Semirelativistic potential model for heavy quarkonia

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The $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ spectra are investigated with the use of a semirelativistic potential model described in an earlier paper. Results for the energy levels, leptonic widths, and $E1$ transition widths are compared with the experimental data for $c\bar{c}$ and $b\bar{b}$ and predicted for $t\bar{t}$. We also find that the quark-antiquark interaction can best be described by a quasistatic rather than a momentum-dependent potential, and propose a theoretical justification for this surprising conclusion.

I. INTRODUCTION

Recently we described a semirelativistic potential model[1] for quarkonia to improve upon the more commonly used nonrelativistic models. We found that the semirelativistic treatment considerably differs from the nonrelativistic treatment[2] for $c\bar{c}$, while the difference between the two treatments is less significant for $b\bar{b}$. We, therefore, provided results only for the $c\bar{c}$ system.

Spectroscopy of heavy quarkonia is particularly suitable for a confrontation of quantum chromodynamics with the experimental data. Therefore, we have now carried out a more rigorous investigation of the $c\bar{c}$ and $b\bar{b}$ systems with the use of the semirelativistic treatment, and we have also extended our treatment to $t\bar{t}$ in view of the current enhanced interest in the top quark.[3] As in our earlier paper, we have used a quark-antiquark potential consisting of a perturbative part, which includes the complete one-loop radiative correction to the one-gluon-exchange interaction, and a linear scalar-exchange confining part. Moreover, in order to clarify the role of momentum dependence in the quark-antiquark potential, we have explored both the quasistatic and the momentum-dependent forms of our potential, and our conclusions are interesting as well as unexpected.

Besides giving our results for the $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ spectra in Secs. II–IV, we discuss the correlation of quarkonium parameters in Sec. V and compare the quasistatic and momentum-dependent potentials in Sec. VI. Our conclusions and the significance of our results are summarized in Sec. VII.

II. $c\bar{c}$ SPECTRUM

Our semirelativistic model[1] is based on a Hamiltonian of the form

$$\mathscr{H}=2(m^2+\mathbf{p}^2)^{1/2}+\mathscr{V}_p(\mathbf{r})+\mathscr{V}_c(\mathbf{r}) , \tag{2.1}$$

where $\mathscr{V}_p$ and $\mathscr{V}_c$ are the perturbative and the confining potentials. It should be noted that while this Hamiltonian includes the relativistic kinetic energy of the system, both $\mathscr{V}_p$ and $\mathscr{V}_c$ represent nonrelativistic potentials, which will be discussed in Sec. VI.

The mathematical formalism required for obtaining the quarkonium energy levels and wave functions with the semirelativistic treatment is fully described in Ref. 1. Our revised results[4] for the $c\bar{c}$ energy levels below the charm threshold as well as the values of the parameters are given[5] in Table I. The splittings of the energy levels are

$$\begin{aligned} &M(\psi')-M(\psi)=589 \text{ MeV} ,\\ &M(\psi)-M(\eta_c)=116 \text{ MeV} ,\\ &M(\psi')-M(\eta_c')=96 \text{ MeV} ,\\ &M(\chi_{\text{c.o.g.}})-M(\psi)=429 \text{ MeV} , \end{aligned} \tag{2.2}$$

TABLE I. $c\bar{c}$ spectrum with $m_c=1.32$ GeV, $\mu=1.94$ GeV, $\alpha_s=0.36$, and $A=0.15$ GeV^2. Theoretical and experimental masses are given in MeV.

State	Mass	Mass (expt)	State	Mass	Mass (expt)
$1\,^3S_1(\psi)$	3097	3097	$2\,^3P_2(\chi_2)$	3558	3556±1
$1\,^1S_0(\eta_c)$	2981	2981±6	$2\,^3P_1(\chi_1)$	3510	3510±1
			$2\,^3P_0(\chi_0)$	3414	3415±1
$2\,^3S_1(\psi')$	3686	3686	$2\,^1P_1$	3528	
$2\,^1S_0(\eta_c')$	3590	3594±5			

TABLE II. $c\bar{c}$ leptonic widths in keV without and with radiative correction.

State	$\Gamma_{ee}^{(0)}$	Γ_{ee}	Γ_{ee} (expt)
$1S$	7.17	4.47	4.6 ± 0.4
$2S$	4.12	2.57	2.0 ± 0.2

$$M(\chi_2)-M(\chi_1)=48 \text{ MeV} ,$$
$$M(\chi_1)-M(\chi_0)=96 \text{ MeV} ,$$

which are in excellent agreement with experiments.[6]

The leptonic widths, obtained with the use of the formula[7]

$$\Gamma_{ee}^{(0)}(^3S_1\to e^+e^-)=\frac{16\pi\alpha^2 e_Q{}^2}{M^2(Q\bar{Q})}\,|\,\psi(0)\,|^2 , \tag{2.3}$$

are given in Table II, and they are larger than the experimental values. It is possible to reduce them by including the radiative correction, which leads to the modification of (2.3) to[8]

$$\Gamma_{ee}=\Gamma_{ee}^{(0)}(1-16\alpha_s/3\pi+\cdots) . \tag{2.4}$$

However, the lowest-order correction term in (2.4) is so large that the unknown higher-order terms evidently cannot be neglected. We have found that if we assume the higher-order corrections to be such that (2.4) becomes

$$\Gamma_{ee}=\frac{\Gamma_{ee}^{(0)}}{1+16\alpha_s/3\pi} , \tag{2.5}$$

our results, shown in Table II, are in good agreement with experiments.[6,9]

Our theoretical $E1$ transition widths

$$\Gamma_{E1}(^3S_1\to {}^3P_J)=\frac{4}{9}\frac{2J+1}{3}\alpha e_Q{}^2 k_J{}^3\,|\,r_{fi}\,|^2 ,$$
$$\Gamma_{E1}(^3P_J\to {}^3S_1)=\tfrac{4}{9}\alpha e_Q{}^2 k_J{}^3\,|\,r_{fi}\,|^2 , \tag{2.6}$$

are given in Table III. They are about twice as large as the experimental values,[10] which may indicate[11] the inadequacy of the unperturbed wave functions for the treatment of $E1$ transitions in $c\bar{c}$.

III. $b\bar{b}$ SPECTRUM

The S and P levels of $b\bar{b}$ below the bottom threshold, obtained with the use of the semirelativistic model, are given in Table IV, while the leptonic widths, the matrix elements $\langle P\,|\,r\,|\,S\rangle$, and the $E1$ transition widths are given in Tables V–VII. We also note that, according to Table IV,

$$\begin{aligned}
&M(\Upsilon')-M(\Upsilon)=553 \text{ MeV} ,\\
&M(\Upsilon'')-M(\Upsilon)=896 \text{ MeV} ,\\
&M(\chi_{b,\text{c.o.g.}})-M(\Upsilon)=441 \text{ MeV} ,\\
&M(\chi_{b2})-M(\chi_{b1})=16 \text{ MeV} ,\\
&M(\chi_{b1})-M(\chi_{b0})=25 \text{ MeV} ,\\
&M(\chi'_{b,\text{c.o.g.}})-M(\Upsilon)=800 \text{ MeV} ,\\
&M(\chi'_{b2})-M(\chi'_{b1})=14 \text{ MeV} ,\\
&M(\chi'_{b1})-M(\chi'_{b0})=21 \text{ MeV} ,
\end{aligned} \tag{3.1}$$

and

$$\frac{M(\chi_{b2})-M(\chi_{b1})}{M(\chi_{b1})-M(\chi_{b0})}=0.64 ,\qquad \frac{M(\chi'_{b2})-M(\chi'_{b1})}{M(\chi'_{b1})-M(\chi'_{b0})}=0.67 . \tag{3.2}$$

The only striking difference between the semirelativistic results and the earlier nonrelativistic results[2] for $b\bar{b}$ is that the semirelativistic model yields larger values for wave functions at the origin. This leads to larger hyperfine splittings, and it also becomes necessary to use the leptonic width formula with radiative correction in the form (2.5) to obtain reasonable theoretical values.

Considering the fact that we are dealing with strong interactions, the overall agreement between the theoretical and experimental results[6,12] is gratifying.

IV. $t\bar{t}$ SPECTRUM

There is some indication that the mass of the top quark is in the range $30\le m_t\le 50$ GeV, and we give the low-lying S and P energy levels of $t\bar{t}$ for $m_t=40$ and 45 GeV in Tables VIII and IX. For $m_t=40$ GeV, we also give the

TABLE III. $E1$ transition widths for $c\bar{c}$ in keV.

Transition	$\langle P\,|\,r\,|\,S\rangle$ (GeV^{-1})	J	Γ_{E1}	Γ_{E1} (expt)
$2S\to 2P_J$	-2.54	2	32.0	17 ± 5
		1	50.5	19 ± 5
		0	62.0	21 ± 6
$2P_J\to 1S$	2.15	2	652.7	330 ± 170
		1	468.1	<700
		0	212.5	97 ± 38

TABLE IV. $b\bar{b}$ spectrum with $m_b=4.78$ GeV, $\mu=3.65$ GeV, $\alpha_s=0.28$, and $A=0.18$ GeV^2. Theoretical and experimental masses are given in MeV.

State	Mass	Mass (expt)	State	Mass	Mass (expt)
$1\,^3S_1(\Upsilon)$	9460	9460	$2\,^3P_2(\chi_{b2})$	9910	9913 ± 1
$1\,^1S_0(\eta_b)$	9416		$2\,^3P_1(\chi_{b1})$	9894	9893 ± 1
			$2\,^3P_0(\chi_{b0})$	9869	9865 ± 2
$2\,^3S_1(\Upsilon')$	10 013	10 023	$2\,^1P_1$	9901	
$2\,^1S_0(\eta_b')$	9987				
			$3\,^3P_2(\chi_{b2}')$	10 268	$10\,271\pm5$
$3\,^3S_1(\Upsilon'')$	10 356	10 356	$3\,^3P_1(\chi_{b1}')$	10 254	$10\,254\pm3$
$3\,^1S_0(\eta_b'')$	10 336		$3\,^3P_0(\chi_{b0}')$	10 233	$10\,233\pm3$
			$3\,^1P_1$	10 260	

leptonic widths, the matrix elements $\langle P\,|\,r\,|\,S\rangle$, and the $E1$ transition widths in Tables X, XI, and XII.

According to Table VIII, the splittings among the S and P energy levels, for $m_t=40$ GeV, are

$$\begin{aligned}
&M(2\,^3S_1)-M(1\,^3S_1)=755\text{ MeV}\,,\\
&M(3\,^3S_1)-M(2\,^3S_1)=299\text{ MeV}\,,\\
&M(4\,^3S_1)-M(3\,^3S_1)=206\text{ MeV}\,,\\
&M(2\,^3S_1)-M(2\,^3P_{\text{c.o.g.}})=64\text{ MeV}\,,\\
&M(3\,^3S_1)-M(3\,^3P_{\text{c.o.g.}})=48\text{ MeV}\,,\\
&M(4\,^3S_1)-M(4\,^3P_{\text{c.o.g.}})=43\text{ MeV}\,,
\end{aligned}\tag{4.1}$$

which show that the $1S$ level is well separated from all other levels. The $1S$ state also differs from other states with regard to spin splitting since its hyperfine splitting is

$$M(1\,^3S_1)-M(1\,^1S_0)=32\text{ MeV}\,,\tag{4.2}$$

while the hyperfine and fine-structure splittings of other states are from 3 to 9 MeV. The changes in the energy-level splittings when m_t increases from 40 to 45 GeV can be seen by comparing Tables VIII and IX.

It is hoped that $t\bar{t}$ might provide a sensitive test for the validity of various potential models at short range, and it is interesting to compare our results with those obtained by others with the use of different potential models. We note that our energy-level splittings and $E1$ transition widths are considerably smaller than those obtained recently by Moxhay and Rosner,[13] while our leptonic widths are in reasonable agreement with theirs. Our energy-level splittings are also smaller than those of Buchmüller and Tye[14] corresponding to $\Lambda_{\overline{\text{MS}}}=500$ MeV (where $\overline{\text{MS}}$ is the modified minimal subtraction scheme), while the leptonic widths are again in reasonable agreement. When compared with the Buchmüller-Tye results corresponding to $\Lambda_{\overline{\text{MS}}}=200$ MeV, neither our energy levels nor our leptonic widths agree with theirs.

Our results are subject to some uncertainty because of the need for extrapolation of the values of α_s and A for $t\bar{t}$ from those for $c\bar{c}$ and $b\bar{b}$ as described in Sec. V, but this uncertainty cannot account for the differences between our results and those of earlier authors.

V. CORRELATION OF QUARKONIUM PARAMETERS

Besides choosing the parameters so as to make the overall agreement between the theoretical and available experimental results for $c\bar{c}$ and $b\bar{b}$ as close as possible, other important considerations have been taken into account.

Our values of α_s for $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ satisfy the quantum-chromodynamic transformation relation

$$\alpha_s'=\frac{\alpha_s}{1+(\alpha_s/12\pi)(33-2n_f)\ln(\mu'^2/\mu^2)}\,,\tag{5.1}$$

and since our perturbative potential includes only the one-loop radiative correction, we have used the one-loop formula for the transformation of α_s. Moreover, the value of μ for each quarkonium is subject to the condition[2] that for all S states

$$|\xi|\ll 1\,,\tag{5.2}$$

where

$$\xi=\frac{(\alpha_s/12\pi)(33-2n_f)\langle\ln(\mathbf{k}^2/\mu^2)\rangle}{\langle 1\rangle}\,,\tag{5.3}$$

TABLE V. $b\bar{b}$ leptonic widths in keV without and with radiative correction.

State	$\Gamma_{ee}^{(0)}$	Γ_{ee}	Γ_{ee} (expt)
$1S$	1.64	1.11	1.22 ± 0.07
$2S$	0.84	0.57	0.53 ± 0.04
$3S$	0.61	0.41	0.40 ± 0.03

TABLE VI. $b\bar{b}$ matrix elements $\langle P\,|\,r\,|\,S\rangle$ in GeV^{-1}.

	$2P$	$3P$
$1S$	1.16	0.22
$2S$	-1.62	1.93
$3S$	0.04	-2.56

TABLE VII. $E1$ transition widths for $b\bar{b}$ in keV.

	$J=2$	$J=1$	$J=0$	Total	Total (expt)
$2S\to 2P_J$	1.71	1.59	0.94	4.24	4.9±1.0
$3S\to 3P_J$	2.68	2.49	1.47	6.64	8.4±1.4
$3S\to 2P_J$	0.09	0.06	0.02	0.17	
$2P_J\to 1S$	44.38	39.79	33.30	117.47	
$3P_J\to 2S$	22.18	18.78	14.22	55.18	
$3P_J\to 1S$	8.84	8.40	7.74	24.98	

which puts a reasonable restriction on the value[15] of μ.

It should be noted that the radiative correction included in our perturbative potential corresponds to the Gupta-Radford (GR) renormalization scheme,[16] which is a momentum-space subtraction scheme equally applicable to light and heavy quarks. According to the parameter values in Tables I, IV, and VIII, the values of the QCD parameter

$$\Lambda=\mu\exp\left[-\frac{6\pi}{(33-2n_f)\alpha_s}\right] \tag{5.4}$$

for $n_f=3$, 4, and 5 are given in the GR scheme by

$$\Lambda_{\rm GR}^{(3)}=279\text{ MeV},\quad \Lambda_{\rm GR}^{(4)}=247\text{ MeV},\quad \Lambda_{\rm GR}^{(5)}=198\text{ MeV}\ . \tag{5.5}$$

The corresponding values in the modified $\overline{\rm MS}$ scheme, obtained from the relation

$$\Lambda_{\overline{\rm MS}}=\Lambda_{\rm GR}\exp\left[-\frac{49/2-5n_f/3}{33-2n_f}\right], \tag{5.6}$$

are

$$\Lambda_{\overline{\rm MS}}^{(3)}=136\text{ MeV},\quad \Lambda_{\overline{\rm MS}}^{(4)}=121\text{ MeV},\quad \Lambda_{\overline{\rm MS}}^{(5)}=98\text{ MeV}\ , \tag{5.7}$$

which are consistent with the generally accepted values of Λ.

Despite our best efforts we were unable to equalize the values of A for $c\bar{c}$ and $b\bar{b}$. Our rigorous semirelativistic treatment leads to the conclusion that $A_{c\bar{c}}$ is smaller than $A_{b\bar{b}}$. Since Λ depends on n_f, it is not surprising that A is also n_f dependent. In order to estimate the value of A for $t\bar{t}$, we observe that when n_f increases from 3 to 4, A increases by a factor $A_{b\bar{b}}/A_{c\bar{c}}=1.2$. Assuming that a similar increase occurs when n_f increases from 4 to 5, we arrive at our value $A_{t\bar{t}}\approx 0.22$ GeV^2. We have also verified that a small ambiguity in the value of $A_{t\bar{t}}$ does not present a serious problem because the energy level splittings of $t\bar{t}$ are not very sensitive to variations in A.

VI. COMPARISON OF QUASISTATIC AND MOMENTUM-DEPENDENT POTENTIALS

Quark-antiquark nonrelativistic potentials have been used by various authors either in the momentum-

TABLE VIII. $t\bar{t}$ spectrum with $m_t=40$ GeV, $\mu=17$ GeV, $\alpha_s=0.184$, and $A=0.22$ GeV^2.

State	Mass (GeV)	State	Mass (GeV)
$1\,^3S_1$	79.113	$2\,^3P_2$	79.808
$1\,^1S_0$	79.081	$2\,^3P_1$	79.802
		$2\,^3P_0$	79.795
$2\,^3S_1$	79.868	$2\,^1P_1$	79.805
$2\,^1S_0$	79.859		
		$3\,^3P_2$	80.121
$3\,^3S_1$	80.167	$3\,^3P_1$	80.118
$3\,^3S_0$	80.162	$3\,^3P_0$	80.113
		$3\,^1P_1$	80.119
$4\,^3S_1$	80.373		
$4\,^1S_0$	80.369	$4\,^3P_2$	80.332
		$4\,^3P_1$	80.329
		$4\,^3P_0$	80.326
		$4\,^1P_1$	80.330

TABLE IX. $t\bar{t}$ spectrum with $m_t=45$ GeV, $\mu=19$ GeV, $\alpha_s=0.179$, and $A=0.22$ GeV^2.

State	Mass (GeV)	State	Mass (GeV)
$1\,^3S_1$	89.045	$2\,^3P_2$	89.776
$1\,^1S_0$	89.013	$2\,^3P_1$	89.771
		$2\,^3P_0$	89.764
$2\,^3S_1$	89.834	$2\,^1P_1$	89.773
$2\,^1S_0$	89.825		
		$3\,^3P_2$	90.091
$3\,^3S_1$	90.135	$3\,^3P_1$	90.088
$3\,^1S_0$	90.130	$3\,^3P_0$	90.084
		$3\,^1P_1$	90.089
$4\,^3S_1$	90.338		
$4\,^1S_0$	90.335	$4\,^3P_2$	90.300
		$4\,^3P_1$	90.297
		$4\,^3P_0$	90.294
		$4\,^1P_1$	90.298

TABLE X. $t\bar{t}$ leptonic widths in keV, for $m_t=40$ GeV, without and with radiative correction.

State	$\Gamma_{ee}^{(0)}$	Γ_{ee}	$\Gamma_{ee}(nS)/\Gamma_{ee}(1S)$
$1S$	7.70	5.87	1
$2S$	2.06	1.57	0.27
$3S$	1.25	0.95	0.16
$4S$	0.94	0.72	0.12

TABLE XI. $t\bar{t}$ matrix elements $\langle P|r|S\rangle$ in GeV^{-1} for $m_t=40$ GeV.

	$2P$	$3P$	$4P$
$1S$	0.28	0.10	0.06
$2S$	-0.62	0.57	0.15
$3S$	0.04	-1.10	0.85
$4S$	-0.02	-0.07	1.52

dependent form by retaining terms to order $\mathbf{p}^2$ or in the quasistatic form. In particular, the linear scalar-exchange confining potential in the momentum-dependent form is expressible as[17]

$$\mathscr{V}_c=Ar-\frac{A}{2m^2}\left[\frac{1}{r}+2r\mathbf{p}^2+\frac{1}{r}\mathbf{L}\cdot\mathbf{S}\right], \tag{6.1}$$

while in the quasistatic form

$$\mathscr{V}_c=Ar-\frac{A}{2m^2r}\mathbf{L}\cdot\mathbf{S}. \tag{6.2}$$

In our earlier papers,[1,2] only the quasistatic form of the scalar-exchange confining potential was employed, but we have now investigated the $c\bar{c}$ and $b\bar{b}$ spectra by using both the quasistatic and the momentum-dependent forms of the quark-antiquark potential in the Hamiltonian (2.1). We were surprised to find that while the quasistatic potential yields very good overall results for the energy levels, this is not the case with the momentum-dependent potential.[18,19] We have, therefore, provided the results only with the use of the quasistatic potential in Secs. II—IV.

Recently it has been shown by Gupta and Radford[20] that quark confinement can be understood as a consequence of the fact that quarks and antiquarks can exchange only hard gluons. We believe this also helps to explain the success of the quasistatic quark-antiquark potential even when $\mathbf{p}^2/m^2$ is appreciably large for a quarkonium[21] as in the case of $c\bar{c}$. Let us consider the scattering of a quark and an antiquark in the center-of-mass frame, and let $\mathbf{p}$ and $\mathbf{p}'$ be the initial and the final momenta of the quark. For this quark-antiquark system, the Fourier transform of the momentum-dependent potential can be converted into the quasistatic form by putting

$$\mathbf{p}^2=\tfrac{1}{4}\mathbf{k}^2+\tfrac{1}{4}\mathbf{s}^2, \tag{6.3}$$

where

$$\mathbf{k}=\mathbf{p}'-\mathbf{p},\quad \mathbf{s}=\mathbf{p}'+\mathbf{p}, \tag{6.4}$$

and then dropping the $\mathbf{s}^2$ term. But, according to (6.3), if $\mathbf{k}^2$ is allowed to take only large values, $\mathbf{s}^2$ can be treated as small, which provides a justification for the quasistatic approximation.

VII. CONCLUSION

By using a semirelativistic potential model, we have presented results of experimental interest for heavy quarkonia as well as analyzed the nature of the quark-antiquark potential.

Considering the fact that we are dealing with strong interactions, our overall results for $c\bar{c}$ and $b\bar{b}$ are gratifying

TABLE XII. $E1$ transition widths in keV for $t\bar{t}$ with $m_t=40$ GeV.

	$J=2$	$J=1$	$J=0$	Total
$2S\to2P_J$	0.20	0.15	0.07	0.42
$3S\to3P_J$	0.29	0.21	0.09	0.59
$3S\to2P_J$	0.20	0.13	0.04	0.37
$4S\to4P_J$	0.38	0.27	0.11	0.76
$4S\to3P_J$	0.18	0.11	0.04	0.33
$4S\to2P_J$	0.13	0.08	0.03	0.24
$2P_J\to1S$	38.62	37.72	36.59	112.93
$3P_J\to2S$	7.68	7.39	7.02	22.09
$3P_J\to1S$	13.96	13.82	13.65	41.43
$4P_J\to3S$	4.70	4.49	4.22	13.41
$4P_J\to2S$	3.37	3.32	3.25	9.94
$4P_J\to1S$	8.33	8.27	8.21	24.81

with the exception that our $E1$ transition widths for $c\bar{c}$ are about twice as large as the experimental values. For $t\bar{t}$, our results for the energy levels and the $E1$ transition widths are considerably smaller than those obtained earlier by others, but our leptonic widths are in agreement with their findings.

We have also found that the quark-antiquark interaction can best be described by a quasistatic rather than a momentum-dependent potential, and we have proposed a theoretical justification for this surprising conclusion.

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[4]These results for the $c\bar{c}$ energy levels slightly differ from those in Ref. 1 because we have eliminated a computational error in our earlier work and also carried out a more rigorous search for the optimum values of the input parameters.

[5]In accordance with the standard spectroscopic notation, we have denoted the lowest P states as $2P$.

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[15]It is interesting that our values of $|\mathbf{p}|/\mu$ for the ground states of $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ are 0.36, 0.36, and 0.32, respectively, which indicates that μ is closely related to the quark momentum $|\mathbf{p}|$ rather than its mass m.

[16]S. N. Gupta and S. F. Radford, Phys. Rev. D **25**, 2690 (1982).

[17]For the derivation of nonrelativistic potentials from the scattering operator, we have followed the treatment of S. N. Gupta, Nucl. Phys. **57**, 19 (1964). The linear scalar-exchange potential takes the form (6.1) when the on-shell quark-antiquark scattering matrix element in the center-of-mass frame is expressed in the simplest possible form, while other forms can be obtained by adding on-shell vanishing terms. See, for instance, T. Barnes and G. I. Ghandour, Phys. Lett. **118B**, 411 (1982), and references therein.

[18]We obtained unsatisfactory results with the momentum-dependent scalar-exchange confining potential when used either in the form (6.1) or in the Barnes-Ghandour form cited in Ref. 17.

[19]We have also looked at the $c\bar{c}$ and $b\bar{b}$ spectra with the use of the quasistatic and the momentum-dependent forms of a linear vector-exchange confining potential. We found unacceptably large spin splittings of energy levels, and concluded that vector-exchange component of the confining potential, if any, is quite small compared with the scalar-exchange component.

[20]S. N. Gupta and S. F. Radford, Phys. Rev. D **32**, 781 (1985).

[21]Our values of $\mathbf{p}^2/m^2$ for the ground states of $c\bar{c}$, $b\bar{b}$, and $t\bar{t}$ are 0.284, 0.076, and 0.018, respectively.