

Implementing Technology in a Geometry Curriculum

Stephanie Zimmerman

The College at Brockport, State University of New York (SUNY)

Abstract

This curriculum project was designed to assist teachers to implement technology into the Common Core Geometry Curriculum and was created by a second career teacher who supported Geometry classes as a teaching assistant. Technology has become deeply rooted in the lives of students from birth and studies show that using technology in the classroom can impact their learning. This curriculum project includes lessons from across the geometry curriculum using various technological tools.

Keywords: Digital Natives, Active Learning, Instructional Technology, Technological Pedagogical Content Knowledge (TPACK)

Table of Contents

1. Abstract.....	2
2. Introduction.....	4
3. Purpose Statement.....	5
4. Literature Review.....	5
a. Technology & Education.....	5
b. Implementing Technology & Technological Pedagogical Content Knowledge	6
c. Standards for Mathematics Teaching.....	8
5. Curriculum.....	10
a. Identifying Congruent Triangle Methods.....	12
b. Practice with Rigid Motions.....	19
c. Triangle Theorems.....	29
6. Conclusion.....	39
7. References.....	40
8. Appendices – Student Answer Key.....	44
a. Appendix I: Identifying Congruent Triangle Answer Key.....	44
b. Appendix II: Practice with Rigid Motions Answer Key.....	47
c. Appendix III: Triangle Theorems Answer Key.....	51

Implementing Technology in a Geometry Curriculum

Introduction

Today's society relies on different forms of technology in every aspect of life. Education is no different. As time goes on, new tools and resources are available to teachers, but they are not always prepared to use instructional technologies effectively (Önal, 2016). In math especially, it is becoming increasingly important that teachers are well versed in using technology in the instructional process to create an interactive learning environment for students (Önal, 2016). Active learning is a broad term that encompasses different pedagogical methods, like implementing technology, to “engage students in their learning and help to improve and retain higher level learning” (Nicol, Owens, Le Coze, MacIntyre & Eastwood, 2018, p. 254). Research shows that by redesigning classroom practices to include well-implemented technology and tools, students display more favorable attitudes towards the class and may help foster their motivation to engage with the material (Nicol et al., 2018).

Even with so much technology available to teachers, some are slow to adopt these tools into their classroom (Hora & Holden, 2013). Studies indicate that this is happening because teachers do not know how to adapt different tools for their own teaching styles (Hora & Holden, 2013). Common Core State Standards for Mathematics (CCSSM) “calls for students to learn mathematics through a variety of practices including looking for and making use of structure” and “using appropriate tools strategically” (Contreras, 2015, p. 26). Since this is required of students, teachers must also be prepared to accomplish the tasks set forth by CCSSM. It is beneficial if both pre-service and in-service teachers work collaboratively to stay current with technology designed for implementation into the mathematics classroom (Dalal, Archambault &

Shelton, 2017). If educators do not have the ability to use technology themselves they will not be able to apply them efficiently to support student learning (Dalal et al., 2017).

Purpose Statement

The purpose of this curriculum project is to provide teachers with lessons that include technology as a tool to support student learning across the geometry curriculum. Technology is a significant factor in everyday life and there is a considerable number of new resources available to teachers. Such technological tools can assist teachers in building students' conceptual understanding, mathematical reasoning and procedural fluency.

Literature Review

Technology & Education

Teachers are facing a new generation of students in their classrooms that use technology in many aspects of their lives (Saralar, Işıksal-Bostan & Akyüz, 2018). Students are now considered “digital natives” who are born into technology rather than learning how to use technology like students of the past (Saralar et al., 2018, p. 3). This makes integrating technology “an important part of effective teaching programs” (Kirişçılar & Yıldız, 2018, p. 101). Educators use technology as a tool to foster student motivation (Nicol et al., 2018) and can be very effective when implemented correctly in a mathematics classroom. Technology can also be used to support students' development of mathematical thinking and problem-solving skills (Kirişçılar & Yıldız, 2018). For example, a computer can easily create a graph or figure that students can manipulate and interact with, allowing them to push their mathematical reasoning deeper than before (Schoenfeld, 2014).

There are a significant number of paid and free resources available to teachers that give students the opportunity to “concretize the abstract concepts” in their mathematics courses

(Kirikçilar & Yildiz, 2018, p. 113). This can make a significant and positive difference for students when used in ways to “enhance students’ experience of the discipline” (Schoenfeld, 2014, p. 742). Quizlet, Kahoot!, and Nearpod are all online applications available to teachers, and according to some research “students were passionately involved when their teachers used these applications (Hoffman & Ramirez, 2018, p. 55). Interactive Geometry Software (IGS) like Geometer’s Sketchpad and GeoGebra is one tool that can be used to facilitate the execution of “important mathematical habits, processes, and practices” (Contreras, 2015, p. 26). Saralar et al. (2018) found the use of a dynamic geometry software, like GeoGebra, can help students who have difficulty visualizing concepts engage with mathematics and begin to overcome such difficulties. When students have more experience using technology like “dynamic diagrams to visualize relationships and patterns, they may overcome some of the visual obstacles associated with static diagrams” (Contreras, 2015, p. 32).

Appropriate technology can also act to even the playing field for students with disabilities (Bouck & Flanagan, 2009). Bouck and Flanagan (2009) found that assistive technology, like calculators and computer-assisted instruction, “may be a means of providing assistance by increasing access to mathematical ideas and helping them experience higher levels of success” (p. 18). Technology has major potential to enhance student learning, but teachers need to know how to implement it in meaningful ways to have the biggest impact on students (Saralar et al., 2018).

Implementing Technology & Technological Pedagogical Content Knowledge

Technological Pedagogical Content Knowledge, or TPACK, has been “accepted as a framework to understand and describe types of knowledge required by teachers to teach specific content with technology effectively” (Patahuddin, Lowrie & Dalgarno, 2016, p. 863). This

framework is comprised of the intersections between seven different sub-knowledge categories shown in Figure 1: “Technological Knowledge (TK), Pedagogical Knowledge (PK), Content Knowledge (CK), Technological pedagogical Knowledge (TPK), Technological Content Knowledge (TCK), Pedagogical Content Knowledge (PCK), and Technological Pedagogical Content Knowledge (TPACK)” (Kirikçilar & Yildiz, 2018, p. 102).

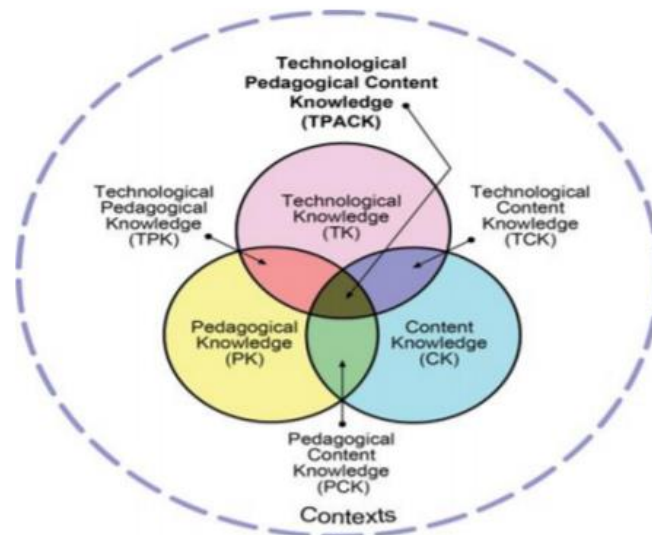


Figure 1. Illustration of the TPACK Framework (Koehler & Mishra, 2008)

There is a significant amount of enthusiasm and desire for teachers to use TPACK more in their classrooms (Schmidt et al., 2009), however, teachers need to be prepared to teach with technology effectively since the presence of technology doesn't guarantee any change or improvement in student learning (Mishra & Koehler, 2006; Schoenfeld, 2014). TPACK “emphasizes the connections, interactions, affordances, and constraints between and among content, pedagogy, and technology” (Mishra & Koehler, 2006, p. 1025). It can be used to “unpack the complexities of teaching with technology” (Foulger, Buss, Wetzel, & Lindsey, 2015, p. 134), but even with the overwhelming support for this definition of TPACK, “there is no consensus on what these components should involve” (Önal, 2016, p. 94). Teachers should be

using the intersection of all these components as they teach, rather than using them as separate entities (Saralar et al., 2018).

Schmidt et al. (2009) describe TPACK as a method of “designing and evaluating teacher knowledge that is concentrated on effective student learning in various content areas,” including mathematics (p. 125). Educators should be practicing TPACK when developing lessons so that the technology being used complements their teaching methods to help overcome any challenges that students may face (Patahuddin et al., 2016). This plan grants teachers the ability to infuse more technology and tools into their teaching practices more effectively so that students feel supported (Foulger et al., 2015). It is imperative that more professional development opportunities beyond an introduction to TPACK are available so that teachers can continue their evolution through this process (Foulger et al., 2015).

Dalal et al. (2017) found that it was key for teachers to “build their technology skills collaboratively” (p. 117). Studies have found that teachers who work in teams to design lessons that include technology are better able to implement those tools when in front of their students (Schmidt et al., 2009). Teachers working towards better implementing technology also need to be flexible and current with the tools that they are using (Mishra & Koehler, 2006). Technology is constantly updated and training teachers to only use specific tools “makes their knowledge too specific to be applied broadly, but it also becomes quickly outdated” (Mishra & Koehler, 2006, p. 1032). Teachers must find tools or applications that will support their students’ learning rather than hinder it (Hora & Holden, 2013). Fluency in educational technology is more than just competence with the latest tools but having a deeper understanding of the relationships between the technology, tools, users, and teaching practices (Koehler & Mishra, 2005).

Standards for Mathematics Teaching

There are a “plethora of state and national technology standards that have been implemented recently and that emphasize enhancing teachers’ knowledge of current versions of hardware and software” (Mishra & Koehler, 2006, p. 1031). The National Council of Teachers of Mathematics (NCTM) *Standards* provide many standards that are directly related to technology (Bouck & Flanagan, 2009). They began a movement towards using standards in 1989 by releasing *Curriculum and Evaluating Standards*, then updated those standards with the release of 2000’s *Principles and Standards for School Mathematics* (Schoenfeld, 2014). “NCTM’s *Standards* articulated goals and objectives and promoted the council’s vision for K-12 mathematics education” (Dingman, Teuscher & Newton, 2013, p. 542). These standards also stipulate that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (NCTM, 2000, p. 24). Many teachers look to NCTM for material and teaching practices, but the Common Core State Standards for Mathematics (CCSSM) are the current standards in place in education (Schoenfeld, 2014).

CCSSM focuses on “two deeply intertwined aspects of mathematics: the content people need to know, and the knowhow that makes for its successful use, called mathematical practices” (Schoenfeld, 2014, p. 737). However, the ideas behind CCSSM are not new (Schoenfeld, 2014, p. 739). The Common Core standards in effect currently are different from prior standards, and the materials and practices teachers are using must adapt to them (Bostic & Matney, 2013). A major goal of CCSSM is to support teachers as they work to engage students in thinking mathematically about what they know and what they need to know (Schoenfeld, 2014). One strength that these standards have is that in some cases, mathematical topics are given expanded

coverage (Dingman et al., 2013). This means that topics like addition and subtraction of whole numbers, which was included across three grade levels under old standards are now included across five grade levels (Dingman et al., 2013).

Throughout nearly every content strand of CCSSM, there is an emphasis on modeling and how much it can impact a student's understanding and ability to solve problems (Bostic & Matney, 2013). This is where technology and tools can make a difference for students, especially in geometry and measuring tasks (Bostic & Matney, 2013). CCSSM states that students should "use appropriate tools strategically, with technology being one of these tools" (Dingman et al., 2013, p. 555). Students, however, are unable to improve their modeling and technology skills without the opportunity to practice in the classroom where they can receive feedback on their progress (Schoenfeld, 2014). As teachers continue to implement CCSSM into their lessons plans, they must also implement technology as well to give their students more opportunity to learn.

Curriculum

This curriculum plan includes three lessons across the Common Core Geometry standards using technology to engage students. Each lesson uses either laptops or Chromebooks to access the online applications and programs. The goal for this curriculum plan is to show teachers that there are programs available for use throughout the year and how to implement them into lessons. The programs used here give students the opportunity to explore the content more and have reference to look back on if they need that.

These lessons are not from the same unit of study but are meant to show how technology can be included across the Geometry curriculum to promote conceptual understanding. Identifying triangle congruence methods is the focus of the first lesson and acts an introduction to triangle congruence proofs. Both the NCTM Illuminations and NearPod activities give the

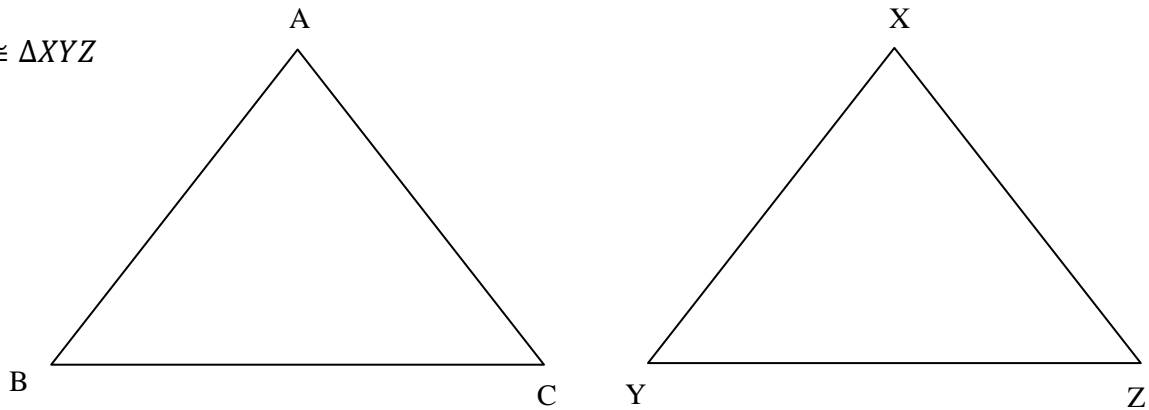
students visuals that they may not see without these programs. The focus of the second lesson is on rigid motions and how to analyze them. This lesson includes an activity from both Interactivate and Kahoot!. Interactivate allows the students to explore what happens to different figures when a rigid motion is applied and attempt to determine the process on their own. The Kahoot! quiz can be accessed from their computers or phones and is an ideal time for any teacher to assess how the students are understanding the materials. Lastly, this plan includes a lesson on the different triangle theorems that students need to learn. Students will use GeoGebra to model how the theorems were created and why they hold true. They are often difficult to visualize independently and GeoGebra is a great tool for students.

Course:	Geometry
Unit:	Triangle Congruence
Lesson Title:	Identifying Congruent Triangle Methods
Standards:	G.CO.B.8 - Explain how the criteria for triangle congruence (ASA, SAS, SSS, AAS and HL (Hypotenuse Leg)) follow from the definition of congruence in terms of rigid motions.
Learning Goals:	I can evaluate if two triangles are congruent and determine the method of congruence.
Technology:	<ul style="list-style-type: none"> • Chromebooks or Laptops • NCTM Illuminations Online Activity <i>https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Congruence-Theorems/</i> • NearPod Online Activity <i>https://app.nearpod.com/presentation?pin=4F2E25A359896E5265987FECABF5A6C4-1</i>
Timeline:	<p>55 Minute Class:</p> <p>5 Minutes - Check and go over homework, Hand out Note Packet</p> <p>20 Minutes - Introduce the 5 methods of congruence (SSS, SAS, ASA, AAS, HL) and the 2 methods that do not work (SSA, AAA) & complete practice problems as a class</p> <p>10 Minutes - Have students complete the NCTM Illuminations activity to test SSA and AAA, and explain why they do not work as methods of congruence</p> <p>10 Minutes - Explain the NearPod activity and have students complete the activity with their groups</p> <p>*Teacher should be moving through the classroom while students are working on both activities to offer support and encouragement to those who need it</p> <p>10 Minutes - Go over activity as a class</p>

Notes - Identifying Congruent Triangle Methods

Name: _____

$\triangle ABC \cong \triangle XYZ$



When two triangles are congruent, there are 6 facts that are TRUE about the triangles:

- There are _____ sets of congruent (equal) _____
- There are _____ sets of congruent (equal) _____

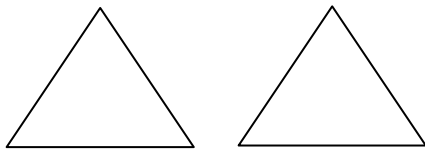
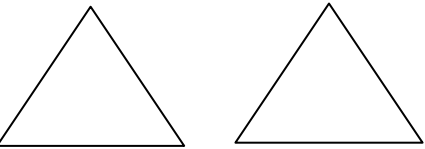
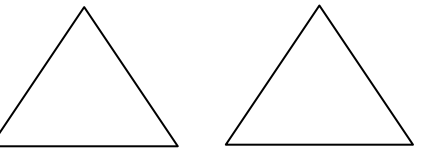
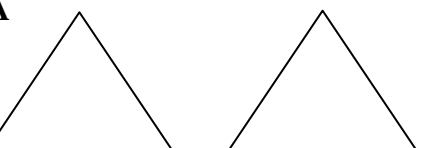
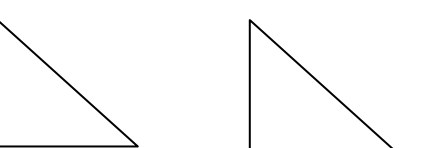
The 6 facts about these two triangles are:

Using your knowledge of congruence answer the following:

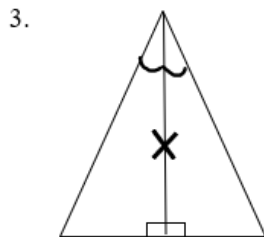
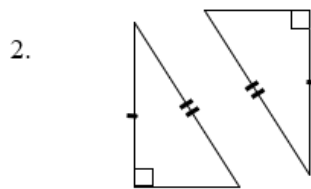
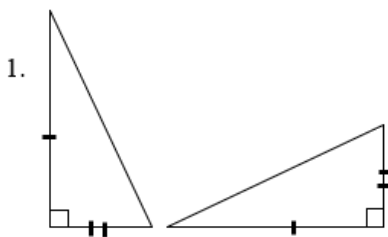
Given: $\triangle ABC \cong \triangle XYZ$

- a. $\sphericalangle A \cong$ _____ b. $\overline{AB} \cong$ _____ c. $\overline{CA} \cong$ _____ d. $\sphericalangle BCA \cong$ _____

The Congruence Methods – 5 ways to Prove Triangles Congruent

<p>SSS</p> 	<p>If 3 sides of one triangle are congruent to 3 sides of another triangle, the triangles are congruent</p>
<p>SAS</p> 	<p>If 2 sides and the INCLUDED angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.</p>
<p>ASA</p> 	<p>If 2 angles and the INCLUDED side of on triangle are congruent to the corresponding parts of another triangle, the triangles are congruent</p>
<p>AAS or SAA</p> 	<p>If two angles and the NON-INCLUDED side of one triangle are congruent to the corresponding parts of another triangle the triangles are congruent</p>
<p>HL</p> 	<p>If the HYPOTENUSE and LEG of one RIGHT triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent</p>

Class Practice: Determine if the triangle pairs are congruent. If they are, determine the method of congruence that would be used to prove it.



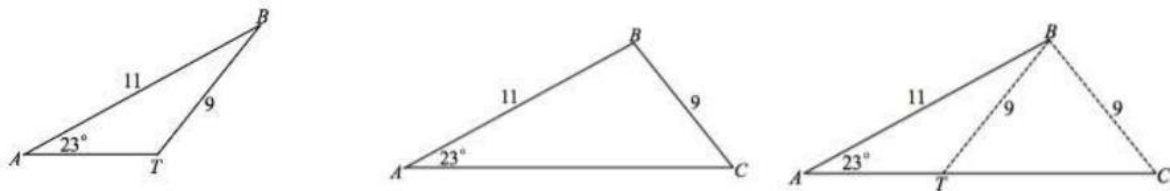
Criteria that **DOES NOT** prove 2 triangles congruent: **SSA/ASS and AAA**

Use this link to try and create congruent triangles using SSA and AAA

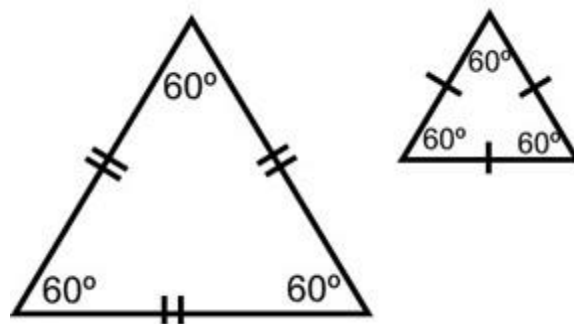
<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Congruence-Theorems/>

Where you able to make congruent triangles using SSA or AAA every time?

Side-Side-Angle (SSA/ASS): Look at the diagrams below. Each triangle has a set of adjacent sides of measures 11 and 9, as well as the non-included angle of 23° . Yet, the triangles are not congruent.



Angle-Angle-Angle (AAA): Look at the diagrams below. Each triangle has congruent angles since they are equilateral but have different side lengths.



Your turn! Now use the link on the Google Classroom to practice identifying these methods of congruence

Online Activity #1: NCTM Illuminations Exploration Screen Shots

Students need to select the needed sides and angles and try to create congruent triangles from the sides and angles pictured. The images are from <https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Congruence-Theorems/>.

SSA Exploration:

The screenshots show the following steps in the SSA Exploration tool:

- Step 1:** A blue triangle is shown with sides AB, BC, AC and angles A, B, C. The instruction is "Connect the elements to make a triangle." The SIDES list has AB, BC, and AC checked. The ANGLES list has A, B, and C unchecked. Buttons for "Flip" and "Reset" are visible.
- Step 2:** A red triangle with sides KT and TG and angle K is shown. A blue triangle with sides CA, AB, BC and angles C, A, B is also shown. The instruction is "Use the congruent elements to make another triangle." The SIDES list has AB, BC, and AC checked. The ANGLES list has A, B, and C unchecked. Buttons for "Flip" and "Reset" are visible.
- Step 3:** A red triangle with sides UN and NG and angle U is shown. A blue triangle with sides CA, AB, BC and angles C, A, B is also shown. The instruction is "Are these triangles congruent?" with "YES" and "NO" buttons. The SIDES list has AB, BC, and AC checked. The ANGLES list has A, B, and C unchecked. Buttons for "Flip" and "Reset" are visible.

Since it is possible to create two different triangles with this combination of congruent sides and angle, SSA is not an option for proving congruence.

AAA Exploration:

The National Council of Teachers of Mathematics (Inc) | <https://www.nctm.org/Classroom-Resources/Illuminations/I...>

Activity

ANGLE - ANGLE - ANGLE

Connect the elements to make a triangle.

ANGLE - ANGLE - ANGLE

Use the congruent elements to make another triangle.

The National Council of Teachers of Mathematics (Inc) | <https://www.nctm.org/Classroom-Resources/Illuminations/I...>

Activity

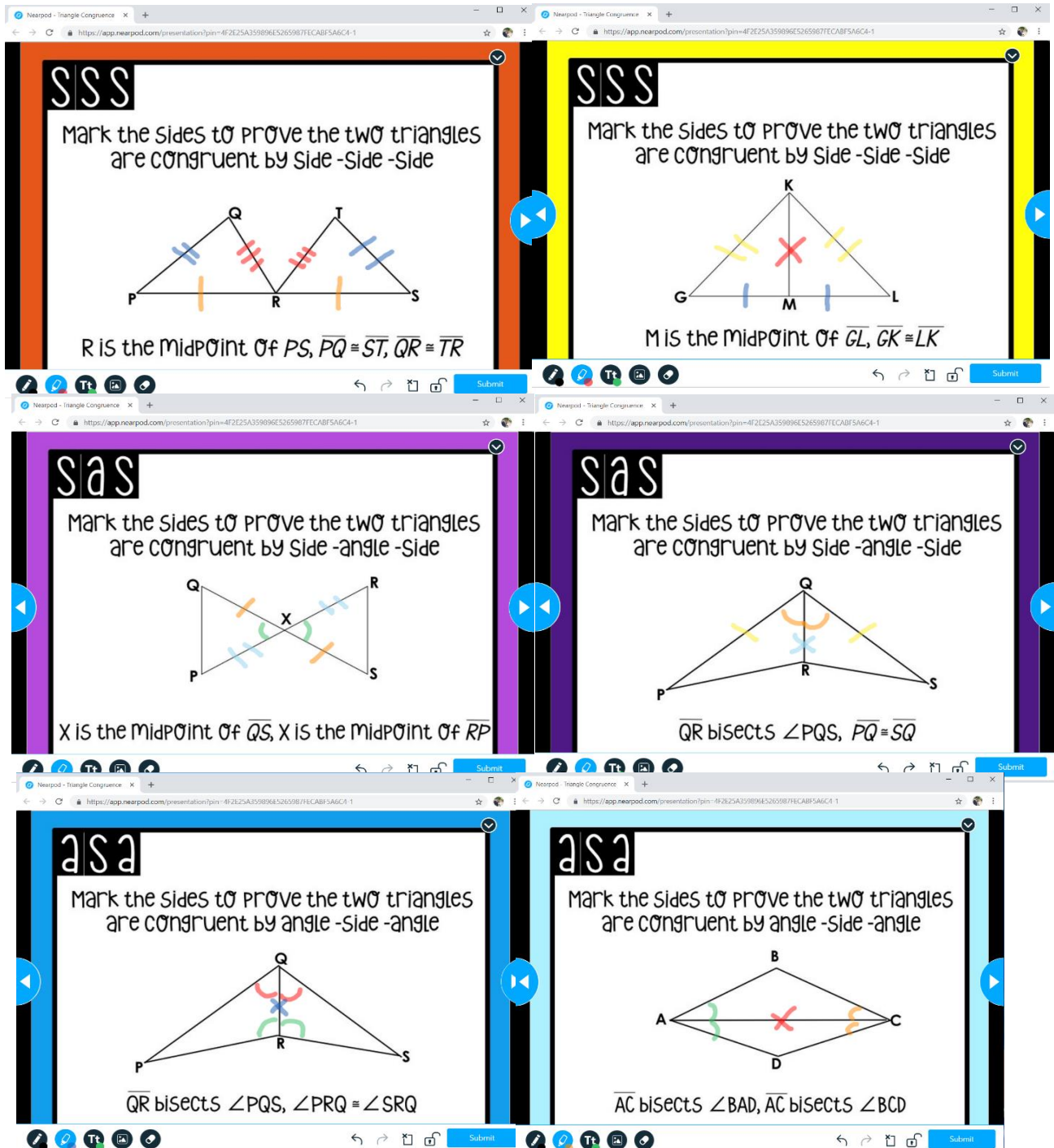
ANGLE - ANGLE - ANGLE

Are these triangles congruent? YES NO

Since it is possible to create two different triangles with this combination of congruent angles, AAA is not an option for proving congruence.

Online Activity #2: NearPod Interactive Presentation Screen Shots

Students are required to use the tools at the bottom of the screen to make markings on the triangles based on the information they are given. This activity allows students to get immediate feedback from the program and the teacher as the teacher moves through the room and supports the students. It also acts as a formative assessment for the teacher to see what students are struggling with and what they understand. The images are from <https://app.nearpod.com/presentation?pin=4F2E25A359896E5265987FECABF5A6C4-1>.



Course:	Geometry
Unit:	Transformations
Lesson Title:	Practice with Rigid Motions
Standards:	G.CO.A.5 - Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto another.
Learning Goals:	I can analyze the pre-image and image of an object and determine the rigid motion used and use the appropriate notation.
Technology:	<ul style="list-style-type: none"> • Chromebooks or Laptops • Interactivate Transmographer Online Activity <i>http://www.shodor.org/interactivate/activities/Transmographer/</i> • Kahoot! Online Quiz <i>https://create.kahoot.it/details/rigid-motions/ebcea73-cb3b-4f6c-93e5-dfca9b660445</i>
Timeline:	<p>55 Minute Class:</p> <p>5 Minutes - Check and go over homework, Hand out Note Packet</p> <p>15 Minutes - Have students get out their Chromebooks and use the link to access the online activity. Instruct them to choose a shape and translate, reflect and rotate the object and write down what they see happen after every rigid motion</p> <p>*Teacher should be moving through the classroom while students are working on the activity to offer support and encouragement to those who need it</p> <p>10 Minutes - Go over activity as a class and discuss their findings after each rigid motion</p> <p>15 Minutes - Review the three different rigid motions and what the notation is for each one. Have the students complete the practice problems in the note packets</p> <p>10 Minutes - Have students log into their Chromebook again and sign into the Kahoot! quiz</p>

Notes - Practice with Rigid Motions

Name: _____

Grab out your Chromebooks and use this link in the Google Classroom!*<http://www.shodor.org/interactivate/activities/Transmographer/>*

1. Select a shape and TRANSLATE it:
 - a. List the Original Points:

 - b. List the Translated Points:

 - c. Can you describe what happened to your shape?

 - d. Can you write this translation in symbolic form?

2. Select a NEW shape and REFLECT it:
 - a. List the Original Points:

 - b. List the Reflected Points:

 - c. Can you describe what happened to your shape?

 - d. Can you write this reflection in symbolic form?

3. Select a NEW shape and ROTATE it:
 - a. List the Original Points:

 - b. List the Rotated Points:

 - c. Can you describe what happened to your shape?

 - d. Can you write this rotation in symbolic form?

Rigid Motions

****Preserve Size and Shape****

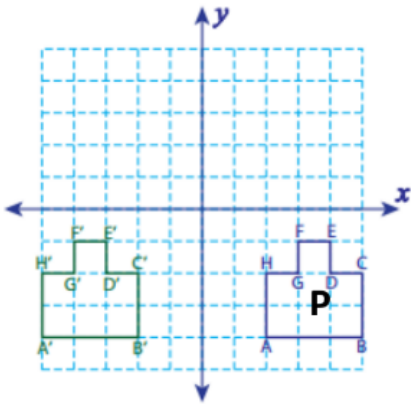
Translations
(Slide)

Reflections
(Flip)

Rotations
(Turn)

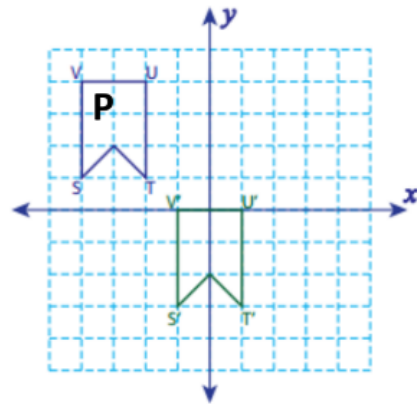
Class Practice:

Write a rule to describe the following translations (Note: The P is the pre-image shape)



Verbal Rule: _____

Symbolic Rule: _____

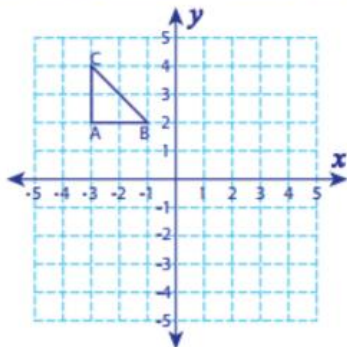


Verbal Rule: _____

Symbolic Rule: _____

Graph the image of each figure after the given translation and write the coordinates of the image:

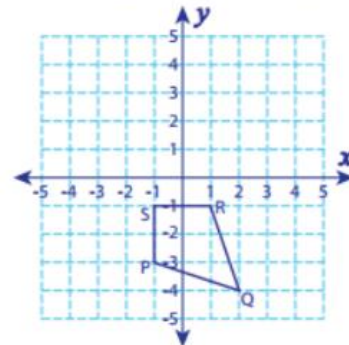
1 unit down and 4 units right



A': _____, B': _____

C': _____

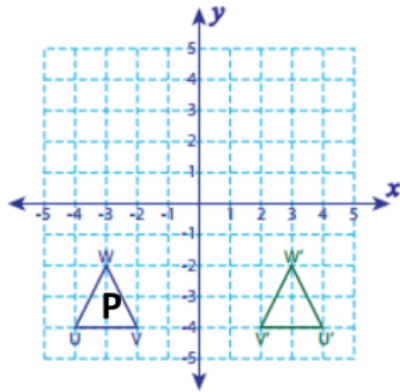
2 units right and 5 units up



P': _____, Q': _____

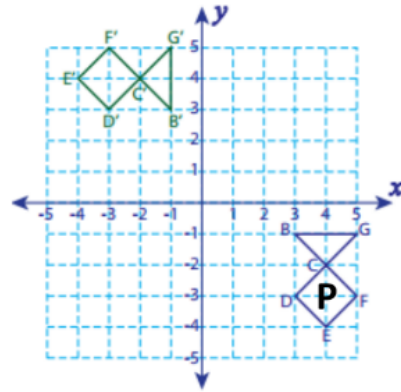
R': _____, S': _____

Write a rule to describe each reflection



Verbal Rule: _____

Symbolic Rule: _____

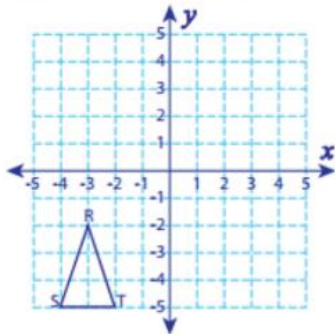


Verbal Rule: _____

Symbolic Rule: _____

Graph the image of each figure after the given reflection and write the coordinates of the image

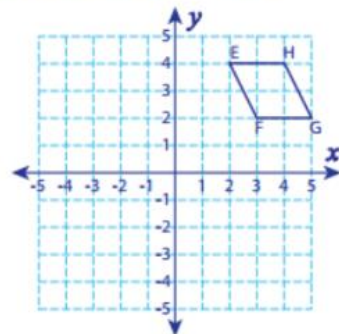
Reflection across the x-axis



R': _____, S': _____

T': _____

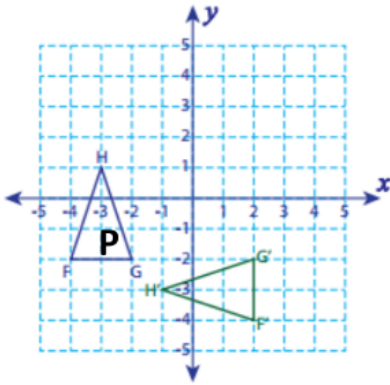
Reflection across the y-axis



E': _____, F': _____

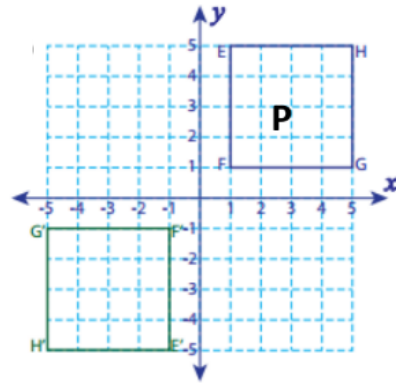
G': _____, H': _____

Write a rule to describe each rotation



Verbal Rule: _____

Symbolic Rule: _____

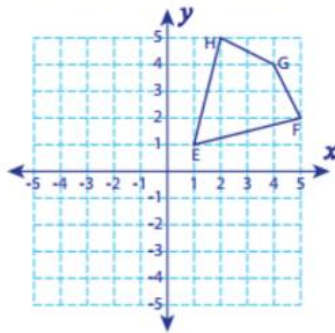


Verbal Rule: _____

Symbolic Rule: _____

Graph the image of each figure after rotating about the origin and write the coordinates of the image

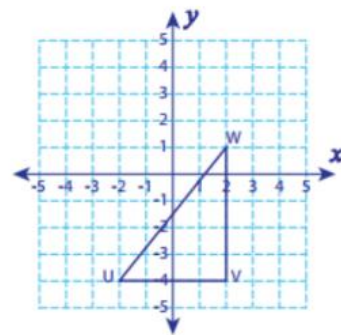
90° counterclockwise rotation



E' : _____ , F' : _____

G' : _____ , H' : _____

270° counterclockwise rotation

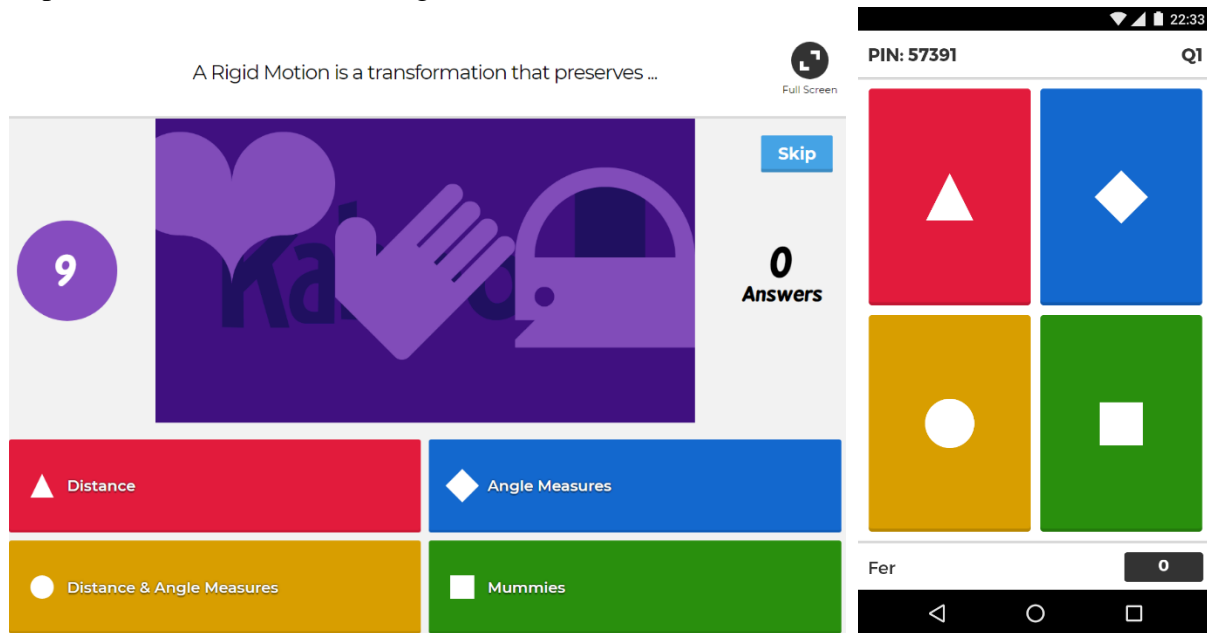


U' : _____ , V' : _____

W' : _____

Kahoot! Screen Shots:

Kahoot! gives every student the same amount of time to answer a question that is projected onto the board. Students need to log into the game by entering the pin generated by the website. They are required to read the question and answers available to select the correct answer. Students get immediate feedback when the time is up if they were correct or not. Teachers can go in and see how each student did individually on each question and assess who may need some more support. The images and questions are from <https://create.kahoot.it/details/rigid-motions/eebcea73-cb3b-4f6c-93e5-dfca9b660445>.



This is projected on the board

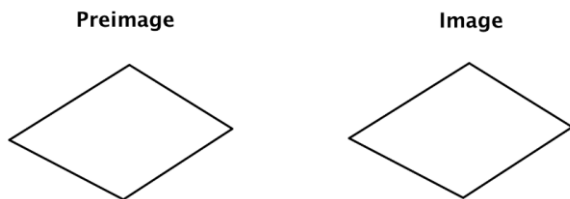
Students see this on their Chromebook

Here are the questions included in this Kahoot! Quiz:

1. A rigid motion is a transformation the preserves:
 - a. Distance
 - b. Angle Measures
 - c. Distance & Angle Measures
 - d. Mummies
2. In a transformation, the original figure is called the:
 - a. Image
 - b. Preimage
 - c. Photograph
 - d. Mapping
3. A transformation of a geometric figure is known as a:
 - a. Function
 - b. Picture
 - c. Mapping

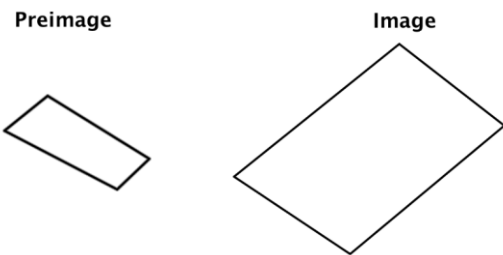
- d. #TransformationTuesday
- 4. Which of the following is NOT a rigid motion?
 - a. Dilation
 - b. Reflection
 - c. Rotation
 - d. Translation

5. Does the transformation appear to be a rigid motion?



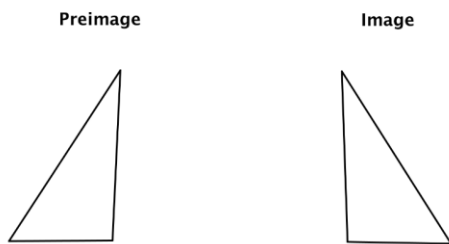
- a. Yaasss
- b. Nah
- c. Not Sure

6. Does the transformation appear to be a rigid motion?



- a. Yaasss
- b. Nah
- c. Not Sure

7. Does the transformation appear to be a rigid motion?



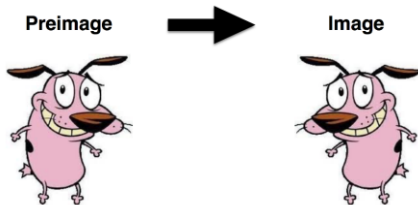
- a. Yaasss
- b. Nah
- c. Not Sure

8. Does the transformation appear to be a rigid motion?



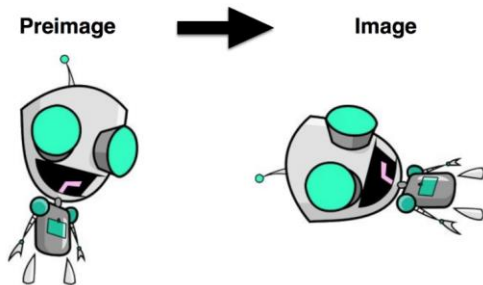
- a. Yaasss
- b. Nah
- c. Not Sure

9. What type of transformation is this?



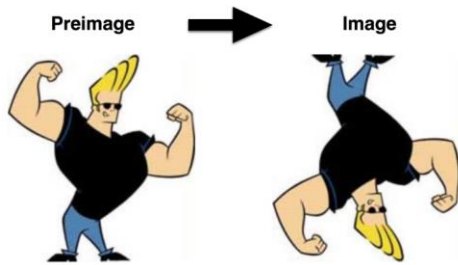
- a. Rotation
- b. Translation
- c. Reflection
- d. Dilation

10. What type of transformation is this?



- a. Rotation
- b. Translation
- c. Reflection
- d. Dilation

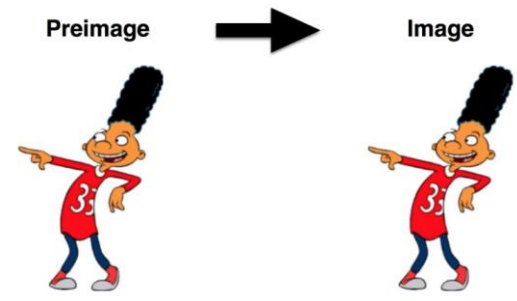
11. What type of transformation is this?



- a. Rotation
- b. Translation
- c. Reflection
- d. Dilation

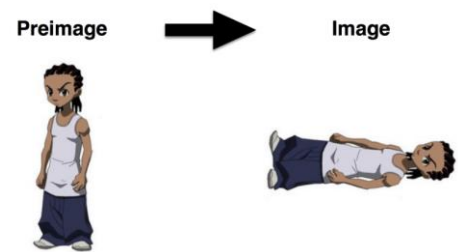
Both occurred here, so either answer is accepted

12. What type of transformation is this?



- a. Rotation
- b. Translation
- c. Reflection
- d. Dilation

13. What type of transformation is this?



- a. Rotation
- b. Translation
- c. Reflection
- d. Dilation

14. If rigid motions preserve distance and angle measures, is the preimage congruent to the image?
- a. True, they are congruent
 - b. False, they are not congruent
 - c. I'm not really sure
 - d. I am completely lost and need extra help A\$AP Rocky, fam
15. Do corresponding point of the preimage and image need to be in the same order?
- a. Always
 - b. Sometimes
 - c. Never
 - d. Not Sure

Interactivate Transmographer Online Activity Screen Shots:

This is the screen that the students are brought to after clicking on the link. The experience of each student will be slightly different depending on what shapes they choose to explore. The images are from <http://www.shodor.org/interactivate/activities/Transmographer/>.

Interactivate Jump To: Browse: Search

Transmographer Find us in the App Store. Change a Function, Change the World

Shodor > Interactivate > Activities > Transmographer

Learner Activity Help Instructor

New Square New Parallelogram New Triangle

Reset

Green Point is at (2.00,6.00)
 Yellow Point is at (6.00,6.00)
 Cyan Point is at (2.00,2.00)
 Black Point is at (6.00,2.00)

Translate
 0 units on the X axis
 0 units on the Y axis

Reflect
 The line $x = 0$ is the y-axis.
 across $x = 0$
 across $y = 0$
 across $y = x$
 The line $y = 0$ is the x-axis.

Rotate
 Around a Point
 x = 0 y = 0 degrees: 0
 Around Center
 The Center of Mass is (0,0)

More Options
 Show Original Polygon
 Show Pre-image

© Shodor

Course:	Geometry
Unit:	Introduction to Geometry
Lesson Title:	Triangle Theorems
Standards:	G.CO.C.10 - Prove and apply theorems about the properties of triangles
Learning Goals:	I can analyze the given figure and determine the theorem or relationship needed to solve the different triangle problems.
Technology:	<ul style="list-style-type: none"> • Chromebooks or Laptops • GeoGebra Explorations <p><i>Triangle Sum Theorem</i> - https://www.geogebra.org/m/FAhtKpR5 <i>Exterior Angle Theorem</i> - https://www.geogebra.org/m/Yf5HV2uK <i>Side-Angle Relationships in Triangles</i> - https://www.geogebra.org/m/AjND4Uma <i>Triangle Side Inequality Theorem</i> - https://www.geogebra.org/m/pNm33AuP</p>
Timeline:	<p>55 Minute Class:</p> <p>5 Minutes - Check and go over homework, Hand out Note Packet</p> <p>20 Minutes - Have students get out their Chromebooks and use the links to access the different online explorations. Instruct them to try and discover the rule that goes with each theorem</p> <p>*Teacher should be moving through the classroom while students are working on the activity to offer support and encouragement to those who need it</p> <p>10 Minutes - Discuss their discoveries and write down the formal rules</p> <p>20 Minutes - Have students complete practice problems in the note packet individually or with a group</p>

Notes – Triangle Theorems

Name: _____

Use the different links in the Google Classroom to answer the following questions:

Triangle Sum Theorem*<https://www.geogebra.org/m/FAhtKpR5>*

- What do you notice happening in the application?
- What do you think the rule is for the Triangle Sum Theorem?

Exterior Angle Theorem*<https://www.geogebra.org/m/Yf5HV2uK>*

- What do you notice happening in the application?
- What do you think the rule is for the Exterior Angle Theorem?

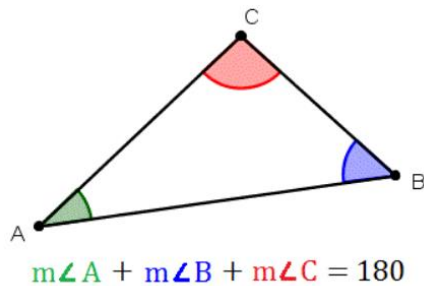
Side-Angle Relationships in Triangles*<https://www.geogebra.org/m/AjND4Uma>*

- What do you notice happening in the application?
- What do you think the rule is for the Side-Angle Relationship Theorem?

Triangle Side Inequality Theorem*<https://www.geogebra.org/m/pNm33AuP>*

- What do you notice happening in the application?
- What do you think the rule is for the Triangle Side Inequality Theorem?

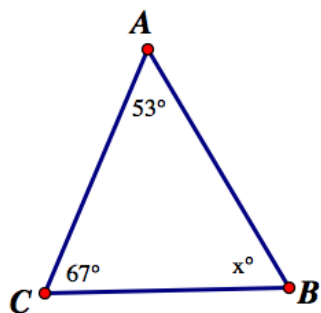
Triangle Sum Theorem: The three angles in a triangle ALWAYS add up to 180° .



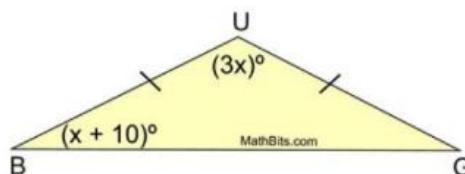
Practice Problems:

Find the value of x :

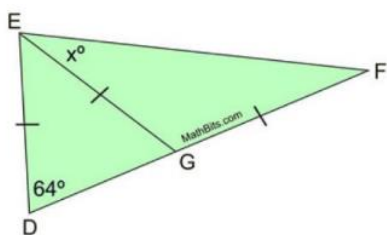
1.



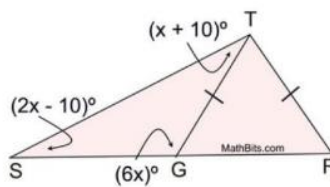
2.



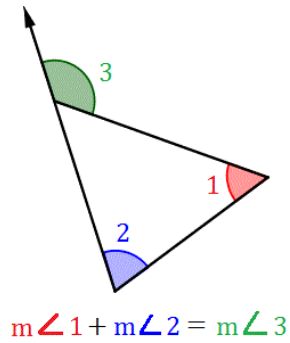
3.



4.

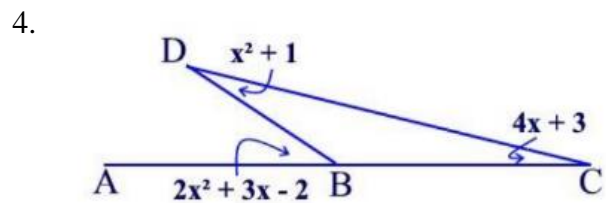
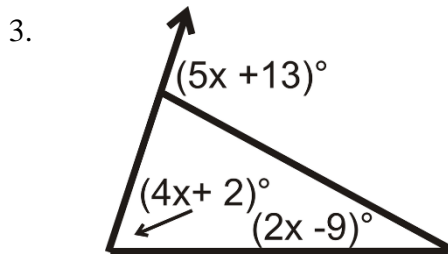
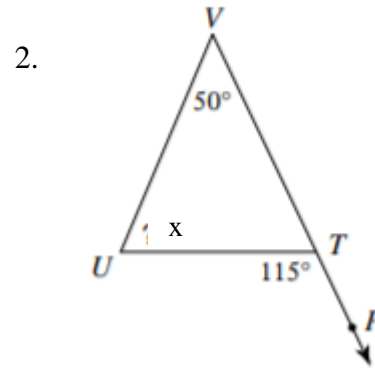
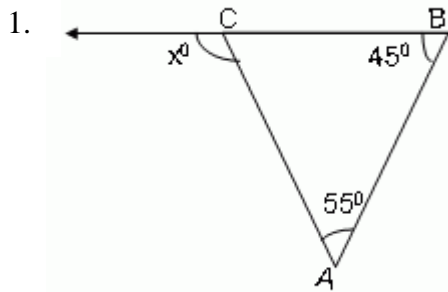


Exterior Angle Theorem: The exterior angle is equal to the sum of the two non-adjacent angles.

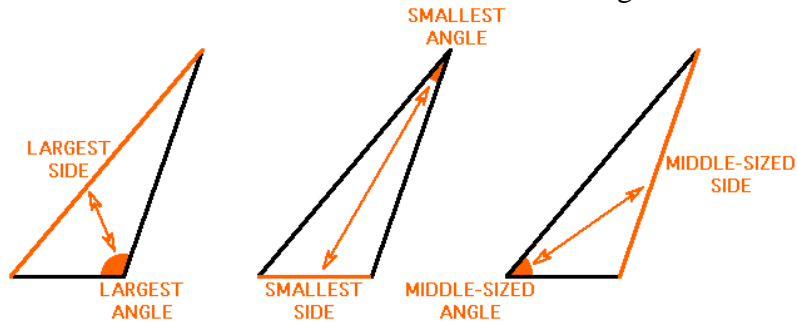


Practice Problems:

Find the value of x :



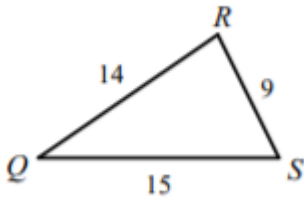
Side-Angle Relationships in Triangles: In a triangle, the longest side is across from the largest angle measure and the shortest side is across from the smallest angle measure.



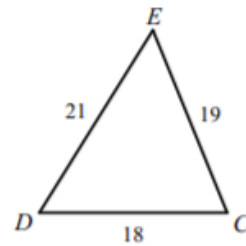
Practice Problems:

List the angles in order from largest to smallest:

1.

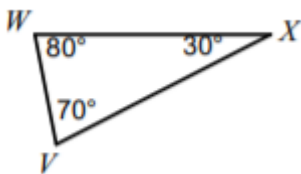


2.

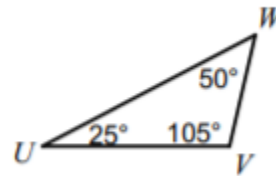


List the sides in order from largest to smallest:

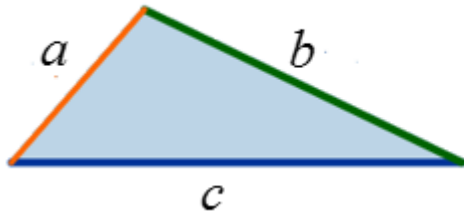
3.



4.



Triangle Side Inequality Theorem: The sum of the lengths of any two sides of a triangle is GREATER than the length of the third side.



$$a + b > c$$

$$a + c > b$$

$$b + c > a$$

Practice Problems:

Can these numbers be the measures of the sides of a triangle?

1. 7, 4, 5

2. 3, 6, 2

3. 8, 2, 8

4. 1, 13, 13

5. 1, 9, 10

6. 5, 8, 4

7. 6, 3, 10

8. 2, 15, 16

Two sides of a triangle have the following measures – Find the range of possible measures for the third side:

9. 9, 5

10. 5, 8

11. 14, 11

12. 11, 8

13. 9, 6

14. 7, 12

15. 4, 6

16. 3, 7

GeoGebra Application Exploration Screen Shots:

These applications allow students to see exactly how these theorems came to be and how to apply them to the problems they will see on homework, notes, or assessments. In every one of these applets, students can manipulate the triangles to see how these relationships behave for different triangles.

1. Triangle Sum Theorem - <https://www.geogebra.org/m/FAhtKpR5>

GeoGebra

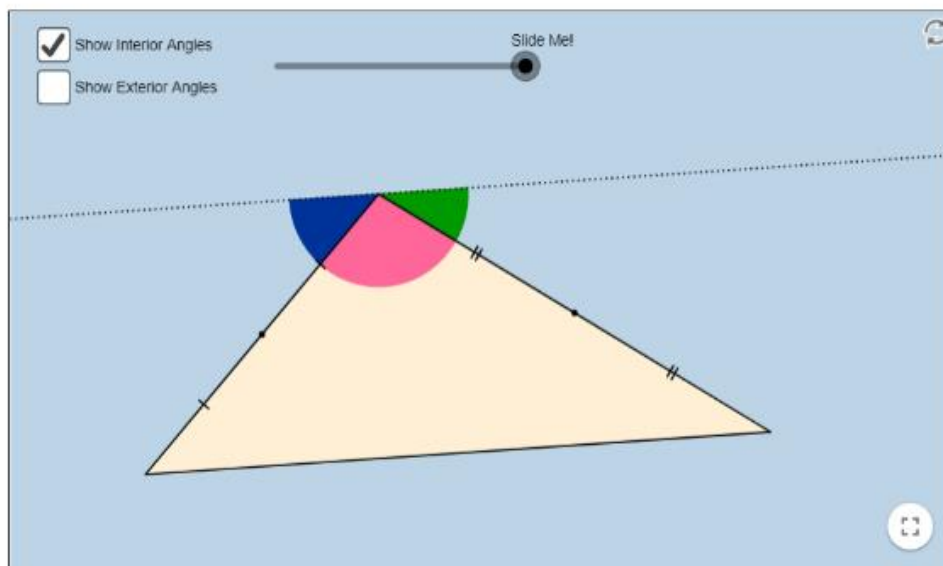
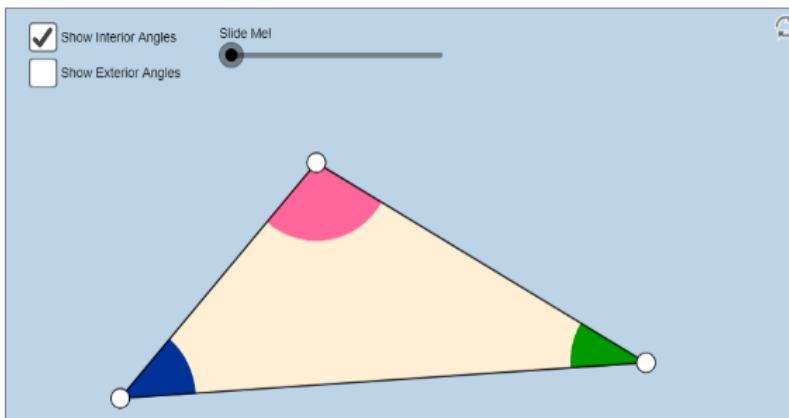
Triangle Angle Theorems (V1)

Author: Tim Brzezinski

Topic: Angles, Triangles

Interact with the applet below for a few minutes.
Then, answer the questions that follow.

Be sure to change the locations of the triangle's WHITE VERTICES each time before you drag the slider!!!



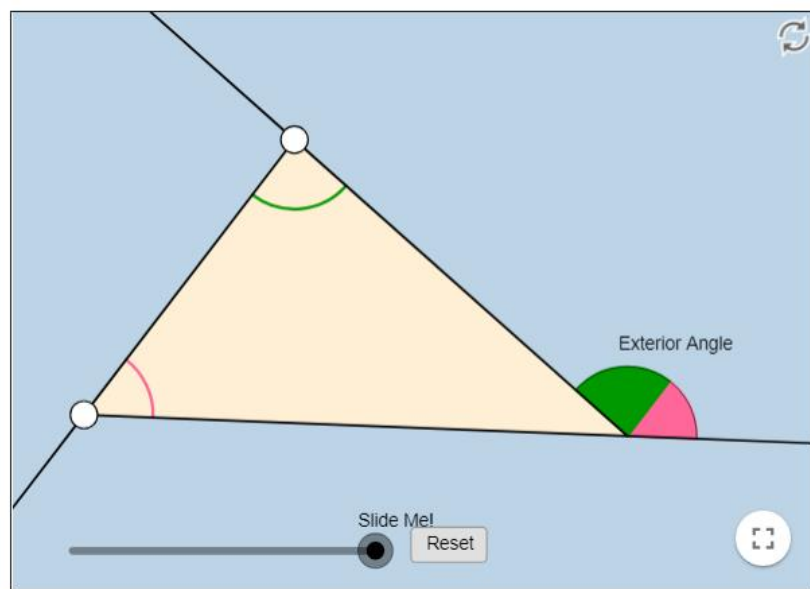
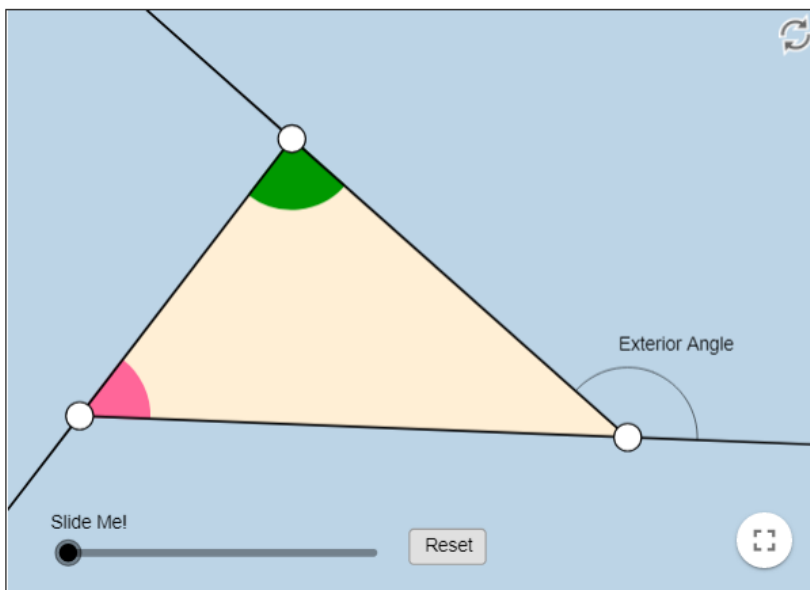
2. Exterior Angle Theorem - <https://www.geogebra.org/m/Yf5HV2uK>



Exterior Angle Theorem

Author: Jill Casey, Tim Brzezinski

Exterior Angle Theorem



3. Side-Angle Relationships in Triangles - <https://www.geogebra.org/m/AjND4Uma>



Angle-Side Relationship Discovery

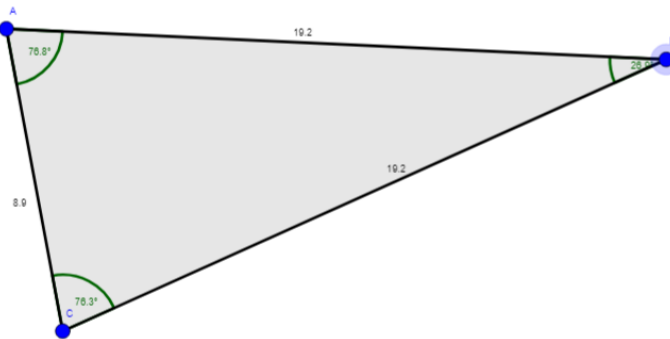
Author: Amanda Cappelli

Angle- Side Relationship

Step 1: Observe the diagram below:
State the the longest side and the largest angle.

Step 2 Move one of the vertices:
Now state the longest side and the longest angle.

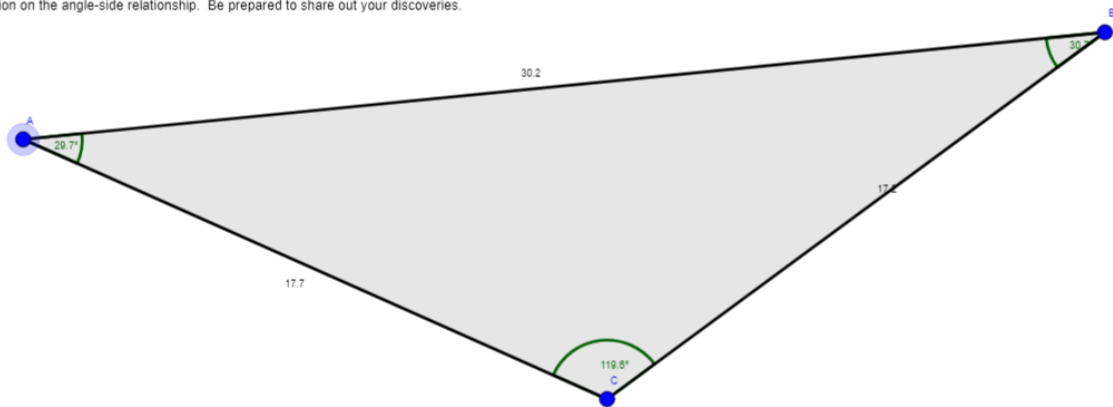
Draw a conclusion on the angle-side relationship. Be prepared to share out your discoveries.



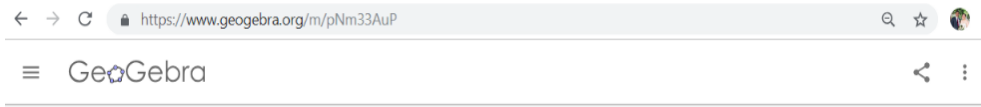
Step 1: Observe the diagram below:
State the the longest side and the largest angle.

Step 2 Move one of the vertices:
Now state the longest side and the longest angle.

Draw a conclusion on the angle-side relationship. Be prepared to share out your discoveries.

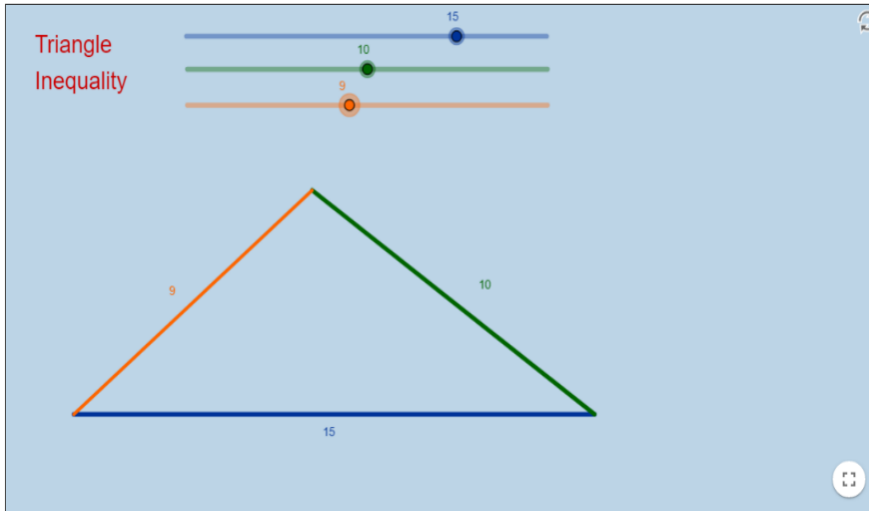


4. Triangle Side Inequality Theorem - <https://www.geogebra.org/m/pNm33AuP>



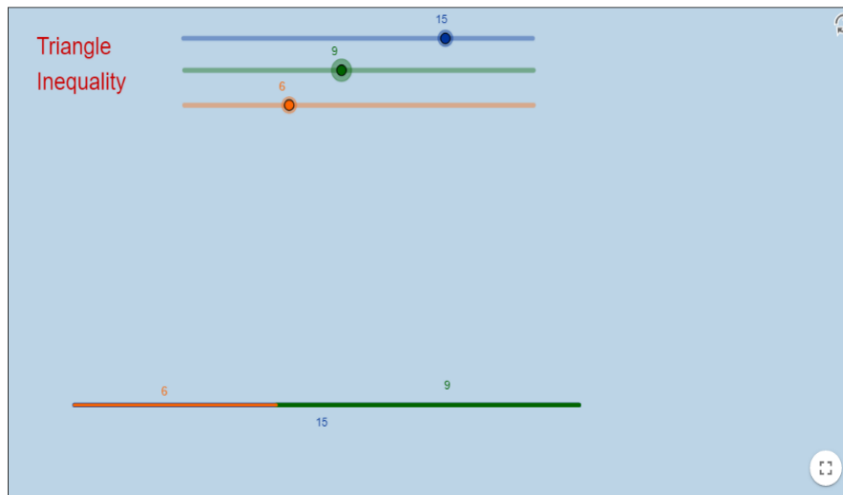
Triangle Inequality (II)

Author: Tim Brzezinski
Topic: Triangles



Triangle Inequality (II)

Author: Tim Brzezinski
Topic: Triangles



Conclusion

Technology is something that is improving and becoming continuously more useful as time goes on. Current and future students have been born into a society that relies heavily on technological advancements in everyday life. While this poses a serious need for adaptation, it can be used to an educators' advantage when implemented appropriately. The goal of this curriculum project is to show teachers how some of the resources available can be used to help support student learning across the geometry curriculum. Any teacher attempting to integrate technology into their classroom should find this curriculum project and the example lessons useful.

The tools used here can be applied to countless other concepts, which is made easier once teachers become familiar with them. Within these lessons, educators need to understand the best method for instruction for their students and what students need to learn. TPACK requires teachers to know their content, how students are going to best learn that content and what technology can make that happen. This curriculum plan uses this concept to outline what the class will include and how technology can help build student knowledge. These lessons are not as effective when the teacher cannot properly use the tools as they were intended or make strong enough connections to the material. It may be difficult to learn about new resources as they come out, but they can benefit the development of students' conceptual understanding of some of the most difficult topics in Geometry. Giving students additional support through instructional technology can help them master what is already considered one of the most difficult content areas.

References

- Bostic, J., & Matney, G. (2013). Overcoming a Common Storm: Designing Professional Development for Teachers Implementing the Common Core. *Ohio Journal of School Mathematics*, (67), 12–19. Retrieved from <https://brockport.idm.oclc.org/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=87432476&site=ehost-live>
- Bouck, E. C., & Flanagan, S. (2009). Assistive technology and mathematics: What is there and where can we go in special education. *Journal of Special Education Technology*, 24(2), 17-30. Retrieved from <https://brockport.idm.oclc.org/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=53163637&site=ehost-live>
- Contreras, J. (2015). Patterns in the pythagorean configuration and some extensions: The power of interactive geometry software. *Journal of Mathematics Education at Teachers College*, 6(1), 25–36. Retrieved from <https://brockport.idm.oclc.org/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=108802844&site=ehost-live>
- Dalal, M., Archambault, L., & Shelton, C. (2017). Professional development for international teachers: Examining TPACK and technology integration decision making. *Journal of Research on Technology in Education*, 49(3/4), 117–133. <https://doi-org.brockport.idm.oclc.org/10.1080/15391523.2017.1314780>
- Dingman, S., Teuscher, D., Newton, J. A., & Kasmer, L. (2013). Common mathematics standards in the united states. *Elementary School Journal*, 113(4), 541–564. <https://doi-org.brockport.idm.oclc.org/10.1086/669939>

- Foulger, T. S., Buss, R. R., Wetzel, K., & Lindsey, L. (2015). Instructors' growth in TPACK: Teaching technology-infused methods courses to preservice teachers. *Journal of Digital Learning in Teacher Education*, 31(4), 134–147. <https://doi-org.brockport.idm.oclc.org/10.1080/21532974.2015.1055010>
- Hoffmann, M. M., & Ramirez, A. Y. (2018). Students' attitudes toward teacher use of technology in classrooms. *Multicultural Education*, 25(2), 51–56. Retrieved from <https://brockport.idm.oclc.org/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=130011889&site=ehost-live>
- Hora, M., & Holden, J. (2013). Exploring the role of instructional technology in course planning and classroom teaching: implications for pedagogical reform. *Journal of Computing in Higher Education*, 25(2), 68–92. <https://doi-org.brockport.idm.oclc.org/10.1007/s12528-013-9068-4>
- Kirikçilar, R. G., & Yildiz, A. (2018). Technological pedagogical content knowledge (TPACK) craft: Utilization of the TPACK when designing the geogebra activities. *Acta Didactica Napocensia*, 11(1), 101–116. <https://doi-org.brockport.idm.oclc.org/10.24193/adn.11.1.8>
- Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research*, 32(2), 131–152. <https://doi-org.brockport.idm.oclc.org/10.2190/0EW7-01WB-BKHL-QDYV>
- Koehler, M., & Mishra, P. (2008). Introducing TPCK. In. AACTE committee on innovation and technology (Eds.), *Handbook of technological pedagogical content knowledge (TPCK) for teaching and teacher educators*, (pp. 3-29). New York and London: Routledge.
- Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A

- framework for teacher knowledge. *Teachers College Record*, 108(6), 1017–1054.
<https://doi-org.brockport.idm.oclc.org/10.1111/j.1467-9620.2006.00684.x>
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Nicol, A. A. M., Owens, S. M., Le Coze, S. S. C. L., MacIntyre, A., & Eastwood, C. (2018). Comparison of high-technology active learning and low-technology active learning classrooms. *Active Learning in Higher Education*, 19(3), 253–265.
<https://doi-org.brockport.idm.oclc.org/10.1177/1469787417731176>
- Önal, N. (2016). Development, validity and reliability of TPACK scale with pre-service mathematics teachers. *International Online Journal of Educational Sciences*, 8(2), 93–107. <https://doi-org.brockport.idm.oclc.org/10.15345/iojes.2016.02.009>
- Patahuddin, S. S. P., Lowrie, T., & Dalgarno, B. (2016). Analysing mathematics teachers' TPACK through observation of practice. *Asia-Pacific Education Researcher (Springer Science & Business Media B.V.)*, 25(5/6), 863–872.
<https://doi-org.brockport.idm.oclc.org/10.1007/s40299-016-0305-2>
- Saralar, İ., Işıksal-Bostan, M., & Akyüz, D. (2018). The evaluation of a pre-service mathematics teacher's TPACK: A case of 3d shapes with geogebra. *International Journal for Technology in Mathematics Education*, 25(2), 3–21. [https://doi-org.brockport.idm.oclc.org/10.1564/tmepass:\[\]v25.2.01](https://doi-org.brockport.idm.oclc.org/10.1564/tmepass:[]v25.2.01)
- Schmidt, D. A., Baran, E., Thompson, A. D., Mishra, P., Koehler, M. J., & Shin, T. S. (2009). Technological pedagogical content knowledge (TPACK): The development and validation of an assessment instrument for preservice teachers. *Journal of Research on Technology in Education*, 42(2), 123–149. Retrieved from

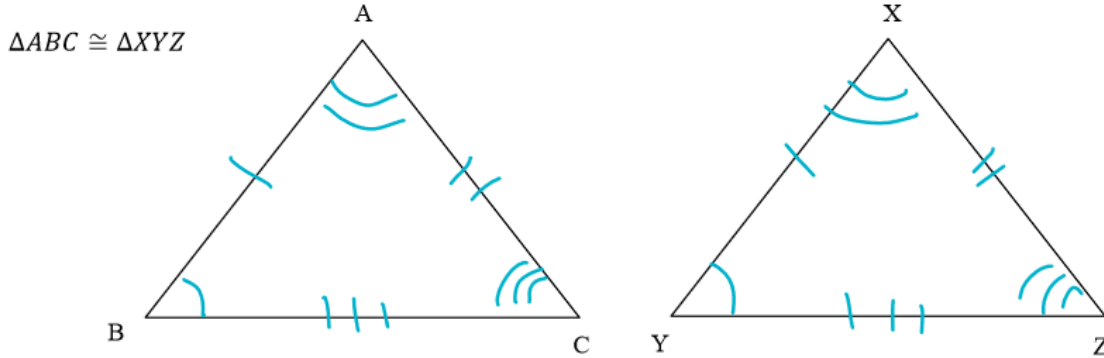
<https://brockport.idm.oclc.org/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=508120396&site=ehost-live>

Schoenfeld, A. H. (2014). Common sense about the common core. *Mathematics Enthusiast*, *11*(3), 737–744. Retrieved from

<https://brockport.idm.oclc.org/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=eue&AN=103710362&site=ehost-live>

Appendices – Student Answer Keys

Name: Key Notes - Identifying Congruent Triangle Methods



When two triangles are congruent, there are 6 facts that are TRUE about the triangles:

- There are 3 sets of congruent (equal) Angles (\sphericalangle)
- There are 3 sets of congruent (equal) Sides

The 6 facts about these two triangles are: When $\triangle ABC \cong \triangle XYZ$

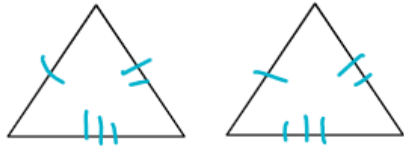
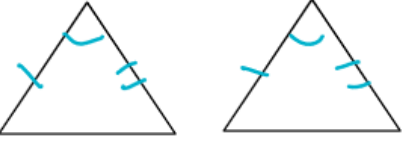
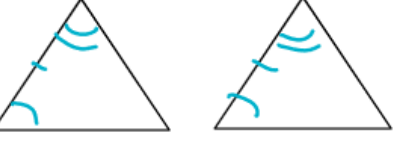
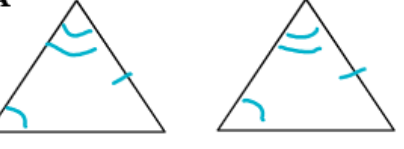

$\sphericalangle A \cong \sphericalangle X$	$\overline{AB} \cong \overline{XY}$
$\sphericalangle B \cong \sphericalangle Y$	$\overline{BC} \cong \overline{YZ}$
$\sphericalangle C \cong \sphericalangle Z$	$\overline{AC} \cong \overline{XZ}$

Using your knowledge of congruence answer the following:

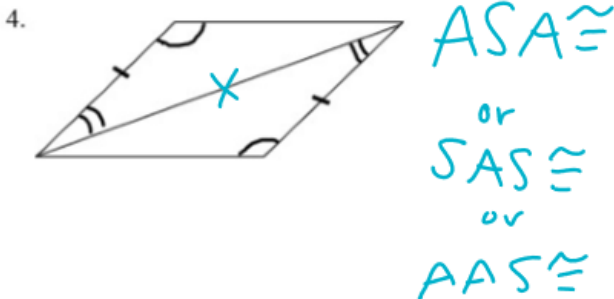
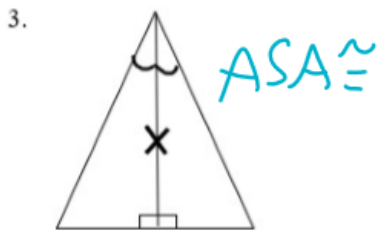
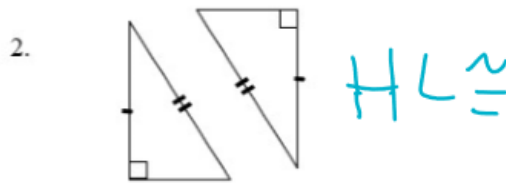
Given: $\triangle ABC \cong \triangle XYZ$

- a. $\sphericalangle A \cong$ $\sphericalangle X$ b. $\overline{AB} \cong$ \overline{XY} c. $\overline{CA} \cong$ \overline{XZ} d. $\sphericalangle BCA \cong$ $\sphericalangle YZX$

The Congruence Methods – 5 ways to Prove Triangles Congruent

<p>SSS</p> 	<p>If 3 sides of one triangle are congruent to 3 sides of another triangle, the triangles are congruent</p>
<p>SAS</p> 	<p>If 2 sides and the INCLUDED angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.</p>
<p>ASA</p> 	<p>If 2 angles and the INCLUDED side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent</p>
<p>AAS or SAA</p> 	<p>If two angles and the NON-INCLUDED side of one triangle are congruent to the corresponding parts of another triangle the triangles are congruent</p>
<p>HL</p> 	<p>If the HYPOTENUSE and LEG of one RIGHT triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent</p>

Class Practice: Determine if the triangle pairs are congruent. If they are, determine the method of congruence that would be used to prove it.



Criteria that **DOES NOT** prove 2 triangles congruent: **SSA/ASS and AAA**

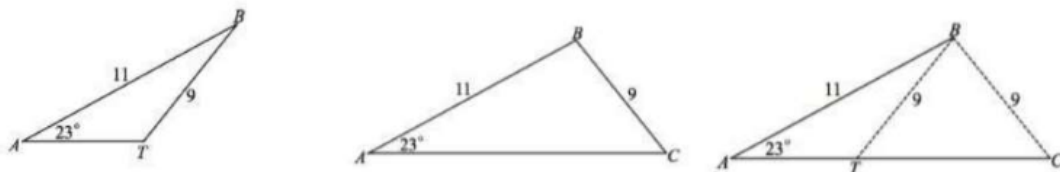
Use this link to try and create congruent triangles using SSA and AAA

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Congruence-Theorems/>

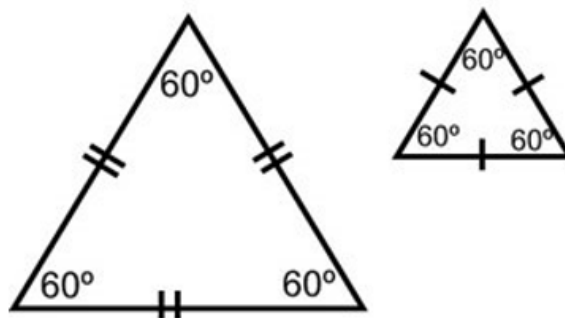
Where you able to make congruent triangles using SSA or AAA every time?

No, it is possible, but Not Always the case

Side-Side-Angle (SSA/ASS): Look at the diagrams below. Each triangle has a set of adjacent sides of measures 11 and 9, as well as the non-included angle of 23° . Yet, the triangles are not congruent.



Angle-Angle-Angle (AAA): Look at the diagrams below. Each triangle has congruent angles since they are equilateral but have different side lengths.



Your turn! Now use the link on the Google Classroom to practice identifying these methods of congruence

Notes - Practice with Rigid Motions

Name: _____

Grab out your Chromebooks and use this link in the Google Classroom!*<http://www.shodor.org/interactivate/activities/Transmographer/>*

1. Select a shape and TRANSLATE it:

a. List the Original Points:

*★ Based on Student
Observations ★*

b. List the Translated Points:

c. Can you describe what happened to your shape?

d. Can you write this translation in symbolic form?

2. Select a NEW shape and REFLECT it:

a. List the Original Points:

b. List the Reflected Points:

c. Can you describe what happened to your shape?

d. Can you write this reflection in symbolic form?

3. Select a NEW shape and ROTATE it:

a. List the Original Points:

b. List the Rotated Points:

c. Can you describe what happened to your shape?

d. Can you write this rotation in symbolic form?

Rigid Motions

Preserve Size and Shape

Translations
(Slide)

$T\langle \Delta x, \Delta y \rangle$

Reflections
(Flip)

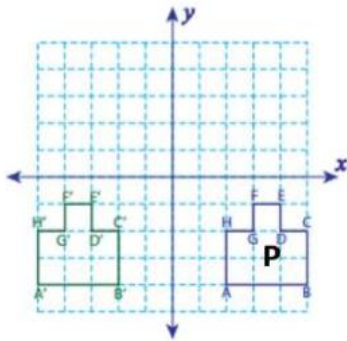
line of reflection

Rotations
(Turn)

Point of Rotation, # degrees

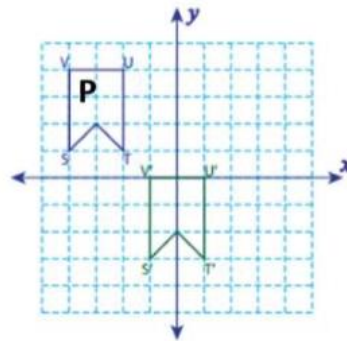
Class Practice:

Write a rule to describe the following translations (Note: The P is the pre-image shape)



Verbal Rule: Slide left 7 units

Symbolic Rule: $T\langle -7, 0 \rangle$

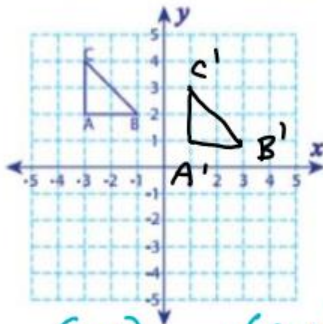


Verbal Rule: Slide Right 3, down 4

Symbolic Rule: $T\langle 3, -4 \rangle$

Graph the image of each figure after the given translation and write the coordinates of the image:

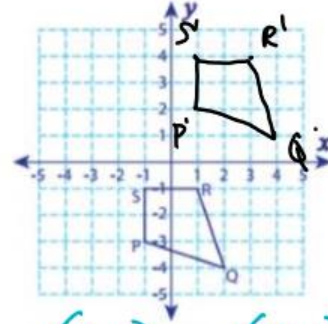
1 unit down and 4 units right



A': (2, 1), B': (3, 1)

C': (2, 3)

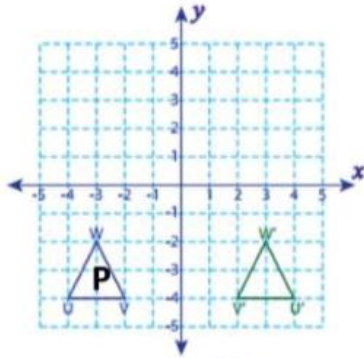
2 units right and 5 units up



P': (3, 2), Q': (6, 2)

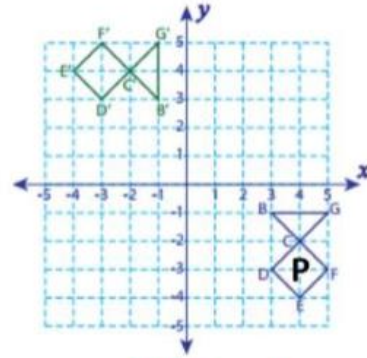
R': (5, 9), S': (3, 9)

Write a rule to describe each reflection



Verbal Rule: Reflection over $x=0$

Symbolic Rule: $r_{y\text{-axis}}$

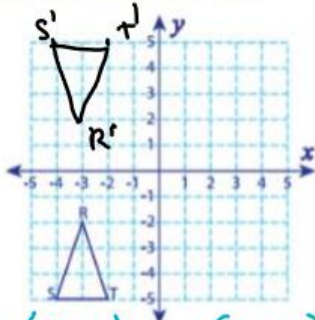


Verbal Rule: Reflection over $y=x$

Symbolic Rule: $r_{y=x}$

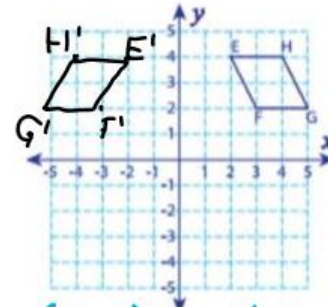
Graph the image of each figure after the given reflection and write the coordinates of the image

Reflection across the x-axis



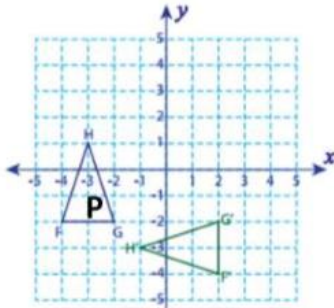
R' : $(-3, 2)$, S' : $(-4, 5)$
 T' : $(-2, 5)$

Reflection across the y-axis

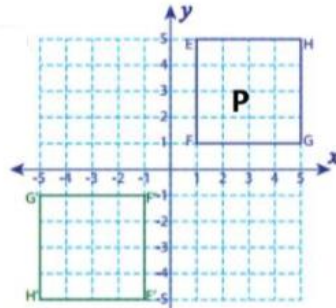


E' : $(-2, 4)$, F' : $(-3, 2)$
 G' : $(-5, 2)$, H' : $(-4, 4)$

Write a rule to describe each rotation



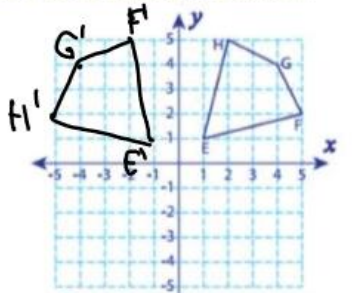
Verbal Rule: Turn 90° around (0,0)
 Symbolic Rule: $R_{(0,0), 90^\circ}$



Verbal Rule: Turn 180° around (0,0)
 Symbolic Rule: $R_{(0,0), 180^\circ}$

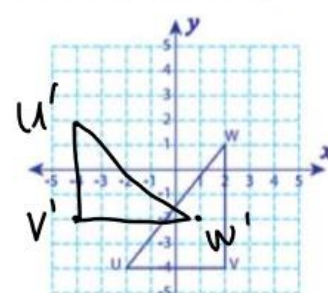
Graph the image of each figure after rotating about the origin and write the coordinates of the image

90° counterclockwise rotation



$E': (-1, 1)$, $F': (-2, 5)$
 $G': (-4, 4)$, $H': (-5, 2)$

270° counterclockwise rotation



$U': (-4, 2)$, $V': (-4, -2)$
 $W': (1, -2)$

Notes – Triangle Theorems

Name: _____

Use the different links in the Google Classroom to answer the following questions:

Triangle Sum Theorem<https://www.geogebra.org/m/FAhtKpR5>

- What do you notice happening in the application?
- What do you think the rule is for the Triangle Sum Theorem?

★ Based
on Student
Observations★

Exterior Angle Theorem<https://www.geogebra.org/m/Yf5HV2uK>

- What do you notice happening in the application?
- What do you think the rule is for the Exterior Angle Theorem?

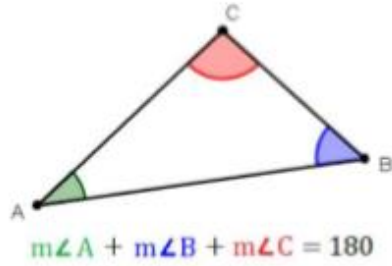
Side-Angle Relationships in Triangles<https://www.geogebra.org/m/AjND4Uma>

- What do you notice happening in the application?
- What do you think the rule is for the Side-Angle Relationship Theorem?

Triangle Side Inequality Theorem<https://www.geogebra.org/m/pNm33AuP>

- What do you notice happening in the application?
- What do you think the rule is for the Triangle Side Inequality Theorem?

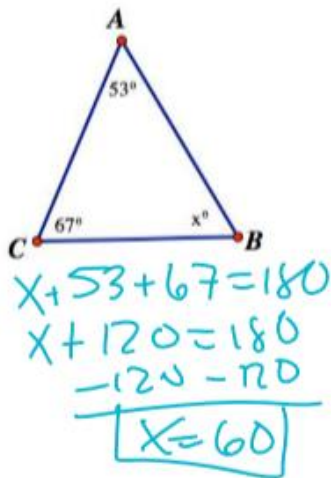
Triangle Sum Theorem: The three angles in a triangle ALWAYS add up to 180° .



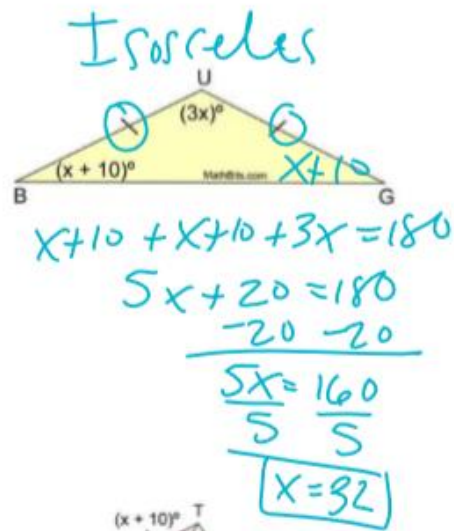
Practice Problems:

Find the value of x:

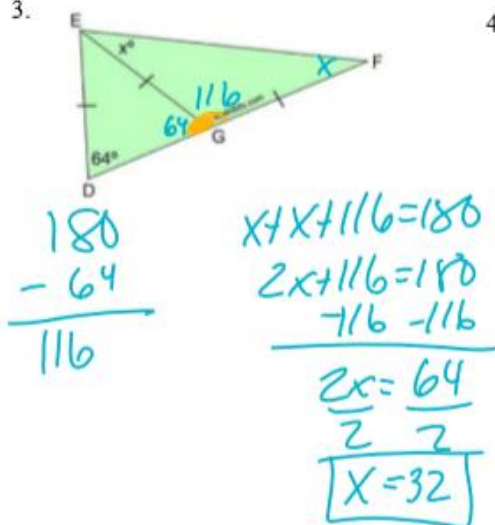
1.



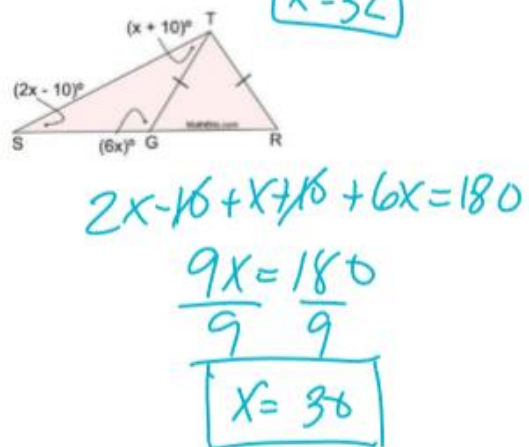
2.



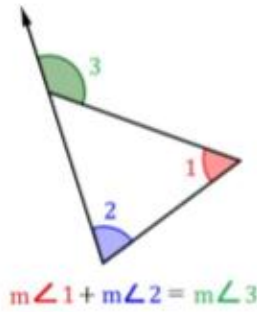
3.



4.



Exterior Angle Theorem: The exterior angle is equal to the sum of the two non-adjacent angles.



Practice Problems:
Find the value of x:

1.
 $55 + 45 = x$
 $100 = x$

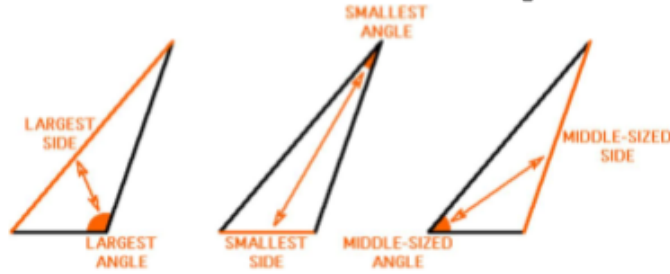
2.
 $50 + x = 115$
 $-50 \quad -50$
 $\hline x = 65$

3.
 $4x + 2 + 2x - 9 = 5x + 13$
 $6x - 7 = 5x + 13$
 $-5x + 7 \quad -5x + 7$
 $\hline x = 20$

4.
 $x^2 + 1 + 4x + 3 = 2x^2 + 3x - 2$
 $-x^2 - 4x - 4 \quad -x^2 - 4x - 4$
 $\hline 0 = x^2 - 1x - 6$
 $0 = (x-3)(x+2)$
 $x-3=0 \quad x+2=0$
 $+3+3 \quad -2-2$
 $\hline x=3 \quad x=-2$

P: -6 | S: -1
 -6, 1
 -2, 3
 -3, 2

Side-Angle Relationships in Triangles: In a triangle, the longest side is across from the largest angle measure and the shortest side is across from the smallest angle measure.



Practice Problems:

List the angles in order from largest to smallest:

1.

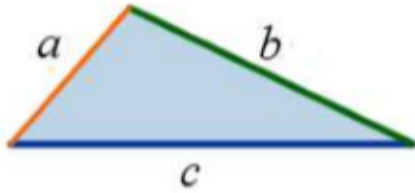
2.

List the sides in order from largest to smallest:

3.

4.

Triangle Side Inequality Theorem: The sum of the lengths of any two sides of a triangle is GREATER than the length of the third side.



$$a + b > c$$

$$a + c > b$$

$$b + c > a$$

Practice Problems:

Can these numbers be the measures of the sides of a triangle?

1. 7, 4, 5

$$4 + 5 = 9$$

Yes

2. 3, 6, 2

$$2 + 3 = 5$$

No

3. 8, 2, 8

$$2 + 8 = 10$$

Yes

4. 1, 13, 13

$$1 + 13 = 14$$

Yes

5. 1, 9, 10

$$1 + 9 = 10$$

No

6. 5, 8, 4

$$4 + 5 = 9$$

Yes

7. 6, 3, 10

$$6 + 3 = 9$$

No

8. 2, 15, 16

$$2 + 15 = 17$$

Yes

Two sides of a triangle have the following measures – Find the range of possible measures for the third side:

9. 9, 5

$$\begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array} \quad \begin{array}{r} 9 \\ +5 \\ \hline 14 \end{array}$$

$4 < x < 14$

10. 5, 8

$$\begin{array}{r} 8 \\ -5 \\ \hline 3 \end{array} \quad \begin{array}{r} 8 \\ +5 \\ \hline 13 \end{array}$$

$3 < x < 13$

11. 14, 11

$$\begin{array}{r} 14 \\ -11 \\ \hline 3 \end{array} \quad \begin{array}{r} 14 \\ +11 \\ \hline 25 \end{array}$$

$3 < x < 25$

12. 11, 8

$$\begin{array}{r} 11 \\ -8 \\ \hline 3 \end{array} \quad \begin{array}{r} 11 \\ +8 \\ \hline 19 \end{array}$$

$3 < x < 19$

13. 9, 6

$$\begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array} \quad \begin{array}{r} 9 \\ +6 \\ \hline 15 \end{array}$$

$3 < x < 15$

14. 7, 12

$$\begin{array}{r} 12 \\ -7 \\ \hline 5 \end{array} \quad \begin{array}{r} 12 \\ +7 \\ \hline 19 \end{array}$$

$5 < x < 19$

15. 4, 6

$$\begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array} \quad \begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$$

$2 < x < 10$

16. 3, 7

$$\begin{array}{r} 7 \\ -3 \\ \hline 4 \end{array} \quad \begin{array}{r} 7 \\ +3 \\ \hline 10 \end{array}$$

$4 < x < 10$