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CERTIFICATION OF PROJECT WORK

We, the undersigned, certify that this project entitled The Positives about Negatives: A Study of Errors and Misconceptions with Integer Operations in Adult Education by Joshua T. Sadler, Candidate for the Degree of Master of Science in Education, Mathematics, is acceptable in form and content and demonstrates a satisfactory knowledge of the field covered by this project.

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THE POSITIVES ABOUT NEGATIVES: A STUDY OF ERRORS AND MISCONCEPTIONS WITH INTEGER OPERARTIONS IN ADULT EDUCATION

By

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Abstract

In this experiment two classes received instruction on integer operations. The first received instruction with the use of technology and the second class was instructed through a traditional approach. The study progressed over a one week span where students began with a five question survey to assess previous knowledge of positive and negative numbers. Following the survey, four days of instruction were provided discussing each operation as its own lesson. After the instruction, students were given a twenty question multiple choice exam that was graded for correctness. Data from the post assessment was also collected to determine if there were any persistent errors. The hypothesis pertaining to the technology enhanced teaching style outperforming the traditional teaching style demonstrates there was not enough evidence, as it is clear that there is no definitive difference when comparing mean scores and p-values.

Table of Contents

Introduction

This research examines the multiple methods involved in solving integer operations as well as the misconceptions in learning these operations. There is no “right” method when instructing/learning integer operations or any type of mathematics for that matter. Students need the opportunity to learn multiple methods. Too many students are not “truly” learning the material. In other words, they are learning the rules of how integer operations work, but they do not know why they work and have no context to support the new knowledge.

I was interested in this topic because I felt integers are an important topic to students in adult learning and that this topic was becoming a misused component in mathematics. Students tend to grasp the wrong ideas when learning the material, but I could not figure out why this was occurring. During my teachings at the Cassadaga Job Corps Academy, I taught two separate sections of adult learners, where one group would meet three times a day, while the other twice a day. Both sections are of similar stature, where the students’ academy math scores fall in the same range. The scores are determined by how well they perform on the Test for Adult Basic Education (TABE), which is the test each student must take in order to attempt to get their GED or High School Diploma, or be placed in their desired trade full time. Though each group would only receive one lesson on integers in the morning, students would have opportunities to ask questions throughout the remainder of the day, as well as take work back to their dorms to practice for the following day. However, although the students had these opportunities, they did not take advantage and would not retain the knowledge from the previous lesson.

My study focuses on giving context to student learning as well as connecting the context to different methods of instruction. The main concept is to attempt to find the reasons behind student misconceptions pertaining to integer operations and how they can overcome them.

It is hypothesized that adult learners, when given a set of problems on integer operations, will perform more efficiently and score higher when instructed with the use of technology versus a traditional teaching approach. Furthermore, students will be likely to make common errors on signed problems due to confusion of operations.

I tested this hypothesis by first assigning both sections of my classes a pre-survey which determined their prior knowledge of integer operations. The survey provided feedback that helped establish what misunderstandings they already possess with these operations, more specifically, those using negative numbers. Next, I selected two separate teaching methods and provided each section with a different method (i.e., one section had an interactive approach with the use of technology and the second section a traditional method). This was to examine how instruction affects these misconceptions and whether the use of an interactive instruction or traditional instruction had a positive effect on learning outcomes. Overall, although the instruction differed, every other aspect of the unit was identical. Despite the fact that different instructions are being imposed, the main purpose of the hypothesis is to gain a deeper understanding of why exactly issues with integer operations occur.

Lit Review

The purpose of this literature review is to examine previous research which refers to methods for solving problems with integer operations and tying in these operations into classes pertaining to adult education. First, the review begins with a background of adult education and different methods that are used to help produce student success. Following this section is an in depth analysis of how negative numbers came to be and why they are important and relevant to everyday life. Next, several methods focused on teaching integer operations are explored and compared. Finally, some misconceptions are explored involving these operations and provide a more profound reason of why these fallacies occur.

History of Adult Education

The history of adult education has come a long way since its recognition to the U.S. educational system in 1966. Rose (1991) states that in 1966 the Adult Education Act (AEA) was the reason adult education became recognized as part of the U.S. educational system. The act was an extension to the Economic Opportunity Act from 1964. The act provided funding to states to "develop, administer, and maintain basic education programs for adults." (p. 8) It is also stated in the research that though the local and state agencies have total administrative control, the federal government provided ninety percent of the program costs. Overall the program has improved drastically throughout the years. In 1965, the program had an estimated 38,000 students, and by 1988, the number of participants grew to about three million. However, though the data shows a large increase during this time period, the program was still a small percentage of the target population. In 1998, the Adult Education Act was repealed and replaced by the Workforce Investment Act (WIA). According to the Employment and Training Administration

(1998), the AEA stated adults qualified for the program if they were economically disadvantaged and if they were of the age of twenty-two or older. The WIA, however, was changed to target all low-income individuals of ages eighteen and up. The AEA also stated that youth included in the program is defined from the age of sixteen to twenty-one, and must be economically disadvantaged. The WIA reformed the eligible age to include individuals of ages fourteen to twenty-one who are low-income.

One major contributor behind the funding and education within the acts listed above is the success of the students and achievements gained by each student while in the given programs. Without the student accomplishment, programs that assist with adult education would obsolete.

Adult Learners and Student Success in Mathematics

Much research has been conducted in regards to adult learners; however, there have not been considerable amounts of data found on characteristics of adult education in mathematics. In general, it could be difficult to teach adult learners because there are too many variables that play a role in their learning success. These variables may include their previous education, home life, culture, etc. These insinuations have been heavily researched to help gather techniques that will assist adult learning (Lipnevich & Beder, 2007; van Groenestijn, 2001; Westberry, 1994)

Some characteristics of adult learners, as stated by Lawrence (2007), can be categorized as: “having internal motivation and a problem-centered orientation.” (p. 3) The researchers also state that adults have a more “pragmatic” approach to learning new material in which they would like to apply knowledge to their own life goals. Although students have this type of mental thought, their ability to want to learn concepts in mathematics may be reduced because they have "math anxiety", in which they have had negative experiences with mathematics in the past and do not have the confidence in their own ability to complete mathematic operations.

The research also explains that some implications adult learners tend to have and suggests how the issues could be resolved. The use of shared learning could be valuable to an adult learner especially due to their low confidence in mathematics. Educators can discuss the mistakes that one may accidently make, but need to explain how these could be beneficial when working with similar problems in the future. Lawrence states this will create more risk taking from the learner because the teacher is vulnerable to making errors as well. Another suggestion the researcher believes is that teachers should approach the concepts of math in which it may be integrated into the learners' personal experience. "Drill and kill" practices are not going to benefit the students because they cannot relate the context to real-world applications. Lastly, a major implication is students' attitude towards the subject matter. If the teacher can provide positive feedback and the learner can endure the "frustrating moments" in their mathematics work, the learner will be able to succeed.

According to van Groenestijn (2001), there are four sets of criteria needed for testing adults in Adult Basic Education (ABE). These standards include:

1. Adult students should be enabled to show the best they can
2. Language in a placement test should not hamper the student from doing the math test
3. Adults, in particular second language learners, should have the chance to apply their own mathematical procedures and the algorithms they are used to
4. The test should yield qualitative information about the adults' mathematical skills in order to enable teachers and program developers to set up adequate and well-turned programs for ABE students (p. 67)

Interpreting the first standard, van Groenestijn states that when adults come back to a classroom setting, they may have forgotten the formal mathematical procedures taught, but they may have developed their own informal methods to solving mathematical concepts. He believes students should be provided with mathematical problems in which they have been derived from real-life situations. The language writes about in the second standard is being described as how mathematical problems should only contain minimum text, but essential information should still be written. van Groenestijn states this is beneficial to learners who may have a language barrier. Some learners included are second language learners, which in regards to the third standard, explains how cultural differences should be allowed in the learning environment as well. Text should be “unambiguous”, i.e., $\frac{1}{5}$ can be pronounced as: “one-fifth”, “one-over-five”, or “one-out-of-five”. These strategies will prevent misunderstandings developed during lessons, but could be difficult to incorporate in placement tests. Finally, the last standard discusses the importance of information acquired from the students’ placement tests. The fact that the learner may have answered a problem incorrectly does not mean they have difficulty with the concept. van Groenestijin explains how educators and program developers must answer these types of questions: “What kinds of computations does the student show in the test?” or “Is the student used to formal algorithms or does he apply informal algorithms?” (p. 3)

One particular placement test that may be used in adult education is The Test for Adult Basic Education (TABE). Similar to standardized tests employed in the public school systems, the adult education system has its own assessments that students will take to advance through the program. Piccone (2006) states the TABE is used to assess common skills in adult basic education including such items as applied mathematics and computation, as well as reading at spelling. All test scores are converted to a grade level equivalency that ranges from 0 to 12.9. A zero would represent a score which is below a first grade level, and a 12.9 would represent a score which means the completion of the twelve grade level, in other words, twelve grade and nine months. Regular testing for students normally occurs approximately three times a year as they are allowed to test every 90 days. The TABE covers a variety of different topics including whole numbers, decimals, fractions, percents, integers, etc. Each topic has its own place in history and for the purpose of this study, the focus will continue with the history of integer operations.

History of the Negative Number

Negative numbers have been around for hundreds of years and have not always been accepted by mathematicians. According to Schwartz et al. (1993), when negatives arouse in Europe around 1500, mathematicians referred to negative numbers as being “debitum”, or more simply, complex numbers. “Positive numbers were referred to as “true solutions” to equations whereas negative numbers were defined as “fictitious solutions.” (p. 41) Previous ideas of negative numbers has led to different interpretations and how negatives have developed to present day (Altiparmak & Ozdogan, 2010; Beswick, 2011; Peled, 2006; Prather, 2008).

Struggles in understanding negative numbers have been involved throughout math expressions for centuries. Heefer (2008) explains how before the sixteenth century, algebra problems that resulted in negative values, were considered absurd or simply impossible. Schwarz et al. (1993) state that negative numbers were first created in China and India in the seventh century. One way these “absurd” values were made plausible, was by interpreting the negative values into some context of debt. Schwartz et al. explained how the Hindu mathematician Brahmagupta was the first to introduce negative numbers this way. Though the numbers were converted, they were not intended to remove the idea of negative values, but since the numbers was now in the form of debt, the use of the negative sign may be removed. Heefer relates today’s education to the barriers that occurred centuries ago. The history of the negative sign prepares educators for potential difficulties they may witness in student understandings of problems with integer operations.

One reason the negative sign may have major implications over the past centuries as well as in today's education may be due to how the symbol is used and its importance. As mentioned previously, negative numbers in the sixteenth century were viewed as "fictitious". Schwartz et al. state that this did not necessarily mean the numbers did not exist, but mathematicians of the sixteenth century did not fully accept that the numbers could represent "nonfictitious phenomena." (p. 41) He also explains how mathematicians like Descartes were forced to use negative numbers when working with the Cartesian plane. However, some mathematicians considered the negative coordinates to be meaningless.

It was not until the nineteenth century that negative numbers were represented as directed magnitudes and Schwartz et al. reported that "integers were defined in a way to give negatives a symmetric status to that of positives". (p. 41) It was further researched by Heeffer (2008) that the number line is a widely accepted method in teaching integer operations, but really has only been in existence since the mid twentieth century, when the "father of New Math", Max Beberman, referred to it then as the number scale. However, the use of the number line may not always be the "primary" method in teaching negative numbers. Heeffer also notes that teaching negative numbers is "no longer allowed in basic education" in Belgium. A variety of methods have been created and demonstrated throughout history, yet there is no perfect instruction for integer operations.

Different Methodologies to Solving Integer Operations

One way to be creative in teaching, which is quite essential to student learning, is to find some process that will describe difficult subjects through an easier technique. Integer operations, like most of mathematics, can be taught through multiple representations. The most common method is the number line, which as stated previously, is a fairly new instrument in mathematics and there are multiple methods used in teaching with this tool. According to Beswick (2011), the number line can be represented horizontally or vertically, and depending on what the context is referenced to, a teacher can determine which is more appropriate for student learning. Beswick explains that the horizontal method is more appropriate for lessons because of “convenience” and that students already have a “familiarity” from previous math lessons. However, the method of the number line may not communicate the knowledge the teacher intends to get out of their students.

Many researchers spend their careers studying methods revolving around integer operations and how these methods can engage or hook the students to learning them. Some methods may include application to real-world content or possibly the use of manipulatives that help solve problems pertaining to integer operations. The following research contains many creative methods to teaching integer operations (Carrerra de Souza, Mometti, Scavazza, & Baldino, N.D.; Cemen, 1993; Flores, 2008; Kennedy, 2000; Lamb & Thanheiser, 2006; Ponce, 2007).

Meaningful Contexts

Real life aspects can be beneficial to student learning as the method may help relate to student interests. Brinkerkent (2000) discussed a few approaches to relating integer operations to real-world content. One strategy she introduces is the topic of “Dry and Wet Numbers” which allows the students to use informal methods to solving adding and subtracting integers. The idea behind the lesson is to stimulate mathematical thinking and to promote discussion amongst the students based on discovery. The two numbers are introduced as numbers above and below the original water level. The change in water level is described as the water gained or lost after a ship enters the lock chamber on a canal. *Dry numbers* are to be represented by numbers above water level (positive), while *wet numbers* are introduced as numbers below water level (negative). Students are to count along the number line to find the distance between different water levels (worksheet was provided with a vertical number line). An example of what a student may have seen in this lesson is the process of a ship’s movement from a lower-level canal to the level of water raised inside the lock chamber to enter the higher water level canal. The main focus of Brinkerkent’s activity’s was to have students notice a need for positive and negative numbers to indicate the change in water levels, but her main intentions was to introduce the concept of integers.

<u>Integer Tiles/Chips</u>

The use of the integer chip model is a very useful manipulative used by many educators in the teaching of integer operations. Pan (2009) offers a brief description of how to use this method and its effectiveness on student achievement. First, he defines the model as requiring two different “chips”. The chips must be represented by two different colors, in this case, black and red. The black is denoted by $+1$, a positive number, and the red is denoted by -1, a negative number. By using one black chip and one red chip, i.e., $+1$ and -1, there would be a net value of zero, “0”, also known as a “zero pair”. Figure 1 below explains the process one must take in order to add and subtract positive and negative integers.

Figure 1

Integer Chip Rules for Integer Addition and Subtraction

Operation	Chip Board analogy
Adding a positive integer, “n”	Case I. Placing, i.e., adding n black chips on the board OR Case IV. Removing, i.e., taking away n red chips from the board.
Adding a negative integer “$-n$”	Case II. Adding n red chips to the board OR Case III. Taking away n black chips from the board
Subtracting a positive integer, “n”	Case III. Removing n black chips from OR Case II. Adding n red chips to the board
Subtracting a negative integer , “$-n$”	Case IV. Removing n red chips from OR Case I. Adding n black chips to the board

To sum up what Figure 1 is describing, the first number, say p, in the expression represents the number of chips on the “Chip Board” while the second number in the operation, n, represents what one would add or remove from the board. Thus for addition, if $n > 0$, one would be adding black chips to the board and either add them to the existing black chips (Case I), or create “zero pairs” with the existing red chips (Case IV) to get the sum. If $n < 0$, one would be adding red chips to the board and either add them to the existing red chips (Case II), or create “zero pairs” with the existing black chips (Case III). For subtraction, if $n > 0$, one must remove the number of black chips from p (Case III). If there are not enough black chips to be removed, “zero pairs” must be brought onto the board until there is a total of n black chips, and then remove the proper amount of chips (Case II). If $n < 0$, one must remove the number of red chips from p (Case IV). Once again, if there are not enough black chips to be removed, “zero pairs” must be introduced to the board until there is a total of n black chips, and then remove the proper amount of chips (Case I).

Pan (2009) also describes how multiplication works with this model as well. He states it is simply just repeated addition or subtraction. This method can be represented by four separate cases as well and is demonstrated in Figure 2 below.

Figure 2
Color Chip Rules of Integer Multiplication

The sign of $m \times q$	Color of Chip (sign of first integer, m)	
	Black ($m > 0$)	Red ($m < 0$)
Addition Operation ($q > 0$) (Sign of second removal integer, q ($q < 0$))	Case A: $(+) \times (+) = (+)$	Case B: $(-) \times (+) = (-)$
	Case C: $(+) \times (-) = (-)$	Case D: $(-) \times (-) = (+)$

Basically, he explains that one would start with no chips on the board and the first integer, m, determines the number of groups that one would be adding or removing and the second integer, q, determines whether one would be adding or removing black or red chips. The following circumstances explain what must be completed to demonstrate integer multiplication

Case A: $m > 0$ and $q > 0$

The integer m will determine the number of groups we are ***adding*** since the number is ***positive***. The integer q is the number of chips that will be contained in each group, which will be represented by black chips since the number is positive.

<u>Example:</u> Let $m = 2$ and $q = 3$, i.e., 2×3. Since 2 is a positive number, we will be adding 2 groups to the board. Since 3 is a positive number, we will be adding groups of 3 which are represented by black chips. Thus we have 2 groups containing 3 black integers chips each, representing a total of 10.

Case B: $m < 0$ and $q > 0$.

The integer m will determine the number of groups we are ***removing***, since the number is ***negative***. Thus one would introduce m “zero pairs”. The integer q in this case will again be represented by black chips and will be removed from the board.

Example: Let $m = -2$ and $q = 3$, i.e., -2×3. Since -2 is a negative number, we will be removing 2 groups from the board. Since 3 is a positive number, we will be adding groups of 3 which are represented by black chips. Thus we have 2 groups containing 3 red integers chips each, representing a total of 10.

Case C: $m > 0$ and $q < 0$.

The integer m will determine the number of groups we are ***adding*** since the number is ***positive***. Since the integer q is negative, the number of chips that will be contained in each group will be represented by red chips.

Example: Let $m = 2$ and $q = -3$, i.e., 2×-3. Since2 is a positive number, we will be adding 2 groups to the board. Since -3 is a negative number, we will be adding groups of 3 which are represented by red chips. Thus we have 2 groups containing 3 red integers chips each, representing a total of -6.

Case D: $m < 0$ and $q < 0$

The integer m will determine the number of groups we are ***removing***, since the number is ***negative***. Thus one would create m “zero pairs”. The integer q in this case will again be represented by red chips and will be removed from the board.

The use of manipulatives, especially integer chips can be quite beneficial to “hands-on” learners. However, the use of the chips may not be the proper method to use when working with adult learners as they may find the approach childish or immature. The use of technology may have spark more interest as technology plays a major role in everyday lives.

Computer Software/Games

Though both the number line and integer tiles are appropriate methods to solving problems involving integer operations, they may not always be an efficient method. Educators instead created other methods of teaching these operations which include the use of technology or include games. These methods again may not be the appropriate method, but may be a beneficial review for students who have already been introduced to the concepts of integer operations. Researchers discuss different methods that have been found valuable in classrooms (Burkhart, 2007; Lamb, 2006; Petrella, 2001; Ponce, 2007; Williams, 1997)

In today’s education, technology has a major effect on student education. Lamb (2006) explains a method that may assist students in understanding negative integers with the use of a “Balloons and Weights Software”. She explains how this model consists of balloons and weights where the helium balloons represent positive integers and the weights represent negative integers. Similar to the integer chips, “zero pairs” may be created when we combine one balloon and one weight. The process of adding on balloons or weights represents integer addition, whereas removing balloons or weights indicates subtraction. Essentially, Lamb explains the model being a vertical number line, as the basket will travel up when a weight is removed, and will travel down when a balloon is removed. The mode is quite beneficial to students due to the real-world application and the visual response the students achieve.

Another creative method is discussed by Ponce, in which he uses playing cards to demonstrate positive and negative concepts. He divides the game into five separate levels where level one has a beginner's aspect to the game while level five has a more difficult approach.

Figure 3
Level 1Rules

1. Black beats red, always.
2. Black beats joker.
3. Joker beats red.
4. The same card drawn is a tie
5. Use tally marks to keep track of the number of wins for each player.
6. After all cards are used, count the tally marks and decide who wins.

Level 1:

This level is similar to what one would see in the card game of "war" but the rules are slightly changed. First all face cards and aces must be removed from the deck. Next, students are to be divided into groups of three, and are to follow the rules given in Figure 3. However, students are yet to be told what black, red, or joker cards represent. Ponce then uses the cards to make the comparisons to mathematics and defines integers, i.e., black = positive, red = negative, and jokers = 0.

Figure 4
Level 2 Rules

1. Black Beats red always.
2. Black beats joker.
3. Joker beats red.
4. If both cards are black, the high number wins.
5. If both cards are red, the lower number wins.
6. If the same color is dealt, the same number is a tie.
7. Use tally marks to keep track of the number of wins for each player.
8. After all cards are used, count the tally marks and decide who wins.

Level 2:

This level is an upgrade to what one would learn in Level one, however, now that students possess a basic understanding of positive and negative numbers, the rules slightly change. As seen in Figure 4, the idea of which number is larger or comparing integers come into play. If the colors are the same, instead of a draw, students will have to see which number is larger.

Figure 5
Level 3 Rules

1. To decide who wins each hand, use level 2 rules.
2. Instead of using tally marks, record the score as follows:
   A) If the cards are the same color, add the numbers, and give the points to the winner.
   B) If the cards are different color, subtract the numbers and give the points to the winner.
   C) A joker has no point value.
3. After all cards are used, add the scores to decide who wins.

Level 3:

Level three gets more in depth and adds another component to the game. In this case, students will be following the same rules that were learned in level two, though this time instead of using tallies for points, they take the numbers drawn and either add them or subtract the smaller number from the larger number and give the points to the winner of that round. This is described in Figure 5. What students do not realize is that they are working with absolute values, which Ponce explains in more detail after the level is complete.

Figure 6
Level 4 Rules

1. To decide who wins each hand, use level 2 rules.
2. Record the scores as follows:
   A) If the cards are the same color, add the numbers and write the score with a pen of the same color.
   B) If the cards are different colors, subtract the numbers and write the score with a pen of the same color of the larger number.
   C) Since a joker has no point value, the score is the number of the other card. Write the score using a pen of the color of that card.
3. To decide who wins, determine the scores in the same manner as was done throughout the level.

Level 4:

After the first three levels, students have gathered an understanding of absolute value and distance, but furthermore need to understand how to place the correct sign on a value. Students will now be issued a "red" pen to represent negative, and a "black" pen to identify positive. Students will again follow a new set of rules, as shown in Figure 6. Students win the game by having the larger score at the end of the game (found by totaling the black and red numbers together). Overall, students are working with the addition operator to find the total for each round.

Figure 7
Level 5 Rules

1. For this level you will need to use the aces, kings, queens, and jacks. These cards are considered trap cards.
2. To decide who wins each hand, use level 2 rules.
3. Shuffle the trap cards and place them facedown.
4. Deal two cards, placing one on each side of the trap cards.
5. Turn the top card face-up. If it is red, the card on the right switches color. If it is black, the card on the right stays the same.
6. The trap card has no effect on a joker.
7. Record the scores as you did in level 4
8. To decide who win, determine the scores in the same manner as was done in level 4.

Level 5:

The final level in the method introduces the idea of subtraction. The technique is stimulating as Ponce introduces “trap cards” into the pile. The trap cards are represented by the aces and face cards that were removed for levels 1 to 4. The “trap card” is played as an operator in this case, where if the card is red, the card to the right switches color and if the card is black, the card to the right stays the same. Ponce explains that the key to the discussions is to help “make the transition to the use of mathematical symbols instead of cards and to help (students) understand the logic behind those symbols”.

There are multiple approaches used in solving problems pertaining to integer operations, but the question is, do these methods benefit student learning, or would a traditional “chalk and board method” be more of an advantage from a student’s perspective. Becoming more interactive during methods of teaching may stimulate student learning, but the method may also hinder learning as well if the student is not learning the true meaning of the concept at hand.

Interactive Instruction Effects on Student Achievement

Many mathematics teachers have expressed concerns over student achievement when solving problems involving integers operations and investigate why students experience learning difficulties. One of the biggest issues when working with these operations is when students are introduced to signed numbers, more specifically, negative numbers. Altiparmak & Ozdogan (2010) have done a considerable amount of research on students' difficulties, and states that these difficulties can be described in three separate categories. The first category is the definition of the numerical system and the direction and magnitude of the number. The next category consists of the difficulties experienced when working with different arithmetic operations. Finally, the last difficulty is related to the meaning of the minus sign.

The idea of comparing an interactive instructional approach with a more traditional approach to integer operations can be valuable in determining whether interactive approaches hinder or help student learning. Hayes (1996) researched a similar idea in which he used students from three separate school districts. The control groups either used number lines or the material provided from the class textbooks and the experimental groups either used integer tiles, workbooks that were provided to each student or a read aloud method, in which teachers would ask operation questions aloud and have the students record their answer. Hayes concluded that all subjects had "reasonable knowledge" after the administration of a pre-test. He also discovered that though both the control and experimental groups were taught the material with different methods, most students "automatically" referred to rules of integer operations taught during instruction rather than tiles or diagrams. However, Hayes also stated teacher influence may have affected outcomes due to the fact that teacher and students' attitude and behavior could not be controlled.

Similar to Hayes' study, Altiparmak & Ozdogan (2006) also compared an interactive approach with a more traditional approach to integer operations. The only operations that Altiparmak & Ozdogan focuses on are operations that include addition and subtraction. The researchers state that one of the difficulties for students is the fact that students do not understand the meaning of these operations. These examples include such operations: $2 + 3 = 5$, $2 + (-3) = (-1)$, and $(-2) + (-3) = (-5)$. Altiparmak & Ozdogan further explain that in order for students' to "build the meaning of calculations," the information must be applied to "real-world" applications. (p. 33) However, he also states that "while contexts should be "realistic", they are not only restricted to real-world situations." In other words, the content should be related to student culture, but should also "support new strategies and concepts".

The study Altiparmak and Ozdogan (2010) conducted both a traditional and interactive method to teaching integer operations. The traditional method, the control group, included the use of the number line and white board work while the interactive approach, the experimental group, used Macromedia Flash animations for instruction. The researchers compared the results of both groups to determine if there was a difference in student achievement, and based on the data, found that there was no statistical sign difference between the instructional methods, however, the researchers observed that the instruction supported by the use of the Flash animations did produce better results than the traditional instruction of negative numbers. The researchers also noted that "computer use in teaching enables learners to construct their own learning" and that the concepts constructed, were created towards "real-world contexts" which provides more purpose to the content. (p. 44) Altiparmak & Ozdogan discuss that the students development with the concrete process should come to a point in which they will not need concrete objects for number sense.

Difficulties with the negative sign amongst students may be due to not knowing how to relate a negative number to real-life situations. Students have no issues when working with positive numbers because they understand the meaning behind the number. Conversely, students struggle with the meaning of negatives because they may not have used them in the environment they live. The studies mentioned above provided positive feedback when instructing problems pertaining to integer operations with an interactive approach, however there is not enough evidence through research explaining why students struggle when working with integer operations. If more research were examined, observers could take the data and examine what exactly creates issues involving integer operation problems and how the concerns could possibly be addressed to assist student learning.

Experimental Design and Data Collection

This experiment was designed to test the hypothesis that would help study why students struggle problems relating to integer operations. During the experiment, adult learners were given multiple methods to solving problems pertaining to integer operations. Student's work was collected and analyzed for misconceptions and errors.

Participants

Figure 8
Number of Students by Age

Age	# of Students
16	6
17	48
18	41
19	45
20	47
21	35
22	15
23	16
24	18

This study was conducted at an alternative education program in New York. The academy has roughly 275 students which due to the acceptance of new students, changes from week to week. Approximately 90% of the students enrolled in the program are from areas of New York, with the majority of the population from New York City. The remaining of the population is from other locations in the United States, and nearly 3% originate from the Virgin Islands. Over half (65%) of the students are African American, while Hispanic/Latinos and Whites form 16% each. The remainder of the pupils is of other ethnicity and make up the rest of the population. As seen in the table, the majority of the students fall within the age range of 17-21. The academy consists of 7 academic instructors, 3 managers (known as administrators in a common public school), and 7 trade instructors.

The participants of this study consisted of 24 students where Class A had 13 students and Class B had 11 students. All students were African American but one who was Hispanic and part of Class A. The ages of the students varied, with some as young as 18 to as old as 24. Any student who was under the age of 18, was not included in this study due to the lack of contact with parents to sign consent forms, however, these students still participated in the class lessons.

Figure 9
Number of Students by Gender

Class A				Class B			
Age	**Male**	**Female**	**Total**	**Age**	**Male**	**Female**	**Total**
18	2	1	3	18	2	0	2
19	0	0	0	19	1	2	3
20	2	1	3	20	1	2	3
21	3	1	4	21	1	0	1
22	1	0	1	22	0	0	0
23	0	0	0	23	1	0	1
24	1	1	2	24	0	1	1
25	0	0	0	25	0	0	0
Total	9	4	13	Total	6	5	11

In order for a student to be enrolled into the program they must choose a trade offered at the center, which includes, Carpentry, Certified Nursing Assistant (CNA), Culinary, Electrical, Plumbing, and Painting. Many of these students dislike mathematics and do not see the purpose behind learning fundamental mathematic problems, which seem to be a result from their previous school experiences.

All students enrolled at the institution were administered a TABE in reading and mathematics within the first week of their arrival. A student's schedule is determined based on how well they perform on their TABE scores in basic mathematics and reading. If their TABE scores do not meet the center's requirements, the students are enrolled into Pre-GED mathematics or reading courses and may retest every 90 days. However, if the students complete the material in the given courses, they can earn their way on to a follow-up TABE after 30 days following their previous test.

After a student completes basic mathematics and/or reading, there is one of two scenarios. If the student entered the program with a high school diploma, then they would be placed in a vocational class or their choosing upon arrival. If the student does not have their diploma, they are to enter a GED preparation course or a High School Diploma program offered at the institution.

Design

The experiment was conducted over a one week span during the 2012 academic year. During this period, two classes were involved in the experiment: one group being taught with an interactive approach, Class A, and the other with a traditional method relating to real-world scenarios Class B. Both classes were given a survey before instruction commenced. This was done to determine what prior baseline knowledge the students have retained from previous lessons in their traditional classroom settings before attending the academy. The survey included different problems pertaining to integer operations, but did not ask them to solve any equations. Figure 10 represents the questions asked, and were provided a week before the unit lesson occurred.

Figure 10
Survey

Name: ________________________________

Define Integer:
What sign would result from $(-)+(-)$? $(-)\times(-)$?
Explain how to solve a $(+)+(-)$? Will the answer be positive or negative?
Based on these questions, rate your understanding of integer operations from 1 (least amount of understanding) to 5 (greatest amount of understanding).
What is something you seem to struggle with when using integers?

Figure 11

Lesson Format

Day 1: Integer Addition – Same Sign
Day 2: Integer Addition – Different Sign
Day 3: Integer Subtraction
Day 4: a) Integer Multiplication/Division
Day 4: b) Review game
Day 5: Exam

Figure 11, located to the right, is a representation of the five day unit that took place. Though each group received different instructional approaches, each was given the same topic to work on each day. Days one through three are critical lessons in which students must understand what they learned from the previous lessons in order to help them succeed in the next. Day four is also a significant lesson because students may mix up "rules", and answer the operations with wrong signs (i.e., $(-) + (-) = (+)$, $(-) \times (-) = (-)$). Although only one lesson is provided a day, students are scheduled in class at least twice a day. Thus their final period will be replaced with a Jeopardy review game following the lesson on integer multiplication and division. The exam is used not only as a final assessment of what the students have learned, but also as a way to determine if students still have misconceptions after the instruction has taken place.

The unit exam consists of twenty questions that are all computation problems. The test is based entirely on the TABE, in which students will only examine integer operations as something computational. The problems were designed to assess the students' knowledge of various aspects of integer operations where most are perceived to have fallacy. Overall, the exam was broken up using all four operations, which include six problems in addition, six problems in subtraction, and six problems in division, and two problems in multiplication. The following are explanations of why some questions were chosen for the post assessment. Problem #2 served a dual purpose as it was used to assess the students' knowledge of the rules for the division operation when using integers, but also was used to examine if students would mistake the operation with the rules of addition. Similarly, Problem #8 was used to not only assess the rules of subtracting integers, but also was used to observe if the student could recall the rules for adding integers. (The post-test can be found in the appendix.)

Data Collection

In order to test the hypothesis, students' data from the surveys were first gathered and viewed to determine if there was any prior knowledge when working with integer operations. Although the questions were general, the data was examined to see whether they confused integer operation rules such as integer addition and integer multiplication. The survey was then analyzed based on a scale of one to five to determine how they assess themselves when working with integers. Overall, the data provides a basic understanding of what students have already retained from previous schooling. The data was later used to compare with the unit exam to see if students made the same mistakes or improved based on their survey responses.

The survey was followed by a five day unit lesson which involves integer operations of addition, subtraction, multiplication, and division. Students were classified as two separate groups, Class A, in which they met three times a day and Class B, which met twice a day. Class A was given an interactive approach, in which students had access to a Promethean Board, and Class B was taught using a traditional method relating to real-world scenarios. Each class received class worksheets after every lesson, however, were able to keep for the purpose of studying. Both classes were given a unit exam consisting of similar questions students would see on the TABE in Mathematics following the in-class lessons and review. The data collected from both Class unit exams was compared to see if there was a significant difference in instructional methods and if one method was more superior to the other. Data was analyzed first by the overall score of the test, followed by a question by question analysis, and finally by each operation. All data collected; survey, class worksheets, and unit exam, was graded by the principal investigator.

Methods of Analysis

The data for this experiment was collected by analyzing the results of three separate items; the surveys, the instructional approaches, and the unit exam. Though it is difficult to know whether the students have been previously exposed to integer operations or not, each student has completed up to eighth grade level, which means that most should have encountered these types of problems before in previous schooling.

Though this experiment was mostly of a quantitative nature, there was also some qualitative research as well. Once the data was collected, the students' surveys were first examined to determine what struggles they may have already encountered with integers. The survey was of qualitative nature and the rest of the data analyzed was quantitative. Next, the worksheets collected from each lesson were then compared by instructional approach. In other words, Class A in-class worksheets were compared to those of Class B. The unit exam played two roles in the analysis. Overall, the first were to see if the scores determined if learning was enhanced based on the instructional methods. The second role consisted of a comparison of the unit exam to the student's original thoughts of integer operations. The comparison was done to determine if any misconceptions from the pre-survey took place on the exam. Data was then observed based on student age to examine if the time spent away from school had a direct effect on the unit exams averages.

Results

Answering the original question, “Does technology have an effect on student learning?” shows surprising results. After analyzing the data collected from the two instructional approaches there are three major results:

- ***There were no differences in mean scores when comparing the technology enhanced teaching style to the traditional approach. (Class A: $\bar{x} = 61.15$ and Class B: $\bar{x} = 62.33$) (P-value: 0.624)***
- ***The main reasons for incorrect answers were due to arithmetic errors and incorrect sign use.***
- ***Students were inconsistent in answering operations of the same kind. In one instance, students correctly answered an integer division operation, however, incorrectly answered a question of similar nature.***
- ***The number of years out of school did not directly affect the scores of older students (18 – 19 $\bar{x}$: 65% 20 – 21 $\bar{x}$: 56.82% 22 – 25 $\bar{x}$: 71%). Student gender however, portrays inconsistent scores (Male $\bar{x}$: 62.86%: Female: $\bar{x}$: 58.89%).***
- ***The survey provided very little or no evidence of prior knowledge with integer operations.***

The hypothesis pertaining to the technology enhanced teaching style outperforming the traditional teaching style demonstrates there was not enough evidence, as it is clear that there is no definitive difference when comparing mean scores and p-values.

Result 1: There were no differences in mean scores when comparing the technology enhanced teaching style to the traditional approach. (Class A: $\bar{x} = 61.15$ and Class B: $\bar{x} = 62.33$) (P-value: 0.624)

Based on the data collected, students from both Class A and B had similar results when comparing the mean scores. The results from the unit exam may be examined in Figure 12 (seen on p. 24). However, when comparing exams by question average rather than the exam as a whole, it was easier to see student misconceptions. The column to the far left represents the percentage of students that answered each questions correctly in Class A, the technology approach. Similarly, the column second from the right represents the students from Class B, the traditional approach. The bottom row represents the mean average of correct answers from both Class A and B. Lastly, the column on the far right is the mean scores for each question for both Classes. All data was determined by entering the number of correct responses for each question, for each class, into a Microsoft Excel spreadsheet. After the data for each student was entered, the sums and percentages of the totals based on each class were found. This table was beneficial in which it demonstrated which questions students had difficulties with.

Figure 12
Grade comparisons between Class A and Class B:
Below are the problems used for the post assessment for both the interactive and traditional approaches along with the statistics found when collecting data from Math I classes at Cassadaga Job Corps Academy.

Class A (Technological Approach)	**Test Problem**	**Class B (Traditional Approach)**	$\bar{x}$
61.54%	1. $35 - {}^{-}5$	54.55%	**58.045%**
61.54%	2. ${}^{-}80 \div {}^{-}5$	72.73%	**67.14%**
84.62%	3. ${}^{-}9 + 4$	81.82%	**83.22%**
76.92%	4. $4 - {}^{-}9$	54.55%	**65.74%**
53.85%	5. ${}^{-}99 \div {}^{-}11$	90.91%	**72.38%**
53.85%	6. $7 + {}^{-}9$	54.55%	**54.2%**
69.23%	7. ${}^{-}65 \div {}^{-}5$	63.64%	**66.44%**
46.15%	8. ${}^{-}4 - 18$	45.46%	**45.81%**
100%	9. $5 \times {}^{-}6$	81.82%	**90.91%**
76.92%	10. $8 \times {}^{-}4$	54.55%	**65.74%**
46.15%	11. $60 \div {}^{-}15$	72.73%	**59.44%**
38.46%	12. $5 + {}^{-}4$	27.27%	**32.87%**
69.23%	13. $26 - {}^{-}3$	81.82%	**75.53%**
84.62%	14. ${}^{-}76 \div 4$	72.73%	**78.68%**
53.85%	15. ${}^{-}6 + {}^{-}8$	54.55%	**54.2%**
46.15%	16. ${}^{-}7 - 4$	54.55%	**50.35%**
61.54%	17. $6 + {}^{-}10$	54.55%	**58.05%**
30.77%	18. $84 \div {}^{-}21$	63.64%	**47.21%**
61.54%	19. ${}^{-}3 + {}^{-}11$	64.64%	**63.09%**
46.15%	20. ${}^{-}3 - 11$	45.46%	**45.81%**
61.15%	**Class Averages**	62.33%	**61.74%**

By observing questions 2, 5, 6, and 18, which all revolve around integer division, Class B outperformed Class A by a wide margin. Conversely, when observing questions 7 and 14, Class A outdid Class B using the same operation. Overall, the use of technology in the classroom did not necessarily prompt different or alternative solutions when compared to traditional classroom instruction. It is also worth noting that based on the mean scores for each question, the hardest question for students was question 12 and the easiest question was questions 9.

Overall it was determined that the scores, when Class A and Class B were compared, had no significant results. Class A's had an overall average of 61.92% and Class B had an average of 63.18%. Furthermore, when calculating the p-value in regards to Class A and B, the result was determined to be 0.624. The results revealed that the original hypothesis statement stating that the use of technology would yield more significant results than instruction of the traditional nature wasn't enough to support the hypothesis.

Result 2: The main reasons for incorrect answers were due to arithmetic errors and incorrect sign use.

The exam was initially analyzed for correct and incorrect answers, but for the purpose of this study, incorrect answers were broken down to three major components. These were represented by responses that had wrong signs, wrong values, and wrong signs and values. These components can be seen in Figure 13. The use of the "N/A" was to represent the students who did not answer the questions on the test.

Figure 13

A complete analysis of correct and incorrect answers on the unit exam

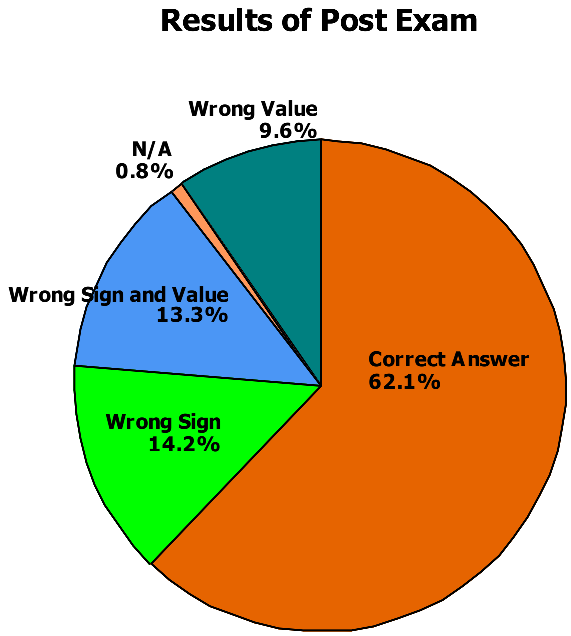
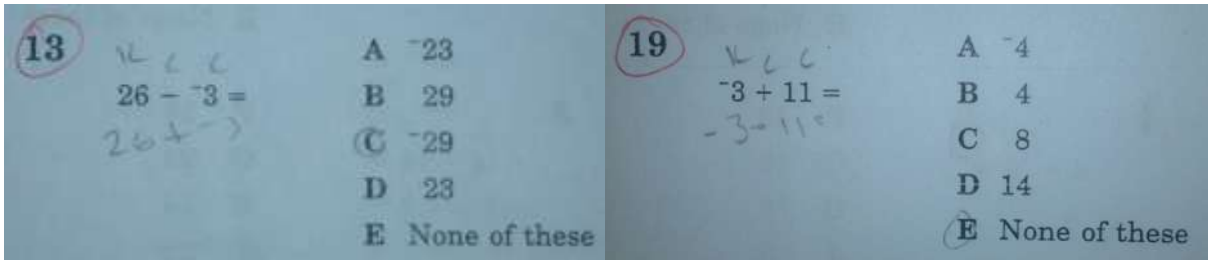


Figure 13 demonstrates that the use of wrong signs were a major contributor for incorrect answers followed closely by wrong signs and values. The reason behind this can be a direct cause of simply confusing the rules for each operation, but could also be due to a student may not having a complete understanding of the operation.

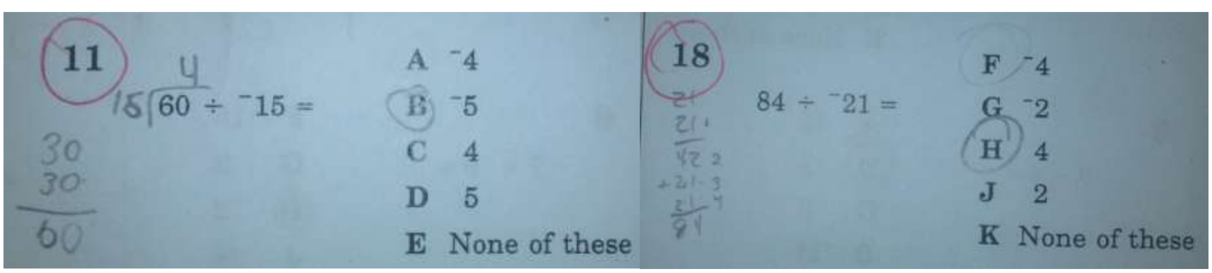
Furthermore, the data was investigated to see what questions on the unit exam were the causes of these errors. Examining the student's responses, it can be seen that some students knew the correct strategies to solving each operation, however, did not complete the strategy properly. The following student responses have been scanned from original raw data without any changes.

Figure 14
This figure demonstrates a students' work that compares integer operations of addition and subtraction from the unit exam.



Notice in Figure 14, this student did use the correct technique when subtracting integers, i.e., the "Keep Change Change" approach, however, did not use it properly. In problem #13, the student did not change the sign of the second integer, -3. Furthermore, the student did not solve the newly created equation correctly either. Question #19, also from this same student, chose to use the rule for subtraction, but this time was used when it was not needed. This shows that the student failed to gain an understanding of when and how to use the rule for subtraction.

Figure 15
This figure demonstrates a students' work that compares integer operations of division from the unit exam and the misunderstanding of the negative sign.



Similarly, it can be seen in Figure 15 that some errors may just occur due to simple mistakes. In question #11, this student correctly solved the problem, but answered incorrectly because they circled the wrong multiple choice letter. Question #18, again had all the correct arithmetic, however, the student failed to understand the rule of dividing integers. This is demonstrated in the example as the student first circled the correct answer, though second guessed themselves, and selected the wrong answer.

Result 3: Students were inconsistent in answering operations of the same kind, i.e., students answered an integer operation question correctly, however, incorrectly answered a question of similar nature.

Figure 16 demonstrates the comparison of correct and wrong answers when viewing questions 9 and 10. Both problems involve integer multiplication and both are formatted as a positive integer multiplied by a negative integer.

Figure 16

The given data represent the percentage of correct answers versus wrong answers in the multiplication operations given on the unit exam for questions 9 and 10

Question #9: $5 \times {}^{-}6$		Question # 10: $8 \times {}^{-}4$	
Correct Answer	91.7%	Correct Answer	70.8%
Wrong Answer	8.3%	Wrong Answer	29.2%

However, it can be seen that students from both Class A and Class B struggled more with question 10 than question 9. Furthermore, when examining the students data, question 9 and 10 were broken down by correct and incorrect answers.

Figure 17
The graphs below represent the reason for incorrect answers.

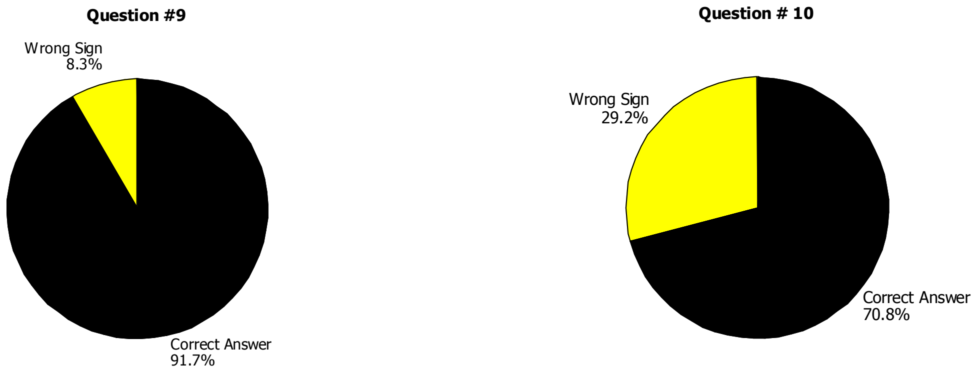


Figure 17 demonstrates that for the students who answered incorrectly, was due to the use of a wrong sign. The data explains that not all students may have completely understood the concept of integer multiplication. The reason behind the data could be because students confused the rules of integer addition with those of integer multiplication.

Result 4: The number of years out of school did not directly affect the scores of older students (18 – 19 $\bar{x}: \mathbf{65}\%$ ***20 – 21*** $\bar{x}: \mathbf{56.82}\%$ ***22 – 25*** $\bar{x}: \mathbf{71}\%$***). Student gender however, portrays inconsistent scores (Male*** $\bar{x}: \mathbf{62.86}\%$***: Female:*** $\bar{x}: \mathbf{58.89}\%$***).***

The data from the unit exams were also analyzed to see if there was a significant difference when comparing the scores by student age. This was done to examine if younger students outperformed the older students based how long each student has been away from school. Figure 18 is a representation of the average score based on student age and gender.

Figure 18
Comparison of mean scores based on Age and Gender for both Class A and Class B

Age	Male	Female	Average Score by Age
18 – 19	5	3	65%
20 – 21	7	4	56.82%
22 – 25	3	2	71%
Average Score by Gender	62.86%	58.89%	

The participants were grouped into three age groups, 18 – 19, 20 – 21, and 22 – 25. Surprisingly, the outcome of the data shows that the number of years out of school did not have a direct effect on the scores of the participants as those who fell in the 22 – 25 range scored higher than those in the other age groups. The age group of 18 – 19, has averages that fall closer in scores unlike those from the age group of 20 – 21, which averages are very inconsistent.

Another aspect of the data examined was student gender and the average scores collected from the post-tests. Figure 16 reveals that there was a difference in average scores between genders, in which males did score slightly higher than females. Similar to the comparison of age, the data demonstrates the scores are quite inconsistent. Though males did score higher than females in average scores, the only perfect score recorded on the unit exam was from a female in Class B.

Result 5: The survey provided very little or no evidence of prior knowledge with integer operations.

Figure 10:
Survey (as seen before)

1. Define Integer:
2. What sign would result from $(-) + (-)$? $(-) \times (-)$?
3. Explain how to solve a $(+) + (-)$? Will the answer be positive or negative?
4. Based on these questions, rate your understanding of integer operations from 1 (least amount of understanding) to 5 (greatest amount of understanding).
5. What is something you seem to struggle with when using integers?

Recalling the survey seen in Figure 6, the results show not much can be taken from the data. The first question, "Define Integer", did not produce any correct responses. Of the twenty-three students, only five students provided a response not saying "I don't know". The replies include: "Numbers on a number line", "Negative and positive equations", "Positive and negative numbers, also zero", "Positive and negative numbers", and "A negative and positive number". All responses include a vague understanding of integers; however, none could provide a correct response. All students did give questions for question two, yet only one correctly responded. Others answered either negative or positive to both operations. Question three did not provide any positive results either, as students responded it would be positive or negative. The results seen in Table 9, provides the level of understanding thought by each student before lessons commenced. Most

Table 10: Ratings of Understanding Based on Survey

Level of Understanding	Number of ratings
1	8
2	3
3	6
4	3
5	0
No Response	3

chose numbers that stated they had an average understanding or very little understanding of integers. Lastly, question five only produced two answers: “I don’t know” and “Everything”.

The real question is whether we can generalize enough to determine why negatives get in the way of solving integer operations. There may be some factors that could make the learning which could be as simple as relating integer operations to real life applications. In general, people do not use negative numbers in everyday life. Do students in adult education, or at any level for the matter, understand the use of the negative sign and its purpose? When relating negative numbers to money as something is being “owed” in the lessons, students generally had a better understanding of how to create the solution. However, taking away that context by only giving computation questions with the negative sign seemed to hinder a student’s ability to understand how to solve the problem correctly. This alone could be a specific factor of why misconceptions occur when students work with integer operations.

Implications for Teaching

The hypothesis for this study was that the use of a technology-based lesson involving integer operations would result in a higher level of achievement than a traditional instructional approach. It was believed that the use of a Promethean Board would help the students put forth more effort as well as create a more interactive environment that would increase interest in student learning. When comparing the two classes and the approach used for each, it was found that the use of the board did not provide a difference in student learning when compared to the traditional chalk and board method when working with integer operations. However, the interest of the students in the technology class did create more note taking and in-class discussions.

The use of technology is a method that is being implemented into many classrooms and the question that arises is if it really benefits student learning or is it a classroom distraction that may hinder learning? In the case of integer operations, there has been much research that shows a variety of results of different instruction strategies and how it affects a classroom. Some researchers have shown the use of a manipulative could be beneficial, while others find games could peak interest and improved results. There is no “definite” or “perfect” way to answer the question ‘What is the greatest way to instruct the topic of integer operations’. The fact that there is no impeccable method to this question leads to several implications of teaching.

Avoid extensive use/reliance on multiple-choice problems in the mathematics classroom

One of the most important implications that results from this study is the use of a multiple choice test. Though the reason behind the test was to replicate the TABE, many students did not show any work and it could not be seen if they simply answered incorrectly or if they simply just circled an answer. Furthermore, some students' answers resulted in demonstrating correct work, but the circling of wrong answers.

The use of work helps demonstrate if students have an idea of whether they had a basic understanding of the problem at hand. This factor may be overcome if the multiple-choice were replaced by a short-answer or open ended test, in which it may have been more beneficial to the results. The use of an exam of this sort would remove answers that may have been falsely given. In other words, students who may have just circled an option or may have guessed a response would be forced to show some type of work or just leave it blank all together. This would give a visual to better understand why students struggle and would help provide what they are actually thinking.

Implementation of "Career-based" lessons may have a significant effect on the unit exam

Another implication that was a result of this study was the relation of integer operations to their everyday lives. Students' main focus at this particular institution is to train for the career they have chosen upon entry. This creates a difficult teaching environment due to the lack of interest in mathematics and their beliefs that they will not require any information learned in their future careers. Although integer operations are difficult to relate to the careers they have chosen, other concepts in mathematics should be taught to peak students interests in their specific chosen field.

As far as the concept of integer operations, the lessons did relate the use of accounting and banking to allow for a better understanding of problems. The lesson mentioned a positive integer representing an amount already owned or gained and a negative integer as something owed. However, though this concept was mentioned, the lesson did not go into detail or have any problems relating to money. If students were to receive content including problems mentioning banking, not only are they learning something that would be beneficial to aspects in life they will encounter in the future, but the students would also observe a representation that would help them understand integer operations.

<u>Place a focus on “basic” mathematic skills</u>

Lastly, another implication that results from this study is the poor use of arithmetic used when solving operations. Many students struggled with their multiplication and division facts which were a direct result to incorrect answers on the post-test. A possible fix to this implication may be to spend more time with basic math operations.

Students are required to move on from one topic to another in such a reckless pace, that not all students are completely efficient in understanding how to solve certain types of operations. Time is not necessarily the only answer for this issue, but “practice” may also be a reason for lack of abilities. The math curriculum contains much content of all varieties, but how much is really needed in order to be applied to the real world? This question could be debatable in any situation; however, in preparation for the TABE test, as well as preparing students for their chosen trade, many topics become irrelevant to some students. The use of whole numbers can be seen as a “pre-requisite” to integer operations, and can play a major factor in providing correct responses to integers.

Suggestions for Future Research

Although this research did not yield conclusive results regarding if a technology-based method to teaching is more efficient than a traditional approach, further research in this area may generate more interesting results. One might consider examining a larger group size by implementing more classes or possibly schools in the experiment. Also, a review with basic mathematic arithmetic should be provided before the unit on integer operations partakes. A review would have beneficial for many students who struggle in their arithmetic skills and may have provided more resourceful data as a result. Conducting a similar study in regards to technology or non-technology methods in the classroom may also be favorable if the content were related to a different content area in mathematics.

Concluding Remarks

The purpose of studying the topic of integer operations was not only to determine if classroom instruction would play different roles in student learning, but was to also gather data that would provide information that would explain the misconceptions when using integer operations. Based on the results of this study, it appears that instruction of integer operations with the use of technology does not provide higher scores when compared to a class that is instruction using a traditional approach. One major concern based on the results is the sample size. In order to gather a more accurate answer to the hypothesis, a greater abundance of students needs to be intact. As with any instructional method, every teacher must choose the appropriate strategies for their students to maximize learning and achievement.

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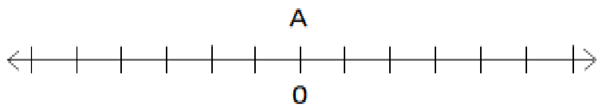
Appendix A

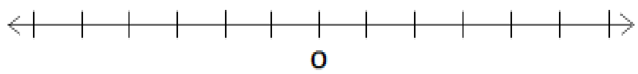
Name: ____

Integers: Absolute Value and Same Sign Addition

1. **Write the letters above the number line to show where each number belongs.**
   A. 0
   B. 3
   C. -4
   D. -6
   E. 5



2.



   a) **Write the letter F above the point -2 on the number line below**

   b) **Write the letter G above the point that is 4 units to the right of point F.**

   c) **Write the letter H above the point that is 3 units to the right of the point F.**

   d) **Write the letter I above the point that is 9 units to the left of H.**

   e) **What number does each letter represent?**

3. **Circle the greatest number in each group**

a.	$-3, 0, 2$	$4, 0, -2$	$5, 3, -7$	$-4, -2, -8$
b.	$-1, -5, -6$	$3, -1, -5$	$-6, -2, 0$	$-5, 3, -1$

4. **Circle the least number in each group**

a.	$-8, -10, 0$	$7, 0, 2$	$8, -1, 0$	$-2, 6, -4$
b.	$-4, 7, -5$	$0, 3, -2$	$-2, 4, 0$	$0, 1, -1$

5. **Write the absolute value.**
   a. $|-3|$
   b. $|5|$
   c. $|-345|$
   d. $|32|$

Appendix A continued

6. **Compare the values in each pair. Write $<$, $>$, $or =$**
   a. $|32|$ $|-32|$
   b. $|-12|$ $|-15|$
   c. 10 $|-20|$
   d. $|3|$ -9
   e. -6 $|-10|$

7. **Add the following integers**

   a. $3 + 5 =$

   b. $6 + 4 + 10 =$

   c. $(-7) + (-2) =$

   d. $|-10| + 3 =$

   e. $(-1) + (-1) =$

   f. $(-1) + (-3) + (-8) + (-4) =$

8. **To add – 5 to – 4 on a number line, start at – 5 and move 4 spaces to the ______.**

9. **A restaurant served 56 pizzas yesterday. It expects to serve 64 pizzas today. How many pizzas will it serve in total?**

10. **In Buffalo, New York, the temperature was – 14°F in the morning. If the temperature went down another – 7°F, what is the new temperature?**

Appendix B

Name: ________________________________

Integers: Adding Integer Operations

1. Compare the values in each pair. Write $<, >, or =$

a. -12 -26

b. 23 $|-16|$

c. 94 $|-94|$

d. -62 62

e. 0 240

Add the following integers

Same Signs

a. $(-19) + (-15) =$

b. $12 + 18 =$

c. $16 + 24 =$

d. $(-27) + (-16) =$

e. $(-50) + (-50) =$

f. $0 + (-14) =$

g. $48 + 31 =$

h. $(-38) + (-30) =$

i. $(-1) + (-46) =$

j. $(-10) + (-46) + (-13) =$

Different Signs

a. $(-18) + 9 =$

b. $(-24) + 12 =$

c. $38 + (-38) =$

d. $(-17) + 16 =$

e. $72 + (-54) =$

f. $(-94) + 108 =$

g. $(-27) + 27 =$

h. $(-12) + (-7) + 13 =$

i. $16 + (-16) + 16 =$

j. $(-23) + 16 + (-12) =$

Appendix C

Name: ________________________________

Adding and Subtracting Integers

1. Compare the values in each pair. Write $<$, $>$, $or =$

a. -15 13

b. 9 $|-5|$

c. 35 $|-35|$

d. -13 -4

e. -100 0

2. Add the following integers

a. $(-3) + 6 =$

b. $(-8) + (-4) =$

c. $1 + 7 =$

d. $9 + (-3) =$

e. $14 + (-2) + 4 =$

f. $9 + (-5) + (-16) =$

g. $76 + (-15) =$

h. $11 + (-14) + (-20) =$

i. $(-27) + (-12) =$

j. $21 + (-5) + 13 =$

3. Subtract the following integers.

a. $5 - (-8) =$

b. $(-7) - 8 =$

c. $(-9) - 4 =$

d. $9 - (-2) =$

e. $80 - (-16) =$

f. $25 - (-15) =$

g. $10 - (-7) - (-9) =$

h. $6 - (-9) + 7 =$

i. $6 + (-8) - 7 =$

j. $14 - (-40) =$

Appendix D

Name: ______________________________________

Integer Operations

Add or subtract.

1. $-13 + -34 =$
2. $21 - (-43) =$
3. $22 + (-21) =$
4. $15 + 26 =$
5. $32 - 45 =$
6. $-10 - (-12) =$
7. $12 + -16 + -10 =$
8. $14 - 30 - (-21) =$

Multiply.

9. $15 \text{ x } 3 =$
10. $10 \text{ x } (-7) =$
11. $-8 \text{ x } (-9) =$
12. $-2 \text{ x } 17 =$
13. $16 \text{ x } (-4) =$
14. $-14 \text{ x } (-5) =$
15. $12 \text{ x } 0 =$
16. $-12 \text{ x } (-3) \text{ x } 5 =$

Divide.

17. $28 \div 4 =$
18. $-30 \div 5 =$
19. $-64 \div -8 =$
20. $132 \div -11 =$
22. $-56 \div -4 =$
23. $45 \div 15 =$
24. $156 \div -12$
21. $-72 \div 9 =$

Appendix E

Jeopardy

Addition

100 - -15+18=

200 - -31+-14=

300 - 10+22+-13=

400 - -16+|-12|+-13=

500 - |-20+-13|=

Subtraction

100 - 14-(-21)=

200 - -17-5=

300 - -5-9-(-12)=

400 - |14|-8-|-9|=

500 - |-17-24-(-11)=

Multiplication

100 - 6 ×-8=

200 - -7×-9=

300 - 11×3×-2=

400 - -10×-5×-4=

500 - |-12|×-5×3=

Division

100 - -64÷8=

200 - -132÷-12=

300 - -81÷9÷-3=

400 - -108÷-9÷-4=

500 - |-75|÷-3÷|5|=

Which is Larger?

100 - |-15| 14

200 - 12-(-6) 20+-7

300 - 5×4 14+|-8|

400 - |-6+-16| |-30|-|-11|

500 - 17+-4+-8 81÷9

Appendix F

Name: ______________________________

1. $35 - {}^{-}5 =$
   A. 25
   B. 35
   C. −25
   D. −35
   E. None of these

2. ${}^{-}80 \div {}^{-}5 =$
   A. −14
   B. 14
   C. −16
   D. 16
   E. None of these

3. ${}^{-}9 + \;\; 4 =$
   A. 13
   B. −5
   C. −7
   D. −11
   E. None of these

4. $4 - {}^{-}9 =$
   A. 13
   B. −13
   C. −5
   D. 5
   E. None of these

5. ${}^{-}99 \div {}^{-}11 =$
   A. −10
   B. −11
   C. 10
   D. 11
   E. None of these

6. $7 + {}^{-}9 =$
   A. −16
   B. 2
   C. −2
   D. 16
   E. None of these

7. ${}^{-}65 \div {}^{-}5 =$
   A. −13
   B. 12
   C. −12
   D. 13
   E. None of these

8. ${}^{-}4 - \;\; 18 =$
   A. −14
   B. 22
   C. 14
   D. −22
   E. None of these

9. $5 \times {}^{-}6 =$
   A. −1
   B. 1
   C. −30
   D. 30
   E. None of these

10. $8 \times {}^{-}4 =$
   A. −2
   B. 32
   C. −32
   D. 2
   E. None of these

Appendix F Continued

11. $60 \div {}^{-}15 =$
    A. -4
    B. -5
    C. 4
    D. 5
    E. None of these

12. $5 + {}^{-}4 =$
    A. 0
    B. -9
    C. 9
    D. -1
    E. None of these

13. $26 - {}^{-}3 =$
    A. -23
    B. 29
    C. -29
    D. 23
    E. None of these

14. ${}^{-}76 \div \ 4 =$
    A. -14
    B. 14
    C. -18
    D. 18
    E. None of these

15. ${}^{-}6 + {}^{-}8 =$
    A. -4
    B. 14
    C. -8
    D. 2
    E. None of these

16. ${}^{-}7 - \ 4 =$
    A. -11
    B. 11
    C. -3
    D. 3
    E. None of these

17. $6 + {}^{-}10 =$
    A. -4
    B. -16
    C. 4
    D. 16
    E. None of these

18. $84 \div {}^{-}21 =$
    A. -4
    B. -2
    C. 4
    D. 2
    E. None of these

19. ${}^{-}3 + {}^{-}11 =$
    A. -4
    B. 4
    C. 8
    D. 14
    E. None of these

20. ${}^{-}3 - \ 11 =$
    A. -8
    B. -12
    C. 8
    D. 14
    E. None of these

Appendix G

Name: _______________________________

Survey

1. Define Integer:

2. What sign would result from $(-) + (-)$? $(-) \times (-)$?

3. Explain how to solve a $(+) + (-)$? Will the answer be positive or negative?

4. Based on these questions, rate your understanding of integer operations from 1 (least amount of understanding) to 5 (greatest amount of understanding).

5. What is something you seem to struggle with when using integers?

Appendix H

TO: Students in Math I

FROM: Joshua Sadler

RE: Consent Form

Purpose, Procedures, and Benefits

Purpose: The purpose of this study is to determine what students struggle with when working with integer operations, and what factors cause these misconceptions.

Procedures: Students will be given a five day unit lesson which consists of integer operations. Students will first receive a survey one week prior to the start of the lesson asking how to solve certain integer operations. During the unit lesson, students will receive one worksheet per day and each will be corrected right or wrong. Students will have one opportunity to correct their work before their final hand in. On the final day of the unit, students will be given a 20 question exam based on the content they received during instruction. One week after the exam, students will receive another survey, identical to the first, to see if the information they learned is retained.

Benefits: Through this study, I hope to determine the misconceptions students receive when working with integers.

Related Information

- You have been asked to be in this study because you are a student in Mr. Sadler's Math I class.
- Protecting Confidentiality: your name will not be used in any materials related for this project.
- Participation in this study is voluntary; you are free to withdraw from the study at any time with no associated penalty. However, be aware you must still complete this unit as the content being taught is still a TABE requirement.
- Data will be collected from the graded worksheets, the completed surveys, and the unit exam made during the study
- There is no cost to you for participating in this research study.
- If you have any additional information, please feel free to contact me by Cassadaga Job Corps phone at 1204321; or by e-mail at: Sadler.Joshua@jobcorps.org.

Appendix H Continued

I would appreciate your collaboration in this very important project. Please sign below to indicate your agreement to participate in this study. You may retain a copy of this letter for your own files. Thank you for giving this request your full consideration.

Student Consent Form

Cassadaga Job Corps Academy

Voluntary Consent: I have read this memo. My signature below indicates that I freely agree to participate in this study. If I withdraw from the study, I understand there will be no penalty assessed to me. I understand that my confidentiality will be maintained. I understand that if I have any questions about the study, I may reach Joshua Sadler by e-mail at: Sadler.Joshua@jobcorps.org.

Student Name (Please Print): ____

Student Signature: ____

Date: _________