# CMST CHALLENGE PROJECT 2005

## NEWTON'S LAW OF COOLING

**By**

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# Newton's Law of Cooling

## *Project explanation*

Students use mathematical modeling and multiple representations to provide a means of interpreting, communicating and connecting mathematical information and relationships. They used the graphing calculator to model the data and Stella to simulate multiple cases of the data. Real world data most often cannot be modeled precisely with a single equation, so mathematicians, scientist, and engineers find functions to fit the data. Students were faced with the challenge of collecting data from an experiment, graphing the data, writing an equation to fit the data and then interpreting the results. Newton's law of Cooling was chosen because of the exponential decay the data produces. Students get an opportunity to model what they learned to a real life situation then see how technology helps simplify the process.

After completing a chapter on Exponents and Logarithms in PreCalculus (grade 11-12), students completed an experiment on Newton's Law of Cooling. In the experiment, students took room temperature and the temperature of hot water in intervals of 5 minutes until the water reached the temperature of the room. Students made sure that the hot water had a temperature of at least 120˚ so that enough data could be collected. They then used the equation $T(t) = (T(0) - T_r)e^{-kt} + T_r$ where T(0) is the initial water temperature, $T_r$ is the room temperature and *k* is the cooling rate, to model the curve of best fit for the data. Next they graphed the data and the function on the TI/83 calculator to verify that the equation fit the data. Next a Stella model was built to demonstrate how much easier the experiment could have been done for multiple cases of the same experiment. The model was then presented to class on the view screen.

### *Why Stella*

Stella was chosen because it is a visual modeling language that creates a model that uses a set of quantities that change over time. We needed a model that represented the changing water temperature, the decreases to these quantities over time, and the outside parameters that affect those changes. Stella provided a model that represented Newton's Law of Cooling not only visually but mathematically.

### *Prerequisite Knowledge*

- Students should know how to use the TI-83/89 calculator to plot data and to find the equation of a function from the data.
- Students should know the properties of the exponential and logarithmic functions and how to solve algebraically.
- Students should know how transformations involving reflections, translations, and stretching affect the exponential function.

***Computational Tools/Materials Needed***

- TI-83/89 graphing calculator
- Computer with Stella program
- Stella model of Newton's Law of Cooling
- Excel

## *NCTM Standards*

***Algebra***

- Students will analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Students should use symbolic algebra to represent and explain mathematical relationships.
- Students should use a variety of symbolic representations, including recursive and parametric equation, for functions and relations.

***Measurement***

- Students should make decisions about units and scales that are appropriate for problem situations involving measurement.

***Problem Solving***

- Students should apply and adapt a variety of appropriate strategies to solve problems.

***Representation***

- Students should use representation to model and interpret physical, social, and mathematical phenomena.

## *NYS Standards - Mathematics*

***Key Idea 3: Operations***

- Students use mathematical operations and relationships among them to understand mathematics
  - Students use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.
  - Students will explore and use negative exponents on integers and algebraic expressions.

***Key Idea 4: Modeling/Multiple Representation***

- Students should use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

- Students represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.
- Students manipulate symbolic representations to explore concepts at an abstract level.
- Students choose appropriate representations to facilitate the solving of a problem.
- Students investigate transformations in the coordinate plane.
- Students use graphing utilities to create and explore geometric and algebraic models.
- Students determine the effects of changing parameters of the graphs of functions.
- Students use polynomial, rational, trigonometric, and exponential functions to model real world relationship.

***Key Idea 7: Patterns/Functions***

- Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patters simply and efficiently.
    - Students represent and analyze functions using verbal descriptions, table, equations, and graphs.
    - Students apply linear, exponential, and quadratics functions in the solution of problems.
    - Students apply and interpret transformations to functions.
    - Students model real-world situations with the appropriate function.
    - Students use computers and graphing calculators to analyze mathematical phenomena.

***Summary of Procedures***

Newton's law of Cooling was presented to the class as an extra credit assignment. About 10 students took advantage of the project. They had to perform the experiment as indicated in the opening statement. Next they had to calculate the function that fit the data. They graphed the data collected and function on the TI-83/89 calculator and wrote a report on their findings.

Stella was introduced to the class as modeling language that visually modeled the same experiment and had capability of changing parameters to produce multiple conclusions. The Stella model was built by the teacher but refined with student assistance. Students then had to complete a worksheet provided by teacher using the Stella model.

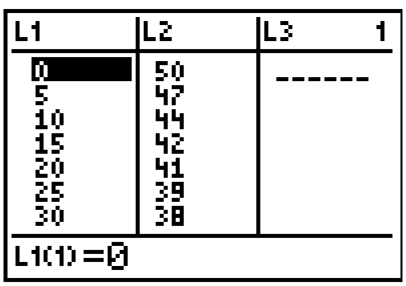
Students were assigned various jobs to complete this CMST project Andrew Wright was assigned the task of doing the power point presentation. Jamie Colon was assigned the task of input the data and equation in the graphing calculator so that we could do a scene capture for the report. Joseph McGinnis was assigned to writing the introduction to the project and finding clip art for the poster. The other students were assigned the task of retyping their reports and helping put the poster together.

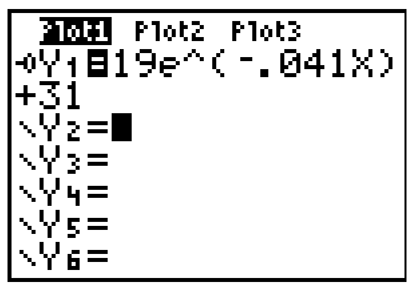
The biggest problem we encountered was finding a common time to work together on the project. The students seem to enjoy working on the project, but was upset to find out that they didn't have to do all that work on the initial experiment to obtain the same results.

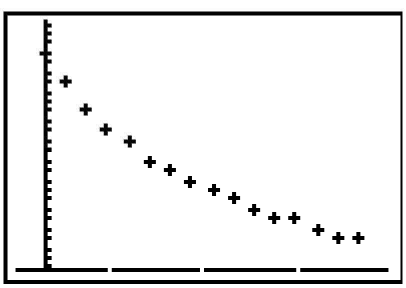
***Conclusion***

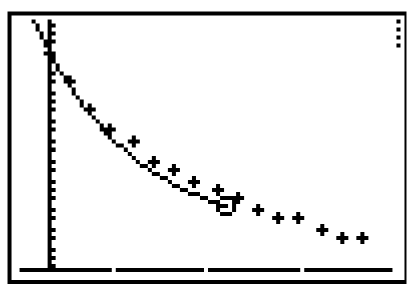
Newton's law of cooling is proportional to the difference in temperature between an object and its surrounding. That means that the larger the difference in temperature between an object and its surroundings, the more rapid the cooling. Conversely, the smaller the difference in temperature between an object and its surroundings, the slower the rate of cooling. Students discovered this fact when comparing the graphs of "temperature vs ambient temp change" and "temperature vs refrigerator temperature change". The temperature dropped faster in the refrigerator because there was a greater gap between the object and its surroundings. Stella allowed us to cover a lot of ground in a very small amount of time. It allowed the student to move beyond the simple application of a formula and gain a deeper understanding of the underlying concepts.

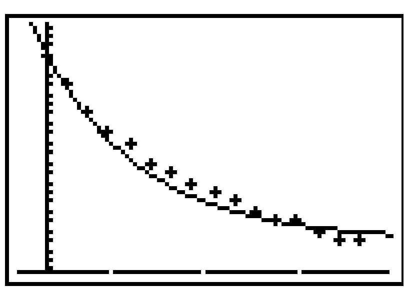
# TI-84 Models of Data and Graphs



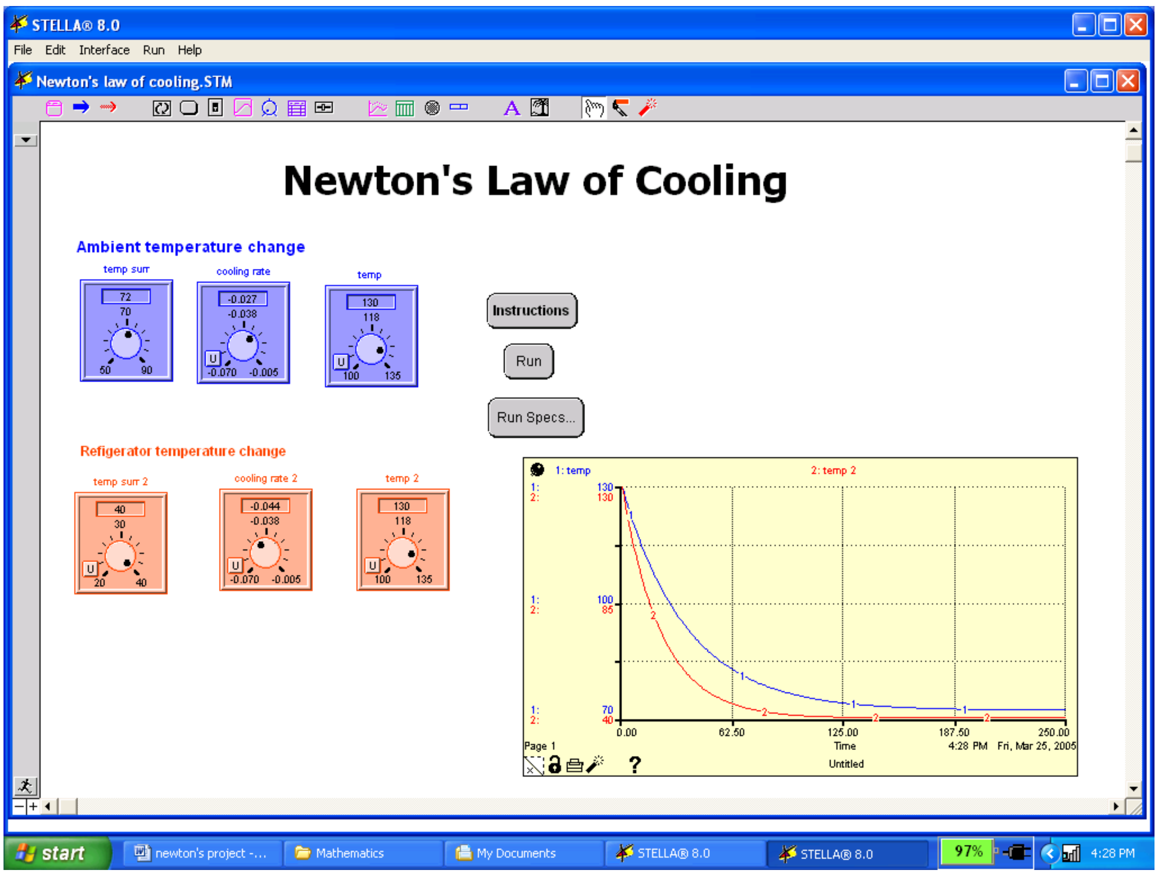



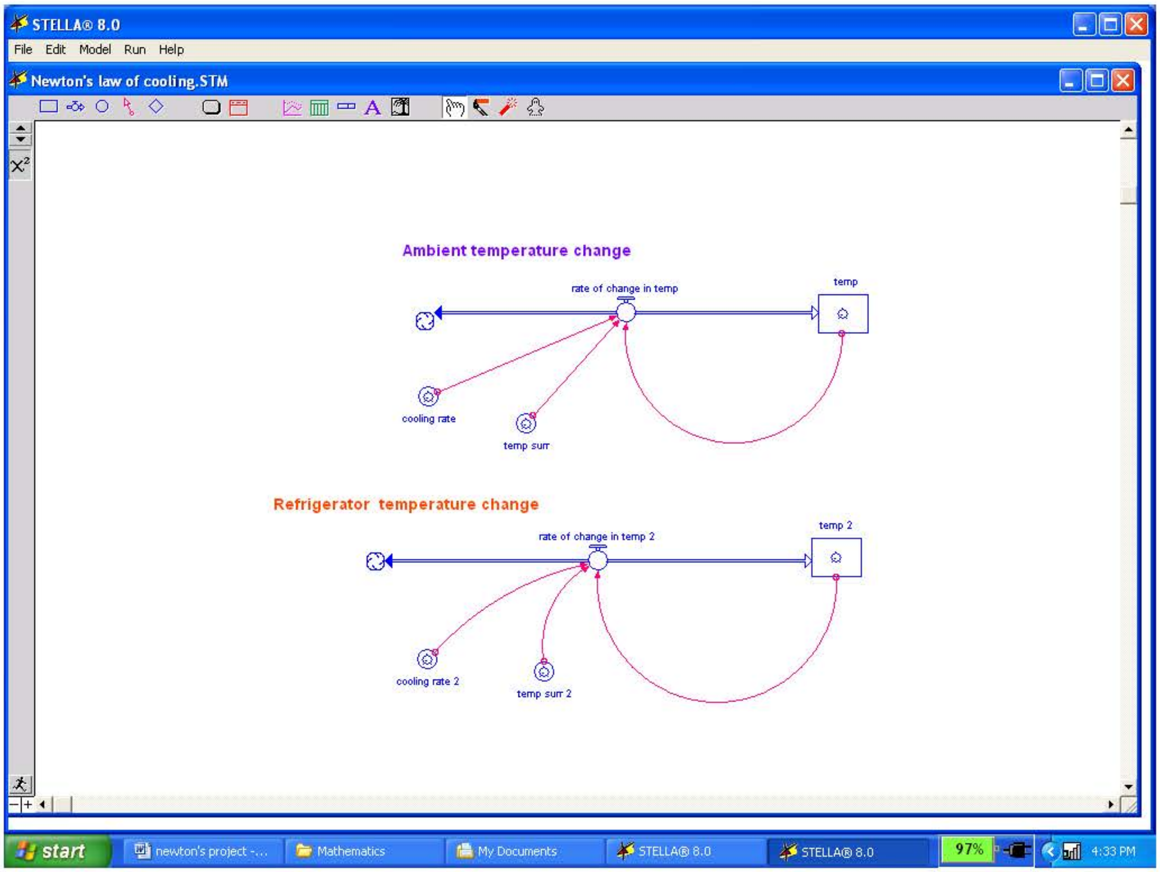







# Stella Models for Newton's Law of Cooling





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# Newton’s Law of Cooling Worksheet

Suppose that a cup of water cooled from 120° F to 90° F in 35 minutes in a room whose temperature was 70° F. How much longer would it take the soup to cool to 70° F?

Instead of being left to stand in the room, the cup of water 120° F is put into a refrigerator whose temperature is 40° F. It takes 10 minutes to cool to 90˚ F. How much longer will it take the water to cool from 90° F to 70° F

Use the **Stella model** to calculate the values.

## Ambient Temperature Change

| **Time (min)** | **Temperature** (˚F) | **Cooling Rate (k)** |
|---|---|---|
| **35** | **90** | |
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## Refrigerator Temperature Change

| **Time(min)** | **Temperature(˚F)** | **Cooling Rate (k)** |
|---|---|---|
| **10** | **90** | |
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