THE GREAT DIVIDE: A STUDY THAT EXAMINES THE UNDERSTANDING OF LONG DIVISION ACROSS MULTIPLE GENERATIONS

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CERTIFICATION OF PROJECT WORK

We, the undersigned, certify that this project entitled THE GREAT DIVIDE: A STUDY THAT EXAMINES THE UNDERSTANDING OF LONG DIVISION ACROSS MULTIPLE GENERATIONS by Steven A. Sturm, Candidate for the degree of Master of Science in Education, Mathematics Education, is acceptable in form and content and demonstrates a satisfactory knowledge of the field covered by this project.

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Abstract

This research explores the understanding of the long division algorithm across multiple generations. It was hypothesized that over time, people either forget how to complete long division problems, or become more inaccurate when asked to solve a long division problem. Specifically, it was hypothesized that students between the ages of 12 and 17 would be more accurate than those between 18 and 23, and adults 24 or older. The results of this study indicate that students between the ages of 12 and 17 and adults 24 and older outperformed students between the ages of 18 and 23. However, there was no significant difference between 12 to 17 year olds and adults 24 or older as well as no significant difference in gender as a whole. Student work samples were collected and analyzed to observe the common mistakes made when dealing with the long division algorithm and inferences were made about how educators can combat these mistakes and misconceptions.

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Introduction

This research explores the understanding of the long division algorithm across multiple generations. Specifically, it examines the common misconceptions of the long division algorithm, teaching of long division throughout history, and the idea that long division is a mathematics topic that qualifies as ‘use it or lose it.’ While the teaching of long division has changed throughout history, the common misconceptions remain the same, leaving long division to be portrayed as a topic that is unnecessary throughout school and life. The important lesson for educators to learn from these common misconceptions is to understand where and how students become confused within the long division algorithm, however, perhaps the more interesting thing is; should the long division algorithm still be taught?

The notion among some adults today that mathematics, or some portion of mathematics, was unnecessary to learn in school directly led to my interest in this topic. The idea that mathematical topics, especially long division, are a strategies that must be learned in school, but is never needed after that is very intriguing. Many adults today are indifferent towards knowing or remembering any mathematics from school. In fact, some see the failure to remember any mathematics as the ‘cool thing.’ Imagining the same perception for something like English or History is a scary thought. My experience teaching in a non-traditional school that offers free education to students seeking their high school equivalency gave me the opportunity to teach the long division algorithm. It also allowed me to explore multiple long division teaching methods including the use of models to aid student learning of the algorithm.

I am interested in examining the difference in performance on long division problems between students and adults as well as exploring the many teaching styles that cater to common misconceptions of the division algorithm. It is important for educators to understand these misconceptions and use or develop teaching strategies that will work best for their students.

It is hypothesized that over time, people either forget how to complete long division problems, or become more inaccurate when asked to solve a long division problem. Students between the ages of 12 and 17 will perform better than college students and adults 24 years old, or older.

This hypothesis was tested using an assessment and survey given to sample students and adults from the population. The assessment was graded using a rubric that scored the participant based on performance and identified types of mistakes in each solution. After completing the assessment, participants were asked to complete a survey which asked them questions regarding the assessment and their mathematical ability. The literature review examines the history of teaching long division throughout history and the common misconceptions of the algorithm. The literature review also explores long division as a mathematical tool that is used throughout school, but is forgotten as a person ages.

Literature Review

This literature review was conducted to explore existing research and literature related to long division. This review covers the history of teaching long division throughout the early 1900s, the "New Math" period, the resurgence of long division in the '80s and '90s, and modern day teaching techniques. Understanding the common misconceptions is an important factor in developing teaching techniques of the long division algorithm. The last section of the review covers the idea that long division is a strategy that is learned in the early school years and is a strategy that is lost as an individual progresses through school and life.

The History of Teaching Long Division

Long division is a strategy that is often overlooked by students and educators. The teaching of long division has changed throughout the past 100 years. Early methods of long division focused mostly on condensing the algorithm to make it easier to teach and learn (Dederick, 1926, pp. 144). Another way that was used to prepare students for the division algorithm was to make sure the students had prior adequate knowledge (Holland, 1942, pp. 590). As the New Math period from 1960 to 1970 came and passed, the common perception of some skills like long division was, "whether a school has or has not a special method for teaching long division is of no significance, for long division is of no importance except to those who want to learn it. And those who want to learn long division will learn it no matter how it is taught" (Klein, 2003). Following New Math, long division has made a minor resurgence in standards and curricula. A popular topic of discussion today involves the effectiveness of reform methods in teaching long division.

Teaching of Long Division in the Early 1900s

Early strategies used to teach long division mostly dealt with the algorithm itself since teachers did not have the technology or knowledge of teaching styles that are used today. Often times this involved creating shortcuts or strategies that made the algorithm shorter (Escott, 1937; Kearney, 1930). Methods of condensing the work, such as not writing partial products, only increase mental operations slightly (Dederick, 1926, pp. 143). Another system commonly used in the early 1900s used ‘rividing’ and ‘multividing’ for small divisors (Fletcher-Jones, 1936, pp. 331). Between the years 1930 and 1950 before the New Math period, an effective strategy eliminated confusion by examining the difficulties with long division and incorporating concepts and strategies into teaching to rival these difficulties (Holland, 1942, pp. 585-96).

A common technique in the early teaching of long division involved a process called ‘rividing’ that shortened the long division algorithm for problems with small divisors (Fletcher-Jones, 1936, pp. 331). Rividing is defined as continued division by a certain integer. Figure 1 below represents the use of this method for the problem

.

Example. To divide 12345 by 29.
“ Rivide ” by 3.

3) 1 2 3 4 5
7 16 20 2 2 1
4 2 5 · 6 8 9, etc., etc.

Figure 1. Using ‘rividing’ to solve . Fletcher-Jones, A.A. (1936). A method of long division for small divisors. *The Mathematical Gazette, 20*(241), 331-332.

Since the divisor ends in a 9, use continued division by 3, or rivide by 3. The process is as follows: Divide 3 into 12, which yields a quotient of 4 and remainder of 0. Next, add the quotient 4 and the value 3 above it from the number 12345 to get the next dividend 7, which appears between the 4 and 3. Then, divide 3 into 7 which gives a quotient of 2 and a remainder of 1. Again, the quotient 2 is added to the above 4 in the number 12345. This time, since the remainder is 1, and not 0, it is added to the front of the result of adding the quotient and digit from above (in this case 6), which yields the next dividend, 16. The process is repeated to the required number of decimal places. The method is slightly more difficult when the last digit on the divisor is not a 9. Figure 2 below shows the process for the problem .

Again an example, $\frac{1}{78}$.

8) 1·0   22 64 16 4 40 10 22 64

0·0   1 2 8 2 0 5 1 2, etc. (the figures recurring).

Figure 2. . Fletcher-Jones, A.A. (1936). A method of long division for small divisors. *The Mathematical Gazette, 20*(241), 331-332.

The method above uses rividing by 8 and multividing by 2. The process is the same except for when combining the remainder and quotient. Multividing by two means the remainder is continually combined with twice the quotient to get the next dividend. The general rule for rividing and multividing is: divide by , rivide by and multivide by (Fletcher-Jones, 1936, pp. 332). While early methods such as this method required prior knowledge or multiplying and adding, it was not a point of emphasis among educators.

An article written by Holland (1942) enforced the necessity for students to have adequate prior knowledge before learning long division. It is important to identify the difficulties that may arise with the algorithm and come up with strategies to combat these problems. Holland (1942) identified four necessary requirements students must master before delving into long division. These are: an understanding of the decimal system and place value of numbers, an understanding of the concept of division as a whole and long division as a process of shortened subtraction, an understanding of the relations between divisor, dividend and quotient, and an understanding of the concepts and processes involved in long division (pp. 592-6). Once students are familiar with the prior knowledge identified above, they are then ready to work through the division algorithm.

Holland (1942) makes teaching suggestions to help students master the prior knowledge required to perform long division. Using visuals such as tiles, beans or other objects is an effective way to develop an understanding of place values and the decimal system (Holland, 1942, pp. 594). Portraying division as a story allows students to explore it as repeated subtraction. Holland (1942) suggests giving a student a stack of papers to hand out equally among the students in the room. As the student is passing them out, write the story of what is happening on the board. A similar strategy can be used to develop an understanding of the terms divisor, dividend and quotient(pp. 594-5). Holland (1942) suggests splitting a box of cookies up among the class and explaining that the number of cookies each student receives is the quotient, the number of students in the class is the divisor, the number of cookies is the dividend, and the leftover cookies are the remainder (pp. 595). This example can be used to develop students' understanding of the concepts and processes used in the long division algorithm. The story on the board will show that the number of students multiplied by the number of cookies each student gets plus the leftover cookies equal the amount of cookies in the box originally. Holland

(1942) emphasizes that there are many ways to tell stories that will help teach the children to meet the difficulties with long division. While this emphasis of prior knowledge was used throughout the early 1900's, educators began to explore other ways to introduce and teach the long division algorithm that required

Long Division During the New Math Period

While long division was a focus of education in the early 1900's, the New Math period from 1960 to 1970 placed little emphasis on basic mathematical skills. Early 1900's research provided multiple ways to teach the long division algorithm as well as condense the work needed to complete division problems. New Math introduced curricula that was excessively formal, programs that dealt with bases other than base ten and heavy emphasis on set theory among other confusing topics. Basic skills such as long division and multiplication were almost non-existent (Klein, 2003). A number of mathematicians were outspoken about the shortcomings of the new math curriculum, and by the early 1970's New Math was almost non-existent.

The Resurgence of Long Division

As the New Math period came to an end in the early 1970's, long division did not initially make an impactful resurgence. The new approach to teaching mathematics let students decide what they wanted to learn and when. The problem with the "free math" period was that it was devastating to students who did not have access to supplemental education outside of school (Klein, 2003). The early 1980's called for a period of 'Back to Basics' mathematics and the creation of national standards was underway. By the early 1990's, long division was required in the curriculum. The resurgence of long division brought new ideas and new teaching methods such as breaking down, divide and conquer, and the distributive property (Israeli Institute of Technology, 1992).

Modern Day Long Division Teaching Strategies

There is an abundance of modern day long division teaching strategies that simplify the algorithm or use reform methods to enhance student learning. Effective pedagogical tools, meaningful tasks and dynamic class discussions allow students to come to a strong understanding of the long division algorithm (Lee, 2007, pp. 49). Similar to Holland's (1942) findings, the meaning of the algorithm can be developed using stories and concrete examples (Wickelgren, 2013). Double division is a strategy that improves student performance on large numbers involving 1-digit and 2-digit divisors (Arzemi & Sivasubramaniam, 2010). Modern day teaching strategies of long division address the most common misconceptions and difficulties with the algorithm among students.

The use of discussions, pedagogical tools, and meaningful tasks as an effective way to teach long division is well documented in a study completed by Ji-Eun Lee (2007). Seven students between the ages of eight and ten took part in a three-year classroom action research project. The students followed a Russian developmental mathematics curriculum developed by Daydov and his colleagues (Lee, 2007, pp. 50). Two of the students had public school experience in the past, while for the others; it was their first form of formal schooling. Lee (2007) used a series of class episodes to observe the effectiveness of the employed curriculum. According to Lee (2007), before beginning to teach long division, it is necessary for the students to master their understanding of place value (pp.51). To do this, Lee (2007) taught the students to solve problems in multiple bases using a chip trading method. Figure 3 on the following page represents in base 4.

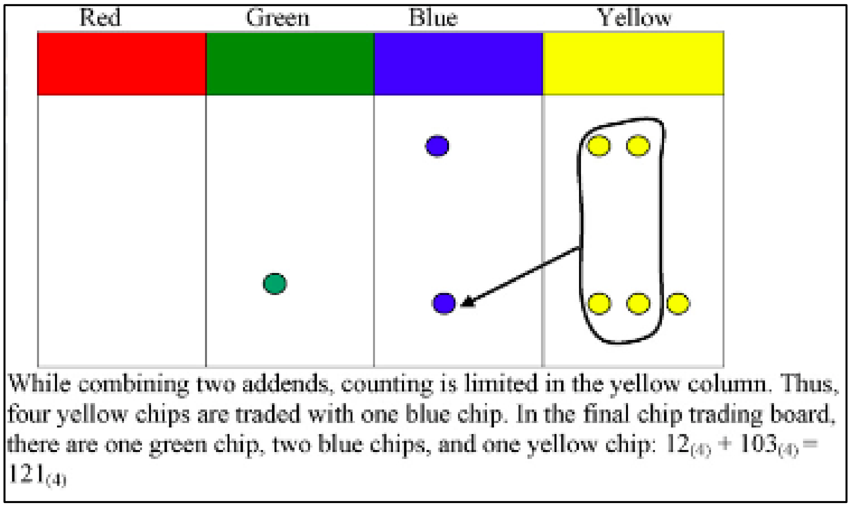

Figure 3. Visual representation of in base 4. Lee, J. (2007). Making sense of the traditional long division algorithm. *Journal of Mathematical Behavior, 26*(1), 48-59.

The green chips represent the hundreds place, the blue chips represent the tenths place and the yellow chips represent the ones place. Since base 4 is used, the four yellow chips are replaced by a single blue chip, which yields 121. Once the students mastered this chip-trading concept, Lee (2007) introduced division to the students by asking them to solve simple division problems using basic division facts as reverse multiplication, estimation, and using the distributive property to break down problems (pp.51). After this, the students were introduced to a more difficult problem, , and the student interactions and thoughts were recorded as episode one (pp. 51). One student instantly suggested the distributive property to break down the algorithm. Instead of completing the problem this way, Lee (2007) introduced the division notation and tried to make a connection between the two as seen on the next page in Figure 4.

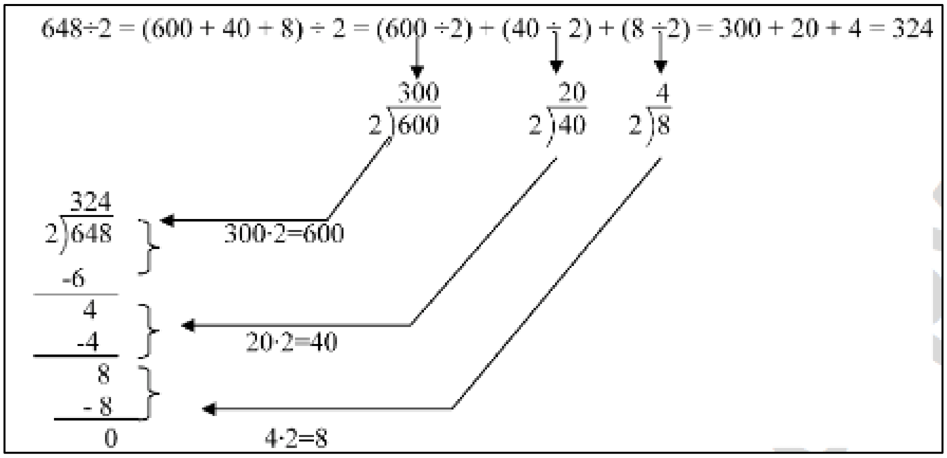

Figure 4. The distributive property and division algorithm. Lee, J. (2007). Making sense of the traditional long division algorithm. *Journal of Mathematical Behavior, 26*(1), 48-59.

The students agreed that the algorithm was much more efficient as it combined three division problems into one. Episode two was dedicated to developing the use of the algorithm with one-digit divisor problems (pp. 52). Students were able to complete division with remainder problems in the third episode, but were unable to explain what the quotient meant during the fourth episode (pp. 52). The fifth and sixth episodes introduced manipulatives such as blocks to represent the meaning of the quotient and place values (pp. 53-6). Figures 5 and 6 on the following page show the first and last steps to solving the problem using blocks. The intermediate steps involve breaking down the bigger blocks into equivalent expressions using the smaller blocks and properly trading and dividing the blocks.

Figure 5. Block representation of 726. Lee, J. (2007). Making sense of the traditional long division algorithm. *Journal of Mathematical Behavior, 26*(1), 48-59.

Figure 6. Representation of dividing 120 into three equal groups. Lee, J. (2007). Making sense of the traditional long division algorithm. *Journal of Mathematical Behavior, 26*(1), 48-59.

Figure 6 shows the result of as three equal groups of 242 each. The students were able to identify the equal groups shown with the blocks as the quotient. The most difficult part of the block process according to Lee (2007) is the trading and breakdown of shapes into other equivalent shapes (pp.56). Lee (2007) used the last two episodes to have students translate the block method to the writing of dots on a place value chart (pp.56-7). The episodes documented in Lee's (2007) research supports reform methods as an effective way to teach the long division algorithm.

There is much research today that supports Lee's (2001) findings with respect to manipulatives and reform methods in the teaching of long division. Ford (2014) suggested teaching division in more than one way and using manipulatives as a building block for the algorithm. He noted that students are less likely to pay attention to manipulatives if they are given the algorithm first. Wickelgren (2013) suggests using a story to learn the division algorithm through guided discovery, a technique also mentioned by Holland (1942). Other modern teaching techniques include the double division method (Arzemi & Sivasubramaniam, 2010) and the development of an understanding of division using the calculator (Ledesma, 2011). These techniques were developed to cater to students' needs based on the common misconceptions and difficulties with the long division algorithm.

Misconceptions of the Long Division Algorithm

To better understand how to teach long division, it is important to know the common misconceptions involving the algorithm and the process itself. Today, some students believe they are unable to do long division because they are not able to achieve mastery due to the reform teaching methods being used (Rogers, 2012). Both teachers and students alike view the algorithm as a set of procedures and not as a way to make sense of what division is (Sellers, 2010). An article from Worth Osburn (1946) identifies six levels of difficulty within long division problems. The levels of difficulty include problems where; placement and a new operation is the only difficulty, a remainder is the only difficulty, quotients containing zeros, examples that call for carrying multiplication, problems with borrowing subtraction and problems that have non-apparent quotients (pp. 441). Jessie Voigt's (1938) master's thesis from the University of Arizona examined some of the most common misconceptions among long division problems. Students often struggle with the operations within long division and long division with remainder problems (Li, 2001). Figure 7 below shows some of the most common mistakes made throughout the use of the long division algorithm.

Common Mistakes Made Within the Long Division Algorithm

- Failure to subtract correctly
- Failure to write a zero in the quotient
- Writing the first quotient figure in the wrong place
- Incorrect multiplication
- Failure to compare after subtracting (Finding the next

Figure 7. Most common mistakes made on long division problems. Voigt, J. (1938). *An analysis of errors in long division* (Master's thesis). University of Arizona. http://arizona.openrepository.com/arizona/bitstream/10150/553400/1/AZU_TD_BOX346_E9791_1938_68.pdf

A study completed by Jessie Voigt (1938) broke down common misconceptions with the long division algorithm. A 30-minute test was administered to 164 students that included division problems containing: a remainder, no remainder, one zero in the quotient, two zeros in the quotient, a final zero in the quotient, scattered zeros in the quotient and, trivial quotients. A breakdown of errors in problems given by Voigt can be seen on the next page in Figure 8.

THE NUMBER AND PER CENT OF ERRORS MADE BY 164 PUPILS IN THE FUNDAMENTAL PROCESSES INVOLVED IN ISOLATED SITUATIONS AND IN THE TOTAL LONG DIVISION TEST

	Addition	Subtraction	Multiplication	First Quotient Figure	Total Long Division Process
Number of Errors	36	206	89	920	433
Per cent of Errors	1.29	4.83	1.26	17.00	26.40

Figure 8. A breakdown of errors in long division problems. Voigt, J. (1938). *An analysis of errors in long division* (Master's thesis). University of Arizona. http://arizona.openrepository.com/arizona/bitstream/10150/553400/1/AZU_TD_BOX346_E9791_1938_68.pdf

As seen in this table, problems with the division algorithm are the cause of the highest percent of errors, but students also made mistakes with subtraction, addition and multiplication within the algorithm itself. Voigt's (1938) analysis of individual problems in the assessment identifies the main problems within the algorithm as subtraction, failure to write a zero in the quotient, incorrect placement of the first quotient figure, multiplication, and failure to compare after subtracting (pp. 26). To correct these errors, teachers must place extra emphasis on the first time they teach the algorithm and make sure students have the proper prior knowledge of operations (Voigt, 1938, pp. 32). Placing this extra emphasis on the long division algorithm may also change the perception that long division is a skill that is needed in school, but not required for use after that.

Long Division: Use it or Lose it?

The common notion today is that as a person gets older, they are no longer required to be able to complete basic skills such as long division. While the New York State Common Core Standards for mathematics contain standards that require students between grades three and seven to be able to complete long division, they do not cover the material throughout later grades (National Governors Association Center, 2010). In fact, long division has been referenced to as becoming "dead as a dodo bird" (Maier, 2012). Perhaps so 'dead' that American adults rank 21st out of 23 advanced economies when it comes to mathematics (Jeffrey, 2013). Adults not only struggle to remember how they were taught division, but also are unable to understand the new reform methods being taught in schools today (Eastaway, 2010).

Division is addressed as early as third grade in the New York State Common Core Standards for Mathematics, however the terminology "long division" only appears twice throughout the entire standards. The third grade standards require students to understand the relationship between division and multiplication and be able to compute division involving integers between 0 and 100. Grade 4 reinforces the importance of understanding the relationship between multiplication and division and grade 5 introduces the division of fractions (National Governors Association Center, 2010). The term "long division" or the long division algorithm is not mentioned at all throughout grades 3 to 5. In fact, the first time long division appears in the Common Core standards is grade 6 in the standard that requires students to convert a rational number into a decimal using long division.

With the amount of reform techniques being used today and lack of attention to the long division algorithm in standards, parents not only fail to remember the long division algorithm, but also do not understand the techniques their children are taught (Eastaway, 2010). When if students are asked to complete long division, they view it as a meaningless chore. Eastaway (2010) writes, “Ask most adults today to carry out a long multiplication or division sum and they will look blankly at you. They may have, sort of, got it once, but they can’t remember how to do it. And anyway, we have calculators now, don’t we?” This is a common notion among many adults today and it poses the question whether or not long division is remembered over time.

Today, many basic mathematical calculations are performed on a calculator by people of all ages. It is also common that adults and older people openly express their inability to remember basic mathematical procedures that they learned in school while students throughout elementary and middle school are still taught long division regularly. The purpose of this study is to determine if the ability to do long division is lost over time and to introduce the discussion of including or removing long division as part of the current mathematics curriculum.

Experimental Design

The experiment was designed to test the hypothesis that middle school students (12-17 years old) would outperform college students (18-23) and adults (24+) on long division problems. The assessment consisted of nine problems that reflected the uses of the long division algorithm in different capacities such as word problems, problems with quotients that contain trailing zeros and problems that contain quotients with place holding zeroes. After completion of the assessment, participants were given a seven-question survey to assess their experience using long division and their feelings towards its usefulness. Assessment problems were designed to provide an overall score for each participant that directly relates to their ability to complete and use the long division algorithm.

Participants

Subjects that participated in the study were all from the Western New York area. A breakdown of participants can be seen below in Figure 9.

	12-17	18-23	24+	Total
Male	20	9	15	44
Female	12	14	15	41
Total	32	23	30	85

Figure 9. Breakdown of participants.

Most of the participants in the 12 to 17 year old age group were recruited from a public rural school in the Northeastern United States. At the time of the assessment, these students were enrolled in an Algebra I class. The remaining 12 to 17 year olds who agreed to participate were children of adults who also agreed to participate in the study. Participants from the 18 to 23 year old age group were recruited from a university Survey of Calculus class taught by a friend of the principal investigator from a comprehensive, selective, public, residential, liberal arts university located in the Northeast. The rest of the participants, those 24 years old and older, were recruited throughout the Northeast. The principal investigator recruited participants from the surrounding neighborhood. The remaining participants of this age group were recruited from friends of an acquaintance of the principal investigator at a hospital, also in the Northeast. Participants were ensured that the study was completely confidential and were given the option to remove themselves from the study at any time.

Design

The assessment tested the hypothesis that middle school students (12-17 years old) would outperform college students (18-23) and adults (24+) on long division problems. Before the assessment was administered, written consent was obtained from each participant in the study. Participants were also informed that the study was voluntary and that they could opt out at any point in time. Figure 10 below briefly covers the procedure and design of the study.

Study Procedure

- Select a group of middle school students, college students, and adults to assess.
- Administer the nine-problem long division assessment to each group of participants.
- Have participants complete a seven-question survey regarding the assessment.
- Collect and grade the assessment.
- Examine the performance as a whole by each group of participants.
- Examine the performance on each problem by group of participants.
- Determine which group of participants is actually the most proficient in solving long division problems.

Figure 10. Long division experimental design.

Participants were given fifteen minutes to complete a nine-problem assessment. The problems were carefully chosen so that the assessment directly reflects a participant's ability to complete long division problems. The problems contained a combination of strictly computational and word problems. Students were directed to complete each problem and show all of their work for all problems. After completing the assessment, the students were given a seven-question survey.

Instrument Items and Justification

The assessment was administered to a sample of middle school students, college students and adults. After participants complete the assessment, it was collected and problems were graded individually based on a series of evaluations. Mistakes in each problem were tallied in a corresponding table and partial credit was determined based on those mistakes. Final scores on the assessment were determined based on the correctness of the answer as well as the work shown. The scores from each group of participants as well as scores on each individual problem were compared head to head to determine which group performed the best.

The assessment that the participants received included problems that assessed the long division itself as well as operations used within the algorithm. Figure 11 below shows the problems included in the assessment as well as the reason for including each subsequent problem. The full assessment is included in Appendix A.

Problem	Reason
	Subtraction with borrowing
	Basic division
	Place holding zero within the quotient
	Basic multiplication
5) If you want to split up a box of 36 cookies among 9 students, how many cookies does each students receive?	Basic division
6) Use long division to solve	Trailing zero with remainder
	Zero quotient
	Trailing zero without remainder
9) Use long division to solve	Division with borrowing subtraction

Figure 11. Breakdown of assessment problems.

The goal of this selection of problems was to provide an accurate representation of a participant's ability to perform long division. Problems 1 and 4 were both intended to determine if a participant had an understanding of the operations that are required within the long division algorithm. If a participant was unable to correctly multiply or subtract, it was expected that they would struggle when doing long division. Problems 2 and 4 were included to provide basic feedback on a participant's ability to understand and complete division. Problem 3 assessed a participants' ability to effectively place a place-holding zero in the quotient when the divisor was larger than the dividend. It was expected that the most common mistake in this problem would be a missing zero in the quotient. Problems 6 and 8 required the participants to place trailing zeros in the quotient. Participants who believed they are done once they can no longer divide the divisor into the dividend would finish with answers that are off by a factor of 10. Similar to a place holding zero, problem 7 required the participants to notice that the quotient was zero and the remainder in this case was the dividend. Problem 9 was a more in-depth division problem that required intermediate level subtraction and multiplication.

This selection of problems best represents each group's ability to comprehend and complete the long division algorithm. The problems were examined individually and scores and mistakes for each problem were tallied in corresponding tables. The scores and mistakes of each group were compared to one another.

After completing the assessment, participants were given a seven-question survey. The contents of the survey can be seen on the following page in Figure 12.

1. Did any question on the assessment stand out as being more difficult than the others? Why so?

2. What is your gender?

3. When is the last time you remember using long division? (Select one)
   [ ] This year
   [ ] Last year
   [ ] 3-5 years ago
   [ ] 5 or more years ago

4. Circle the operation that you consider most challenging:
   Multiplication
   Addition
   Division
   Subtraction

5. On a scale from 1 to 10 where 1 means very little and 10 means a lot, how much do you enjoy math?

6. What is your age?

7. Do you think long division is an important skill to know? Why or why not?

Figure 12. Post-assessment survey.

The purpose of asking the participants which problem stood out as being the hardest was to reveal which type of division problem is most difficult among the three groups being studied. Participants who checked one of the last two boxes in survey question 3 were expected to be the adults or college students. This may reinforce the idea that the ability to do long division is lost over time. Participants were asked to elaborate what they thought about long division so that an observation could be made between the level of importance of long division of each of the three groups of people being studied. The survey questions were evaluated on a participant-by-participant basis before examining if there was any association between survey, assessment, and participant group.

Methods of Data Analysis

The data for this assessment were generated a third of the way through the university's spring semester of 2016. Participants were given 15 minutes to complete the assessment and were then given the survey to complete. Each problem was assigned a number of possible points that could be earned by a participant based on the number of steps required and the difficulty of the problem. For example, problem 3 required 3 steps of long division, accounting for 3 points, and one point for a correct answer. Figure 13 below shows the breakdown of possible points on the assessment.

Number	Number of Possible Points
1	1
2	1
3	4
4	1
5	1
6	3
7	2
8	3
9	3
Total	**19**

Figure 13. Allocation of assessment points.

After grading the assessment based on the table above and calculating a score for each participant out of 19 points, solutions were examined individually to determine what types of mistakes each person made. A breakdown of mistakes by problem can be seen below in Figure 14.

Problem	No Mistake	Division	Multiplication	Subtraction	Remainder	Other (Specify)
1						
2						
3						

Figure 14. Breakdown of assessment mistakes.

Tallies were taken in the above table for each problem on the assessment. After taking a general tally to examine the difficulties within each problem, the table was then split up into three more identical tables, one each for middle school students, college students, and adults. The results from the assessment on a right or wrong basis as well as the difficulties in each problem were examined for each age group to determine statistical significance.

Descriptive and Inferential Statistics

Once the assessments were completely graded, the results were placed in an Excel file and exported into Minitab to determine if there were significant differences in scores among the age groups used in the experiment. Inferential and descriptive statistics were calculated to analyze each participants' performance on the assessment. Figure 15 below breaks down the process that was performed using Minitab:

Breakdown of Statistical Analysis

- Analysis of scores by each age group
- Comparison of scores by gender
    - Comparison of scores by age and gender
- Ranking of each problem from easiest to hardest
    - Ranking of each problem from easiest to hardest by age group
- Breakdown of survey question responses by age group

Figure 15. Breakdown of statistical analysis.

First, an Analysis of Variance (ANOVA) and General Linear Model for the three age groups were analyzed to generate p-values to determine whether or not there was a significant difference in performance by age group. After examining the scores based on age group, the data was further broken down by gender to examine if there was a statistically significant difference in performance by gender. Assessment problems were ranked by difficulty for all participants and then broken down by age group. Student work samples were analyzed to determine the common misconceptions and mistakes made throughout assessment problems. Select survey questions were examined to determine if there was a difference in response by age group.

Results

After examining the tests performed to determine whether or not long division is lost over time, the data produced two major results:

1) ***There was no statistical difference in performance between 12-17 year olds and adults, however there was a significant difference between both adults ( ) and 12-17 year olds ( ) and 18-23 year olds ( ).***

After running an Analysis of Variance Test and General Linear Model between the three age groups, it was determined that the 12-17 year old participants did not significantly outscore adults ($p = .104$). However, the 12-17 year old participants did outscore the 18-23 year old participants ($p = .023$) and the adults 24 years old and older significantly outperformed the 18-23 year old participants ($p = .046$). As a whole, there was no significant difference in performance by gender between males and females ($p = .961$), but there was a significant difference in performance between adult males and adult females ($p = .033$).

Analyzing problems individually revealed the problems in which participants struggled on the most. Figure 16 below ranks each problem from easiest to hardest using mean score for the assessment as a whole (Note that the mean is calculated such that 1 is a maximum score for each problem).

Problem	Mean (All participants)
5) If you want to split up a box of 36 cookies among 9 students, how many cookies does each students receive?	0.990829 (Highest Overall)
2)	0.937250
4)	0.880960
1)	0.846765
8)	0.644785
3)	0.525232
6) Use long division to solve	0.418085
9) Use long division to solve	0.395193
7)	0.195719 (Lowest Overall)

Figure 16. Ranking of assessment problems from easiest to hardest.

Examining Figure 16, problems 1,2,4, and 5 were clearly the easier problems on the assessment, while problem 7 was the hardest. It is intriguing to see the similarity of the scores on the word problems. After ranking the problems from easiest to hardest for every participant, the table above was broken down further to rank the same difficulty by age group, which can be seen below in Figure 17.

Problem	Mean (12-17)	Problem	Mean (18-23)	Problem	Mean (24+)
5	1.0000	**5**	0.9565	**5**	1.0000
2	0.9697	**1**	0.9130	**2**	1.0000
4	0.8788	**2**	0.8261	**4**	0.9655
1	0.8182	**4**	0.7826	**1**	0.7931
8	0.7475	**8**	0.4928	**8**	0.6782
3	0.7272	**3**	0.3152	**3**	0.5172
6	0.5152	**9**	0.2609	**6**	0.5058
9	0.4950	**6**	0.2174	**9**	0.4138
7	0.2188	**7**	0.1522	**7**	0.2069

Figure 17. Ranking of assessment problems from easiest to hardest by age group.

The order of 'most difficult' to 'least difficult' problems matched for each age group closely to the rankings when all age groups were combined. The only glaring difference between the 12-17 year olds and adults (disregarding the college aged participants) was the difference in mean score on problem 3, which turned out to be significant at the .05 significance level (p = 0.02). After examining the assessments, it appears as though more adults 24 and older forgot about the place holding zero in the quotient. College students aside, the difference in performance between adults and 12-17 year olds can be seen on each problem except for problems 5, 2, and 4. In other words, the adults performed much better on the easier problems, two of which did not contain long division, than the 12-17 year olds. However, the 12-17 year olds outperformed adults on the remaining questions, most of which required long division. After removing problems 1, 2, 4, and 5 from the scoring, the mean score for adults on the remaining problems out of 15 was 7.3, while the 12-17 year olds averaged 8.6 out of 15 possible points. After running an Analysis of Variance between these means which reflected the participants performance on the long division problems themselves, there was almost a significant difference between adults and middle school students at the level (p = .053). While these two groups did not show significantly different results on either the full assessment or the selection of long division problems, the differences between the 18-23 year old participants and the other two groups can be clearly seen in Figure 17. College aged students performed much better on the first problem, which was a subtraction with borrowing problem than both of the other groups, however the college students' scores were inferior to the other groups scores' on all of the remaining questions.

2) ***Many mistakes throughout the long division algorithm occurred due to two factors:***
   - ***Lack of understanding of the division algorithm (place holding zeros, trailing zeros, zero quotients, or incorrect division.)***
   - ***Non-mastery of previously learned operations such as subtraction or multiplication, leading to mistakes within the division algorithm.***

A breakdown of mistakes by problem can be seen below in Figure 18.

Problem	**No Mistake**	**Division**	**Multiplication**	**Subtraction**	**Remainder**	**Other (Specify)**
1	76			9		
2	80	5				
3	44	14	18	3	5	
4	78		7			
5	83	2				
6	34	22	13	9	7	
7	16	7			1	61 (Decimal Answer or no answer)
8	54	20	9		2	
9	33	19	23	6	4	

Figure 18. Analysis of mistakes in assessment problems.

Among errors in problems, mistakes with division itself were most prominent. This included incorrect division in a part of the problem (which led to an incorrect digit(s) in the quotient), lack of a trailing zero, and lack of a place holding zero. The second most common mistake among problems involved multiplication. These mistakes were often made when participants correctly divided, but then proceeded to multiply back through in the algorithm incorrectly. The third most common mistake made throughout problems dealt with subtraction. Participants who made subtraction mistakes did so when borrowing was required within the division algorithm. Remainder mistakes were not as prominent as others, but these participants either did not include a remainder, or did not include a correct remainder even though the rest of their work was correct. Student work samples were gathered in order to examine the mistakes identified in the figure above. Common mistakes were expected throughout the assessment based on the problem. For example, the most common mistake expected on the third problem,

, was the lack of a zero placeholder in the quotient. It turned out that this was in fact the most common mistake in the problem and Figure 19 below represents participant work on this specific problem.

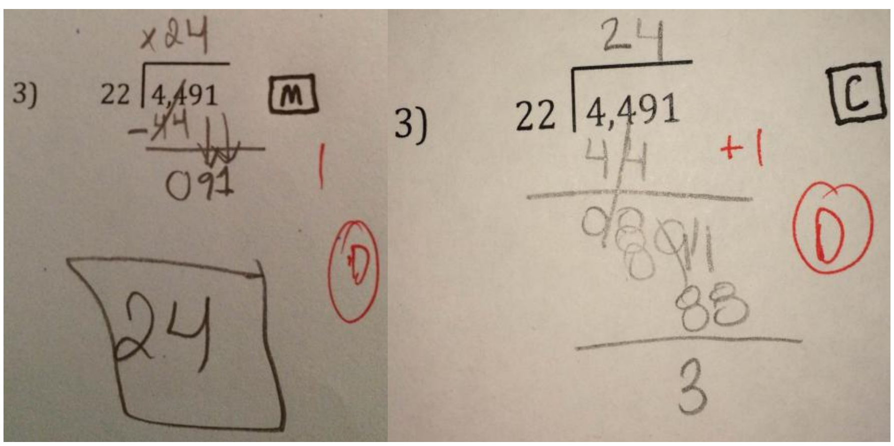

Figure 19. Two participant responses to Problem 3.

In the above sample, the participants failed to include the place holding zero in the quotient. The most common incorrect answer to this problem was a quotient of 24 instead of the correct quotient, 204. This is reflected in both the left participant and right participant's work. Each of them correctly began the long division algorithm by dividing 44 by 22 and getting a quotient of 2. After this, the participants disregarded the step

9 that was required next and instead brought down both the 9 and 1 in the dividend

and divided 91 by 22, yielding a quotient of 24. Of the participants who made mistakes on this problem, failure to complete the step was the most common. Participants of all age groups also did not have a sound understanding of trailing zeros as seen in the figures of student work samples below in Figure 20.

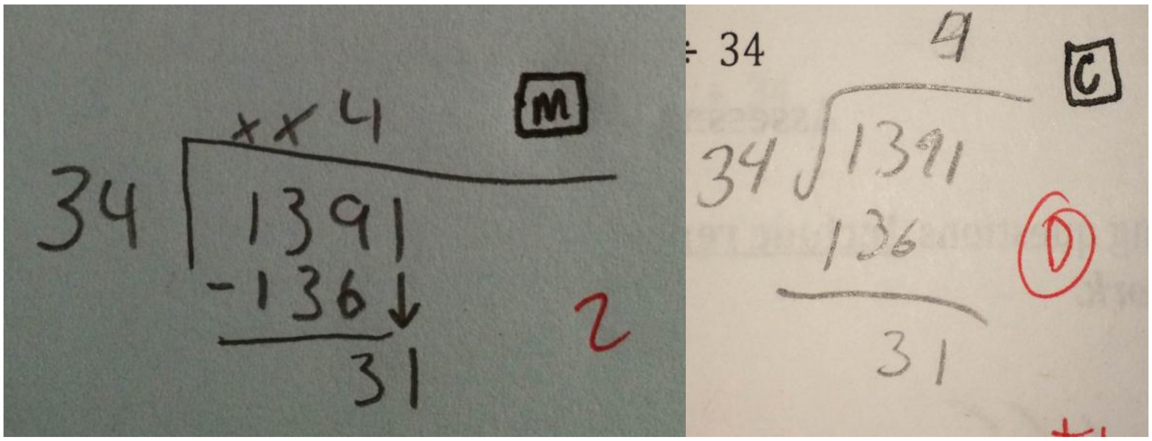

Figure 20. Two participant solutions to Problem 6.

Many participants who did not receive full credit on this problem had work similar to the work shown above. Both participants correctly began the long division algorithm and correctly carried out , which yielded a quotient of 4 and a remainder of 3. Both participants also correctly carried down the 1 from the dividend. However, at this step, both participants displayed their lack of understanding of trailing zeros and zero quotients as they did not carry out , which left the problem incomplete. Altogether, I was surprised not to see more participants check their work when they came to an answer of 4 remainder 31 since this answer is easy to identify as being far too small. As expected, the most difficult problem on the assessment was problem 7 and participants provided a multitude of answers that displayed their misunderstanding or lack of knowledge about a zero quotient. It is important to remember that on all problems, I asked for a quotient and a remainder rather than a decimal answer, and this problem produced an overwhelming number of decimal responses. Figure 21 below shows some of the most frequent incorrect answers.

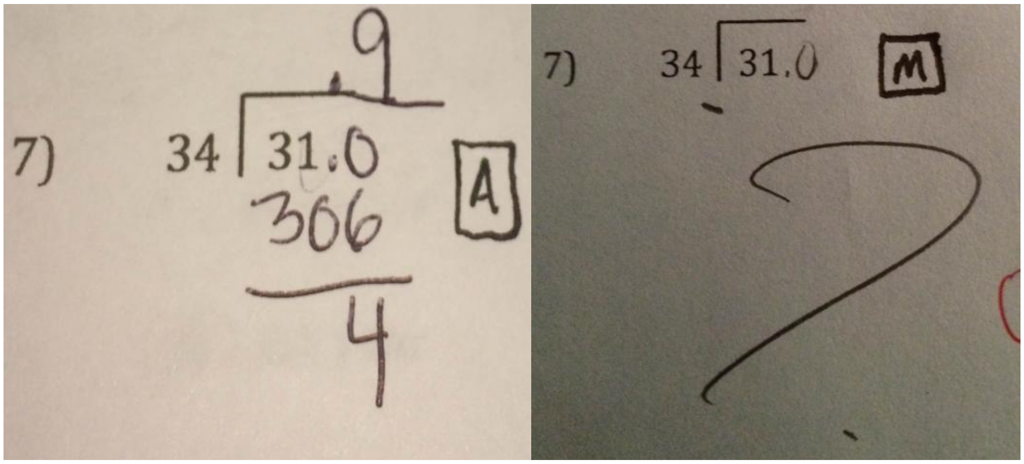

Figure 21. Two participant responses to Problem 7.

The participant on the left above correctly recognized that 34 did not divide 31, but proceeded directly to including a decimal. Many incorrect answers had a quotient that began with .9 even though the directions clearly stated not to use decimal. Among the other incorrect answers were answers like the participant's on the right. Many participants did not even recognize that a zero quotient could exists and either left it blank or wrote and answer such as a question mark or a comment stating that they did not know how to complete it. While the participant on the left did attempt an answer, both participants and most participants who did not correctly complete this problem showed a similar lack of understanding of zero quotients. Throughout these and other problems on the assessment, participants often made mistakes that were the byproduct of incorrectly performing another operation within the long division algorithm. Figure 22 on the following page shows work samples in which participants did not multiply correctly.

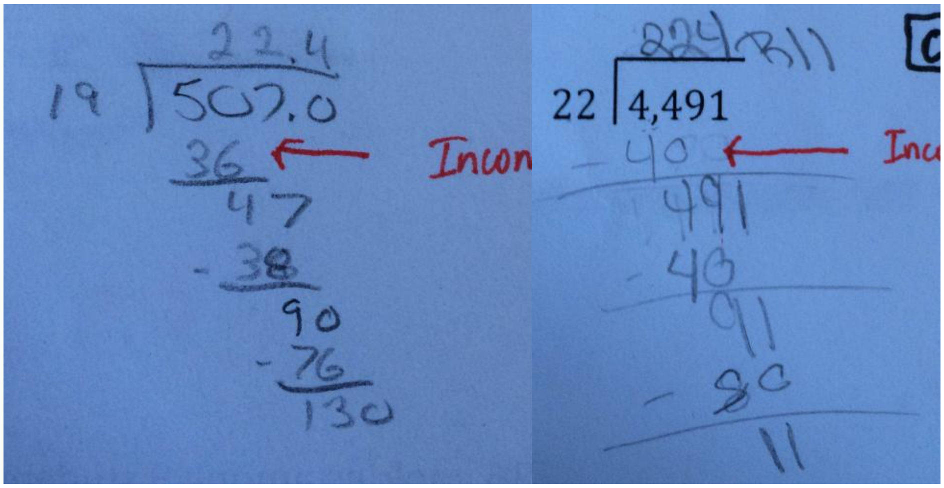

Figure 22. Two participant solutions that contain multiplication errors.

Both participants above did not multiply correctly even though they correctly found the first digit in the quotient. Participants who made multiplication mistakes often made them early in the long division algorithm. As seen in the samples above, making a multiplication mistake early in the algorithm causes the whole answer to be incorrect even if the algorithm is performed correctly from there on out. Following the assessment, participants took a survey that asked them when the last time they remembered using long division was. While there was no significant difference between assessment performance and length of time using long division, the answer to the survey question provided intriguing results. Most participants ages 12-17 remembered using long division 3-5 years ago, but also accounted for the least number of participants who recalled using it more than 5 years ago. Perhaps the most surprising part of this data is that more college students recalled using long division 5 or more years ago than adults. Also, a larger percent of adults have used long division this year than middle school students. Since adults claim to have used long division more recently, it could provide an explanation as to why there was not a significant difference in performance between the adults and the 12-17 year old participants on the assessment. Despite the fact that there was no conclusive evidence that the ability to do long division is lost over time, examining the previously shown work samples as well as the rest of the assessments and surveys reveled necessary points of emphasis teachers must make so that the long division algorithm is understood by students.

Implications for Teaching

This study tested the performance of multiple age groups on long division problems. Through scoring and examination of the assessments, the study also revealed some of the underlying misconceptions and common mistakes within the long division algorithm. The survey completed by the participants following the completion of the assessments collected information about the individuals' background, mathematics experience, profession, and dispositions about long division in the curriculum. Based on participant performance on the assessment and their comments on the survey, three implications for educators emerged.

Implication #1: Make sure students achieve mastery of other operations before trying to master long division.

The key to completing the long division algorithm correctly depends on a student's ability to complete operations such as multiplication, addition, and subtraction. Though the overwhelming majority of participants chose division as the operation that they felt was the most challenging, about a quarter of the mistakes made throughout the assessment were the results of incorrect multiplication, subtraction, or addition.

For teachers, it is important that students are exposed early to the operations that will be used in their future for long division. The idea of subtraction and addition appear as early as Pre-K in the New York State Common Core Standards for Mathematics and are formally introduced as problems by the kindergarten level. Foundations for multiplication are covered in the standards at the second grade level and multiplication is introduced at the third grade level. The concept of division is also introduced within the third grade standards.

For teachers, it is important to make sure students have an adequate understanding of each preceding operation before division. This means making sure that learners of all learning styles understand multiplication, addition, and subtraction. There are many hands on ways to teach these operations today such as integer chips and multiplication circles. Integer chips are a great way to teach both addition and subtraction by using colored chips that represent positives and negatives. The chips give students a visual aid to see how subtraction or addition is completed. Multiplication can be taught by using multiplication tables, flashcards or games. A card game called multiplication war where students follow traditional 'war' rules, but throw down two cards and must find the multiple of the two to determine who's is larger is an interactive way for students to master multiplication. Furthermore, it is important for the educator to assess the student's skills on each of the operations before moving to the next. This may include giving formal assessments in the form of quizzes, tests or homework or using informal assessments in class by asking students to solve problems or observing how quickly and efficiently students work on given problems.

Implication #2: Ensure students have a sound understanding of place values within the division algorithm and "place holding zeros."

Another common mistake throughout the assessment involved place holding zeros and trailing zeros within the quotient of division problems. Participants across all age levels often forgot to put a place holding zero in the quotient when the divisor was larger than the dividend at certain point within the long division algorithm. Place value is first addressed in the Common Core Standards at the first grade level. A basic understanding of place value should allow students to notice that a place holding zero must be placed in the quotient. As a teacher it is important to emphasize that it is possible for a divisor to divide a dividend zero times and that the zero can not be skipped in the quotient. The same can be said for trailing zeros. It is important for students to understand that the division process is not over until all of the numbers in the dividend have been accounted for or used in the division process.

To promote understanding of these concepts among students, a good method is having them guess an answer and then check their work after completing the problem. Not only does having students make an educated guess promote number sense, it allows them to make a connection with their guess and their final answer and evaluate how accurate their answer is. For example, if a student were asked to solve the problem , a good starting guess would be 40 since is 1200. The students should then recognize that since the dividend is greater than 1200, then the quotient should be slightly greater than 40 or the quotient is 40 and there is a remainder. In this specific problem, a trailing zero is required in the quotient, 40, and there is a remainder. For students who do not include the trailing zero, it will be easy to see that their answer is far smaller than their guess. In fact for any answer where a student forgets a trailing zero or place holding zero, they will be able to recognize that their answer is far smaller than their initial guess. This causes the student to evaluate where they went wrong or if the answer is off by a multiple of 10, students may be able to recognize that they forgot to include a trailing zero(s).

Implication #3: Should the long division algorithm still be taught?

After analyzing data gathered from assessments, it is clear that some people from each generation do not recall how to do long division. Moreover, when asking participants on the survey whether they thought long division is an important skill to know or not, participants of all ages pointed out that long division should not be necessary anymore because problems can be solved using calculators.

Today, long division is targeted for de-emphasis from the curriculum mostly due in part to reform mathematics that teaches the concept of division in multiple different ways. Methods like chunking and the one seen on the following page in Figure 23 are intended to show students a clearer meaning of division and what division really is. However, an important purpose of mathematics is to complete problems as efficiently as possible. The division algorithm is the most efficient way to solve division problems and the bottom line is whether teachers and educators value understanding or efficiency more when it comes to long division.

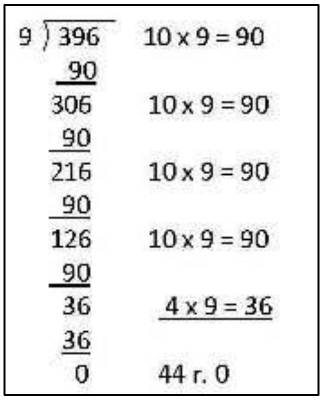

Figure 23. Division reform method. Ford, B. (2014, February 26). Why should I teach more than one way to do computations? The division procedure, part four. [Web log post]. http://www.aimsedu.org/2014/02/26/why-should-i-teach-more-than-one-way-todo-computations-the-division-procedure-part-four/

A reform method like the method above, which involves continually subtracting by multiples of 9, may help students conceptualize the idea of division as repeated subtraction. This idea can be translated to show why the general long division algorithm works. While educators can use such reform methods to develop an understanding of division, students should not be encouraged to continue to use them throughout their schooling. Students should be taught the long division algorithm, as it is the quickest, most efficient way to carry out division of integers and division of polynomials students may encounter later into the math curriculum.

Suggestions for Future Research

Although this research did not yield many conclusive results regarding the idea that long division is forgotten over time, further research may potentially yield interesting results. Gathering a sample of participants country-wide, or perhaps, world-wide may yield different, more significant results. Moreover, a larger, more diverse sample would allow the researcher to further examine the results and make more conclusions about misconceptions and common mistakes throughout the long division algorithm. This data could be broken down into country, region, or state to see if the misconceptions revealed in this study hold true throughout other areas of the world. Age groups can also be examined by area to determine if there is any area in which younger students outperform adults on long division. The focus of future research, does not need to focus on memory however, as it can focus on long division performance itself through the country or for a certain sample. It may be important for educators and administrators to know where their students stand when it comes to long division compared to other districts or students in other areas. The misconceptions and mistakes among division problems can also be examined throughout different areas to determine what a certain area needs to focus on, or if there are common teaching points that must be focused on when teaching the long division algorithm.

Concluding Remarks

Long division is becoming a math pastime with new reform methods and its devalue from the curriculum, causing people to question its necessity. It is, however, still being taught throughout the education system. It may seem like long division is a useless skill to know and could be something that is easily forgotten over time, but there was no difference in performance between 12-17 year olds and adults 24 years old and older on the long division based assessment. Adults and 12-17 year old students did outperform the college students by a significant margin, but it would take further research to back this result and show that the college students were not just showing little effort on the assessment. The underlying question surrounding long division is whether or not it should still be included in the curriculum. If so, educators must make extra effort to ensure that their students are well prepared to take on the long division algorithm. This is done by ensuring students achieve mastery of basic operations like addition, subtraction, and multiplication as well as have a sound understanding of place values and the basic process of the long division algorithm.

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Appendix A

Assessment

Answer the following questions. Include remainders, do not compute decimals. Show all of your work.

1)

2)

3)

4)

5) If you want to split up a box of 36 cookies among 9 students, how many cookies does each student receive?

6) Use ***long division*** to solve

7)

8)

9) Use ***long division*** to solve

Appendix B

Survey

1. Did any question on the assessment stand out as being more difficult than the others? Why so?

2. What is your gender?

3. When is the last time you remember using long division? (Select one)

   [ ] This year

   [ ] Last year

   [ ] 3-5 years ago

   [ ] 5 or more years ago

4. Circle the operation that you consider most challenging:

   Multiplication

   Addition

   Division

   Subtraction

5. On a scale from 1 to 10 where 1 means very little and 10 means a lot, how much do you enjoy math?

6. What is your age?

7. Do you think long division is an important skill to know? Why or why not?

Appendix C

Parental Consent Form

Fredonia

Hi, my name is Steve Sturm and I am a graduate student in the Math 7-12 program at Fredonia. I am conducting this study to complete a portion of my master's thesis. The purpose of this study is to evaluate the understanding of the long division algorithm across three generations.

Your participation in this study is greatly appreciated. Please read the information on the following page that outlines your participation this study. Print and sign your name on the third page to indicate your agreement to participate. Feel free to retain a copy of this letter for your files. Thank you for your full cooperation regarding this request.

TO: Parents/Guardians of Students
FROM: Steve Sturm
RE: Consent Form

Purpose, Procedures, and Benefits

- The purpose of this study is to determine the knowledge and perception of the long division algorithm across multiple age groups.

- Students will be given an assessment and survey to complete containing questions directly related to long division.

- This study is designed to not only provide potential benefits for your student, but to math education and its' best practices as a whole.

Related Information

- To maintain confidentiality, neither your student's name nor yours will be used at any point throughout the study. The materials will be kept in the researcher's office at all times and will be destroyed within three years of completion of the study.

- There is no cost to participate in this study and students will not be compensated for participation.

- Participation in this study is voluntary. Students may opt out or withdraw at any time. If at any time students become uncomfortable or feel the need to talk to someone, they will have the ability to discuss the matters with their teacher or principal. The study will last around a month, however your students' required participation is just a single assessment.

- There are no risks associated with this study as it is conducted for educational purposes only.

- Please read over and discuss the information with your students to make sure everyone is fully aware of the information included in this study.

- For additional information regarding this study or any other questions you may have, please contact any of the following by phone or email:
  - Mr. Sturm, 716-880-0708, stur5365@fredonia.edu
  - Dr. Keary Howard: Mr. Sturm's college advisor, 716-673-3873, keary.howard@fredonia.edu
  - Judith Horowitz: Human Subjects Protection Administrator at Fredonia, 716-673-3335, Judith.horowitz@fredonia.edu

Thank you in advance for your participation in this study. Please complete the attached consent form and return with your student.

Voluntary Consent: I have read this memo and have been fully apprised of this study and all that it entails. My signature indicates that I agree to allow my son/daughter to participate in this study. If I withdraw my son/daughter from this study, I understand that there will be no penalty assessed to him/her. I understand that my son/daughter's confidentiality will be maintained throughout the study's entirety. I understand that if I have any questions about the study, I may contact Steve Sturm at stur5365@fredonia.edu

Parent/Guardian Name (please print):____________________________________

Parent/Guardian Signature:____

Date:______________________

Appendix D

Student Consent Form

Fredonia

Hi, my name is Steve Sturm and I am a graduate student in the Math 7-12 program at Fredonia. I am conducting this study to complete a portion of my master's thesis. The purpose of this study is to evaluate the understanding of the long division algorithm across three generations.

Thank you for participating in this study. Please read the information on the following page that entails your participation in this study. Print and sign your name on the following page to show that you agree to be a part of this study. Remember that signing the form allows Mr. Sturm to use your data for this research project.

TO: Students
FROM: Steve Sturm
RE: Consent Form

- You are being asked to participate in a research study.

- To participate, you will be asked to complete a nine-question assessment and a following survey regarding the material being studied.

- By signing the consent form, you are allowing the researcher to examine your work on the assessment and survey.

- Your name will never be used in any way. Your assessment will be kept in Mr. Sturm's office at all times and will be destroyed within three years of completion.

- You will not be allowed to seek assistance or work with others throughout the entirety of this study.

- There are no risks to you involved in this study. The main focus of the study is to make math education better as a whole.

- There is no penalty for not signing the consent form. You are also allowed to back out of the study at any point without penalty (If you decide you don't want to do it, it's fine).

- You will not get any rewards for participating in the study.

- Your participation in the study is only needed for one day to complete the assessment.

Please discuss this with your parent or guardian. If you or they have any questions, feel free to ask. Please sign and return the original consent form on the following page as soon as possible.

Voluntary Consent: I have read this memo and I am fully aware of all that this study involves. My signature below shows that I freely agree to participate in this study. I understand that there will be no penalty for not participating and that I may withdraw from the study at any time. I understand that my name or any personal information will not be included in the study at any time. I understand that if I have any questions about this study, I may contact Mr. Sturm at stur5365@fredonia.edu

Student Name (please print):____

Student Signature:____

Date:________________________

Parent/Guardian Signature:____

Date:________________________

Appendix E

Consent Form

Fredonia

Hi, my name is Steve Sturm and I am a graduate student in the Math 7-12 program at Fredonia. I am conducting this study to complete a portion of my master's thesis. The purpose of this study is to evaluate the understanding of the log division algorithm across three generations.

Your participation in this study is greatly appreciated. Please read the information on the following page that entails your participation in the study. Print and sign your name on the third page to indicate your agreement to participate in the study. Feel free to retain a copy of this letter for your files. Thank you for your full cooperation regarding this request.

TO: College Participants
FROM: Steve Sturm
RE: Consent Form

Purpose, Procedures, and Benefits

- The purpose of this study is to determine the knowledge and perception of the long division algorithm across multiple age groups.

- You will be given an assessment and survey to complete containing questions directly related to long division.

- This study is designed to not only provide potential benefits for you, but to math education and its' best practices as a whole.

Related Information

- To maintain confidentiality, your name will not be used at any point throughout the study. The materials will be kept in the researcher's office at all times and will be destroyed within three years of completion of the study.

- There is no cost to participate in this study and you will not be compensated for participation.

- You will not be allowed to seek assistance or work with others throughout the entirety of this study.

- Participation in this study is voluntary. You may opt out or withdraw at any time. If at any time you become uncomfortable or feel the need to talk to someone, they will have the ability to discuss the matters with their teacher or principal.

- There are no risks associated with this study as it is conducted for educational purposes only.

- Please read over and discuss the information to make sure you are fully aware of the information included in this study.

- For additional information regarding this study or any other questions you may have, please contact any of the following by phone or email:
    - Mr. Sturm, 716-880-0708, stur5365@fredonia.edu
    - Dr. Keary Howard: Mr. Sturm's college advisor, 716-673-3873, keary.howard@fredonia.edu
    - Judith Horowitz: Human Subjects Protection Administrator at Fredonia, 716-673-3335, Judith.horowitz@fredonia.edu

Thank you in advance for your participation in this study. Please complete the attached consent form and return.

Voluntary Consent: I have read this memo and have been fully apprised of this study and all that it entails. My signature indicates that I agree to participate in this study. If I withdraw from this study, I understand that there will be no penalty assessed. I understand that my confidentiality will be maintained throughout the study's entirety. I understand that if I have any questions about the study, I may contact Steve Sturm at stur5365@fredonia.edu .

Participant Name (Please Print):____

Participant Signature:____

Date:________________________________