THE USE OF VIRTUAL MANIPULATIVES IN FOURTH GRADE TO IMPROVE MATHEMATIC PERFORMANCE

by

Jaimie Morris

A Master's Project

Submitted in Partial Fulfillment

Of the Requirements for the Degree of

Master of Science in Education

Curriculum and Instruction Inclusive Education

Curriculum and Instruction

State University of New York at Fredonia

Fredonia, New York

December 2013

State University of New York at Fredonia
Department of Curriculum and Instruction

CERTIFICATION OF PROJECT WORK

We the undersigned, certify that this project entitled THE USE OF VIRTUAL MANIPULATIVES IN FOURTH GRADE TO IMPROVE MATHEMATIC PERFORMANCE by Jaimie Morris, Candidate for the Degree of Masters of Science in Education, Curriculum and Instruction Inclusive Education, is acceptable in form and content and demonstrates a satisfactory knowledge of the field covered by this project.

[redacted]
Janeil C. Rey, PhD
Master's Project Advisor
EDU 691
Department of Language, Learning, and Leadership

12/30/13
Date

[redacted]
Mira Berkley, PhD
Department Chair
Department of Curriculum and Instruction

1/2/14
Date

[redacted]
Dean Christine Givner, PhD
College of Education
At SUNY Fredonia

1/28/14
Date

Abstract

Virtual manipulatives are mathematical tools recommended by the National Council of Teachers of Mathematics (NCTM), which are underutilized within elementary schools. This study investigated the impact of virtual manipulatives on fourth-grade students' mathematic performance. Students in one general education math class were randomly assigned to either a treatment group or one of two control groups. Together the three groups were comprised of twelve fourth-grade students who were taught by the same math teacher. The treatment and both control groups studied adding and subtracting three to six digit whole numbers. The treatment group used virtual manipulatives to practice the concepts from the lesson, while one control group used concrete manipulatives and the other control group used paper and pencil worksheets to practice the concepts. An identical paper and pencil pre-test was given prior to instruction to all groups as well as an identical paper and pencil post-test after the unit of adding and subtracting whole numbers. The findings showed that all three groups scores improved between the pre-test and post-tests. However, there was a significant improvement with the students who participated in the virtual manipulative group. Students' mathematical performance was positively impacted when students used virtual manipulatives during the adding and subtracting whole numbers math unit.

Table of Contents

Introduction

What is the impact on mathematic performance for 4th grade students when teachers use virtual manipulatives? To answer this question I conducted an empirical quantitative study at a rural elementary school in Western New York State using two control groups and a treatment group. The results from this study presents evidence for teachers deciding whether to incorporate virtual manipulatives within their lessons to impact student learning. Technology is growing within schools and can help teachers differentiate instruction in their lessons. Representing concepts in different ways helps teachers reach all types of learners. "The power of virtual manipulatives is in combining several representations in ways that support the learner in connecting multiple aspects of mathematical concepts and ideas" (Moyer-Packenham, Salkind, &Bolyard, 2008, p. 204). With technology and the accessibility of internet in classrooms, teachers can take advantage of free web-based resources to support instruction.

There are several reputable websites that offer free virtual mathematic manipulatives for teachers. The National Library of Virtual Manipulatives is a website that allows students to interact and explore virtual manipulatives in conjunction with completing activities provided by teachers who create an account ("National Library of Virtual Manipulatives," 2010). The National Council of Teachers of Mathematics' (NCTM) *Illuminations* website provides a variety of mathematic resources, including virtual manipulatives that are aligned with the NCTM Principles and Standards for School Mathematics ("Illuminations," 2013). When using these tools, teachers have to consider the new Common Core State Standards during planning and implementation of lessons.

Elementary teachers in New York State are now incorporating the Common Core State Standards (CCSS) into their daily instruction of mathematics and English Language Arts. The CCSS mathematics standard number four, “model with mathematics” (“National Governors Association Center for Best Practices,” 2010) highlights students’ use of models to represent their mathematical reasoning. Standard number five, “use appropriate tools strategically” (http://www.corestandards.org/Math/Practice) highlights that students are able to use technology as a tool to explore and deepen mathematic concepts. Internet resources and computers are now being used more frequently by teachers within lessons and have come to be expected (Steen, Brooks, & Lyon 2006). According to the CEO Forum (2001) 67% of teachers use computers daily in lessons. More recently Gray, Thomas, and Lewis (2010) discussed that 97% of teachers have computers within their classrooms and 54% have access to laptops.

Computers are an important tool to support students with the exploration and discovery of mathematical concepts (Burns & Hamm 2011). Burns & Hamm (2011) found little research that supports the use of virtual manipulatives over concrete manipulatives. Similarly, research on the impact of virtual manipulatives is limited due to the use of virtual manipulatives being fairly new in classrooms (Steen, Brooks, & Lyon 2006). Due to the limited research in the area of virtual manipulatives, I have become very curious about their impact on mathematic performance in fourth grade students.

I was a substitute teacher for the school where the study was conducted, and I saw many different ways that teachers taught mathematical concepts to their students. I saw students interacting with concrete manipulatives during math lessons, but I did not typically see the use of virtual manipulatives incorporated within lessons. Technology is growing within schools. I have noticed more computers in classrooms, and classrooms containing two to five ipads that students use for a number of activities, including math. It is important that these tools are used to help students perform better in mathematics.

In the future, I will take advantage of the technology and free resources available to students and teachers that will help improve learning. Incorporating these tools can help me as a teacher comply with the Common Core State Standards for mathematical practice and allow students to model with mathematics to solve, analyze, and explore concepts. It will also allow students to use appropriate tools such as virtual manipulatives to help visualize and deepen their understanding of concepts.

The following question will be used to guide this study: What is the impact on mathematic performance for 4th grade students' when teachers use virtual manipulatives? Previous research and the research provided by this study indicated whether using virtual manipulatives will help students perform better in mathematics.

Literature Review

In the following literature, the impact of virtual manipulatives is reviewed and explored. This literature review will explore standards that govern teaching mathematics at an elementary level, technologies available within classrooms, and instructional strategies for teaching mathematical concepts. Two specific instructional strategies reviewed within this paper are the use of concrete and virtual manipulatives. Benefits and drawbacks of both concrete and virtual manipulatives will be addressed. Students' learn mathematics in a number of different ways. One way that students can learn mathematics is through the use of concrete and virtual manipulatives, which is called mathematical modeling and using multiple representations.

Theoretical Framework

Mathematic concepts should not be taught simply as a concept within the classroom but should be tied in with authentic, real life experiences. One way to allow students' to experience real world situations with mathematics is through modeling (Durmus. & Karakirk., 2006). According to Durmus and Karakirk (2006), mathematical modeling is used to "understand, to explain, to describe, and to predict the different aspects of the real world" (p.118). Teachers bring to the class their own fluidity to master mathematical skills with numbers, equations, and processes. In the classroom teachers model interactively how to solve the problem and show relationships between the numbers to help student explain how they came to their answer, not just the mechanics of the problem. One approach with modeling is having students learn with different forms of manipulatives. This approach can be used in classrooms to encourage students to use models to solve problems so that the solution is scaffolded, visualized, and reflected. Models or representation serves the purpose of allowing students to formulate and understand an end-product, not just a tool used to interpret the problem (Durmus & Karakirk., 2006). The students are able to interact and touch the problem, which allows the students to construct their own knowledge about the math and make connections to the world. In addition, Boaler 2001 states that students should be engaged in their learning to help develop the meaning of the content. When students are engaged in their learning they are motivated to learn and they are creating opportunities for deeper understanding of the content being taught.

Two types of representations that can be used as modeling tools are concrete and virtual manipulatives. Concrete manipulatives as a modeling tool appeals to all types of learners (Durmus & Karakirk., 2006).Virtual manipulatives as a modeling tool also appeals to all types of learners along with providing immediate feedback to ensure mathematical concepts are being reinforced correctly (Burns & Hamm, 2001; Durmus & Karakirk., 2006; Suh & Moyer-Packenham, 2008). Boaler 2001 mentions in his article that mathematical modeling can help students enhance their understanding of the content but can also "provide students with opportunities to engage in practices that are represented and required in everyday life" (p.126). When trying to incorporate virtual manipulatives into mathematic instruction, teachers must relate the use of virtual manipulatives to the standards that guide the mathematic curriculum.

Standards

Common Core State Standards. Many state standards are being replaced by the Common Core State Standards (CCSS). This is a federal initiative that has established a single set of standards that states can adopt voluntarily. These standards are for kindergarten through 12^{th} grade in English language arts (ELA) and mathematics. The CCSS ensure that there is consistency across participating states with clear expectations to prepare students for college and career readiness. Through a collaborative effort, the CCSS were created by teachers, school administrators, and content experts ("National Governors Association Center for Best Practices," 2010). By 2013 forty-five states, the District of Columbia, and four territories had adopted the CCSS. New York State adopted the Common Core Learning Standards (CCLS) in January 2011 ("National Governors Association Center for Best Practices," 2010). The CCSS for mathematical practice contain eight standards. These standards are geared to help students not just retain math concepts test to test, but retain information throughout grades and build upon math concepts.

There are two specific mathematical standards that address modeling and the use of tools. Standard number four "Model with mathematics," highlights that students should use models to communicate mathematical problems. Standard number five "Use appropriate tools strategically," highlights that students need to consider the tools that are available to them to help in solving a mathematical problem ("National Governors Association Center for Best Practices," 2010). According to the National Governors Association Center for Best Practices (2010), fourth grade mathematic instruction should focus on students becoming fluent place value, multi-digit addition and subtraction, multi-digit multiplication and division, fraction equivalence, addition and subtraction of fractions, and geometric figure classification based on their properties. Students should understand these concepts and be able to practically apply their knowledge when focusing on mathematics. The objective of the present study was for students to perform multi-digit (three to six digits) arithmetic. Along with the CCSS, teachers' mathematic instruction is also guided by another form of standards called the Principles and Standards for School Mathematics.

National Council of Teacher of Mathematics Standards. Teachers also use the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (PSSM) to guide instruction in math. NCTM supports teachers to make sure that mathematics learning is of the highest quality for all students. These NCTM principles and standards guide instruction to help students become mathematically competent. Mathematics is used in everyday tasks and students need to be able to use math in their own lives.

The PSSM have six principles that present high-quality instruction in mathematics. Technology is one of the principles, which highlights the importance of incorporating technology, specifically calculators and computers, within mathematical lessons (NCTM, 2000). NCTM (2000) states that technology incorporated in mathematical instruction can provide students with multiple perspectives that enrich the quality of exploration. Multiple representations throughout mathematical lessons are encouraged by the PSSM (Moyer-Packenham, Ulmer, & Anderson, 2012).

Representation is defined as a "configuration of signs, characters, icons, or objects that stand for something else" (Moyer-Packenham & Suh, 2011). There are a variety of representations that are used in classrooms including physical, visual, symbolic, and virtual (Moyer-Packenham, Salkind, & Bolyard, 2008). Technology such as computers can expand the representations, which the students use to flip, stretch, and zoom mathematical representations (NCTM, 2000). These representations should support students' understanding of mathematics and should help students communicate mathematic concepts and arguments (NCTM, 2000).

As with the CCSS, the PSSM also emphasize that teachers and students should model mathematics. Modeling mathematical problems with technological tools allows the students to interact, explore, and develop mathematical understanding (NTCM, 2000). Taking these standards and principles into consideration, teachers need to evaluate their instructional strategies and technology use within their classroom. Mathematics should be supported by technological tools to help students' mathematical learning (Polly, 2011).

New York State Mathematic Standards. Curriculum and instruction in schools are guided by state standards throughout the United States. Standards are what students are expected to learn grade to grade throughout their school years. All subjects have set standards that teachers use to help students learn what they need to know to be successful. Just recently states have been adopting a new form of standards into their curriculums. According to Engage NY (2012) New York State P-12 Common Core Learning Standards for Mathematics, the main purpose of the Common Core Learning Standards (CCLS) for mathematics is to provide required principles that allow students to deepen understanding, gain greater skill, and enhance fluency. The site further notes that the focus of the curriculum is to give students opportunities to understand mathematical concepts and practice the concepts to deepen understanding and fluency to prepare students to be college and career ready ("Engage NY," 2012). Many math programs being used in classrooms today are not aligned to the CCLS due to these programs being published before CCLS were initiated. To help teachers meet the CCLS, curriculum modules have been created to align with each New York State Standard.

New York State Curriculum Modules. New York State is now moving towards the uses of curriculum modules in both English Language Arts and mathematics within schools to help align the curriculum to the Common Core Learning Standards (CCLS). School districts can either adopt, adapt or ignore these modules to assist with the implementation of the Common Core. According to "Engage NY," 2012, the mathematic modules are created to focus more in-depth on fewer topics. The purpose of these curriculum modules are to allow more classroom time for the students to practice and reflect on the problems. The overall goal is to have students achieve mastery level in the mathematic content by practicing and focusing deeply on fewer topics throughout the academic school year ("Engage NY," 2012). These mathematic curriculum modules were created for grades pre-K through twelfth grade. They connect to the NCTM Principles and Standards for Mathematical Practice to guide the mathematical content within the classrooms. Curriculum modules were just recently introduced in the 2012-2013 academic year. The modules are divided into units and lessons that follow the standards that are to be taught and provide a very detailed lesson plan for the teacher to follow. Certain districts have already begun to use these modules in schools. Other districts are waiting to introduce the modules due to their newness and the lack of training provided for teachers on implementing the curriculum modules.

With the transition to common core standards there has also been transition and increase with the uses of technology within classrooms.

Technology

Along with the shift to Common Core Learning Standards there is also another shift within schools known as the 21st century learner. According to Blair 2012, these 21st century learners are being asked throughout school to quickly access new knowledge. This is where technology in the classroom lends to the easy and quick access of new knowledge at students' fingertips.

Technology is rapidly growing in classrooms. According to Gray, Thomas, and Lewis (2010), 97% of teachers in public schools had at least one or more computers within their classrooms and 54% of teachers had access to computers that could be brought into the classroom. Of the computers located in the classroom, 93% of them have Internet access (Gray, Thomas, & Lewis, 2010). The technology present in classrooms today includes: Smart Boards, Document Cameras, desktop computers, laptops, ipads, and tablets. With all this technology available to students and teachers, it is important to allow students to use and practice skills with technology that will help students in their technology infused lives, work places, and learning environments (Blair 2012).

This new technology is being incorporated within mathematics lessons with greater regularity (Moyer-Packenham et al., 2008). The students in classrooms today are the new technological generations and it is the teachers challenge to involve students in lesson (Allen, 2007). Technology can help student achievement in different areas including mathematics for both special needs students and general education students (CEO Forum, 2001). In an NCTM article discussing the use of technology, Polly (2011) stated that technology could possibly improve students' mathematical learning if it is used to develop higher- order thinking skills and is used to support exploration of tasks while allowing students to communicate mathematical ideas. When students use technology in the classroom they become explorers and mathematicians. Allowing students to explore as mathematicians they would "try to understand, analyze, and evaluate their experience to answer posed questions" (Blair, N., 2012, p.10). In this way the students can take ownership of their learning and gain real-world, problem solving experience by using technology. With the use of technology in classrooms, teachers need to explore instructional strategies that could help incorporate the technology within mathematical instruction.

Instructional Strategies

Huang, Liu, and Chang (2012) mentioned that the assessment of students' mathematical understanding should not be solely based on their writing of the problems but also on their demonstration and oral interpretation. Instructional strategies for mathematical instruction should also be differentiated to help different ability groups. One instructional strategy that is used in mathematical instruction is the use of manipulatives, both concrete and virtual.

Concrete Manipulatives. Manipulatives are concrete objects used as tools that allow students to experiment and explore mathematical concepts (Burns & Hamm, 2011). Boggan, Harper, and Whitmire (2010) state that manipulatives have been used for many years and from several different civilizations to solve mathematical problems that they had encountered every day. Some of these ancient manipulatives included counting boards, abacus and, knotted string called quipu. Today manipulatives can range from bottle caps, Unifix cubes, base-ten blocks, clocks, and money, just to name a few. According to Burns and Hamm (2011), students who use manipulatives have a better understanding, create mental images, and represent abstract ideas.

When students employ concrete manipulatives in long-term use during the early elementary level they have greater mathematical achievement than students who have not used concrete manipulatives (Burns & Hamm, 2011). These concrete tools can be manipulated in many ways such as stacking, sorting, classifying, and weighing (Boggan, Harper, &Whitmire, 2010). "Concrete objects that resemble everyday items should assist students in making connections between abstract mathematical concepts and the real world" (Carbonneau, K., Marley, S., & Selig, J., 2013, p.381). As with any type of instructional strategy, concrete manipulatives have both benefits and drawbacks.

One study looked specifically at the benefits and drawbacks of concrete manipulatives. Hunt, Nipper, and Nash (2011), found many perceived benefits on the use of concrete manipulatives. The benefits included opportunities to experience trial and error, view mathematic information visually and kinesthetically, break down mathematical concepts, and actively engage in the math lesson (Hunt et al., 2011). Trial and error has the student try multiple times to find the correct answer allowing students to learn from their mistakes. Using concrete materials benefits students with different learning styles, such as students who learn from visuals or students who learn kinesthetically. Students can see and touch concrete manipulatives to practice math concepts and to understand how a problem is solved. "A good manipulative bridges the gap between informal math and formal math." (Boggan et al., 2010, p.2).

The Hunt, Nipper, and Nash (2011) study also found the perceived drawbacks of using concrete manipulatives. One of the drawbacks of concrete manipulatives is that they do not provide the student with feedback if they have solved the problem correctly or incorrectly; the teacher has to monitor the class and provide the students with feedback (Hunt et al., 2011). Without feedback, students do not know if they are completing the mathematical problem correctly or using the manipulatives correctly. In addition, many teachers do not use concrete manipulatives due to the time investment and difficulty (Puchner, Taylor, O'Donnell, & Fick, 2010). According to Uribe-Flórez and Wilkins (2010), teachers believe that it is important for students to explore mathematical concepts with hands-on activities such as manipulatives. They also found that teachers who teach older students used manipulatives less often believing it was less necessary to use concrete manipulatives (Uribe-Flórez& Wilkins, 2010). Other studies have explored the implementation of virtual manipulatives in mathematical instruction.

Virtual Manipulatives. Advances in technology have allowed the properties of concrete manipulatives to be combined with computer technology to create a new type of manipulative called a virtual manipulative (Burns & Hamm, 2011). A virtual manipulative as defined by Moyer, Bolyard, and Spikell (2002) is "an interactive, web-based virtual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Unlike concrete manipulatives, virtual manipulatives include additional features and expand on the capacity of the concrete manipulatives (Steen et al., 2006). With just a click of the mouse, virtual manipulatives are readily available with unlimited access (Moyer-Packham, Salkind, & Bolyard., 2008). They are cost-effective, accurate, flexible, and beneficial (Hwang, Su, Huang, & Dong, 2009). Use of virtual manipulatives can develop an understanding of mathematical concepts and visualization skills in students (Moyer-Packenham et al., 2012).

A study conducted by Steen, Brooks, and Lyon, (2006) investigated the impact of virtual manipulatives on first grade students' academic achievement and behaviors. They collected data on 31 first-grade students who were randomly placed in a treatment or control group. Each group was structured the same except that the treatment group used virtual manipulatives for practice, the control group used concrete manipulatives and worksheets for practice. Pre-tests and post-tests were used to show results of the study. Their study found that the students in the treatment group had significant gains on the tests. Steen, et al., (2006) found that instructional time was saved by using virtual manipulatives because students did not have to spend time cleaning up and this increased the time-on-task and repetition of a practice activity. They also found in this study that every student had an equal opportunity to access the same high-quality lesson and activities without having to take turns as with concrete manipulatives (Steen et al., 2006; Moyer et al., 2002).

A benefit when working with students who have trouble with fine motor skills, specifically with writing, was that they were able to use the virtual manipulatives easily and were able to focus on the math task not their difficulty with writing or using concrete manipulatives (Steen et al., 2006). In another study, Suh and Moyer-Packenham (2008) employed qualitative action research using records from planning, teaching, observing, and reflecting. Their participants consisted of 19 fourth-grade students in a diverse population. Ten students from the 19 were identified with a special need and had an Individual Education Plans (IEPs). Suh and Moyer-Packenham (2008) stated that the features of virtual manipulatives provided students with special needs time to focus more on the mathematical concepts within the activity and to formulate and make sense of the mathematical rules.

Other studies conducted also found benefits of using virtual manipulatives in mathematics instruction. Burns and Hamm (2010) examined concrete manipulatives compared to virtual manipulatives and their effectiveness with the use of fraction concepts in third-grade and symmetry concepts in fourth-grade. Their design was pre-tests and post-tests scores of 91 third-grade students and 54 fourth-grade students. The results of the post-tests suggested that there was no change on the students' performance by the lesson condition. Burns and Hamm

(2011) found that virtual manipulatives provide students with immediate feedback on the computer after the students have entered or moved a manipulative allowing the student to self-correct and assuring the student is getting the best practice when completing independent classwork or homework. In addition, another benefit that Burns and Hamm (2011) found was that the majority of virtual manipulatives are free of charge when using the internet and students could use the same virtual manipulatives from class at home for reinforcement of the mathematical lesson.

In a study with third graders, Reimer and Moyer (2005) explored the effects of using several virtual manipulative applets during fraction instruction. The participants from their study included 19 third-grade students. They participated for two weeks during a fraction unit and data was collected using pre-tests and post-tests of the students' conceptual knowledge. They also collected data using student interviews and a student attitude survey. The results of Reimer and Moyer's (2005) study suggested that there was a significant improvement in the students' scores from pre-tests to post-tests. The student interviews and attitude surveys indicated that virtual manipulatives were helpful when learning concepts, easier and faster than paper and pencil, and enjoyable for the students. Additionally Reimer and Moyer (2005) found benefits of using virtual manipulatives to teach fractions. Reimer and Moyer (2005) also found that virtual manipulatives allowed differentiated instruction where students worked at their own pace within their different ability levels.

A study conducted in Taiwan by Lin, Shao, Wong, Li, and Niramitranon (2011) examined a virtual Tangram puzzle to improve students' geometry skills in a collaborative learning environment. The researchers designed the collaborative Chinese Tangram activity that incorporated problem-solving learning strategies. Participants in this study included 25 sixthgraders from a suburb elementary school of Tia-Chung City. Data collection for this study consisted of pre-tests, post-tests, student questionnaires, and student interviews.

Lin et al (2011) suggested that the rotation and space of shapes from the Tangram puzzle improved students' scores. They stated that using a virtual Tangram manipulative "could reduce the gap between high-ability and low-ability students" (p. 256). Students' questionnaires and interviews indicated that their confidence was strengthened and they were motivated and interested in math after using the virtual tangram puzzle.

The majority of the studies on virtual manipulatives explored for this literature review showed gains in students' performance within mathematical concepts. In studies that noted benefits, they also noted drawbacks to the use of virtual manipulatives. Burns and Hamm (2010) noted one drawback being that virtual manipulatives required teachers to plan in advance for set up and equipment such lap tops with wireless Internet access. With the uses of computers and technology, there is always a drawback that is a concern to any teacher. When using virtual manipulatives, there is a risk of technology problems such as computers not working or low batteries and a risk of interrupted Internet on class computers and lap tops (Burns & Hamm, 2011; Reimer & Moyer, 2005).

Teachers may not be comfortable with the technology within classrooms or the applets used to provide the virtual manipulatives through the Internet (Reimer & Moyer, 2005). The NCTM (2000) noted that the effectiveness of technology in the class depends on the teacher's ability to use it well. Hunt et al., (2011) conducted a three-year study that looked at the effectiveness of virtual manipulatives in conceptual understanding of number concepts compared to concrete manipulatives with pre-service middle grade teachers. Seventy-eight middle grade teacher candidates participated in this study using various concrete and virtual manipulatives to study integers, fractions, and non-decimal-bases numbers.

These teacher candidates met for 75 minutes twice a week for their course. Qualitative data was collected using surveys. Their responses were compared and the results suggested that concrete manipulatives were easier to use and they were considered more helpful than virtual manipulatives. Additionally Hunt et al. (2011) noted perceived drawbacks of virtual manipulatives such as the students are not able to touch the manipulative, some of the applets do not provide instructions on how to begin the mathematical problem, and students cannot layer certain pieces to see how they are equivalent to each other.

The literature reviewed for this study has provided many possible benefits and some drawbacks of using virtual manipulatives. Studies reviewed used both qualitative and quantitative data collection. Some of the studies compared the effectiveness of virtual manipulatives when compared to concrete manipulatives. Many studies reviewed suggested improvements in students' performance on a variety of math concepts when using virtual manipulatives. After reviewing literature on virtual manipulatives, the literature and the following research question will guide this proposed study. What is the impact on mathematic performance for 4^{th} grade students' when teachers use virtual manipulatives?

Methodology

The research question that guided this study was: What is the impact on mathematic performance for 4^{th} grade students when teachers use virtual manipulatives? This study took place over four weeks. The treatment group completed practice activities after whole group math instruction using virtual manipulatives. One of the control groups completed practice activities after whole group math instruction using concrete manipulatives and the second control group completed practice activities after whole group math instruction using traditional worksheets. Participants were assessed using pre-tests and post-tests that were collected and analyzed to identify the impact that virtual manipulatives had on students' mathematic performance.

Setting

This study was conducted in a rural elementary school in Western New York State. The elementary school was comprised of 95% White, 1% American Indian or Alaska Native, 1% Black or African American, 1% Asian or Native Hawaiian/Other Pacific Islander, and 2% Multiracial. Approximately 34% of the students at the elementary level are eligible for free or reduced lunch (New York State Education Department Report Card, 2011). During this study, lessons were conducted in one fourth-grade classroom.

The fourth-grade classroom had six desktop computers and 20 laptops available, so every student was equipped with a computer or laptop for this unit. For this study, only one of the four fourth-grade classes participated. The class was split into three groups. One group was the treatment group and two were the control group. Consent was obtained from both students and their parents by the teacher assistant.

The study began in October and lasted for four weeks. The students and teacher had a classroom routine set and implemented by October, but the students had a fresh start to the fourth-grade math concepts. Having a fresh start using virtual manipulatives may encourage students to use this tool after the study has been completed. Students participated in a four week-long study during the math unit of adding and subtracting whole numbers. One teacher taught the three groups during the study to reduce teacher effects.

Participants

Participants of this study were fourth-grade students from one classroom. The treatment group consisted of four fourth-grade students and the control groups consisted of four fourth-grade students each. Twelve 4th grade students (6 F, 6 M) within one mathematic class at this rural school in western New York State participated in the study. Participants ranged in age from 9 to 10 years old. Two of the participants had Individualized Education Plans (IEPs). At the time of this study, students learned how to add and subtract whole numbers. There were two control groups and one treatment group where the treatment group practiced mathematical concepts with virtual manipulatives. The one control group practiced mathematical concepts with paper and pencil worksheets provided by the McGraw-Hill math textbook and the other control group practiced mathematical concepts with concrete manipulatives. Formal data was only collected from the students who provided individual and parental consent.

Design

One regularly scheduled math class met each day for 50 minutes. The general education teacher taught whole class instruction on adding and subtracting whole numbers to the entire class. The purpose of this study was to examine the impact virtual manipulatives have on 4th grade mathematic performance. These virtual manipulatives were compared to typical mathematic instruction guided by the school district's math text, McGraw-Hill a national text publisher. The Common Core State Standards are the foundation of the math text.

Students were given a week to bring back consent forms from parents. Once consent forms were collected, the math teacher gave students in both control groups and treatment group a pre-test for the adding and subtracting whole numbers unit. Testing accommodations were provided to students with testing accommodations on their IEPs. The following day after the pre-test was completed, all three groups of students received instruction as outlined in the McGraw-Hill *My Math* grade 4 volume 1, a nationally published math textbook. The objectives that were covered in this unit include: addition properties, subtraction rules, addition and subtraction patterns, estimate sums and differences, adding whole numbers, subtracting whole numbers, and solving multi-step word problems.

Students in the control group 1 used their student textbooks during whole group instruction and corresponding practice worksheets for independent practice during the entire study. Students in the control group 2 used their student textbooks during whole group instruction and concrete manipulatives for independent practice. The students in the treatment group used the same student textbooks during whole group instruction, but instead of using practice worksheets or concrete manipulatives, students used virtual manipulatives for independent practice from the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) and Illuminations (http://illuminations.nctm.org/). If a virtual manipulative was not available from either website on a specific objective, the students practiced the math concepts on the *My Math* website aligned with the *My Math* textbook published by McGraw-Hill that provides virtual manipulatives for each unit. The students in the treatment group used the classroom computers and laptops that were connected to the Internet to complete virtual manipulative activities.

Data Collection

This study collected quantitative data. The data collected from this study were the test scores from the student participants. The scores were collected from both control groups and treatment group students. During the first class of the unit, students from all three groups completed the same pre-test prior to instruction, provided by the publishers of the textbook. The pre-test contained 10 questions that addressed all the objectives of the unit.

Following the instruction and practice activities, students in the three groups were given a post- test, provided by the publishers of the textbook. Testing accommodations were provided on the post-test to students with testing accommodations on their IEPs. The post-test contained ten questions that addressed all objectives from the unit. For the tests, students in the treatment group completed the test just as the control groups, without using computers.

Pre-and post-test scores and answers from the post-tests were graded by the teacher and given to the investigator. The investigator kept these unidentifiable scores and answers on an excel worksheet on the investigator's personal laptop that was protected by a secure password and the hard copy test scores were kept in a locked filing cabinet with no identifying information. The scores collected were compared between control group 1, control group 2, and the treatment group. The pre- and post-test scores in each group were compared for each student. This showed if virtual manipulatives during this unit helped impact 4th grade math performance. The answers collected from the post-tests were tallied to see what answers were given by students if they answered the problem incorrectly. The answers showed how many answered a question incorrectly and if there were commonalities between the students' incorrect answers. This allowed the researcher to diagnose where the students' understandings failed. After the study was concluded, the data remained in the locked filing cabinet and password secured laptop. Three year from the conclusion of the study, data will be destroyed and shredded.

Data Analysis

The scores collected from control group 1, control group 2, and the treatment group were calculated to summarize the data. Data was analyzed using mean and standard deviation. The post-test answers were tallied to analyze the number of questions that were answered incorrectly and to compare the incorrect responses to the correct answer to see where students struggled.

Using the pre- and post-test scores, an ANOVA was conducted to examine changes from pre- to post-tests.

Limitations

There were a few limitations inherent in this study design. The population of students receiving instruction in math was limited to a small sample size consisting of fourth-grade students. The sample size consisted of 12 fourth-grade students. The population of fourth-grade students was also limited to the students being enrolled from one elementary school. Small sample size and the students being enrolled from a single elementary school limit the ability to make generalizations about the effects virtual manipulatives may have on other fourth-grade students in other classrooms or schools. Keeping these limitations in mind, the researcher analyzed and interpreted students' pre-tests and post-test.

Findings

The data collected from the study was analyzed and organized into tables. An ANOVA was conducted to analyze the results between the pre-tests and post-test of the twelve participants. There was also a student response tally sheet that tallied the number of incorrect responses. The incorrect responses were analyzed to interrupt the responses compared to the correct answers on the post-test. When reviewing and analyzing the data, there were three major findings which helped answer the guiding question for this study.

Group Findings

The mean pre-test and post-test math scores of the fourth grade students are reported in Table 1. When looking at the data of the groups using the virtual manipulatives, concrete manipulatives, and no manipulatives all showed improvement from the pre-test to the post-test scores. The virtual manipulative group had a mean of 23.3 on pre-test scores whereas the concrete manipulative group had a mean of 36.5 and no manipulatives group had a mean of 53.8 on the pre-test scores. The no manipulatives group had a much higher mean due to the fact that two of the students scored two of the highest scores on the pre-test with a 62 % and a 69%. Each individual group made improvement between the pre-test and post-test scores.

The group that used the virtual manipulatives showed the most improvement compared to the other two groups. Additionally, there was a significant difference within improvement between the virtual and concrete manipulatives pre-test and post-test scores. Pre-test and post-test for the virtual manipulative increased 52 points and for the concrete manipulatives there was a 40 point increase between the pre-test and post-test. On the post test, the virtual manipulative group received a mean of 75.5, the concrete manipulative group received a mean of 76.8, and the no manipulative group received a mean of 79. The data showed there was significant difference within improvement between the virtual manipulative group and the no manipulative group when comparing their pre-test and post-test scores.

Table 1.

Group Findings

Group	**Pre-test Mean (SD)**	**Post-test Mean (SD)**	**Difference between Pre-test/ Post-test**
Virtual Manipulatives	23.25 (9.93)	75.25 (13.48)	52
Concrete Manipulatives	36.5 (14.92)	76.75 (16.89)	40.25
No Manipulatives	53.75 (12.34)	79 (6.63)	25.25

*SD= Standard Deviation

Students' incorrect responses were tallied from the post-tests to show the number of students that answered each question incorrectly and their incorrect answers were compared to the correct answers as shown in Table 2.

Table 2.

Student Incorrect Responses on the Post Test

Math Problem	Number of students who answered incorrectly	Student incorrect answer	Correct answer (provided by My Math)
Write each number.			
4. 1,000 less than 27,421	I	26,417	26,421
Estimate each sum or difference.			
6. 772 – 334 =	IIIIIIIIII	438; 1,100	400 or 500 **sample answers**
7. 923 + 75 =	IIIIIIIIII	998; 849; 1,700	980 or 1, 000 **sample answers**
Find each sum or difference.			
8. 251,905 + 44,590 =	III	207,315; 246,495	296,495
9. 5,000 – 3,139 =	IIII	1,681; blank response; 2,139; 1,860	1,861
Write an equation using a variable for the unknown.			
10. Amelia had $10. She bought a banana for $2 and a carton of milk. She now has $7. How much did the carton of milk cost?	III	m=$8; m=$0; m=$17	$10 - $2 – m = $7; m= $1

One finding from the post-tests incorrect responses in Table 2 concerned question number 9 where the students had to find the difference. This problem required the students to subtract across zeros. There were four out of the twelve students who answered this question incorrectly. When looking at the post-test, the incorrect responses were given by four students who came from all three groups. One of the four was from the virtual manipulative, two students from the concrete manipulatives, and one from the traditional worksheets. An interesting finding was that half of the concrete manipulative participants answered this question incorrectly. In addition to each group improving between the pre-tests and post-tests, each student within the three groups' improved between their pre-tests and post-tests scores.

Individual Findings

When looking at the students' individual performance on the pre-test and post-test as seen in Table 3, the students in the no manipulative group had the higher scores ranging from 38.0 to 69.0 on the pre-test out of all twelve students. Two of the students in this group had the highest scores out of all twelve students. As shown in Table 3, all of the twelve students across the three groups had an increase between the pre-test and post-test scores. Due to the no manipulative group having higher pre-test scores, the improvement was not as large as the groups using virtual manipulatives and concrete manipulatives.

Table 3.

Individual Findings

Student	Pre-test Score	Post-test Score	Mean	Difference between Pre-test/ Post-test
1	23%	62%	43	39
2	31%	92%	62	61
3	46%	85%	66	39
4	8%	62%	35	54
5	15%	54%	35	39
6	31%	85%	58	54
7	62%	69%	66	7
8	38%	77%	58	39
9	69%	85%	77	16
10	31%	69%	50	38
11	46%	85%	66	39
12	54%	99%	77	45

As shown in Table 2, students' incorrect responses were tallied from the post-tests to show the number of students who answered which question incorrectly. Questions number six and seven asked students to estimate the difference of each sum or difference. Out of the 12 participants, 10 of the students answered number six incorrectly. When looking at their responses, the majority of the students did not follow the directions and they found the actual difference not the estimated difference. This was similar for question number seven, where 10 of the students answered number seven incorrectly. As with question number six all the students who answered number seven incorrectly did not follow the directions. They found the actual sum of the problem not the estimated sum. However, even though the question was marked as incorrect due to the students not following the directions, the answers provided were solved correctly. One interesting finding was that when looking at the pre-tests, all ten of the students who had not estimated on the post-test, had estimated the sum or difference for numbers six and seven on the pre-tests.

Students' with Disabilities Findings

Out of the twelve students who participated in this study, two of the students have Individualized Education Plans (IEPs) but are not serviced in math. However the pre-test and post-test were read to both students. The two students with IEPs were both in the virtual manipulative group based on random selection. As shown in Table 4, student 1 received a 23% on the pre-test and a 62% on the post-test, this was a 39 point increase between the tests. Student 2 received a 8% on the pre-test and a 62% on the post-test, this was a 54 point increase between the tests. The 54 point increase between the pre-test and post-test scores was the second largest gain among all twelve students. The 39 point gain was the third largest gain between pre-test and post-test scores. Out of the twelve students, five of them had a 39 point gain between the test scores.

Table 4.

Special Education Findings

Student	Pre-test Score	Post-test Score	Mean	Difference between Pre-test/ Post-test
1	23%	62%	43	39
4	8%	62%	35	54
5	15%	54%	35	39

A third student in the concrete manipulative group receives academic intervention services in mathematics and is being evaluated for a disability. As shown in Table 4, student 5 received a 15% pre-test score and a 54% post-test score. Between the pre-test and post-test the student gained 39 points.

The ANOVA results showed that there was a statistically significant difference between the three groups pre-test scores p= 0.043, where p=0.01 indicates statistically significant difference. The results also showed that there was a statistically significant difference between the three groups post-test scores p=0.940.

Discussion

The guiding question for this study was: what is the impact on mathematic performance for 4th grade students' when teachers use virtual manipulatives? As the results show, the students in all three groups showed improvement between the pre-test and post-test on the operation of adding and subtracting whole numbers. The instruction using virtual and concrete manipulatives and worksheets all positively impacted the 4th grade mathematical performance. However, virtual manipulatives had the greatest impact as evidenced by student gains among all groups between pre and post test scores.

Interpreting Findings

The findings of this study revealed that virtual manipulatives had a positive impact on the 4th grade students' mathematical performance. The findings of the differences between the pre-test and post-tests indicate that virtual manipulatives and concrete manipulatives can reinforce math concepts in addition and subtraction of whole numbers consisting of three to six digit whole numbers. An indirect finding indicated was that combinations of the two types of manipulatives can reinforce mathematical concepts in addition and subtraction of whole numbers.

As mentioned in a study conducted by Reimer and Moyer (2005), virtual manipulatives allowed differentiated instruction and enabled students to work at their own pace within their ability levels. In this study, the opportunity to differentiate the math instruction was beneficial for the students with IEP's and the student who was receiving academic intervention services (AIS) in mathematics in both the virtual manipulative and concrete manipulative groups. Another explanation for the findings is that the teacher who participated in the study is an excellent instructor who has a strong educational background in mathematics and his pedagogy helped students to learn the mathematical concepts on top of the instructional tools used during the mathematic instruction.

The findings showed that there was not a major significant difference between the virtual and concrete manipulative groups' pre-test and post-test means. However, the use of virtual manipulatives was a very effective instructional tool for the treatment group, more effective than the paper and pencil activities used by the non-manipulative control to practice addition and subtraction. The positive gains of the students using virtual and concrete manipulative as evidenced in their pre-test and post-test scores confirm the findings in other studies that show positives gain with the use of virtual manipulatives (Moyer-Packenham & Suh, 2012; Reimer & Moyer, 2005; Moyer-Packenham, Ulmer, & Anderson 2012).

Using virtual manipulatives was preferred to concrete manipulatives by the participating teacher in this study because it was easier to clean up and did not take a long time to set up. This was similar to the finding noted in an article by Moyer, Bolyard, & Spikell (2002) where they mentioned the management of the computer manipulatives was easier and thus more beneficial. When working with manipulatives, the best manipulative to use with students is a manipulative that the teacher will use on a regular basis and is more time efficient. The ease of application of the virtual manipulatives may make teachers more willing to incorporate manipulatives leading to increased time of use by students. Students may use virtual manipulatives for longer periods of time and use them on more frequent occasions in school and/or at home.

In this study, the group using virtual manipulatives improved slightly more than the group using concrete manipulatives on the test scores, which contradicts the findings from the fourth-grade study conducted my Burns and Hamm (2011). The Burns and Hamm (2011) study found that between their fourth-grade pre-test and post-test scores, the students' had larger gains in the concrete manipulative group. This study also contradicted what Steen, Brooks, and Lyon (2006) found which was that "virtual manipulatives were as effective as the traditional text activities" (p.386). The virtual manipulative group had larger gains between the pre-test and post-test scores than the non-manipulative group. However, the non-manipulative group had higher pre-test scores; two of the students did not show a huge gain when looking at the post-test scores. The other two students had an average gain between the test scores when compared to the other groups.

All three groups, the treatment and two control groups, improved in their mathematical performance between the pre-tests and the post tests on whole number addition and subtraction. When working with students' with a wide range of ability level, a variety of instructional tools can help the teacher differentiate instruction to help students succeed in mathematics. The use of virtual manipulatives, concrete manipulatives, and traditional worksheets combined can positively impact mathematical performance. However, the present study showed that the greatest impact on students' mathematical performance is the use of virtual manipulatives or concrete manipulatives or the combination of both. The use of manipulatives can address all ability levels within the classroom.

Limitations

There were many limitations to this study that could impact the data. One limitation was the sample size consisting of 12 fourth-grade students from one rural elementary school. Another limitation found was that with system updates within the school district, some of the virtual manipulatives took longer to retrieve from the websites so students wasted time waiting for virtual manipulatives instead of practicing as those in the concrete and non-manipulative groups were. One last limitation of this study was that there were only two students with Individualized Education Plans (IEPs) who participated in this study. In this study, the virtual manipulatives did impact the two students with IEPs positively. Nevertheless, a study with more students with IEPs may show different results. If these limitations are addressed in future research then the study may have results that contradict or confirm this studies results.

Implications for Practice

There are many implications for practice when talking about virtual manipulatives in elementary schools. School administrators can encourage teachers at the elementary level to use virtual manipulatives during mathematic instruction. They can also provide teachers with professional development to help teachers explore virtual manipulative websites and learn how to address technological issues that they may encounter when using computer or other technology with virtual manipulatives. Professional development can also be provided for teachers in how to integrate technology across the mathematics curriculum

There are many implications for practice for elementary teachers. As a result of this study, teachers can use either virtual manipulatives and/or concrete manipulatives as an effective instructional tool to help improve students' mathematic performance. These instructional tools can support students who benefit from visualizations and can help develop students' visualization skills. Teachers can introduce or increase the use of virtual manipulatives to practice and model a variety of mathematic concepts. Having students visualize the relationships in math and make connections using manipulatives can help low achieving students' visualize the mathematical representations within the lessons.

Teachers can incorporate virtual manipulatives in a number of ways within their classrooms. They can incorporate them into lessons as this study did where the virtual manipulative is used as a practice after the concept is taught. Another option is teachers can have a virtual manipulative center during their math centers within their classrooms. They also can use the manipulative to introduce and teach the concept, to allow the teacher to model how to use the manipulative and have a meaningful discussion to expand students' visualization understanding. These implications for practice are suggestions for administrators and teachers on how to use and incorporate virtual manipulatives in to elementary schools and classrooms based on the study conducted.

Another implication for practice is for elementary school parents. Parents who have children in classrooms using virtual manipulatives can have their child access the same free resources at home for further practice and reinforcement of the mathematic content taught that day or to review previous content taught. Allowing students to continue their learning and practice at home may be a motivational tool to help student complete homework. These free resources available online can help parents support their child's learning if they do not understand or have difficulty supporting their child with homework. One downside of this implication for practice is that not all households are equipped with technology such as computers, tablets, or ipads especially if the district is located in a low socio economic status. However, the increase in public technology available at libraries and the almost ubiquitous use of smart phones could enable students to access virtual manipulative applications outside of school.

Further Research

Further research should examine the teacher and students' perceptions of the use of virtual manipulatives. Certain perceptions can impact the students' performance when using manipulatives. Interviews could be used to further reveal students' and teachers' perceptions of the practice of using virtual manipulatives and to probe the comfort level of the teacher with technology. In addition, research characterized by observations to see the implementation of the virtual manipulatives could be conducted. Another suggestion for further research is to increase the sample size and conduct the study in different schools across socio-economic statuses to see if the results would be similar or different.

Student surveys could be implemented within the study to gather students' opinions on the use of virtual manipulatives during mathematic instruction. Additionally, qualitative data could be collected based on the feedback of students and teachers using virtual manipulatives.

This could help teachers gear the use of virtual manipulative that will be beneficial to student performance and developers of virtual manipulatives could use the feedback to create more effective manipulatives that will benefit teaching and learning of mathematical content.

Further research can look at how virtual manipulatives can be incorporated into the new curriculum modules. These modules are very specific on how to teach a lesson and further research can explore if virtual manipulatives can be used within these modules and how they impact students learning when paired with a guided lesson plan. Teachers are constantly reflecting on and evaluating their instruction to help students succeed. This study and further research can help teachers reflect and evaluate on their mathematic instructional practices.

References

Allen, C. (2007). An action based research study on how using manipulatives will increase students' achievement in mathematics. Retrieved from http://files.eric.ed.gov/fulltext/ED499956.pdf

Blair, N. (2012). Technology integration for the new 21st century learner. *National Association of Elementary School Principles,* 8-13. Retrieved from http://www.naesp.org/principal-januaryfebruary-2012-technology/technology-integration-new-21st-century-learner

Boaler, J. (2001). Mathematical modelling and new theories of learning. *Teaching Mathematics and its Applications. 20(3),* 121-128 doi: 10.1093/teamat/20.3.121

Boggan, M., Harper, S., & Whitmire, A. (2010). Using manipulatives to teach elementary mathematics. Retrieved from http://www.aabri.com/manuscripts/10451.pd

Burns, B.A., & Hamm, E.H. (2011). A comparison of concrete and virtual manipulative use in third- and fourth-grade mathematics. *School Science and Mathematics, 111(6),* 256-261 doi: 10.1111/j.1949-8594.2011.00086.x

Carbonneau, K., Marley, S., & Selig, J. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of education Psychology, 105(2),* 380-400. doi: 10.1037/a0031084

CEO Forum. (2001*). Key building blocks for students achievement in the 21st century*. Washington, DC: CEO Forum on Education & Technology.

Durmus, S. & Karakirik, E. (2006). Virtual manipulatives in mathematics education: A theoretical framework. *The Turkish Online Journal of Education Technology, 5(1),* 117-123. Retrieved from http://wwwtojet.net/articles/v5i1/5112.pdf

Engage NY: Our Students. Their Moment. (2012). Retrieved from http://www.engageny.org/

Gray, L., Thomas, N., and Lewis, L. (2010).Teachers' Use of Educational Technology in U.S. Public Schools: 2009 (NCES 2010-040). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.

Huang, T.-H., Liu, Y.-C., & Chang, H.-C. (2012). Learning achievement in solving word-based mathematical questions through a computer-assisted learning system. *Educational Technology & Society, 15 (1),* 248–259. http://www.ifets.info/journals/15_1/22.pdf

Hunt, A. W., Nipper, K. L., & Nash, L. E. (2011). Virtual vs. concrete manipulatives in mathematics teacher education: Is one type more effective than the other?. *Current Issues in Middle Level Education.*16(2).Retrieved from http://www.napomle.org/cimle/fall2011/Hunt.pdf

Hwang, W.-Y., Su, J.-H., Huang, Y.-M., & Dong, J.-J. (2009). A study of multi-representation of geometry problem solving with virtual manipulatives and whiteboard system. *Educational Technology & Society, 12 (3),* 229–247. http://www.ifets.info/others/abstract.php?art_id=970

Illuminations: Resources for Teaching Math (2013). Retrieved April 21, 2013, from http://illuminations.nctm.org/

Lin, C.-P., Shao, Y.-J., Wong, L.-H., Li, Y.-J, Niramitranon, J. (2011). The impact of using synchronous collaborative virtual tangram in children's geometric. *Turkish Online Journal of Educations Technology, 10 (2),* 250-258. http://www.eric.ed.gov/PDFS/EJ932243.pdf

Moyer, P. S., Bolyard, J. J., &Spikell, M. A. (2002). What are virtual manipulatives. *The National Council of Teachers of Mathematics.* Retrieved from http://www.grsc.k12.ar.us/mathresources/instruction/Manipulatives/Virtual Manipulatives.pdf

Moyer-Packenham, P.S., Salkind, G. &Bolyard, J.J. (2008). Virtual manipulatives used by K-8 teachers for mathematics instruction: The influence of mathematical, cognitive, and pedagogical fidelity. *Contemporary Issues in Technology and Teacher Education, 8*(3), 202-218. http://www.editlib.org/p/26057.

Moyer-Packenham, P.S. &Suh, J. (2012). Learning mathematics with technology: The influence of virtual manipulatives on different achievement groups. *Journal of Computers in Mathematics and Science Teaching, 31(1),* 39-59. http://www.editlib.org/p/37643.

Moyer-Packenham, P.S., Ulmer, L.A.,& Anderson, K.L. (2012). Examining pictorial models and virtual manipulatives for third-grade fraction instruction. *Journal of Interactive Online Learning*, 11 (3), 103-120. http://www.ncolr.org/jiol/issues/pdf/11.3.2.pdf

National Council of Teachers of Mathematics. (2013). Retrieved April 21, 2013, from http://www.nctm.org/

National Governors Association Center for Best Practices. (2010). *Common core state standards initiative*. Retrieved from http://www.corestandards.org/the-standards

National Library of Virtual Manipualtives. (2010). Retrieved April 21, 2013, from http://nlvm.usu.edu/

Nctm standards. (2000). *National Council of Teachers of Mathematics*. Retrieved from http://www.nctm.org/

New York, NY: McGraw-Hill.

New York State Education Department (NYSED). (2011). Retrieved from https://reportcards.nysed.gov/

Polly, D. (2011). Technology to develop algebraic reasoning. *National Council of Teachers of Mathematics.*17(8).http://www.nctm.org/eresources/article_summary.asp?URI=TCM2011-04-472a&from=B

Public Schools: 2009 (NCES 2010-040). National Center for Education Statistics, Institute of Education Sciences, *U.S. Department of Education*. Washington, DC.

Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2010).Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7). doi: 10.1111/j.1949-8594.2008.tb17844.x

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching*, *24(1).*5-25.

Steen, K., Brooks, D. & Lyon, T. (2006).The impact of virtual manipulatives on first grade geometry instruction and learning. *Journal of Computers in Mathematics and Science Teaching*, *25*(4), 373-391. http://www.editlib.org/index.cfm?fuseaction=Reader.ViewAbstract&paper_id=19812

Suh, J. M., & Moyer-Packenham, P. S. (2008). Scaffolding special needs students' learning of fraction equivalence using virtual manipulatives. *Proceedings of the International Group for the Psychology of Mathematics Education*, 4. Retrieved from http://mason.gmu.edu/~jsuh4/presentations/01.15.08RR.SuhMoyer2.pdf

Uribe-Flórez, L. J. and Wilkins, J. L. M. (2010), *Elementary School Teachers' Manipulative Use. School Science and Mathematics*, 110 (7). doi: 10.1111/j.1949-8594.2010.00046.x

Appendices

Appendix A
Chapter 3 Pretest
McGraw-Hill *My Math* grade 4 volume 1

Name ______________________________ Date ____________________

Pretest

Find each unknown. Write each property or rule that is used.

1. 12 + 13 = 13 + ☐ **1.** ____________________

2. 45 – ☐ = 0 **2.** ____________________

3. 11 + (☐ + 2) = (11 + 9) + 2 **3.** ____________________

Write each number.

4. 1,000 less than 27,412 **4.** ____________________

5. 100 more than 823,645 **5.** ____________________

Estimate each sum or difference.

6. 772 – 334 **6.** ____________________

7. 923 + 75 **7.** ____________________

Find each sum or difference.

8. 251,905 + 44,590 **8.** ____________________

9. 5,000 – 3,139 **9.** ____________________

Write an equation using a variable for the unknown.

10. Amelia had $10. She bought a banana for $2 and a carton of milk. She now has $7. How much did the carton of milk cost? **10.** ____________________

Appendix B
Chapter 3 Posttest
McGraw-Hill *My Math* grade 4 volume 1

Name ______________________ Date ______________

Posttest

Find each unknown. Write each property or rule that is used.

1. 12 + 13 = 13 + ☐ **1.** ______________

2. 45 – ☐ = 0 **2.** ______________

3. 11 + (☐ + 2) = (11 + 9) + 2 **3.** ______________

Write each number.

4. 1,000 less than 27,412 **4.** ______________

5. 100 more than 823,645 **5.** ______________

Estimate each sum or difference.

6. 772 – 334 **6.** ______________

7. 923 + 75 **7.** ______________

Find each sum or difference.

8. 251,905 + 44,590 **8.** ______________

9. 5,000 – 3,139 **9.** ______________

Write an equation using a variable for the unknown.

10. Amelia had $10. She bought a banana for $2 and a carton of milk. She now has $7. How much did the carton of milk cost? **10.** ______________