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Department of Mathematical Sciences

CERTIFICATION OF PROJECT WORK

We, the undersigned, certify that this project entitled NUMERACY AND MATHEMATICAL PREPAREDNESS OF HIGH SCHOOL STUDENTS FOR INTRODUCTORY LEVEL COLLEGE MATHEMATICS COURSES by Ashley R. Martin, Candidate for the Degree of Master of Science in Education, Mathematics Education, is acceptable in the form and content and demonstrates a satisfactory knowledge of the field covered by this project.

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ARE YOU SMARTER THAN A HIGH-SCHOOLER?  
NUMERACY AND MATHEMATICAL PREPAREDNESS OF HIGH SCHOOL  
STUDENTS FOR INTRODUCTORY LEVEL COLLEGE MATHEMATICS  
COURSES

by

Ashley R. Martin

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## **Abstract**

This research examines the ability of students in introductory level college mathematics courses to recall fundamental information they learned in high school mathematics courses. During the first week of the Spring 2012 semester, students from three college mathematics classes were given a nineteen-problem quiz that consisted of problems on high school mathematics topics. Immediately following the quiz, the students were asked to complete a six-question survey which was used to measure students' prior mathematical knowledge, their outlook on mathematics, and how easily the students felt they could complete the quiz based on their ability to recall previously learned material. Results from the quiz and survey were compared and analyzed to draw conclusions. At the conclusion of this research study, it was determined that a significant difference existed in the students' scores on individual questions based on the type of mathematics problem and a significant difference existed in the students' total quiz scores based on their previous mathematics experience.

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## **Introduction**

This research examines the ability of students in introductory level college mathematics courses to recall fundamental information they learned in high school mathematics courses. It is a requirement in New York State colleges that each student must complete three credits of college mathematics. As a result of this requirement, college professors have expectations for their students and students' abilities to recall information they learned in high school.

However, students in college level mathematics courses may not remember important concepts they learned in high school and as such may not be able to apply knowledge of these concepts to new college mathematics material they learn.

This topic is of interest to me because this year I taught an introductory college mathematics course. At the beginning of the Fall 2011 semester, the mathematics knowledge base that my students had was a prominent question. As I began teaching I found that although most of my students are college freshmen who just graduated from high school, they seemed to have difficulty in successfully completing mathematics assignments for this course. By researching this topic I hoped to gain a better understanding of the preparation of high school students for university mathematics courses. The preparation of college students for university mathematics courses was researched to gain an understanding as to why students in introductory college mathematics courses appear to have difficulty in successfully completing assignments they are given. By having a better understanding of the preparation that high school students receive for university mathematics courses an improvement in teaching practices at both college and high school level mathematics will be able to occur.

The ability to recall information that students have learned in high school is a factor that may affect their performance in core curriculum college classes. By obtaining a knowledge base

from students which provides information regarding their ability to recall from high school, college and high school mathematics instruction will be easier to direct.

This study focuses on determining students' recall of high school mathematics, which helps in determining what information they are able to build on when learning new college mathematics material.

*It is hypothesized that introductory level college students are unable to recall and demonstrate knowledge of many Algebra, Geometry, and Algebra II Trigonometry topics that they learned in high school. Specifically, students will have difficulty solving problems related to Graphing Linear Equations and applying the Quadratic Formula.*

The hypothesis statement was tested by creating a nineteen problem quiz which contains fundamental New York State high school mathematics content including Statistics, Graphing Linear Equations, Solving an Equation for a Variable, Area of a Triangle, Volume of a Box, Order of Operations, Proportions, FOIL, Plotting and Labeling Points, Probability, Quadratic Formula, Pythagorean Theorem, Transformations, Solving a Quadratic Equation, Sequences, Similar Triangles, Exponents, Circle Geometry, and Fractions. This quiz was given to students who currently attend SUNY Fredonia and who are taking introductory level college mathematics courses including MATH 108 and MATH 110. A comparison of students' scores on this quiz and survey responses were determining factors in the level of mathematics training the students had previously completed. By comparing these results, an understanding of which problems and topics students struggle with the most and have the most difficult time reproducing was analyzed.

### **Literature Review**

To examine the existing research pertaining to high school and college level mathematics courses and curriculum a literature review was completed in an attempt to define a context for further research. According to Martin (2007), “Mathematical literacy implies that a person is able to reason, analyze, formulate, and solve problems in a real-world setting”(p. 29). To define a student’s mathematical literacy prior to entering a college class, textbook content and content standards for high school students were studied. Then, textbook curriculum and researchers’ approaches to teaching liberal arts mathematics courses are examined in an attempt to define the type of material that students learn in a liberal arts mathematics course. Next, information provided by researchers regarding GED requirements and Adult Learners mathematics standards are examined in an attempt to understand the students’ knowledge base in the area of mathematics. Then the SAT mathematics curriculum and research regarding test preparation are studied in an attempt to understand the mathematics base that students are expected to have prior to college acceptance. Next, research regarding preparedness of high school students for college curriculum is examined in the hopes of determining the relationship between high school and college level mathematics curricula as well as professor’s expectations of liberal arts mathematics students. Finally, a brief synopsis of essential mathematics topics that appeared in each section of this literature review will be provided to direct focus for this research study.

### **High School Mathematics Curriculum and Standards**

Hundreds of available resources present the Integrated Algebra, Geometry, and Algebra II and Trigonometry curriculum. Several of these textbooks are useful in providing expected learning outcomes, explanations, and examples of the New York State Integrated Algebra,



Geometry, and Algebra II and Trigonometry curriculum that are explored here (Casey(b), 2010; Florio, 2010; Long, 2006; NYSED, 2005; Primiani & Caroscio, 2010; *Teacher Annotated Edition: Integrated Algebra*, 2010).

In the textbook *Teacher Annotated Edition: Integrated Algebra* published by Glencoe/McGraw Hill (2010) the authors begin by separating the Integrated Algebra Curriculum into individual lessons that are divided into thirteen chapters, each lesson directly associated with specific NYS Integrated Algebra Core Curriculum Standards. The chapter content for this textbook includes: Sets and Real Numbers; Expressions, Equations, and Inequalities; Solving Equations and Inequalities; Polynomials and Factoring; Slope and Linear Equations; Linear Functions and Relationships; Systems of Equations and Inequalities; Quadratic and Nonlinear Functions; Rational Expressions and Equations; Right-Triangle Relationships; Measurement and Geometry; Data Analysis and Statistics, and Probability (p. iii-xi). This textbook also includes a section that is titled “How to Master the New York Exam”(p. xiii), which provides students and teachers with a description of the Integrated Algebra Exam as well as sample exam questions that are useful in preparing for the Integrated Algebra Exam.

The textbook *Prentice Hall Brief Review for the New York State Regents Exam Geometry* by Primiani & Caroscio (2010b) separates the Geometry curriculum into nine chapters, each section of a particular chapter directly correlates to specific NYS Geometry Core Curriculum Standards (2010). The chapter titles for this textbook include: Basic Geometry in the Plane, Mathematical Statements, Triangles, Relationships with Triangles, Quadrilaterals and Other Polygons, Coordinate Geometry, Transformational Geometry, Angles in Circles, and Solid Geometry and Its Applications (p. iii-vii). Each of these chapters are then divided into separate lessons that teach Geometry topics such as Constructing Perpendicular and Parallel Lines,

Conditional and Biconditional Statements, Triangle Congruence, Pythagorean Theorem and Its Converse, Proofs on Special Quadrilaterals, Systems of Equations, Proofs with Transformations, Chords of a Circle and Segments Intersected by a Circle, and Perpendicular and Parallel Planes (p. iii-vii). Throughout this textbook, examples of Geometry Exam problems and solutions are also included to provide students with practice for taking the actual Geometry Regents Examination.

In the textbook *Prentice Hall Brief Review for the New York State Regents Exam Algebra II and Trigonometry*, Primiani & Caroscio (2010a) separate the Algebra II and Trigonometry curriculum into sixteen chapters, each section of a particular chapter directly correlating to specific NYS Algebra II and Trigonometry Core Curriculum Standards. The chapter titles for this textbook include: The Real Number System; Exponents and Radicals; Rational Expressions; Polynomial Operations; Linear Equations and Inequalities; Quadratic Functions and Equations; Applications of Quadratic Functions; Relations, Functions, and Graphs; Complex Numbers; Polynomial Functions and Variations; Exponential and Logarithmic Functions; Circular Functions; Trigonometric Equations and Identities; Sequences and Series; Statistics; and Probability (p. iii-x). Throughout this textbook, examples of Algebra II and Trigonometry Exam problems and solutions are also included to provide students with practice for taking the actual Algebra II and Trigonometry Regents Examination.

Through exploring textbook content and curriculum for the Integrated Algebra, Geometry, and Algebra II and Trigonometry exams, a mathematical knowledge base of high school students who completed each of these three exams and courses can be determined. Through understanding content that was previously taught to high school students, a better correlation can be created when comparing content taught in liberal arts mathematics courses.

Figure 1 summarizes the topics that were prominent in the Integrated Algebra Curriculum, the Geometry curriculum, and the Algebra II and Trigonometry curriculum in New York State.

**Figure 1: High School Mathematics Curriculum and Standards**

<b>Integrated Algebra Topics</b>	<b>Geometry Topics</b>	<b>Algebra II and Trigonometry Topics</b>
Sets and Real Numbers	Basic Geometry in the Plane	The Real Number System
Expressions, Equations, and Inequalities	Mathematical Statements	Exponents and Radicals
Polynomials and Factoring	Triangles	Rational Expressions
Slope and Linear Equations	Relationships with Triangles	Linear Equations and Inequalities
Linear Functions and Relationships	Quadrilaterals and Other Polygons	Quadratic Functions and Equations
Systems of Equations and Inequalities	Coordinate Geometry	Polynomial Functions and Variations
Quadratic and Nonlinear Functions	Transformational Geometry	Exponential and Logarithmic Functions
Rational Expressions and Equations	Angles in Circles	Circular Functions
Right-Triangle Relationships	Solid Geometry and its Applications	Trigonometric Equations and Identities
Measurement and Geometry	Pythagorean Theorem and its Converse	Sequences and Series
Data Analysis and Statistics		Statistics
Probability		Probability

After examining the high school mathematics curriculum and standards as they are listed in high school mathematics textbooks, a knowledge base of high school students was determined. To determine the mathematics knowledge base that high school students will need to know as they enroll in college mathematics courses, an analysis of college mathematics curriculum and standards will also be conducted.

### **College Mathematics Curriculum and Standards**

Many resources are available which explain the content that is taught in liberal arts mathematics courses. Several of these textbooks are useful in providing examples and explanations of liberal arts mathematics content that are explored here (Barnett, 2011; Blitzer, 2007; Strasser, 2011; Tannenbaum, 2010).

The textbook that is currently being utilized in Survey of Calculus I at SUNY Fredonia is *Calculus: For Business, Economics, Life Sciences, and Social Sciences* by Barnett et al. (2007). Barnett begins this textbook by separating it into twelve chapters which are titled: Linear

Equations and Graphs, Functions and Graphs, Limits and Derivatives, Additional Derivative Topics, Graphing and Optimization, Integration, Additional Integration Topics, Multivariable Calculus, Trigonometric Functions, Differential Equations, Taylor Polynomials and Infinite Series, Probability and Calculus (p. vii-ix). Several of these chapters are then divided into separate lessons that teach Calculus topics that relate to topics that were taught in high school Integrated Algebra, Geometry, and Algebra II and Trigonometry classes such as Linear Equations and Inequalities, Graphs and Lines, Linear Regression, Functions, Graphs and Transformations, Quadratic Functions, Polynomial and Rational Functions, Exponential Functions, Logarithmic Functions, and many other concepts (p. vii-ix).

The textbook that is currently being utilized in Mathematics in Action at SUNY Fredonia is *Excursions in Modern Mathematics* by Tannenbaum (2010). Tannenbaum begins this textbook by separating it into sixteen chapters which are titled: The Mathematics of Voting; The Mathematics of Power; The Mathematics of Sharing; The Mathematics of Apportionment; The Mathematics of Getting Around; The Mathematics of Touring; The Mathematics of Networks; The Mathematics of Scheduling; Fibonacci Numbers and the Golden Ratio; The Mathematics of Money; The Mathematics of Symmetry; The Geometry of Fractal Shapes; Collecting Statistical Data; Descriptive Statistics; Chances, Probabilities, and Odds; and The Mathematics of Normal Distributions (p. vii-xiv). Several of these chapters are then divided into separate lessons that teach Mathematics in Action topics that include topics that were taught in high school Integrated Algebra, Geometry, and Algebra II and Trigonometry classes such as Percentages, Reflections, Rotations, Translations, Glide Reflections, Symmetry as a Rigid Motion, Patterns, Populations, Sampling, Sampling Terminology and Key Concepts, Graphical Descriptions of Data, Variables,

Permutations and Combinations, Probability Spaces, Normal Curves and Normal Distributions, and Statistical Inference (p. vii-xiv).

The textbook *Finite Mathematics: For Business, Economics, Life Sciences, and Social Sciences* by Barnett et al. (2011) is currently being used in Prize-Winning Mathematics at SUNY Fredonia is. The authors begin this textbook by separating it into eleven chapters which are titled: Linear Equations and Graphs; Functions and Graphs; Mathematics of Finance; Systems of Linear Equations, Matrices; Linear Inequalities and Linear Programming; Linear Programming: Simplex Method; Logic, Sets, and Counting; Probability; Markov Chains; Games and Decisions; and Data Description and Probability Distributions (p. v-vii). Several of these chapters are then divided into separate lessons that teach Prize-Winning Mathematics topics that correlate to topics that were taught in high school Integrated Algebra, Geometry, and Algebra II and Trigonometry classes such as Linear Equations and Inequalities; Graphs and Lines; Elementary Functions: Graphs and Transformations; Quadratic Functions; Exponential Functions; Logarithmic Functions; Systems of Linear Equations; Inequalities in Two Variables; Logic; Sets; Permutations and Combinations; Random Variable, Probability Distribution, and Expected Value; and Normal Distribution (p. v-vii).

Through investigating textbook content and curriculum for Liberal Arts Mathematics classes we are able to compare content taught in college and high school level mathematics classes. Through understanding what mathematics content was taught in high school classes, college professors will be able to compare this knowledge base with new information that student will learn in their college mathematics class. Figure 2 summarizes the topics that were prominent in the Survey of Calculus I Curriculum, the Mathematics in Action curriculum, and the Prize-Winning Mathematics curriculum.

**Figure 2: College Mathematics Curriculum and Standards**

<b>Survey of Calculus I Topics</b>	<b>Mathematics in Action Topics</b>	<b>Prize-Winning Mathematics Topics</b>
Linear Equations and Graphs	Percentages	Linear Equations and Inequalities
Linear Equations and Inequalities	Reflections, Rotations, and Translations	Graphs and Lines
Graphs and Lines	Glide Reflections	Elementary Functions
Linear Regression	Symmetry as a Rigid Motion	Graphs and Transformations
Functions and Graphs	Patterns	Quadratic Functions
Functions	Populations	Exponential and Logarithmic Functions
Graphs and Transformations	Sampling	Systems of Linear Equations
Quadratic Functions	Graphical Descriptions of Data	Inequalities in Two Variables
Polynomial and Rational Functions	Permutations and Combinations	Logic and Sets
Exponential Functions	Probability Spaces	Permutations and Combinations
Logarithmic Functions	Normal Curves and Normal Distribution	Probability
Probability	Statistical Inference	Normal Distribution

After examining the college mathematics curriculum and standards as they are listed in college level mathematics textbooks, a knowledge base of college students was determined. Many students who attend college may not have completed high school and as such, may have obtained only a GED or its equivalent. Therefore, an analysis of GED requirements and Adult Learners Mathematics Standards will also be conducted.

### **GED Requirements and Adult Learners Mathematics Standards**

Many researchers have spent time studying the topics that are taught to students who wish to obtain their GED or attend a school as an Adult Learner. The mathematics content that is taught to these students is examined by many researchers (Comprehensive Adult Student Assessment System, 2008; Goschen, 1994; Ward, 2000; Van Groenestijn, 2001).

In the article by Goschen (1994), lesson plans and applications that were developed for adult learners to obtain mathematics skills that are needed for them to pass the General Educational Development Tests and obtain their GED are provided. The lesson plan that is included in this article which teaches mathematics concepts to Adult Learners who are pursuing their GED includes topics that relate to topics that were taught in high school Integrated Algebra, Geometry, and Algebra II and Trigonometry classes such as Whole Number Operations, Ratios

and Proportions, Exponents and Roots, Signed Numbers and Algebraic Expressions, Volume, Angles, Triangles and Quadrilaterals, Congruence and Similarity, Pythagorean Theorem, and Probability (p. 8-14).

Further research that was examined in this literature review includes research from another state other than New York State in order to gain insight into GED requirements in other states as well as New York State. Ward (2000) examines the education of mathematics students who are pursuing their GED in Arkansas. Through examining the instructional methods of instructors in 57 adult education centers, Ward found that the main focus of mathematics was on teaching manipulation of numbers rather than on problem solving, critical thinking, and mathematical reasoning (p. 6). Ward furthermore states that mathematics skills should encompass more than just repeated problems and should teach students skills that they will need to use in the future (p. 6).

The Comprehensive Adult Student Assessment System presents content standards for mathematics (2008). Content standards in this article include mathematics concepts that are taught to Adult Learners who are pursuing their GED. The mathematics concepts contained in this article relate to topics that were taught in high school Integrated Algebra, Geometry, and Algebra II and Trigonometry classes such as Patterns, Relationships, and Functions; One-Step Equations; Pythagorean Theorem; Solving Systems of Linear Equations and Inequalities; Solving Quadratic Equations; Exponents and Numbers; Graphing Linear and Quadratic Functions; Transforming Shapes to Create Different Figures; Area; Volume; and Probability (p. 3-12).

Through examining GED content and curriculum for adult learners we are able to compare content taught in adult basic education and high school level mathematics classes.

Through understanding what mathematics content is taught to adult learners and how it is related to their everyday life, college professors can compare what these students have learned with the content that they are teaching to their Liberal Arts Mathematics students. Figure 3 summarizes the topics that were prominent in the GED Curriculum, the Arkansas GED curriculum, and the Comprehensive Adult Student Assessment System curriculum.

**Figure 3: GED Requirements and Adult Learners Mathematics Standards**

<b>GED Topics</b>	<b>Arkansas GED Topics</b>	<b>Comprehensive Adult Student Assessment System Topics</b>
Whole Number Operations	Manipulation of Numbers	Patterns, Relationships, and Functions
Ratios and Proportions	Problem Solving	One-Step Equations
Exponents and Roots	Critical Thinking	Transformations
Signed Numbers and Algebraic Expressions	Mathematical Reasoning	Solving Systems of Linear Equations and Inequalities
Volume and Area		Pythagorean Theorem
Triangles and Quadrilaterals		Solving Quadratic Equations
Pythagorean Theorem		Volume and Area
Probability		Probability

In preparation for college entrance, students are required to complete entrance examinations. Examinations that are required by the majority of colleges and universities in the United States are the SAT and ACT. In New York State, student performances on Regent's Examinations are also taken into consideration. Therefore, an analysis of SAT, ACT, and Regent's examination topics will be completed.

### **SAT and Test Preparation for College Mathematics**

Many resources are available to students and educators that are solely devoted to preparing students for the Integrated Algebra, Geometry, Algebra II and Trigonometry, SAT, and ACT examinations. Several textbooks and research articles are useful in providing the content that will be present on these examinations as well as the frequency with which particular questions appear on the examinations that are explored here (Black & Anestis, 2011; Clemens &



Clemens, 2010; Costanzo, 2001; Foley-Peres & Poirier, 2008; Leff, 2009; *The Official SAT Study Guide*, 2009; Tamar, 2003).

The first SAT resource by Black & Anestis (2011) provides a list of mathematics topics that appear on the SAT that students take in order to gain entrance into a college or university (2011). The content that is included in this textbook which is present on the SAT and also contain topics that were taught in high school Integrated Algebra, Geometry, and Algebra II and Trigonometry classes include Finding Patterns; Thinking Logically; Numbers and Operations; Ratios and Proportions; Solving Equations; Working with Exponentials; Factoring; Inequalities; Mean, Median, and Mode; Probability; Triangles; Pythagorean Theorem; Areas and Perimeters; Volume; Sequences; Transformations; and Data Analysis (p. vi-vii).

Another exam resource book by Leff provides a list of topics as well as the frequency of their appearance on Regent's Geometry Examinations (2009). Topics that are included in this textbook that appear frequently on the Regent's Geometry Examination include Logic; Angles and Line Relationships; Parallel and Perpendicular Planes; Angles of a Triangle and Polygon; Triangle Inequalities; Trapezoids and Parallelograms; Proofs Involving Congruent Triangles; Indirect Proofs and Mathematical Reasoning; Ratios and Proportions; Pythagorean Theorem; and Coordinate Geometry (p. vi-vii).

A third exam resource book by Clemens & Clemens provide a list of topics as well as the frequency of their appearance on Regent's Geometry Examinations (2010). Topics that are included in this textbook that appear frequently on the Regent's Algebra II and Trigonometry Examination include Exponents and Radicals; Complex Numbers; Quadratics and Higher-Order Polynomials; Rationales; Absolute Value; Direct and Inverse Variation; Circles; Functions; Exponential and Logarithmic Functions; Trigonometric Functions; Trigonometric Graphs;

Trigonometric Identities; Trigonometric Laws and Applications; Sequences and Series; Statistics; Regressions; and Probability (p. viii-ix).

Through exploring the content and curriculum in SAT and Regent's exam preparation resources, a mathematical knowledge basis of students who completed each of these exams can be determined. Through understanding content that was present on these exams, a better correlation can be created when comparing content taught in liberal arts mathematics courses. Figure 4 summarizes the topics that were prominent on the SAT, Regents Geometry Exam, and the Regents Algebra II and Trigonometry Exam.

**Figure 4: SAT and Test Preparation for College Mathematics**

SAT Topics	Regents Geometry Topics	Regents Algebra II and Trigonometry Topics
Finding Patterns	Logic	Exponents and Radicals
Thinking Logically	Angles and Line Relationships	Complex Numbers
Numbers and Operations	Parallel and Perpendicular Planes	Rationales
Ratios and Proportions	Angles of a Triangle and Polygon	Absolute Value
Solving Equations	Triangle Inequalities	Direct and Inverse Variation
Working with Exponentials	Trapezoids and Parallelograms	Circles
Graphs and Transformations	Proofs Involving Congruent Triangles	Functions
Factoring	Indirect Proofs and Mathematical Reasoning	Quadratics and Higher-Order Polynomials
Inequalities	Ratios and Proportions	Exponential and Logarithmic Functions
Sequences	Pythagorean Theorem	Trigonometric Functions
Pythagorean Theorem	Coordinate Geometry	Statistics and Probability
Probability		Sequences and Series

After examining the curriculum and standards of high school, college, GED, and Adult Learner education as well as examination content and curriculum, an understanding of high school and college mathematics students can be better understood. An analysis of previous research that has been conducted which pertains to the preparedness of high school students for the college curriculum will now be conducted.

## **Preparedness of High School Students for College Curriculum**

Many researchers have spent time studying the preparedness of high school students for college curriculum through test performances, content comparison, and through a comparison of what expectations professors have for Liberal Arts students as well as students who major in Mathematics (Clery, 2006; Kepner, 2010; Rich & Walker, 2009; Shelton & Brown, 2008; Sworder, 2007).

The first researchers who spent time studying the preparedness of high school students for college curriculum are Shelton & Brown (2008) who base their research on the fact that although 60% of all community college students are placed into remedial level mathematics courses it is unclear if the students have had the opportunity to learn and demonstrate the skills that are required for success in college level classes. To conduct their research, the authors compared test results of college level mathematics students to their test results from the Algebra 1 and Geometry examinations. The authors found that one-third of the curriculum taught at the college level did not correlate to topics that were taught to high school students (p. 10-11).

In an article by Sworder (2007) a comparison of the success rates in liberal arts mathematics courses in two groups of students was conducted. The first group completed an intermediate algebra course that prepared students for less rigorous college mathematics courses, whereas the second group completed an algebra course which prepared them for more rigorous college mathematics courses. Although the group that took the less rigorous courses had a 90% success rate, and the group that took the more rigorous courses had only an 80% success rate, in a comparison of the curriculum and content covered in the classes, there did not appear to be any higher correlation between the classes that these groups of students enrolled in and their success in college mathematics courses from one group to the other (p. 19-22).

The next researcher who spent time studying the preparedness of high school students for college curriculum is Clery (2006) who examined the percentage of students who attempted and completed both developmental and college-level math in thirty-five Achieving the Dream institutions throughout the United States. Clery found that of the 36,246 students who were referred to developmental math only 6,353 students completed the program. Clery also found that of the 6,353 students who completed the developmental math program only 4,077 students attempted college-level math by the end of their third academic year, and only 3,041 actually completed college level math (p. 3). After researching this topic, Clery determined that students appeared to prolong the fulfillment of their math requirements and that many students might not understand college math requirements (p. 3).

Through investigating previous research that has been completed regarding the preparedness of high school students for college level mathematics classes, we are able to better understand not only what previous knowledge basis that students have, but we are also able to determine if preparedness of high school students for college level mathematics classes is a problem that we can see through comparing students performance in both high school and college level math courses. By understanding what content students have been previously taught and comparing it to the content that students are expected to know and learn when they are in a Liberal Arts Mathematics course, teachers can better guide their instruction in both high school and college level courses.

### **Essential Mathematics Topics**

After examining the existing research pertaining to high school and college level mathematics courses and curriculum, the following figure has been designed. Figure 5 provides a

list of critical and essential topics that have appeared in both the High School Mathematics Curriculum as well as the college liberal arts mathematics curriculum, the GED, Adult Learning curriculum, and the SAT. These topics will be included in a test that will be given to students in college level mathematics classes to attempt to provide an answer to the research question: what effect does the high school mathematics curriculum have on the preparation of high school students for college mathematics courses, specifically in New York State?

**Figure 5: Critical and Essential Mathematics Topics**

Circles	Solving Quadratic Equations	Quadratic Formula
Data Analysis and Statistics	Mean, Median, and Mode	Ratios and Proportions
Exponential and Logarithmic Functions	Normal Curves and Normal Distribution	Relationships with Triangles
Exponents and Radicals	Patterns, Relationships, and Functions	Sequences and Series
Factoring	Permutations and Combinations	Sets and Real Numbers
Linear Equations and Inequalities	Probability	Transformations
Logic	Pythagorean Theorem	Volume and Area

After synthesizing the mathematics topics that were prominent in the High School Mathematics Curriculum, the college liberal arts mathematics curriculum, the GED, Adult Learning curriculum, and the SAT, an understanding of high school and college mathematics students can be better understood. The critical and essential mathematics topics that were determined through conducting this Literature Review will be used as a basis for the experimental design in this study.

### **Experimental Design**

After examining textbook and examination content as well as previous research that has been conducted, careful consideration was given to the experimental design in this study. The participants, experimental design, instrument items, and method of data collection will be discussed in this section.

## Participants

This study was conducted at a four year liberal arts comprehensive university in Western New York. The participants in this study consist of 83 college students including 34 males and 49 females. The participants in this study were predominately Caucasian and were traditional college-age students. The students were enrolled in a section of either Mathematics in Action or Prize-Winning Mathematics. The Mathematics in Action course had 35 students: 24 freshmen, four sophomores, four juniors, and three seniors. The first section of the Prize-Winning Mathematics course had 21 students: eight freshman, six sophomores, two juniors, and five seniors. The second section of the Prize-Winning Mathematics course had 27 students: 11 freshmen, eight sophomores, six juniors, and two seniors. The Mathematics in Action course that the subjects were enrolled in was taught by this researcher.

All three of these courses are college core curriculum courses and as such, students majoring in Mathematics or Adolescence Mathematics Education are typically not permitted to take them. Figure 6 summarizes the course and class year of each of the participants in this study.

**Figure 6: Subject Course and Class Year**

Year	Mathematics in Action	Prize-Winning Mathematics Section 1	Prize-Winning Mathematics Section 2
Freshman	24	8	11
Sophomore	4	6	8
Junior	4	2	6
Senior	3	5	2
Total	35	21	27

**Design**

This experiment was designed to test the hypothesis that introductory level college students are unable to recall and reproduce many Algebra, Geometry, and Algebra II Trigonometry topics that they learned in high school. The experiment consisted of a 19-problem quiz that was followed by a short six-question survey. A copy of the quiz can be found in Appendix A and a copy of the survey is included in Appendix C. The quiz was administered to students in one of the three courses on their first day of class, before they began any college level mathematics instruction to assess their prior knowledge of mathematics and their ability to recall high school mathematics material. The students were given 20 minutes to complete the quiz and 2 minutes to complete the survey before each one was collected. Immediately following the quiz, the students were asked to complete the survey which was used to measure students prior mathematical knowledge, their outlook on mathematics, and an analysis of how easily the students felt they could complete the quiz based on their ability to recall previously learned material.

- 19-problem quiz administered for 20 minutes (Appendix A)
- 6-question survey administered for 2 minutes (Appendix C)

**Instrument Items and Justification of Items**

The quiz items were designed to assess the students' knowledge of various high school mathematics topics. The problems that were included in this quiz were designed using previous New York State Regents Exam problems, SAT problems, problems that were suggested to be included by college professors at SUNY Fredonia, and questions that were previously included on Mathematics in Action Exams and Quizzes during the Fall 2011 Semester.

The data that was given special consideration was the data that was collected from four college professors from Fredonia University regarding the expectations that these professors have for liberal arts mathematics students. These professors were asked the question, “If you could pick two to three mathematics problems that you would expect liberal arts mathematics students to solve based on their high school mathematics knowledge, what would they be?”. Several of the professors who were asked this question gave the response that they expect liberal arts mathematics students to be able to graph a line. The first professor that was asked this question, whose specialty is in analysis, gave the answer that students should know how to graph a line, plot points, solve a linear equation, use the order of operations to solve a problem, compute the area of a triangle, compute the volume of a box, FOIL, use the quadratic formula, and factor a quadratic. The next professor at Fredonia University who was asked this question, whose specialty is in Statistics, gave the answer that students should be able to find the median given a set of data, graph a line, solve problems involving proportions, answer a simple probability question, and solve problems involving similar triangles. Another professor who was asked this question, whose specialty is in mathematics education, gave the answer that he expects students to be able to solve questions involving exponents, circle geometry, and quadratic equations. The last professor who was asked this question, whose specialty is in algebraic analysis, gave the answer that students should be able to graph a line, solve a linear equation for  $x$ , and solve problems involving the quadratic formula. The answers that these professors gave were then recorded and compiled into a list of questions from which the actual quiz questions were made.

The problems that were compiled from college professors at Fredonia University, along with the information that was collected from New York State Regents Exam problems, SAT



problems, and questions that were previously included on Mathematics in Action Exams and Quizzes during the Fall 2011 Semester, became the basis for the quiz design.

Problem #1 was used to assess the students' knowledge of statistics through determining the median in a set of data. Problem #2 was used to assess the students' ability to graph a linear equation. Problem #3 was used to assess the students' ability to solve for  $x$  given the equation  $5x - 8 = 17$ . Problem #4 was used to assess the students' ability to determine the area of a triangle. Problem #5 was used to assess the students' ability to calculate the volume of a box. Problem #6 was used to assess the students' ability to evaluate and simplify the equation  $2 + (-1 + \sqrt{16})^2$  using the order of operations. Problem #7 was used to assess the students' knowledge of proportions by calculating 20% of 180. Problem #8 was used to assess the students' knowledge of FOIL and distribution as well as simplification. Problem #9 was used to assess the students' ability to plot and label points. Problem #10 was used to assess the students' knowledge of probability. Problem #11 was used to assess the students' knowledge of the Quadratic Formula. Problem #12 was used to assess the students' ability to use the Pythagorean Theorem to find a missing side in a given right triangle. Problem #13 was used to assess the students' knowledge of transformations through reflecting a triangle over a given line  $L$ . Problem #14 was used to assess the students' ability to solve a quadratic equation for  $x$ . Problem #15 was used to assess the students' ability to calculate the next three values in a sequence of numbers. Problem #16 was used to assess the students' knowledge of similar triangles. Problem #17 was used to assess the students' knowledge of exponents. Problem #18 was used to assess the students' knowledge of circle geometry. Problem #19 was used to assess the students' ability to simplify and combine fractions. Figure 7 provides a summary of the problems that were used for this quiz.

**Figure 7: Quiz Problem Summary**

<b>Problem</b>	<b>Mathematical Topic Covered</b>	<b>Problem Given</b>
Problem 1	Statistics (Median) (A.S.4)	Given the list of exam scores from students, find the median: 90, 67, 88, 94, 92, 93, 90, 93, 99, 100, 56, 86, 98, 87, 99
Problem 2	Graphing Linear Equations (A.G.4)	Graph $y = 2x + 3$ and label the axes.
Problem 3	Solving an Equation for a Variable (A.A.22)	Solve $5x - 8 = 17$ .
Problem 4	Area of a Triangle (A.G.1)	Given the side lengths in inches, find the area of triangle ABC.
Problem 5	Volume of a Box (A.G.2)	Given the side lengths in inches, find the volume of the given box.
Problem 6	Order of Operations (A.N.2)	Evaluate $2 + (-1 + \sqrt{16})^2$ .
Problem 7	Proportions (A.N.5)	What is 20% of 180?
Problem 8	FOIL (A.A.27)	Multiply and combine like terms: $(3x + 2)(4x - 1)$ .
Problem 9	Plotting and Labeling Points (A.G.3)	Plot and label the following points: $(3, 2)$ , $(6, -1)$ , $(-1, -3)$ , $(-2, 5)$ .
Problem 10	Probability (A.S.20)	Given a fair coin, which of the following is most likely to occur? THHH, HHHH, HTHT, TTHT, THHT, or they are all equally likely.
Problem 11	Quadratic Formula (A2.N.9)	Solve $x^2 + x - 4 = 0$ for $x$ .
Problem 12	Pythagorean Theorem (G.G.48)	Solve for $x$ in the following diagram.
Problem 13	Transformations (Reflection) (G.G.54)	Draw and label the image of triangle ABC on the grid above under the reflection of the line L.
Problem 14	Solving a Quadratic Equation (A.A.27)	Solve $x^2 - 7x = 0$ for $x$ .
Problem 15	Sequences (A2.A.32)	Determine the next 3 values in the sequences.
Problem 16	Similar Triangles (G.G.45)	Find the value of $x$ in the following pair of triangles.
Problem 17	Exponents (A.N.6)	Simplify: $\frac{x^{12}y^5}{x^4y^2}$ .
Problem 18	Circle Geometry (G.G.51)	Which of the following is a chord, but not a diameter?
Problem 19	Fractions (A.N.5)	Solve: $\frac{1}{2} + \frac{1}{16} + \frac{1}{8} =$

The survey questions were designed to obtain information from the students in regard to their prior mathematics experience and knowledge of high school content. Question #1 was

designed to determine what level of mathematics classes students previously completed.

Question #2 was designed to determine the amount of time that has elapsed since the students have taken a mathematics class. Question #3 was designed to determine students' feelings about mathematics. Question #4 was designed to determine how difficult students felt that the quiz was after completing it. Question #5 was designed to determine the amount of mathematics material that students feel they remembered from high school mathematics. Question #6 was designed to determine how dependent students are on the use of a calculator. This quiz was designed so that students could complete every problem without the use of a calculator, however students may have felt that they needed a calculator to complete the quiz. Figure 8 provides a summary of the questions that are included in the post experimental survey that was used for this research.

**Figure 8: Post Experimental Survey Question Summary**

<b>Question Number</b>	<b>Survey Question Asked</b>
Question 1	What mathematics training have you received prior to the current math class that you are taking?
Question 2	How many years has it been since you have taken a mathematics course?
Question 3	On a scale of 1-10, what are your feelings about mathematics?
Question 4	On a scale of 1-10 how difficult was this quiz?
Question 5	On a scale of 1-10 how much do you feel that you remember from high school mathematics?
Question 6	Do you feel that a calculator was necessary for this quiz?

The problems on the quiz and questions on the survey were designed to determine students' knowledge base with regard to high school mathematics as well as to determine why students may have difficulty in completing these problems. Data was then collected and analyzed in order to draw conclusions.

**Data Collection**

The data for this research was collected by analyzing the scores the students received on the quiz and the answers students gave to the post experimental survey. There were a total of 83 quizzes and surveys to examine. The quiz and survey were administered during the first week of the Spring 2012 semester in the students' college core curriculum mathematics course. As a result of this administration, the students received little to no college mathematics instruction prior to completing the quiz. Therefore, the students' only mathematics knowledge base was the information that they remembered from high school mathematics courses unless they had previously completed a college mathematics course, which was indicated on their survey.

**Methods of Data Analysis**

This research was quantitative in nature given that each student's score on the quiz was determined by the amount of points students received on each question. Descriptive statistics were used to draw conclusions based on ANOVA scores that were calculated. The score for each question was calculated using the point value of the question and the answer provided on the answer key, which determined the student's ability to complete each problem. Each problem on the quiz was assigned a point value and after determining how many points each problem received, a percentage was calculated for the total amount of points that each students received out of a total of 50 points. If a student provided a correct answer and showed an alternate method of solving the problem which also proved correct, students still received full credit for a particular problem. The answers that are provided on the answer key are only sample solutions and were used as a guideline for determining the score a student received since many of the

problems could have alternate methods of completion. The answer key that was used as a guideline for grading the students' quizzes is provided in Appendix B.

After students' scores were calculated on the quiz, this score, along with the results of the survey were then compared in an attempt to determine the number of high school mathematics topics the students were able to recall as well as the level of mathematics the students had previously taken, their prior feelings about mathematics, their reliance on a calculator, and the number of years that had passed since they received high school mathematics instruction. This information was then entered in a table that can be seen in the Results section of this research.

## Results

Four primary results were evident following the collection and analysis of the quiz scores and student answers given on the post experimental survey. There is insufficient evidence to reject the original hypothesis. Therefore, introductory level college students are unable to recall and demonstrate knowledge of many Algebra, Geometry, and Algebra II Trigonometry topics that they learned in high school. An analysis of variance (ANOVA) was utilized to determine the significance of the treatment at the 0.05 level with regards to the following results.

- ***A significant difference existed in the students' scores on individual questions based on the type of mathematics problem:*** Students were better able to answer quiz problems based on the mathematical topic that was covered in the question. The mathematics topics that the students were able to complete best included Solving an Equation for a Variable (95.2%), Plotting and Labeling Points (93.7%), and Sequences (88.7%), and the mathematics topics that the students had the most difficulty in completing included the

Quadratic Formula(6.7%), Solving a Quadratic Equation (29.2%), and Similar Triangles (34.8%). ( $p$ -value = 0.000, Solving an Equation for a Variable:  $\bar{x} = 95.78$ , Quadratic Formula:  $\bar{x} = 7.23$ )

- A significant difference existed in the students' total quiz scores based on their previous mathematics experience (specifically in completing a Pre-Calculus course):***

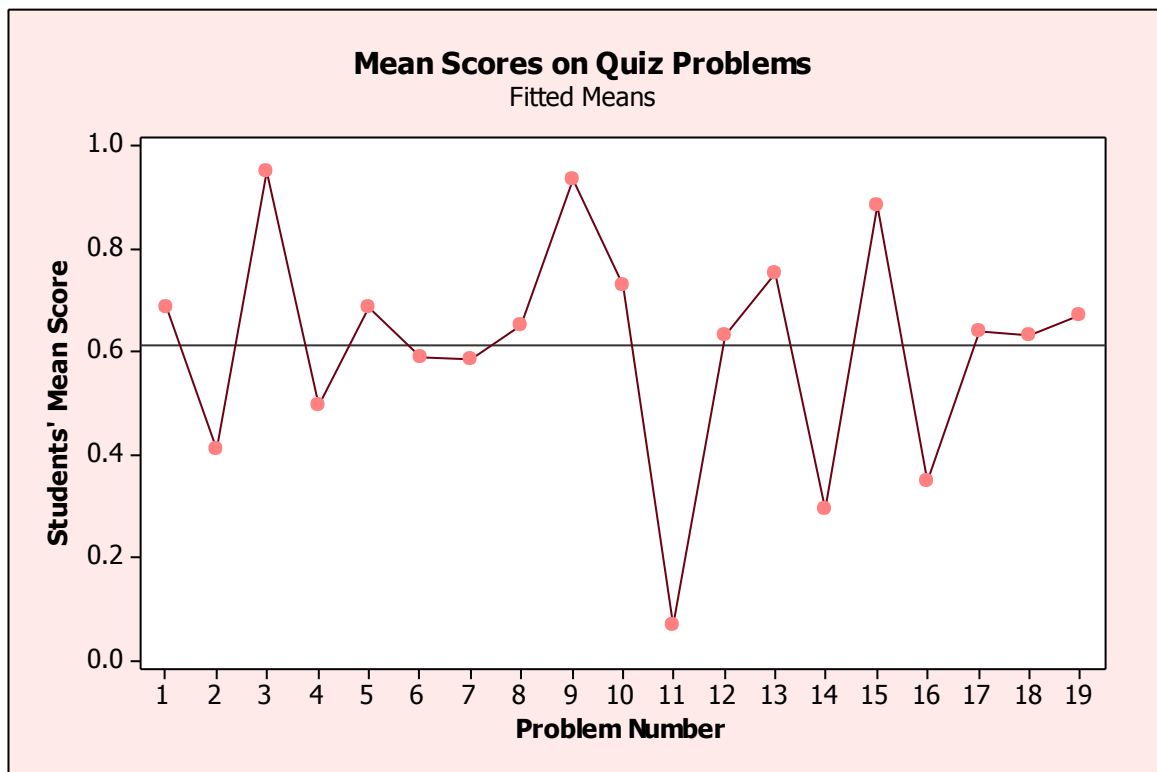
Students who had taken Pre-Calculus received higher quiz scores than those students who had not taken Pre-Calculus. ( $p$ -value = 0.000, No Pre-Calculus Experience Mean Score:  $\bar{x} = 52.24$ , Pre-Calculus Experience Mean Score:  $\bar{x} = 68.57$ )
- A significant difference existed in the students' total quiz scores based on their gender:***

Males answered more questions correctly than females did and therefore received higher total quiz scores. ( $p$ -value = 0.011, Female:  $\bar{x} = 56.37$ , Male:  $\bar{x} = 66.47$ )
- A significant difference existed in the students' total quiz scores based on their feelings about mathematics:*** Students who indicated that they like mathematics received higher overall quiz scores than those students who indicated that they did not like mathematics. ( $p$ -value = 0.000, Students Who Hate Mathematics Mean Score (Score of 1):  $\bar{x} = 46.0$ , Students Who Love Mathematics Mean Score (Score of 10):  $\bar{x} = 76.7$ )

***Result 1: A significant difference existed in the students' scores on individual questions based on type of mathematics problem***

Upon analysis of the individual quiz questions, it was determined that the type of mathematics question that was asked greatly impacted student performance on that particular question. Figure 9 provides a comparison of the mean scores that were received on each quiz question. Figure 10, which was also provided in the Instrument Items and Justification of Items section of this research provides a list of quiz problems.

**Figure 9: Mean Scores on Quiz Problems**



**Figure 10: Quiz Question Summary**

<b>Problem</b>	<b>Mathematical Topic Covered</b>	<b>Percent Correct</b>
Problem 3	Solving an Equation for a Variable	95.2%
Problem 9	Plotting and Labeling Points	93.7%
Problem 15	Sequences	88.7%
Problem 13	Transformations (Reflection)	75.4%
Problem 10	Probability	73.0%
Problem 5	Volume of a Box	68.8%
Problem 1	Statistics (Median)	68.8%
Problem 19	Fractions	67.0%
Problem 8	FOIL	65.1%
Problem 17	Exponents	63.9%
Problem 12	Pythagorean Theorem	63.4%
Problem 18	Circle Geometry	63.4%
Problem 6	Order of Operations	58.8%
Problem 7	Proportions	58.5%
Problem 4	Area of a Triangle	49.7%
Problem 2	Graphing Linear Equations	41.1%
Problem 16	Similar Triangles	34.8%
Problem 14	Solving a Quadratic Equation	29.2%
Problem 11	Quadratic Formula	6.7%

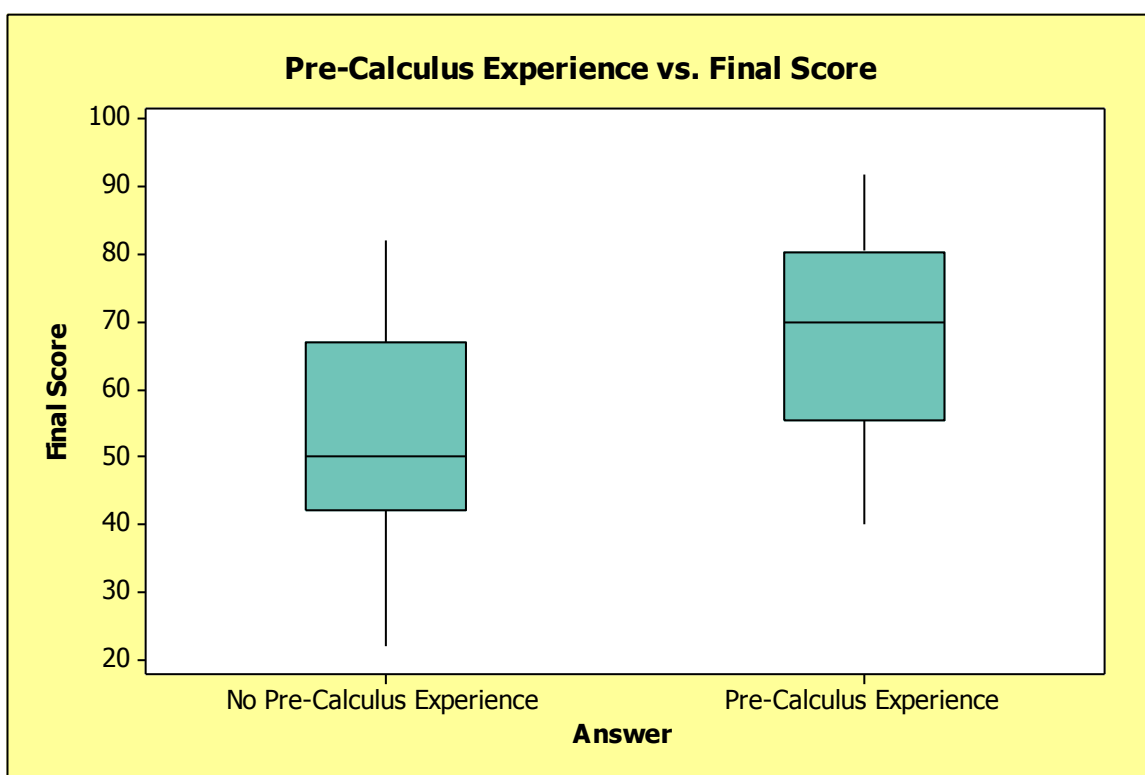
The resulting  $p$ -value of the ANOVA test ( $p$ -value = 0.000) indicates that the mathematics topic that was covered on each quiz problem was significant in student performance. After comparing the mean scores for each quiz problem, it was determined that the mathematics topics that the students were able to complete best included Solving an Equation for a Variable (95.2%), Plotting and Labeling Points (93.7%), and Sequences (88.7%). Likewise, based on the mean scores for each quiz question, it was determined that the mathematics topics that the students had the most difficulty in completing included the Quadratic Formula (6.7%), Solving a Quadratic Equation (29.2%), and Similar Triangles (34.8%).



***Result 2: A significant difference existed in the students' total quiz scores based on their previous mathematics experience (specifically in completing a Pre-Calculus course)***

In addition to analyzing the score on each individual quiz problem, the overall quiz score was analyzed. Upon analysis of the students' overall quiz scores, it was determined that whether the students had taken Pre-Calculus previously or not did impact the overall score that each student received. In Figure 11 are compared overall quiz scores with the students' Pre-Calculus experience.

**Figure 11: Pre-Calculus Experience vs. Final Grade**



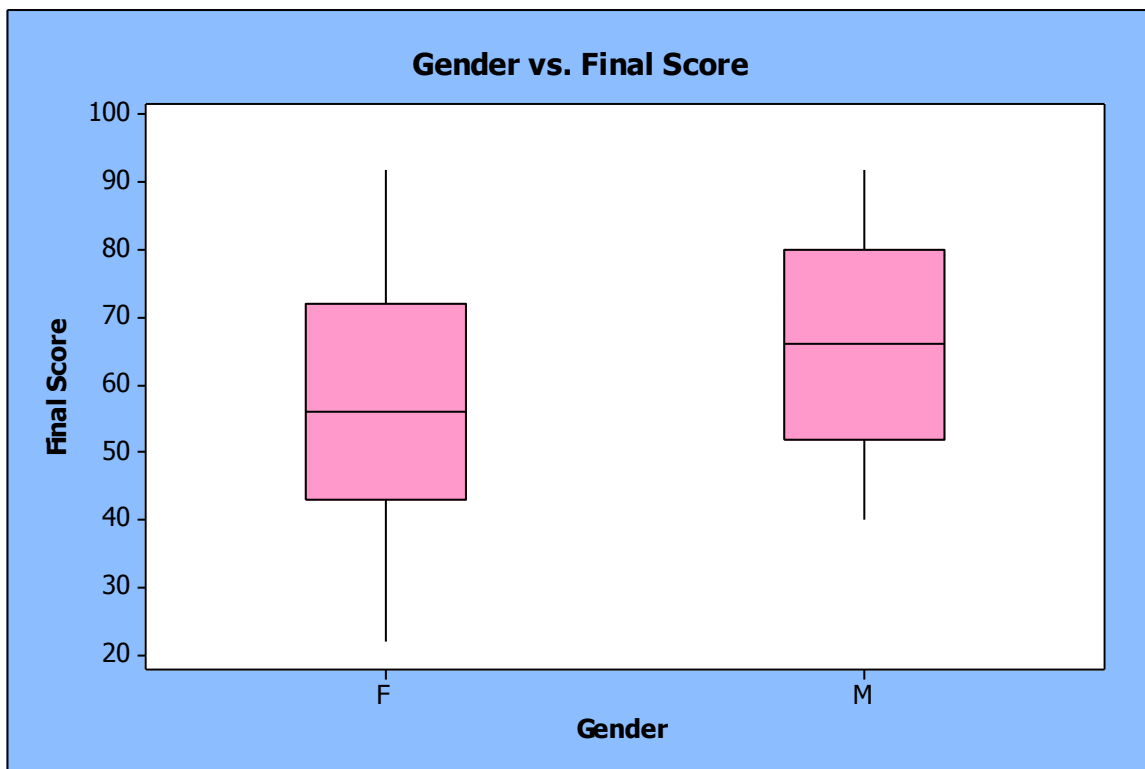
The resulting  $p$ -value of the ANOVA test ( $p$ -value = 0.000, No Pre-Calculus Experience Means Score:  $\bar{x} = 52.24$ , Pre-Calculus Experience Mean Score:  $\bar{x} = 68.57$ ) indicates that the

Pre-Calculus experience that each student had was significant in student performance. After comparing the mean scores for each group of students, it was determined that the group of students that completed Pre-Calculus performed better overall on the quiz. The mathematics experience that students previously had was determined to be a contributing factor to the students' ability to recall high school mathematics material.

***Result 3: A significant difference existed in the students' total quiz scores based on their gender***

Attention was also given to the results of the subjects grouped by gender. Upon analysis of the overall quiz scores, it was determined that a significant difference did exist between males and females in this study. Figure 12 compares overall quiz scores based on gender (male:  $n = 34$ , female:  $n = 49$ ).

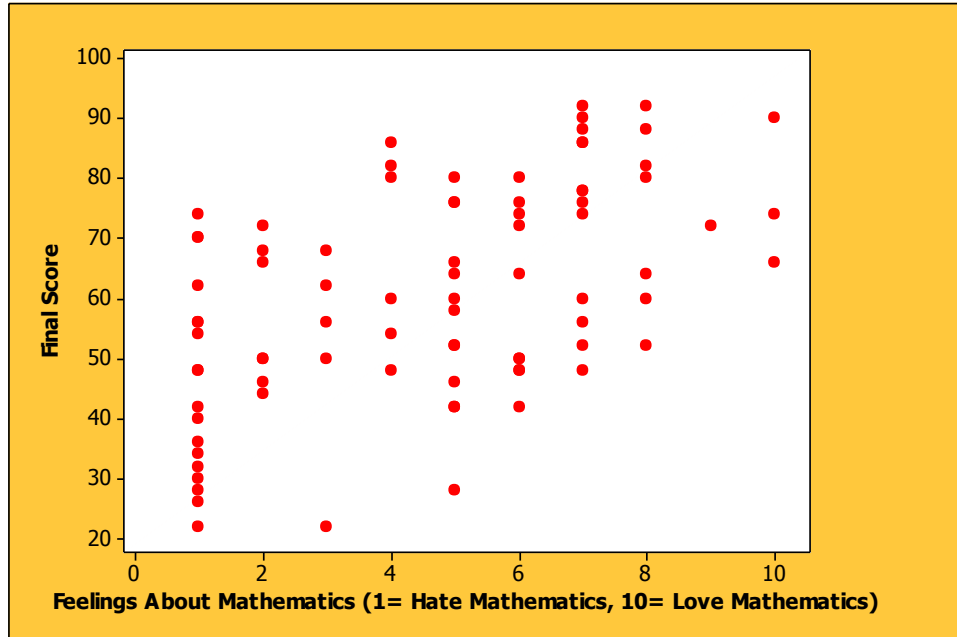
**Figure 12: Gender vs. Final Grade**



The resulting  $p$ -value of the ANOVA test ( $p$ -value = 0.011, Female:  $\bar{x} = 56.37$ , Male:  $\bar{x} = 66.47$ ) indicates that the gender of each student was a significant factor when comparing overall quiz scores. After comparing the mean scores for each group of students, it was determined that the males performed better than the females overall on the quiz. The gender of each student was determined as a contributing factor in the students' ability to recall high school mathematics material.

***Result 4: A significant difference existed in the students' total quiz scores based on their feelings about mathematics***

In addition to analyzing the results by performance on each individual quiz question, previous mathematical experience, and gender, recognition was also given to the survey question: On a scale of 1-10, what are your feelings about mathematics? Upon analysis of the students answers to this survey question, it was determined that the students' feelings about mathematics greatly impacted student performance on the quiz. Figure 13 compares the overall quiz scores with the students' feelings about mathematics.

**Figure 13: Student Feelings About Mathematics vs. Final Score**

The resulting  $p$ -value of the ANOVA test ( $p$ -value = 0.000, Students Who Hate Mathematics Mean Score (Score of 1):  $\bar{x} = 46.0$ , Students Who Love Mathematics Mean Score (Score of 10):  $\bar{x} = 76.7$ ) indicates that the students' feelings regarding mathematics was a significant factor when comparing overall quiz scores. After comparing the mean scores, it was determined that students who indicated that they hate mathematics scored much lower than students who indicated that they love mathematics. Each student's feelings regarding mathematics were determined to be contributing factors in the students' ability to recall high school mathematics material.

After determining the four results that were discussed in this section, implications for teaching can be made. These implications include classroom suggestions as well as suggestions for further research.

## **Implications for Teaching**

The hypothesis for this study was that introductory level college students are unable to recall and demonstrate knowledge of many Algebra, Geometry, and Algebra II Trigonometry topics that they learned in high school. Specifically, it was predicted that students would have difficulty completing problems regarding Graphing Linear Equations and the Quadratic Formula. During the course of this study, student performance on each specific quiz problem as well as their performance on the given quiz was compared. It was determined that the final score that each student received was impacted by students' prior mathematics experience, gender, and their feelings about mathematics. The results that were determined in this study will definitely have implications in my classroom. These implications as well as suggestions for how the results of this experiment may lead to further research are offered in this section.

### ***Classroom Implications***

The implications for classroom instruction parallel three of the principal results of the research that was discussed in the previous section.

- **Teachers need to place a greater focus on mathematical understanding rather than rote memorization.**

The first implication for teaching in my classroom that results from this study is that placing a greater focus on mathematical understanding as opposed to rote memorization may increase student performance in the future.

Through analyzing the students' scores on each quiz question, it can be seen that the questions in which students seemed to struggle most were those that involved the

ability of students to recall a mathematics formula that they learned in high school. For example, the question that seemed to give students the most trouble was a problem involving the Quadratic Formula. An example solution of this problem can be seen in Figure 14. After scoring each of the students' quizzes, in answering this problem, several times students were able to recognize that they needed to use the quadratic formula but then they indicated that they did not remember the formula to be able to use it. Many other students also recognized that this problem involved a quadratic so they tried to factor the problem and did not recognize that the problem was not able to be easily factored using integers. Another problem in which a misconception occurred was in Problem 4 which asked the students to find the area of a triangle. Many students forgot parts of this formula or tried to use another formula they remembered that was related to polynomials. This misconception can be seen in Figure 15.

The situation of being unable to recall a necessary formula to correctly complete the quiz problems occurred very often throughout the quiz that was given to the students which seemed to indicate that the students had memorized the formula while they were in high school but that they may not have completely understood the reason for using that formula and how to derive the formula on their own.

After analyzing the students' scores on their quizzes and noticing that students struggled with correctly answering questions that require the use of a formula, implementation of a greater focus on mathematical understanding may be necessary in teaching mathematics students.

**Figure 14: Quadratic Formula Misconceptions**

11. (4 points) Solve  $x^2 + x - 4 = 0$  for  $x$ .

$(x+2)(x-2)$   
 $2x$   
 $-2x$   
 $-4$   
 $x=2$   
 $x=-2$

-4

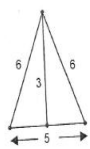
11. (4 points) Solve  $x^2 + x - 4 = 0$  for  $x$ .

$x^2 + x - 4 = 0$   
 $+4$   
 $x^2 + x = 4$   
 $-x^2 - x$   
 $x = 4 - x^2$

-4

**Figure 15: Area of a Triangle Misconception**

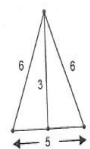
4. (3 points) Given the side lengths in inches, find the area of the given triangle (figure not drawn to scale).



$L \cdot H \cdot W$   
 $6 \cdot 3 \cdot 5$   
 $18 \cdot 5$   
 $90$

-3

4. (3 points) Given the side lengths in inches, find the area of the given triangle (figure not drawn to scale).



$A = \frac{1}{2}bh$   
 $A = 5 \cdot 3$   
 $A = 15$

-2

- **The mathematics curriculum may be too broad and not deep enough. Teachers may need to place a greater focus on fundamental mathematics topics to increase mathematical numeracy.**

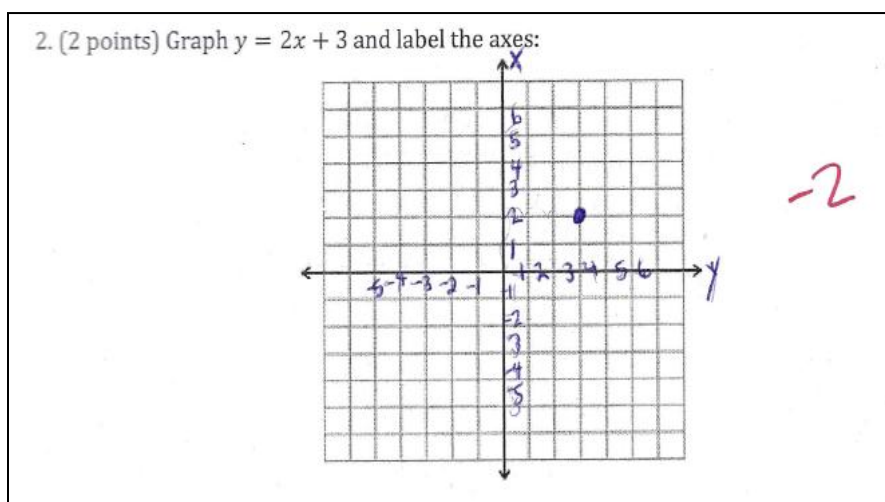
The second implication for teaching in my classroom that results from this study is that placing a greater focus on fundamental mathematics topics will help to increase mathematical numeracy.

After analyzing the students overall quiz scores, it was determined that the students in this study have an overall mathematical numeracy of 60.51% which would indicate that many of the students in this study lack basic pre-requisite skills that are needed for college mathematics courses. All of the quiz questions that were included in

this study were determined to be fundamental mathematics skills that are required for college mathematics courses.

The fact that the mathematical numeracy of these students is below the passing average for high schools and colleges indicates that a greater focus should be placed on teaching fundamental skills before moving into new material. An example of a problem on this quiz that presented a problem for students was Problem 2, in which the students were asked to graph a linear equation. This misconception can be seen in Figure 16. Many students who attempted to complete this problem could not remember which line represented the  $x$ -axis and which line represented the  $y$ -axis. Students also forgot which number represented the slope and  $y$ -intercept in the formula. Another example of a problem in which students struggled to remember fundamental mathematics concepts was Problem 6. In this problem students needed to know the order of operations and without that knowledge were not able to complete this problem. This misconception can be seen in Figure 17.

**Figure 16: Graphing Linear Equations**





**Figure 17: Order of Operations Misconception**

6. (4 points) Evaluate  $2 + (-1 + \sqrt{16})^2$ .

Handwritten work shows the student incorrectly simplified the expression as follows:

$$2 + (1 + \sqrt{32})$$

$$(17) \quad (19) \quad -4$$

The student's work indicates a misunderstanding of the order of operations, specifically in handling the exponent and the square root.

The pressures of state testing and college preparation make it very difficult for teachers to spend an enormous amount of time on one topic, however without knowing fundamental mathematics topics well enough, students will be unable to build on their previous knowledge base. Spending a few extra minutes each day to review what was learned the previous day and to review what was learned throughout the entire class period at the conclusion of each class may be a start in reviewing fundamental concepts and increasing mathematical numeracy.

- **Teachers may need to find ways to make mathematics fun and innovative for students to improve their feelings about mathematics.**

The last implication for teaching in my classroom that results from this study is that finding ways to make mathematics fun and innovative may improve students' feelings about mathematics.

After examining the overall quiz scores with students' feelings about mathematics it was determined that a significant impact was present. Mathematics is a subject in which many students struggle and often dislike, thus by finding ways to make mathematics fun and interesting for students, student performance levels may increase as well.

Mathematics students often ask the question of “Where am I ever going to use this stuff?” and often are reluctant to learn because they do not find mathematics to be pertinent to their everyday lives. By finding ways to relate mathematics topics to everyday life students may be more apt to learn and may increase their performance level. If a student is reluctant to learn and does not find any interest in what they are learning about they often will perform poorly. However, if teachers can inspire their mathematics students and encourage them to want to learn through the use of interesting word problems, games, puzzles, and other innovative techniques, students may have higher performance rates and may also increase their love of mathematics.

### ***Suggestions for Further Research***

Although this research offered many insightful answers and implications, there are still a few questions to be answered and investigated. This study was conducted using college mathematics students at the beginning of their participation in a college core curriculum mathematics courses. It would be interesting to perform this same study with high school seniors and juniors to determine their current mathematical ability. Also, this study focused on whether students were able to recall high school mathematics coursework but did not investigate how to increase this recall ability. Conducting further research regarding specific teaching methods that could be used to assist students in recalling fundamental mathematics material would also be very interesting to focus attention toward.

***Concluding Remarks***

The motivation behind this study was to determine if students in college level mathematics courses are able to remember basic concepts they learned in high school and are able to apply these concepts to new college mathematics material they learn. Based on the results of this study, it appears that students in college level mathematics courses are only able to remember an average of 60% of the mathematics material that they learned in high school, and as such, they may not be able to build on these fundamental concepts in college mathematics courses. Through implementing a greater focus on mathematical understanding, fundamental mathematics topics, as well as fun and innovative ways to present mathematics concepts, teachers may improve their students' mathematical ability and may also help their students to better succeed in future mathematics classrooms.

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**Appendix A: Quiz (Student Copy)**

Name: \_\_\_\_\_

Class: \_\_\_\_\_

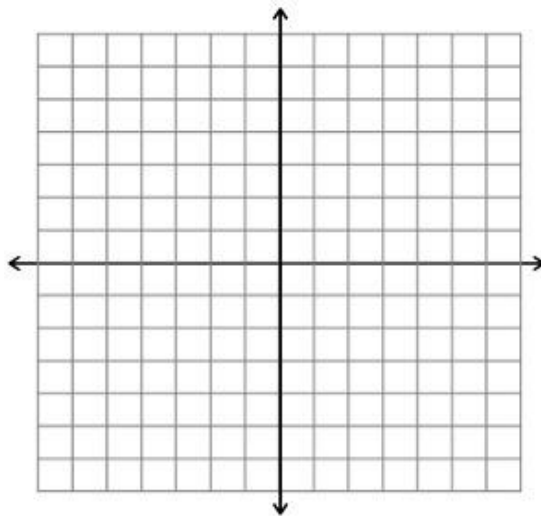
**Are You Smarter Than a High Schooler?**

Please answer the following questions without the use of a calculator and **show all of your work**.

1. (2 points) Given the list of exam scores from students, find the median.

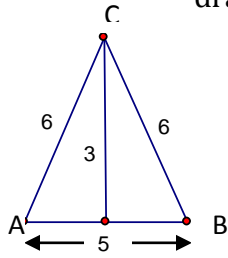
90, 67, 88, 94, 92, 93, 90, 93, 99, 100, 56, 86, 98, 87, 99

2. (2 points) Graph  $y = 2x + 3$  and label the axes:

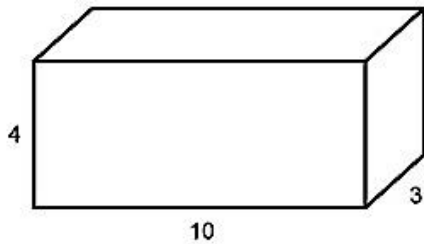


3. (2 points) Solve  $5x - 8 = 17$ .

4. (3 points) Given the side lengths in inches, find the area of triangle ABC (figure not drawn to scale).



5. (2 points) Given the side lengths in inches, find the volume of the given box.



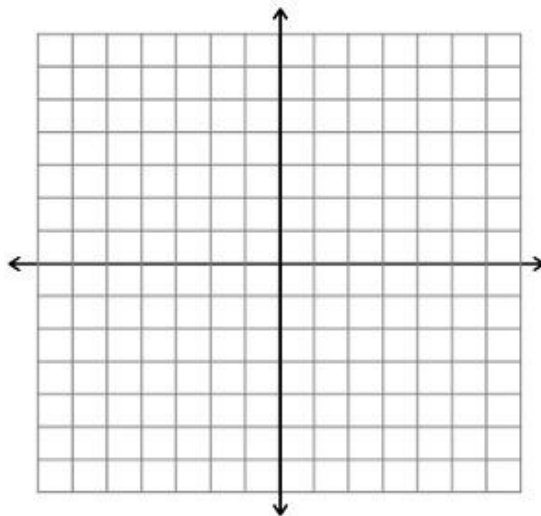
6. (4 points) Evaluate  $2 + (-1 + \sqrt{16})^2$ .

7. (3 points) What is 20% of 180?



8. (2 points) Multiply and combine like terms:  $(3x + 2)(4x - 1)$ .

9. (5 points) Plot and label the following points:  $(3, 2)$ ,  $(6, -1)$ ,  $(-1, -3)$ ,  $(-2, 5)$ .

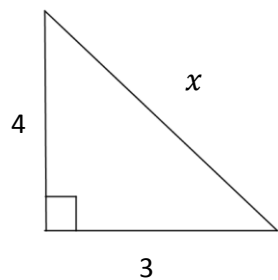


10. (2 points) Given a fair coin, which of the following is most likely to occur?

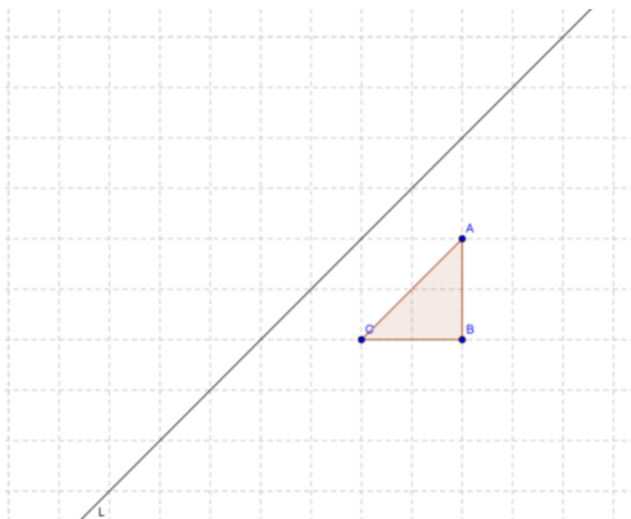
- a.) THHH
- b.) HHHH
- c.) HTHT
- d.) TTHT
- e.) THHT
- f.) they are all equally likely

11. (4 points) Solve  $x^2 + x - 4 = 0$  for  $x$ .

12. (4 points) Solve for  $x$  in the following diagram:



13. (2 points) Draw and label the image of triangle ABC on the grid above under the reflection of the line L.

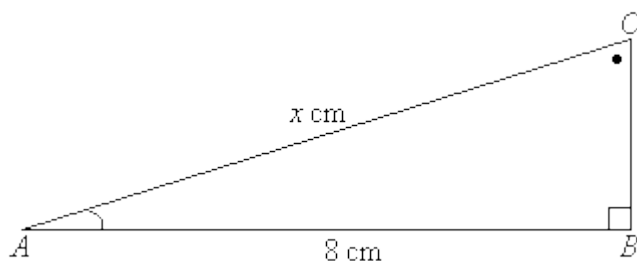
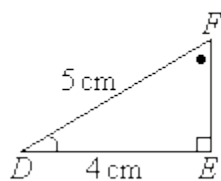


14. (3 points) Solve  $x^2 - 7x = 0$  for  $x$ .

15. (3 points) Determine the next 3 values in the sequence.

2                      4                      8                      16                      \_\_\_\_                      \_\_\_\_                      \_\_\_\_

16. (3 points) Find the value of  $x$  in the following pair of triangles.



17. (2 points) Simplify:  $\frac{x^{12}y^5}{x^4y^2}$

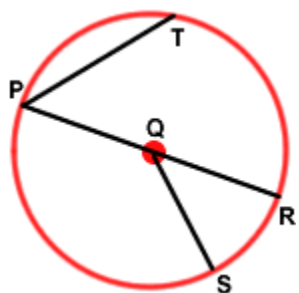
18. (1 point) Which of the following is a chord, but not a diameter?

a)  $\overline{PT}$

b)  $\overline{PR}$

c)  $\overline{QS}$

d) None of the above.



19. (1 point) Solve:  $\frac{1}{2} + \frac{1}{16} + \frac{1}{8} =$

**Appendix B: Quiz (Teacher Copy)**Name: Answer Key

Class: \_\_\_\_\_

**Are You Smarter Than a High Schooler?**

Please answer the following questions without the use of a calculator and **show all of your work**.

1. (2 points) Given the list of exam scores from students, find the median.

90, 67, 88, 94, 92, 93, 90, 93, 99, 100, 56, 86, 98, 87, 99

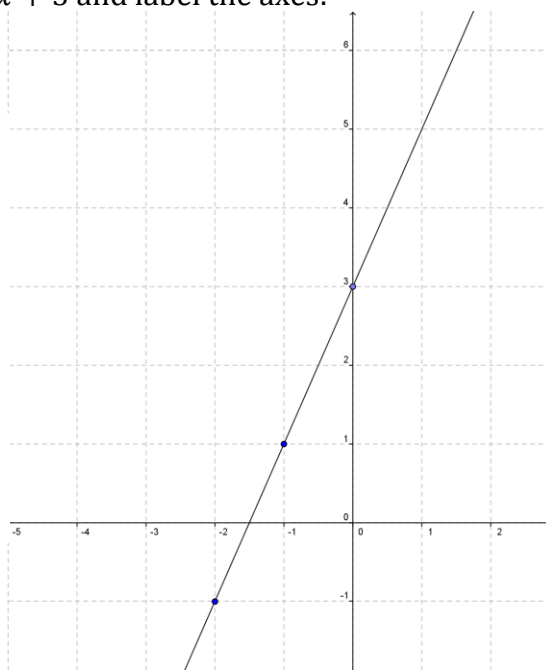
56, 67, 86, 87, 88, 90, 90, 92, 93, 93, 94, 98, 99, 99, 100 (1 point)

Median=92 (1 point)

2. (2 points) Graph  $y = 2x + 3$  and label the axes:

(1 point for labeling the axes)

(1 point for correctly graphing the line)



3. (2 points) Solve  $5x - 8 = 17$ .

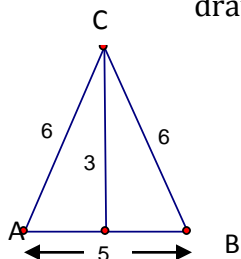
$$5x - 8 = 17$$

$$+8 \quad +8$$

$$\frac{5x}{5} = \frac{25}{5} \text{ (1 point)}$$

$$x = 5 \text{ (1 point)}$$

4. (3 points) Given the side lengths in inches, find the area of triangle ABC (figure not drawn to scale).



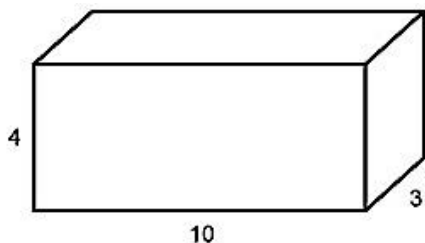
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5)(3) \text{ (1 point)}$$

$$A = \frac{1}{2}(15) \text{ (1 point)}$$

$$A = 7.5 \text{ (1 point)}$$

5. (2 points) Given the side lengths in inches, find the volume of the given box.



$$V = l * w * h$$

$$V = (4)(10)(3) \text{ (1 point)}$$

$$V = (40)(3) \text{ (1 point)}$$

$$V = 120 \text{ (1 point)}$$

6. (4 points) Evaluate  $2 + (-1 + \sqrt{16})^2$ .

$$= 2 + (-1 + \sqrt{16})^2$$

$$= 2 + (-1 + 4)^2 \text{ (1 point)}$$

$$= 2 + (3)^2 \text{ (1 point)}$$

$$= 2 + 9 \text{ (1 point)}$$

$$= 11 \text{ (1 point)}$$

7. (3 points) What is 20% of 180?

$$\frac{20}{100} = \frac{x}{180} \text{ (1 point)}$$

$$\frac{3600}{100} = \frac{100x}{100} \text{ (1 point)}$$

$$x = 36 \text{ (1 point)}$$

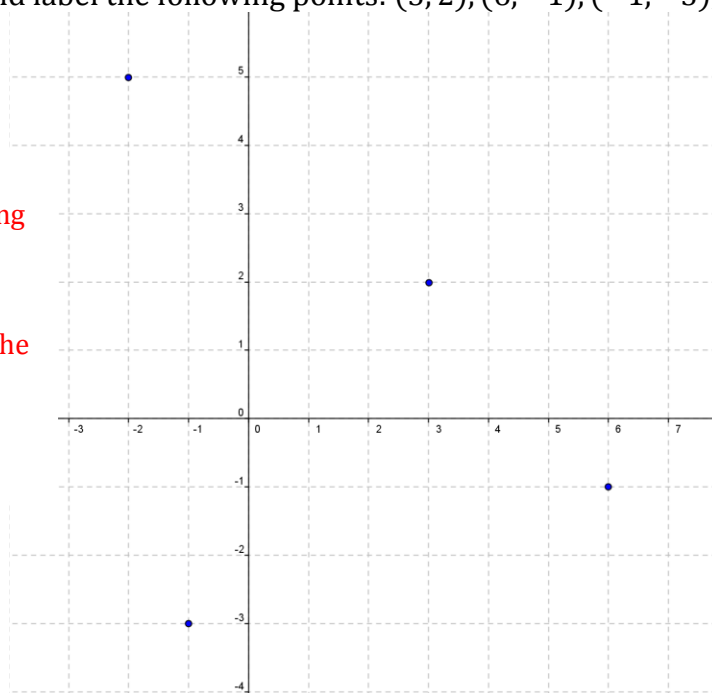
8. (2 points) Multiply and combine like terms:  $(3x + 2)(4x - 1)$ .

$$\begin{aligned}(3x + 2)(4x - 1) \\&= 12x^2 - 3x + 8x - 2 && (1 \text{ point}) \\&= 12x^2 + 5x - 2 && (1 \text{ point})\end{aligned}$$

9. (5 points) Plot and label the following points:  $(3, 2)$ ,  $(6, -1)$ ,  $(-1, -3)$ ,  $(-2, 5)$ .

(1 point for  
correctly placing  
each point)

(1 point for  
labeling all of the  
points)



10. (2 points) Given a fair coin, which of the following is most likely to occur?

a.) THHH

b.) HHHH

c.) HTHT

d.) TTHT

e.) THHT

f.) they are all equally likely (2 points)

11. (4 points) Solve  $x^2 + x - 4 = 0$  for  $x$ .

$$x^2 + x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

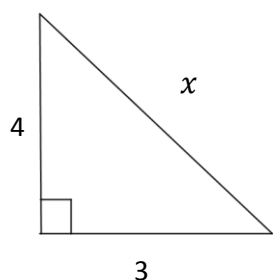
$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-4)}}{2(1)} \quad (1 \text{ point})$$

$$x = \frac{-1 \pm \sqrt{1+16}}{2} \quad (2 \text{ points})$$

$$x = \frac{-1 \pm \sqrt{17}}{2} \quad (1 \text{ point})$$

$$x = \frac{-1 + \sqrt{17}}{2}, \frac{-1 - \sqrt{17}}{2}$$

12. (4 points) Solve for  $x$  in the following diagram:



$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = x^2 \quad (1 \text{ point})$$

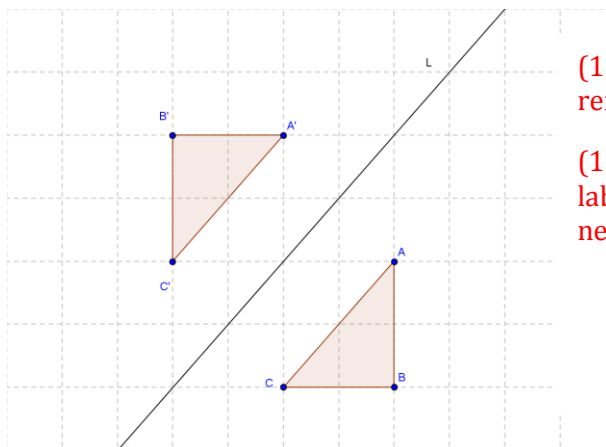
$$9 + 16 = x^2 \quad (1 \text{ point})$$

$$25 = x^2$$

$$\sqrt{25} = x \quad (1 \text{ point})$$

$$x = 5 \quad (1 \text{ point})$$

13. (2 points) Draw and label the image of triangle ABC on the grid above under the reflection of the line L.



(1 point for correctly reflecting the triangle)

(1 point for correctly labeling the points in the new triangle)

14. (3 points) Solve  $x^2 - 7x = 0$  for  $x$ .

$$x^2 - 7x = 0$$

$$x(x - 7) = 0 \quad (1 \text{ point})$$

$$x = 0 \quad x - 7 = 0 \quad (1 \text{ point})$$

$$\quad \quad +7 \quad +7$$

$$\quad \quad x = 7$$

So,  $x = 0, x = 7$ . (1 point)

15. (3 points) Determine the next 3 values in the sequence.

2

4

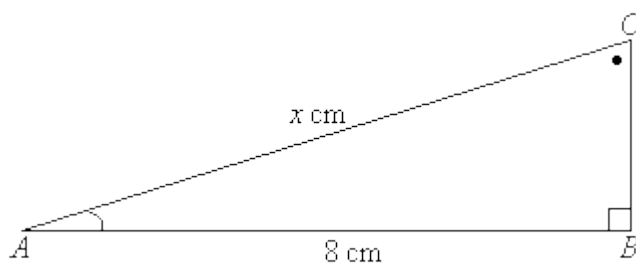
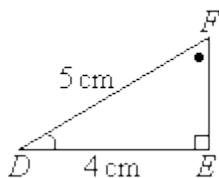
8

16

3264128

(1 point each)

16. (3 points) Find the value of  $x$  in the following pair of triangles.



$$\frac{5}{4} = \frac{x}{8} \quad (1 \text{ point})$$

$$\frac{40}{4} = \frac{4x}{4} \quad (1 \text{ point})$$

$$10 = x \quad (1 \text{ point})$$

17. (2 points) Simplify:  $\frac{x^{12}y^5}{x^4y^2} = x^8y^3$  (1 point for  $x^8$  and 1 point for  $y^3$ )



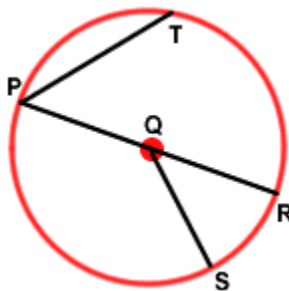
18. (1 point) Which of the following is a chord, but not a diameter?

a)  $\overline{PT}$  (1 point)

b)  $\overline{PR}$

c)  $\overline{QS}$

d) None of the above.



19. (1 point) Solve:  $\frac{1}{2} + \frac{1}{16} + \frac{1}{8} = \frac{11}{16}$  (1 point)

## Appendix C: Post Experimental Survey

Name: \_\_\_\_\_

Class: \_\_\_\_\_

### Are You Smarter Than a High Schooler?

#### Post Experimental Survey.

1. What mathematics training have you received prior to the current math class that you are taking? Please circle all that apply.

a.) Integrated Algebra

b.) Geometry

c.) Algebra II and Trigonometry

d.) Pre-Calculus

e.) Other (Please Specify) \_\_\_\_\_

2. How many years has it been since you have taken a mathematics course?

\_\_\_\_\_

3. On a scale of 1-10, what are your feelings about mathematics?

(I hate it) 1    2    3    4    5    6    7    8    9    10 (I love it!)

4. On a scale of 1-10 how difficult was this quiz?

(Very Easy) 1    2    3    4    5    6    7    8    9    10 (Very Difficult)

5. On a scale of 1-10 how much do you feel that you remember from high school mathematics?

(Very Little) 1    2    3    4    5    6    7    8    9    10 (Everything)

6. Do you feel that a calculator was necessary for this quiz?

a.) Yes

b.) No