A Study of the Performance of Deaf/Hard of Hearing Students in High School Mathematics on Conceptual Understanding, Procedural Fluency, and Mathematical Reasoning Tasks

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Abstract

Mathematics tends to be a subject where many students struggle and that struggle becomes especially prevalent as students make the transition from concrete mathematics to abstract mathematics, or from elementary or middle school to high school mathematics. With students that are deaf, the learning of mathematics becomes more complicated. Many barriers to learning present themselves as deaf students work their way through school and as they go through school, the performance gap between hearing and deaf students begins to grow. This thesis discusses the language barrier as one possible contributor as well as other factors like teacher preparedness and pedagogical practices. This study focuses on the comparison of students that are deaf and their hearing equivalents and how they display conceptual understanding, procedural fluency, and mathematical reasoning on an assessment with New York State Regents Algebra I questions.
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Chapter One: Introduction

In areas with a large Deaf community, almost everyone knows or has met someone who is Deaf or Hard of Hearing (D/HH) but very few people understand the obstacles that deaf or hard of hearing persons face. Deaf individuals have severe hearing loss and hard of hearing individuals have hearing loss that falls on a spectrum from mild to profound. Typically, deaf students are academically behind their hearing peers and mathematics is one of the areas where they face learning challenges. For example, if a deaf student goes to a public state or local school for the deaf the teacher may be fluent in American Sign Language (ASL) but may lack pedagogical content knowledge (PCK) of how to teach mathematical concepts for understanding (Lang & Pagliaro, 2007). Also, a deaf student could be mainstreamed in a mathematics class with an excellent teacher but have an interpreter that cannot communicate the mathematics being taught effectively. Such common communication challenges may begin to explain why the deaf or hard of hearing students are academically behind their hearing peers. It is well known that mathematical concepts become more abstract and harder to comprehend in middle and high school, and for deaf individuals the language barrier may be a factor that makes this abstraction even more difficult. For the remainder of this paper, DHH will be used to refer to the population of Deaf and Hard of Hearing students.

List of Terms

To guide the reader through the content in this work, the following terms are defined:

Deafness – Defined by the Individuals with Disabilities Education Act (IDEA), it is a hearing impairment that is so severe that the child is impaired in processing linguistic information through hearing, with or without amplification. Deafness falls on the extreme end of the spectrum as severe.
Hard of hearing – A hearing impairment that falls between mild and profound on the spectrum

Deaf culture – A set of social beliefs, behaviors, art, literary traditions, history, values, and shared institutions of communities that are influenced by deafness and which use sign languages as the main means of communication. When used as a cultural label especially within the culture, the word deaf is often written with a capital D and referred to as "big D Deaf" in speech and sign. When used as a label for the audiological condition, it is written with a lower case d. For example: “He is Deaf”, means that he is a member of the Deaf Community while “He is deaf” means that he is lacking the sense of hearing.

Residential schools – An institution where students typically go and live full time or during the week while attending school. These can be private or state schools. All the students in the school are deaf or hard of hearing. They are often educated by deaf teachers or teachers who are trained in deafness. Some residential schools offer day-only options for students that are able to commute from home.

Conceptual Understanding – Demonstrated by recognizing, labeling, and generating examples of concepts; using and interrelating models, diagrams, manipulatives, and representations of concepts, identifying and applying principles, comparing and contrasting, and integrating relating concepts and principles

Procedural Fluency - the ability to apply procedures accurately, efficiently, and flexibly; transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another.

Mathematical Reasoning – The ability to think logically about the relationships among
concepts. Such reasoning is valid and stems from careful consideration of alternatives, and includes knowledge of how to justify conclusions.

**Pedagogical Content Knowledge** - The overlap of information about subject knowledge, that is knowledge of the subject being taught, and the knowledge of how to teach. It includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to learning.

**Knowledge of Content and Students** – The intertwining of a teacher’s knowledge of a subject content and their students. It is the understanding of how the students will interact with the content.

**Performance Gap** – Students classified as hearing impaired are generally 2 years behind their hearing peers. As the students progress, the performance gap grows.

**Common Core State Standards (CCSS)** – A set of academic standards in mathematics and English that were created to ensure that all students graduate from high school with the necessary skills and knowledge to succeed in the future. In 2010, the CCSS were adopted by much of the United States. The provided link shows where the standards have been implemented:


**National Council of Teachers of Mathematics (NCTM) standards** – A set of academic standards that was used to inform state standards before the implementation of the Common Core.

**Analysis of Variance (ANOVA)** – A statistical method used to analyze areas of significant differences among group means. It provides a test of whether or not the means of several groups are equal.
Purpose

Research in the field of deafness and deaf culture show that DHH students are approximately two years behind their hearing equivalents. Specifically, in mathematics, DHH students graduate at a fifth or sixth grade comprehension level (Pagliaro & Kritzer, 2013). The research done by many has targeted specific mathematical topics that are typically the most difficult. These topics include problem solving, measurements, estimation, patterns, and more (Kritzer, 2009). However, this data only looks at young DHH children and because the majority of deaf studies include pre-kindergarten through second grade (ages 3 to 8), there is a great lack of information for DHH students past the eighth grade. It could be assumed that the performance gap for DHH pre-kindergarten and kindergarten children extends to DHH high school students. Previous research leads into an examination of the widening performance gap for DHH in high school mathematics. The performance gap is approximately two years and seemingly continues to grow as students continue through high school. It is not clear if the performance gap centers around problem solving specifics such as conceptual understanding, procedural fluency, and mathematical reasoning. Thus, the purpose of this research is to determine how conceptual understanding, procedural fluency, and mathematical reasoning contribute to DHH students’ learning of mathematics in comparison to their hearing peers.

Chapter Two: Literature Review

Research provides a background on mathematics performance for both hearing and DHH students. The results from the 2003 National Assessment of Educational Progress (NAEP) report that students without hearing loss in fourth and eighth grade show increasing average mathematics scores. On the other hand, the Standard Achievement Test-9 (Traxler, 2000) showed that 80% of DHH students in fourth and eighth grade score below the average level in
procedural performance. Half of those DHH students fell below a third and fifth grade level respectively. In problem solving, similar results were reported. For fourth graders, 80% scored average or below average, with half scoring just above a second grade level. For eighth graders, 80% scored average or below and half of eighth graders at only a fourth grade level (Pagliaro, 2006).

Many researchers in the field of deaf studies are concerned with why and when the performance gap begins (Pagliaro & Kritzer, 2013). It is known that about 50% of DHH students have a co-occurring disability (Caemmerer et al., 2016). Some professionals may be hesitant to diagnose other disabilities in a student who is DHH because of the difficulty in ruling out the student’s hearing loss and reduced exposure to language and communication models as a primary cause of a disability (Caemmerer et al., 2016). However, ruling out any co-occurring disabilities, there is no difference in cognitive abilities between deaf and hearing students (Nunes & Moreno, 1998).

Instead, it is possibly the language barrier, experiences, and instruction in a child’s life that play a role in their mathematics performance (Pagliaro & Kritzer, 2013). Humans incidentally learn mathematical concepts at a young age. For example, a parent may count toes or use words like “big” or “little” to identify a sibling. Mathematical concepts, like quantity, develop from infancy and children begin to mathematize between ages 3 and 6. Children intuitively develop concepts from numbers to geometry (Pagliaro & Kritzer, 2013). DHH children most often lack those experiences and parental instruction because of the language barrier assuming the parents do not sign.

DHH children who know American Sign Language (ASL) show average or better skills in object counting. These children understood a one-to-one correspondence between object and
sign. Research by Lang and Pagliaro (2007) examined the predictors of geometry terms recall. They chose to study familiarity, imagery, signability, and concreteness. Of the four factors, imagery proved to be the best predictor of term recall. This supports research that argues that terms represented by a single sign are recalled better than terms represented by compound signs (Lang and Pagliaro, 2007, p. 457). But, further research on other mathematical concepts did not show similar results. Young DHH children scored average or below average in many other categories, especially story problems, also known as word or applied problems (Pagliaro & Kritzer, 2013). At any age or grade level, story problems tend to be a large part of mathematics and classroom instruction. Typically, an individual without DHH can pick up necessary vocabulary as well as numbers to correctly solve the problem, but this is more of a challenge for DHH students. A study done on young DHH children examined their ability to solve word problems. Ansell and Pagliaro (2006) found that the children did not connect the story language to arithmetic functions important in solving the problem. Even when the story was presented in ASL the children did not view the story as having any links to the numbers. In general, the children were missing the linguistic cues that would make the problem easier to solve (Ansell & Pagliaro, 2006).

**Pedagogical Best Practices**

The discussions of the pedagogical best practices are not unique for the field of DHH education. There is framework surrounding what pedagogical content knowledge (PCK) is and how it relates to students’ mathematical outcomes. Hill et al. (2008) research leads to an examination and conceptualization of teachers’ knowledge of content and students (KCS). Both PCK and KCS are argued to be crucial in a classroom, for both DHH and hearing students. KCS
is defined as “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content” (Hill et al., 2008, p. 375).

Using pedagogical content knowledge best practices can have a tremendous affect on student performance. Schoenfeld (2002) conducted a study of 40,000 students without hearing loss and showed that when a teacher followed the National Council of Teachers of Mathematics (NCTM) mathematical standards, students performed better than when a teacher did not implement the standards as strongly. Similar research has found that in the field of deaf education, schools for the deaf use mostly traditional teaching methods (Pagliaro & Kritzer, 2005, NCTM, 2000). One study showed that in a deaf education classroom, teachers do not base instruction on national, professional recommendations (Pagliaro & Kritzer, 2005). Beyond the instructional method, many educators responsible for teaching mathematics in schools for the deaf do not have a degree or certification in mathematics education (Kelly, Lang & Pagliaro, 2003). A survey in 2003 found that in a residential school for the deaf only 39% of those teaching mathematics held a mathematics education teacher certification. In an inclusive school setting, 67% had this certification. Within that same survey, certified mathematics teachers supported the idea that, “…Preparation and certification in mathematics makes a difference in instruction particularly in the kinds of word problem solving challenges provided to deaf students” (Kelly, Lang, & Pagliaro, 2003).

**Teacher Preparedness**

Evidence has shown that U.S. teachers were lacking in essential knowledge to teach mathematics and that lack of knowledge was impacting their students’ learning (Hill et al., 2005). A study (Hill et al., 2005) supports the idea that teachers’ mathematical knowledge for teaching positively predicted student gains in mathematics achievement and also suggest that
knowledgeable teachers may provide better mathematical explanations, construct better representations, and have a clearer understanding of the structures essential to mathematics and how they relate. The amount of DHH students across the country has increased considerably, yet the number of teachers prepared to teach those students has remained the same (Johnson, 2004). It is unknown exactly how many teachers are educating DHH students without the appropriate certifications. Overall, the amount of teachers in special education without appropriate preparation is about 10% (Johnson, 2004).

A 2002 study found significant changes in teacher preparation programs in the past 2 decades. There was a reported 46 Council on Education of the Deaf (CED) approved programs, a decline of 18% since a similar 1988 study. The 2002 study also found that only 61% of known teacher preparation programs in deaf education have CED approval (Jones & Ewing, 2002). The focus of CED-approved programs has changed and despite the growing numbers of deaf students with multiple disabilities (Karchmer & Allen, 1999), the percentage of programs offering "multi-handicapped" specializations has dropped. (Jones & Ewing, 2002).

In their research, Kelly et al. (2003) found that in terms of education preparation and certification, mainstream classrooms had the most qualified educators. Center schools and self-contained classrooms are receiving mathematics instruction from teachers not certified in mathematics. Concerning problem solving, all types of schools spent similar time on word problems. Most teachers spent time focused on procedures and practice rather than true problems as well as emphasizing visualizing strategies rather than analytical strategies. Kelly et al. (2003) believed such emphasis stemmed from the teachers’ perceptions of DHH students’ capabilities in mathematics. There are many teacher perceptions revolving around DHH students’ abilities to solve word problems which leads some to argue that the teachers role heavily influences DHH
students’ learning (Kelly et al., 2003). Mainstream teachers have a higher perception of DHH students than teachers in center schools and self-contained classrooms (Kelly et al., 2003).

**Understanding Mathematics Conceptually, Procedurally, and Critically**

Problem solving in mathematics requires conceptual understanding, procedural fluency, and mathematical reasoning, and the NCTM Standards states there should be a balance between them (NCTM, 2000; Wade, 2011). Mathematics word problems in secondary education, often referred to as applied problems, require conceptual understanding, procedural fluency, and critical (or mathematical) reasoning. Such problems are often addressed in the new Common Core State Standards (CCSS) mathematics standards. In the field of DHH mathematics education there has been an emphasis on problem solving skills (including conceptual understanding and procedural fluency) when teaching mathematics. DHH students struggle with problem solving tasks and often achieve below their hearing peers when solving applied problems (Kelly et al., 2003). DHH students may conceptually understand mathematics better because of the link between ASL being a conceptual language and conceptually understanding problem solving. On the other hand, DHH students may struggle with using correct procedures and reasoning when solving problems (Lang & Pagliaro, 2007).

Skemp (2006) referred to discontinuities in learning as cognitive conflict, and according to this notion, if students fail to understand mathematical concepts, or if they grasp concepts but cannot connect them to relevant procedures, those flawed procedures develop into what Clark and Lovric (2009) referred to as a synthetic model. This model represents the misconceptions of mathematics learned that do not assimilate to other future mathematics courses. While much is yet to be researched, it is known that many students find developing appropriate new framework for higher levels of learning very challenging. Such performance discrepancies are thought to be
stemming from linguistic, cognitive, and experiential factors (Kelly et al., 2003). Language content has been considered the dominant factor in DHH students’ difficulties with mathematics. DHH students tend to display a greater difficulty with English thus the English-language structure used in mathematics causes more difficulty for this population of students (Kelly et al., 2003).

Kelly et al. (2003) also looked at the strategies used for problem solving. Their results suggested that teachers of DHH students give substantial attention and time to comprehension and pre-problem set up with much less focus on the aspects of solving and analysis of the mathematical strategies. The instruction used by educators is insufficient for advanced problem solving. The results of the survey show that teachers of DHH students tend to avoid more challenging aspects of word problem solving. This could be due to the students’ English-language abilities as well as a lack in teacher preparation in mathematics (Kelly et al., 2003).

**Chapter Three: Method**

To better understand the performance gap in mathematics of DHH students, this study investigated the constructs of conceptual understanding, procedural fluency, and mathematical reasoning during problem solving. To do this, 10 problems from past New York State Regents Algebra exams were chosen that had specific problem solving tasks that aligned with the procedural fluency, conceptual understanding, or mathematical reasoning constructs. In all, there were five conceptual understanding tasks, six procedural fluency tasks, and five mathematical reasoning tasks from the solutions in the 10-problem assessment. The classifications of these constructs were based on the NCTM (2000) definitions. Some problems were multi-stepped involving two or more of the constructs while other problems had solutions that aligned with only one of the constructs. For the questions that were broken up into parts or had more than one
classification, each individual piece was graded. The possible grades for all questions were 0, 1, or 2. A grade of 0 was given if the question was not attempted; a grade of 1 was given for an incorrect answer; a grade of 2 was given for a correct answer.

Table 1 shows the constructs and how they were aligned (and defined) for each problem on the assessment

**Sample**

To investigate if the constructs of conceptual understanding, procedural fluency, and mathematical reasoning can be used to better understand the mathematical performance gap for

Table 1: *How the Constructs Conceptual Understanding, Procedural Fluency, and Mathematical Reasoning were mapped to the solutions of the 10 problems on the assessment.*

<table>
<thead>
<tr>
<th>Problems on the Assessment</th>
<th>Conceptual Understanding</th>
<th>Procedural Fluency</th>
<th>Mathematical Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
<td>Mental methods for finding products, sums, and differences</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>Using examples of models or representation of the concept</td>
<td>None</td>
<td>Think logically about relationships among concepts and situations</td>
</tr>
<tr>
<td>3</td>
<td>None</td>
<td>Knowledge of procedures, when and how to use them appropriately</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>Generate models, identify and apply principles; know and apply facts and definitions</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>5</td>
<td>None</td>
<td>None</td>
<td>Thinking logically among concept; navigate through concepts and solutions methods to see that they fit together in some way; knowledge of how to justify the conclusion</td>
</tr>
<tr>
<td>6</td>
<td>Interpret and apply the signs symbols and terms</td>
<td>None</td>
<td>Careful consideration of alternatives and knowledge of how to justify the conclusion</td>
</tr>
</tbody>
</table>
### DEAF STUDENTS IN MATHEMATICS

<table>
<thead>
<tr>
<th></th>
<th>None</th>
<th>Mental methods for finding products, quotients, sums, and differences</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>None</td>
<td>Mental methods for finding sums and using calculators</td>
<td>Think logically about the relationships among concepts and situations</td>
</tr>
<tr>
<td>8</td>
<td>Recognize, label and generate examples of concepts using models or diagrams</td>
<td>Mental methods for finding products, quotients, sums, and differences</td>
<td>None</td>
</tr>
<tr>
<td>9</td>
<td>Recognize, label and generate examples of concepts using models or diagrams</td>
<td>Methods for using calculator</td>
<td>Think logically about the relationships among concepts and situations</td>
</tr>
<tr>
<td>10</td>
<td>Recognize, label and generate examples of concepts using models or diagrams</td>
<td>Methods for using calculator</td>
<td>Think logically about the relationships among concepts and situations</td>
</tr>
</tbody>
</table>

DHH students, the 10 problem assessment was given to a group of DHH students and students without DHH. Analyzing performance across both groups can inform if the performance gap centers around procedural fluency, as Kelly et al. (2003) posed. It may also inform if DHH students struggle with overall critical thinking and problem solving (Lang & Pagliaro, 2007). The assessment was given to the two different groups of students (DHH and without DHH) in two different schools. Students without DHH were 9th graders from a public high school in upstate NY (n=11). Students that were DHH were from a public School for the Deaf, also in upstate NY (n=5). Due to the limited number of students within the School for the Deaf, the group of 5 students are from grades 9 through 12. The groups in both schools were given the same assessment and had about 45 minutes to complete it. All assessments were graded equally.

**Analysis**

To evaluate the problem solving differences across groups (students with and without DHH) and across constructs (conceptual understanding, procedural fluency, and mathematical reasoning) were analyzed using analysis of variance (ANOVA).
Chapter Four: Results

All data is based on student performance between DHH students and students without hearing loss. Gender was not recorded in this study. The data was analyzed to understand whether or not there is a significant difference in conceptual understanding, procedural fluency, and mathematical reasoning between the two groups. Figure 1 compares the mean difference in problem solving across the three constructs for the two groups. There is a significant difference for the conceptual understanding construct because the error bars do not overlap. There is also a significant difference in performance across the other two constructs, evidenced by the error bars not overlapping. Interestingly, students with DHH performed significantly higher on the

![Figure 1](image_url)

*Figure 1: Mean performance in groups of students with DHH and without DHH for the three construct of conceptual understanding, procedural fluency, and mathematical reasoning.*
mathematical reasoning tasks. To investigate this further, analysis was run across the group of ninth graders and students that were in higher grades. Although the latter was a much smaller group, Figure 2 shows that the difference in performance in the mathematical reasoning group is not because of the older students being in the group of students with DHH.

![Figure 2: Mean performance in groups of 9th grade students and students in higher grades.](image)

**Chapter 5: Conclusion**

This study was completed in upstate New York with a small sample size. It is important to note that no student in the entire sample got every answer correct. It is also important to note that some students did not complete all of the problems. It is unknown if those students did not complete all problems because they did not have enough time or did not know how to do the problem.

This study shows the level of performance of DHH students as compared to their hearing peers. It analyzed how DHH students and students with no hearing loss performed on conceptual,
procedural, and reasoning tasks. Based on the results that students without hearing loss performed significantly higher on conceptual and procedural tasks it can be thought that those students are more likely to maneuver through mathematics by applying concepts to procedures. The results that DHH students performed significantly higher on mathematical reasoning tasks lends to the idea that they are more likely to complete mathematics problems by using critical thinking skills and reasoning.

Despite the results showing a significant difference in performances between DHH and students with no hearing loss, this study was limited to two small groups in one state. Therefore, this study reveals the importance to continue researching what and how to assist the education of DHH students. The fact that DHH students performed better on mathematical reasoning tasks points to the need for more research in this area. Conceptual understanding, procedural fluency, and mathematical reasoning are the three key components of problem solving (NCTM, 2000), yet a great deal of research only focuses on conceptual understanding and procedural fluency. It seems that mathematical reasoning tends to get thrown in at the end without much substantial background information. Mathematical reasoning is the ability to think logically about mathematical concepts in order to complete procedures correctly and effectively. This pushes forward the idea that mathematical reasoning in DHH students should be researched more formally to understand and raise student performance not only in mathematical reasoning but also in conceptual understanding and procedural fluency. Concerning the low numbers of teachers prepared to teach mathematics to DHH students, such research could impact how teachers implement standards and navigate topics. It is important to know how DHH students use mathematical reasoning and critical thinking skills in order to develop conceptual understanding.
and procedural fluency of various mathematical topics. This in turn may lead to closing the performance gap between DHH students and their hearing peers.
References


Appendix

Assessment

1.

The expression $3(x^2 - 1) - (x^2 - 7x + 10)$ is equivalent to

(1) $2x^2 - 7x + 7$ \hspace{1cm} (3) $2x^2 - 7x + 9$
(2) $2x^2 + 7x - 13$ \hspace{1cm} (4) $2x^2 + 7x - 11$

2.

A public opinion poll was taken to explore the relationship between age and support for a candidate in an election. The results of the poll are summarized in the table below.

<table>
<thead>
<tr>
<th>Age</th>
<th>For</th>
<th>Against</th>
<th>No Opinion</th>
</tr>
</thead>
<tbody>
<tr>
<td>21–40</td>
<td>30</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>41–60</td>
<td>20</td>
<td>40</td>
<td>15</td>
</tr>
<tr>
<td>Over 60</td>
<td>25</td>
<td>35</td>
<td>15</td>
</tr>
</tbody>
</table>

What percent of the 21–40 age group was for the candidate?

(1) 15 \hspace{1cm} (3) 40
(2) 25 \hspace{1cm} (4) 60

3.

The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of $r$, in terms of $h$ and $V$, is

(1) $r = \sqrt{\frac{V}{\pi h}}$ \hspace{1cm} (3) $r = 2V\pi h$
(2) $r = \sqrt{V\pi h}$ \hspace{1cm} (4) $r = \frac{V}{2\pi}$
4. Which table represents a function?

(1) \[
\begin{array}{|c|c|c|c|c|}
\hline
x & 2 & 4 & 2 & 4 \\
\hline
f(x) & 3 & 5 & 7 & 9 \\
\hline
\end{array}
\]

(3) \[
\begin{array}{|c|c|c|c|c|}
\hline
x & 3 & 5 & 7 & 9 \\
\hline
f(x) & 2 & 4 & 2 & 4 \\
\hline
\end{array}
\]

(2) \[
\begin{array}{|c|c|c|c|}
\hline
x & 0 & -1 & 0 & 1 \\
\hline
f(x) & 0 & 1 & -1 & 0 \\
\hline
\end{array}
\]

(4) \[
\begin{array}{|c|c|c|c|}
\hline
x & 0 & 1 & -1 & 0 \\
\hline
f(x) & 0 & -1 & 0 & 1 \\
\hline
\end{array}
\]

5. A student invests $500 for 3 years in a savings account that earns 4% interest per year. No further deposits or withdrawals are made during this time. Which statement does not yield the correct balance in the account at the end of 3 years?

(1) \[500(1.04)^3\]

(2) \[500(1 - .04)^3\]

(3) \[500(1 + .04)(1 + .04)(1 + .04)\]

(4) \[500 + 500(.04) + 520(.04) + 540.8(.04)\]
6. Which inequality is represented in the graph below?

\begin{align*}
(1) \ y & \geq -3x + 4 \\
(2) \ y & \leq -3x + 4 \\
(3) \ y & \geq -4x - 3 \\
(4) \ y & \leq -4x - 3
\end{align*}

7. If the difference \((3x^2 - 2x + 5) - (x^2 + 3x - 2)\) is multiplied by \(\frac{1}{2}x^2\), what is the result, written in standard form?
8. Jacob and Zachary go to the movie theater and purchase refreshments for their friends. Jacob spends a total of $18.25 on two bags of popcorn and three drinks. Zachary spends a total of $27.50 for four bags of popcorn and two drinks.

Write a system of equations that can be used to find the price of one bag of popcorn and the price of one drink.

Using these equations, determine and state the price of a bag of popcorn and the price of a drink, to the nearest cent.

9. A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by $x$, and the area of the garden is 108 square meters.

Determine, algebraically, the dimensions of the garden in meters.
10. A football player attempts to kick a football over a goal post. The path of the football can be modeled by the function \( h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x \), where \( x \) is the horizontal distance from the kick, and \( h(x) \) is the height of the football above the ground, when both are measured in feet.

On the set of axes below, graph the function \( y = h(x) \) over the interval \( 0 \leq x \leq 150 \).

Determine the vertex of \( y = h(x) \). Interpret the meaning of this vertex in the context of the problem.

The goal post is 10 feet high and 45 yards away from the kick. Will the ball be high enough to pass over the goal post? Justify your answer.