and word side, he could correctly tell me the amount. When I would still have the picture side face up, and the student would tell me the wrong amount, I asked him to look at the card to compare. However, the student was not able to make the association between the replica coins when they are on the card versus when they are off of the card.

Since student “A” was much further along than student “E,” I decided to work with him on using the least amount of coins to reach a designated amount. To do this, I will use the copy, cover, compare method. When the student was asked to count to forty-three cents without using dimes, he resorted to using nickels instead. I used the copy, cover, compare method with this student by showing him the answer pictured on the back of the card. After I showed the student the answer, I asked him to count the coins and add them up as I pointed to each coin. Then, I flipped the card over and asked him to get to forty-three cents the way that the card showed him. He correctly pulled out one quarter, one dime, one nickel, and three pennies. For sixty cents, the student looked at the back of the card for a few seconds, then flipped it back over and started pulling out coins. He chose one quarter, three dimes, and one nickel.

When the student was asked to count to ten cents using the fewest coins, he started to pull out a dime, but then looked at the back of the card which said one nickel and five pennies so he answered this way. After I checked his answer, he asked me if it was correct. I told him that the card showed him one way, but there are many ways he could count to ten cents. I asked him if

there was a way to count to ten cents using fewer coins. The student thought about it for a second or two, and then pulled out two nickels. He then said he could also use only one dime. For twelve cents, the student just added two pennies to the dime without needing to look at the picture answer on the back of the flash card. He also answered twenty-five cents without assistance as well by selecting one quarter.

The next amount that the student was asked to count to was seventy cents. At first, he pulled out seven dimes. I asked him if there was another way to get to seventy cents using fewer coins. He pulled out two quarters, but became very confused. After I told him that he needed twenty more cents, he pulled out two dimes. The next question was sixty-three cents. Seeing that the student was having a tough time thinking on his own, I reminded him that he could use the copy, cover, compare strategy and look at the back of the card to see how to answer it. He then answered the question the way the card showed him, two quarters, one dime, and three pennies.

When the student was asked to count out fifteen cents, he pulled out three nickels. I then told him to look at the back of the card and he saw one dime and one nickel. For fifty-six cents, the student immediately selected two quarters and six pennies. After he looked at the back of the card, he changed his answer to one quarter, three dimes, and one penny. I noted to the student that the card did not show the fewest amount of coins to get to fifty-six cents. At that point, I showed him two quarters, one nickel and one penny as I counted out fifty-six cents. Lastly, the student was asked to count out sixty-five cents. He pulled out six dimes and five pennies. I reminded him to look at the back of the card and he saw two quarters and three nickels, but knew that he could substitute one dime for two of the three nickels.

In the final trial with student “E,” the student correctly identified one penny and one cent without any visual or verbal prompts. When the student was asked to count out four cents, he put all five pennies from the pile in front of him onto the money flash card. I asked him if he had four cents and he said “no.” I then asked him once more to show me four cents. He was still unable to do this so I counted out four cents with the student. I then flipped the card over to show him the picture answer on the back. He matched the replica coins with the pictures and said “four cents” when he finished. I asked him to match the replica coins to the picture coins two more times and each time he was able to match independently. I then asked him to flip the card over so the numerical side was facing up and told the student to show me four cents. This time he correctly counted out four cents.

When the student was required to count out five cents, he pulled out all seven pennies that were in front of him (I added two coins to see if the student would select the entire pile or correctly count out only five pennies). I then flipped the card over to the picture side and asked him to match the replica pennies to the picture pennies. I asked the student to repeat this process three times. He was able to match the coins correctly each time. Then I asked him to show me five cents after I flipped the card over to the number side. Once again, he included all seven pennies when he was asked to count out five cents.

65 cents: 2 quarters, 1 dime, 1 nickel	56 cents: 2 quarters, 1 nickel, 1 penny
15 cents: 1 dime, 1 nickel	63 cents: 2 quarters, 1 dime, 3 pennies
25 cents: 1 quarter	70 cents: 2 quarters, 2 dimes
12 cents: 1 dime, 2 pennies	10 cents: 1 dime
60 cents: 2 quarters, 1 dime	43 cents: 1 quarters, 1 dime, 1 nickel, 3 pennies

Figure 6

\$1.25: 5 quarters	\$2.25: 9 quarters	\$4.50: 18 quarters
\$1.36: 5 quarters, 1 dime, 1 penny	\$3.71: 14 quarters, 2 dimes, 1 penny	
\$4.94: 19 quarters, 1 dime, 1 nickel, 4 pennies		

Figure 7

Next, I had the student use the names of coins to count out a designated amount of coins. The results for a trial with word coins are below.

\$2.67: 10 quarters, 1 dime, 1 nickel, 2 pennies
\$1.45: 5 quarters, 2 dimes
\$3.16: not enough quarters the first try, then 10 quarters, 6 dimes, 1 nickel, 1 penny
\$2.35: 9 quarters, 1 dime

Figure 8

Seeing that the student had a solid grasp of counting money with replica coins and with the words of the coins, I had the student count out money without using any manipulatives. I wrote an amount on a white board and asked the student to write what coins he would use and how much of each coin would be needed. His results for this trial are below.

36 cents: 1 quarter, 1 dime, and 1 penny	77 cents: 3 quarters, 2 pennies
\$1.42: 5 quarters, 1 dime, 1 nickel, 2 pennies	\$3.05: 12 quarters, 1 nickel
\$4.29: 17 quarters, 4 pennies	\$5.01: 20 quarters, 1 penny
\$10.00: 40 quarters	

Figure 9

In student “D’s” final trial, I asked him to try using the fewest amount of coins. To help him, I taught him to use the copy, cover, compare method. His results are below.

12 cents: 1 dime, 2 pennies	
70 cents: 3 quarters, 1 nickel (counted correctly as 80 cents) 2 quarters, 1 dime, 2 nickels (with assistance)	
25 cents: 1 quarter	63 cents: 2 quarters, 1 dime, 3 pennies
15 cents: 1 dime, 5 pennies (instead of pennies, he also used 1 nickel)	
56 cents: 2 quarters, 1 nickel, 2 pennies	
65 cents: 3 quarters, 1 nickel (recounted and had too much), 2 quarters, 1 dime, 1 nickel (with assistance)	
10 cents: 1 dime	60 cents: 2 quarters, 2 nickels
43 cents: 4 dimes, 3 pennies (reminded student to use CCC method) 1 quarter, 1 dime 1 nickel, 3 pennies	

Figure 10

This student had trouble counting mixed sets of coins. The implementation of the copy, cover, compare strategy did seem to help the student with using the fewest amount of coins, but it did not help the student with counting from one type of coin to the next.

Overall, the results that can be taken from the experimental phase of this research project are that the use of manipulatives is highly beneficial to students with disabilities. As far as the evidence-based strategies that were tested; self-regulation/ self-monitoring, concrete-representational-abstract, and copy, cover, compare; are assessed, the most successful were concrete-representational-abstract and copy, cover, compare. Furthermore, the positive effects of the strategies were increased when the two strategies are combined. It is worth noting that time was a negative factor in this experiment. Ideally, such an investigation into a particular

strategy's effectiveness would take several months if not an entire school year. In addition, due to the support services that many of the students received, testing/ instruction did not occur as frequently as intended. However, when examining the increase in the students' ability to find the correct amounts each time and gradually work towards using the fewest coins possible was impressive. Moreover, even after the testing has concluded, the students repeatedly ask to continue the testing process and methods.

### **Part III**

Throughout the entire experimental phase of this research study, one thing that has remained constant among all of the participants is that the use of manipulatives has been beneficial. Furthermore, the students thoroughly enjoyed using them. Not only were the manipulatives used to grab the students' attention and make the lesson topic more meaningful for the students, but they were also used because they were essential in the concrete-representational-abstract strategy testing. The manipulatives gave the students the ability to see and touch a "concrete" object and then "manipulate" it as they added the values of the coins in front of them. This is an important advantage for students with disabilities who need to see what they are doing and who are generally more tactile learners.

The manipulatives gave the students a concrete object that they could physically move around to give them a 3-D picture of what they were doing. I began using the plastic, replica coins because of the reasons mentioned. The plastic coins made it easier for the students to determine if they had too little, enough, or too much money in front of them. Furthermore, they could see the coins accumulating as they worked their way towards the designated amount on the flash cards. In addition, I used the plastic coins to determine the students' level of abstract

thinking. All of the students started by using the replica coins, and then worked towards using letter representations on a white board. Having the replica coins in front of them made it easier for the students to determine what coins to use because they were able to look at the coin and figure out what coin it was and how much it was worth.

Once the students showed mastery using the plastic, replica coins, they moved onto using computer generated pictures of coins. Although they still looked very similar to the replica coins and the students could still manipulate them, the pictures made it slightly more challenging. The nickel, dime, and quarter looked more similar because the size difference wasn't so profound. Moreover, because they were pictures, the students were not able to flip the coins over to look at the reverse side, therefore the students had to know what the front and back of each coin looked like. Students who mastered the picture coins progressed towards using pieces of paper with the names of coins written on them. While the students could still manipulate these as well, the appearance of the coin was completely eliminated. The students now had to read the name of the coin and know how much that coin is worth. The last step was similar but added new challenges. The students were then asked to use letter symbols to show what coins they would use. For example, a "P" stands for a penny, a "N" stands for a nickel, etc.

The final component of this research study was using the copy, cover, compare method to teach the students to use the fewest amount of coins possible. Many of the participants in this study favored using dimes and pennies. The most likely explanation for this is because the students earn dimes for completing classroom jobs, which they then add up and use at the "school store." As for the favoritism towards using pennies, since pennies are each worth one cent, the students who know one-to-one correspondence can easily calculate them. Therefore, in order for the students to learn to use the fewest coins possible, I would show them the back of the

flash cards which displayed picture answers to the amount on the opposite side. The students could then see what coins to use. Next, I would flip the card over, showing the numerical value they had to calculate and asked the students to show me that amount using the plastic, replica coins. Students who could do this with no errors moved onto the picture coins and then the words and letter symbols.

## Treasury Coin Assortment - Set Of 460

### Teach Monetary Value And Math Skills

Set of 460 realistic looking and sized plastic coins. Includes 100 each of pennies, nickels, dimes and quarters, 50 half-dollars and 10 Sacagawea dollar coins. Comes in a convenient storage tub.



## Really Good Tug Of War - Money

### Support Basic Money Skills With This Familiar Card Game

Designed to be played like War, this game is great for money-counting practice in three formats. Students play a card and whoever has the greatest amount of money on their card wins the round. Each game

includes instructions, an answer key, and an Activity Guide with variations for differentiated play. Grades 1-4



## Numeracy Center-In-A-Bag™: Saving Money

**Reinforce Coin-Value Understanding And Practice Adding Money With This Interactive Center**

Students take turns rolling the dice, deciding where to write that number on their work mats, and then totaling the value of the columns. The goal is to be the one who “saves” the most in his or her “bank.” This center can be easily varied to provide differentiated practice. Corresponds directly to NCTM standards.



## U.S. Money Bulletin Board Kit

**Illustrate Money sense in a big way!**

Includes 3 header panels, a front and back diecut of 1.00, 5.00, 10.00, 20.00 50.00 and 100.00. Quarter, Dime, Nickel, Penny and half-dollar. 2 labels for each in written and numeric form. 53 pieces total. Paper Currency measures 10"x 4" Coins up to 4" diameter.



## Big Money Magnetic Coins And Bills Set

### Oversized Plastic, Magnetic Money Helps Students Make Sense Of Dollars And Cents

These extra-large magnetic coins and bills are perfect for classroom lessons. Textured plastic coins are 2½ times the actual size and molded in realistic colors. Full-color bills are 1½ times the actual size and display full-color, up-to-date images. Includes 10 pennies, 10 nickels, 10 dimes, five quarters, two half-dollars, five \$1 bills, two \$5 bills, two \$10 bills, two \$20 bills, one \$50 bill, one \$100 bill, and an activity guide.



### Teaching Supply Websites

[www.reallygoodstuff.com](http://www.reallygoodstuff.com)

[www.lakeshorelearning.com](http://www.lakeshorelearning.com)

[www.discountschoolsupply.com](http://www.discountschoolsupply.com)

\*the teaching supplies were selected from the websites in this box

A major component of this research study was the use of manipulatives as a way to gradually get students with disabilities to think abstractly. From an observational stand point, using manipulatives gave the students guidance in their counting of money. Furthermore, it sparked an interest in them wanting to learn the skill of counting money. Boggan, et. al. state that “it is important for children to have a variety of materials to manipulate and the opportunity to sort, classify, weigh, stack and explore if they are to construct mathematical knowledge.” Moreover, she states that manipulatives are “physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics.” They can be made of or from almost anything at home or can be bought in a store. They can also be as simple as a bag of lima beans or as complex as Unifix cubes or base-ten blocks. However, the manipulatives should be appropriate for the child’s developmental level. A student who is in kindergarten should be using individual counters, while older students should be using blocks that represent multiple numbers.

Manipulatives can be used to teach objectives from any of the five NCTM standards. Counters, base-ten blocks, etc. can all be used to teach numbers and operations such as addition and subtraction. Other examples of uses for manipulatives are: geoboards to learn to identify geometric shapes, rulers and measuring cups to measure length and volume, spinners for learning probability, and fraction strips for learning to add and subtract fractions. The possibilities of uses for manipulatives are endless. Some teachers also use manipulatives to get parents involved in their child’s learning as well. Often times, teachers will send home math games with all the needed materials and the student can play and learn at home with their parents or guardians. Teachers have found positive results with this idea.

While manipulatives can be very beneficial to students, it is important that they are used correctly. The students need to understand the idea behind the skill being taught as opposed to just moving objects around. The manipulatives should be used with the understanding of meeting a specific objective or standard of the curriculum. Furthermore, teachers should allow extra time for the students to explore their manipulatives and play around with them. “After the students have explored the manipulatives, ‘the materials cease to be toys and assume their rightful place in the curriculum’” (Smith, 2009). Moreover, NCTM recommends the teachers use manipulatives because they support learning theory as well as educational research in the classroom. “Manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning” (unknown). The manipulation of objects guides students towards understanding math processes and procedures. Using manipulatives can help students gain a deeper understanding of mathematics by connecting ideas and integrating prior knowledge.

Research conducted over the decades in several grade levels of multiple countries has revealed that achievement in mathematics increases with the proper use of manipulatives. Additional studies have suggested that manipulatives help to improve students’ short- and long-term retention of mathematics. Also, Cain-Caston’s research in 1996 found that manipulatives help to improve the math classroom environment. Other research had found that not only is mathematical understanding enhanced, but anxiety is greatly reduced. Moreover, In 2008 Kenneth Chang evaluated the findings of research scientist Jennifer Kaminski and concluded that when students use concrete examples, they have a better understanding of math. Lastly, additional studies revealed that students who use manipulatives are more likely to achieve higher results than students who don’t have the opportunity to use manipulatives. Research also

supports the use of manipulatives for students with various types of disabilities, finding it to be highly effective for this population of students.

### Manipulatives from A-Z

**ACES** Never throw away a deck of playing cards. They have dozens of math applications: counting, sorting, probability, and card-house engineering, just to name a few.

**BOLTS** Stock up at the hardware store. Nuts, bolts, and screws are perfect for counting, weighing, and sorting.

**CDS** Save the free ones you get in the mail. Trace for perfect circles, or challenge kids to create a giant, symmetrical design for your bulletin board.

**DRUMS** Play various beats and talk about rhythm, counting, and patterns. (Make your own by stretching a balloon across an oatmeal canister.)

**EYE CHARTS** Measure and count the letters. You can also measure the distances from which kids can read the different lines on the chart.

**FLASHLIGHTS** Have kids bring one from home. Then turn off the lights and play with shadows. Talk about symmetry and shape.

**GOOGLE** FYI, this popular search engine is also a calculator. Enter a problem (such as  $5 + 7$ ) into the search field and the answer pops up.

**HOLOGRAMS** Pick up one or two hologram postcards and use them as a starting place to talk about 2-D and 3-D.

**ICE CUBE TRAYS** Use for sorting small objects, measuring, or counting by 2s (have kids put a bean in every other slot).

**JUNK MAIL** Try catalog math. Invite kids to calculate various purchases. Or host an engineering challenge--who can build the strongest bridge from credit card offers?

**KISSES** (The chocolate kind.) Not for every day, but a fun Valentine's Day treat. Estimate how many are in a jar, or do some nutrition math.

**LUNCH TRAYS** Borrow some from your cafeteria. Then use them as a surface to create simple graphs with string and counters.

**MAGIC EIGHT BALL** A fun way to talk about probability. How many possible answers are there? How many are "good" answers? How many are "bad"?

**NOTEBOOKS** Invite kids to keep math journals--a place to work out their thinking and keep track of what they find interesting and confusing.

**ORIGAMI PAPER** This stuff is bright, colorful, and cheap--and a great way to talk about 2-D and 3-D shapes, folding, and symmetry.

**PHONES** Bring in an old cell, handheld, or even rotary phone. Put it in your math center and invite kids to practice writing and dialing phone numbers.

**Q-TIPS** Use as cheap, disposable paintbrushes. Write math problems using invisible ink.

**RACE CARS** Invite kids to bring in toy cars from home. See how far they can go with a gentle push. How many red cars do students own? Blue? Green?

**STAMPS** Cut canceled ones off your mail. Sort by color and shape, or add up how much they all cost together.

**TOILET PAPER TUBES** You're probably already saving these. Use them to talk cylinders or as sorting containers.

**UTENSILS** Plastic silverware, potato mashers, and other kitchen gizmos make for great pointers and fun tools for your math center.

**VEGETABLES** Sugary treats are so last century. Count tomato seeds or weigh green beans. Edamame (soybeans) are perfect counters.

**WOODEN BLOCKS** Endless opportunities here: Who can build the tallest tower? How many blocks tall is the teacher? How much does a block weigh?

**X-RAYS** Doctors and hospitals are often willing to donate old films. Count those ribs and measure that ulna!

**YARN** Love those craft store teacher discounts. Use yarn as a flexible ruler. Weave on a cardboard loom and make patterns. Glue onto paper to make shapes or numbers.

**ZITI** Pasta shapes equal hours of math learning. Make patterned necklaces, measure and pour, or build noodle sculptures.

~~~~~

By Hannah Trierweiler

COMMON CORE MODULE:

## Counting Groups of Mixed Coins

### MODULE SUMMARY

**Content area focus:** Counting Groups of Mixed Coins

**Priority standards:** Count groups of mixed coins using strategies such as self-regulation, Copy, Cover, Compare, and Concrete-Representational-Abstract.

Students will count group of like objects by using 1:1 correspondence while incorporating skip counting when using mixed groups of coins. (e.g.,  $\$0.01+\$0.01+\$0.01=\$0.03$ ); (e.g.,  $\$0.01+\$0.01+\$0.01+\$0.01=\$0.04$ )

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **(1.OA.1)**

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? **(2.MD.8)**

## **Supporting Standards:**

Recognize and identify coins, their names, and their value. **(1.MD.3)**

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **(1.NBT.1)**

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **(1.NBT.4)**

**Domain:** 1.OA Operations and Algebraic Thinking

**Instructional time:** 13 days.

There are four main coins in the United States monetary system; penny, nickel, dime, and quarter. Although there is also the half dollar and the golden dollar, due to the rare use of those coins, they will not be taught in this unit.

There are many ways to count out a designated amount of money by using the four major coins. This unit is designed to first get the students to feel comfortable using the plastic coins as

concrete objects, students who master use of the plastic coins will move on to using pictures of coins, and then progress towards more abstract thinking as they count money. Also, this unit will teach the students that although there are several ways to count any amount of money, using the fewest amount of coins is preferred.

This module has been designed to develop and guide students with disabilities in their learning to count mixed groups of coins. They will be using three different evidence-based practices to ease the learning process. These strategies will help them to first learn to double check their own work before they say that they are ready to turn it in, correct their own errors, develop their abilities to think abstractly when counting mixed groups of coins by first using concrete objects, and learning to use the fewest amount of coins with the aid of the copy, cover, compare method. Ultimately, the goal for the end of the unit is for the students to develop a deep understanding of counting mixed groups of coins and become proficient in using the fewest amount of coins possible to arrive at a designated amount.

Our monetary system requires students to develop a deep conceptual understanding of the identification and value of each coin as well as become fluent in the operation of addition as it relates to counting mixed groups of coins. The intention for this unit is to take place once the students have developed a solid understanding of basic addition and subtraction. This includes single and double digit operations. In addition, the students must know or at least have a basic understanding for skip counting by 5s, 10s, and 25s.

The unit assessment will seek to determine if the students have developed a deep understanding of counting mixed groups of coins as well as fluency and accuracy in using the fewest amount of coins possible.

## **CONTENTS**

- I. Module overview
  - a. Content area focus and priority standard
  - b. Instructional time
  - c. Assessment goals
  - d. Assessment tools
- II. Integrated tasks
  - a. Representational/Geometric Tasks
  - b. Measurement Tasks
  - c. Computational Tasks
  - d. Writing tasks
- III. Module outline
  - a. Section1
    - i. Section 1 summary
    - ii. Section 1 pre-assessment
  - b. Section2
    - i. Section 2 summary

- ii. Activities that emphasize conceptual understanding of section 2 content
    - iii. Activities that emphasize fluency of section 2 content
    - iv. Section 2 assessment
  - c. Section3
    - i. Section 3 summary
    - ii. Activities that emphasize conceptual understanding of section 3 content
    - iii. Activities that emphasize fluency of section 3 content
    - iv. Section 3 assessment
- V. Appendices
  - a. self-regulation checklist
  - b. Assessment flash cards

## MODULE OVERVIEW

**Content area focus:** Counting Groups of Mixed Coins

**Priority standards:** Count groups of mixed coins using strategies such as self-regulation, Copy, Cover, Compare, and Concrete-Representational-Abstract.

Students will count group of like objects by using 1:1 correspondence while incorporating skip counting when using mixed groups of coins. (e.g.,  $\$0.01+\$0.01+\$0.01=\$0.03$ ); (e.g.,  $\$0.01+\$0.01+\$0.01+\$0.01=\$0.04$ )

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **(1.OA.1)**

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? **(2.MD.8)**

### **Supporting Standards:**

Recognize and identify coins, their names, and their value. **(1.MD.3)**

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **(1.NBT.1)**

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction;

relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **(1.NBT.4)**

**Domain:** 1.OA Operations and Algebraic Thinking

**Instructional time:** 13 days.

The timeframe can be adjusted depending on student needs based on mastery of prior knowledge concepts.

**Prior knowledge skills required:** The most important prior knowledge students must possess is the ability to add single- and double-digit numbers. If students have not mastered this skill, they will be given instruction at their ability level.

**Students with Disabilities (SWD)** will benefit from structured/extended work with manipulatives and visual models before relying on abstract representations of money.

**Assessment goals:** Accuracy and conceptual understanding

**Accuracy:** By the end of this module, students will be able to quickly and accurately count a mixed group of coins using the fewest amount of coins possible.

Conceptual understanding: Students will be able to efficiently count money using the fewest coins to get a specific amount.

**Assessment tools:** This module contains a pre-assessment, several formative assessments, and a summative assessment.

Pre-assessment – The intention is for the teacher to determine the knowledge gaps and misconceptions that students have regarding the content. It is crucial that these gaps and misconceptions are identified and addressed throughout instruction of this unit. Failure to do so will result in frustration for both students and instructors.

The pre-assessment also contains material that will be covered in this module. If a student has already mastered some or all of the material that is contained within this unit, accommodations should be made for the advanced student as well allowing this student to progress with new or more challenging material. Differentiation of material is important so that students do not become frustrated because the material is too tough or bored because the material is not challenging enough. In this case, differentiation is very manageable – offer the student material that is more abstract and thus, requires deeper thinking.

Formative assessments – These are concise assessments, intended to measure the learning progress and accuracy of each student throughout this unit. Throughout the unit there are descriptions of in-class, formative assessments that the teacher can do, both formal and informal. In the appendix there is an assessment for section 1 that comprises most of the module. This assessment can be used at intervals throughout the module.

Summative assessment – By the end of the module students are expected to demonstrate a mastery of the basic material presented on this assessment with 100% accuracy. Student performance on the final assessment should continue to notify the teacher of any gaps and/ or misconceptions that students may still have regarding this material. Additional material may be included in this assessment for students who have gone beyond the targeted expectations.

### **Integrated tasks**

The following tasks will integrate skills that overlap with some of the other core standards from this domain as well as other Common Core standards. A majority of attention will be focused on the priority standard.

### **Representational/Geometric Tasks:**

Students will represent each of the four major coins through plastic coins, pictures of coins, coin names, and letter representations of each coin. This process will allow the students to create a deeper meaning of each coin and its value as well as the process of adding multiple coins together. For example, at the beginning of the module a student will be using plastic coins, but at the conclusion, they will be using letters such as “N” to represent a nickel. They will know that when they see “Q, 2D, 1N, 3P” they will know that they will have fifty-three cents. Furthermore, students will have created a visual representation of a set of mixed coins.

Students will also experience the value of coins through hands-on experiences such as using the plastic coins, the pictures of coins, and the pieces of paper with the names of the coins on

them. The teacher might say “if we have two quarters, one nickel, and four pennies, how much do we have?” In addition, students will experience these coins in as many different concrete scenarios as possible so that the concept of monetary value eventually transcends the physical objects and students become truly fluent with the idea of monetary value the same way they become fluent with the words in a book.

### **Measurement Tasks:**

Using mixed groups of coins the students will measure the amount of coins/money they will need to purchase a specific item. They will be able to count groups of coins and determine the existing coin value. In order to measure the value of coins that they will then need to obtain to purchase the item they will deduct the amount they currently have from the total amount of the object to be purchased. Throughout the process of obtaining money they will need to gauge and measure the number of jobs/tasks required to obtain the given amount of money needed to make the purchase of the given item.

### **Computational Tasks:**

Students will be asked to perform addition and subtraction when adding the coin values together. These four coins (penny, nickel, dime, and quarter) are the main coins used in our monetary system. Once these four coins are mastered, the students will be able to count any amount of money and be on their way to learning how to make change for a given amount.

### **Writing Tasks:**

The students will reflect on the value of money and develop a budget for obtaining and spending their money. They will write about items that they want to spend their money on and how they will obtain the money in order to purchase the items. They will develop a plan for ways in which they will work for the given amount of money. The students will also determine how to spend their money on items of priority.

**Module Outline:** This unit can be broken down into three sections of instruction, first of which will incorporate one of the three evidence-based practices mentioned at the beginning of this module.

**SECTION 1      What are the four main coins in the U.S. monetary system and what are their values?**

Section 1 summary: This section serves as a brief introduction to the unit and focuses on assessing if the students know the specific coins as well as their values. It also focuses on teaching students to use the self-regulation strategy.

The cumulative nature of mathematics demands that the teacher thoroughly assesses whether or not students have mastered concepts that the students should have learned previously. The goal of this section is to see if the students know the coins and their values or not. If not, it is critical that the teacher identifies any deficiencies or misconceptions that may exist. Without this remedial instruction, students will only fall further behind as they continue to grasp the concept.

Section 1 Pre-Assessment: Students will be asked to perform the following task:

1. Visualization: given an amount of money, students will be asked to show that amount using plastic coins
2. Visual representation: accurately draw a picture that represents two groups of objects, for example:  $\$0.05 + \$0.05 = \$0.10$
3. Written description: write a simple sentence or two that accurately describes the relative number of objects depicted in the drawing, using phrases such as, “more than” and “less than”.  
You can use this as item #1 is more expensive/costs more than item #2
4. Verbal: use skip counting skills by 5’s to count up to 100. Counting by 5’s is beneficial for counting groups of coins.

Additionally, there is a possibility that material on the pre-assessment will be unfamiliar to the some of the students. The expectation is that students will ask to skip these questions, essentially leaving the answer blank. This material appears here for two reasons:

First, if students don’t know this material, they will simply leave it blank or guess and the teacher will use this as baseline data to compare against the final assessment. This comparison will allow the teacher and student to clearly see what progress the student made throughout the unit.

The second reason concerns the students who are able to correctly fill in all of the questions. If there are students in the class who already know the material that will be taught, they should not be required to sit through the lessons. Rather, the teacher should spend time with these students to determine if they truly have a deep conceptual understanding of the material. If there are students who have mastered more advanced material, then they should be offered more challenging content.

Also in this section will be the introduction to the self-regulation strategy. The students will be given a checklist that they will be asked to fill out once they have completed a worksheet and before they are ready to turn in the worksheet to a teacher. Students should be taught to properly use the checklist and each step should be clarified by the teacher.

## **SECTION 2      How does CRA look?**

Section 2 summary: Students will begin using the plastic coins to answer the same questions asked in the pre-assessment. Once they master the plastic coins, they will move onto using the picture coins to once again complete the same questions. Next they will use the coin names and lastly the letter symbols.

Activities that emphasize conceptual understanding of section 2 content:

- Students will be asked to calculate the amount of coins needed to reach a given amount of money.
- Students will be asked to count out the money that they have chosen as a way of double checking their work prior to giving their final answers.
- Students will practice skip counting by 5s, 10s, and 25s.
- The skip counting will be practiced and mastered by counting groups of the same coin. For example, counting a set of nickels, and then separately counting sets of dimes and quarters.

- The students will discuss how they arrived at a designated amount of money by first explaining the coins that they had chosen, then counting out those coins to the teacher as they verify that they have the correct amount.

Activities that emphasize fluency of section 2 content:

- “Skip counting” will be an important skill emphasized in this section. As an example, students will be asked to, “Skip count a set of each type of coin up to \$1.00.” The students will be taught how to start at either 5, 10, or 25 depending on what coin they are using. Students who are advancing quickly can then be taught how to skip count backwards, (i.e., “skip count 100, 90, 80, 70, etc.”)
- Students will begin with the smallest value coin which is the nickel, assuming that all students can count by 1s. Initially, the skip counting will be carried out in unison with the replica coins until the students begin to show understanding.

Section 2 Assessment: Students will be asked to perform the following task:

1. Count out the correct amount of money for the amount indicated on a flash card. Students will be assessed on how efficient they are able to arrive at the specific amount and how much help they require from the teacher. They will also be assessed on their accuracy in answering the questions correctly.

**SECTION 3      How can Copy, Cover, Compare help student efficiency?**

Section 3 summary: This section will be a brief summary of how the students will use the copy, cover, compare method to help them figure out how to use the fewest amount of coins possible to arrive at a designated amount. The students will be using flash cards that have a numerical amount of money and the written form of that same amount on one side. Then on the back, there are pictures of each coin that is needed to reach the amount on the reverse side. The students will be given the opportunity to look at the amount they need to count up to, and then look at the picture answer for a brief amount of time, depending on ability level in terms of comprehension. Next, they will be asked to arrive at the amount of the front while using the same coins that were pictured on the back.

Activities that emphasize conceptual understanding of section 3 content:

- Students will be given approximately fifteen of each coin (ex: fifteen pennies, fifteen nickels, etc.). They will also have the stack of ten flash cards that are used for the assessments. Once they know what the amount being asked is and the coins that the flash card says they need, they will flip the card back over and arrive at the designated amount using their comprehension skills to help them remember what coins to use. After they have chosen their set of coins, the teacher will hide the card and ask the student to count out the coins that they have in front of them. Successful completion of this task will demonstrate understanding of the content.

Activities that emphasize fluency of section 3 content:

- Counting by fives, and tens, is really no different than counting by ones, will be an important emphasis of this section. In addition, counting by twenty-fives will also be heavily encouraged.

- Students will understand that using the fewest amount of coins possible is more simple and easier to manage.

Section 3 Assessment: Students will be asked to perform four tasks:

1. Verbal: Students will count by fives, tens, and twenty-fives to one hundred.
2. Visual representation: Students will represent a specific amount of money while using the fewest amount of coins after they have looked at a picture answer of the intended amount.

## Conclusion

The evidence from this research study finds that the evidence-based practices of concrete-representational-abstract, and copy, cover, compare are successful for use with students with disabilities. In terms of the CRA, students are motivated to use manipulatives and those manipulatives make it easier for them to develop abstract thinking. While the ability levels of the participating students were a factor in the progress that they made, there is overwhelming proof that this strategy is effective for students with disabilities. As for the copy, cover, compare strategy, it helped students to figure out how to arrange the least amount of coins to get to a designated amount of money. The strategy helped the students to see what coins were needed and where they could substitute for fewer coins. With extended practice these two strategies would continue to help the students develop not only their money skills, but other math skills as well.

## Appendices

I. Self-regulation checklist

II. Assessment flash cards

Appendix I.

| Make Corrections |                                                                                                           |   |                          |
|------------------|-----------------------------------------------------------------------------------------------------------|---|--------------------------|
| 1                | double check<br>         | = | <input type="checkbox"/> |
| 2                | erase<br>                | = | <input type="checkbox"/> |
| 3                | re-read directions<br> | = | <input type="checkbox"/> |
| 4                | re-write answer<br>    | = | <input type="checkbox"/> |
| 5                | raise hand<br>         | = | <input type="checkbox"/> |

|                                     |                                     |                                     |
|-------------------------------------|-------------------------------------|-------------------------------------|
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |
| <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> | <input checked="" type="checkbox"/> |

## Appendix II: Assessment flash cards

65 cents

56 cents

15 cents

63 cents

25 cents

70 cents

12 cents

10 cents

60 cents

43 cents