Decreasing Math Anxiety Through a Quadratics Unit

by

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May 2013

A thesis submitted to the

Department of Education and Human Development of the

State University of New York College at Brockport

In partial requirements for the degree of

Master of Science in Education

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Abstract

Anxiety related to the learning of mathematics is referred to as *math anxiety* and has been shown to have a negative influence on student performance. Research reveals that math anxiety is something that can be unlearned and informs about the potential causes and treatments of math anxiety in the mathematics classroom. A majority of math anxiety experienced by students has been caused by teachers' repetitious teaching styles. Research presents various teaching strategies that have helped teachers when working with students who have math anxiety. These strategies include writing, class discussions, cooperative groups, kinesthetic activities, use of manipulatives, and various, frequent, and untimed assessments. Based on these findings, a unit plan on quadratics has been created to utilize all of these strategies and to demonstrate to the reader and to mathematics teachers that there are specific pedagogical strategies that teachers can use to help decrease math anxiety in their students.

Keywords: math anxiety, quadratics

Chapter One: Introduction

Many people have negative connotations towards learning mathematics and can pinpoint certain subjects or facets in mathematics that still cause feelings of apprehension. Some people blame this apprehension on the instructor, other students, or simply the course content. Another source of apprehension that is becoming more apparent in the classroom is the anxiety associated with learning mathematics. This is often referred to as *math anxiety*.

Richardson and Suinn (1972) defined math anxiety as a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations. After this definition was published, others began to provide their own interpretations. Legg and Locker (2009) defined math anxiety as a general fear or tension associated with anxiety-provoking situations that involve interactions with the learning of mathematics. Cates and Rhymer (2003) stated that math anxiety is a condition in which students experience negative reactions to mathematical concepts and evaluation procedures. These definitions provide the foundation for further investigation of math anxiety. Legg and Locker (2009) referred to math anxiety as a fear of mathematics while Stodolsky (1985) proposed that math anxiety led to an aversion and avoidance of mathematics entirely. However, fear of mathematics is a side topic of math anxiety. Math anxiety deals with tension that interferes with doing mathematical work while the fear of mathematics deals with its complete avoidance. Cates and Rhymer's (2003) definition also brings up another topic when they discuss anxiety during evaluation procedures. This type of anxiety is considered as test anxiety. Test anxiety and math anxiety are two different topics; however a student may experience both. Therefore, based on these discrepancies and because Richardson and Suinn

(1972) were the first to coin the term math anxiety, their definition is the most widely referenced and will be the foundation of this thesis.

Since the awareness of math anxiety has become a larger concern in the classroom, the purpose of this thesis project is to develop a unit that attempts to lower math anxiety during the learning of mathematics in an algebra classroom. Teaching students how to control math anxiety is important because if left untreated or unaddressed, math anxiety may lead to the avoidance of learning mathematics, to low confidence in students' abilities to learn mathematics, as well as low competence and achievement in the mathematical situations both in and out of the classroom (Ashcraft, 2002). The unit presented in this thesis will utilize the various teaching styles, strategies, and methods previous research has shown to be effective in dealing with math anxiety.

Chapter Two: Literature Review

History of Math Anxiety

Until the 1950s discussions about anxiety in the educational setting was usually related to a general overarching theory of anxiety or as test anxiety. Even if a student struggled with anxiety in a specific subject it was attributed to one of these two theories. However, two reports, one an anecdotal record by Gough (1954) and the other a report on numerical anxiety by Dreger and Aiken (1957), triggered the idea of having content specific anxiety. This was the beginnings of the investigation of math anxiety. Dreger and Aiken (1957) created a scale to measure the observed anxiety associated with the learning of mathematics by students and referred to their scale as the Numerical Anxiety Scale. However, it was not until 1972 that math anxiety took center stage thanks to the article "The Mathematics Anxiety Rating Scale: Psychometric Data" by Richardson and Suinn (1972).

Since Richardson and Suinn's (1972) seminal work, research in math anxiety extended to focus on several branches of anxiety. The most common topics were: (1) the causes and effects of math anxiety; (2) solutions/remedies to alleviate math anxiety; and (3) the effect of math anxiety on student performance. One of the leading researchers on the aforementioned topics is Ray Hembree (1990) who synthesized the majority of previous data collected on math anxiety as well as collected his own data about students' math anxiety. As a result, he was able to conclude that the higher anxiety that students have, the lower their performance, which aligned previous research findings. Hembree was able to conclude that math anxiety is learned and can be relieved. Students who showed symptoms of high math anxiety went through psychological treatments such as desensitization and anxiety management training. After such treatments, most students showed significantly lower levels of math anxiety. This finding by Hembree (1990) generated more math anxiety research. The focus of current math anxiety research is on solutions to this phenomenon (Geist, 2010).

While some researchers still focused on solutions to math anxiety, Mark H. Ashcraft (1996; 2001; 2002) began to look at math anxiety through another lens: the cognitive effect of math anxiety. Ashcraft (1996; 2001; 2002) was able to research topics such as working memory and mental arithmetic and how they were limited by math anxiety. Through his work Ashcraft was able to find that students with high anxiety had lower working memory capacities and ended up making considerably more mistakes than less anxious students. If students had significantly higher math anxiety most of them simply skipped or avoided attempting to solve mathematics problems.

From these four researchers, Richardson (1972), Suinn (1972), Hembree(1990), and Ashcraft (1996; 2001; 2002), the majority of the research in math anxiety is derived. Their work has been expanded upon by others, but these individuals are seen as the foundations and are referenced in almost all math anxiety articles. Thanks to their contributions, mathematics teachers now have available to them resources and information that can be referenced as they seek to support their students' in the mathematics classroom.

Math Anxiety and Performance

Hembree (1990) presented one of the foundational math anxiety research studies. Within this one study, Hembree (1990) concluded that: (1) high anxiety leads to low performance; (2) math anxiety is related to general anxiety; and (3) females tend to have higher math anxiety than males. Of these three constructs, math anxiety and performance is the most popularly researched. Hembree (1990) concluded that math anxiety and performance have an inverse relationship. The more anxious a student is, the lower their performance will be and vice-versa. This conclusion sparked future research on this topic by numerous researchers (Khatoon & Mahmood, 2010; Engelhard Jr., 1990; Faust, Ashcraft, & Fleck, 1996; Ashcraft & Kirk, 2001; Ashcraft, 2002). Khatoon and Mahmood (2010) completed a study concerning the relationship anxiety has on performance in the six major secondary school systems in India. Based on their findings, Khatoon and Mahmood agreed with Hembree's (1990) initial findings that there is an inverse relationship between math anxiety and performance in each school system. Though the degree of this inverse relationship differed from school to school it was still evident. Likewise, Engelhard Jr. (1990) completed a study comparing 13 year old students from the United States and Thailand. In this study the inverse relationship between math anxiety and performance was discovered again, however, it was noted that the United States negative correlation between math anxiety and performance was much stronger than that of Thailand. Based on this research, it is apparent that math anxiety may have debiliatiting effects on student achievement. Therefore in order to combat math anxiety in the classroom, reasearchers needed to determine the factors that may cause math anxiety.

Potential Causes of Math Anxiety

Many of the studies on potential causes of math anxiety are qualitative based, but there is agreement among researchers concerning the causes of math anxiety. Stodolsky (1985), Furner and Duffy (2002), Miller and Mitchell (1994), and Geist (2010) all agree that the main source that can cause math anxiety is mathematics teachers. For some people, the roots of their math anxiety were caused by a teacher's repetitious teaching style (Furner & Duffy, 2002). Many mathematics teachers tend to use the same outline and agenda each day. Typically this agenda consists of a warm-up, direct instruction, and then student independent practice. For many students, this is not the way that they learn best. However, many teachers continue this monotonous pattern and it creates an environment that increases a students' math anxiety (Furner & Duffy, 2002).

Another way mathematics teachers may contribute to a student's math anxiety is by their attitude and behavior in the classroom. Jackson and Leffingwll (1999) stated that being hostile towards students, exhibiting an uncaring attidue, having unrealistic expectations and embarrassing students in front of the class are all ways to increase a students math anxiety. All of these behaviors demonstrate to the student that the teacher is an obstacle to their learning rather than a resource to help them succeed and overcome their anxiety.

Besides the mathematics teacher, society is another factor that may contribute to a student developing math anxiety. Through parents, friends, and media, the learning of mathematics has been given a negative connotation. Many people believe that the ability to do mathematics is a talent similar to those who are artists, musicians and athletes rather than a subject to learn (Stodolsky, 1985; Furner & Duffy, 2002). Parents who show or tell their children that they have anxiety when dealing with mathematics may pass that belief on to their children (Furner & Duffy, 2002). Also, students who come from families where the parents have lower levels of education achievement tend to have higher anxiety because parents do not emphasize the importance of mathematics or simply have less knowledge of mathematical concepts (Geist, 2010).

Treatments for Math Anxiety

Hembree (1990) stated that math anxiety is learned over time, and therefore can be treated or unlearned. This led researchers to analyze potential treatments for math anxiety. Since the mathematics teacher is usually the cause of math anxiety, the majority of the treatments are associated with the teacher. The first strategy to help overcome math anxiety is to use a variety of teaching strategies. Through the use of manipulative, discussions, writing, kinesthetic activities, and questioning techniques, students will be able to learn mathematics in a variety of other styles, rather than the typical drill and practice (Furner & Duffy, 2002; Miller & Mitchell, 1994). These strategies allow for students to experience and view mathematics through a different lens. All students learn in different ways and teachers need to give every student equal opportunity to learn in their preferred style. Researchers found that if a student is comfortable in the style of learning, they work harder and perform at higher rates than other students (Furner & Duffy, 2002). One specific teaching strategy that has shown to decrease math anxiety in students is cooperative grouping (Miller & Mitchell, 1994; Furner & Duffy, 2002). There are many benefits to students working in groups. First, students that have math anxiety may feel more comfortable asking their peers questions about the lesson rather than the teacher. Having the students in groups allows this type of communication to occur without being as distracting to the whole class when compared to a direct instruction/lecture approach to teaching. Another benefit of cooperative groups is it gives students who understand the material an opportunity to help their fellow classmates. If a student is able to teach someone else the material, it demonstrates to the instructor that the student has mastered the topic. One last benefit of working in groups is that it allows the students to practice working together with others as a team. This is an important life skill that will be used both in and out of school.

Another way that teachers can help decrease math anxiety is through different assessments. Today's education system has become very reliant upon high stakes timed tests (Geist, 2010). These types of tests severely affect students with math anxiety. Therefore it is important to use a variety of assessment tools rather than single paper-pencil exams. Miller and Miller (1994) suggest using alternate methods of evaluation such as various questioning activities, writing activities, and short quizzes to conclude lessons. If these types of assessments are used on a more frequent basis as compared to an end of the unit test, students will feel less pressure in the classroom. Unit tests are still encouraged to be used, just not as the lone source of assessing a student's knowledge. Another factor when dealing with assessments is time. Having time limits on assessments has a negative effect on math anxiety (Geist, 2010). Though time limits are a great way to give students a sense of urgency, students with math anxiety tend to focus more on the time rather than the assessment. To combat this struggle with time limits, assessments should either not have a time limit or be short enough where a time limit is not a problem.

The last way teachers can help alleviate math anxiety is by having a positive classroom environment and attitude (Furner & Duffy, 2002). If students know that the teacher's classroom is a place of learning and that people can make mistakes, students with math anxiety will feel more comfortable. This is important because if a student is comfortable in their environment, they are more likely to give their best effort. Teachers should allow students to make mistakes and even let the students know that they make mistakes as well (Miller & Mitchell, 1994). Just knowing that it is okay to mess up takes a great deal of pressure off of many students with math anxiety. Along with having a positive environment, teachers need to develop and maintain relationships with the students. Getting to know the student allows the teacher deeper insight into the struggles students are having and can help teachers design, develop, and tailor strategies to alleviate math anxiety for each student (Miller & Mitchell, 1994).

Chapter 3: Unit Plan with Lessons

When creating a unit to decrease or prevent math anxiety, it is important to align the instruction and resources to what previous research has proved to work. For this unit plan, the author used various teaching styles and strategies, manipulatives, and various assessment tools that align with current math anxiety research. In the discussion section of chapter 4, each lesson will be discussed to demonstrate to the reader how a lesson can be designed in an attempt to decrease math anxiety.

It is to be noted that some of the worksheets and activities that accompany each lesson do not follow APA format, for example, the margins and font may vary. The reason for this is so that teachers can maximize the amount of information on a single page, which in turn will save paper.

Table 1

Unit Calendar

Day	Topic	Day	Topic
1	Graphing Quadratics	7	Completing the Square
2	More Graphing of Quadratics	8	Solving Quadratics Using all Methods
3	Finish Quadratic Graphs and Characteristics/Beginning of Solving by Factoring	9	Quadratic Applications
4	Solving Quadratics by Factoring	10	Quadratic Applications
5	The Nature of Roots: Using the Discriminant	11	Review
6	The Quadratic Formula	12	Quadratics Assessments

Lesson Plans

Day 1: Graphing Quadratics

Objective:
We shall determine characteristics of quadratic functions through their graphs and we will share our knowledge by graphing and labeling these characteristics independently.

Warm-up:
Make tables for the following functions:
1) $y = 2x + 3$
2) $y = -2x + 3$
3) $y = x^2 + 3x + 2$
4) $y = -x^2 + 3x + 2$

Vocabulary:
Quadratic Function, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Roots, Zeros, X-Intercepts, Solutions, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) Graphing Quadratics (Direct instruction/Modeling)
   - Write down and discuss vocabulary in notebooks
   - Describe differences between standard and vertex form
   - Using graphing quadratic functions worksheet, do a few complete examples with the students
2) Independent Practice – Finish Graphing Quadratic Functions Worksheet
3) Closure Activity

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Notebooks
Copies of "Graphing Quadratic Functions"

Closure:
In their own words, have the students define the vocabulary words axis of symmetry, vertex, and roots/zeros.

Name ______________________________ Date ______________ Period ____________

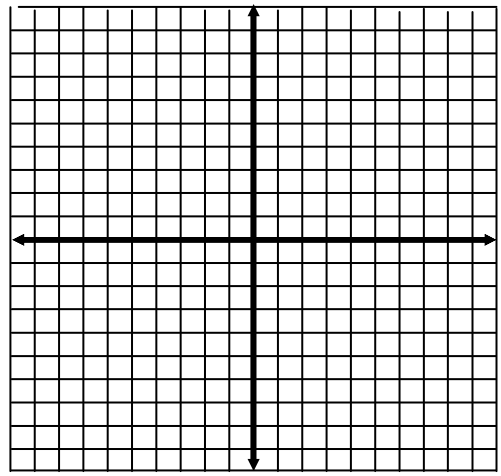
Graphing Quadratic Functions

Directions: Fill out the table and graph each of the following quadratic functions. Fill in the indicated information. Label the axis of symmetry, vertex, and roots on the graph.

1. $y = x^2 - 4x + 6$

x	y

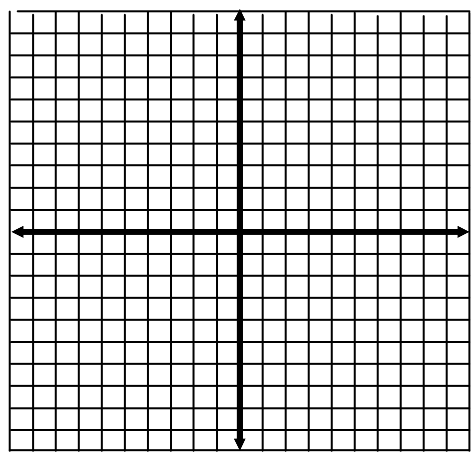
Opens Up / Opens Down
Vertex:__________
Axis of Symmetry:________
Roots:__________
Domain:_____________
Range:____________



2. $y = (x - 5)(x - 5)$

x	y

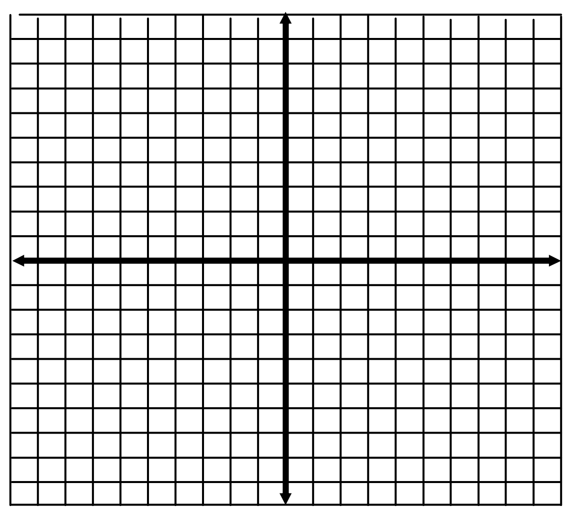
Opens Up / Opens Down
Vertex:__________
Axis of Symmetry:________
Roots:__________
Domain:_______________
Range:____________



3. $y = 2(x+1)^2 - 4$

x	y

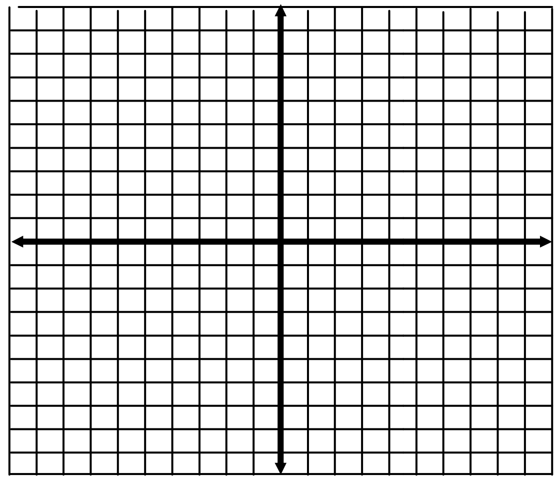
Opens Up / Opens Down
Vertex:__________
Axis of Symmetry:________
Roots:__________
Domain:_____________
Range:____________



4. $y = x^2 - 4$

x	y

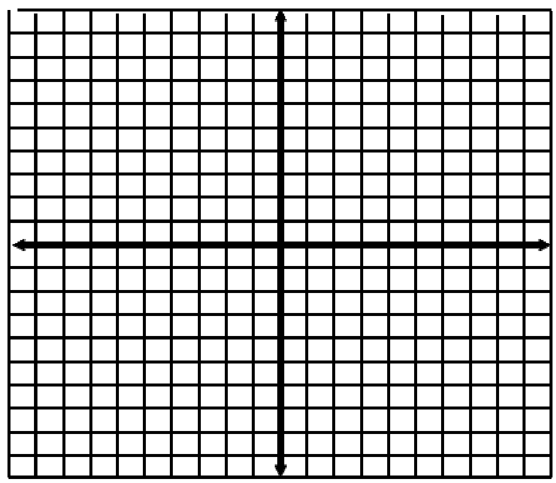
Opens Up / Opens Down
Vertex:___________
Axis of Symmetry:________
Roots:___________
Domain:______________
Range:_____________



5. $y = -2(x-2)^2 + 1$

x	y

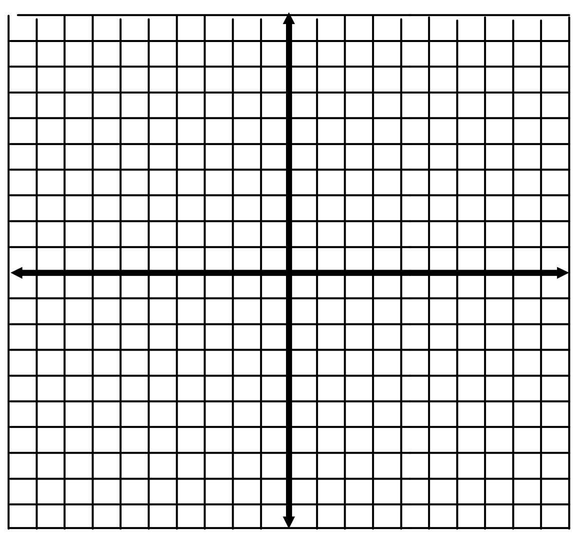
Opens Up / Opens Down
Vertex:___________
Axis of Symmetry:________
Roots:___________
Domain:______________
Range:_____________



6. $y = -2(x-1)(x-5)$

x	y

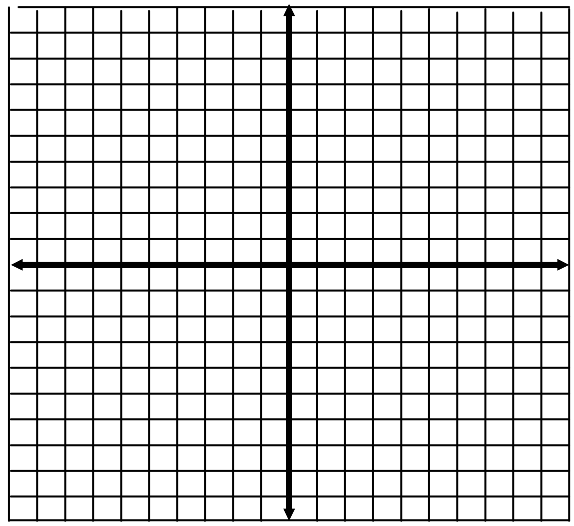
Opens Up / Opens Down
Vertex:___________
Axis of Symmetry:________
Roots:___________
Domain:______________
Range:_____________



7. $y = -4x^2 - 12x - 3$

x	y

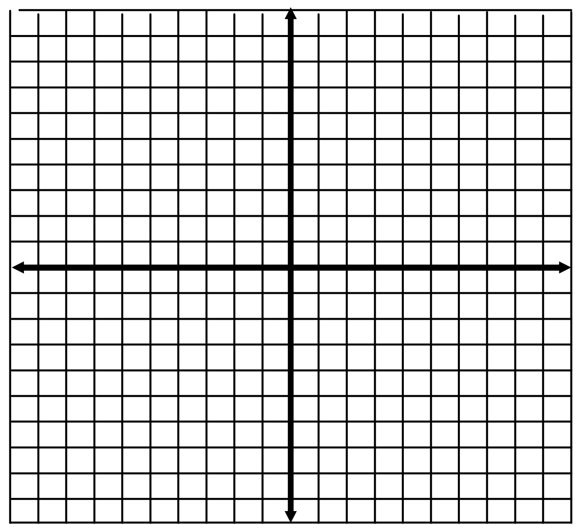
Opens Up / Opens Down
Vertex:___________
Axis of Symmetry:________
Roots:___________
Domain:______________
Range:_____________



8. $y = 2(x+2)^2 - 5$

x	y

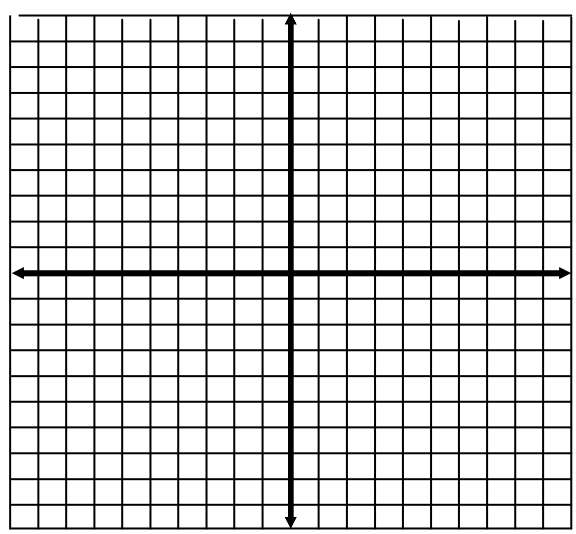
Opens Up / Opens Down
Vertex:___________
Axis of Symmetry:________
Roots:___________
Domain:______________
Range:_____________



9. $y = -(x-3)(x+1)$

x	y

Opens Up / Opens Down
Vertex:___________
Axis of Symmetry:________
Roots:___________
Domain:______________
Range:_____________



_______ 10. The graph of a function is shown below.

What is the value of ***y*** when ***x*** is –1?

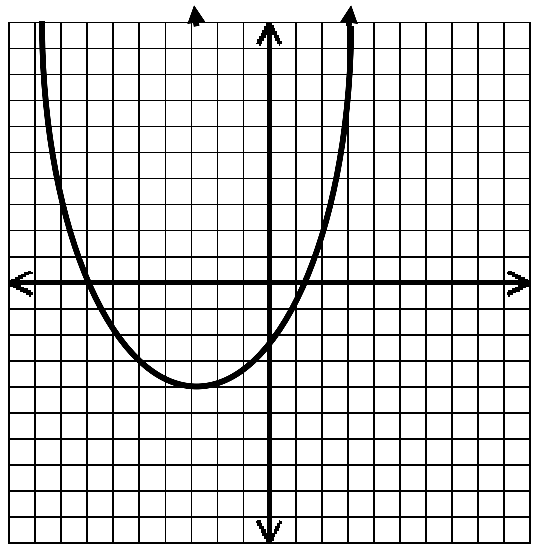
A. –3

B. $-2\frac{1}{2}$

C. –2

D. 2

E. 3



_______11. A part of the graph of the equation $y = -\frac{1}{2}x^2 + 4x - 2$ is shown on the coordinate grid.

Between which 2 integers will the graph again cross the x-axis?

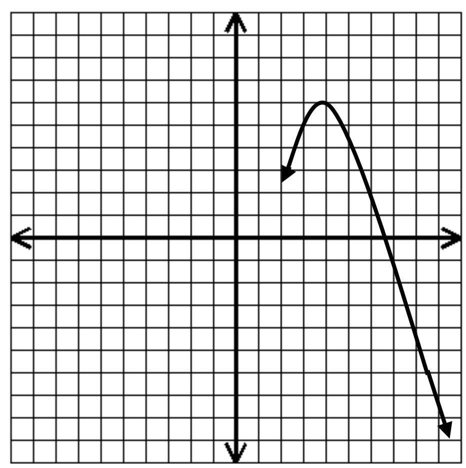
A. Between –2 and –1

B. Between –1 and 0

C. Between 0 and 1

D. Between 1 and 2

E. Between 2 and 3



Day 2: More Graphing of Quadratics

Objective:
We shall determine characteristics of quadratic functions through their graphs and we will share our knowledge by graphing and labeling these characteristics in cooperative groups.

Warm-up:
Graph the following quadratic and list its characteristics: $y = x^2 - 8x + 12$

Vocabulary:
Quadratic Function, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Roots, Zeros, X-Intercepts, Solutions, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) Warm-up
2) Debrief "Graphing Quadratic Functions" Packet from previous class
3) Cooperative Groups using "More Graphing of Quadratic Functions" Worksheet
- In groups of three, have students work together solving the problems on the "More Graphing of Quadratic Functions" worksheet.
- Each student of the group is responsible to answer two problems on their own.
- After each student has finished their two, each member must go to students from other groups who answered the same problems. Students will then check their answers to make sure that everyone agrees with the work. If there is a disagreement, students must work together to come to the correct solution.
- Once solutions are confirmed, students will go back to their original group.
- In their original group, students will present the findings on the two problems that they solved.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Copies of "More Graphing of Quadratic Functions"

Closure:
Students present their findings of their two problems with other members of their group.

Name ____ Date ____________________ Period __________

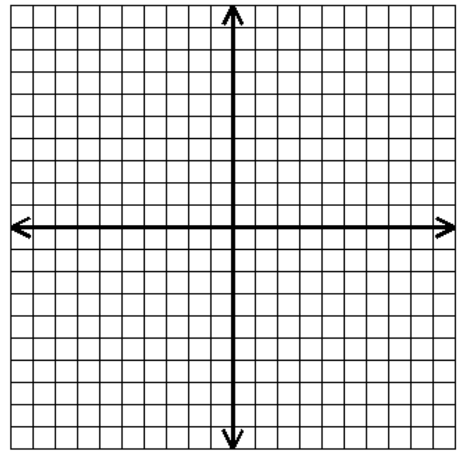
More Graphing of Quadratic Functions

Directions: Fill in the indicated information. Make a table and use it to graph each quadratic. Label the vertex, axis of symmetry, and roots on the graph.

1. **$y = 2(x - 2)^2 - 2$**

Opens Up / Opens Down
Vertex:__________
Axis of Symmetry:__________
Roots:__________
Domain:__________________
Range:___________________

x	y



2. **$y = -2(x + 3)(x - 1)$**

Opens Up / Opens Down

Vertex:__________

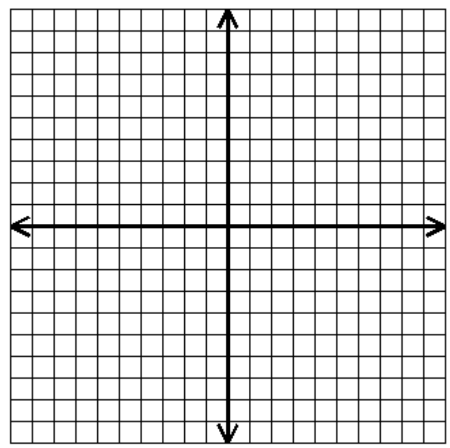
Axis of Symmetry:__________

Roots: ______________

Domain:__________________

Range:___________________

x	y



3. **$y = -2x^2 + 4x$**

Opens Up / Opens Down

Vertex:__________

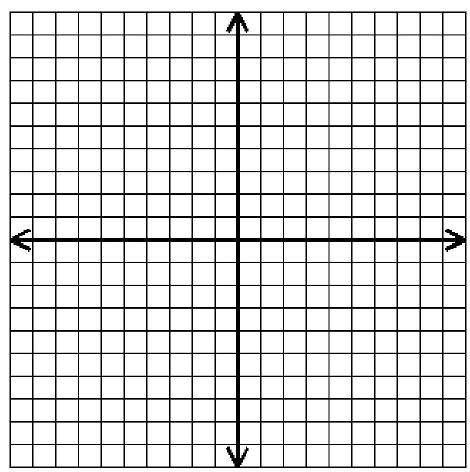
Axis of Symmetry:__________

Roots: ________________

Domain:__________________

Range:___________________

x	y



4. **$y = (x - 5)(x - 3)$**

Opens Up / Opens Down

Vertex:__________

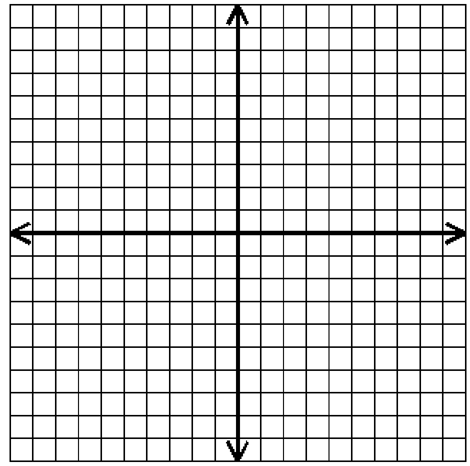
Axis of Symmetry:__________

Roots: ______________

Domain:________________

Range:__________________

x	y



5. **$y = -x^2 + 9$**

Opens Up / Opens Down

Vertex:__________

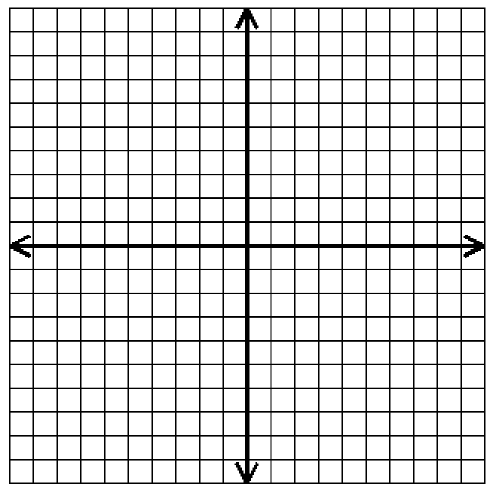
Axis of Symmetry:__________

Roots: ______________

Domain:________________

Range:__________________

x	y



6. **$y = (x + 3)^2 - 9$**

Opens Up / Opens Down

Vertex:__________

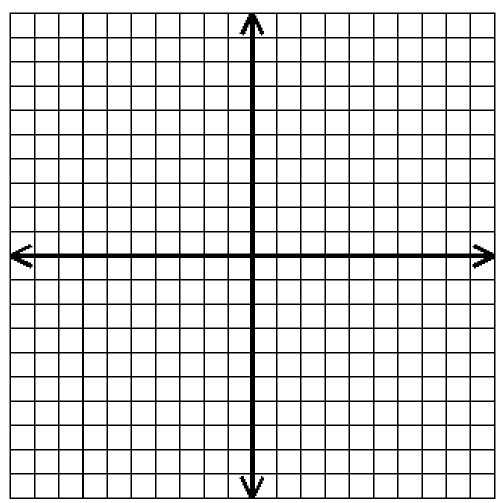
Axis of Symmetry:__________

Roots: ______________

Domain:________________

Range:__________________

x	y



Day 3: Finishing Quadratic Graphs and Characteristics/Beginning of Solving by Factoring

Objective:
We shall analyze characteristics of quadratic functions as well as review the process of factoring polynomials and we will share our knowledge through a matching activity and factoring "dominoes."

Warm-up:
Graphing the following quadratic and list its characteristics: $f(x) = (x - 3)^2 + 1$

Vocabulary:
Quadratic Function, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Roots, Zeros, X-Intercepts, Solutions, Y-Intercept, Maximum, Minimum, Domain, Range, Polynomial, Trinomial, Binomial

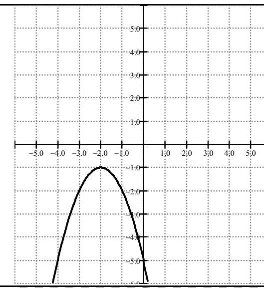
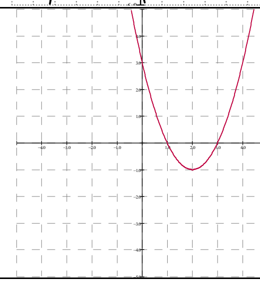
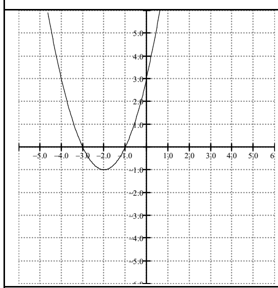
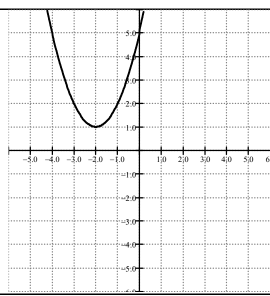
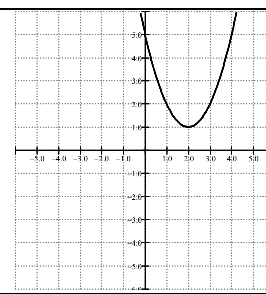
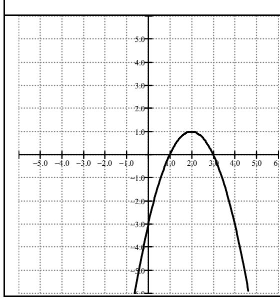
Instructional Plan:
1) Warm-up
2) Students complete "Mix and Match: Quadratic Equations"
- This will be the culminating activity for graphing quadratics.
- Students will work with a partner.

3) Students work on "Factoring Dominoes"
- Since students have previously completed a unit on factoring, this activity is review.
- Students will be able to work with partners if they would like, but each student must still complete their own set of dominoes.
- To complete this activity, students must factor on the polynomial side by using factoring techniques. On the factor side, students must use the process of FOIL to come up with the original polynomial.
- This activity is a precursor to the following day when students will be learning to solve quadratics by factoring.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Copies of "Mix and Match: Quadratic Equations"
Copies of "Factoring Dominoes"
Scissors
Glue/Tape

Closure:
Students will complete their factoring dominoes. If done correctly, they will make a loop

<u>Mix and Match: Quadratic Equations</u>

$x = 2$	$x = -2$	$(2, 1)$	$x -$ intercept(s): $(3,\ 0), (1,\ 0)$ $y -$ intercept(s): $(0,\ -3)$	
$(-2, -1)$	$x -$ intercept(s): None $y -$ intercept(s): $(0,\ 5)$	$(-2,\ -1)$	$x = -2$	
	$x = -2$	$(2,\ -1)$	$x -$ intercept(s): $(-3,\ 0), (-1,\ 0)$ $y -$ intercept(s): $(0, 3)$	$(2, 1)$
$x -$ intercept(s): $y -$ intercept(s): $(0,\ -5)$		$(-2, 1)$	$x -$ intercept(s): None $y -$ intercept(s): $(0,\ 5)$	
	$x = 2$	$x = 2$		$x -$ intercept(s): $(1,\ 0), (3,\ 0)$ $y -$ intercept(s): $(0,\ 3)$

<u>Directions:</u> Cut out the following cells. Match the correct axis of symmetry, vertex, intercepts, and graph to each equation. Glue/tape the cells to the blank chart.

Equation	Axis of Symmetry	Vertex	Intercepts	Graph
$y = x^2 + 4x + 3$				
$y = -x^2 - 4x - 5$				
$y = x^2 + 4x + 5$				
$y = x^2 - 4x + 3$				
$y = x^2 - 4x + 5$				
$y = -x^2 + 4x - 3$				

Factoring Dominoes

Cut on the solid lines, not on the dashed lines. Match the factors with the polynomials. Show all work. If done correctly, it will make a loop.

Factors	Polynomial	Factors	Polynomial
$(x - 4)(4x - 1)$	$3x^2 - 2x - 5$	$(5x + 4)(x+3)$	$2x^2 + 5x + 2$
$3(3x + 1)(x+7)$	$5x^2 - 18x + 9$	$(x - 7)(4x - 7)$	$6x^2 + 7x - 49$
$(3x - 2)(x - 2)$	$2x^2 + 3x - 9$	$4(x+5)(4x - 5)$	$6x^2 + 5x - 6$
$(5x - 3)(x - 3)$	$4x^2 - 35x + 49$	$(3x - 5)(x + 1)$	$3x^2 - 8x + 4$

Factors (2x – 3) (x + 3)	Polynomial $5x^2 + 19x + 12$	Factors (2x + 1)(x+2)	Polynomial $9x^2 + 66x + 21$
Factors (3x – 7)(2x + 7)	Polynomial $16x^2 + 60x - 100$	Factors (2x + 3)(3x – 2)	Polynomial $4x^2 – 17x + 4$

Day 4: Solving Quadratics by Factoring

Objective:
We shall investigate the process of solving quadratics by factoring and we will share our knowledge through cooperative group practice.

Warm-up:
Factor the following polynomials:
1) $4x^2 + 10x$
2) $x^2 + 14x + 24$
3) $2x^2 - 9x + 7$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Polynomial, Trinomial, Binomial, Zero Product Property

Instructional Plan:
1) Warm-up
2) Debrief "Factoring Dominoes" Activity
3) Solving Quadratic Equations by Factoring (All in Student Notebook)
- Discuss with students the Zero Product Property
- Show students with various examples how it solves the quadratic.
- Confirm that factoring can find the roots just like graphing/making a table.

4) Cooperative Groups – "Matching Zeros"
- Students work in groups of three solving the problems on the worksheet.
- If the group wants to get a 100, every problem needs to be done. However, each group member must finish eight problems each.
- If a group does not have equal distribution of completion of the problems, grades will be lowered.
- Since everyone is expected to contribute equally and not all students learn at the same rate, problems start easier and then increase in difficulty.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Notebooks
Copies of "Matching Zeros"

Closure:
Students will complete the "Matching Zeros" Activity.

Matching Zeros

Match the zeroes with the correct quadratic equation.

1. $x^2 + 3x = -2$	9. $x^2 + 5x = -6$	17. $x^2 + 7x = -10$
2. $x^2 + 10x = -16$	10. $x^2 + 15y = -36$	18. $x^2 + 22x = -40$
3. $x^2 - 3x = -2$	11. $x^2 - 13x = -12$	19. $x^2 - 11x = -18$
4. $x^2 - 10x = -24$	12. $x^2 - 1x = 27$	20. $x^2 - 13x = -36$
5. $x^2 - 5x = 14$	13. $x^2 + x = 20$	21. $x^2 - 3x = 40$
6. $x^2 + 2c = 63$	14. $x^2 + 10x = 75$	22. $x^2 - 7x = 44$
7. $3x^2 + 31x = -36$	15. $2x^2 - 19x = -24$	23. $5x^2 + 23x = -26$
8. $2x^2 - 11x = -15$	16. $5x^2 + 28x = -32$	24. $2x^2 - 27x = -36$

Answers
12, 3/2
3 , 9
-4 , -8/5
4 , 6
3 , 5/2
2 , 9
-2, -13/5
12 , 1
8 , 3/2
1 , 2
-9, -4/3
-2 , -20
11, -4
-3 , -12
-15 , 5
-2 , -8
-9 , 7
-2 , -5
8 , -5
-2, -3
-5 , 4
-1, -2
7 , -2
4 , 9

Work with a group of three. Fill out the form below. Turn in all scratch paper to show work.

Name:	Which problems did you do?	Grade:

Day 5: The Nature of Roots: Using the Discriminant

Objective:
We shall analyze the process of finding the nature of roots for a quadratic by using the discriminant and we will share our knowledge through a ticket out the door.

Warm-up:
Find the zeros of the following quadratics by factoring:
1) $x^2 - x - 56$
2) $3x^2 + 16x + 5$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Polynomial, Trinomial, Binomial, Zero Product Property, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots

Instructional Plan:
1) Warm-Up
2) Nature of Roots/Discriminant Notes (All in Student Notebook)
   - Describe what the discriminant is and what it shows us.
   - Explain the different types of roots a quadratic can have.
   - Do several examples showing how to use the discriminant to describe the roots.
3) "Real or Complex Roots?" Ticket out the Door Activity

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Notebooks
Copies of "Real or Complex Roots?"

Closure:
"Real or Complex Roots?" Ticket out the Door Activity

Real or Complex Roots?

Name: ____ Date: ___________ Period: ________

Use the graphing program of your calculator to determine if each quadratic has real or complex (non-real) roots.

1. Which quadratic equation has solutions that are complex numbers? Sketch each to prove your answer.

$x^2 + 8x - 3 = 0$ $2x^2 + 4x + 3 = 0$ $3x^2 - 3x - 1 = 0$ $x^2 + 4x - 4 = 0$

2. Which quadratic equation has solutions that are complex numbers? Sketch each to prove your answer.

$x^2 + 7x - 18 = 0$ $x^2 - 5x - 126 = 0$ $3x^2 + 19x - 14 = 0$ $3x^2 + 5x + 17 = 0$

3. Which quadratic equation has solutions that are complex numbers? Sketch each to prove your answer.

$2x^2 + 3x + 5 = 0$ $x^2 + 2x - 24 = 0$ $x^2 - x - 6 = 0$ $2x^2 - 11x - 63 = 0$

4. Which quadratic equation has solutions that are complex numbers? Sketch each to prove your answer.

$3x^2 - 11x - 70 = 0$ $4x^2 - 7x - 15 = 0$ $x^2 - 25x + 150 = 0$ $x^2 + 2x + 65 = 0$

5. Use the discriminant to describe the roots.

   $3x^2 + 5x + 17 = 0$

6. Use the discriminant to describe the roots.

   $x^2 + 4x + 25 = 0$

7. Create two quadratic functions that have real roots and create two quadratic functions that have complex (non-real) roots. Sketch your answer to prove your work.

Day 6: The Quadratic Formula

Objective:
We shall apply the process of finding the nature of roots by using the discriminant as well as analyze the process of solving quadratics using the quadratic formula and we will share our knowledge through the completion of a scavenger hunt.

Warm-up:
Use the discriminant to determine the types of roots the following quadratics will have.
1) $x^2 + 10x + 2 = 0$
2) $2a^2 – x + 2 = 0$
3) $b^2 – 6b = -9$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula

Instructional Plan:
1) Warm-Up
2) Debrief the "Real or Complex Roots?" ticket out the door activity
3) Complete guided practice of solving quadratics using the quadratic formula. (In Student Notebooks)
   - Using the same three problems from the warm-up, we will find their roots using the quadratic formula.
   - It will be pointed out how the discriminant is a part of the quadratic formula and that it shows what the roots will be/look like.
4) Complete the Discriminant/Quadratic Formula Scavenger Hunt
   - Place the posters around the room.
   - To start, students answer one of the questions. Their answer will be at the top of another poster around the room.
   - When they find their answer, students then solve the question that is below that first question.
   - Continue this process until all of the questions are answered. If done correctly, the last answer will be at the top of the poster where they started.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Notebooks
Scavenger Hunt Posters
Scavenger Hunt Work Papers

Closure:
Students will complete the scavenger hunt activity.

Scavenger Hunt Work Page

Quadratic Equation	**Discriminant** (Type of Solutions) $b^2 - 4ac$	**Quadratic Formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Roots/Zeros**
$x^2 + 8x = -12$			

Answer:

Two Complex Roots

$$1 \pm i\sqrt{2}$$

Question:

$$x^2 + 8x = -12$$

Answer:

Two Real, Rational Roots

-6,-2

Question:

$$x^2 + 10x = -25$$

Answer:

One Real Root

-5

Question:

$$x^2 = 3x - 1$$

Answer:

Two Real, Irrational Roots

$$\frac{3 \pm \sqrt{5}}{2}$$

Question:

$$x^2 = -6x + 5$$

Answer:

Two Real, Irrational Roots

$$-3 \pm \sqrt{14}$$

Question:

$$x^2 - 5x = -4$$

Answer:

Two Real, Rational Roots

1,4

Question:

$$x^2 = 2x - 5$$

Answer:

Two Complex Roots

$$1 \pm 2i$$

Question:

$$x^2 - 2x + 3 = 0$$

Day 7: Completing the Square

Objective:
We shall explore the process of solving quadratics through completing the square and we will share our knowledge through a PowerPoint and independent practice.

Warm-up:
Describe the roots and solve using the quadratic formula for each of the following.
1) $4x^2 - 28x = -49$
2) $2x^2 + 5 = 2x$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula, Completing the Square

Instructional Plan:
1) Warm-Up
2) Debrief the Discriminant/Quadratic Formula Scavenger Hunt
3) "Solving Quadratic Equations by Completing the Square" PowerPoint
   - Go through the PowerPoint explaining the process of solving quadratics using completing the square.
   - Allow students time to ask questions to the teacher or, if they prefer, other students.
   - Give students time to complete the five problems on the last slide independently.
4) Closure Activity

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Copies of "Solving Quadratic Equations by Completing the Square" PowerPoint

Closure:
Write down, in your own words, the steps to solving a quadratic by completing the square. Share what you wrote with the person sitting next to you.

Day 8: Solving Quadratics Using All Methods

Objective:
We shall apply our knowledge of solving quadratics using all of the methods learned and we will share our knowledge through guided and group practice.

Warm-up:
Solve the following quadratics using completing the square.
1) $4x^2 - 28x = -49$
2) $2x^2 + 5 = 2x$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula, Completing the Square, Parabola, Axis of Symmetry, Vertex, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) Warm-Up
2) Guided Practice
   - Using the "Solving Quadratics Four Ways", lead the students through each method for the equations:
     $f(x) = x^2 + 5x - 6$ and $g(x) = 3x^2 - 4x - 1$
   - Emphasize how, no matter what method is used, the roots will always be the same.
3) Group Practice (Groups of Four)
   - Each member of the group is responsible to solve two problems.
   - The groups will answer the following questions:
     - $f(x) = x^2 + 7x + 10$
     - $h(a) = 3a^2 - 5a$
     - $g(x) = 2x^2 - 19$
     - $d(b) = 2b^2 - 2b + 3$
     - $s(t) = t^2 + 6t + 12$
     - $f(x) = 3x^2 + 4x + 3$
     - $p(x) = x^2 + 4x + 10$
     - $f(x) = 10x^2 + 7x + 4$
4) Closure Activity

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Copies of "Solving Quadratics Four Ways"

Closure:
Write down which method of solving quadratics you like the best and the worst. Explain why you have these feelings for these methods.

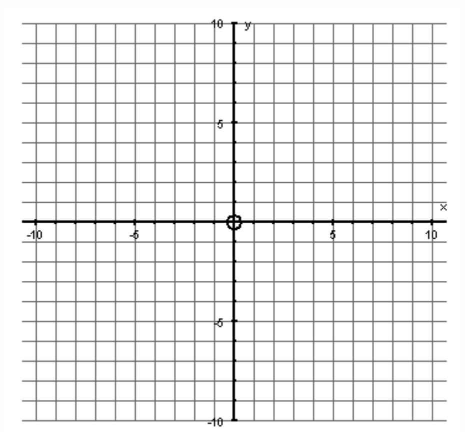
Solving Quadratics Four Ways

Problem: ______________________________

Graphing

Opens Up / Opens Down
Vertex: __________
Axis of Symmetry: ___________
Roots/Zeros: ___________________
Domain: ______________
Range: _____________

X	Y



a	b	c	b^2	4ac	2a

The Discriminant

$b^2 - 4ac$

Type of Roots: _____________________

Factoring

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the Square

Day 9: Quadratic Applications Day 1

Objective:
We shall explore quadratic application word problems and questions associated with their graphs and tables and we will share our knowledge through team practice.

Warm-up:
Solve the following quadratic using all methods.
1) $3x^2 + 9x = 12$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula, Completing the Square, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) Warm-Up
2) Team Practice
   - Students will work in teams answering the questions based on the graphs and tables.
   - Each team will start with one problem. When they finish their problem, the team will move to the next group and answer the next question.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Copies of "Quadratic Applications Word Problems"

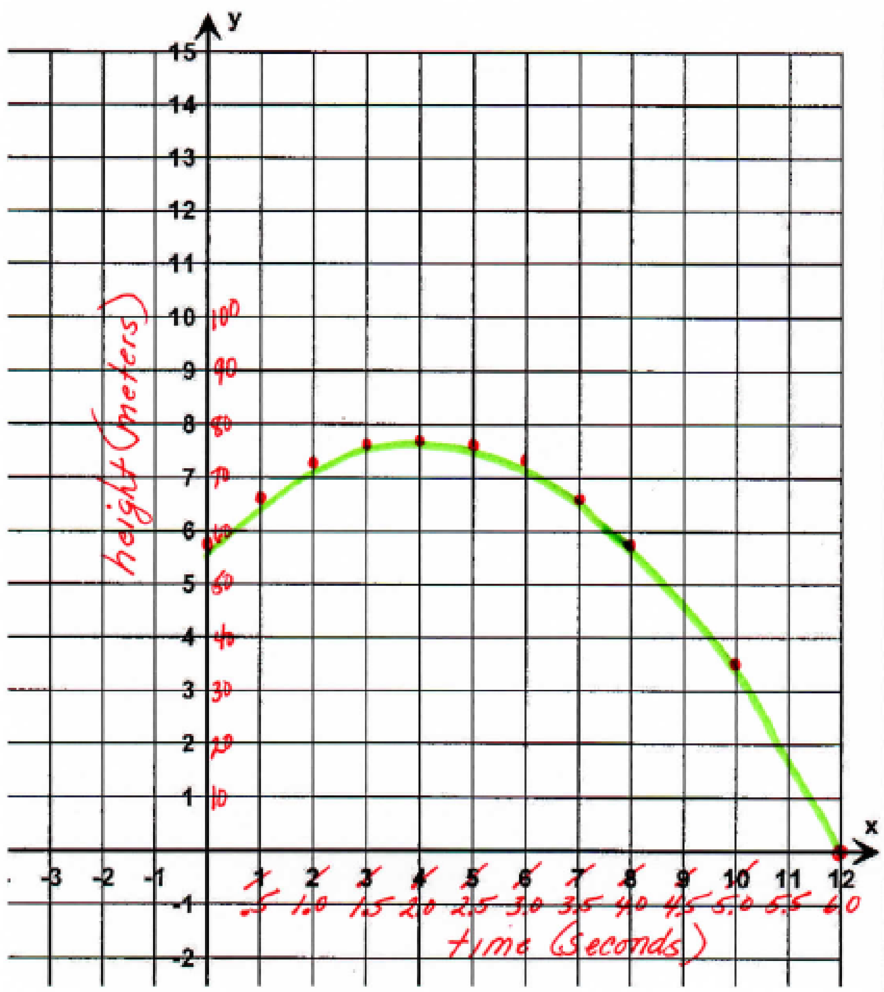
Closure:
Finish solving the quadratic application word problems.

Quadratic Application Word Problems

An object is launched at 19.6 meters per second (m/s) from a 58.8 meter tall platform. The equation for the object's height h (meters) at time t (seconds) after launch is $h(t) = -4.9t^2 + 19.6t + 58.8$. Tabular and graphic representations are below.

1. How long does it take for the object to strike the ground?
2. How high is the object at 4 seconds?
3. Why does the graph not start with (0,0)?
4. Are there two roots for this equation? Why is only one logical?

Problem A

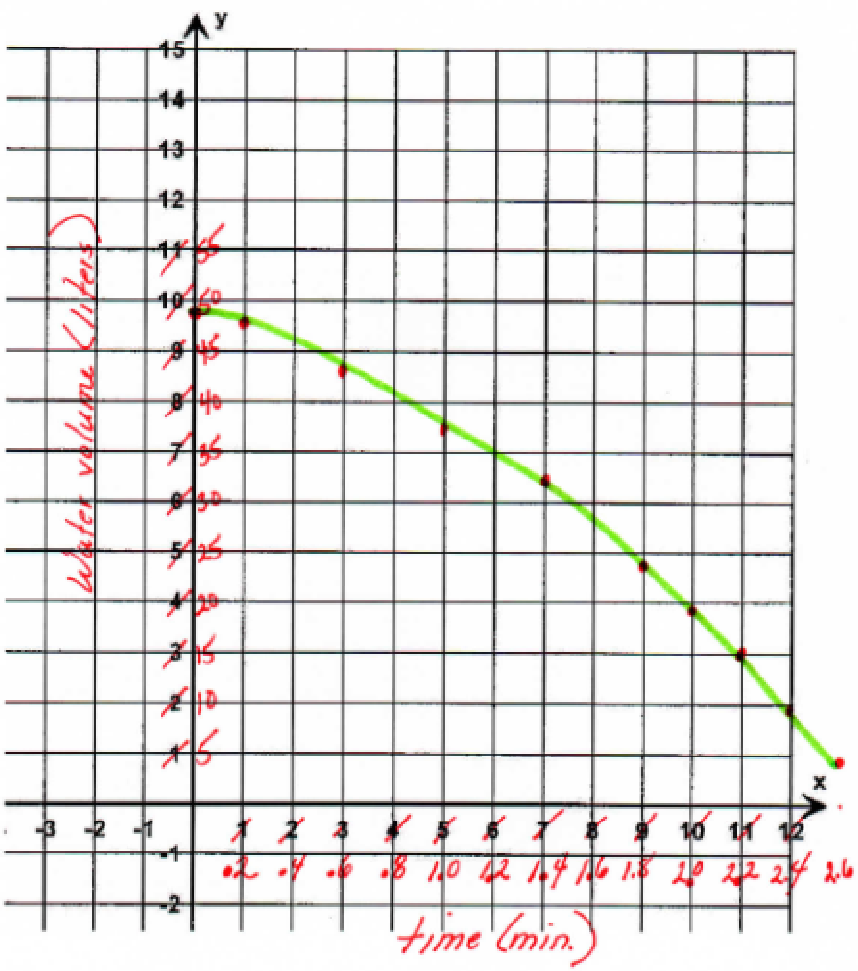


x time (sec)	y height (m)
0	58.8
.5	67.4
1.0	73.5
1.5	77.2
2.0	78.4
2.5	77.2
3.0	75.5
3.5	67.4
4.0	58.8
5.0	34.3
6.0	0

An experiment of the relationship between the amounts of water draining from a bathtub as it relates to time resulted in the following data. A model of the data resulted in the function $f(x) = -4x^2 - 6.8x + 49.2$ where $f(x)$ is the amount of water in liters and x is time in minutes. Graphic and tabular representations are below.

1. How much water was in the tub at the beginning of the experiment?
2. Do you see something in the function equation that would have told you the amount of water in the tub without seeing the graph or table?
3. How long did it take the tub to empty?
4. Use your graphing calculator to find the maximum of this function. Is this a useful piece of information? Why?

Problem B

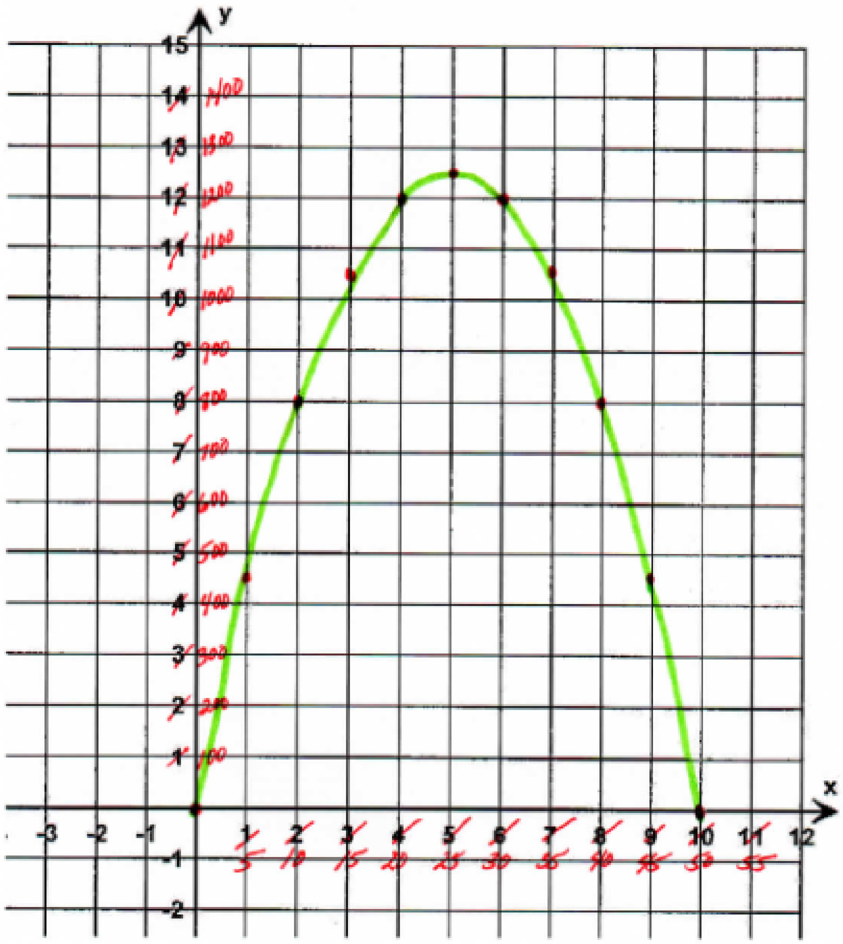


x time (min)	y water (liters)
0	49.2
.2	47.7
.6	43.7
1.0	38.4
1.4	31.8
1.8	24
2.0	19.6
2.2	14.9
2.4	9.8
2.6	4.5
2.76	0

The members of the Young Entrepreneurs Club decide to sell T-shirts in their school colors for Spirit Week. Club members find that at a price of \$20 they would sell 60 T-shirts. For each \$5 increase in price, they would sell 10 fewer T-shirts. A model for this situation yields $r(p) = -2p^2 + 100p$ where r represents revenue in dollars and p represents price of the shirts in dollars. Graphic and tabular representations are below.

1. If club members want to receive at least \$1,050 in revenue what price should they charge for the T-shirts?
2. What is the maximum revenue they can receive?
3. Why would the revenue decline as the price increases?

Problem C

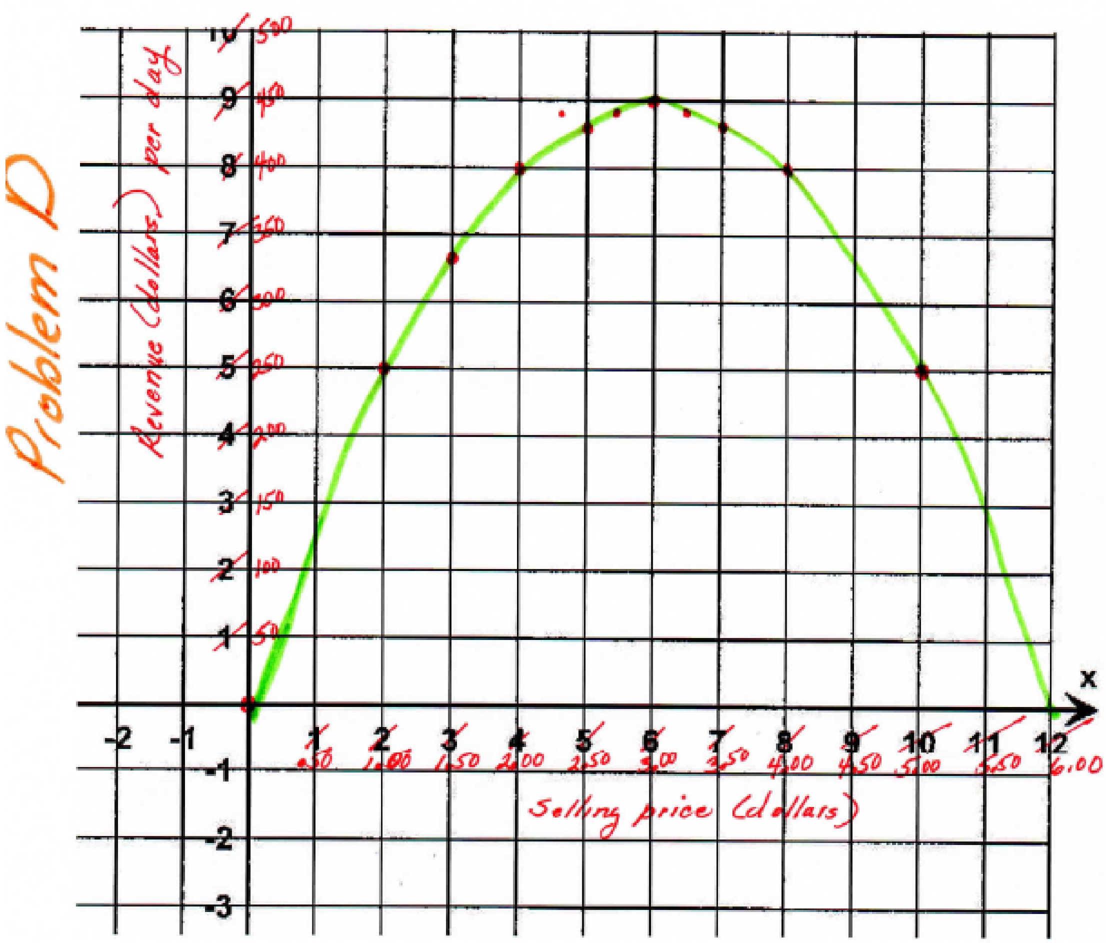


price x (dollars)	revenue y (dollars)
0	0
5	450
10	800
15	1050
20	1200
25	1250
30	1200
35	1050
40	800
45	450
50	0

A local outlet store charges $2.00 for a pack of four AA batteries. On an average day, 200 packs are sold. A survey indicates that the number sold will decrease by 5 packs per day for each $0.10 increase in price. The situation can be modeled by $r(p) = -50p^2 + 300p$ where r is revenue in dollars and p is price in dollars. The relationship is graphed below.

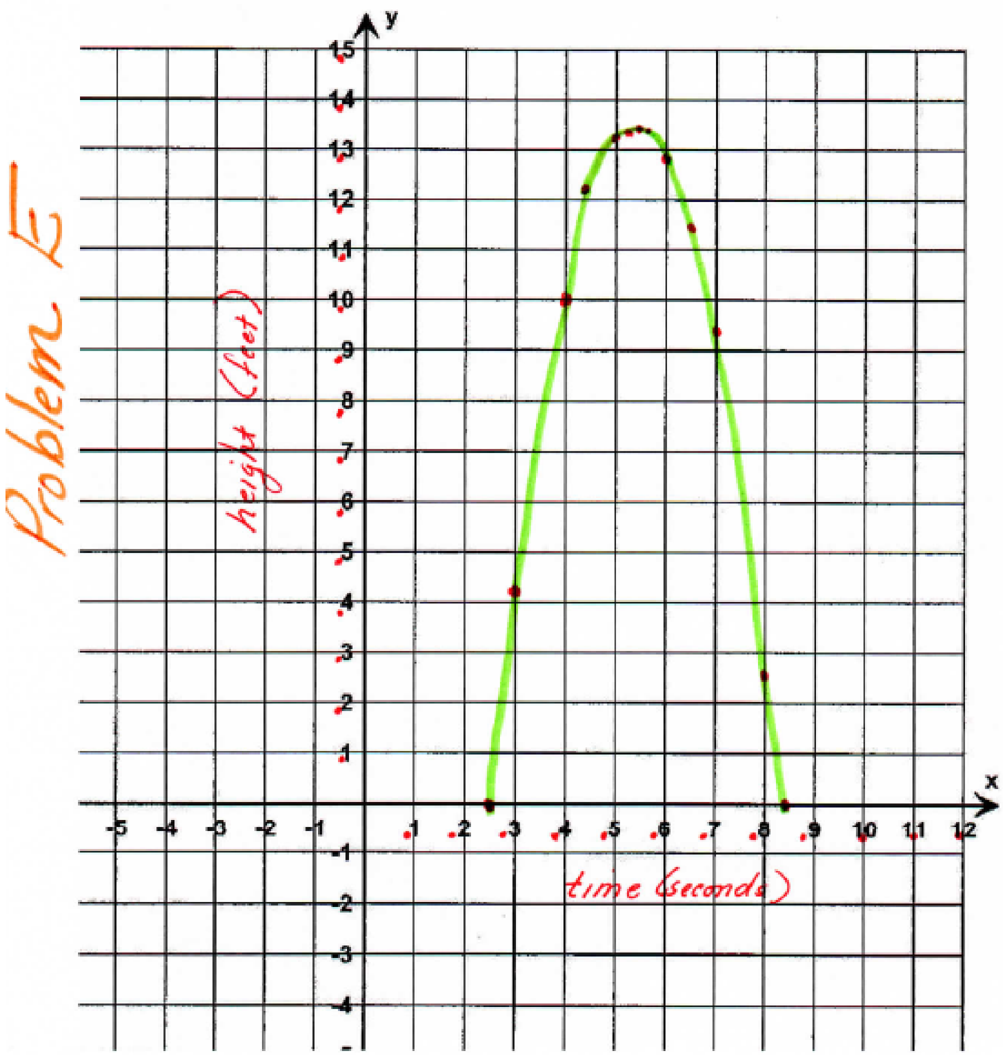
1. What is the selling price that provides the maximum revenue?
2. What is the maximum revenue?
3. Why does the revenue decrease is the price is increased?

price x $	0	1	1.5	2	2.5	2.7	2.8	2.9	3	3.1	3.2	3.5	4.0	5.0	6.0
revenue y $	0	250	337	400	438	445	448	449.5	450	449.5	448	437.5	400	250	0



A stopwatch records that when Julie jumps in the air, she leaves the ground at 0.25 seconds and lands at 0.83 seconds. The data was used to model the situation and the resulting equation was $h(s) = -16\,(s - 0.25)(s - 0.83)$ where h is height in feet and s is seconds. The relationship is graphed below.

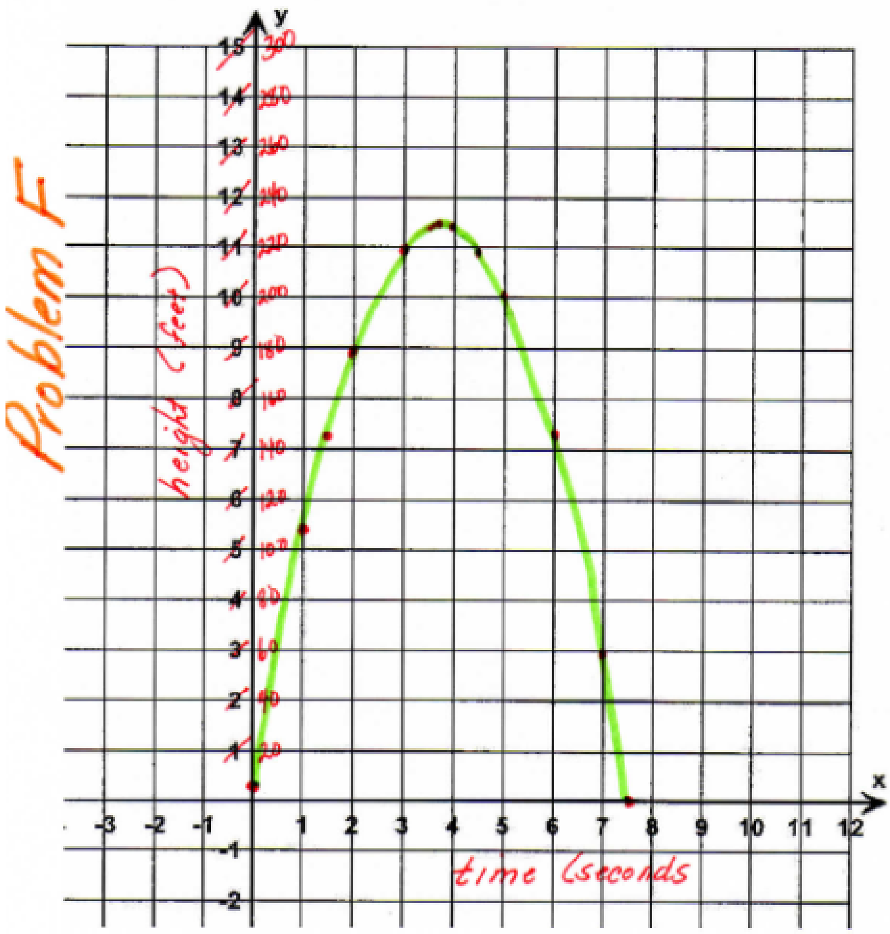
1. What was Julie's maximum height reached?
2. How long was she in the air?
3. Downward acceleration is $-16 ft/s^2$ which is the -16 in the model equation. What other relationship do you see in the model equation?



x	y
.25	0
.30	.42
.40	1.0
4.5	1.22
5.0	1.32
.53	1.34
.54	1.35
.55	1.34
.60	1.29
.65	1.15
.70	.94
.80	.26
.83	0

Nora hits a softball straight up at a speed of 120 ft/s . If her bat contacts the ball at a height of 3 ft above the ground, the situation can be modeled by $h(t) = -16t^2 + 120t + 3$ where h is height in feet and t is time in seconds. The relationship is graphed below.

1. When does the ball reach its maximum height?
2. How high does the ball travel?
3. When will the ball reach the ground?
4. What does the 3 at the end of the equation represent?
5. Why does the graph not start with (0,0)?



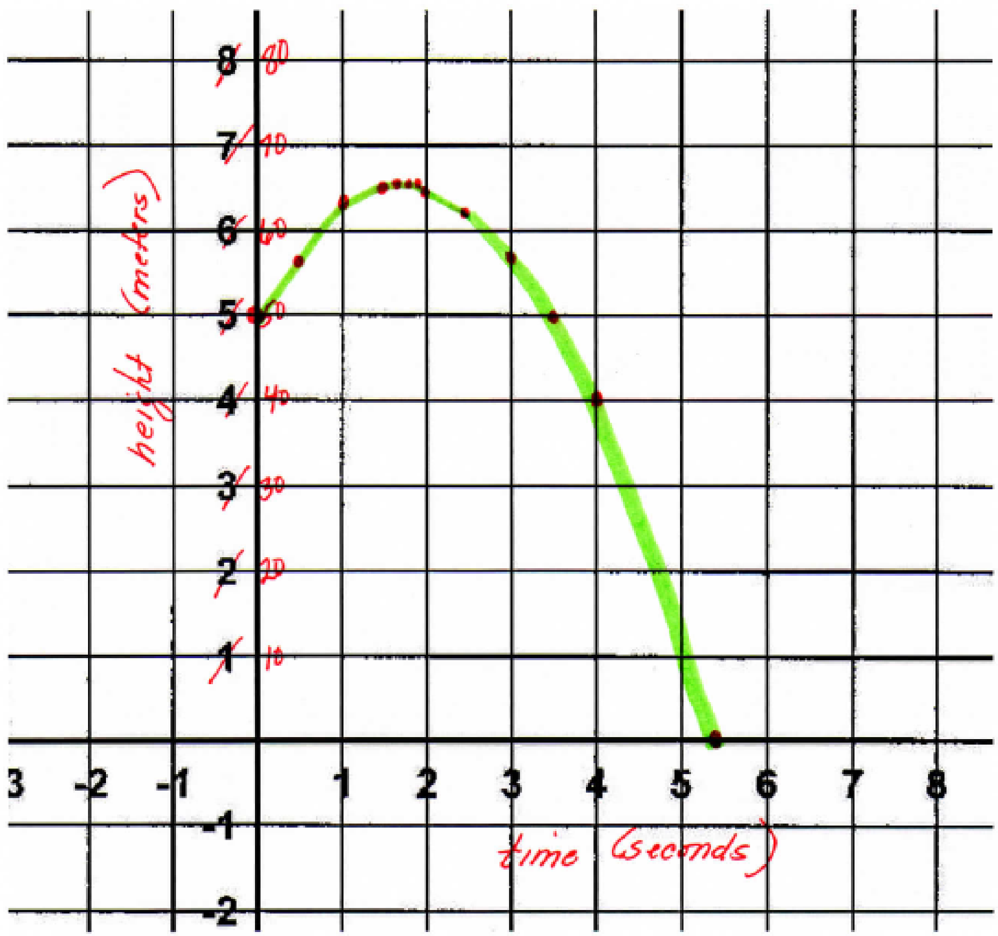
x time (s)	y height (ft)
0	3
1	107
1.5	147
2.0	179
3.0	219
3.4	226
3.5	227
3.75	228
4.00	227
4.5	219
5	203
7	59
7.5	0

A rock is thrown upward from the edge of a cliff overlooking Lake Superior, with an initial velocity of 17.2 m/s. A model of this situation can be reflected as $h(t) = -4.9t^2 + 17.2\,t + 50$ with h representing height in meters and t representing time in seconds. This function is graphed below.

1. How long does it take the rock to reach its maximum height?
2. What is the maximum height the rock reaches?
3. How long will it take to hit the ground?
4. How high is the cliff that the rock is thrown from?

x time (sec)	0	.5	1	1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4.0	5.4
y height (m)	50	57	62	65	65.0	65.1	65.1	65.0	65	62	57	50	40	0

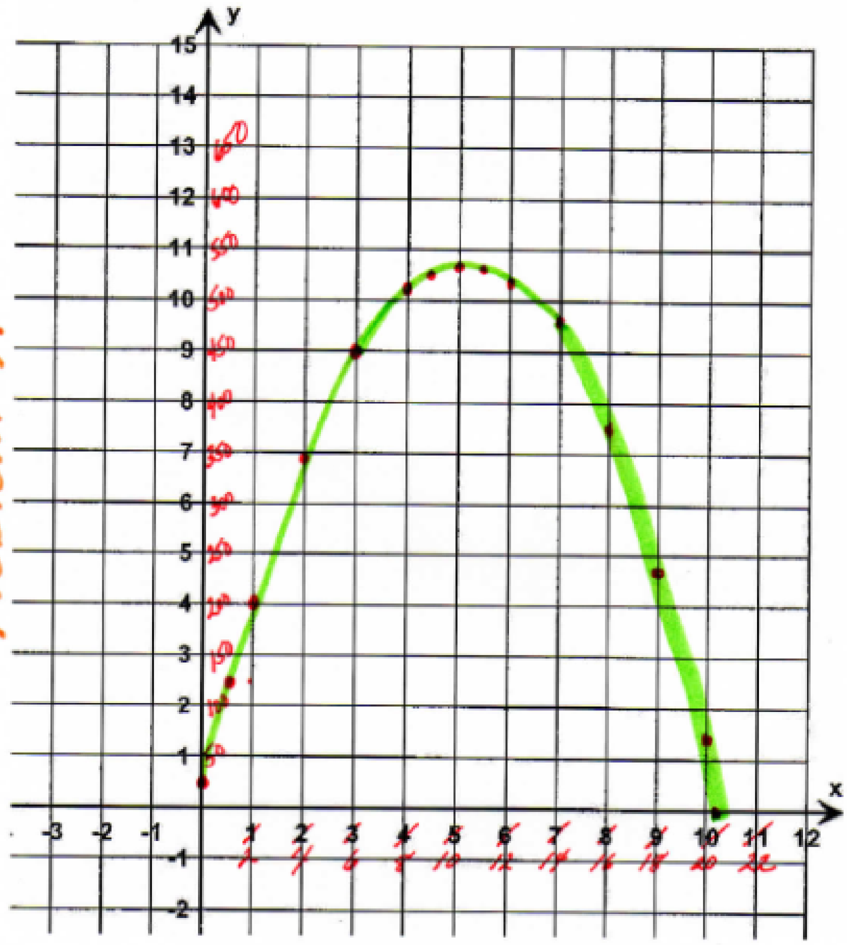
Problem G



Problem H

An object is projected upward, and this data was collected. A model indicates a relationship $h(t) = -4.9t^2 + 100t + 25$ where h represents height in meters and t represents time in seconds. A graph of the relationship is below.

1. What was the initial height of the object (at time = 0 seconds)?
2. What is the maximum height the projectile reaches?
3. How long will it take for the object to reach the maximum height?

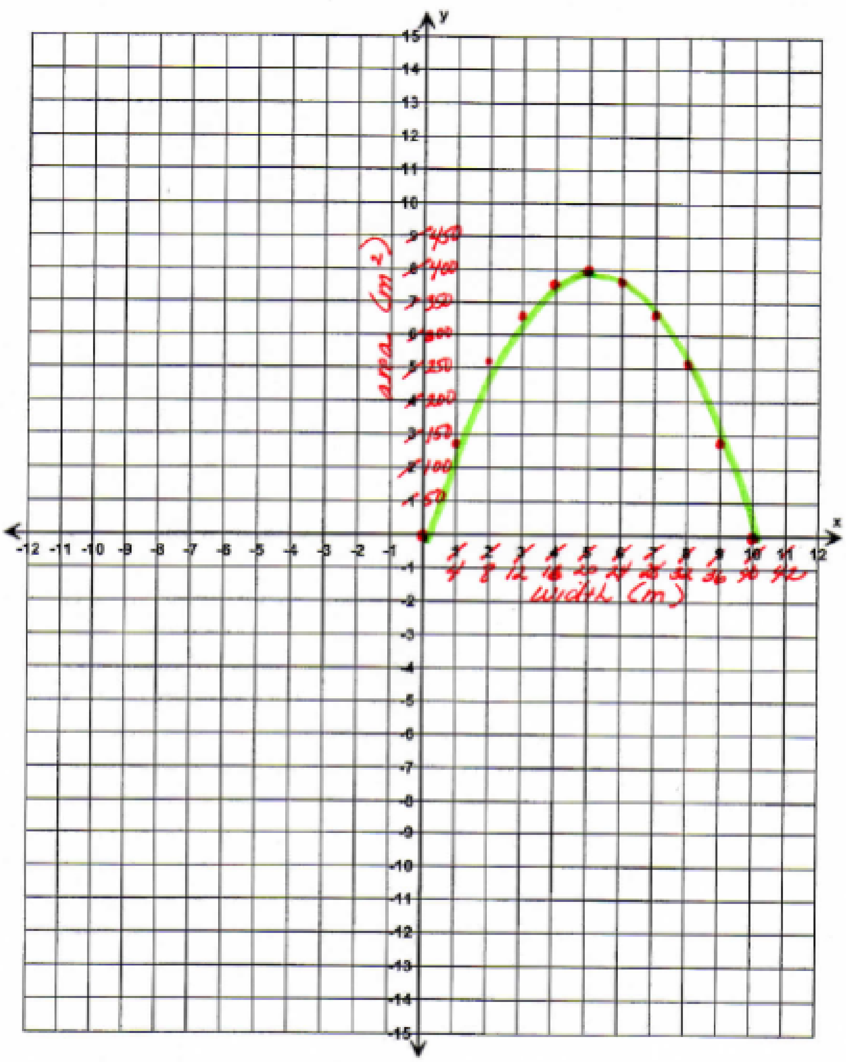


x time (sec)	y height (meters)
0	25
1	120
2	205
6	449
7	485
8	511
9	528
10	535
11	532
12	519
14	465
18	237
20	65
20.66	0

Problem I

Delores has 80 meters of fence to surround an area where she is going to plant a vegetable garden. She wants to enclose the largest possible rectangular area. The model for this situation is $a(w) = -w^2 + 40w$ where a is area and w is width. The relationship is graphed below.

1. Which width provides the largest possible area?
2. What is the largest possible area?
3. Which width will result in an area of 256 meters squared?
4. Why are there no negative points to plot on the graph?

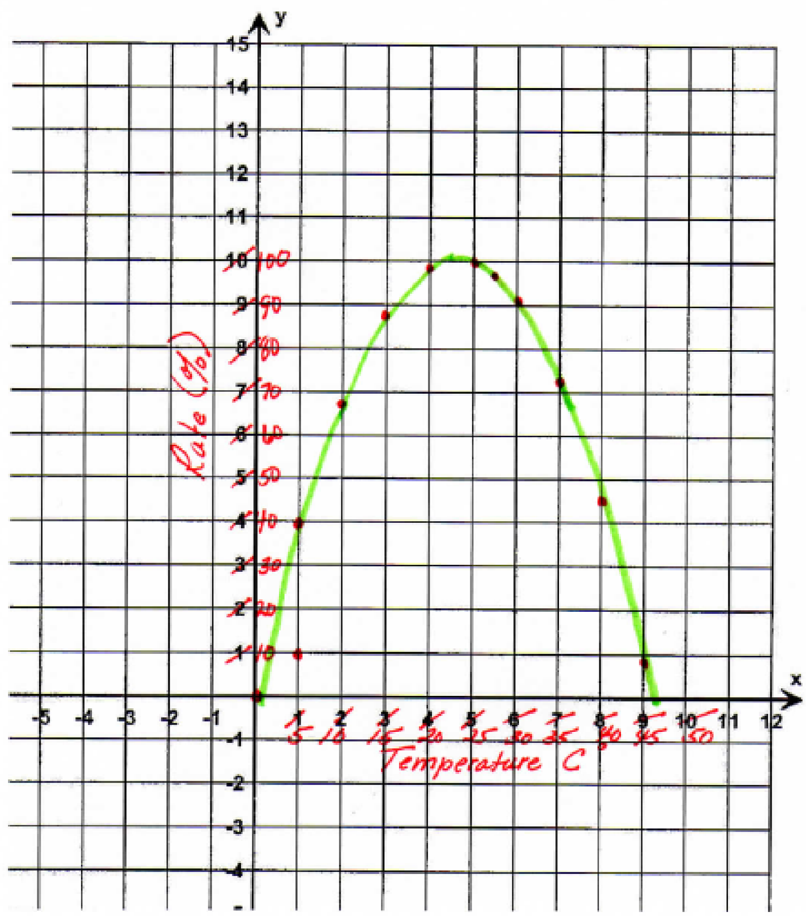


x width (m)	y area (m²)
0	0
4	144
8	256
12	336
16	384
20	400
24	384
28	336
32	256
36	144
40	0

Problem J

Photosynthesis is the process by which plants use energy from the sun, together with CO_2 (carbon dioxide) and water, to make their own food and produce oxygen. Various factors affect the rate of the photosynthesis, such as light intensity, light wavelength, CO_2 concentration, and temperature. The situation can be modeled with $r(T) = -0.19\,(T - 23)^2 + 100$ where r is the rate of photosynthesis and T is the temperature in C for a particular plant with all other factors held constant. The relationship is graphed below.

1. Based on this model, what is the optimum temperature for photosynthesis?
2. At what temperature(s) does the rate of photosynthesis fall to zero?
3. What temperature range would keep the rate of photosynthesis in the optimum rant of 80% to 100%?
4. Which resource is easier to use to answer the questions on this problem, the graph of the table?



x Temp (C)	y Rate (%)
0	0
5	39
10	68
15	88
20	98.8
21	100.3
23	100.5
24	100.3
25	99.8
30	91
35	73
40	46
45	9
46	0

Day 10: Quadratic Applications Day 2

Objective:
We shall analyze quadratic application word problems and the process of solving them and we will share our knowledge through guided and independent practice.

Warm-up:
Solve the following quadratic using all methods.
1) $2x^2 = 20x - 48$

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula, Completing the Square, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) Warm-Up
2) Guided Practice
   - Guide students through several problems in "Solving Quadratic Application Problems"
   - Allow students to read and choose a couple of problems that they want to be guided through.
3) Independent Practice
   - Have students finish the "Solving Quadratic Application Problems" Packet
   - Answer any and all question students may have in the packet.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Copies of "Solving Quadratic Application Problems"

Closure:
Finish "Solving Quadratic Application Problems" Packet.

Name: ________________________________ Date: _______________ Period: _____

Solving Quadratic Application Problems

1. A model for a company's revenue is $R = -15p^2 + 300p + 12{,}000$ where p is the price in dollars of the company's product. What price will maximize revenue? What is the maximum revenue?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 30	y_{max} 20,000

2. The equation for the motion of a projectile fired straight up at an initial velocity of 64 ft/s is $h = 64t – 16t^2$, where h is the height in feet and t is the time in seconds. Find the time the projectile needs to reach its highest point. How high it will go?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 20	y_{max} 100

3. The equation for the cost in dollars of producing automobile tires is $C = .000015x^2 - .03x + 35$, where x is the number of tires produced. Find the number of tires that minimizes the cost. What is the cost for that number of tires?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 2000	y_{max} 100

4. Suppose you want to frame a collage of pictures. You have a 9-ft strip of wood for the frame. What dimensions of the frame give you the maximum area for the collage? What is the maximum area?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 10	y_{max} 10

Area = (length)(width) and
Area = (length)(4.5-length)
Area = $4.5L – L^2$

5. Suppose you are standing 10 feet from a fence and you throw a ball over the fence. Barely clearing the fence, the ball reaches its highest point directly above the fence. A model of the flight of the ball is $y = -.1x^2 + 10$. What is the height of the fence? Where does the ball land?

Estimate Domain	Estimate Range
x_{min} -15	y_{min} 0
x_{max} 20	y_{max} 20

6. A rock club's profit from booking local bands depends on the ticket price. Using past receipts, the owners find that the profit p can be modeled by the function $p = -15t^2 + 600t + 50$, where t represents the ticket price in dollars. What price yields the maximum profit? What is the maximum profit?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 30	y_{max} 10,000

7. A landscape architect is planning a playground. She wants to fence a rectangular space using an existing wall. What is the greatest area she can fence in using 100 ft of donated fencing?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 50	y_{max} 5000

Area = (length)(width)
= (length)(100 – 2lengths)
= L(100 – 2L) = $100L - 2L^2$

What is the maximum area? What is the length to create the maximum area?

8. The Big Brick Bakery sells more bagels when it reduces its prices, but then its profit changes. The function $y = -1000(x - .55)^2 + 300$ models the bakery's daily profit in dollars, from selling bagels, where x is the price of a bagel in dollars. The bakery wants to maximize the profit. What price should the bakery charge to maximize its profit from bagels? What is the maximum profit?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 3	y_{max} 1000

9. A smoke jumper jumps from a plane that is 1700 feet above the ground. The function $y = -16t^2 + 1700$ gives the jumper's height y in feet at t seconds during free fall. How long is the jumper in free fall if the parachute opens at 1000 feet?

Estimate Domain	Estimate Range
x_{min} 0	y_{min} 0
x_{max} 20	y_{max} 2000

10. A ball is thrown straight up at an initial velocity of 54 feet per second. The height of the ball t seconds after it is thrown is given by the formula h(t) = 54t – 12t2. How many seconds after the ball is thrown will it return to the ground?

11. A rock is thrown vertically from the ground with a velocity of 24 meters per second, and it reaches a height of 2 + 24t – 4.9t2 after t seconds.
    a. How many seconds after the rock is thrown will it reach maximum height and what is that maximum height, in meters?
    b. How many seconds after the rock is thrown will it hit the ground? Round your answer to the nearest hundredth.

12. A baseball player throws a ball from the outfield toward home plate. The ball's height above the ground is modeled by the equation $y = -16x^2 + 48x + 6$, where y represents the height, in feet, and x represents time, in seconds. The ball is initially thrown from a height of 6 feet.
    a. How many seconds after the ball is thrown will it again be 6 feet above the ground?
    b. What is the maximum height, in feet, that the ball reaches?

13. A model for a company's revenue is $R = -15p^2 + 300p + 12{,}000$ where p is the price in dollars of the company's product.
    a. What price will maximize revenue?
    b. What is the maximum revenue?

14. A rock club's profit from booking local bands depends on the ticket price. Using past receipts, the owners find that the profit p can be modeled by the function $p = -15t^2 + 600t + 50$, where t represents the ticket price in dollars.
    a. What price yields the maximum profit?
    b. What is the maximum profit?

Day 11: Quadratics Review

Objective:
We shall review all quadratics topics and we will share our knowledge through a scavenger hunt.

Warm-up:
Solve the following quadratic application problem:

A skating rink manager finds that the revenue, R, based on an hourly fee, F, for skating is represented by the function $R = -480f^2 + 3120f$.

a. What hourly fee will produce the maximum revenue?
b. What is the revenue at $5.00 per hour?

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula, Completing the Square, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) Warm-Up
2) Scavenger Hunt
   - Allow students to begin the scavenger hunt on whatever problem they are comfortable with.
   - Students will use regular lined, notebook paper to show their work.

Materials/Resources:
Starboard/Smart Board
Document Camera/Elmo
Notebook Paper
“Quadratics Review” Scavenger Hunt Posters

Closure:
Finish the “Quadratics Review” Scavenger Hunt.

Answer:

$$\frac{3 \pm \sqrt{5}}{2}$$

Question:

What type of roots will you get from the following quadratic?

$$5x^2 + x + 2 = 0$$

Answer:

Two Complex Roots

Question:

What type of roots will you get from the following quadratic?

$-3x^2 - 2x = -7$

Answer:

Two Real, Irrational Roots

Question:

What are the roots of the following quadratic?

$x^2 + 18 = 11x$

Answer:

$x = 2$ and $x = 9$

Question:

If the height of a rock dropped from a cliff is modeled by $H = -16t^2 + 64t + 75$, where height is feet and t is seconds, what is the height of the rock after 4 seconds?

Answer:

75

Question:

What type of roots will you get from the following quadratic?

$x^2 = -10x - 25$

Answer:
One Real Root

Question:
What type of roots will you get from the following quadratic?

$$x^2 - 16 = 0$$

Answer:

Two Real, Rational Roots

Question:

What are the roots of the following quadratic?

$$5x^2 + x + 2 = 0$$

Answer:

$$\frac{-1+i\sqrt{39}}{10} \text{ and } \frac{-1-i\sqrt{39}}{10}$$

Question:

What are the roots of the following quadratic?

$-3x^2 - 2x = -7$

Answer:

$$\frac{-1+\sqrt{22}}{3} \text{ and } \frac{-1-\sqrt{22}}{3}$$

Question:

What is the vertex, axis of symmetry, roots, domain, and range of the following quadratic?

$$x^2 + 6x + 8 = 0$$

Answer:

Vertex: (-3,-1)

Axis of Symmetry: x = -3

Roots: x = -2 and x = -4

Domain: (-∞,∞)

Range: [-1,∞)

Question:

What is the vertex, axis of symmetry, roots, domain and range of the following quadratic?

$$-x^2 - 6x + 8 = 0$$

Answer:

Vertex: (-3,17)

Axis of Symmetry: $x = -3$

Roots: $x = -7.12$ and $x = 1.12$

Domain: $(-\infty,\infty)$

Range: $(-\infty,17]$

Question:

If the cost function for making a product is $C = x^2 - 3x + 12$ where C is in thousands of dollars and x is in hundreds of units, what amount of units will result in the lowest cost?

Answer:
150

Question:
Make a table of values for the following quadratic:

$$-x^2 - 6x + 8 = 0$$

Answer:

X	Y
-5	13
-4	16
-3	17
-2	16
-1	13

Question:

Make a table of values for the following quadratic:

$$x^2 + 6x + 8 = 0$$

Answer:

X	Y
-5	3
-4	0
-3	-1
-2	0
-1	3

Question:

Find the roots of the following quadratic:

$$x^2 = 3x - 1$$

Day 12: Quadratics Assessments

Objective:
We shall assess student knowledge of all quadratic topics and we will share our knowledge through an in class and take home paper-based assessment.

Warm-up:
None

Vocabulary:
Quadratic Function, Roots, Zeros, X-Intercepts, Solutions, Discriminant, Real Roots, Irrational, Rational, Imaginary/Complex Roots, Quadratic Formula, Completing the Square, Parabola, Standard Form, Vertex Form, Axis of Symmetry, Vertex, Y-Intercept, Maximum, Minimum, Domain, Range

Instructional Plan:
1) In Class Assessment "In Class Quadratics Test"
2) Take Home Assessment
   - Students will choose one of the following quadratics and solve it using the "Solving Quadratics Four Ways" Worksheet from Day 8.
     - $f(x) = 3x^2 – 4x – 2$
     - $f(x) = 2x^2 – 2x – 1$
     - $f(x) = 2x^2 + 6x + 5$
     - $f(x) = 2x^2 + 3x – 1$
     - $f(x) = 3x^2 – 5x – 4$
     - $f(x) = 4x^2 + 4x - 15$

Materials/Resources:
Copies of "In Class Quadratics Test"
Copies of "Solving Quadratics Four Ways"

Closure:
Students begin working on the take home assessment.

Name: ______________________________ Date: ___________ Period: _________

In Class Quadratics Test

BE SURE TO SHOW ALL WORK TO RECEIVE FULL CREDIT!

1. Find the discriminant value of the following quadratic **and** describe the roots.

$$\mathbf{2x^2 + 6x = -12}$$

2. Find the roots of the quadratic by using the quadratic formula.

$$\mathbf{x^2 + 10x = -2}$$

3. Find the zeros of the quadratic by factoring.

$$\mathbf{x^2 + 8x - 9 = 0}$$

4. Find the roots of the quadratic by completing the square.

$$\mathbf{x^2 = 27 - 6x}$$

5. List/fill in the characteristics of the following quadratic.

$$\mathbf{x^2 + 7x + 10 = 0}$$

Opens Up / Opens Down   Vertex: _____________________

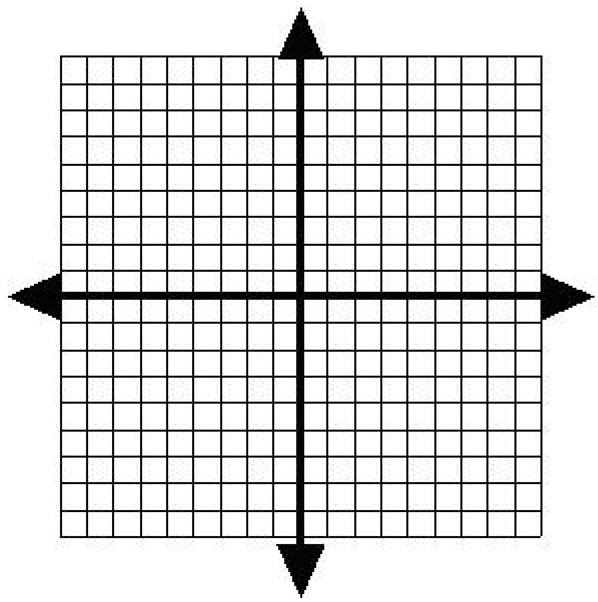
Axis of Symmetry: ___________________   Roots: ____________________

Domain: ______________________   Range: _______________________

6. Make a table of x and y values for the following quadratic. Use the table to graph the quadratic.

$$\mathbf{-x^2 - 5x + 6 = 0}$$

X	Y



7. For a model rocket, the altitude h, in meters, as a function of time t, in seconds, is given by the function $h(t) = 68t - 4.9\,t^2$.

   a. Find the amount of time it takes the rocket to return to the ground.

   b. How high is the rocket at 3 seconds?

8. A lighting fixture manufacturer has daily production costs of $C = 0.25n^2 - 10n + 800$, where C is the total daily cost in dollars and n is the number of light fixtures produced.

   a. How many fixtures should be produced to yield a minimum cost?

   b. What would the daily cost be for making 25 fixtures a day?

Chapter 4: Discussion

For the reader, there may be questions to how and why this unit plan decreases math anxiety in the classroom. The activities and assessments used throughout this unit align with research concerning how to decrease math anxiety in students. Starting with day1, the difference from what a typical lesson may be is the closure activity. Students are asked to write, in their own words, the definitions to the vocabulary used in the lesson. Incorporating writing allows the student to think through what they have just learned, deepening their understanding of the concept and solidifying it in their long term memory (Furner & Duffy, 2002). Another reason why this activity may decrease math anxiety is because the teacher can use this closing activity as a form of assessment without the students even knowing. Finding discrepancies and correcting them early in the student's learning process will save the student from having to relearn material in the future. This activity will also inform the teacher of any concepts that will need to be retaught or corrected before moving on to the next topic.

In the lesson plan for Day 2, the debriefing time is very important for students with math anxiety. Having class discussions about a mathematical topic provides students who have math anxiety insight into what peers as well as teachers perceive about a topic. It is even more beneficial when students hear from their peers who are having the same struggles because they then know that they are not alone (Furner & Duffy, 2002). During this time, the teacher can also address positive and negative results and observations from the previous day's closure activity as well as reteach any topics that seemed lacking. After this debriefing time, students will work in cooperative groups. As previously mentioned, cooperative groups significantly reduce math anxiety because students tend to feel more comfortable asking peers' questions and this will allow them to learn and understand the material better (Miller & Mitchell, 1994). This activity also provides the opportunity for students to learn through presenting and teaching their fellow group mates. This aligns with using a variety of teaching strategies, which is one of the proposed methods of reducing math anxiety.

Day 3 used two activities with manipulatives. Students who are taught using manipulatives have an easier time developing concrete understanding of the topic (Stodolsky, 1985; Miller & Mitchell, 1994; Furner & Duffy, 2002). This is especially important for students who have math anxiety because their memory tends to shut down when confronted with math problems (Ashcraft & Kirk, 2001). The matching activity also helped decrease math anxiety because students were working with partners.

After the warm-up in Day 4, students were again given the opportunity to debrief and have a class discussion about the domino activity from the previous class. After the discussion, the teacher then led the class through some direct instruction. Though commonly found in the typical mathematics classroom, direct instruction also has its rightful place in the classroom that is attempting to decrease math anxiety. Direct instruction is simply another teaching style and some students truly learn at the optimum rate when it is used. With that said, direct instruction should not be the dominant teaching style used, as shown throughout this unit plan. The "Matching Zeros" activity that follows the direct instruction serves two purposes. First, students are again working in cooperative groups. Secondly, this activity can again be used as an informal assessment. Students may just think they are completing another group activity, but this activity will inform the teacher concerning student understanding of the material. Likewise, Day 5 may be a typical mathematics lesson with direct instruction. However, the ticket out the door activity can again be used to assess the students understanding midway through the unit.

Day 6 begins with another debriefing, this time about the previous day's ticket-out-the-door activity. It then leads to a short amount to direct instruction and a scavenger hunt activity. Scavenger hunts are great for decreasing math anxiety because they are kinesthetic activities and student paced. Rather than sitting in their desks all period, students are up moving around the room *doing* mathematics. Also, since this is a student-paced activity, students who have math anxiety may not feel the pressure to keep up with the rest of the class. Lastly, the advantage of the scavenger hunt is that the answers are somewhere around the room. This gives students immediate feedback into if they are correctly solving the problems. If they are not, they can then ask for assistance from the teacher or another student.

The remaining lessons of the unit are mostly composed of strategies that have been previously discussed. Days 7 and 8 utilize discussions, group practice, and writing, which are key components to disabling math anxiety. Day 9 is another group activity, which is more along the exploring side of learning rather than simply practicing. Students are interpreting graphs and tables without being given any previous instruction. This type of activity may be helpful to students with math anxiety because it starts them on a level playing field with the other students. No one has been taught how to solve the problems so everyone in the group needs to work together to come up with the answers and teach themselves. Though they may not be as quick to learn the concept as other students, it still shows them that their peers can have the same struggles that they have. For days 10 and 11, students have a warm-up, direct instruction, and independent practice. While this is typical for the mathematics classroom, it is not recommended that teachers use such pedagogical practices every day if the teacher has a goal to decrease math anxiety.

The final day of the unit is the assessment. What makes this assessment different from a typical assessment is that there is a short in class portion and a take home portion. As discussed earlier in this paper, assessments given to students with math anxiety should not have a time limit. This is true for this assessment. As it is, time should not even be a factor since the in class portion has eight questions. Secondly, for the take home portion of the exam, students are allowed to choose which two problems they would like to complete. Giving the students a choice relieves the pressure of not knowing what to expect on the assessment. Having the take home portion of the exam also eliminates the problem of time limits. Combining this complete assessment with the previous closure activities, teachers should have a well-rounded view of what the students know about quadratics.

In conclusion, math anxiety, if left untreated, can create obstacles for students trying to learn mathematics and can have negative effects on their performance. For many, this leads to a diminished understanding of mathematical concepts and low achievement. However, through the use of class discussions, writing, various teaching styles, and informal and untimed assessments, math anxiety has been shown to decrease in students. Mathematics teachers should include these strategies in their repertoire as math anxiety continues to be an increasing problem in the classroom.

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