Applying the 5E Instructional Model to Systems of Linear Equations

Lucas G. Craner

State University of New York (SUNY) Brockport

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Abstract

This curriculum project applies the 5E instructional model and presents four lessons on solving systems of linear equations designed to promote student-centered lessons and problem-solving within the mathematics classroom. The 5E model consists of five phases: Engage, Explore, Explain, Elaborate, and Evaluate, and provides a structure to the four included lessons designed to improve student engagement and understanding of systems of linear equations. The materials were designed to elevate the importance of the exploration and explanation phases. The 5E instructional model provides a framework that can utilize complex tasks to encourage student engagement and discovery-based learning to further students' understanding of systems of linear equations. The keys for all student materials can be found in the appendix.

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Introduction

This curriculum project focuses on incorporating the 5E instructional model to teach Systems of Linear Equations to Algebra 1 students. The 5E instructional model incorporates a systematic approach to lesson composition that includes five phases: engage, explore, explain, elaborate, and evaluate. It has been acknowledged around the world that student-centered lessons impact students' engagement with mathematical content. Student-centered activities can lead to mathematical discourse which can further deepen students' understanding of the content. It can be challenging for many teachers to engage students with the content and effectively create and utilize student-centered lessons to deepen student understanding while maintaining engagement with the materials.

The 5E instructional model encourages student-centered activities with stages of engagement, exploration, and explaining thinking. Students are at the core of these lessons and their creative minds and discovery drive the learning. The 5E model challenges students to use problem-solving skills while engaging with rich mathematical tasks to further create a strong comprehension of the content at hand. Challenges will arise when utilizing the 5E model, but there are some strategies teachers can use to combat these obstacles and further support students to be creative and deep thinkers of mathematics. Going along with that, the 5E instructional model harmonizes well with other educational strategies that can exponentially increase the impact of student-centered lessons.

Literature Review

This curriculum project applies constructivist learning practices developed from the 5E model which ensures engaging classrooms that require students to think, discover, and learn. The 5E model was developed to align with the constructivism learning cycle developed by Atkin and

Karplus (1960). This cycle includes three phases: exploration, invention, and discovery. Turan and Matteson (2021) described the three phases: exploration is the introduction to new content where students “learn through their own actions in a new situation”; inventory is where students “perpetuate their learning through studying” or rather defining the “new terms”; and discovery is where students extend “the range of applicability of the new concept” by applying the new skills to problems and tasks (p. 23-25). The Atkin and Karplus (1960) learning cycle was initially created for teaching science but was adapted by Bybee (2006) who modified the cycle by adding more stages to create the 5E learning cycle. The model includes “five key elements for effective instruction: engagement, exploration, explanation, elaboration, and evaluation” with the three middle stages aligning with Atkin and Karplus’ original stages of exploration, invention, and discovery (Turan & Matteson, 2021, p. 22-39). When implementing the 5E model into mathematics, the challenge is for teachers to develop learning tasks that will allow for growth and learning. Research has shown that the 5E model improves student performance by actively engaging with learning mathematics (Turan & Matteson, 2021). The most typical troubling part for teachers using the 5E model has been the explanation phase where students justify their conclusions. Many teachers utilize vertical surfaces for students to show their work such as whiteboards and here students can present their thoughts to their group or class as a whole and explain their thinking. This encourages mathematical discourse when students explain their problem-solving.

Many of the practices that teachers implement that encourage thinking and reasoning in the classroom align with the 5E model and mathematical discourse, which is one of the benefits of using the 5E model. According to Kilic et al. (2010) researchers found five practices to enhance mathematical discourse in the classroom. The first is having students explain their thinking to further deepen their understanding. The second is giving feedback whenever the teacher can during the exploration and explanation phases. Another strategy is to encourage alternative solutions, which can increase students' mathematical sense making and understanding. The next strategy is to listen to students' thoughts without evaluating their work. Here the teacher can always recommend that students check their answers. The last strategy is to not be afraid to analyze incorrect answers or errors. Being able to draw students' attention to look at common misconceptions is important for students to further deepen their understanding of mathematical content. This is similar to the idea of flawed reasoning (Russell, 1999). During mathematical reasoning and problem solving, students try a strategy and sometimes it seems to work, when in fact it does not. Even though the incorrect process did not further the solution, the student can learn from the flawed reasoning process (Wade, 2011). The 5E model and these strategies can be implemented into a classroom to support mathematical discourses and to enhance student learning. Van Merriënboer et al. (2006) defined a complex task as including real-world applications, as having multiple solutions (such as algebraic, graphic, analytical, etc.), as requiring time to learn, and as creating a high cognitive load. The 5E model provides a definitive way to scaffold complex tasks and this curriculum project presents four exemplar lessons that implement the 5E instructional model to scaffold complex tasks and support the learning of mathematics.

Curriculum

Course Name: Algebra I **Time Estimate:** 55 minutes

Lesson 1:

PLANNING

Lesson Topic / Title of Lesson
Using the 5 E model to solve Systems of linear equations graphically.
Central Focus, Lesson Purpose and Rationale
The topic of systems of linear equations delves deeper into the possibilities of what you can do with linear equations, looking at them graphically and algebraically, going back to solving linear equations. Students apply their skill of graphing linear equations to then solving systems of linear equations graphically.
Pre-Assessment Data
Students recently finished the linear equations unit, so they are expected to be able to graph linear equations using slope-intercept, graph inequalities, and were also introduced to point-slope form. Students also have knowledge of solving linear equations.

PREPARATION

Lesson Objectives
Students apply their knowledge of graphing linear equations to solve systems of linear equations graphically and understand the solution is the point of intersection of the two lines.
Next Generation Learning Standards
6a. Solve systems of linear equations in two variables both algebraically and graphically.

LESSON SCRIPT

Engage

Recalling previous skills that were taught, the students are tasked with graphing two linear functions given in the form of slope-intercept form. Only thing different now is that they are graphing two linear functions on the same graph.

The two functions are as on the notes:

$f(x) = \frac{1}{2}x + 2$ and $g(x) = 2x - 1$

While students are working on the do now, the teacher should walk around to aid any students, offering support like asking them what is their slope and what is their y-intercept for both of the functions. The teacher should also note any missing gaps in the learning.

The teacher will then ask "what do we notice about these lines? Why do you think we graphed them on the same coordinate graph?" It is expected that students respond with either "they intersect," "they cross," or something along those lines. The teacher agrees, and then asks the students to write where the points cross/intersect at.

Once students find the point, the teacher will ask for a volunteer to share.

Explore

Students will then work on the given problems to explore the possibilities of systems of linear equations, and work on the associated questions with the problems. Students will then be asked leading questions to make conjectures about systems of linear equations. The teacher will go around assisting students, helping them with their work, and may have questions furthering from the conjecture questions. For the second question, the teacher can ask why the students believe the lines don't intersect on part (c)? Will the functions ever intersect? What makes this pair of equations different from the other equations?

It is also possible to give a question with equivalent lines that look different because of a multiplier. For instance, $y = \frac{3}{2}x - 4$ and $2y = 3x - 8$.

Explain

Once students seem to have finalized their conjectures, the teacher will have them discuss with their group members and then will go over it as a class.

The teacher will have volunteers share what they discovered in their research trying to get as much input from a variety of volunteers. The teacher will then explain how these are what we call systems of equations, and systems of equations can be used together to find the solution for two or more variables. The teacher will explain that graphically, the solution is the point of intersection, and algebraically, the point works for both equations.

The teacher can also ask students to volunteer to share about the parallel lines example, and why they believe or don't believe the lines will intersect.
Elaborate
Students will then continue in the notes, trying the more challenging problems. These challenging problems require manipulation of the equations so that they are in standard form. The students will graph the equations using slope-intercept form and will have to identify the solution of the system of linear equations. The teacher will go around checking on students' progress, making sure students are being careful with the equations they are given as they aren't all immediately in slope-intercept form. This will require some refocusing on solving linear equations and how to isolate a variable, y. Question 4 is unique as it is a problem that requires understanding of how the solution relates to the specific equations in the system of linear equations. If we plug in our value of x from our supposed solution, then we should obtain the same output for both equations, or if we plug in our x and y value, we shall observe that both equations turn out to be true as in both sides of the equations are equivalent. The teacher should then utilize the error of Question 4's solution to allow students to solve the system of linear equations using a graphing calculator.
Evaluate
Students will be evaluated using an exit ticket given with 5 minutes left in class. Students will be assessed with the skill of graphing a system of linear equations and identifying the solution to the system of linear equations.

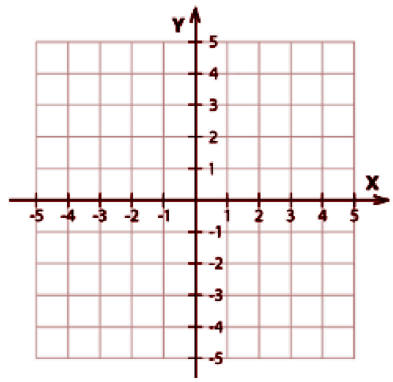
MATERIALS

NOTES

Do Now: Graph the following linear functions on the same graph.

$f(x) = \frac{1}{2}x + 2$

$g(x) = 2x - 1$

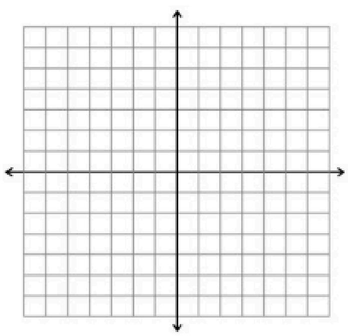


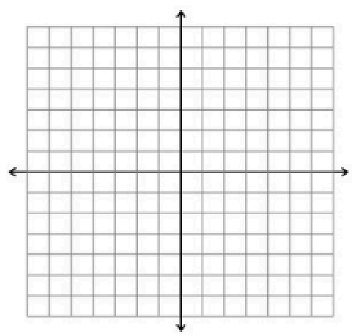
Graph the following system of linear equations and answer the questions:

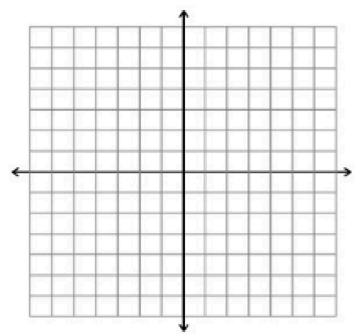
a. $f(x) = -2x + 1$
$g(x) = \frac{1}{2}x - 4$

b. $y = 2x + 1$
$y = \frac{1}{4}x - 6$

c. $y = \frac{3}{2}x + 1$
$y = \frac{3}{2}x - 2$







1. How many times do the lines intersect?

2. Could you explain what is unique about c. and why is it unique?

3. For example (a), evaluate the following.

$f(2) =$ $g(2) =$

4. Why do you think I had you find $f(2)$ and $g(2)$?

Systems of linear equations are multiple linear equations that are used together to solve for the solution.

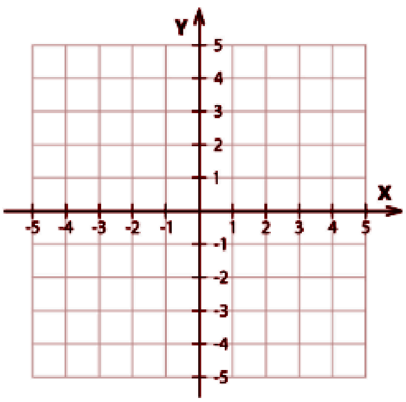
The solution to a system of equations is the _________ ___ ______________.

Algebraically, the x-value results in the _________ ___________ when plugged into both equations.

Practice:

1. $y = 3$

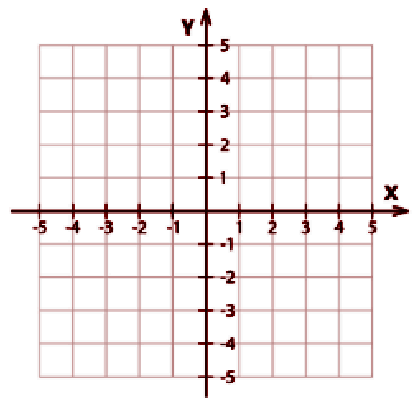
   $y = 4x - 5$



(______, _______)

2. $y - 3 = -2x$

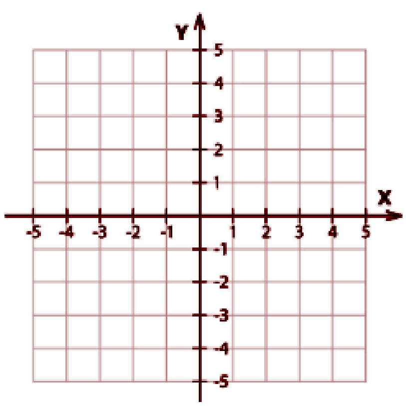
   $y = \frac{1}{2}x - 2$



(_______, _______)

3. $2y = 3x - 6$

   $x = 2$



(_______, _______)

4. Josh believes that the solution to the following system of equations is $(3, 1)$. Check whether he is correct or not algebraically. Confirm your answer using a graphing calculator.

$$3y = 4x - 9$$
$$y = \frac{-1}{6}x + 6$$

Exit Ticket:

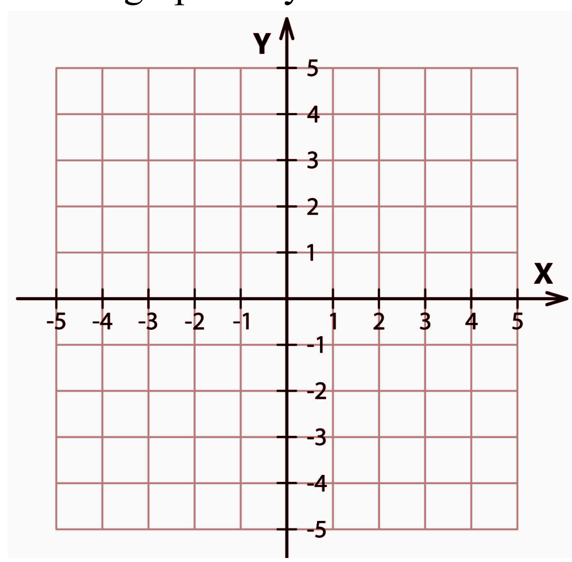
Name: ________________________________

Solve systems of linear equations graphically.

Solve the following system of linear equations graphically.

$$y = 3x - 3$$

$$y = \frac{-1}{2}x + 1$$



What does this solution mean?

Course Name: Algebra I **Time Estimate:** 55 minutes

Lesson 2:

PLANNING

Lesson Topic / Title of Lesson
Using the 5 E model to solve Systems of linear equations algebraically by substitution.
Central Focus, Lesson Purpose and Rationale
Students solve systems of linear equations algebraically by substitution and using their skills of solving linear equations.
Pre-Assessment Data
Students recently finished the linear equations unit, so they are expected to be able to graph linear equations using slope-intercept, graph inequalities, and were also introduced to point-slope form. Students also have knowledge of solving linear equations. In lesson 1 they learned about systems of linear equations and practiced how they could solve them graphically.

PREPARATION

Lesson Objectives
- Students solve systems of linear equations algebraically by substitution.
Next Generation Learning Standards
6a. Solve systems of linear equations in two variables both algebraically and graphically.

LESSON SCRIPT

Engage
The students have a Do Now to engage themselves in the topic by offering an easy system of linear equations as the initial problem to get the idea of solving systems of linear equations algebraically. Students would identify that if x=1 and y=x+2, then y=3 and they are completing substitution without realizing it. The teacher can walk around to assist students on the do now, possibly reading the question out loud "if I know x is 1 and y is x plus 2, what is y?" The teacher will then be able to go over the steps for substitution as students prepare to tackle more challenging problems. The rules are: "1. Solve one of the equations for one of the variables. Either x =... or y =... 2. Substitute in for the variable in our other equation. 3. Solve for the remaining variable. 4. Plug back into the equation to find the other variable." And the example substitution is complete at the bottom of the page.
Explore
Students will then work on the problems continuing the notes. The following problems will be equations that are both in $y = mx + b$ form, hoping for the students to realize that if the equations are in slope-intercept form, you can just set the two equations equal to each other. Students will be recommended to check their solutions either by plugging the solution back into both equations or checking graphically. The teacher will walk around to assist students, always falling back on the steps for solving these systems of linear equations. The teacher may need to help students with the substitution part, and can use a highlight to have the students focus on the parts they're working with. i.e For example a. $y = 2x + 2$ and $y = 3x - 1$, the teacher will highlight y = 2x + 2 ; y = 3x-1 and draw an arrow from one to the other.
Explain
The teacher will repeat what the statement at the bottom of the page says, how if they are both y= or x=, they can just set each equation equal to each other and solve for x. The teacher will complete 2 of the examples, **a** and **c** and have students lead them through the problem. Once going through the problems, the teacher will then instruct the students to continue working on the back page. The teacher will model initially how to do the substitution with the highlighter technique on the first example **e**. y = 4x -7 ; 2y + 3x = 14

Elaborate
Students will then continue working on the problems on the back. As students are working, the teacher will walk around to assist the students with their work. Some questioning to follow is: "Do we have either of our equations that say *x*= or *y*= ? We do, now where do you see *that variable* in the other equation (have students point it out). Okay, so, what is that equal to again (referring to the other equation). So, since it's equal to *other equation*, then we can just place that in for it in this equation." Before students work on the exit ticket, the teacher will go over the example **e** and will set up the two equations for the **challenge**. Since neither of our equations are *y*= or *x*=, we will have to initially solve one of our equations for one of the variables and then use substitution. As one last clarifier, the teacher must stress that our solution is a coordinate point, relating to the graph, so we should have an *x* and *y* value.
Evaluate
Students will be given an exit ticket to complete within the last six minutes of class. Students will be assessed on their skill of solving systems of linear equations algebraically by substitution.

MATERIALS

NOTES

Do Now: Solve the following system of linear equations.

$x = 1$ $\qquad\qquad$ $y = 3$

$y = x + 2$ $\qquad\qquad$ $y = 2x - 1$

For today's lesson, we will be focusing on the property of substitution to solve systems of linear equations algebraically.

Rules for Substituting: ____

1. Solve one of the equations for _______ of the ___________.
2. ______________ in for the variable in our other __________.
3. Solve for the __________________ _______________.
4. Plug back into the equation to find the __________ ____________

Example of substitution:

$y = 2x + 2 \quad ; \quad y = 3x - 1 \qquad \Rightarrow$

Let's try some examples:

a. $y = 2x + 2$

$y = 3x - 1$

b. $y = 3x + 5$

$y = 5x - 3$

c. $y = 6x + 2$

$y = -3x + 11$

d. $y = 10x - 3$

$y = \frac{1}{2}x + 16$

So, when both of the equations are $y =$… (or $x =$…), you can set the two equations equal to each other and solve for x. But sometimes, only one of the equations will be $y =$… or $x =$… and we will have to be careful where we substitute.

Practice continue:

e. $y = 4x - 7$

$2y + 3x = 14$

f. $4y + 2x + 2 = 12$

$y = \frac{1}{2}x - 4$

CHALLENGE: Solve the following system of linear equations.

$3x - 2y = 4$ and $x + 3y = 5$

Exit Ticket

Solve the following system of linear equations algebraically.

$y = x + 8$ and $y = 4x - 1$

Course Name: Algebra I **Time Estimate:** 55 minutes

Lesson 3:

PLANNING

Lesson Topic / Title of Lesson
Using the 5 E model to solve Systems of linear equations algebraically by elimination.
Central Focus, Lesson Purpose and Rationale
Students solve systems of linear equations algebraically by elimination and using their skills of solving linear equations.
Pre-Assessment Data
Students recently finished the linear equations unit, so they are expected to be able to graph linear equations using slope-intercept, graph inequalities, and were also introduced to point-slope form. Students also have knowledge of solving linear equations. In lesson 1 they learned about systems of linear equations and practiced how they could solve them graphically. In lesson 2, they solve systems of linear equations algebraically by substitution.

PREPARATION

Lesson Objectives
- Students solve systems of linear equations algebraically by elimination.
Next Generation Learning Standards
6a. Solve systems of linear equations in two variables both algebraically and graphically.

LESSON SCRIPT

Engage
. Students are directed to complete the do now, solving the linear equation by substitution. As review, depending on the students' performance on the exit ticket, the teacher will go around to those who struggled to assist them on the do now, walking the student through the steps. A useful thing to do would be to highlight y=-2x+4 and 2y=2x+8, so they know they substitute in -2x+4 in for y and solve for x. The teacher will then work on the problem, going through the steps established for solving systems of linear equations algebraically by substitution. The teacher will fill in the blanks in the notes above "Before, we solved systems of linear equations algebraically by using the principle of substitution. Now, we will use a technique called elimination." The teacher will perform the example that is the same question as the do now, but will complete it doing elimination, and will write the steps on the side. The steps for elimination are as follows: 1. Look for opposites: Are x's opposites? Are y's opposites? (If not, then multiply or divide to make opposite) * the teacher explains that they will explain what that means later on. 2. Add down each term. 3. Solve for the remaining variable. 4. Plug into one of the equations to find the other variable.

Explore
Students will then be tasked with working on the given problems on the next page. Students will use the steps to solve for x and y in the following problems. Questions 1-3 will have opposite x or y terms, so there will be no manipulation required, but question 4, you don't have visible opposites. The teacher will have to go to each table to explain that to be able to "make" opposites, you can multiply a whole equation by an integer to make opposites. For question 4, I have a +y and +y in both equations. I can multiply one of those by a -1 to have a +y and -y, then we would have opposites. On the other questions, the teacher will explain to each group of students that you basically target either the x or y terms and multiply the equations by a number to obtain opposites. To assist the students, the teacher can help walk the students through the steps, especially on questions 1-3 to get them in the routine of elimination. Asking "do we have opposites? We do, which terms are my opposites? Ok, since I have opposites, I can add the two equations together, combining the like terms, and I would end up with a one variable equation."

Explain
The teacher will go over questions 1, 2, 5, and 7. For questions 1 and 2, the teacher will efficiently go through the steps that were established earlier in class. The teacher will verbally identify the opposites and write them on the side, i.e for question 1, write "-y and y are opposites." For question 5, the teacher will explain that they have the power to manipulate either of the equations and multiply by any number so that they can get opposites. For question 5, the teacher will identify that you have +2y and +2y, to have opposites, I just need one of them to be -2y, so I multiply by -1. After doing this and then completing the problem, the teacher will then rewrite the question on the side, with the original problem and the resulting one variable equation below. The teacher can then ask, after multiplying by -1 and then adding, what am I really doing here? Hopefully, students will be able to see that we are just subtracting. The teacher could have already shown this to some higher level students before this discussion. The teacher will then complete question 7, explaining that 16x and -8x can be easily multiplied to get opposites. Students will resume completing the problems on their own.
Elaborate
.The teacher will then ask the students to put their pencils down for a moment, and bring their attention to the board. The teacher will have the following problem written on the board, question 2 from the notes. $$-4x - 2y =- 12$$ $$4x + 8y =- 24$$ The teacher will then explain that we are going to further try and understand how elimination works. The teacher will ask "what do we know about 4x+8y and -24?" pointing at the bottom equation on the board; "what's the relationship between those?" "They are equal, yes. Now, we are taking two equal expressions, and adding it to both sides of an equation." The teacher will pause, ask "does that ring a bell? Adding or subtracting the same value to both sides of an equation keeps both sides equal. The whole math behind elimination is rooted in the property of equality. So just like solving linear equations, and we add the same thing to both sides to cancel out terms and isolate a variable, we are doing the same thing here, just the two things we're adding don't look the same. A strategy to help students identify opposite terms could be highlighting the opposite terms. This can allow students to then focus on the remaining terms in the equation. A connection can also be made to the exploration phase of lesson 1, where students compare two functions that are equivalent but with different multipliers. Note: this elaboration may be just for the higher level students.
Evaluate
Students will be given an exit ticket on their ability to solve a system of linear equations with opposite coefficients on one of their variables already, testing the minimum requirements.

MATERIALS

NOTES

Before, we solve systems of linear equations algebraically by using the principle of ___________________. Now, we will use a technique called

______________________.

Do Now: Solve the following system of equations using substitution.

$2y = 2x + 8$

$y = -2x + 4$

Example:

$2y = 2x + 8$

$y = -2x + 4$

Steps:

Try some:

Solve the following systems of linear equations by elimination.

1. $2x - y = 13$

   $4x + y = 17$

2. $-4x - 2y = -12$

   $4x + 8y = -24$

3. $2x + 3y = -5$

   $-6x - 3y = 21$

4. $8x + y = -16$

   $-3x + y = -5$

5. $8x + 2y = 30$

$7x + 2y = 24$

6. $2x + 3y = -5$

$3x - y = 9$

7. $16x - 10y = 10$

$-8x - 6y = 6$

8. $-3x + 7y = -16$

$-9x + 5y = 16$

Exit Ticket

Solve the following system of linear equations algebraically.

$$4x + 8y = 20$$

$$-4x + 2y = -30$$

What does this solution mean?

Course Name: Algebra I **Time Estimate:** 55 minutes

Lesson 4:

PLANNING

Lesson Topic / Title of Lesson
Using the 5 E model to create and solve systems of linear equation word problems.
Central Focus, Lesson Purpose and Rationale
Students create and solve systems of linear equations from word problems either by graphing, substitution, or elimination.
Pre-Assessment Data
Students recently finished the linear equations unit, so they are expected to be able to graph linear equations using slope-intercept, graph inequalities, and were also introduced to point-slope form. Students also have knowledge of solving linear equations. In lesson 1 they learned about systems of linear equations and practiced how they could solve them graphically. In lesson 2, they solve systems of linear equations algebraically by substitution. In lesson 3, they solved systems of linear equations algebraically by elimination.

PREPARATION

Lesson Objectives
- Students create and solve systems of linear equations from word problems.
Next Generation Learning Standards
AI-A.CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

LESSON SCRIPT

Engage

To engage the students, they have an initial word problem and a graphic organizer to help guide them through the problem. The students are tasked with completing the top two quadrants of the organizer, asking “what we want to know” and “what we already know” which will guide the students into creating the variables and our system of equations.

The teacher will then ask for students to volunteer to share their results for the do now. For the example, the teacher explains that we assign a value to our variables to try and decipher what the operations in our equations are like, so let’s say the cost of a marker is $2.00 and the cost of a pencil is $1.00. Try and figure out the total cost of 3 markers and 2 pencils with those values. Then find the cost of 4 markers and 6 pencils with those values. It doesn’t come to our given totals, but that’s expected, but with this, we are able to reason with the numbers and logic of the question to figure out the operations in the equations.

So, in the square we will put our system of linear equations:

$$3x + 2y = 1.80$$

$4x + 6y = 2.90$, where *x*-$ of marker, and *y*- $ of pencil

We can then solve this either graphically or algebraically. The teacher will encourage the students to solve this graphically using desmos graphing calculator. The teacher will then solve the system of linear equations algebraically by elimination, sharing their thoughts out loud. The solution for *x* and *y* is then stated in the solution box and are checked using the two equations in the check quadrant.

Explore

The teacher will hand out 2 bags to each group of students (students are in groups of 3). One bag says 150 cal. containing three regular oreo cookies, and the other says 210 cal. containing 3 double stuffed oreos. To start off the problem, the teacher will just illustrate the deconstruction of an oreo, defining the two distinct parts to be the outside crackers and the filling inside. The outside crackers have a certain amount of calories to them, and the filling inside has a certain amount of calories to it. The students will then work in their groups to create systems of linear equations to model the calorie breakdown of oreos.

The teacher can encourage the students to go through the graphic organizer that was used in the do now, going through the steps of defining your variables, establishing what you know, maybe doing an example, and then arriving at your solution.

To guide students, the teacher can help them focus on the example part to figure out the operations for the equations. The teacher can say, “let’s say 1 cracker has 50 calories and 1 filling has 60 calories, how many calories would a regular oreo have? 160 calories? How did

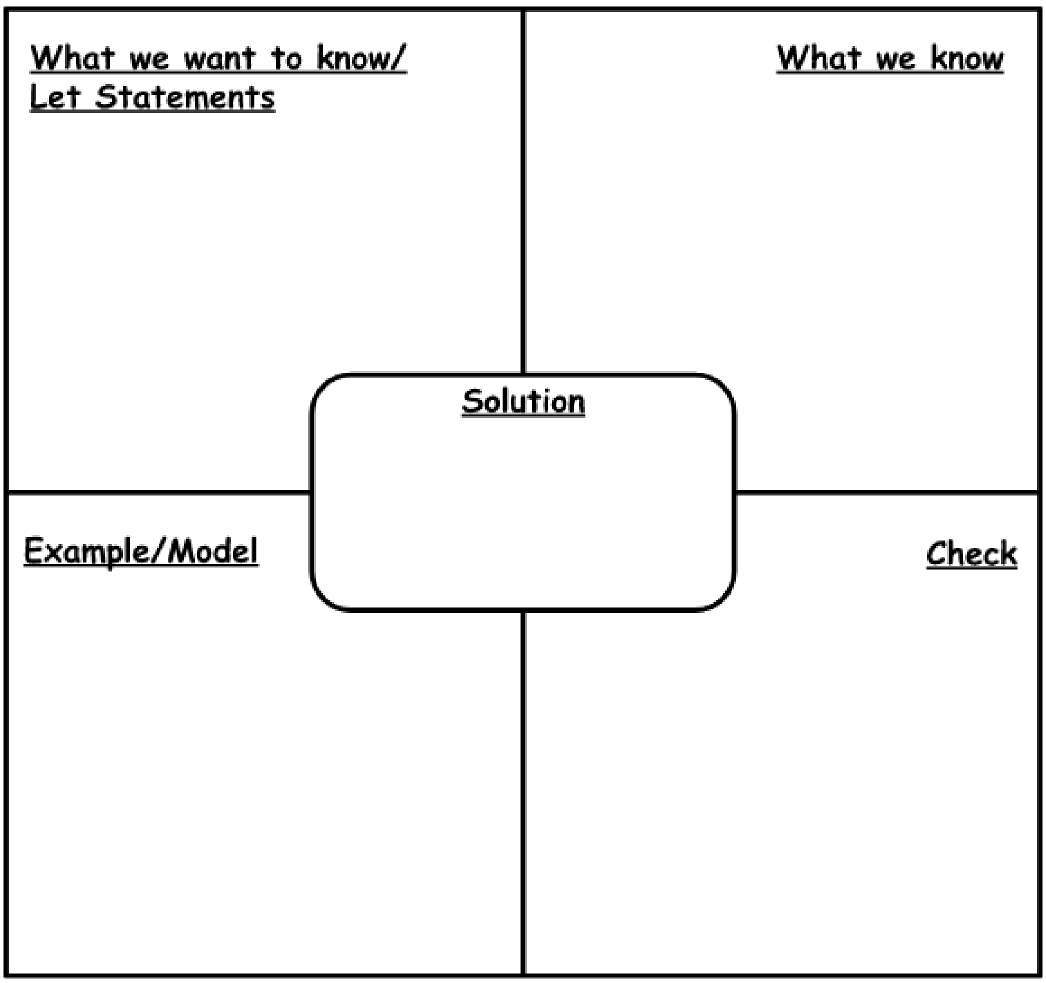
you calculate that?" We multiply the number of crackers by the amount of calories in the crackers and add to that the product of the number of fillings and the calories of the fillings. Now, we just have to do that for 3 regular and 3 double stuff cookies.
Explain
The teacher can then go over the oreo problem, defining one of the equations to be the calories of the regular oreos and the other to be the calories of the double stuffed oreos. Since there's 3 cookies, that means there's 6 crackers and 3 fillings for the regular. So, we have $6x + 3y = 150$ And for the double stuff, still 6 crackers but now 6 filling sizes since it's doubled. So, we have $6x + 6y = 210$ We can now use this system to solve for the calories of a cracker and the calories of a filling. The teacher can ask for volunteers on how to solve this system. "Should we use substitution or elimination? Why elimination? If we subtract the equations, we can cancel out the x-terms, leaving us a one variable equation. Now that we have solved for x and y, how many calories would a triple stuffed oreo have? What about a club sandwich style oreo where there's a middle cracker? Once finished with the oreo example, the teacher will let the students continue to work on the individual practice.
Elaborate
The teacher will go over the 2 individual practice examples, explaining that not all problems will be like the initial ones. Some problems will have equations in slope intercept form. Referring to our last unit, when we are given a rate and a starting value, we can write the equation in slope-intercept form. So, for the question 4, the equations will be $y = 25x$ and $y = 20x + 30$. The teacher will also go over again how to check their answer, they plug in our values into the two equations, and they should both come out to be true. The teacher can then relate this to graphing, and graph the system from question 3 on desmos graphing calculator. The solution we get is given as a point and it's where the two lines intersect. So, when we are solving these, we are still solving for the point of intersection on a graph.
Evaluate
Students will be given an exit ticket to assess their skills of creating and solving a system of linear equations from a word problem.

MATERIALS

NOTES

Do Now: Read the following question and fill in the "What we want to know/Let Statements" and "What we know" quadrants below.

1. The cost of 3 markers and 2 pencils is $1.80. The cost of 4 markers and 6 pencils is $2.90. What is the cost of each item?



2. Oreos are a popular brand of cookies, with the double stuffed oreo being a popular variation. You are given 3 cookies of each and told the calories of each.

We are going to deconstruct to find the calories of any possible oreo cookie.

Individual Practice.

3. Journey bought 3 slices of cheese pizza and 4 slices of mushroom pizza for a total cost of $12.50. Rebekah bought 3 slices of cheese pizza and 2 slices of mushroom pizza for a total cost of $8.50.
What is the cost of one slice of mushroom pizza?

Practice continues.

4. The gym teacher is racing their students but is giving them a head start. The gym teacher runs 25 feet per second and their students run 20 feet per second but he's giving them a head start of 30 feet. Write equations that represent the gym teacher's and their students' position, y, in respect to time, x.

How long does it take for the gym teacher to catch up to his students?

If the gym teacher finishes the race in 10 seconds, how far ahead are they?

Exit Ticket

Julio buys 4 apples and 2 bananas for $5.50. Chris went to the same farmer's market and bought 2 apples and 2 bananas for $3.50. Create a system of equations representing this situation.

How much would 7 apples and 3 bananas cost?

Validity

To validate this project's effectiveness, an experienced mathematics teacher reviewed the curriculum project and was asked specific questions to offer feedback. The teacher was previously an Algebra II teacher for many years, but has recently taken the role as a Multi-Tiered System of Supports Teacher focusing on supporting struggling students at all grade levels. The questions and a summary of their answers are listed below.

1. *Based on your experience with students as a tenured teacher, how effective do you think the implementation of the 5E model would be in meeting the needs of students?*

The teacher stated that they admire the ideas of the 5E Instructional Model and how it encourages the constructivist approach. The teacher also stated that the 5E Model would be effective to engage the students in the learning and follows a common lesson planning template such as Ignite, Chunk, Chew, and Review.

2. *Do you think all lessons provide opportunities for each stage of the 5E model: Engage, Explore, Explain, Elaborate, and Evaluate?*

The teacher shared that the consistency of the lesson format allows for each stage of the 5E model to be utilized within the lesson. They also shared how the Engage stage of the 5E Model mirrors the idea of "igniting" learning to pique the interest of the learners. The Explore phase allows for a constructivist approach to be utilized in the lessons, focusing on learners' conceptual understanding.

3. *What do you think are the strengths of these activities and the 5E Instructional Model?*

The teacher stated that the consistent lesson structure should meet the needs of all learners and the exploration section of the 5E model allows for learning accommodations. The teacher pointed out in Lesson 1, during the Elaborate phase, Question 4 is important as it questions the students conceptual understanding of what the solution to a system of linear equations is, and why the intersection point of the two functions is called the solution. The teacher also shared their admiration for the highlighting technique in lesson 2 during the explore phase, to help students visualize the substitution technique and how it works.

4. *What do you think are the weaknesses of these activities and the 5E Instructional Model?*

The teacher shared that the Explore phase aligns nicely with the constructivist approach of learning, but constructivism requires more time than “sit and get” teaching. It is important that the teacher is patient with their learners to allow for the discovery of the learning. The teacher also suggests making the learning relevant to real life in order to engage the students’ during the Engage phase.

In summary, the teacher believes these lessons would be effective in an Algebra 1 classroom, and the 5E Instructional model is practical to engage the students and encourage a constructivist approach to learning. Their feedback was taken into consideration when finalizing the lessons in this curriculum project to further improve its effectiveness.

Conclusion

The 5E instructional model is an effective framework for lessons that has been effective in the science classroom and has a lot of potential to be used in the math classroom. Through the use of the exploration and explanation phases, students can further deepen their understanding of mathematics through constructivist learning cycles and prompts to engage in mathematical discourse. It is the author’s hope that other mathematical teachers will use the lessons from this project to promote engaging student-centered learning environments with the use of complex tasks, discovery-based learning, and mathematical discourse.

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Appendix

Lesson 1 Key for Materials:

Notes

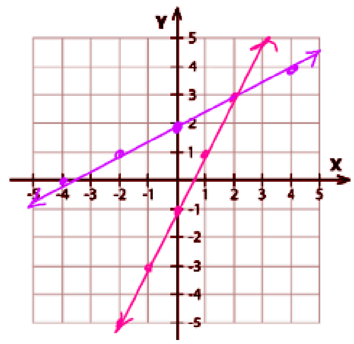
Do Now: Graph the following linear functions on the same graph.

$f(x) = \frac{1}{2}x + 2$

$m = \frac{1}{2}$ $b = 2$

$g(x) = 2x - 1$

$m = \frac{2}{1}$ $b = -1$



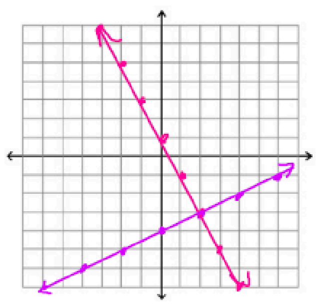
They intersect at (2, 3)

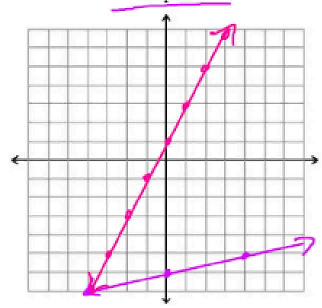
Graph the following system of linear equations and answer the questions:

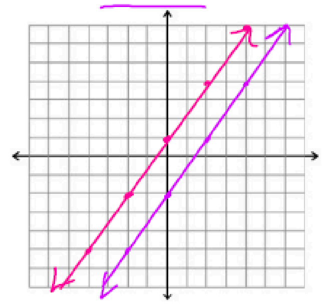
a. $f(x) = -2x + 1$
$g(x) = \frac{1}{2}x - 4$

b. $y = 2x + 1$
$y = \frac{1}{4}x - 6$

c. $y = \frac{3}{2}x + 1$
$y = \frac{3}{2}x - 2$







1. How many times do the lines intersect?

a. and b. intersect once each

c. the lines don't intersect at all.

2. Could you explain what is unique about c. and why is it unique?

The lines don't intersect. It could be because they have the same slope. They are parallel lines.

3. For example (a), evaluate the following.

$f(2) = -2(2)+1$
$= -3$

$g(2) = \frac{1}{2}(2) - 4$
$= -3$

4. Why do you think I had you find $f(2)$ and $g(2)$?

They have the same output. They both have the point (2, -3), it's where they intersect.

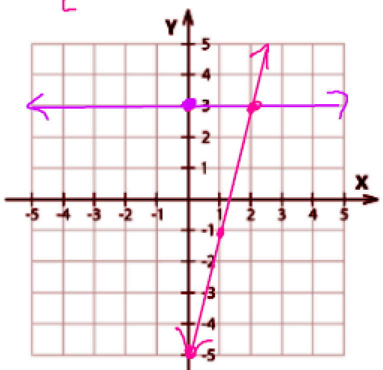
Systems of linear equations are multiple linear equations that are used together to solve for the solution.

The solution to a system of equations is the point of intersection.

Algebraically, the x-value results in the same output when plugged into both equations.

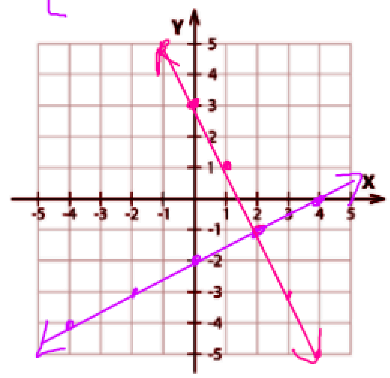
Practice:

1. $\begin{cases} y = 3 \\ y = 4x - 5 \end{cases}$



(2 , 3)

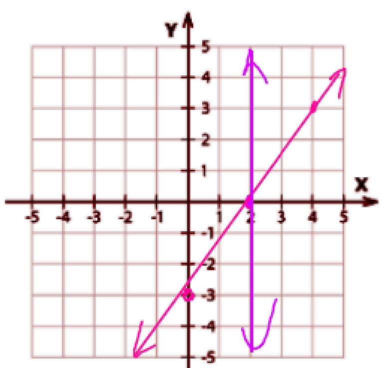
2. $\begin{cases} y - 3 = -2x \Rightarrow y = -2x + 3 \\ y = \frac{1}{2}x - 2 \end{cases}$



(2 , -1)

3. $2y = 3x - 6 \Rightarrow y = \frac{3}{2}x - 3$

$x = 2$



(2 , 0)

4. Josh believes that the solution to the following system of equations is $(3, 1)$. Check whether he is correct or not algebraically. Confirm your answer using a graphing calculator.

$$3y = 4x - 9$$
$$y = \frac{-1}{6}x + 6$$

$3(1) = 4(3) - 9$ $\quad$ $1 = -\frac{1}{6}(3) + 6$

$3 = 12 - 9$ $\quad$ $1 = -\frac{1}{2} + 6$

$3 = 3$ ✓ $\quad$ $1 \neq 5\frac{1}{2}$ ✗

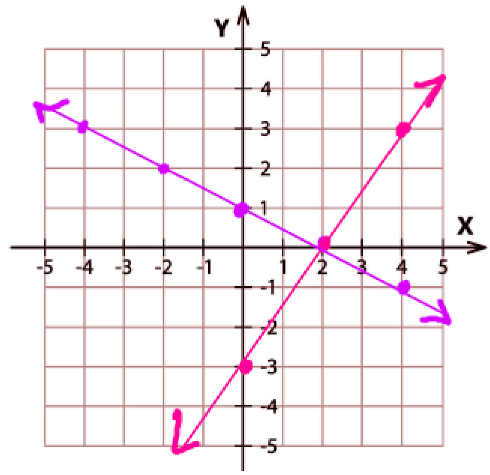
Not True

Lesson 1 Exit Ticket:

Solve the following system of linear equations graphically.

$$y = \frac{3}{2}x - 3$$

$$y = \frac{-1}{2}x + 1$$



(2,0)

What does this solution mean?

It's the point of intersect. When $x = 2$, the two equations have the same output.

Lesson 2 Key for Material:

NOTES

Do Now: Solve the following system of linear equations.

$x = 1$ | $y = 3$
$y = x + 2$ | $y = 2x - 1$

$y = 1+2$
$y = 3$

$3 = 2x - 1$
$+1 \quad +1$
$4 = 2x$
$\frac{4}{2} = \frac{2x}{2}$
$2 = x$

For today's lesson, we will be focusing on the property of substitution to solve systems of linear equations algebraically.

Rules for Substituting: want $x=$ or $y=$

1. Solve one of the equations for one of the variables.
2. Substitute in for the variable in our other equation.
3. Solve for the remaining variable.
4. Plug back into the equation to find the other variable

Example of substitution:

$y = 2x + 2 \quad ; \quad y = 3x - 1 \quad \Rightarrow$

$3x - 1 = 2x + 2$
$-2x \quad -2x$
$x - 1 = 2$
$+1 \quad +1$
$x = 3$

$y = 2x + 2$
$y = 2(3) + 2$
$y = 8$

$(3, 8)$

Let's try some examples:

a. $y = 2x + 2$

$y = 3x - 1$

$3x - 1 = 2x + 2$

$-2x \quad -2x$

$x - 1 = 2$

$+1 \quad +1$

$x = 3$

$y = 3x - 1$

$y = 3(3) - 1$

$y = 9 - 1$

$y = 8$

$(3, 8)$

b. $y = 3x + 5$

$y = 5x - 3$

$5x - 3 = 3x + 5$

$-3x \quad -3x$

$2x - 3 = 5$

$+3 \quad +3$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$y = 3x + 5$

$y = 3(4) + 5$

$y = 12 + 5$

$y = 17$

$(4, 17)$

c. $y = 6x + 2$

$y = -3x + 11$

$-3x + 11 = 6x + 2$

$+3x \quad +3x$

$11 = 9x + 2$

$-2 \quad -2$

$\frac{9}{9} = \frac{9x}{9}$

$1 = x$

$y = 6(1) + 2$

$y = 8$

$(1, 8)$

d. $y = 10x - 3$

$y = \frac{1}{2}x + 16$

$\frac{1}{2}x + 16 = 10x - 3$

$-\frac{1}{2}x \quad -\frac{1}{2}x$

$16 = \frac{19}{2}x - 3$

$+3 \quad +3$

$\frac{19}{\frac{19}{2}} = \frac{\frac{19}{2}x}{\frac{19}{2}}$

$x = 2$

$y = 10(2) - 3$

$y = 17$

$(2, 17)$

So, when both of the equations are $y = \ldots$ (or $x = \ldots$), you can set the two equations equal to each other and solve for x. But sometimes, only one of the equations will be $y = \ldots$ or $x = \ldots$ and we will have to be careful where we substitute.

Practice continue:

e. $y = 4x - 7$

$2y + 3x = 14$

$2(4x - 7) + 3x = 14$

$8x - 14 + 3x = 14$

$-14 + 11x = 14$

$+14 \quad +14$

$11x = 28$

$x = \frac{28}{11}$

$y = 4\left(\frac{28}{11}\right) - 7$

$y = \frac{112}{11} - 7$

$y = \frac{112}{11} - \frac{77}{11} = \frac{35}{11}$

$\left(\frac{28}{11}, \frac{35}{11}\right)$

f. $4y + 2x + 2 = 12$

$y = \frac{1}{2}x - 4$

$4\left(\frac{1}{2}x - 4\right) + 2x + 2 = 12$

$\frac{4}{2}x - 16 + 2x + 2 = 12$

$\frac{8}{2}x - 14 = 12$

$+14 \quad +14$

$4x = 26$

$x = \frac{26}{4}$

$x = \frac{13}{2}$

$y = \frac{1}{2}\left(\frac{13}{2}\right) - 4$

$y = \frac{13}{4} - \frac{4}{1} = \frac{13}{4} - \frac{16}{4}$

$y = \frac{-3}{4}$

$\left(\frac{13}{2}, \frac{-3}{4}\right)$

CHALLENGE: Solve the following system of linear equations.

$3x - 2y = 4$ and $x + 3y = 5$

$x = 5 - 3y$

$3(5 - 3y) - 2y = 4$

$15 - 9y - 2y = 4$

$15 - 11y = 4$

$-15 \quad -15$

$-11y = -11$

$y = 1$

$x = 5 - 3(1)$

$x = 5 - 3 = 2$

$(2, 1)$

Lesson 2 Exit Ticket

Solve the following system of linear equations algebraically.

$y = x + 8$ and $y = 4x - 1$

$-8 \quad -8$

$x = y - 8$

$y = 4(y - 8) - 1$

$x = 11 - 8 = 3$

$y = 4y - 32 - 1 = 4y - 33$

$y = 4y - 33$

$-4y \quad -4y$

$\frac{-3y = -33}{-3}$ $y = 11$

$(3, 11)$

$x + 8 = 4x - 1$

$+1 \quad +1$

$x + 9 = 4x$

$-x \quad -x$

$\frac{9 = 3x}{3}$ $x = 3$

$y = x + 8$

$y = 3 + 8 = 11$

$(3, 11)$

Lesson 3 Key for Materials:

NOTES

Before, we solve systems of linear equations algebraically by using the principle of substitution. Now, we will use a technique called elimination.

Do Now: Solve the following system of equations using substitution.

$$2y = 2x + 8$$

$$y = -2x + 4$$

$2(-2x+4) = 2x+8$

$-4x+8 = 2x+8$

$+4x \quad +4x$

$8 = 6x+8$

$-8 \quad -8$

$0 = 6x$

$x = 0$

$y = -2(0)+4$

$y = 0+4$

$y = 4$

$(0,4)$

Example:

$$2y = 2x + 8$$

$$+ \quad y = -2x + 4$$

$3y = 12$

$y = 4$

2x and -2x are opposites.

$4 = -2x + 4$

$-4 \quad -4$

$0 = -2x$

$0 = x$

$(0,4)$

Steps:

1. Identify the opposite x- or y-terms. If you don't have opposites, make some by multiplying one of the equations.
2. Add down each term in the equations.
3. Solve for the remaining variable.
4. Plug back into one of the original equations to solve for other variable.

Try some:

Solve the following systems of linear equations by elimination.

1. $2x - y = 13$

   $4x + y = 17$

   $6x = 30$, $x = 5$

   $4(5) + y = 17$

   $20 + y = 17$

   $y = -3$

   $(5, -3)$

   -y and y opposites

2. $-4x - 2y = -12$

   $4x + 8y = -24$

   $-2y = -12$

   $+8y = -24$

   $6y = -36$

   $y = -6$

   $4x + 8(-6) = -24$

   $4x - 48 = -24$

   $4x = 24$, $x = 6$

   $(6, -6)$

3. $2x + 3y = -5$

   $-6x - 3y = 21$

   $-4x = 16$

   $x = -4$

   $2(-4) + 3y = -5$

   $-8 + 3y = -5$

   $3y = 3$

   $y = 1$

   $(-4, 1)$

4. $8x + y = -16$

   $-1(-3x + y = -5)$

   $3x - y = 5$

   $11x = -11$

   $x = -1$

   $-3(-1) + y = -5$

   $3 + y = -5$

   $y = -8$

   $(-1, -8)$

5. $8x + 2y = 30$

$- 7x + 2y = 24$

$1x = 6$

$x = 6$

$8(6) + 2y = 30$

$48 + 2y = 30$

$-48 \quad -48$

$\frac{2y}{2} = \frac{-18}{2}$

$y = -9$

$(6, -9)$

6. $2x + 3y = -5$

$3(3x - y = 9)$

$+ 9x - 3y = 27$

$\frac{11x}{11} = \frac{22}{11}$

$x = 2$

$2x + 3y = -5$

$2(2) + 3y = -5$

$4 + 3y = -5$

$3y = -9$

$y = -3$

$(2, -3)$

7. $\not{1}6x - 10y = 10$

$2\left(-8x - 6y = 6\right)$

$+ \quad -16x - 12y = 12$

$-22y = 22$

$y = -1$

$16x - 10(-1) = 10$

$16x + 10 = 10$

$-10 \quad -10$

$\frac{16x}{16} = \frac{0}{16}$

$x = 0$

$(0, -1)$

8. $3(-3x + 7y = -16)$

$-9x + 21y = -48$

$+ \quad -9x + 5y = 16$

$\frac{26y}{26} = \frac{-32}{26}$

$y = \frac{-16}{13}$

$-9x + 5\left(\frac{-16}{13}\right) = 16$

$-9x - \frac{80}{13} = 16$

$+\frac{80}{13} \quad +\frac{80}{13}$

$\frac{-9x}{-9} = \frac{288}{13} / -9$

$x = -\frac{288}{117}$

$\left(-\frac{288}{117}, \frac{-16}{13}\right)$

Lesson 3 Exit Ticket

Solve the following system of linear equations algebraically.

$$4x + 8y = 20$$
$$+ \quad -4x + 2y = -30$$

$$10y = -10$$
$$\frac{10y}{10} = \frac{-10}{10}$$
$$y = -1$$

$$4x + 8(-1) = 20$$
$$4x - 8 = 20$$
$$+8 \quad +8$$
$$4x = 28$$
$$\frac{4x}{4} = \frac{28}{4}$$
$$x = 7$$

(7, -1)

What does this solution mean?

It's the point of intersection on the graph.

Lesson 4 Key for Materials:

NOTES

Do Now: Read the following question and fill in the "What we want to know/Let Statements" and "What we know" quadrants below.

1. The cost of 3 markers and 2 pencils is $1.80. The cost of 4 markers and 6 pencils is $2.90. What is the cost of each item?

What we want to know/ Let Statements

$ of marker – m

$ of pencil – p

What we know

3 markers & 2 pencils = 1.80

4 markers & 6 pencils = 2.90

Solution

$3m + 2p = 1.80$

$4m + 6p = 2.90$

Example/Model

Say m = 2 p = 1

3 markers & 2 pencils

⇓

$3(2) + 2(1)$

Check

$m = .50$ $p = .15$

$3(.50) + 2(.15) = 1.80$

$1.80 = 1.80$ ✓

$4(.50) + 6(.15) = 2.90$

$2.90 = 2.90$ ✓

$$3(3m + 2p = 1.80) \rightarrow 9m + 6p = 5.40$$

$$- \quad 4m + 6p = 2.90$$

$$\frac{5m}{5} = \frac{2.50}{5}$$

$$m = .50$$

$$3m + 2p = 1.80$$

$$3(.5) + 2p = 1.80$$

$$1.50 + 2p = 1.80$$

$$-1.50 \qquad -1.50$$

$$\frac{2p}{2} = \frac{.30}{2}$$

$$p = .15$$

Marker cost 50¢

pencil cost 15¢

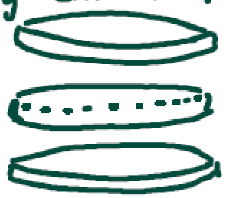
2. Oreos are a popular brand of cookies, with the double stuffed oreo being a popular variation. You are given 3 cookies of each and told the calories of each.

We are going to deconstruct to find the calories of any possible oreo cookie.

Regular Oreo

x - calories outside cookie

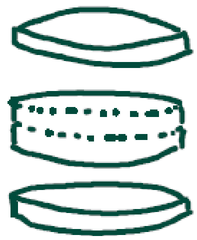
y - calories filling.



3 cookies

$6x + 3y = 150$

Double Stuff



$6x + 6y = 210$

$$\begin{array}{r} 6x + 3y = 150 \\ - 6x + 6y = 210 \\ \hline -3y = -60 \\ \frac{-3y}{-3} = \frac{-60}{-3} \\ y = 20 \end{array}$$

$$\begin{array}{r} 6x + 3y = 150 \\ 6x + 3(20) = 150 \\ 6x + 60 = 150 \\ -60 \quad -60 \\ \hline 6x = 90 \\ x = 15 \end{array}$$

1 cracker : 15 calories

1 filling : 20 calories

Triple stuffed: $2(15) + 3(20) = 90$ calories

Club sandwich style $3(15) + 2(20) = 85$ calories

Individual Practice.

3. Journey bought 3 slices of cheese pizza and 4 slices of mushroom pizza for a total cost of $12.50. Rebekah bought 3 slices of cheese pizza and 2 slices of mushroom pizza for a total cost of $8.50.
What is the cost of one slice of mushroom pizza?

What we want to know

$ mushroom slice – m

$ cheese slice – c

What we know

4 mushroom, 3 cheese = 12.50

2 mushroom, 3 cheese = 8.50

$$4m + 3c = 12.50$$
$$2m + 3c = 8.50$$

Example (optional)

m = 3   c = 2

4(3) + 3(2)

Check

m = 2.00   c = 1.50

4(2.00) + 3(1.50) = 12.50

12.50 $\overset{\checkmark}{=}$ 12.50

2(2.00) + 3(1.50) = 8.50

8.50 $\overset{\checkmark}{=}$ 8.50

Solve

$$
\begin{array}{r}
4m + 3c = 12.50 \\
-\;\; 2m + 3c = 8.50 \\
\hline
2m \;\;\;\;\;\; = 4.00 \\
m = 2.00
\end{array}
$$

Mushroom slice cost $2.00

$$
\begin{array}{r}
2(2.00) + 3c = 8.50 \\
4.00 + 3c = 8.50 \\
-4.00 \;\;\;\;\;\;\;\; -4.00 \\
\hline
\frac{3c}{3} = \frac{4.50}{3} \\
c = 1.50
\end{array}
$$

cheese slice cost $1.50

Practice continues.

4. The gym teacher is racing their students but is giving them a head start. The gym teacher runs 25 feet per second and their students run 20 feet per second but he's giving them a head start of 30 feet. Write equations that represent the gym teacher's and their students' position, *y*, in respect to time, *x*.

$y = mx + b$

Teacher:
$m = 25$
$b = 0$

$y = 25x$

Student:
$m = 20$
$b = 30$

$y = 20x + 30$

How long does it take for the gym teacher to catch up to his students?

$25x = 20x + 30$
$-20x \quad -20x$

$\frac{5x}{5} = \frac{30}{5}$

$x = 6$

It takes the gym teacher 6 seconds

If the gym teacher finishes the race in 10 seconds, how far ahead are they?

$y = 25(10)$
$y = 250$ Feet.

Lesson 4 Exit Ticket

Julio buys 4 apples and 2 bananas for \$5.50. Chris went to the same farmer's market and bought 2 apples and 2 bananas for \$3.50. Create a system of equations representing this situation.

Apples = A
Bananas = B

Julio:
$4A + 2B = \$5.50$

Chris:
$2A + 2B = \$3.50$

$$4A + 2B = 5.50$$
$$-\ 2A + 2B = 3.50$$
$$\frac{2A}{2} = \frac{2.00}{2}$$
$$A = 1.00$$
$$2(1.00) + 2B = 3.50$$
$$\frac{2B}{2} = \frac{1.50}{2}$$
$$B = .75$$

How much would 7 apples and 3 bananas cost?

$$7(1.00) + 3(.75)$$
$$7 + 2.25$$
$$\$9.25$$