

Determining the Acceleration of a System of 2 Objects

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Grade level(s)/Subject taught: AP Physics/ IB Physics (11/12)

Objectives:

Students will be able to use Newton's second law ($\Sigma F = ma$) to calculate the acceleration of a block on a table via another block hooked to a pulley and the tension needed to accelerate the block.

This lesson addresses the following standards in the IB Curriculum Guide:

2.2.2 Each force should be labeled by name or given a commonly accepted symbol. Vectors should have lengths approximately proportional to their magnitudes. Identify the forces acting on an object and draw free-body diagrams representing the forces acting.

2.2.11 Solve problems involving Newton's second law.

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The Science Concept or "key idea" that modeling will be used to teach:

The sum of the forces acting on an object is equal to the product of the mass and the acceleration of the object ($\Sigma F = ma$). When there is more than one object involved, a separate equation must be written for each object and then the equations must be solved simultaneously to find the acceleration of the system. **This lesson will take one 40 minute period.**

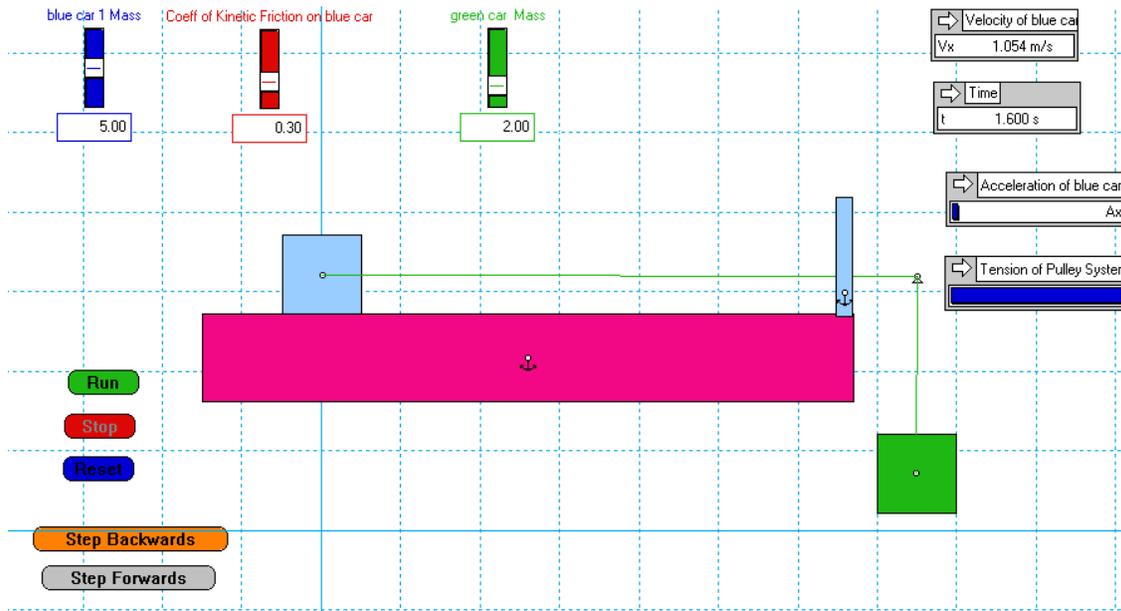
Materials:

A computer, a computer projector, the Integrated Physics program (with a previously constructed model), and a calculator.

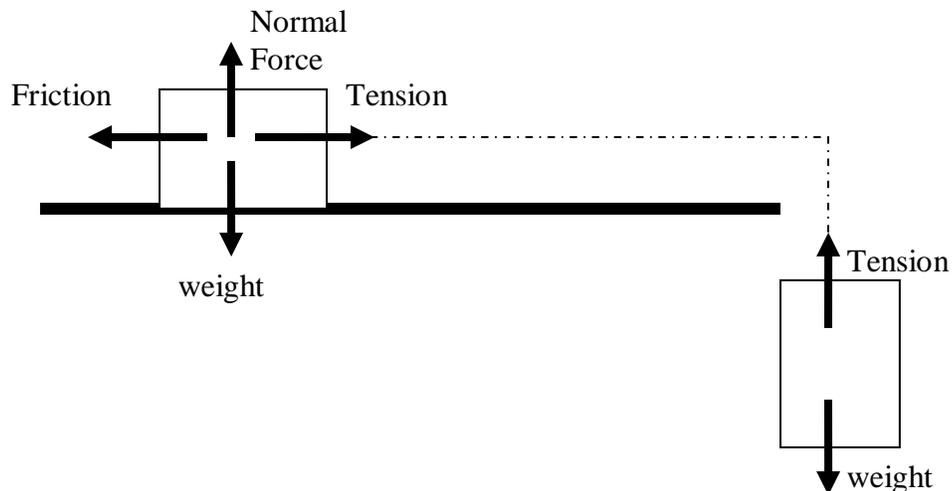
Answers for student work are in the lesson. A quiz with answers is also attached to the lesson.

The Lesson:

- Review Newton's Second Law:
the sum of the forces acting on an object is equal to the product of the object's mass and acceleration or $\Sigma F = ma$.
- Run Interactive Physics and open up the **Block on Table** file. You should see this:



- **Ask:** How would we determine the acceleration of this situation where one block is pulling another? **Run simulation.** Students should notice that the Acceleration and Tension force (indicated by the blue bar graphs) remain relatively constant and that the velocity should increase. (Hopefully students recall drawing vector diagrams for single objects involving friction.)
- Draw vector diagrams on the picture above by using a Smartboard (or overhead)



- Discuss how to use Newton's Second Law to set up equations for each object: Remember that we write the forces in such a way that the direction that the object will move is the positive direction (in this case, in the direction of T for the blue object and the direction of the weight vector for the green object)

For the blue object (its mass will be denoted m_{bl}): $\Sigma F = ma \rightarrow T - F_f = m_{bl} a$
 Since $F_f = \mu F_N$ and in this case $F_N = W_{bl} = m_{bl} g$, we then can write the following:

$$T - \mu m_{bl} g = m_{bl} a$$

For the green object (its mass will be denoted m_{gr}): $\Sigma F = ma \rightarrow W_{gr} - T = m_{gr} a$
 which becomes: $m_{gr} g - T = m_{gr} a$

- The next step is to solve the equations simultaneously so that can get the acceleration of the system. Solve both of the equations for T and set them equal to each other.

$$T = m_{bl} a + \mu m_{bl} g \quad \text{and} \quad T = m_{gr} g - m_{gr} a$$

Now solve for a:

$$m_{bl} a + \mu m_{bl} g = m_{gr} g - m_{gr} a$$

$$m_{bl} a + m_{gr} a = m_{gr} g - \mu m_{bl} g$$

$$a (m_{bl} + m_{gr}) = m_{gr} g - \mu m_{bl} g$$

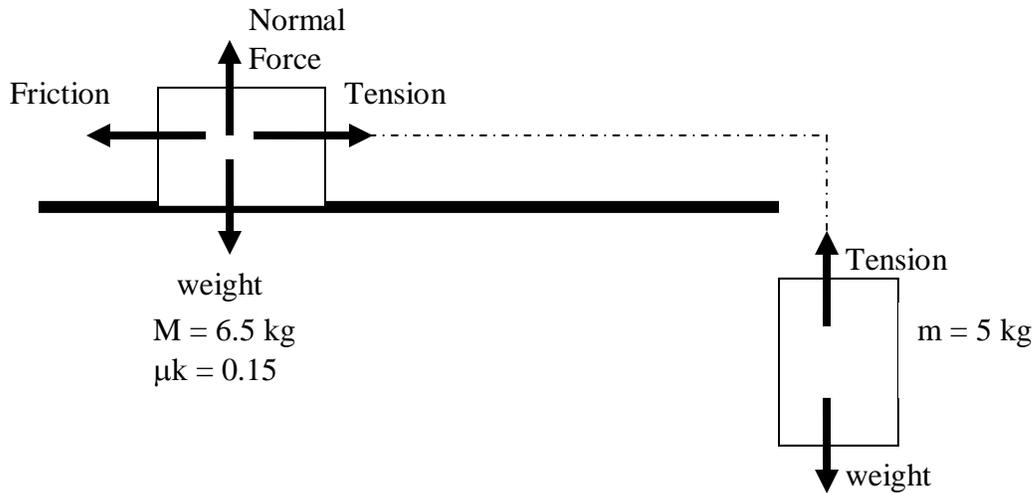
$$a = (m_{gr} g - \mu m_{bl} g) / (m_{bl} + m_{gr})$$

Using the values on the model ($m_{bl} = 5\text{kg}$, $m_{gr} = 2\text{kg}$, $\mu_k = 0.3$, and $g = 9.8 \text{ m/s}^2$), $a = 0.73 \text{ m/s}^2$ To solve for the tension in the rope, plug $a = 0.73 \text{ m/s}^2$ into the equation $T = m_{gr} g - m_{gr} a$ and you will get $T = 18.214 \text{ N}$

- On the model the exact values of the Acceleration and Tension can be known by clicking on the arrow in the top left of each box. The model gives an acceleration of 0.702 m/s^2 and a Tension of 18.238N . The model is accurate to within 0.25% , which these numbers fall within.
- Do another example with $m_{bl} = 2\text{kg}$, $m_{gr} = 3 \text{ kg}$, and $\mu_k = 0.2$. The acceleration of this system is 5.1 m/s^2 and the Tension on the rope is 14.14 N (**be sure to hide the answers before you run it.**)
- Have the students try these on their own:
 1. $m_{bl} = 5 \text{ kg}$, $m_{gr} = 2.5 \text{ kg}$, and $\mu_k = 0.25$. **Answers: $a = 1.64 \text{ m/s}^2$ $T = 20.45 \text{ N}$**
 2. $m_{bl} = 8 \text{ kg}$, $m_{gr} = 4 \text{ kg}$, and $\mu_k = 0.1$. **Answers: $a = 2.62 \text{ m/s}^2$ $T = 28.78 \text{ N}$**

- **Evaluation:**

There will be a quiz in two days on the material. The answers are below, the quiz is on the following page.



$$T - \mu Mg = Ma$$

$$mg - T = ma$$

$$T = Ma + \mu Mg \quad \text{and} \quad T = mg - ma$$

Now solve for a:

$$Ma + \mu Mg = mg - ma$$

$$Ma + Ma = mg - \mu mg$$

$$a(M + m) = mg - \mu Mg$$

$$a = (mg - \mu Mg) / (M + m)$$

$$a = 3.43 \text{ m/s}^2$$

To solve for the tension in the rope, plug $a = 3.43 \text{ m/s}^2$ into the equation $T = mg - ma$ and you will get $T = 31.89 \text{ N}$

Name: _____

Newton's 2nd Law Quiz

On the diagram below, draw (and label) all of the forces acting in the system below. Then determine the acceleration of the system and the Tension on the rope using the values in the diagram below.

