Lesson 1: Connection to Linearity 60 minutes

Goals

Focus Question: How is an arithmetic sequence similar to a linear relationship? How is it different?

Common Core Standard(s):

- HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs
- HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Objectives:

Students will be able to:

- Work with, create, and make connections between various forms of a linear relationship (tables, graphs, equations)

Students will understand that:

- An arithmetic sequence is a linear relationship with a restricted domain

Assessment:

- Group discussions about the task
- Extensions/Homework section

Design

Launch:

- Engage students in a discussion about their previous knowledge of linear relationships
  - $y = mx + b$
  - Constant rate of change
  - Graphs of lines with a y-intercept and a constant slope

Explore:

- Allow them to work in their collaborative groups to complete this lesson
- Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions:
  - *How does George's pay relationship relate to something you have seen before?*
  - *What are the key parts of George and Michelle's pay?*
  - *How can we see these key features of their pay in the various representations?*
  - *How are George and Michelle's pay relationships similar? Different?*

Summarize:

- If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task.
- Discuss their thoughts on part C as a class. Tie this section back to the similarities and differences between the two pay relationships shown in previous parts of the problem.

Lesson 1: Connection to Linearity

A. George works at an ice cream stand during the summer. He gets paid an initial amount each day and then an hourly rate. The table below represents his daily pay, G, after h hours of work.

Hours of work	1	2	3	4	5	6	7	8
Daily Pay (in dollars)	23	31	39		55			

1. Complete the table above to continue the pattern of (h, G) coordinates.

2. Describe the relationship between the number of hours George works, h, and the amount of money he earns each day, G. Include the values for the initial pay and hourly pay in your response.

3.

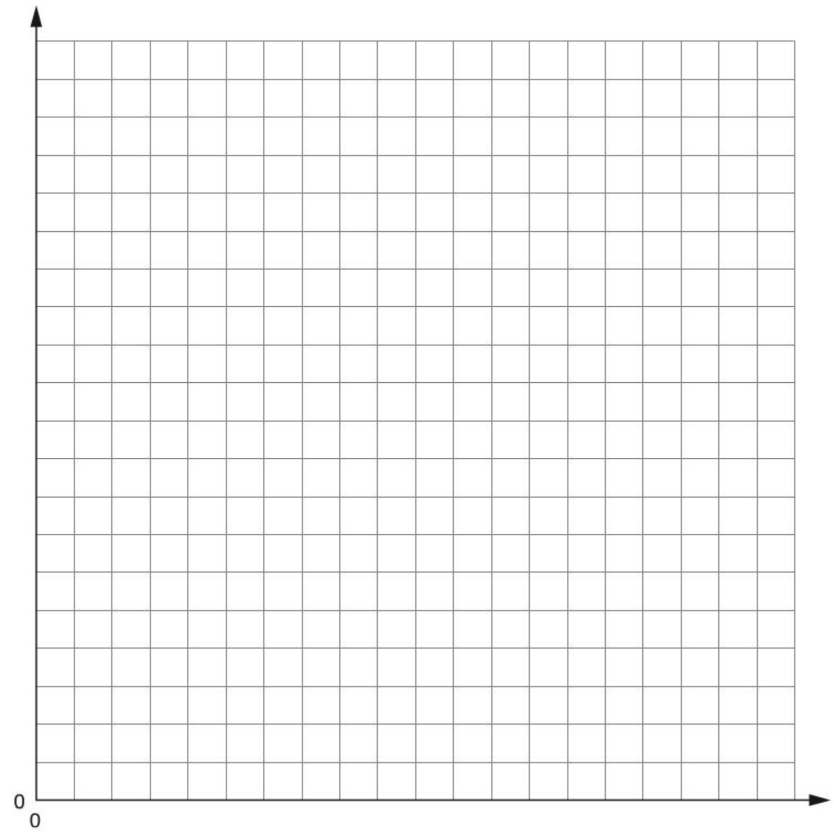
a. Write an equation to represent George's daily pay, G, after h hours of work.

b. Explain how your equation in **(a)** represents the relationship you described in question **2.**

c. Determine the amount of money George would earn if he worked a 12-hour shift.

4.

a. Graph George's daily income relationship on the grid below.



b. Explain how your graph in represents the relationship you described in question **2.**

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

d. Describe what the coordinate point (1.5, 27) means in this situation.

B. Michelle works at a lemonade stand by the beach. She is paid an initial amount each day and then earns commission from each drink that she sells. The table below represents her daily pay, M, after n lemonades are sold.

Lemonades Sold	1	2	3	4	5	6	7	8
Daily Pay (in dollars)	15.5	18	20.5	23				

1. Complete the table above to continue the pattern of (n, M) coordinates.

2. Describe the relationship between the number of lemonades Michelle sells, n, and the amount of money she earns each day, M. Include the values for the initial pay and amount earned per lemonade sold in your response.

3.

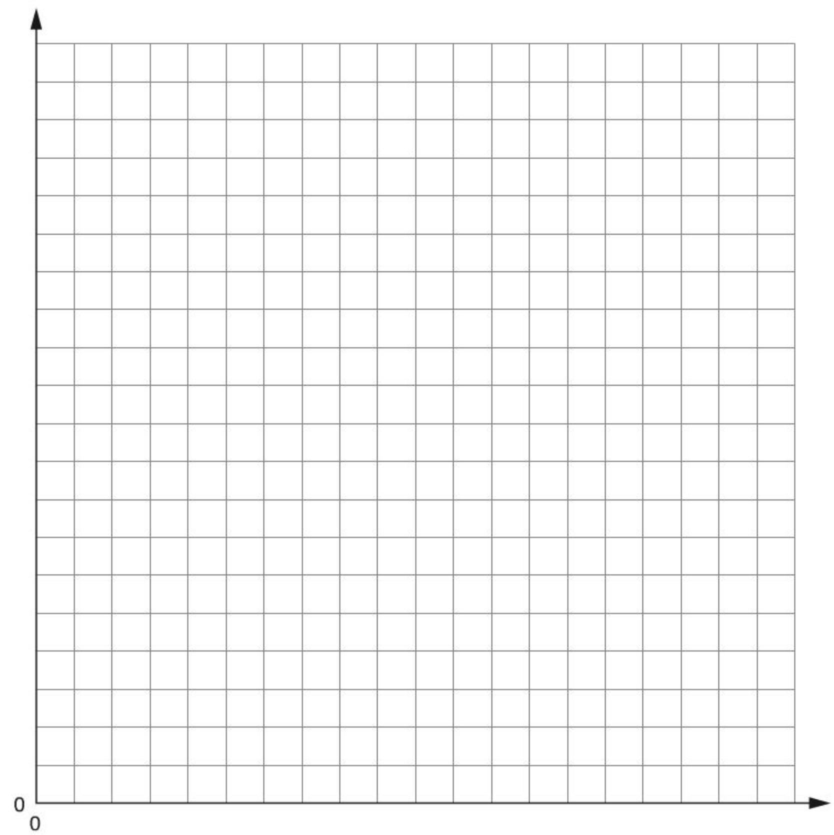
a. Write an equation to represent Michelle's daily pay, M, after n lemonades are sold.

b. Explain how your equation in **(a)** represents the relationship you described in question **2.**

c. Compare Michelle's daily earnings to George's. What similarities exist and how are the two situations different?

4.

a. Graph Michelle's daily income relationship on the grid below.



b. Explain how your graph in represents the relationship you described in question **2.**

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

d. Describe what the coordinate point (1.5, 16.75) would mean in this situation. Is this realistic?

C. Both George's and Michelle's pay are examples of linear relationships. Michelle's is also an example of an arithmetic sequence. Based on your work in parts **A** and **B**, what do you think makes a linear situation an arithmetic sequence? Why is George's pay not an example of an arithmetic sequence?

Arithmetic Sequences

Extensions/Homework

1. Jane works at a nail salon and is paid $12 an hour. Would her pay represent an arithmetic sequence? Explain.

2. Write an equation for the relationship in problem **1**. Compare this relationship to George's pay relationship in part **A.**

3. Would the relationship shown in the table below represent an arithmetic sequence?

Hours worked	1	2	3	4	5
Pay ($)	2	4	8	16	32

Lesson 2: Explicit Formulas 60 minutes

Goals	
Focus Question: How can we apply our knowledge of linearity to write explicit formulas for arithmetic sequences?	
Common Core Standard(s): ▪ HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs ▪ HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ▪ HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.	
Objectives: *Students will be able to:* ▪ Work with, create, and make connections between various forms of a linear relationship (tables, graphs, equations) ▪ Write explicit formulas of the form $y = mx + b$ and $a_n = a_1 + (n-1)d$	*Students will understand that:* ▪ An arithmetic sequence is a linear relationship with a restricted domain ▪ An arithmetic sequence can be written using various equivalent forms
Assessment: ▪ Group discussions about the task ▪ Extensions/Homework section	
Design	

Launch:
▪ Engage students in a discussion about the previous lesson's introduction of arithmetic sequences (Lesson 1 part C).
  ○ *How were arithmetic sequences similar to linear relationships? How were they different?*
  ○ *How did we see these similarities and differences in the various representations of George and Michelle's pay?*
Explore:
▪ Allow them to work in their collaborative groups to complete this lesson
▪ Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions:
  ○ *How does your equation relate to the linear equation $y = mx + b$?*
  ○ *What connections can you make between the various equations?*
  ○ *Where do we see the y-intercept and slope in each form of the equation?*
  ○ *Is the y-intercept shown in each representation? Why or why not?*
 	
Summarize:
▪ If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task.
▪ Summarize with C and D as a class to emphasize the connections between the forms.	

Lesson 2: Explicit Formulas

A. Michael, Michelle's brother, works at an iced tea stand near his sister's stand by the beach. Michael's boss pays him more each day but he earns less for each drink he sells. He earns a daily pay of $20 and then earns $1 for each iced tea he sells.

1. Complete the table below to represent how much Michael earns in one day of sales.

Iced Teas Sold	0	1	2	3	4	5	6	7
Daily Pay (in dollars)								

2.

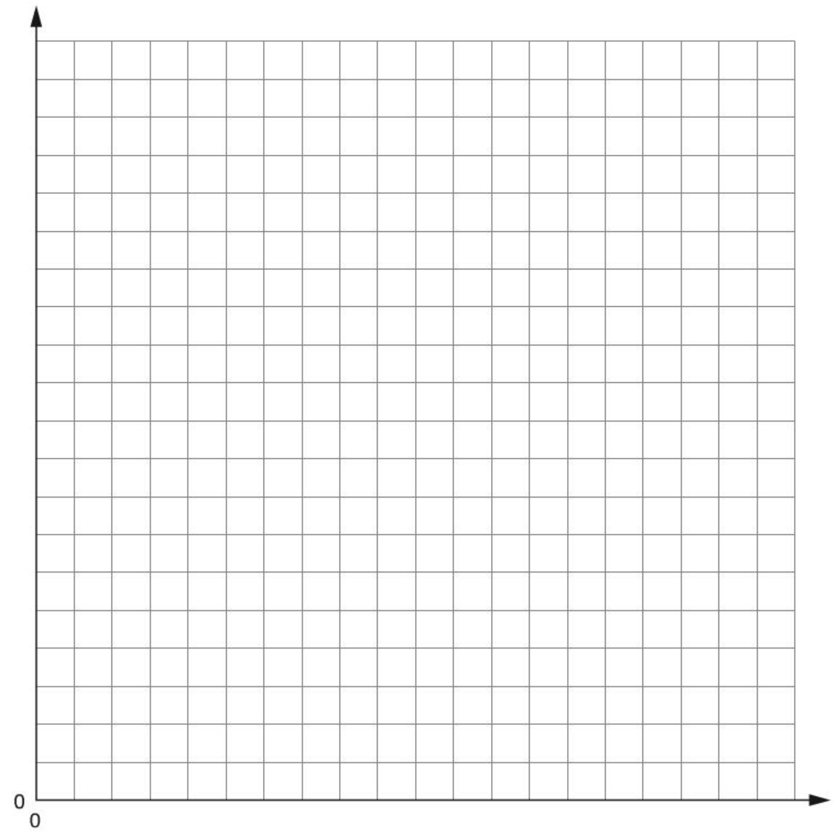
a. Write an equation to represent the amount of money Michael earns, M, in one day after he sells t iced teas.

b. Label how each variable and number in your equation represents this situation.

3. Does Michael's daily pay relationship represent an arithmetic sequence or is it just a linear relationship? Explain.

4.

a. Graph Michael's daily income relationship on the grid below.



b. Explain how your graph in represents your equation from part **2a.**

c. Did you connect your data points on your graph? In this context, would it make sense to do so?

B. Any arithmetic sequence can be written as an equation in the form

$$a_n = a_1 + (n - 1)d$$

This is often how reference sheets display the generic equation. The variables are defined as follows:

- a_n is the n^{th} term of the sequence. For example, a_6 is the 6^{th} term.
- n represents the number of the current term. For a_6, $n = 6$.
- a_1 is the term when $n = 1$ which is usually the first term of the sequence.
- d is the common difference. It represents how much the sequence changes from one term to the next.

1. Suppose we apply this new equation format to Michael's daily pay relationship from part **A.** What would a_6 represent in this case?

2. Michael says that a_1 represents the y-intercept. Whereas Michelle says that a_0 represents the y-intercept. Who do you agree with? Explain.

3. What is the value of d for Michael's pay situation? What is another name we typically use for this value?

4. Write an equation using the new format to represent Michael's daily pay.

5. Simplify your equation from part **4**. How does this equation relate to the one you wrote in part **2a**?

C. Arithmetic sequences can be written as equations using the traditional linear format of $y = mx + b$ or the new format $a_n = a_1 + (n-1)d$.

1. Which variable in the linear equation corresponds to the a_n variable from the new format?

2. Which variable in the new format corresponds to the m variable from the linear format?

3. Is there a variable in the new format that corresponds to the variable b from the linear format? If not, how could we write it using the new notation?

D. For each sequence shown below, write an equation using the traditional $y = mx + b$ format and an equation using the new $a_n = a_1 + (n-1)d$. Simplify all equations completely and draw arrows to show the relationship between the two equations.

1. y-intercept of 6, slope of -3.

2. $a_1 = \frac{1}{2}, d = 2\frac{1}{2}$

3.

x	1	2	3	4	5
y	14	9	4	-1	-6

E. For each sequence from part **D**, determine the value of a_{10}.

Extensions/Homework

1. Use the equation of an arithmetic sequence show below to answer the following questions.

$$a_n = 6 + (n - 1)(4)$$

a. What is the first term, a_1? What is the common difference, d?

b. Simplify the equation completely.

c. What is the value of the y-intercept of this relationship?

d. What is the value of a_4?

For # below, write an equation using the traditional $y = mx + b$ format and an equation using the new $a_n = a_1 + (n - 1)d$. Simplify all equations completely and draw arrows to show the relationship between the two equations.

2. y-intercept of 2, slope of 6.

3. $a_0 = 3, d = -2$

4.

x	1	2	3	4	5
y	7	11	15	19	23

Lesson 3: Recursive Formulas 60 minutes

Goals

Focus Question: How can we apply our knowledge of linearity to write recursive formulas for arithmetic sequences?

Common Core Standard(s):
- HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs
- HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Objectives:

Students will be able to:
- Use and write $NOW - NEXT$ statements to illustrate a sequence
- Write recursive formulas of the form $a_n =$ or $a_{n+1} =$ for a given arithmetic sequence
- Use recursive formulas to find values of a sequence

Students will understand that:
- A $NOW - NEXT$ statement shows the pattern of change between two consecutive terms of a sequence
- Recursive formulas can be written using various syntax but all are of the $NOW - NEXT$ form

Assessment:
- Group discussions about the task
- Extensions/Homework section

Design

Launch:
- Engage students in a discussion about the previous lesson's connection between linear formulas $y = mx + b$ and the formula for an arithmetic sequence $a_n = a_1 + (n-1)d$.
  - *What connections did we make between the two explicit formulas?*
  - *How were the two formulas different from each other?*

Explore:
- Allow them to work in their collaborative groups to complete this lesson
- Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions:
  - *How would the $NOW - NEXT$ change if Michelle made $3 per drink?*
  - *How does the $NOW - NEXT$ form relate to the $a_{n+1} = a_n + d$ form?*
  - *How can we use recursive formulas to find sequence values?*
  - *When is a recursive formula more useful? When is an explicit more useful?*

Summarize:
- If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task.
- Summarize with C and D as a class to emphasize the strengths of each type of formula and more examples of writing recursive formulas.

Lesson 3: Recursive Formulas

In the previous two lessons we discussed how both Michelle's and Michael's daily income situations were examples of arithmetic sequences. We used the general linear equation $y = mx + b$ and the arithmetic sequence equation $a_n = a_1 + (n - 1)d$ to represent these relationships.

Michelle's Daily Pay

Lemonades Sold	1	2	3	4
Daily Pay (in dollars)	15.5	18	20.5	23

Michael's Daily Pay

- Initial daily pay of $20
- $1 earned for each iced tea sold

A. We can use $NOW - NEXT$ statements to illustrate how to create a sequence of values **recursively**. The NOW variable represents the current term and the $NEXT$ variable represents the very next term. An example is shown below.

Sequence: 4, 7, 10, 13, …

$$NEXT = NOW + 3$$

Starting at: 4

This example says to start at a value of 4 and then add 3 to the current term in order to get the next term.

1. Write a $NOW - NEXT$ statement for both Michelle and Michael's daily pay. Use the y-intercept (for 0 drinks sold) as the starting value for each.

2. Which number in each of your equations would be the slope, m, if the equation was in the form $y = mx + b$?

3. Which number in each of your equations would be the y-intercept, b, if the equation was in the form $y = mx + b$?

B. Another similar method of writing these equations recursively uses different notation to represent the same situation. An example of this notation is shown below.

Sequence: 4, 7, 10, 13, …

$$a_{n+1} = a_n + 3 \text{ for } n \geq 1$$
$$a_1 = 4$$

1. Match up each part of this new formula with the previous one.

a. $NOW =$______________

b. $NEXT =$______________

c. Starting at =______________

2. Why does this formula need to include the $n \geq 1$ part? Would you be able to create the sequence without this? Explain.

3. Rewrite your $NOW - NEXT$ statements from **A1** using this new format.

4. What did you label your starting values in part **3** as? Explain your decision.

5. Using your formula for Michelle's daily pay from part **4**, list the steps needed to find the value of a_4.

C. Using your work from this lesson and previous lessons, answer the following questions.

1. If you had to find the value of a_{20}, which formula would you use? Explain.

Option 1	**Option 2**
$a_{n+1} = a_n + 3$ for $n \geq 1$ $a_1 = 5$	$a_n = 3n + 2$

2. Was your choice in part **1** an explicit or a recursive relationship?

3. When is an explicit formula for an arithmetic sequence (in the form $y = mx + b$ or $a_n = a_1 + (n-1)d$) more helpful than a recursive one?

4. When is a recursive formula more useful than an explicit one?

D. Write an explicit and a recursive equation for each situation described below.

1. Joan makes $10 per day plus an additional $1.30 for each basket she sells.

2. A stack of boxes at a grocery store consists of 22 boxes on first row on the bottom, 18 in the next row up, and so on forming an arithmetic sequence until the top has 2 boxes.

Extensions/Homework

1. At a large grocery store they are having a promotion to give away free samples of pasta sauce. On the first day, they start with 200 cans of sauce. They plan on giving away 12 cans of sauce per day.

    a. Write a recursive formula to represent this situation.

    b. Write an explicit formula to represent this situation.

    c. Find how many cans they will have left after 8 days.

    d. Determine on which day of the promotion they will run out of sauce.

Write an explicit and a recursive equation for each situation described below.

2. To help study for an upcoming test, Julia decides to study each night more than she studied the previous night. She plans on studying for 10 minutes the first night and will increase by 5 minutes each night.

3. James is paying off his student loans every month. In the first month, his loans start at a value of $30,000. He pays them off by writing checks for $250 each month.

Lesson 4: Explicit and Recursive Using Various Syntax 60 minutes

Goals

Focus Question: How can we write explicit and recursive formulas for arithmetic sequences using a variety of syntax?

Common Core Standard(s):

- HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs
- HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Objectives:

Students will be able to:

- Write explicit and recursive formulas using a variety of syntax and variables
- Use explicit and/or recursive formulas to find values of a sequence
- Make connections between different formulas for the same relationship

Students will understand that:

- The variables and syntax used in defining arithmetic sequences in interchangeable
- Function notation can be used to rewrite sequence equations just like how $f(x)$ is interchangeable with the variable y in $y = mx + b$

Assessment:

- Group discussions about the task
- Extensions/Homework section

Design

Launch:

- Engage students in a discussion about how to write recursive and explicit formulas for a given sequence of values.
  - *How would we write a recursive formula for 3, 8, 13, 18, …? An explicit one?*
  - *What are the key features of a sequence needed to write any formula for it?*

Explore:

- Allow them to work in their collaborative groups to complete this lesson
- Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions:
  - *How can we determine if two formulas represent the same sequence?*
  - *How does function notation change our previous formulas?*
  - *Which notation makes sense in each situation? Is there one perfect style for all?*

Summarize:

- If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task.
- Summarize with D as a class to discuss how different syntax can represent the same relationship.

Lesson 4: Explicit and Recursive Using Various Syntax

Just like how we can use the variables M and t in place of y and x in $y = mx + b$, we can also use different variables in place of a_n and n in $a_n = a_1 + (n-1)d$.

A. Josh has a job selling tie-dye T-shirts at the beach. His pay is represented by the equation show below:

$$a_n = 8 + (n-1)(3)$$

1. Simplify this equation completely.

2. Rewrite the equation using different variables that better align with the context. Briefly explain why you chose your variables.

3. Could this equation represent this situation? Explain.

$$J_t = 3t + 5$$

B. Function notation is also often used to represent a linear relationship. The form $f(x) = mx + b$ can be used in place of $y = mx + b$.

1. Rewrite your equation from **A2** using function notation.

2. Explain how a_n and $f(n)$ notations are alike.

C. Recursive sequences can also be written using various forms. Two recursive formulas for Josh's T-shirt income are shown below.

(a)	**(b)**
$J_{n+1} = J_n + 3$ for $n \geq 0$	$J_n = J_{n-1} + 3$ for $n \geq 1$
$J_0 = 5$	$J_0 = 5$

1. Compare these formulas. Are they both accurate ways of showing Josh's income relationship?

2. Using both formulas, generate the first five terms of each sequence. Does this support or change your answer from part **1**?

3. Rewrite the equations using $NEXT - NOW$ statements.

4.

a. If J_n is the NOW, what is the $NEXT$?

b. If J_n is the $NEXT$, what is the NOW?

5. Rewrite formula **(a)** using function notation like $J(n + 1)$.

D. Nicole works for the same T-shirt company as Josh and has to put up posters around town to advertise for an upcoming sale. She starts the day with 240 posters and is able to put up 3 every minute.

1. Write an explicit equation for this sequence using p_m as the number of posters left after m minutes.

2. Rewrite your equation from part **1** using function notation.

3. Write a recursive formula for this sequence using p_m as the number of posters left after m minutes.

4. Rewrite your recursive formula in a different but equivalent way. Use part **C** as an example.

5. Rewrite your formula from part **3** using function notation.

6. Rewrite your formula from part **4** using function notation.

7. Out of all equations written in parts **1-6**, which is the easiest one for you to understand. Justify your choice.

Extensions/Homework

1. Wood is often stacked like the picture shown at right with the bottom layer having the most and the top layer having the least. Suppose the first layer on the bottom has 48 logs. Each additional layer has 4 less logs than the layer below it.

a. Write an explicit equation for this relationship. Explain why you chose the variables you did.

b. If you answer in part **a** is not in function notation, rewrite it so that it is. Otherwise, rewrite it so that the equation is in a_n notation.

c. Write a recursive formula for this situation.

d. Write a different, yet equivalent, recursive formula for this situation.

e. Using any of your formulas, find the number of logs in the first 6 layers.

Lesson 5: Applying the Various Syntax — 60 minutes

Goals

Focus Question: How can we write, interpret, and utilize explicit and recursive formulas for arithmetic sequences using a variety of syntax?

Common Core Standard(s):

- HSF-LE.A.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs
- HSF-BF.A.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
- HSF-IF.A.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Objectives:

Students will be able to:

- Rewrite explicit and recursive formulas using a variety of syntax and variables
- Interpret given formulas and rewrite them in equivalent forms
- Use explicit and/or recursive formulas to find values of a sequence
- Make connections between different formulas for the same relationship

Students will understand that:

- The variables and syntax used in defining arithmetic sequences in interchangeable
- Due to the context, some sequences need to start with a 0th term while others need to start with a 1st term

Assessment:

- Group discussions about the task
- Extensions/Homework section

Design

Launch:

- Engage students in a discussion about how to rewrite a given sequence formula in as many different ways as possible.
  - *How many formulas can we write for the sequence* $2, -3, -8, \ldots$*?*
  - *Which formula makes the most sense to you? Why?*

Explore:

- Allow them to work in their collaborative groups to complete this lesson
- Provide them with supporting questions but do not provide concrete answers or affirmations. Example questions:
  - *How can we think of this recursive relationship as a* $NOW - NEXT$*?*
  - *Should this sequence start with a 0th term or a 1st term? Why does it matter?*
  - *Is there a better way to write this formula? What makes that way more useful?*

Summarize:

- If necessary, a summary can be done before the entire lesson has been completed in order to split up the parts of the task.
- Summarize with parts of C and all of D as a class to discuss the different equation forms and the difference between a 0th first term and a 1st first term.

Lesson 5: Applying the Various Syntax

A. Jeff creates a plan to raise money to clean up the beach. He receives an initial donation and then receives a donation for each T-shirt sold at Josh's T-shirt store. The store offers Jeff a few options to choose from.

For each function below, define what each variable represents. Also, determine how much the initial donation and how much the donation per T-shirt would be.

1. $f(0) = 40$
$f(x+1) = f(x) + 2$ for $x \geq 0$

2. $d_t = 30 + 2.25t$

3. $x_n = x_{n-1} + 2.75$
$x_0 = 25$

4. $g(y) = 35 + 2.1y$

5. For this context, is the first term of the sequence the 0^{th} term or the 1^{st} term? Explain.

B. Upon hearing about Jeff's fundraiser, the town decides to help with cleanup on the first day. As a result, the group was able to remove 15 barrels of garbage on the first day. After that, the remaining cleaners were able to remove 4 barrels of garbage a day.

1. Complete the table below to represent this situation.

Day	1	2	3	4	5	6
Barrels of Garbage Removed						

2. One of Jeff's workers, Molly, determines that a recursive formula could represent this relationship. Describe what her formula represents.

$$G(d+1) = G(d) + 4 \text{ for } d \geq 0$$
$$G(0) = 15$$

3. Does her equation accurately depict this situation? If yes, explain. If no, fix her formula so it is accurate.

4. Write an explicit formula using g_d notation for this relationship.

5. Rewrite your formula from part **4** using function, $f(x)$, notation.

6. For this context, is the first term of the sequence the 0^{th} term or the 1^{st} term? Explain.

C. For each arithmetic sequence defined below, do the following:

- State the first term and the common difference
- Write the first five terms of the sequence
- State whether the formula is explicit or recursive.
- If the formula is explicit, write a recursive formula. If it is recursive, write an explicit equation.
- Write the relationship as a $NEXT - NOW$ statement.

1. $f(1) = 12$
$f(x) = f(x-1) + 4$ for $x \geq 2$

2. $b_i = 4 + (n-1)(-2)$

3. $x_{n+1} = x_n - 2$ for $n \geq 1$
$x_1 = -5$

D. Throughout these problems you have seen some sequences that start with the first term as a_1 and others that start with a_0. Describe the differences between these two types of sequences. Use examples to show how each one may be used.

Extensions/Homework

Jane decides to do her own fundraiser to help with the beach cleanup. Her results from the cleanup form an arithmetic sequence and are shown in the table.

Day	1	2	3	4	5	6
Barrels of Garbage Removed	10	16	22	28	34	40

1. What is the initial value of the sequence? What is the common difference?

2. Write an explicit equation for this relationship. Explain why you chose the variables you did.

3. Josh defines Jane's sequence recursively as shown below. Does his sequence accurately create her results? If yes, explain. If not, what should he fix?

$j_{n+1} = j_n + 10$ for $n \geq 1$

$j_1 = 6$

Summative Assessment

Name: ____________________

Multiple Choice #1-5

1. What is the common difference of the sequence $5, 2, -1, -4, \ldots$?

(1) 3 (2) -3

(3) 5 (4) -5

2. Which formula accurately represents the sequence from question **1**?

(1) $a_{n+1} = a_n + 3$ for $n \geq 1$
$a_1 = 5$

(2) $a_{n+1} = a_n - 3$ for $n \geq 1$
$a_1 = 5$

(3) $a_{n+1} = a_n + 5$ for $n \geq 1$
$a_1 = -3$

(4) $a_{n+1} = a_n + 5$ for $n \geq 1$
$a_1 = 3$

3. What is the fifth term of the sequence described below?

$$f(n) = f(n-1) + 4 \text{ for } n \geq 2$$
$$f(1) = -3$$

(1) 13 (2) 5

(3) 1 (4) 17

4. Which equation would produce the same sequence as $a_n = 2 + (n-1)(-3)$?

(1) $y = 2x + 3$ (2) $y = -3x + 2$

(3) $y = -3x - 1$ (4) $y = -3x + 5$

5. Which explicit equation would represent the sequence defined below?

$$x_{n+1} = x_n + 3 \text{ for } n \geq 1$$
$$x_1 = 7$$

(1) $x_n = 3n + 7$ (2) $x_n = 3n + 4$

(3) $x_n = 7n + 3$ (4) $x_n = 7n - 4$

Short Answer – Show all work to support your solutions.

6. Bailey creates a pattern using blocks. In the first design, she uses 5 blocks. In the second, she uses 11 blocks. In the third, she uses 17 blocks.

a. Assuming the pattern continues in the same way. Explain why this situation represents an arithmetic sequence.

b. Write an explicit equation to model this sequence.

c. Write a recursive formula to model this sequence.

d. Determine the number of blocks Bailey would use in the 50^{th} design.

e. Which design number would need 35 blocks? Show your work to support your answer.

7. Josie is collecting donations for a walkathon. Her donors pledge a certain amount up front and then more for each mile she walks. Her donors provide her with the equations below to represent their pledges. For each equation, determine the initial donation, amount pledged per mile, and first five amounts for walking 1-5 miles.

a. $f(x) = 4x - 2$

Initial Donation	Pledge Per Mile	First Five Terms

Mile	1	2	3	4	5
Donation Total					

b. $b_{n+1} = b_n + 3$ for $n \geq 1$
$b_1 = 5$

Initial Donation	Pledge Per Mile	First Five Terms

Mile	1	2	3	4	5
Donation Total					

c. $a_n = 4 + (n-1)(3)$

Initial Donation	Pledge Per Mile	First Five Terms

Mile	1	2	3	4	5
Donation Total					

d. Are any of the plans from **a-c** unrealistic? Explain.

8. If Y_n represents the current term of a sequence, what notation would be used to represent the **next** term?

9. If $f(n)$ represents the current term of a sequence, what notation would be used to represent the **previous** term?

10. Robert is planning on running a marathon of about 42 kilometers. To build up his endurance, he plans on running 1.5 km the first day and then running an additional 0.75 km each day.

a. Write an explicit formula to represent the distance he runs each day D_n after n days.

b. Will Robert meet his goal of 42 km after a month (30 days)? If not, how close will he be?

c. On what day will Robert reach a distance of 42 km?