Multiple Representations, Technology, and Purposeful Questioning:

An Exemplar of Teaching 6th Grade Ratios During the Pandemic

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Abstract

The aim of this curriculum project is to provide educators with a comprehensive set of lessons and materials that support the teaching of ratios and related concepts remotely using multiple representations and purposeful questioning. Students’ formal education in ratios often begins in the 6th grade and will continue throughout their study of mathematics and the learning of ratios represents an important milestone in a student’s mathematical education. The concept of ratios will appear in various mathematics classes in the form of rates, unit rates, slope, constant rate of change, average rate of change, the derivative and trigonometric ratios to name a few. A strong understanding of ratios is also important in the fields of science and engineering because of their numerous real-world applications. It is thus vital that students gain a strong conceptual understanding of ratios early on through multiple representations such as tape diagrams, double number lines, coordinate planes, and ratio tables. The lessons within this curriculum project incorporate free online learning platforms as well as other learning tools to ensure easy access to material and provide ample opportunity for student engagement. The six remote lessons that accompany this thesis, created during the 2020 COVID-19 pandemic, can be brought into the physical classroom and used in place of more traditional lessons.
# Table of Contents

**Introduction** .......................................................................................................................... 5

**Literature Review** ..................................................................................................................... 7

  Why Emphasize the Ratio ........................................................................................................ 7
  Concepts and Misconceptions ..................................................................................................... 8
  Correcting Misconceptions Through Multiple Representations ............................................... 12
    Ratio Tables ............................................................................................................................. 12
    Tape Diagrams ......................................................................................................................... 12
    Double Number Lines ............................................................................................................. 13
    Coordinate Plane ..................................................................................................................... 14
  Remote Learning ........................................................................................................................ 15

**Curriculum** ............................................................................................................................. 17

  Introduction to curriculum ......................................................................................................... 17
  Lesson One – Ratios from Tables ............................................................................................... 18
  Lesson Two – Ratios from Double Number Lines ................................................................... 25
  Lesson Three – Ratios from Tape Diagrams ............................................................................ 33
  Lesson Four – Ratios from the Coordinate Plane .................................................................... 41
  Lesson Five – Unit Rates .......................................................................................................... 53
  Lesson Six – Constant of Proportionality ................................................................................. 62

**Integration into the Classroom** .............................................................................................. 69

  Teaching Ratios Remotely ......................................................................................................... 69

**Conclusion** ............................................................................................................................. 72

**References** ............................................................................................................................... 74

**Appendix** ................................................................................................................................ 76
Lesson One – Ratios from Tables – Answer Key ............................................................... 76
Lesson Two – Ratios from Double Number Lines – Answer Key ........................................ 77
Lesson Three – Ratios from Tape Diagrams – Answer Key ................................................ 78
Lesson Four – Ratios – Answer Key ................................................................................. 80
Lesson Five – Unit Rates – Answer Key ........................................................................... 83
Lesson Six – Constant or Proportionality – Answer Key ................................................. 84
Introduction

The purpose of this curriculum project is to introduce 6th graders to the topic of ratios using multiple representations and purposeful questioning supported with instruction that incorporates free online learning platforms. The instructional aim is to expose students to multiple representations of ratios including ratio tables, graphs, double number lines and tape diagrams with the goal to build and strengthen a deep understanding of ratios. Ratios and proportional reasoning present a thread in mathematics that follow students throughout their mathematical education. Ratios are present in most mathematics classes represented by rates, unit rates, slope, constant rate of change, average rate of change, the derivative, trigonometric ratios, and more. Ratios are also used heavily in the sciences such as engineering and physics and also have countless real-world applications. Because of this, it is of great importance that students not only understand the procedures behind solving ratio problems, but develop a deep and meaningful understanding of ratios. Exposing students to multiple representations of ratios is one way to build a solid understanding which can be accomplished through the use of technology.

While teaching remotely presented many challenges to teachers and students alike, it also provided insight into how technology can be used to support instruction and learning. The use of technology can provide electronic manipulatives such as interactive cartesian planes that allow students to develop a deeper understand of the material being taught. Online learning platforms often provide videos and worked examples that students and teachers can use to support learning. For teachers, utilizing online platforms provides a number of benefits that are instrumental to educational success. A major issue in education today is differentiated learning.
Online platforms allow teachers a great amount of flexibility by assigning students specific exercises and lessons that target their individual needs. If a student is struggling with ratio tables because their multiplication facts are not on grade level, the teacher can quickly locate and assign an exercise that will help the student strengthen that foundational concept. For individual or groups of students who are accelerated learners, teachers can quickly locate more challenging exercises or move those students through the lessons at a quicker pace. A draw-it function is often included in online learning plate forms and allows students to use an e-pen or mouse to write out their work on an electronic device rather than using a sheet of paper and pencil. Draw-it functions provide a real time glimpse of the student’s thought process as they work through a problem and they make pinpointing misconceptions and mistakes much easier. If used efficiently, draw-it functions allow teachers to watch and monitor multiple students’ progress simultaneously. Also, online learning tools provide teachers with digital manipulatives that they most likely could not create on their own such as interactive graphs and tables. Lastly, online learning platemforms provide teachers a way of strengthening their own content knowledge through videos and exercises by giving a helpful and easy way to review material before presenting. Teachers can look ahead to see the progression of material from grade to grade which will help them build a solid foundation of concepts and make them more flexible in their own presentations and strengthen their ability to address students’ needs.
Why Emphasize the Ratio?

Because of the various applications of ratios across multiple fields of study and in the real-world, it is important that students first exposure to ratios, usually in the 6th grade, is as meaningful and thorough as possible. Understanding of ratios is critical to student success across multiple topics including mathematics, science and ratios have countless applications in the real world (Daughtry, Bryant, Bryant & Shin, 2017). Ratios and proportional reasoning represent an essential milestone in the mathematical education of students (O’Keeffe & White, 2018). In the topic of mathematics, proportions and ratios provide a key component in building skills surrounding slope and rates of change (Daughtry et al., 2017). These ideas and topics flow into the algebra classroom where trigonometric ratios are explored such as sine, cosine, and tangent (Daughtry, et al., 2017). Ratios are also prevalent in trigonometry and serve as a link between algebraic and geometric thinking and is a prerequisite for many advanced fields such as physics, architecture and engineering (Koyunkaya, 2016). In the science classroom, ratios are used when dealing with density and acceleration as well as other areas in which units of measurement are compared (Daughtry, et al., 2017). In the real-world, ratios are applied when using recipes, calculating gas mileage and any other places where relationships between quantities are being described (Daughtry, et al., 2017). They are also used in practical activities such as scale drawings of homes, schools, playgrounds and any other large scale construction projects that involve a floor plan or layout (Dole, 2008).

New York State requires students have a strong procedural and conceptual understanding of ratios. In the 6th grade curriculum, the ratios and proportional relations
module is one of the longest and most thorough (New York State Next Generation Mathematics Learning Standards [nysed.gov]). The standards outlined by New York State require students to understand the concept of a ratio and use ratio language to describe a relationship between two quantities, understand the concept of a unit rate a/b associated with a ratio a:b with (b not equal to zero), and use rate language in the context of a ratio relationship, and use ratio and rate reasoning to solve real-world problems. The ratio topics learned in 6th grade follow students throughout their academic education into algebra, trigonometry and calculus as well as the sciences (Daughtry, et al., 2017). As students progress through middle and high school, their understanding of ratios must increase and will be pushed ever further (Daughtry, et al., 2017).

**Concepts and misconceptions**

The ability to correctly and consistently use ratios and proportional reasoning is something that takes time and effort (Dole, 2008). Although it is well documented that proportions and ratios are of vital importance, students regularly struggle with topics surrounding ratios and teachers sometimes struggle to communicate those concepts effectively (O’Keeffe, White, 2018). As teachers well know, a deep level of content knowledge can help the presentation ideas, make them more flexible in answering student questions, and increase their ability to help students make connections between concepts (Ekawati, Lin, Yang, 2018).

Proportional reasoning and the ability to understand ratios requires being able to compare quantities in multiplicative terms (Dole, 2008). For students to fully understand and use ratios when solving problems, they must first learn some important concepts (Daughtry, et al., 2017). First, ratios and the relationship between their quantities are multiplicative
Second, unit rates can be found even if one number is not a factor of another, which creates decimals or fraction answers (Daughtry, et al., 2017). Third, for a ratio to be equivalent, it does not necessarily require integral ratios in common (Daughtry, et al., 2017). Proportional reasoning also requires a strong understanding of fractions and decimals (O’Keeffe & White, 2018). Unfortunately many students do not have a solid understanding of the concepts referred to above which may lead to learning challenges in more advanced topics (Daughtry, et al., 2017). Many of these struggles begin early on (such as in 6th grade) when misconceptions arise due to a disconnect between educator and learner (Daughtry, et al., 2017). One such misconception is the additive versus multiplicative comparison (Daughtry, et al., 2017). Many students instinctively rely on addition rather than multiplication when solving ratio problems which is incorrect and students’ tendency to use additive strategies when solving ratio problems creates difficulty for teachers (O’Keeffe & White, 2018). A way of illustrating this difficulty involves ingredients used while baking. For example if a recipe requires 3 cups of flour for every 2 cups of sugar, how many cups of sugar will be needed if 4 cups of sugar are used? Students who correctly understand that ratios are multiplicative in nature know that because 4 cups of sugar is 2 times the original amount of sugar used, they must also multiply the cups of flour by 2 in order to keep the ratio equivalent and keep the recipe intact (Daughtry, et al., 2017). Students who misunderstand this by thinking of ratios in terms of addition might have noticed that 3 cups of flour is 1 more than 2 cups of flour. Then when asked how many cups of flour would be needed if 4 cups of sugar were used, they would have counted from 2 cups of sugar to 4 cups of sugar and then simply counted up from 3 cups of flour to 5 cups of flour (Daughtry, et al., 2017). This misconception can, however, be effectively
remedied and, with proper scaffolding and support, even children in the first grade can understand the concept of ratios and proportions (O’Keeffe & White, 2018). Although learning ratios and understanding proportions takes time, if students are given the chance to explore ratios in a variety of ways and given the chance to experiment and play with ratios and proportions, it can be done effectively (Dole, 2008).

The second misconception stems from an incorrect understanding of fractions and how to represent a ratio as a fraction (Daughtry, et al., 2017). When teachers are mindful of the language they use when teaching ratios they can limit confusion when using terms such as “fraction”, “ratio” and “proportion”. The term “fraction” usually refers to a divided quantity (Litwiller, B., Bright, G., 2002). When a quotient refers to the multiplicative relationship between two numbers we can refer to is as a ratio (Litwiller, B., Bright, G., 2002). The terms proportion and proportionality refer to reasoning with ratios (Litwiller, B., Bright, G., 2002). Fractions can be ratios, but not all ratios are fractions, again highlighting the importance of language and how ratios are discussed. For example, instructors often teach students that a fraction such as 5/8 should be read as “five out of eight” (Daughtry et al, 2017). Instead, because the whole is 8, it is better if the fraction 5/8 is read as “five, one-eighths” (Daughtry, et al., 2017). This misunderstanding related to fractions can lead to misconceptions about ratios (Daughtry, et al., 2017).

A third misconception is a lack of covariational thinking (covariance is a measure of the joint variability of two random variables) (Daughtry, et al., 2017). An example of this can be seen in Table 1. Rather than looking row to row, students should go back to the original ratio in the first row, 3:1, to solve for the missing values. If they try to go from the first row to the row
immediately following it, they will accidentally come upon different constants of proportionality (Daughtry, et al., 2017).

Table 1: Ratio problem solving ratio Example

<table>
<thead>
<tr>
<th>Cups of flour</th>
<th>Cups of sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>21</td>
<td>7</td>
</tr>
</tbody>
</table>

One reason students develop misconceptions is because of inadequate exposure to ratios in different forms (Daughtry, et al., 2017). Along with this comes too much emphasis on procedural fluency and not enough focus on a solid conceptual understanding of ratios (Daughtry, et al., 2017). Procedural fluency is based around the use of mathematical symbols to represent mathematical ideas while conceptual understanding is the relationship between ideas and information (Koyunkaya, 2016). Skemp described procedural and conceptual as instrumental and relational understanding (Koyunkaya, 2016). He described instrumental thinking as “rules without reason” and relational understanding as knowing what to do and knowing why you are doing it (Koyunkaya, 2016). Although procedures are important and should be taught in the classroom, teachers should also spend time building a conceptual understanding through targeted and purposful questioning (Daughtry, et al., 2017). To many educators, both types of thinking are essential in the teaching of mathematics (Koyunkaya, 2016).
Correcting Misconceptions Through Multiple Representations

Multiple representations can be used during instruction to present ratios in various ways and support student understanding of ratio concepts (Daughtry, et al., 2017). Some of these different representations include tables, tape diagrams, double number lines and the coordinate plane among others (Daughtry, et al., 2017).

Ratio Tables

Ratio tables are an effective way for students to develop strategies for solving ratio problems (Dole, 2008). Ratios tables provide students with a convenient and understandable way to view multiplicative relationships between numbers and promote the use of certain number strategies (Dole, 2008). Ratio tables do, however, come with drawbacks that can be overcome but need to be taken into account (Dole, 2008). Ratio tables take time and practice to construct, can extend infinitely causing confusion, ratio tables sometimes show numbers out of order, and students need to remember to check each ratio against the original to ensure the ratios are equivalent (Dole, 2008). Despite these challenges, ratio tables demonstrate the linear quality of proportional relationships that can be shown by ordering values within the ratio table (Dole, 2008).

Tape Diagrams

A tape diagram consists of strips of paper or drawings used to display relationships between numbers (Ding, 2018). Using tape diagrams in a purposeful way along with meaningful teacher/student discourse and purposeful questioning will allow the opportunity for students to reason about additive and multiplicative relationships among numbers (Ding, 2018). When used with proper supports and scaffolds, tape diagrams can be used to help students solve
problems by modeling quantitative relationships (Ding, 2018). Tape diagrams have an advantage over other modeling tools such as counters and cubes because they can extend student thinking beyond the specific problem (Ding, 2018). Common Core State Standards expect students to use tape diagrams by the 6th grade to model and solve real-world problems which involve ratios and proportional relationships (Ding, 2018). Exposing students to tape diagrams in earlier grades is not discouraged and makes their use in 6th grade and beyond more meaningful (Ding, 2018). Early exposure to tape diagrams can lay a foundation for students’ education in ratios and proportional relationships in later grades (Ding, 2018).

**Double number lines**

For students to fully understand mathematics they must utilize and generate a variety of mathematical representations, one of these useful representations is the double number line (Orrill, Brown, 2012). Double number lines build on students’ knowledge of multiplication as a stretching and shrinking of quantities (Orrill, Brown, 2012). The lines in a double number line are stretched and squeezed by the same factor in order to maintain a proportional relationship between quantities (Orrill, Brown, 2012). The double number line allows a visualization of quantities to create rates such as mile and hours (Orrill, Brown, 2012). Rates are created by connecting values of quantities such as miles and hours to create speed (Orrill, Brown, 2012). Double number lines are similar to the coordinate plane in that the top and bottom line are not arranged on the same scale, instead they demonstrate the relationship between two numbers (Orrill, Brown, 2012). Double number lines can support learners and are a useful tool for teachers who are demonstrating ideas surrounding ratios, rates and other proportional relationships and their benefits in helping understand rates extend through highschool and into
Like ratio tables, double number lines are useful in that they keep numerical relationships constant and extend the concept of the ratio table by giving visual aid for comparing two quantities within a single diagram (Orrill, Brown, 2012). The ability of double number lines to represent rates and unit rates is an important feature that extends well beyond elementary math. Double number lines also serve as a useful tool in helping students understand unit conversion which is an important skill that follows students through their academic career and into the real-world. Unit Conversion is covered by the Common Core standards 6.RP.1 and 7.RP.1 and require students to demonstrate an understanding of ratios (Anticole, 2012).

**Coordinate Plane**

Graphing ratios on the coordinate plane allows students one of their first formal experiences with the idea of slope (Deniz, Uygur-Kabael, 2017). Slope or steepness can and should be viewed as a ratio of the vertical distance relative to the horizontal distance (Deniz, Uygur-Kabael, 2017). Slope can be calculated and graphed as the ratio “rise over run”, giving students another way of visualizing ratios (Deniz, Uygur-Kabael, 2017). The concept of slope evolves into constant rate of change in middle school, then average rate of change in high school and finally instantaneous rate of change as the derivative once a student enters the calculus classroom (Deniz, Uygur-Kabael, 2017). Students will recognize the line tangent to a curve as a ratio when finding derivatives in Calculus. By building a strong conceptual foundation of slope through the graphing of ratios, students will learn to understand slope as a ratio thus helping them as they advance through their mathematical education (Deniz, Uygur-Kabael, 2017).
Remote Learning

The COVID-19 international health crisis and resulting school closures and modified schedules led to a large increase in online learning (Hong, Lee, Ye, 2021). In 2020, during the lockdown caused by COVID-19, over 130 countries closed their schools and other educational buildings to help prevent infection (Hong, et al., 2021). In 2021 many schools continued using distance learning to not only stop the spread of the virus but also to ensure that learning is not disrupted in the event of future outbreaks (Hong, et al., 2021). The effectiveness of online learning is still being studied and debated and will be for years to come (Hong, et al., 2021). While online learning and online learning platforms present some benefits to student learning, there are also drawbacks that can be predicted by a student’s ability to self-regulate and their predisposition to procrastination (Hong, et al., 2021). There are six sub-constructs that determine a student’s ability to avoid procrastination which include task strategy, mood adjustment, self-evaluation, environmental structure, time management and help seeking (Hong, et al., 2021). How effective online learning is depends on the student’s attitude and personality rather than their familiarity with technological devices (Hong, et al., 2021). For a student to be successful in an online setting, they need the ability to demonstrate self-control while online to overcome obstacles such as isolation and distractions which can lead to procrastination (Hong, et al., 2021). A successful student’s learning process has been described in three phases which include forethought, performance and self-reflection (Hong, et al., 2021). The forethought phase involves task analysis and self-motivation to set goals and make plans. The performance phase includes the fulfilment of their plans which requires self-control and self-observation. The self-reflection phase involves students evaluating their learning through
self-evaluation and instructor feedback (Hong, et al., 2021). All of these behaviors are thought to be essential for the success of learning online (Hong, et al., 2021).

The use of technology in the classroom to support remote learning was gaining momentum well before the COVID-19 pandemic forced schools to close their doors (Panigrahi, Srivastava, Sharma, 2016). Online or distance learning has many benefits including allowing students to be flexible with the time they engage with classroom materials and has the ability to alleviate spatial problems sometimes associated with traditional, in-class education (Panigrahi, et al., 2016). Online learning is presented in one of two ways, either synchronous where the teacher is giving instruction in real-time or asynchronous where materials, including videos, are posted online where students have access to them anytime (Panigrahi, et al., 2016). Asynchronous allows students to access learning anytime and anyplace and allows for materials to be distributed to a larger group of individuals (Panigrahi, et al., 2016). Asynchronous learning also allows students to learn at their own pace rather than trying to keep on pace with their classmates (Panigrahi, et al., 2016).

Despite several advantages, online learning does present challenges as well (Panigrahi, et al., 2016). Keeping students engaged is a major challenge in online learning (Panigrahi, et al., 2016). Briefing, buddy-ing and the ability to provide feedback are all measures used to combat the issue of engagement (Panigrahi, et al., 2016). Online learning, especially asynchronous learning, also requires that students be more self-motivated and self-disciplined (Panigrahi, et al., 2016).
Introduciton to the Curriculum Project

Teaching remotely presented many challenges but also provided myself and others with insight into how to best engage students. During remote learning I was fortunate enough to have a small class of eight students. Also, student engagement was strong partially because of solid relationships built with families throughout the course of the school year. The ability of students to communicate privately via chat also worked to my advantage. Conversations through private chat allowed students who were uncertain of their own comprehension the ability to ask questions without fear of ridicule. The lessons I created to teach the ideas and concepts related to ratios relied heavily on purposeful questioning in order to challenge students’ ability and deepen their conceptual understanding. These purposeful questions are included in each lesson and also serve as a way to maintain student engagement.

Many teachers, myself included, do not plan to return to the model of instruction used before the pandemic. Instead, many plan on incorporating online learning platforms used during the COVID-19 pandemic as a way to differentiate lessons according to student progress and achievement. The following practice sets, quizzes and tests, which are aligned with New York State standards, are an excellent resource that provides instructional tools and resources to address individual student needs.

Please notice the purposeful questions provided in each of the lessons. These questions are representative of what was used to engage my students during remote instruction. Learning needs vary for each class, so these questions should be considered as a guide and modified as needed to best meet your own student learning needs. In the lessons T represents purposeful questioning provide by the teacher.
Lesson one – Ratios from Tables

<table>
<thead>
<tr>
<th>Unit Title:</th>
<th>Ratios and Rates</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>CCS Standards:</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
</tr>
<tr>
<td>6.RP.A.3.a - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Learning Targets:</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can use a table to organize my thinking, identify ratios and solve for missing values</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vocabulary:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio - the relation between two amounts showing the number of times one value contains or is contained within the other.</td>
</tr>
<tr>
<td>Equivalent - equal in value, amount, function, and/or meaning</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nearpod Link</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="https://share.nearpod.com/e/XStb2E8O7fb">https://share.nearpod.com/e/XStb2E8O7fb</a></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Opening with Purposeful Questioning:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ratio is a relation between two amounts and shows the number of times one value contains or is contained within another. If I have two rabbits and three dogs, my ratio of rabbits to dogs is 2:3 or 2 to 3.</td>
</tr>
<tr>
<td>Can you come up with your own example of a ratio?</td>
</tr>
<tr>
<td>Ratio tables provide us with equivalent ratios (remind students what an equivalent ratio is).</td>
</tr>
<tr>
<td>Let’s take a look at an example of ratio and use it to build a ratio table</td>
</tr>
<tr>
<td>Derick takes one sip of soda for every 4 pretzels he eats.</td>
</tr>
<tr>
<td>What ratio represents the number of sips of soda to the number of pretzels Derick eats?</td>
</tr>
</tbody>
</table>
We can use this information in order to build a ratio table

<table>
<thead>
<tr>
<th>Sips of Soda</th>
<th>Pretzels Eaten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

If Derick takes two sips of soda he will eat eight pretzels. Let’s add another row to our table

<table>
<thead>
<tr>
<th>Sips of Soda</th>
<th>Pretzels Eaten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Notice we can write these ratios as fractions and both fractions are equivalent:

\[
\frac{1}{4} = \frac{2}{8}
\]

How can we confirm both fractions and their associated ratios are equivalent?

Ans: By treating each fraction as a division problem. Have students divide 1 by 4 then divide 2 by 8. Have them come to the conclusion that both ratios are equivalent because the decimal answer is the same.

Are the fractions \( \frac{1}{4} \) and \( \frac{2}{8} \) equivalent? If so then we have equivalent ratios.

If Derick takes 3 sips of soda, how many pretzels has he eaten? He has eaten 12 pretzels. Let’s continue our table:

<table>
<thead>
<tr>
<th>Sips of Soda</th>
<th>Pretzels Eaten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Notice again that if we write our ratios as fractions, all of the fractions are equivalent:

\[
\frac{1}{4} = \frac{2}{8} = \frac{3}{12}
\]

Guided Practice:

Problem one:

Claudia works for a pizzeria. She uses 50 slices of pepperoni for every pizza she makes.

Complete the table using equivalent ratios
Begin by having students identify the ratio given in the problem. 50:1 or 50 to 1.

Have students write this ratio on a sheet of paper and label the ratio.

Remind students that for a ratio to be equivalent, both sides of the ratio must be multiplied by the same number.

T: What number did we multiply to 50 to get to 100?

If we multiplied the 50 by two, what number do we have to multiply to the other side of the ratio to keep it equivalent?

*If students have trouble answering the question “what number did we multiply to 50 to get to 100?” remind them that they can use the inverse of multiplication (division) to find the answer. 100 divided by 50 equals two.

Fill in the missing value in the third row with the number 2.

T: Now let’s work the other way. If Claudia wants to make three pizzas, how many slices of pepperoni will she use? Remember, we have to ask ourselves, what number did I multiply to one to get to three?

Ans: Three.

T: So what number do I have to multiply to the other side of the ratio?

Ans: 3

T: What is 3 times 50?

Ans: 150

T: So how many slices of pepperoni will Claudia use?

Ans: Claudia will use 150 slices of pepperoni to make three pizzas.

Fill in the missing value in the fourth row with the number 150

*S For additional clarification point out that we asked “what number did I multiply to one to get to three?” for simplicities sake. Would have asked “what number did I multiple to 2 to get to 3?” Students could use division or prior knowledge to tell you that you would have to multiply 2 by 1.5 to get to three. They could then multiply 100 by 1.5 to come to the answer needed for the blank in the fourth row.
**Problem two:**

Franklin uses 7 cups of flour for every 2 batches of cupcakes he makes.

Complete the table using equivalent ratios:

<table>
<thead>
<tr>
<th>Cups of Flour</th>
<th>Batches of Cupcakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

Begin by having students identify the ratio given in the problem. 7:2 or 7 to 2.

Have students write this ratio on a sheet of paper and label the ratio.

Remind students that for a ratio to be equivalent, both sides of the ratio must be multiplied by the same number.

*T: What number did we multiply to 2 to get to 4?*

Ans: Two

*T: If we multiplied the number of batches of cookies by two, what number do we have to multiply to the other side of the ratio to keep it equivalent?*

Ans: Two

*T: So how cups of flour will it take to make 4 batches of cupcakes?*

Ans: 14 cups of flour

*T: Let’s find the last missing value by working in the other direction. What number did we multiply to seven to get to 35?*

Ans: Five

*T: Good. What number do we have to multiply to the other side of the ratio to keep it equivalent?*

Ans: Five

*T: So how many batches of cupcakes could be make with 35 cups of flour?*

Ans: Franklin could make 10 batches of cupcakes with 35 cups of flour

*To push students thinking about equivalent ratios ask the following question:*

*T: How could we find how many cups of flour would it take to make one batch of cupcakes? (Students should write out the original ration 7:2 again and label then place the number “1” under the “2”. If needed, ask them how we went from 2 to 1. Some might tell you that we subtracted one from two. This is a common mistake. Remind them that we used multiplication*
earlier to fill out the table and now, since we are going in reverse, we need to use the inverse of multiplication.

Ans: We would need to divide the number of batches of cupcakes in the first row by itself to get to one. Then we need to divide the 7 by two to keep the ratio equivalent.

T: Good! And what is seven divide by two?

Ans: 3.5

*Use this time to explain to the students that they just discovered the idea of a unit ratio. A unit ratio can be written as a fraction with a denominator of one. A unit rate is also called a unit ratio. For example, 45 miles per hour (or per every one hour) is a unit ratio.

**Work Time:**

30 to 45 mins

**Additional Support**


**Online Problem Sets**

Below is a link for the problem set on KhanAcademy.org. Examples from the problem set are included in the nearpod for teacher and student benefit.


**Closing and Assessment:**

Remember, to keep our ratios equivalent, we must multiply or divide both sides of our ratio by a constant number. You can use any two numbers with in the table that are above or below one another in order to determine which number we must multiply or divide by.
1. Maria is having a picnic after school at a local park. She plans on making Kool Aid for her guests to drink during the picnic. Maria uses 3 cups of sugar for every one pitcher of Kool-Aid to make sure the Kool Aid tastes just right. Complete the table below showing the ratio of sugar to pitchers of Kool Aid:

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>Pitchers of Kool Aid</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

2. Maria figures that for every four guests, she’ll need one pitcher of Kool Aid. Complete the table below showing the ratio of guests to pitchers of Kool Aid needed for the picnic:

<table>
<thead>
<tr>
<th>Number of Guests</th>
<th>Pitchers of Kool Aid</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

3. Now Maria wants to know how many cups of sugar she’ll need to purchase for the party. Complete the table below by first figuring out how many cups of sugar she’ll need per guest.

<table>
<thead>
<tr>
<th>Number of Guests</th>
<th>Cups of Sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
1. Julian is making orange paint by mixing red paint and yellow paint together. For every 3 quarts for red paint, Julian uses 4 quarts of yellow paint.

Complete the table using equivalent ratios.

<table>
<thead>
<tr>
<th>Red Paint</th>
<th>Yellow Paint</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
</tr>
</tbody>
</table>
Lesson two – Ratios from Double Number Lines

<table>
<thead>
<tr>
<th>Unit Title:</th>
<th>Ratios and Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCS Standards:</td>
<td></td>
</tr>
<tr>
<td>6.RP.A.1 - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</td>
<td></td>
</tr>
<tr>
<td>6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td></td>
</tr>
<tr>
<td>6.RP.A.3.a - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td></td>
</tr>
<tr>
<td>Learning Targets:</td>
<td>I can construct a model of a ratio using a double number line and solve problems by reading a double number line.</td>
</tr>
<tr>
<td>Vocabulary:</td>
<td>Ratio - the relation between two amounts showing the number of times one value contains or is contained within the other.</td>
</tr>
<tr>
<td></td>
<td>Equivalent - equal in value, amount, function, and/or meaning.</td>
</tr>
<tr>
<td></td>
<td>Equivalent ratios – ratios that are equal ratios are two ratios that express the same relationship between numbers.</td>
</tr>
<tr>
<td></td>
<td>Double number line - A double number line is a representation of a ratio relationship using a pair of parallel number lines. One number line is drawn above the other so that the zeros of each number line are aligned directly with each other.</td>
</tr>
<tr>
<td>Nearpod Link:</td>
<td><a href="https://share.nearpod.com/e/QUtiOvqP7fb">https://share.nearpod.com/e/QUtiOvqP7fb</a></td>
</tr>
<tr>
<td>Opening and Lesson:</td>
<td>Ratio – A ratio is a relation between two amounts and shows the number of times one value contains or is contained within another. If I have two rabbits and three dogs, my ratio of rabbits to dogs is 2:3 or 2 to 3</td>
</tr>
<tr>
<td></td>
<td>Quick review/warm up of previous day’s lesson regarding ratio tables</td>
</tr>
</tbody>
</table>
Warm up: Claudia uses two cups of mozzarella cheese for every pizza she makes. Complete the table using equivalent ratios

<table>
<thead>
<tr>
<th>Number of Pizzas</th>
<th>Cups of cheese</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

*For students that struggle, remind them that equivalent ratios are those that express the same relationship between numbers. A ratio can be written as a fraction and if the two fractions are equivalent (ie. $\frac{1}{2}$ and $\frac{2}{4}$) then the corresponding ratios are equivalent. To find the missing value in the first row, we must first find what value we multiplied 2 to get to 8. Students should ask themselves “what did I multiply to 2 to get to 8 or what is 8 divided my 2?” To fill out the missing value in the third row, students can use the same process but for the other column.

Lesson:

T: Today we will continue our exploration of ratios. Today, however, we will be using double number line in order to demonstrate and model ratios. A double number line is a representation of a ratio relationship using a pair of parallel number lines. One number line is drawn above the other so that the zeros of each number line are aligned directly with each other. Unlike the ratio table, a double number line gives us a visualization of ratio relationships.

Ex 1

Let’s look at an example of a ratio table from yesterday and see if we can use it to create a double number line.

<table>
<thead>
<tr>
<th>Slices of pepperoni</th>
<th>Pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>150</td>
<td>3</td>
</tr>
</tbody>
</table>

Notice that for every 50 slices of pepperoni, we can make 1 pizza.

What ratio could we use to describe the relationship between slices of pepperoni and pizzas?

Ans: 50:1 or 50 to 1

Remember, a double number line is a representation of a ratio relationship using a pair of parallel number lines. Let’s start by drawing two parallel lines.
These two lines will be used to compare two quantities from the table above. We’ll use the top line to represent the number of pizzas and the bottom line to represent the number of pepperoni. Let’s label it now using the values from the table.

T: Now let’s label the bottom line of our double number line with the corresponding number of pepperonis needed to make each pizza.

T: What do you notice about the double number line? *Wait for students to answer but give plenty of time for numerous responses. The main thing we want students to take away is that the number of pizzas on the top line, lines up with or corresponds with the number of pepperoni displayed by the bottom line.

T: Why do you think double number lines are beneficial while exploring ratios? *again, give students time and discuss any answers they may give. Ask questions to lead students to the idea that double number lines give a visual aid or representation of the ratio.
Ex 2

T: Let’s look at another example from yesterday

Derick takes one sip of soda for every 4 Pretzels he eats.

<table>
<thead>
<tr>
<th>Sips of Soda</th>
<th>Pretzels Eaten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

T: Again, we will draw two parallel lines

T: Next we will label the top line as “sips of soda” and write in our values

T: Lastly we will label the bottom line with the number of pretzels eaten. Remember to write the corresponding number of pretzels directly below the number of sips of soda.

T: Now we can easily see the relationship between the quantities “Sips of Soda” and “pretzels Eaten”.
*Go around the room and call on students to ask the following questions regarding the double number line

If Derrick drank one sip of soda, how many pretzels did he eat?
Ans: 4

If Derrick drank two sips of soda, how many pretzels did he eat?
Ans: 8

If Derrick drank three sips of soda, how many pretzels did he eat?
Ans: 12

If Derrick ate 4 pretzels, how many sips of soda did he take?
Ans: 1

If Derrick ate 8 pretzels, how many sips of soda did he take?
Ans: 2

If Derrick ate 12 pretzels, how many sips of soda did he take?
Ans: 3

T: Digging deeper, why do you suppose we marked each end of our double number lines with arrows? *Give students time to answer and address all relevant responses. Students should come to the conclusion that the ratio is not constrained by the values on the double number line but that it extends indefinitely.

T: Could we figure out how many pretzels Derrick ate if he took 6 sips of soda? If so, how?
Ans: Yes. Since each sip of soda corresponds with 4 pretzels being eaten, we could multiply 6 x 4 to get to 24. If he drank 6 sips of soda, he must have eaten 24 pretzels.

T: Good. Is there another way we could have solved this problem? If so, how?
Ans: We could have extended the double number line in order to show the values needed.

**Work Time:**
30 to 45 mins

**Additional Support**
Online Problem sets:

Below is a link for the problem set on KhanAcademy.org.

https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/visualize-ratios/e/create-double-number-lines?modal=1


1. Debbie is running in a 5 mile race to raise money for the American Cancer Society. A generous donor has decided to donate 100 dollars for every 1 mile that Debbie runs. Complete the double number line below that shows the amount of money she will make for every mile she runs:

```
| 0 |  |  |  |  | 100 |
```

2. To train for the race, Debbie ran 6 miles every week. Complete the double number line below to show how many miles she ran leading up to the race.

```
| 0 |  |  |  |  | 6 |
```

3. Use the double number line and extend your thinking further. If Debbie started training 8 weeks before the race, how many training miles did she run?

4. How many weeks did it take for Debbie to run 42 miles?
1. John uses 90 strawberries for every 2 strawberry pies he’s baking. Complete the table of equivalent ratios below and use it to construct a double number line showing the relationship between strawberries and pies.

<table>
<thead>
<tr>
<th>Strawberries</th>
<th>Pies</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>2</td>
</tr>
</tbody>
</table>

How many strawberries does John need to bake 5 strawberry pies?
Lesson 3 – Ratios with Tape Diagrams

**Unit Title:**
Ratios and Rates

**CCS Standards:**

6.RP.A.1 - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3.a - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

**Learning Targets:**

I can construct a model of a ratio using a tape diagram and solve problems by reading and interpreting a tape diagram.

**Vocabulary:**

Ratio - the relation between two amounts showing the number of times one value contains or is contained within the other.

Equivalent - equal in value, amount, function, and/or meaning.

Equivalent ratios – ratios that are equal ratios are two ratios that express the same relationship between numbers.

Tape Diagram – Tape diagrams are visual representations of a ratio that represent the sections of a ratio by using rectangles.

**Nearpod Link**

https://share.nearpod.com/e/KYELg9tP7fb
Lesson:
Ratio – A ratio is a relation between two amounts and shows the number of times one value contains or is contained within another. If I have three carrots and four potatoes, my ratio of carrots to potatoes is 3:4 or 3 to 4.

Warm up:
David is making Kool-aid. For every 4 cups of water, David uses one packet of Kool-aid. Complete the double number line below remembering to properly label each line. What unit ratio will we use to construct our double number line? How many packets of cool aid will David need if he uses 12 cups of water.

Answer:
The ratio of packet of Kool-Aid to Water is 1:4 or 1 to 4.
If David wishes to keep his ratio equivalent, he will have to use 3 packets of Kool-Aid for a 12 cup batch.

Lesson:
T: Today we will be looking at a new way visualize ratios using Tape Diagrams. A tape diagram is a rectangular visual mode that is used visual representation of ratios and solve problems using ratios. It is similar to a double number line in some ways but unlike a double number line, the two “tapes” are not always equal in length.
At the beginning of class we discussed a ratio of carrots to potatoes. We said we had 3 carrots for every 4 potatoes. Let’s look at how we would have represented this ratio using tables and double number lines. Then we will construct a tape diagram using this information:

Table method (Review from lesson one)

<table>
<thead>
<tr>
<th>Carrots</th>
<th>Potatoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

Double number line (review from lesson two)

Now let’s look at a double number line. We will use rectangles and show each quantity with the lowest possible number or whole rectangles. Let’s start with the carrots.

Notice I made a rectangle for each carrot in our ratio.

Now let’s add another group of rectangles to represent the potatoes. We will place that group directly beneath the group we made for the carrots.

T: We now have a visual representation of a ratio that differs from the double number line. How is it different? *Give students ample time to answer. Take all genuine responses seriously and discuss as time permits.
T: How could we use this tape diagram to figure out the amount of potatoes we have if we six carrots? *Again. Give students ample time to answer. Take all genuine responses seriously and discuss as time permits. Ask students questions to lead them to the idea that if we want six carrots, we have to multiply the number of carrots (three) by two. If we are to keep our ratio equivalent, we must also multiply the number of potatoes by two.

T: Let’s try some more problems

Ex 1

Toni is practicing for her high school basket team’s tryouts. Below is a tape diagram showing the amount of shots she made to the shots she attempted during practice.

```
[Diagram: Shots made: 3 boxes; Shots attempted: 6 boxes]
```

Write down the ratio shown with the tape diagram. Assuming the ratio of shots made to shots attempted remained equivalent, how many shots to Toni make if she made 12 attempted shots.

Ex 2

For every four questions that Casey answered during a test, she answered one incorrectly. Below is a tape diagram showing the ratio of questions attempted to questions answered incorrectly.

```
[Diagram: Questions attempted: 3 boxes; Questions answered incorrectly: 1 box]
```

Assuming the ratio of “questions attempted” to “questions answered incorrectly” remains equivalent, how many questions must Casey have attempted if she answered 8 incorrectly?

Ex 3
Jalynn works at an ice cream shop near her home. For every 2 customers that order vanilla ice cream, 5 order chocolate. A tape diagram showing the ratio of customers ordering vanilla to the customers ordering chocolate is shown below.

Assume the ratio of customers ordering vanilla to the number ordering chocolate remains equivalent. If 20 customers order vanilla ice cream, how many ordered chocolate?

**Work Time:**
30 to 45 mins

**Additional Support**
https://youtu.be/suRIY3ULrQo

**Online Problem Sets**
Below is a link for the problem set on KhanAcademy.org.

**Closing**
T: Tape diagrams allow us another way to visualize ratios. In our previous lessons, we looked at ratio tables and double number lines as a way to represent ratios. Each method allows us a way to not only visualize ratios, but answer questions related to those ratios. We can expand on the ratios shown to model situations that may occur in the real world.
1. Karen is making ice cream sundaes for her friends during a sleep over. For every 3 scoops of ice cream, she uses 2 tablespoons of chopped peanuts.

   a. Draw a tape diagram showing the ratio of scoops of ice cream to tablespoons of chopped peanuts.

   b. If Karen uses 8 tablespoons of chopped peanuts, how many sundaes did she make for her friends?

2. Daniel is building a patio on the back of his house. For every piece of lumber he needs 4 galvanized screws.

   a. Draw a tape diagram showing the ratio of lumber to galvanized screws.

   b. Daniel figures that he will need 40 pieces of lumber to build his deck. How many screws will Daniel need to buy?
3. Mr. Marlowe LOVES his morning coffee and he likes it STRONG! For every 2 cups of water he uses 3 scoops of coffee.
   
a. Draw a tape diagram showing the ratio of cups of water to scoops of coffee.

b. This morning Mr. Marlowe is particularly tired and wants to make 12 cups of coffee. How many scoops of coffee will he need?
Below is a tape diagram showing the number of boys to the number of girls at Discovery Charter school.

Use the tape diagram to fill out the ratio table and double number line below

<table>
<thead>
<tr>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Unit Title:</strong> Ratios and Rates</td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>CCS Standards:</strong></td>
<td></td>
</tr>
</tbody>
</table>
| 6.RP.A.1 - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.  
6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.  
6.RP.A.3.a - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. |
| **Learning Targets:** |
| I can represent ratios by plotting equivalent ratios on a coordinate plane |
| **Vocabulary:** |
| Ratio - the relation between two amounts showing the number of times one value contains or is contained within the other.  
Equivalent - equal in value, amount, function, and/or meaning  
Equivalent ratios – ratios that are equal ratios are two ratios that express the same relationship between numbers.  
Double number line - A double number line is a representation of a ratio relationship using a pair of parallel number lines. One number line is drawn above the other so that the zeros of each number line are aligned directly with each other.  
Coordinate plane - The coordinate plane is a two-dimension surface formed by two number lines. One number line is horizontal and is called the x-axis. The other number line is vertical number line and is called the y-axis. The two axes meet at a point called the origin. We can use the coordinate plane to graph points, lines, and more. |
| **Nearpod Link** |
| https://share.nearpod.com/e/1BKkT5HP7fb |
Opening:

T: In the last three lessons we have looked at describing ratios in a number of different ways. We have used tables, double number lines, and tape diagrams in order to describe ratios and give them a visual representation. Today we will be looking at another way to describe ratios by plotting the values of equivalent ratios on the coordinate plane.

Remember the coordinate plane is a two-dimensional surface formed by two interesting number lines. One line is vertical (up and down) which we call the y-axis. The second is horizontal (left to right) which we call the x-axis. Where the two lines meet is called the origin. We describe the origin as the point (0, 0).

Plotting ratios on the coordinate plane is arguably the most important visual representation of a ratio. It gives us a strong mathematical tool for problem solving and making predictions when it comes to ratios. Plotting a ratio on the coordinate plane also gives a visual comparison of two quantities through the steepness of slope of the graph we plot. Another term for this is the constant of proportionality. The constant of proportionality is the ratio between two directly proportional quantities. Two quantities are directly proportional when they increase and decrease at the same rate.

Let’s go back to our ratio tables from lesson one and use them to plot our ratios on the coordinate plane.

<table>
<thead>
<tr>
<th>Pretzels Eaten</th>
<th>Sips of Soda</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

By looking at this table we can see that for every one sip of soda, two pretzels are eaten. If we take two sips of soda, we eat four pretzels. Going the other direction, if we eat six pretzels we take three sips of soda.

Now let’s take these values and plot them on our coordinate plane.
Most times the coordinate plane will be given to you but let’s create our own today for some extra practice. Start by drawing both your x and y-axis. Remember to label the horizontal line with an “x” and the vertical line with a “y”.

Next, let’s label each axis with the column headings from our ratio table and number each line with a scale that’s appropriately in line with the values in the ratio table.
Notice that the x and y-axis are numbered differently. The y-axis increased by one while the x-axis increases by two. This is ok as long as both axis are labeled consistently meaning that if the x-axis is increasing by two, it always increases by two. It doesn’t suddenly change value.

Now we are ready to plot the values from our ratio table.

The first set of values in our ratio table are 2 pretzels eaten for 1 sip of soda. We could write this ratio as 2:1 or by using the coordinate pair (2, 1). Remember when using coordinate pairs, the x-value is first then the y-value (x, y).
We plotted a point at (2, 1) which represents 2 pretzels for every one sip of soda. We can see visually that for every 2 pretzels, we take one sip of soda. Now let’s plot the rest of the points.
We’ve now plotted the points (4, 2) and (6, 3).

Could we keep going? *give students ample time to think about the question and discuss their responses.

The neat thing about plotting ratios on a coordinate plane is we can easily extend the ratios and also see visually see how quickly they are increasing. Let’s keep going by plotting more points.

Now we can plainly see that for every 8 pretzels, there are 4 sips of soda, for every 10 pretzels, there are 5 sips of soda and for every 12 pretzels there are 6 sips of soda. The other neat thing about plotting our ratios on the coordinate plane is we can start to visualize the constant of proportionality.

The constant of proportionality is the relation between quantities. Two quantities are directly proportional when they increase and decrease at the same rate. The constant of proportionality \( k \) is given by \( k = \frac{y}{x} \) where \( y \) and \( x \) are two quantities that are directly proportional to each other. We’ve showed this already in previous lessons when writing our ratios as fractions. We know the first value in our ratio table 2:1 can be written as an ordered pair (2, 1) where 2 is our \( x \)-value and 1 is our \( y \)-value. Using the definition of the constant of proportionality, \( k = \frac{y}{x} \), we can represent see that \( k \), or constant of proportionality is equal to \( \frac{1}{2} \). All of the ratios in our ratio table can be represented by an equivalent fraction.

Lastly we can draw a line through the ratios we plotted to get a visual sense of the constant of proportionality.
What if our ratio increased at a faster or slower rate? How would that change the shape of our graph and also the constant or proportionality?

What if for every four pretzels eaten we took one sip of soda? What would that look like?

<table>
<thead>
<tr>
<th>Pretzels Eaten</th>
<th>Sips of Soda</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
</tbody>
</table>

We’ll plot these new points on the same graph, (4, 1), (8, 2), (12, 3). This time we will use the color green so we can easily distinguish between the two graphs.
T: What do you notice about the new set of ratios we plotted on our graph? *give students ample time to think and discuss all suggestions. Ask questions that lead students to the idea that the graph is increasing at a slower rate.

Tell students that the next lesson we will be discussing constant or proportionality further.

**Work Time:**
30 to 45 mins

**Additional Support**
https://youtu.be/yVYJRT5hTS0

**Online Problem Sets:**
Below is a link for the problem set on KhanAcademy.org. Examples from the problem set are included for teacher and student benefit.

https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-ratio-word-problems/e/ratios-on-coordinate-plane?modal=1
Closing and Assessment:

Today we learned another way of representing ratios by plotting them onto a coordinate plane. This gives us a visual representation of the ratio and better visual understanding of how the quantities in the ratio relate to one another. We also touched on the idea of a constant of proportionality and what that means. In our next lesson we will continue to work on the idea of the constant of proportionality and how it describes the slope of a line.
1. Brandon is having a Memorial Day BBQ and is planning on grilling chicken legs for his guests. Brandon figures that each guest will consume 4 chicken legs.

   a. Complete the ratio table below which showing the ratio of chicken legs to the number of guests attending.

<table>
<thead>
<tr>
<th>Guests</th>
<th>Number of Chicken Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

   b. Use the table above to create a coordinate plane. Plot the ratio from the table on the coordinate plane. Make sure to use an appropriate scale for your x and y axis. Label each axis appropriately with the correct heading from the ratio table.
c. Extend your thinking beyond the graph. Let’s say Brandon has a ton of guests planning on attending his Memorial Day Picnic. If 23 guests attend the picnic, how many pieces of chicken will he need to buy?

d. Uh-oh! Brandon miscalculated and bought 36 chicken legs but at the end of the party there were 8 chicken legs left over. Assuming everyone at the BBQ ate exactly four pieces of chicken, how many people attended the BBQ?

2. Sara owns a flower stand on 5th Avenue. It’s Valentine’s Day and she is selling a dozen roses for 45 dollars.

a. Complete the ratio table below showing the ratio of roses sold by the dozen to the revenue generated

<table>
<thead>
<tr>
<th>Roses sold by the dozen</th>
<th>Revenue Generated</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$540</td>
</tr>
<tr>
<td></td>
<td>$720</td>
</tr>
</tbody>
</table>

b. Use the table above to create a coordinate plane. Plot the ratio from the table on the coordinate plane. Make sure to use an appropriate scale for your x and y axis. Label each axis appropriately with the correct heading from the ratio table.

c. Sara pays the city a monthly rental fee for the spot on 5th Avenue she uses to sell her roses. The rent is $800 per month. How many dozen roses will Sara have to sell to cover her monthly rent?
1. Betty drinks one glass of milk for every three cookies she eats. Complete the table below, plot the ratios as points on the coordinate plane and calculate the constant of proportionality. Make sure to label your graph appropriately.

<table>
<thead>
<tr>
<th>Glasses of Milk</th>
<th>Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
</tbody>
</table>
Lesson five – Unit Rates

**Unit Title:**
Ratios and Rates

**CCS Standards:**
6.RP.A.2 - Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.

**Learning Targets:**
I can understand a unit rate as the comparison of two quantities with two different units of measurement

**Vocabulary:**
Rate - A rate is a ratio that compares two quantities with different units of measure.
Unit Rate - A unit rate of two quantities in a ratio is the number of units of the first quantity for every 1 unit of the second quantity.
Ratio - the relation between two amounts showing the number of times one value contains or is contained within the other.
Equivalent - equal in value, amount, function, and/or meaning.
Equivalent ratios – ratios that are equal ratios are two ratios that express the same relationship between numbers.
Double number line - A double number line is a representation of a ratio relationship using a pair of parallel number lines. One number line is drawn above the other so that the zeros of each number line are aligned directly with each other.
Coordinate plane - The coordinate plane is a two-dimension surface formed by two number lines. One number line is horizontal and is called the x-axis. The other number line is vertical number line and is called the y-axis. The two axes meet at a point called the origin. We can use the coordinate plane to graph points, lines, and more.

**Nearpod Link**
https://share.nearpod.com/e/dRvdv9MP7fb

**Opening:**
T: Today we are going to look at Unit Rates. A unit rate is something you are all probably familiar with. It is a ratio where the number of units of the first quantity for every ONE unit of the second quantity. What does this mean? Well let’s take a ratio with different units of measure that you’re all familiar with. Say, miles and hours. We could write that as:
Miles per hour or miles/hour or use the abbreviation mph.

This tells us the amount of miles we travel in ONE hour. Notice the attention to the word “one”.

Remember, a unit rate of two quantities in a ratio is the number of units of the first quantity for every ONE unit of the second quantity.

Ex1. If I am traveling on a train at 80 miles per hour, I have traveling 80 miles every ONE hour. Or 80mi/1hr

Ex2. If there are 130 calories per serving of potato chips, there are 130 calories per ONE serving. We could write that as 130cal/serving.

Ex3. Doctors often check patients heart rate via their pulse. Heart rates are measured in beats per minute. If my heart is beating at 75 beats per ONE minute, I could write that as 75 beats per minute or 75 beats/1 minute.

Unit rates are useful because they allow you to make direct comparisons between two different units of measurement.

Guided Practice:

Let’s take a look at some problems involving unit rates

**Ex1 Casandra can type 183 words ever 3 minutes. If Casandra types at a constant rates, how many words can she type per minute?**

T: Remember a unit rate is a ratio where the number of units of the first quantity for every ONE unit of the second quantity. We need to keep that in mind as we’re solving all of these problems. Let’s start by writing down the ratio given in the problem and make sure to label each part of the ratio correctly:

\[
\frac{183 \text{ words}}{3 \text{ minutes}}
\]

Notice that I labeled the 183 as “words” and the 3 with “minutes”. Now what I want to do is find our how many words Casandra can type every ONE minute to turn this ratio into a unit rate.

I should now ask myself, “what number to I need to multiply or divide to 3 in order to make it a 1?” Remember I need to make the 3 turn into a 1 because it’s in the minutes column and I want
to find out how many words PER MINUTE Casandra can type. *Give students time to answers. They should tell you that to make the minutes column a 1 they either need to divide by 3 or multiply by 1/3.

\[
\begin{array}{c|c}
\text{words} & \text{minutes} \\
183 & 3 \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{words} & \text{minutes} \\
61 & 1 \\
\end{array}
\]

T: Good! Now if we divided one side of our ratio by 3, what do we have to divide the other side by to keep the ratio equivalent? *Give students time to answers. They should tell you that if they divided one side by 3 they need to divide the other side by three as well to keep the ratio equivalent.

\[
\begin{array}{c|c}
\text{words} & \text{minutes} \\
183 & 3 \\
\end{array}
\]

\[
\begin{array}{c|c}
\text{words} & \text{minutes} \\
61 & 1 \\
\end{array}
\]

T: How many words can Casandra type per minute? She can type 61 words per minute. We could also write that as 61 words/1 minute

**Ex2.** Robbie bought 12 chicken wings for $8. What is the price of one chicken wing?
T: How will this question differ from the first one? *Give students time to respond. Lead them to the idea that when they divide, 12 will not go evenly into 8. Assure them this is ok and explain to them the meaning as you go through the example.

T: Remember a unit rate is a ratio where the number of units of the first quantity for every ONE unit of the second quantity. We need to keep that in mind as we’re this problem. Let’s start by writing down the ratio given in the problem and make sure to label each part of the ratio correctly:

\[
\begin{array}{c|c}
\text{Chicken wings} & \$ \\
12 & 8
\end{array}
\]

Notice that I labeled the 12 as “chicken wings” and the 8 with “$” because it is the symbol for money. Now what I want to do is find out how much it will cost for every ONE chicken wing.

I should now ask myself, “what number to I need to multiply or divide to 12 in order to make it a 1?” Remember I need to make the 12 turn into a 1 because it’s in the “chicken wing” column and I want to find out how much money it costs PER CHICKEN WING. *Give students time to answers. They should tell you that to make the “Chicken wing” column a 1 they either need to divide by 12 or multiply by 1/12. Most students will tell you to divide.
T: Good! Now if we divided one side of our ratio by 12, what do we have to divide the other side by to keep the ratio equivalent? *Give students time to answers. They should tell you that if they divided one side by 12 they need to divide the other side by 12 as well to keep the ratio equivalent.

Let students know that because they are dealing with cost, they need to divide to the thousands place then round to the nearest hundredth. How much is the cost of one chicken wing? The cost of one chicken wing is $0.67.

**Ex3:** A Racecar driver can drive 375 miles in 3 hours. How many miles does the racecar driver travel in 1 hour?

T: Remember a unit rate is a ratio where the number of units of the first quantity for every ONE unit of the second quantity. We need to keep that in mind as we’re solving this problem. Let’s start by writing down the ratio given in the problem and make sure to label each part of the ratio correctly:

\[
\begin{align*}
\text{miles} & : \text{hours} \\
375 & : 3
\end{align*}
\]

Notice that I labeled the 375 as “miles” and the 3 with “hours”. Now what I want to do is find out how many miles the driver can travel in 1 hour.

I should now ask myself, “what number to I need to multiply or divide to 3 in order to make it a 1?” Remember I need to make the 3 turn into a 1 because it’s in the “hours” column and I want...
to find out how far the driver can travel PER hour. *Give students time to answers. They should tell you that to make the “hours” column a 1 they either need to divide by 3 or multiply by 1/3. Most students will tell you to divide.

<table>
<thead>
<tr>
<th>miles</th>
<th>hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>375</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>( \div 3 )</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

T: Good! Now if we divided one side of our ratio by 3, what do we have to divide the other side by to keep the ratio equivalent? *Give students time to answers. They should tell you that if they divided one side by 3 they need to divide the other side by 3 as well to keep the ratio equivalent.

<table>
<thead>
<tr>
<th>miles</th>
<th>hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 375 \div 3 )</td>
<td>( 3 \div 3 )</td>
</tr>
<tr>
<td>( 125 )</td>
<td>( 1 )</td>
</tr>
</tbody>
</table>
T: How many miles did the racecar driver travel in one hour? They traveled 125 miles in one hour. Bonus question. How fast was the racecar driver traveling? They were traveling 125 miles per hour or 125 mph.

<table>
<thead>
<tr>
<th>Work Time:</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 to 60 mins</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional Support</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="https://youtu.be/qGTYSAeLTOE">https://youtu.be/qGTYSAeLTOE</a></td>
</tr>
<tr>
<td><a href="https://youtu.be/Zm0Kalw-35k">https://youtu.be/Zm0Kalw-35k</a></td>
</tr>
<tr>
<td><a href="https://youtu.be/d7rAlcNHDUI">https://youtu.be/d7rAlcNHDUI</a></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Online Problem Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-rates/e/unit-rates?modal=1">https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-rates/e/unit-rates?modal=1</a></td>
</tr>
<tr>
<td><a href="https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-rates/e/rate_problems_0.5?modal=1">https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-rates/e/rate_problems_0.5?modal=1</a></td>
</tr>
<tr>
<td><a href="https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-rates/e/comparing-rates?modal=1">https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-ratios-prop-topic/cc-6th-rates/e/comparing-rates?modal=1</a></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Closing and Assessment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today we learned about unit rates. A unit rate is a ratio where the number of units of the first quantity for every ONE unit of the second quantity. Remember, a unit rate of two quantities in a ratio is the number of units of the first quantity for every <strong>ONE</strong> unit of the second quantity. Unit rates are useful because they allow you to make direct comparisons between two different units of measurement.</td>
</tr>
</tbody>
</table>
1. Jonathan is taking a trip from New York to Switzerland on a Boeing 747 which can fly 2,280 miles every 4 hours.

   a. What is the unit ratio of miles to hours of the Boeing 747?

   b. Assuming the plane travels at a constant speed, how many miles is it to Switzerland if Jonathan is flying for 7 hours?

2. Dillan babysat for 3 hours each night for 12 nights. He earned $540 for his work babysitting.

   a. How much money did Dillan earn per hour babysitting?

   b. Dillan is saving for his fall semester at college where he is studying to be an engineer. If tuition is $1,500 for the semester, what is the minimum hours he will have to work?

3. A baseball team is having a bake sale to raise money for uniforms for the season. The team charged $2 for every cookie that they sold. In 5 hours, they sold 200 cookies. At that rate, how much money could the team earn selling cookies for 12 hours?
1. John Clarke was a speed reader who could read 84 pages every 3 minutes. How many pages could John read in 1 minute?
Lesson six – Constant of Proportionality

<table>
<thead>
<tr>
<th><strong>Unit Title:</strong></th>
<th>Ratios and Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CCS Standards:</strong></td>
<td></td>
</tr>
<tr>
<td>7.RP.A.2 - Recognize and represent proportional relationships between quantities.</td>
<td></td>
</tr>
<tr>
<td>7.RP.A.2b - Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td>
<td></td>
</tr>
<tr>
<td>6.RP.A.1 - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</td>
<td></td>
</tr>
<tr>
<td>6.RP.A.3 - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</td>
<td></td>
</tr>
<tr>
<td>6.RP.A.3.a - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</td>
<td></td>
</tr>
<tr>
<td><strong>Learning Targets:</strong></td>
<td></td>
</tr>
<tr>
<td>I can identify the constant of proportionality through tables, graphs, equations, and lines graphed coordinate planes</td>
<td></td>
</tr>
<tr>
<td><strong>Vocabulary:</strong></td>
<td></td>
</tr>
<tr>
<td>Proportion - a part, share, or number considered in comparative relation to a whole.</td>
<td></td>
</tr>
<tr>
<td>Constant of proportionality - The constant of proportionality is what determines the relationship between y and x. If r is the constant of proportionality then an example is y = rx. The value of y is dependent on how the given value of x is effected by the constant of proportionality.</td>
<td></td>
</tr>
<tr>
<td>Ratio - the relation between two amounts showing the number of times one value contains or is contained within the other.</td>
<td></td>
</tr>
<tr>
<td>Equivalent - equal in value, amount, function, and/or meaning</td>
<td></td>
</tr>
<tr>
<td>Equivalent ratios – ratios that are equal ratios are two ratios that express the same relationship between numbers.</td>
<td></td>
</tr>
<tr>
<td>Double number line - A double number line is a representation of a ratio relationship using a pair of parallel number lines. One number line is drawn above the other so that the zeros of each number line are aligned directly with each other.</td>
<td></td>
</tr>
</tbody>
</table>
Coordinate plane - The coordinate plane is a two-dimension surface formed by two number lines. One number line is horizontal and is called the x-axis. The other number line is vertical number line and is called the y-axis. The two axes meet at a point called the origin. We can use the coordinate plane to graph points, lines, and more.

**Nearpod Link**

https://share.nearpod.com/e/ia6lXMVP7fb

**Opening:**

T: Today we are going to look at examples of our work from the past few days and think about a new concept, the constant or proportionality. The constant of proportionality is what determines the relationship between y and x (or whatever variable you choose). If we have 4 lemons for every 2 apples, we would say we have 2 times as many lemons as apples. We could even write this out mathematically: The amount of lemons = 2x(amount of apples). If we boiled this down even further, we could use the letter “l” to represent lemons and the letter “a” to represent apples. Doing this we would get the following, l=2a.

Let’s go back to our definition for the constant of proportionality and see if we can figure out what its value is in our equation. Our definition stated The constant of proportionality is what determines the relationship between y and x (or in our case, l and a).

What determines our relationship in l=2a? What number or value that makes a different from l? *Give students plenty of time to respond. They should notice that since 2 is being multiplied to a, the answer is 2.

Let’s take a look at some previous days examples and see if we can identify the constant of proportionality.

<table>
<thead>
<tr>
<th>Sips of Soda</th>
<th>Pretzels Eaten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Here we had the table showing equivalent ratios. We could write each of the ratios in our table as the number of pretzels eaten to the number of sips of soda.

4:1
8:2
12:3

As we learned previously, these ratios could be written as fractions

4:1 = 4/1
8:4 = 8/4
12:3 = 12/3
We also have learned that any fraction can be written as a division problem
4/1 = 4÷1
8/2 = 4÷2
12/3 = 12÷3
What do we notice about these three division problems? *Give students plenty of time to respond. Ask questions, if needed, to lead them to the idea that the answer for all three problems is 4.

T: Good! We have just found the constant of proportionality for our ratio table. Remember the constant of proportionality is the number that relates two quantities. In this case, the constant of proportionality is 4 and relates the quantities of sips of soda to the number of pretzels eaten.

T: How could we write this relationship, including the constant of proportionality, as an equation? *Give students ample time to work out the problem on their own.

T: Let’s start by writing down the two things we are relating to one another, sips of soda and pretzels and include the constant of proportionality in a place that makes numerical sense and relates directly to our table of values.

pretzels Eaten = 4x(Sips of Soda)

Now let’s simplify this even further by abbreviating “sips of soda” by just calling it “s” and “pretzels Eaten” by just calling it “t”. Let’s sub these variables in for the words in our original equation. T=4(S)

We can test this equation using values from the table to see if it works.

<table>
<thead>
<tr>
<th>Sips of Soda (S)</th>
<th>Pretzels Eaten (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

The first row give us our unit rate. It says, for every 4 Pretzels Eaten, we are going to take 1 sip of soda. Let’s plug these value into our equation and see if we get a true statement.

Start by writing the equation: T=4(S)
Sub in the appropriate value: 4 = 4(1)

Does 4=4? It sure does! This is a true statement and confirms that our equation is correct along with our constant of proportionality. But we shouldn’t stop there. Always check several value to ensure that your true answer is not a fluke.
Let’s check the equation against the second row of values.

Again start by writing the equation $T=4(S)$

Sub in the appropriate values: $8=4(2)$

Does $8=8$? It sure does. We have a true statement.

At this point we can be reasonably sure that we’ve have found our constant of proportionality but let’s check the equation against the last row just to make sure.

Start by writing our equation $T=4(S)$

Sub in the values from row three $S=3$ and $T=12$

$12=4(3)$

Since $12=12$, we know our equation works for modeling the ratios in the table and our constant of proportionality is 4.

**Work Time:**

2 to 3 class periods of 45 mins depending on how many exercises are necessary

**Additional Support**


[https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/7th-constant-of-proportionality/v/identifying-constant-of-proportionality-graphically#:~:text=The%20constant%20of%20proportionality%20is%20what%20determines%20the%20relationship%20between,by%20the%20constant%20of%20proportionality.](https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/7th-constant-of-proportionality/v/identifying-constant-of-proportionality-graphically#:~:text=The%20constant%20of%20proportionality%20is%20what%20determines%20the%20relationship%20between,by%20the%20constant%20of%20proportionality.)

**Online Problem Sets**

Below is a link for the problem set on KhanAcademy.org.

[https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/7th-constant-of-proportionality/e/constant-of-proportionality-from-graphs](https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratio-proportion/7th-constant-of-proportionality/e/constant-of-proportionality-from-graphs)


Closing and Assessment:

Today we dove deeper into our understanding of ratios by exploring the constant of proportionality. The constant of proportionality is what determines the relationship between $y$ and $x$. If $r$ is the constant of proportionality then an example is $y = rx$. The value of $y$ is dependent on how the given value of $x$ is effected by the constant of proportionality.
1. Which equation below has a constant or proportionality of 3? Choose all that apply.
   a. $Y = 3x$
   b. $3 = yx$
   c. $Y = \frac{12}{3}x$
   d. $Y = 4x$

2. Which equation below has a constant or proportionality of 5? Choose all that apply.
   a. $Y = 10x$
   b. $Y = \frac{10}{2}x$
   c. $Y = 5x$
   d. $5y = x$

3. Sara is training to run a marathon this coming spring. She can run 1 mile every 7.5 minutes. Complete the ratio table below, calculate the constant of proportionality and graph the values on the coordinate plane. Make sure to label the coordinate plane correctly, using appropriate scaling and labels for the x and y-axis.

<table>
<thead>
<tr>
<th>Miles</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Tyler is baking cookies for a school bake sale. For every 1 cup of sugar he uses 3 cups of flour. Complete the ratio table below, calculate the constant of proportionality and graph the values on the coordinate plane. Make sure to label the coordinate plane correctly, using appropriate scaling and labels for the x and y-axis.

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>Cups of Flour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Coordinate plane diagram]
Integrating into the classroom

Teaching Ratios Remotely

As teachers well know, the 2020 COVID-19 pandemic presented many challenges to educators and students alike. Instruction changed from traditional classroom instruction to online learning and student engagement, in many cases, dropped. Teachers worked diligently to embrace technology in order to support student learning while providing instruction remotely. And while many saw this as a roadblock to learning, there are many lessons learned that can and should be brought back into the physical classroom. Many teachers, including myself, relied on programs such as Zoom, Khan Academy and Nearpod. As such, these are integrated into the curriculum section of this curriculum project. The lessons present actual examples from Khan Academy as practice questions along with questions created by myself in order to provide thorough and rigorous lessons.

Khan Academy was used as a supplement in my curriculum in order to provide multiple representations of ratios and interactive exercises. Other exercises including problem sets and exit tickets were created by myself to ensure that each lesson was rich in content. Khan Academy is a free, online program that provides educational resources in mathematics starting at pre-k and going all the way through advanced courses such as differential equations and linear algebra. Most beneficial to school teachers are the curriculums that are developed around the Engage NY standards. Grades 3 through 8 all have curriculums that align to Engage NY Standards making them extremely useful in the mathematics classroom. Nearpod is an online presentation tool used in my classroom and in my lessons in order to share lessons with students. It acts as my white board and allows me to view student progress while working
through practice problems. Nearpod is a teaching tool that allows an entire lesson, including videos, practices sets, teachers examples, online resources, and gamification resources to be nested directly in the presentation. Lessons can be presented as “student paced” in which student control the speed at which they move through a lesson and “live participation” in which the teacher controls the pace of the lesson. The lessons containing new content contain live participation lessons which was guided by myself. Every week the lessons were uploaded into Google Classroom allowing students easy access to the lessons and give them the ability to preview upcoming material. Zoom was used as a way to interact with students and encourage student discourse. One advantage to Zoom is that it allows students the ability to answer questions in chat privately. This gives students that are unsure of their mathematical ability a safe place to ask questions and receive encouragement.

Each Nearpod contains the following slides: A cover slide which included lesson title, learning target and standard, a video to support the days lesson which itself included imbedded questions to maintain student engagement, practice problems that are presented in a “draw it” format allowing for student practice and the ability to monitor for misconceptions, and a practice set that was that was taken from Khan Academy’s 6th grade Engage NY curriculum. After giving my own presentation on the material which included worked examples and watching the video, students would have anywhere from 5 to 10 practice questions that would be completed as a class which I monitored in real time. I could talk with students via Zoom and correct any misconceptions by an individual or group as they were occurring. If I felt students were struggling, I could assign more practice questions to complete as a class. If students found the material easy to complete, I could skip practice questions and head straight to the Khan
Academy practice set. Within the Khan academy practice set I could monitor how many questions students completed and how many they answered correctly. If a student had not completed any questions or was working at a slow pace, I could easily redirect them or clarify misconceptions. If a student answered multiple questions incorrectly I could have them “share screen” via Zoom which would allow me the ability to work through problems with struggling students. Khan academy also has it’s own “draw it” feature allowing me to see a student’s thought process as they completed their practice set. One of my favorite features of Khan academy is their color-coded grade book which allows a quick way to assess how individuals and the group performed on a day’s assignments.

**Conclusion**

The purpose of this curriculum project was to provide teachers and other educators with materials and tips on how use remote instruction with purposeful questioning to teach 6th grade concepts of ratios and proportional reasoning. The concept of ratios and proportional reasoning, first taught in 6th grade, is important as it will follow students through their mathematical education as they progress through algebra, geometry, trigonometry and calculus. In order to provide students with a comprehensive education in ratios, it is important that they be exposed to multiple representations of ratios in the form of ratio tables, double number line, tape diagrams, and examples of ratios on the coordinate plane along with purposeful questioning to push their thinking. This curriculum attempted to reach that goal using technology and methods developed during the 2020 Covid-19 Pandemic and subsequent lockdown of schools around the world. The lessons, which were developed to be used remotely, can also be used in the physical classroom in order to provide students with a
thorough and engaging set of lessons that build procedural fluency and a solid conceptual understanding of ratios and proportional reasoning.


Lesson one – Ratios from Tables – ANSWER KEY

1. Maria is having a picnic after school at a local park. She plans on making Kool Aid for her guests to drink during the picnic. Maria uses 3 cups of sugar for every one pitcher of Kool-Aid to make sure the Kool Aid tastes just right. Complete the table below showing the ratio of sugar to pitchers of Kool Aid:

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>Pitchers of Kool Aid</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
</tr>
</tbody>
</table>

2. Maria figures that for every four guests, she’ll need one pitcher of Kool Aid. Complete the table below showing the ratio of guests to pitchers of Kool Aid needed for the picnic:

<table>
<thead>
<tr>
<th>Number of Guests</th>
<th>Pitchers of Kool Aid</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
</tr>
</tbody>
</table>

3. Now Maria wants to know how many cups of sugar she’ll need to purchase for the party. Complete the table below by first figuring out how many cups of sugar she’ll need per guest.

<table>
<thead>
<tr>
<th>Number of Guests</th>
<th>Cups of Sugar</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>
Exit ticket

Julian is making orange paint by mixing red paint and yellow paint together. For every 3 quarts for red paint, Julian uses 4 quarts of yellow paint.

Complete the table using equivalent ratios.

<table>
<thead>
<tr>
<th>Red Paint</th>
<th>Yellow Paint</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>

Lesson two – Ratios from Double Number Lines – ANSWER KEY

1. Debbie is running in a 5 mile race to raise money for the American Cancer Society. A generous donor has decided to donate $100 dollars for 1 mile that Debbie runs.
   Complete the double number line below that shows the amount of money she will make for every mile she runs:

   ![Double number line](image)

2. To train for the race, Debbie ran 6 miles every week. Complete the double number line below to show how many miles she ran leading up to the race.
3. Use the double number line and extend your thinking further. If Debbie started training 8 weeks before the race, how many training miles did she run?

Ans: Debbie would have run 48 miles if she trained for 8 weeks

4. How many weeks did it take for Debbie to run 42 miles?

Ans: It would take Debbie 7 weeks to run 42 miles

Exit ticket:

John uses 90 strawberries for every 2 strawberry pies he’s baking. Use the table of equivalent ratios below to construct a double number line showing the relationship between strawberries and pies. How many strawberries does John need to bake 5 strawberry pies?

Ans: John needs 225 strawberries to make 5 strawberry pies

Lesson three – Ratios from Tables – ANSWER KEY

1. Karen is making ice cream sundaes for her friends during a sleep over. Every sundae uses 3 scoops of ice cream, she uses 2 tablespoons of chopped peanuts.

   a. Draw a tape diagram showing the ratio of scoops of ice cream to tablespoons of chopped peanuts.
b. If Karen uses 8 tablespoons of chopped peanuts, how many sundaes did she make for her friends?

Ans: If she used 8 tablespoons of chopped nuts, she made 4 sundaes

2. Daniel is building a patio on the back of his house. For every piece of lumber he needs 4 galvanized screws.

a. Draw a tape diagram showing the ratio of lumber to galvanized screws.

b. Daniel figures that he will need 40 pieces of lumber to build his deck. How many screws will Daniel need to buy?

Ans: If Daniel uses 40 pieces of lumber, he will need 160 screws

3. Mr. Marlowe LOVES his morning coffee and he likes it strong! For every 2 cups of water he uses 3 scoops of coffee.

c. Draw a tape diagram showing the ratio of cups of water to scoops of coffee.

4. This morning Mr. Marlowe is particularly tired and wants to make 12 cups of coffee. How many scoops of coffee will he need?

Ans: To make 12 cups of coffee he will need 18 scoops of coffee.

Exit ticket
1. Below is a tape diagram showing the number of boys to the number of girls at Discovery Charter school.

![Tape Diagram]

2. Use the tape diagram to fill out the ratio table and double number line below.

<table>
<thead>
<tr>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

![Number Line]

3. Brandon is having a Memorial Day BBQ and is planning on grilling chicken legs for his guests. Brandon figures that each guest will consume 4 chicken legs.

ea. Complete the ratio table below which showing the ratio of chicken legs to the number of guests attending.

<table>
<thead>
<tr>
<th>Guests</th>
<th>Number of Chicken Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>
f. Use the table above to create a coordinate plane. Plot the ratio from the table on the coordinate plane. Make sure to use an appropriate scale for your x and y axis. Label each axis appropriately with the correct heading from the ratio table.

\[\begin{array}{cc}
6 & 24 \\
\end{array}\]

\[\begin{array}{cccc}
g & 1 & 2 & 3 \\
\hline
chicken legs & 12 & 16 & 20 \\
\end{array}\]

\[\begin{array}{cccc}
g & 1 & 2 & 3 \\
\hline
chicken legs & 12 & 16 & 20 \\
\end{array}\]

g. Extend your thinking beyond the graph. Let’s say Brandon has a ton of guests planning on attending his Memorial Day Picnic. If 23 guests attend the picnic, how many pieces of chicken will he need to buy?

Ans: If 23 guests attend, Brandon will need 92 chicken legs

h. Uh-oh! Brandon miscalculated and bought 36 chicken legs but at the end of the party there were 8 chicken legs left over. Assuming everyone at the BBQ ate exactly four pieces of chicken, how many people attended the BBQ?

Ans: 7 people attended the BBQ

4. Sara owns a flower stand on 5th Avenue. It’s Valentine’s Day and she is selling a dozen roses for 45 dollars.
d. Complete the ratio table below showing the ratio of roses sold by the dozen to the revenue generated

<table>
<thead>
<tr>
<th>Roses sold by the dozen</th>
<th>Revenue Generated</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$180</td>
</tr>
<tr>
<td>8</td>
<td>$360</td>
</tr>
<tr>
<td>12</td>
<td>$540</td>
</tr>
<tr>
<td>16</td>
<td>$720</td>
</tr>
</tbody>
</table>

e. Use the table above to create a coordinate plane. Plot the ratio from the table on the coordinate plane. Make sure to use an appropriate scale for your x and y axis. Label each axis appropriately with the correct heading from the ratio table.

Sara pays the city a monthly rental fee for the spot on 5th Avenue she uses to sell her roses. The rent is $800 per month. How many dozen roses will Sara have to sell to cover her monthly rent?

Ans: She will have to sell at least 18 dozen roses to cover her rent

Exit Ticket:
Betty drinks one glass of milk for every three cookies she eats. Complete the table below, plot the ratios as points on the coordinate plane and calculate the constant of proportionality. Make sure to label your graph appropriately.

<table>
<thead>
<tr>
<th>Glasses of Milk</th>
<th>Cookies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Lesson five – Unit Rates – ANSWER KEY

1. Jonathan is taking a trip from New York to Switzerland on a Boeing 747 which can fly 2,280 miles every 4 hours.

   c. What is the unit ratio of miles to hours of the Boeing 747?

      Ans: The ratios of miles to hours is 570:1

   d. Assuming the plane travels at a constant speed, how many miles is it to Switzerland if Jonathan is flying for 7 hours?
Ans: he will travel 3,990 miles in 7 hours

a. Dillan babysat for 3 hours each night for 12 nights. He earned $540 for his work babysitting. How much money did Dillan earn per hour babysitting?

Ans: Dillan earned $15 per hour babysitting

c. Dillan is saving for his fall semester at college where he is studying to be an engineer. If tuition is $1,500 for the semester, what is the minimum hours he will have to work?

Ans: Dillan will have to work 100 hours to afford his tuition

2. A baseball team is having a bake sale to raise money for uniforms for the season. The team charged $2 for every cookie that they sold. In 5 hours, they sold 200 cookies. At that rate, how much money could the team earn selling cookies for 12 hours?

Ans: At that rate the team would make $480 in 12 hours

Exit Ticket:

John Clarke was a speed reader who could read 84 pages every 3 minutes. How many pages could John read in 1 minute?

Ans: John can read 28 pages in 1 minute

Lesson six – Constant of Proportionality – ANSWER KEY

1. Which equation below has a constant or proportionality of 3? Choose all that apply.

a. \( Y = 3x \)
b. \( 3 = yx \)
c. \( Y = \frac{12}{3}x \)
d. \( Y = 4x \)

2. Which equation below has a constant or proportionality of 5? Choose all that apply.

   b. \( Y = 10x \)
   c. \( Y = \frac{10}{2}x \)
   d. \( Y = 5x \)
   e. \( 5y = x \)

3. Sara is training to run a marathon this coming spring. She can run 1 mile every 7.5 minutes. Complete the ratio table below, calculate the constant of proportionality and graph the values on the coordinate plane. Make sure to label the coordinate plane correctly, using appropriate scaling and labels for the x and y-axis.

<table>
<thead>
<tr>
<th>Miles</th>
<th>Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>22.5</td>
</tr>
</tbody>
</table>
Exit Ticket

Tyler is baking cookies for a school bake sale. For every 1 cup of sugar he uses 3 cups of flour.

Complete the ratio table below, calculate the constant of proportionality and graph the values on the coordinate plane. Make sure to label the coordinate plane correctly, using appropriate scaling and labels for the x and y-axis.

<table>
<thead>
<tr>
<th>Cups of Sugar</th>
<th>Cups of Flour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Ans: The constant of proportionality is 3