Utilizing Cross Curricular Learning to Incorporate Computer Science Skills in the Circles Unit of Geometry Classrooms

Kate Isaksen

State University of New York (SUNY) Brockport

A thesis submitted to the Department of Education and Human Development

State University of New York (SUNY) Brockport in partial fulfillment of the requirements

for the Degree of Master of Science in Education

December 2023

Abstract

This curriculum was designed to use cross curricular learning by introducing students to computer science skills while reinforcing geometry content. The geometry lessons cover topics including circumference, area, arc length, and sector area of a circle. The computer science cross curricular design guides students in creating an analog clock utilizing the BlocksCAD program. Keys for all instructional materials and computer science applications are included in the appendix. The lessons are aligned with the New York State Next Generation Mathematics Learning Standards.

Table of Contents

Introduction

Educators today are trying to change the way that we teach students. We are moving away from lecturing students to creating more interactive and hands-on learning. When students just complete simple practice problems going along with the unit, they struggle to see the importance of the material. By applying these topics to other disciplines and real-world examples, students are able to see the purpose behind what they are learning.

This curriculum project is designed around the circles unit in geometry and aimed to utilize computer science programs to apply the skills in measuring the circumference, area, arc length, and sector area of circles. This teaching method is also known as cross curricular teaching, which is “instruction within a field in which subject boundaries are crossed and other subjects are integrated into the teaching” (Beckmann, 2009, 6). This method provides students with an opportunity to apply what they are learning to another discipline and solidify their subject knowledge they were just taught.

Literature Review

Students often ask teachers the question, “when are we ever going to use this?” They struggle to see the importance of the skills they are learning and how they can apply them. When mathematics classes consist of lectures and drills, students can feel unmotivated to learn the material and complete the work. Research shows that students favor learning about topics that interest them and that relate to their everyday lives (Zhan et al, 2017). By using problems that are more practical and connect with the outside world, they can make sense of why educators are teaching the material. One way to connect problems to the real world is to use interdisciplinary, or cross-curricular learning. This type of learning “encourages the interconnection between individuals’ prior knowledge and novel learning tasks. Then it provides individuals with more opportunities to deal with ill-structured problems in real life which would motivate them to learn and advance their high order thinking skills" (Zhan et al, 2017).

By utilizing cross curricular teaching, educators can integrate other subjects into instruction and broaden students' learning experiences (Ward-Penney, 2010). This instructional method emphasizes applying their content knowledge to other disciplines and real-world experiences and connecting the skills to tangible concepts. When educators use cross curricular teaching, students learn how to apply the knowledge and skills and reflect on the problem-solving process (Ivanitskaya et al., 2002). For this to happen, students must have a foundational understanding of the mathematics topic before teachers integrate them into applications. When implementing cross curricular instruction into the classroom, it is also beneficial to have students complete the tasks in group settings so students can work collaboratively to solve the problems. Educators can prompt, encourage, and motivate students, but they should allow the students to explore connections in cross curricular instruction. Students can use trial and error to navigate problems they encounter.

Implementing cross curricular teaching into the classroom may not be possible in all educational settings. It does take additional resources and time that educators may not have access to. It requires educators to work collaboratively to create material that integrates both subjects. This requires common planning time to ensure the success of the cross curricular instruction. It can also cause frustration in students if they are not familiar or confident with the corresponding subject. If the process of the activity holds students back instead of the material, the function of the cross curricular learning is lost.

The purpose of this curriculum project is to provide mathematics teachers with an interactive way to teach and reinforce geometry content while introducing students to computer science skills. Shamir et al. (2017) explains that "teaching math with computer programming can give mathematical concepts context and relevance while still requiring the same amount of rigor as traditional mathematics instruction" (2). Computer programs also allow students to talk about the problems easier because they can discuss the structure, development, and relationships. This is very beneficial because students often have difficulties talking about the problem-solving process (Foerster, 2016). The science, technology, engineering, and mathematics (STEM) industry is ever growing, and it is important to expose students to cross curricular concepts in better preparation for college or career in STEM.

Curriculum

Lesson 1

Lesson 1 reminds students about the circumference and area formulas for circles and applies them to higher-level examples. The lesson starts with an activity designed for students to work individually or in small groups to discover the meaning of pi. This lesson then focuses on calculating the circumference and area of circles with examples and applications, respectively. The circumference and area formulas should be a review for students but are foundational and preparatory for the following lessons. Then the clock activity should be introduced to students, and they should be given time to explore the BlocksCAD website (https://www.blockscad3d.com/editor) and familiarize themselves with the functions. Students should then complete part 1 of the activity by creating the clock body.

Lesson Outline

Lesson Title	Circumference & Area of Circles
Standards	**GEO-G.GMD 1**- Provide informal arguments for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
Learning Objectives	- Students will be able to formulate where the circumference of a circle originated from. - Students will be able to evaluate the circumference of circles and execute problems using the formula. - Students will be able to evaluate the area of circles and execute problems using the formula. - Students will create a clock body in the BlocksCAD program
Materials	- 3 circular objects (if multiple objects are not available, teachers can use 3 printed circles of different sizes) - String/twine/ribbon - Scissors - Lesson 1 Notes - Laptop/Desktop - Part 1 of Clock Activity (Clock Activity Packet attached in appendix)
Timeline (Estimates)	50 min class 10 min- Discovering Pi Activity Video explanation: https://youtu.be/WPuzpggl9Z4 10 min- Circumference 10 min- Area 5 min- Introduction to Clock Activity and explore BlocksCAD website 15 min- Part 1 Clock Activity

Name: ____ Date: ________

Lesson 1- Circumference & Area of Circles

Activity

Today you are going to experiment with different size circles and compare their circumferences and diameters.

Steps

1. Pick one circular object.
2. Cut a piece of string equal to the object's circumference.
3. Cut a second piece of string equal to the object's diameter.
4. Record the lengths of the circumference and diameter in the table below.
5. Describe the relationship between the circumference and diameter.
    a. If you fold the longer piece of ribbon in half, does it fit the length of the other piece?
    b. If you did not find a good fit by folding the longer piece in two equal sections, see if you get a better fit by folding it into three, four, or five equal sections.
    c. Do you get an exact or an approximate fit? How would you describe your findings in words like "twice as long" or "three times as long"?
6. Repeat the steps above with the other two objects.
7. Calculate the ratio of the circumference to the diameter. What do you notice about the three ratios?

Object	**Circumference**	**Diameter**	**Relationship**	**Ratio**
1				
2				
3				

Circumference

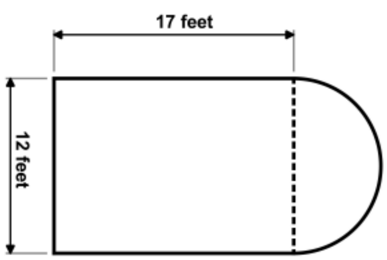
Recall from middle school the formula for circumference

The Circumference of a Circle
For a circle whose diameter is d and whose radius is r, the circumference, C is: $C = \pi d$ or $C = 2\pi r$

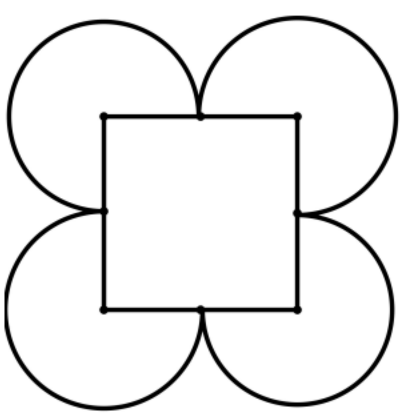
Ex 1: For the circle whose equation is $x^2 + y^2 = 36$, what is the circumference?

Ex 2: Given the circumference is 5ft, what is the diameter (in inches)?

Ex 3: A garden is enclosed by a combination of a rectangle and a semicircle as shown. Find the amount of fencing that needed to be put around the garden.



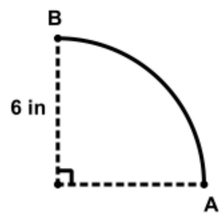
Ex 4: A metal ornament is being designed such that its perimeter is created by four identical three-quarter circles as shown below whose centers are connected to form a square. The square has sides that are 4 inches long. Determine the total perimeter of the 4 circles in terms of pi.



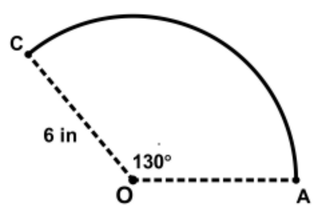
Semicircles and quarter circles are types of arc lengths. Recall an arc is simply a part of a circle.

Ex 5:

a) Determine the arc length $\widehat{AB}$



b) Explain why the following proportion would find the length of $\widehat{AC}$



$$\frac{x}{12\pi} = \frac{130}{360}$$

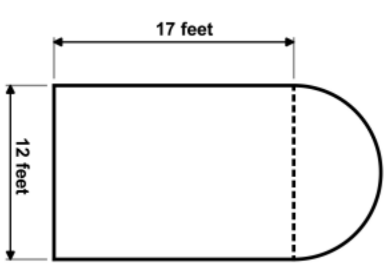
c) Solve the proportion in (b).

Area

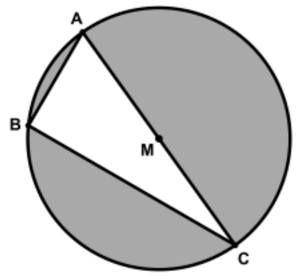
Recall from middle school the formula for area of a circle

The Area of a Circle
The area of a circle whose radius is r is $A = \pi r^2$

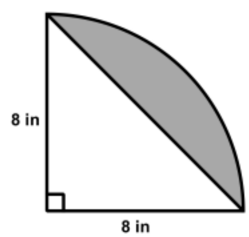
Ex 1: Going back to the fencing problem from earlier, what is the area of the garden?



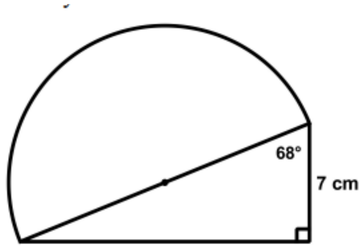
Ex 2: In the diagram below, ΔABC is inscribed in circle M. $\overline{AC} = 26, \overline{BC} = 24$. Determine the area of the shaded region.



Ex 3: Find the area of the shaded region.



Ex 4: Determine the area of the semicircle.



Lesson 2

Lesson 2 introduces how to calculate the arc length of circles. The lesson starts with an example from the previous notes to connect the proportional relationship between the circumference and angle measure to calculate the arc length. The notes are followed with some examples for students to complete in a pair or small group. Then students should move into part 2 of the clock activity where they will add the numbers to the face of the clock.

Lesson Outline

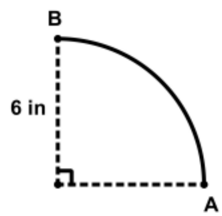
Lesson Title	Arc Length
Standards	**GEO-G.C**- Understand and apply theorems about circles. 2a. Identify, describe, and apply relationships between the angles and their intercepted arcs of a circle. **GEO-G.C-** Find arc lengths and area of sectors of circles. 5. Using proportionality, find one of the following given two others; the central angle, arc length, radius or area of sector.
Learning Objectives	- Students will be able to evaluate the arc length of a circle when given its radius and central angle. - Students will create the numbers of the clock in the BlocksCAD program.
Materials	- Lesson 2 Notes - Laptop/Desktop - Part 2 of Clock Activity (Clock Activity Packet attached in appendix) Videos Loops Part 1: https://www.youtube.com/watch?v=OQeyRl3j3FU Loops Part 2: https://www.youtube.com/watch?v=LSPY_KG8hSQ
Timeline (Estimates)	50 min class 20 min Arc length Notes 40 min Part 2 Clock Activity

Name: ____ Date: ________

Lesson 2- Arc Length

Arc Length

Last Class we determined the arc length $\overset{\frown}{AB}$

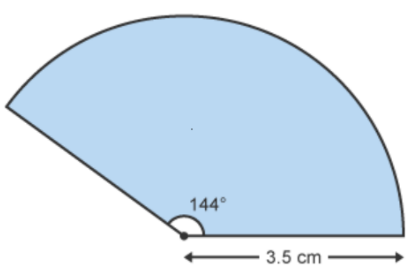


This proportion will work for calculating all arc lengths

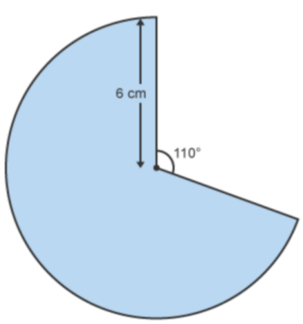
Arc A piece of the circumference of a circle $Arc\ length = \frac{x}{360} * 2\pi r$

Ex 1: Find the arc length

a)

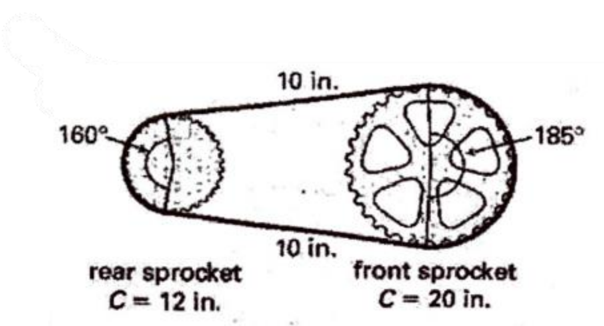


b)



Ex 2: At Mickey's Mechanic Shop a pulley system is used to life engines from cars. The pulley system consists of a cable that goes around a pulley with a radius of 1ft. To the nearest degree, how many degrees of rotation are required for an engine to be lifted 10ft?

Ex 3: The chain of a bicycle travels along the front and rear sprockets, as shown. The circumference of each sprocket is given. About how long is the chain?



Lesson 3

Lesson 3 introduces how to calculate the sector area of circles. The lesson starts by walking students through how the sector area formula is generated. Basic examples are given next and then students are given some word problems to complete. The last example is a challenge problem that students could complete individually, with a partner, or with a small group. Once finished, students should move into part 3 of the clock activity where they will add the hour and minute hands to the clock.

Lesson Outline

Lesson Title	Sector Area
Standards	**GEO-G.C**- Understand and apply theorems about circles. 2a. Identify, describe, and apply relationships between the angles and their intercepted arcs of a circle. **GEO-G.C-** Find arc lengths and area of sectors of circles. 5. Using proportionality, find one of the following given two others; the central angle, arc length, radius or area of sector.
Learning Objectives	- Students will be able to evaluate the area of a sector when given its radius and central angle. - Students will create the hour and minute hands in the BlocksCAD program.
Materials	- Lesson 3 Notes - Laptop/Desktop - Part 3 of Clock Activity (Clock Activity Packet attached in appendix)
Timeline (Estimates)	50 min class 30 min Sector Area Notes 20 min Part 3 Clock Activity

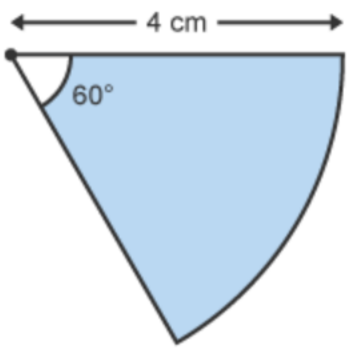
Name: ____ Date: ________

Lesson 3- Area of Sectors

Sector Area

We can also find the area of part of a circle.

Ex 1: Calculate the area of the shaded region



60 degrees is _________ of 360 degrees

Therefore the shaded area is ________ of the full area

Area of circle formula: ____________

Shaded region area =

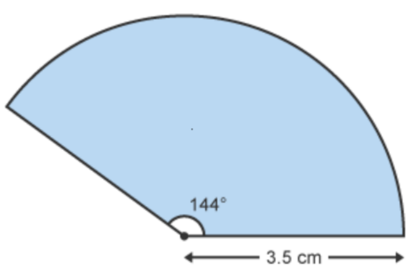
Sector

A slice of the circle bounded by 2 radii and an arc

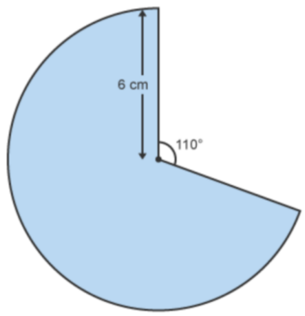
$$Sector\ Area = \frac{x}{360} * \pi r^2$$

Ex 2: Find the sector area

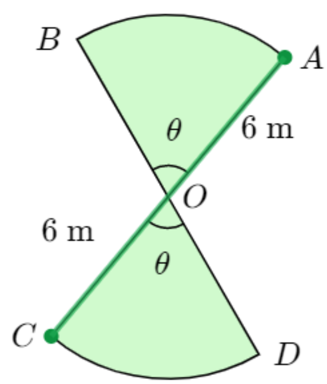
a)



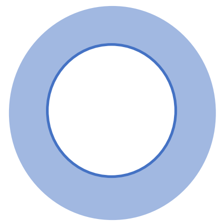
b)



Ex 3: Two cows, Gauri and Bindiya are grazing in a field. They are tethered using ends of the same rope fixed at point O. Gauri moves from A to B and Bindiya moves from C to D. Total area grazed by the cows is 10π. The length of the rope is 12m. Find the angle made by each cow at the center.

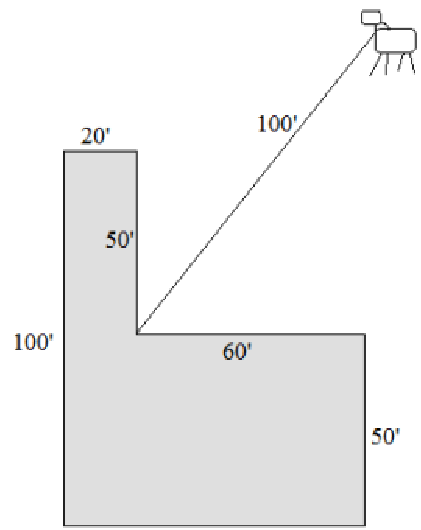


Ex 4: An ice cream shop wants to design a super straw to serve with its extra thick milkshakes that is double both the width and thickness of a standard straw. A standard straw is 4mm in diameter and 0.5mm thick. What is the area of the cross-section of the thickness of the straw (the shaded area)? Round to the nearest hundredth



Challenge Problem

A cow is tethered toa 100-ft rope, attached to the inside corner of an L-shaped building (as shown in the diagram). Find the grazing area of the cow. Hint: think about the places the cow can go to the sides of the building.



Lesson 4

Lesson 4 continues the work started in Lesson 2 and 3 as students will continue to solve problems on arc lengths and area of sectors. The lesson begins with giving students a sequence of statements where they have to decide if they are trying to solve for arc length or sector area. Next the students will complete a mixture of application problems and higher-level thinking problems. Once finished, students will move to the final part of completing their clock, animating the hour and minute hands.

Lesson Outline

Lesson Title	Arc Length & Sector Area Practice
Standards	**GEO-G.C-** Find arc lengths and area of sectors of circles. 5. Using proportionality, find one of the following given two others; the central angle, arc length, radius or area of sector. **GEO-G.MG-** Apply geometric concepts in modeling situations. 1. Use geometric shapes, their measures, and their properties to describe objects.
Learning Objectives	- Students will be able to decide when to use when to use the arc length and sector area formulas - Students will be able to construct a strategy to solve problems of unknown length and area using their knowledge of arc length and area of sectors. - Students will animate the hour and minute hands in the BlocksCAD program.
Materials	- Lesson 4 Notes - Laptop/Desktop - Part 4 of Clock Activity (Clock Activity Packet attached in appendix) Videos Using Variables: https://www.youtube.com/watch?v=I3OeE52zIns
Timeline (Estimates)	50 min class 5 min- Arc length vs Sector area scenarios 20 min- Practice Problems 25 min- Part 4 Clock Activity

Name: ____ Date: ________

Lesson 4- Arc Length & Sector Area Practice

Arc Length vs Sector Area Scenarios

1. Amount of space a slice of pizza takes up (AL/SA)
2. How many miles a plane flies (AL/SA)
3. The area of a clock from 1pm to 3pm (AL/SA)
4. The amount of chain needed for a bicycle (AL/SA)
5. The amount of space covered by a water sprinkler (AL/SA)
6. The path a dog walks while on a leash (AL/SA)
7. How many feet a wheel travels when it makes 150 revolutions (AL/SA)

Mixed Examples

Ex 1: A horse is attached to an 8ft long rope and can rotate a distance of 28ft.

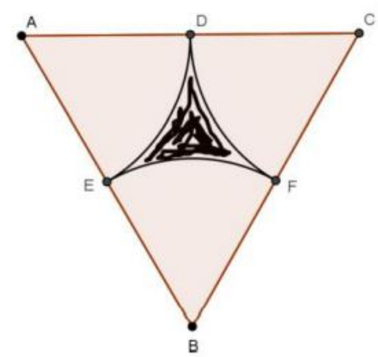
a) What angle can the horse walk about?

b) How much area does the horse have to wonder?

Ex 2: Peter and his partner are conducting a physics experiment on pendulum motion. Their 30 cm pendulum traverses an arc of 15 cm. To the nearest degree, how many degrees of rotation did the pendulum swing? What is the area of the sector?

Ex 3: Triangle ABC is an equilateral triangle with edge length 20cm. D, E, F are midpoints of the sides. The vertices of the triangle are the centers of the circles creating the arcs shown. Find the following (round to the nearest hundredth).

a) Area of the sector with center A



b) Area of triangle ABC

c) Area of the shaded region

d) Perimeter of the shaded region

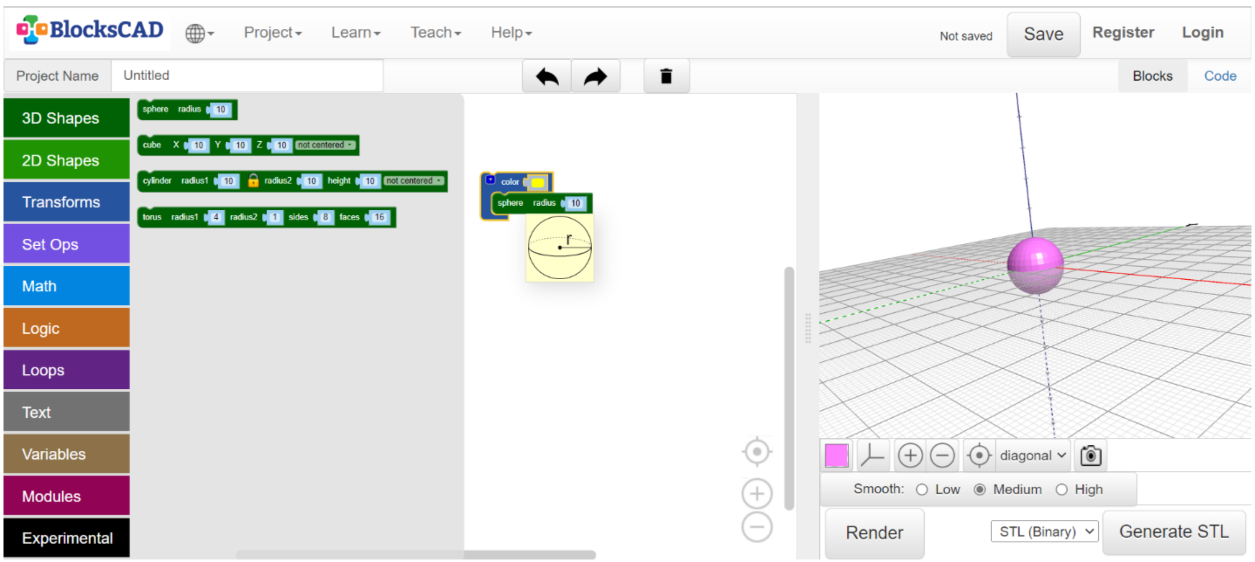
Name: ____ Date: ________

Clock Activity

Building an Analog Clock

Introduction to BlocksCAD

This activity is going to introduce you to block programming through a website called BlocksCAD. The block hides the written code to make this a nice introduction to something a computer scientist might do.



On the left is the bank on blocks available to you.

Your workplace is the middle section. You drag and drop blocks from the bank and they can connect together if you want them to be part of the same group.

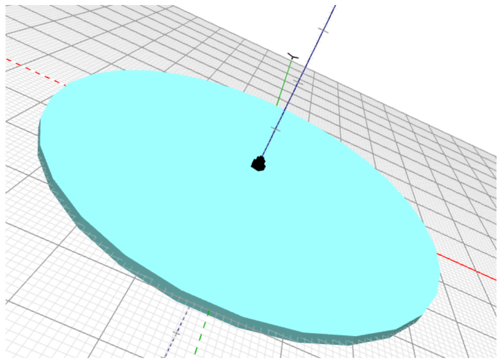
If you hover over a block, it gives you a description of what it does.

On the right it displays what you have done. To refresh the display, hit the render button.

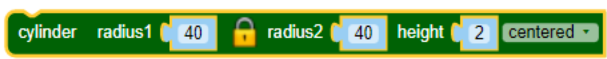
Task: Your task is to build an analog clock that adjusts its hands based on the input values

Part 1: Clock Body

We are going to experiment with the cylinder, union, and color blocks to build our clock body.



We have to use 3D blocks for this project. So, to create the face of the clock we are going to use a cylinder block with a radius of 40 and a height of 2. Make sure to center the cylinder. You can choose the color if you would like.

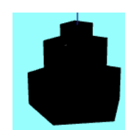


What is the circumference and the area of the face of the cylinder?

Now we need to add the nobs in the center of the face for the hands to turn on. For this create three cylinders that are not centered.

Cylinder 1: r = 1.5, h= 2 Cylinder 2: r = 1, h = 3 Cylinder 3: r = 0.5, h = 3.5

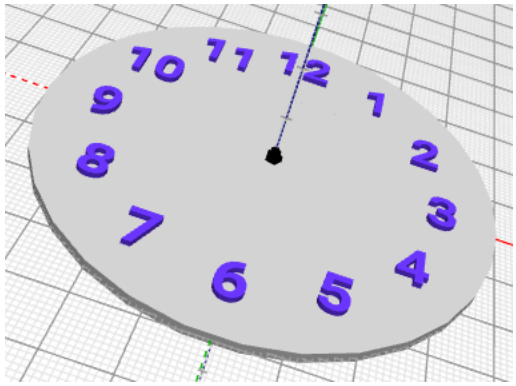
What is the circumference and area of each of the faces of the cylinders?



The union block combines 2 or more objects. Use the block to combine all of the cylinders to create the clock body.

Part 2: Numbers

We are going to add the numbers to our clock.



It would take a while and would be repetitive to do each number separately. There is a way to make things simpler by using a block called a loop block or count with/do block.

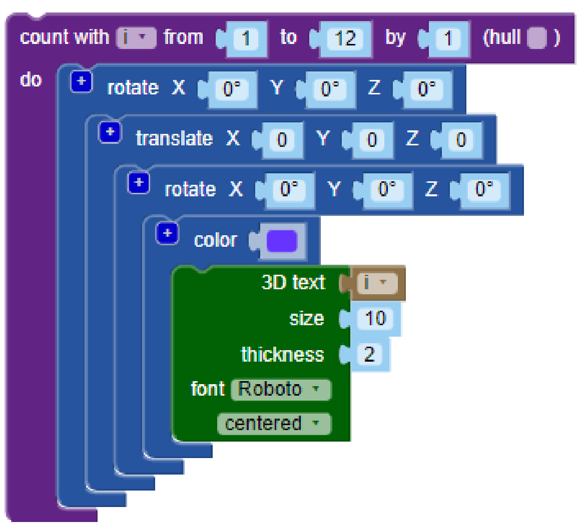
Watch these videos to learn more

Loops Part 1: https://www.youtube.com/watch?v=OQeyRl3j3FU

Loops Part 2: https://www.youtube.com/watch?v=LSPY_KG8hSQ

Start with the 3Dtext block and the count with I block to get your numbers. You will then need to rotate, translate, and rotate the numbers to position them correctly. Use the math blocks to help you with the spacing.

You will rotate in the Z direction. You should translate in the Y direction to move the numbers to the outer edge.



Part 3: Creating the Hour & Minute Hand

We are going to create the hour and minute hands. There is no 3D line in BlocksCAD, so we have to create one by using the hull block to combine the cylinders. The hull block combines blocks by wrapping them together.

Minute Hand

Start with 2 cylinder blocks with radius = 1 and height = 0.1 not centered.

You need one at the center and one to be near the numbers. You can do this by translating the cylinders

To connect the cylinders, use a hull block.

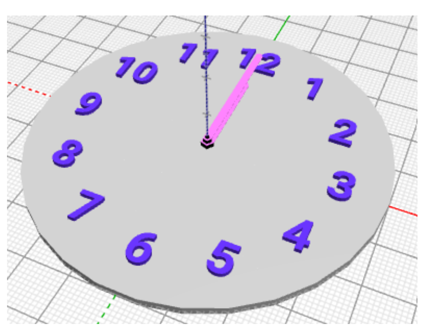
Hour Hand

Duplicate the minute hand blocks by right clicking on the hull block.

Change the radius of the cylinders to 1.5.

Change the translate Z values to 2.

Change the translate Y value from 30 to 20.



Part 4: Animating the Clock Hands

The final step is to make the clock hands rotate.

To start we are going to create hour and minute variables so that we don't have to adjust the code in multiple locations when we want to change the time.

Watch this short video to learn more about variables in computer science.

https://www.youtube.com/watch?v=I3OeE52zIns

Creating the Variables

Create a minute variable and hour variable, and place a math block in it so you are able to change the input.

Minute Hand

To make the minute hand rotate, you just need to add a rotate block outside of the hull block and change the Z value.

What would the angle measure be for 1 minute (Knowing that there are 360 degrees in a circle and 60 minutes in one complete circle)?

Use the math block in the Z value to help you.

Hour Hand

The hour hand is a little tricky because it is determined by both the hour and minute.

We are going to start with just the hour part. What would the angle measure be for 1 hour?

Eq1 =

Now we need to add the minute effect.

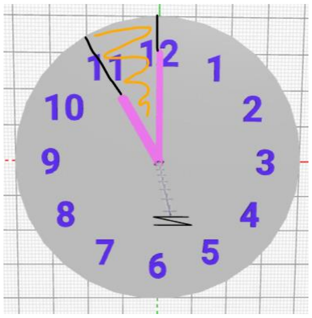
We know that $hour \times \frac{360}{12}$ We also know $hour = \frac{minutes}{60}$

Eq2 = Plug the second equation in the first equation to get an equation for the movement of the hour hand with respect to the minute value.

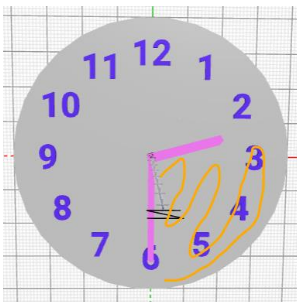
So, our final equation for the Z value would be $Eq1 + Eq2$

Practice finding arc length & sector area

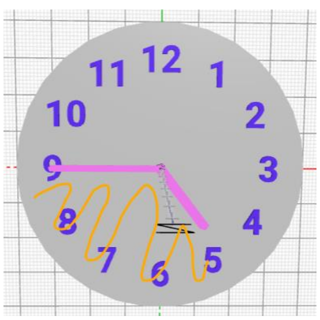
1. Find the arc length and sector area of the section between the hands when the time is 11:00.



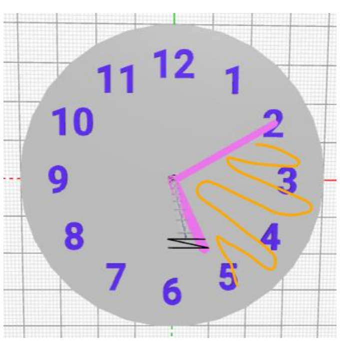
2. Find the arc length and sector area of the section between the hands when the time is 2:30.



3. Find the arc length and sector area of the section between the hands when the time is 4:45.



4. Find the arc length and sector area of the section between the hands when the time is 7:10.



Validity

Teachers have a lot of influence on students and have the opportunity to expose students to new ideas and skills. We have the ability to manipulate the curriculum to incorporate outside examples and projects so that students can apply what they are learning to the real world. Teachers also have the responsibility to help students navigate their lives and their future goals and dreams.

When I was in high school, my sophomore mathematics teacher recommended that I take a computer science course the next year. At that time, I had no clue what computer science was and just thought it was about fixing computers. After taking an introductory course though, I realized it was much more than just the typical computer nerd portrayed in tv shows and movies. I found out that it was all about problem solving and using mathematics and other skills to implement functions. The programs we utilized used the building blocks code, like in BlocksCAD, that made developing the code much easier. I enjoyed the class so much I decided to take AP Computer Science my senior year and then that led me to double majoring in Computer Science and Mathematics for my undergraduate degree. Now as a future educator, my inspiration for this curriculum project was to combine my passion for mathematics and computer science by having a way for students to reinforce the skills they were just taught and giving them an opportunity to be exposed to computer science at an earlier age.

When designing the lesson plans, I wanted to make sure that students could see how the mathematics skills were incorporated in the project. By utilizing the BlocksCAD building blocks, the blocks hide away the complicated written code and just allow the students to puzzle together the functions they want to use. This lets the students not worry about syntax errors and focus on the math behind the problem. The building blocks also make for easy manipulation and the program lets the students run the program as frequently as they would like, allowing them to see their progress and make any changes without the need of an adult.

Conclusion

Utilizing cross curricular teaching allows students to show their understanding of the material while applying it to real world problems. These lesson plans encouraged students to integrate their mathematics skills into solving computer science programs. The intention of using computer science as the other discipline was to provide students an opportunity to learn a discipline that is not always taught or even offered in school districts. The goal of this curriculum project was to make learning more relatable to students and provide teachers with an easy way to introduce computer science skills into their classrooms. The lesson plans, worksheets, and answer keys can all be found in the appendix. It is my hope that other teachers can use these lessons in their classroom.

References

Beckmann, A. (2009). A conceptual framework for cross-curricular teaching. The Mathematics Enthusiast, 6(4), 1-58.

Foerster, K. T. (2016, September). Integrating programming into the Mathematics curriculum: Combining Scratch and Geometry in grades 6 and 7. In Proceedings of the 17th annual conference on information technology education (pp. 91-96).

Ivanitskaya, L., Clark, D., Montgomery, G., & Primeau, R. (2002). Interdisciplinary learning: Process and outcomes. Innovative higher education, 27, 95-111.

Looking to use blockscad in the classroom?. BlocksCAD. (2019). https://www.blockscad3d.com/

Shamir, M., Kocherovsky, M., & Chung, C. (2019, March). A paradigm for teaching math and computer science concepts in k-12 learning environment by integrating coding, animation, dance, music and art. In 2019 IEEE Integrated STEM Education Conference (ISEC) (pp. 62-68). IEEE.

Ward-Penny, R. (2010). Cross-curricular teaching and learning in the secondary school... Mathematics. Routledge.

Zhan, Y., So, W. W. M., & Cheng, I. N. Y. (2017). Students' beliefs and experiences of interdisciplinary learning. Asia Pacific Journal of Education, 37(3), 375-388.

Appendix

Lesson 1

Name: Key Date: ________

Lesson 1 - Circumference & Area of Circles

Activity

Today you are going to experiment with different size circles and compare their circumferences and diameters.

Steps

1. Pick one circular object.
2. Cut a piece of string equal to the object's circumference.
3. Cut a second piece of string equal to the object's diameter.
4. Record the lengths of the circumference and diameter in the table below.
5. Describe the relationship between the circumference and diameter.
    a. If you fold the longer piece of ribbon in half, does it fit the length of the other piece?
    b. If you did not find a good fit by folding the longer piece in two equal sections, see if you get a better fit by folding it into three, four, or five equal sections.
    c. Do you get an exact or an approximate fit? How would you describe your findings in words like "twice as long" or "three times as long"?
6. Repeat the steps above with the other two objects.
7. Calculate the ratio of the circumference to the diameter. What do you notice about the three ratios?

Object	Circumference	Diameter	Relationship	Ratio
1	25.12	8	circumference is about 3 times the diameter	$\frac{25.12}{8} \approx 3.14$
2	18.85	6	" "	$\frac{18.85}{6} \approx 3.1417$
3	37.70	12	" "	$\frac{37.70}{12} \approx 3.14158$

Circumference

Recall from middle school the formula for circumference

The Circumference of a Circle
For a circle whose diameter is d and whose radius is r, the circumference, C is: $C = \pi d$ or $C = 2\pi r$

Ex 1: For the circle whose equation is $x^2 + y^2 = 36$, what is the circumference?

$r^2 = 36$ $\quad C = 2\pi r$

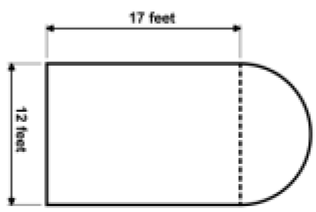
$r = 6$ $\quad C = 2\pi(6)$

$C = 12\pi \approx 37.70$

Ex 2: Given the circumference is 5ft, what is the diameter (in inches)?

$C = 5ft$ $\quad C = \pi d$ $\quad \frac{5}{\pi} = \frac{\pi d}{\pi}$ $\quad d = 1.59ft \cdot \frac{12in}{1ft} \approx 19.10in$

Ex 3: A garden is enclosed by a combination of a rectangle and a semicircle as shown. Find the amount of fencing needed to be put around the garden.



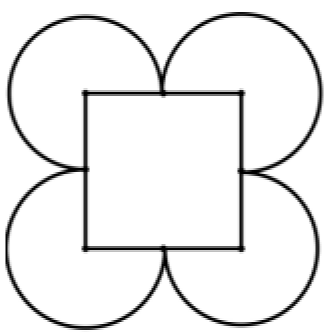
$P_{rec} = 12 + 2(17) = 46ft$

$P_{semi} = \frac{1}{2}\pi d = \frac{1}{2}\pi(12) = 18.85$

$P_{TOT} = P_{REC} + P_{semi} = 46 + 18.85$

$P_{TOT} = 64.85ft$

Ex 4: A metal ornament is being designed such that its perimeter is created by four identical three-quarter circles as shown below whose centers are connected to form a square. The square has sides that are 4 inches long. Determine the total perimeter of the 4 circles in terms of pi.



$C_{whole} = \pi d = 4\pi$

$C_{3/4\ circ} = \frac{3}{4} \cdot 4\pi = 3\pi$

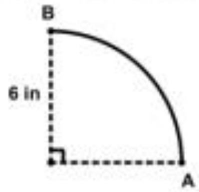
$P_{TOT} = 4(3\pi) = 12\pi$

Semicircles and quarter circles are types of arc lengths. Recall an arc is simply a part of a circle.

Ex 5:

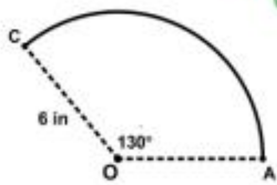
$C_{circ} = 2\pi r = 2\pi(6) = 12\pi$

a) Determine the arc length $\widehat{AB}$



$\widehat{AB} = \frac{1}{4}(12\pi) = 3\pi$

b) Explain why the following proportion would find the length of $\widehat{AC}$



$$\frac{x}{12\pi} = \frac{130}{360}$$

part we want → x; part we have → 130; total degrees of whole circle → 360; Circumference of whole circle → 12π

c) Solve the proportion in (b).

$\frac{x}{12\pi} = \frac{130}{360}$

$360x = 1560\pi$

$\frac{360x}{360} = \frac{1560\pi}{360}$

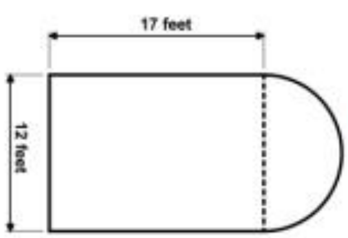
$x = 4.33\pi$

Area

Recall from middle school the formula for area of a circle

The Area of a Circle
The area of a circle whose radius is r is
$A = \pi r^2$

Ex 1: Going back to the fencing problem from earlier, what is the area of the garden?

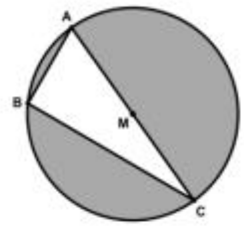


$A_{rec} = lw = 12(17) = 204$

$A_{semi} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(6)^2 = 18\pi$

$A_{TOT} = 204 + 18\pi$

Ex 2: In the diagram below, ΔABC is inscribed in circle M. $\overline{AC} = 26, \overline{BC} = 24$. Determine the area of the shaded region.



$A_{shaded} = A_{circ} - A_{triangle}$

$x^2 + 24^2 = 26^2$

$x^2 + 576 = 676$

$x^2 = 100$

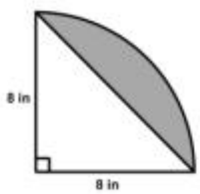
$x = 10$

$A_{shaded} = \pi r^2 - \frac{1}{2} bh$

$= \pi(13)^2 - \frac{1}{2}(10)(24)$

$= 169\pi - 120$

Ex 3: Find the area of the shaded region.

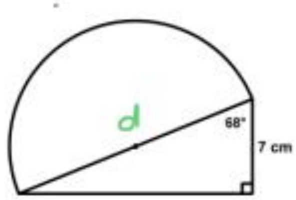


$A_{shaded} = A_{1/4\ circ} - A_{Triangle}$

$= \frac{1}{4}\pi r^2 - \frac{1}{2} bh$

$= \frac{1}{4}\pi(8)^2 - \frac{1}{2}(8)(8) = 16\pi - 32$

Ex 4: Determine the area of the semicircle.



$\cos 68 = \frac{7}{d}$

$d = \frac{7}{\cos 68}$

$d = 18.69$

$A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi\left(\frac{18.69}{2}\right)^2$

$A = 137.18$

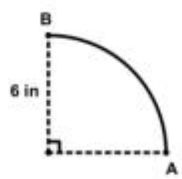
Lesson 2

Name: Key   Date: ________

Lesson 2- Arc Length

Arc Length

Last Class we determined the arc length $\widehat{AB}$



$\widehat{AB} = \frac{1}{4} C_{\text{whole circ}}$

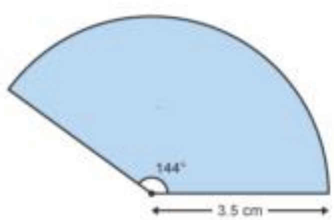
$= \frac{1}{4} 2\pi r = \frac{1}{4} 2\pi(6) = 3\pi$

This proportion will work for calculating all arc lengths

Arc A piece of the circumference of a circle $Arc\ length = \frac{x}{360} * 2\pi r$

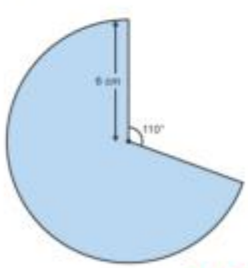
Ex 1: Find the arc length

a)



$arc = \frac{144}{360} \cdot 2\pi(3.5) = 8.8$

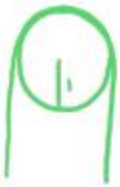
b)



$arc = \frac{250}{360} \cdot 2\pi(6) = 26.2$

$360 - 110 = 250$

Ex 2: At Mickey's Mechanic Shop a pulley system is used to life engines from cars. The pulley system consists of a cable that goes around a pulley with a radius of 1ft. To the nearest degree, how many degrees of rotation are required for an engine to be lifted 10ft?



$$10 = \frac{x}{360} \cdot 2\pi r$$

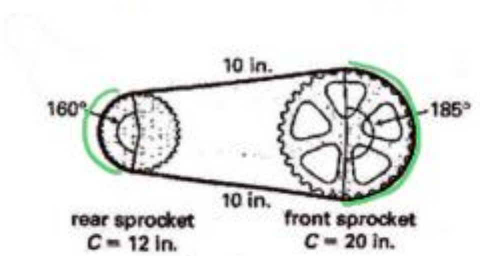
$$10 = \frac{x}{360} \cdot 2\pi(1)$$

$$10 = \frac{x}{180}\pi$$

$$\frac{1800}{\pi} = x \qquad x = 572.96$$

$$x \approx 573°$$

Ex 3: The chain of a bicycle travels along the front and rear sprockets, as shown. The circumference of each sprocket is given. About how long is the chain?



$r = 6 \qquad r = 10$

rear: $\frac{160}{360} = 2\pi(6) = \frac{16}{3}\pi$

front: $\frac{185}{360} \cdot 2\pi(10) = \frac{185}{18}\pi$

Chain length: $10 + 10 + \frac{16}{3}\pi + \frac{185}{18}\pi = 69.04$ in

Lesson 3

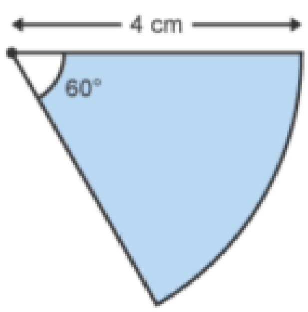
Name: Key ______________________ Date: ________

Lesson 3- Area of Sectors

Sector Area

We can also find the area of part of a circle.

Ex 1: Calculate the area of the shaded region



60 degrees is $\frac{1}{6}$ of 360 degrees $\frac{60}{360} = \frac{1}{6}$

Therefore the shaded area is $\frac{1}{6}$ of the full area

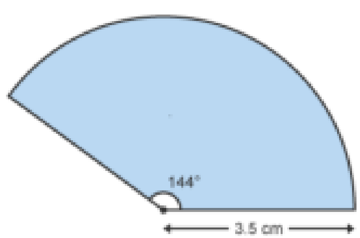
Area of circle formula: πr^2

Shaded region area = $\frac{1}{6}\pi r^2 = \frac{1}{6}\pi(4)^2$

Sector
A slice of the circle bounded by 2 radii and an arc
$Sector\ Area = \frac{x}{360} * \pi r^2$

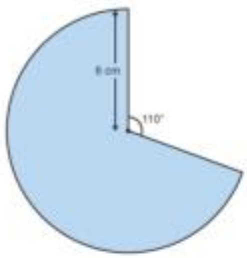
Ex 2: Find the sector area

a)



$\frac{144}{360} \cdot \pi(3.5)^2 = 15.4$

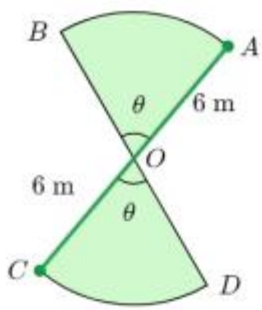
b)



$$\frac{250}{360}\pi(6)^2 = 78.5$$

$360 - 110 = 250$

Ex 3: Two cows, Gauri and Bindiya are grazing in a field. They are tethered using ends of the same rope fixed at point O. Gauri moves from A to B and Bindiya moves from C to D. Total area grazed by the cows is 10π. The length of the rope is 12m. Find the angle made by each cow at the center.



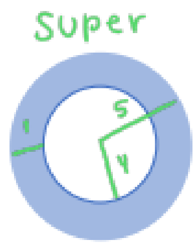
$$\frac{10\pi}{2\pi} = \frac{2\left(\frac{\theta}{360}\right)\pi(6)^2}{2\pi}$$

$$5 = \frac{\theta}{360}(36)$$

$$5 = \frac{\theta}{10}$$

$$\theta = 50^\circ$$

Ex 4: An ice cream shop wants to design a super straw to serve with its extra thick milkshakes that is double both the width and thickness of a standard straw. A standard straw is 4mm in diameter and 0.5mm thick. What is the area of the cross-section of the thickness of the straw (the shaded area)? Round to the nearest hundredth



$A = \pi(5)^2 - \pi(4)^2$

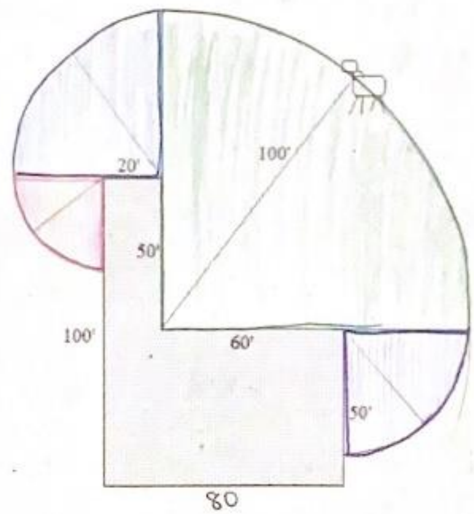
$A = 25\pi - 16\pi$

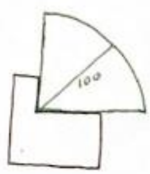
$A = 9\pi$

$A = 28.27\,mm^2$

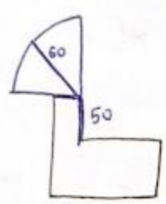
Challenge Problem

A cow is tethered toa 100-ft rope, attached to the inside corder of an L-shaped building (as shown in the diagram). Find the grazing area of the cow. Hint: think about the places the cow can go to the sides of the building.

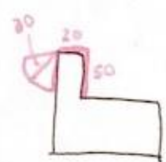




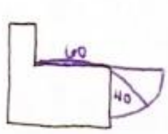
$r = 100$
$\theta = 90$
$A = \frac{90}{360}\pi(100)^2$
$A = 2500\pi \text{ ft}^2$



$r = 50$
$\theta = 90$
$A = \frac{90}{360}\pi(50)^2$
$A = 625\pi \text{ ft}^2$



$r = 30$
$\theta = 90$
$A = \frac{90}{360}\pi(30)^2$
$A = 225\pi \text{ ft}^2$



$r = 40$
$\theta = 90$
$A = \frac{90}{360}\pi(40)^2$
$A = 400\pi \text{ ft}^2$

Total area:

$2500\pi + 625\pi + 225\pi + 400\pi$
$= 3750\pi \text{ ft}^2$

Lesson 4

Name: Key Date: ________

Lesson 4- Arc Length & Sector Area Practice

Arc Length vs Sector Area Scenarios

1. Amount of space a slice of pizza takes up (AL/**SA**)
2. How many miles a plane flies (**AL**/SA)
3. The area of a clock from 1pm to 3pm (AL/**SA**)
4. The amount of chain needed for a bicycle (**AL**/SA)
5. The amount of space covered by a water sprinkler (AL/**SA**)
6. The path a dog walks while on a leash (**AL**/SA)
7. How many feet a wheel travels when it makes 150 revolutions (**AL**/SA)

Mixed Examples

Ex 1: A horse is attached to an 8ft long rope and can rotate a distance of 28ft.

a) What angle can the horse walk about?

$$28 = \frac{x}{360} 2\pi(8)$$

$$x = 200.5°$$

b) How much area does the horse have to wonder?

$$A = \frac{200.5}{360} \pi(8)^2 = 112 \text{ ft}^2$$

Ex 2: Peter and his partner are conducting a physics experiment on pendulum motion. Their 30 cm pendulum traverses an arc of 15 cm. To the nearest degree, how many degrees of rotation did the pendulum swing? What is the area of the sector?

$15 = \frac{x}{360} \cdot 2\pi(30)$

$15 = \frac{\pi}{6}x$

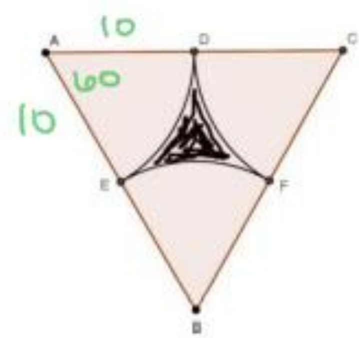
$x = 28.65° = 29°$

$A = \frac{29}{360}\pi(30)^2 = 227.77\text{ cm}$

Ex 3: Triangle ABC is an equilateral triangle with edge length 20cm. D, E, F are midpoints of the sides. The vertices of the triangle are the centers of the circles creating the arcs shown. Find the following (round to the nearest hundredth).

a) Area of the sector with center A

$A = \frac{60}{360}\pi(10)^2 = 16.667\pi \approx 52.36\text{ cm}^2$



b) Area of triangle ABC

$10^2 + x^2 = 20^2$

$x^2 = 300$

$x \approx 17.32$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(10)(17.32)$

$A = 86.6 \times 2$

$A = 173.21\text{ cm}$

c) Area of the shaded region

$A_{shaded} = A_{\Delta} - 3A_{sector}$

$= 173.21 - 3(52.36)$

$= 16.13\text{ cm}^2$

d) Perimeter of the shaded region

Arc length $= \frac{60}{360} \cdot 2\pi(10) = 10.472$

$3 \times 10.472 = 31.42\text{ cm}$

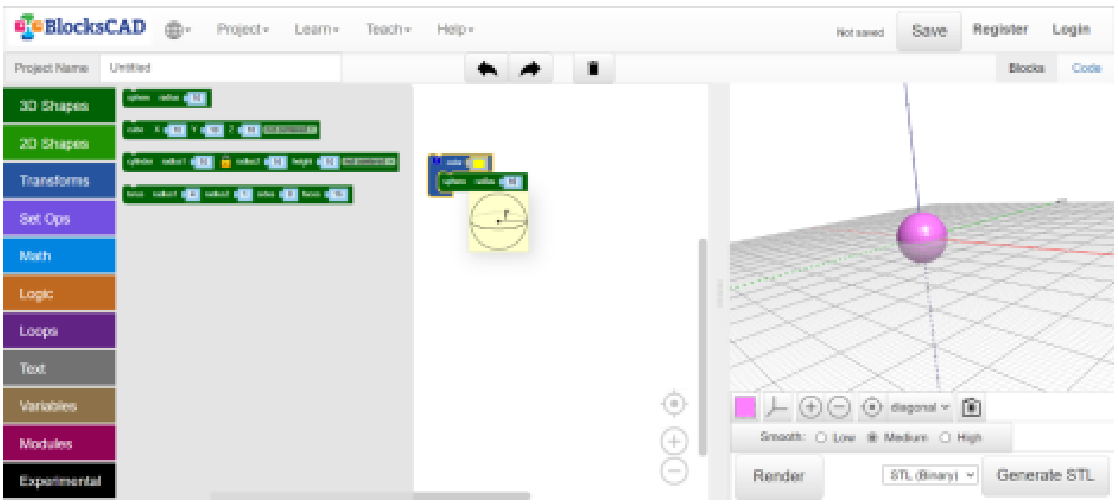
Name: Key______________________________ Date: ________

Clock Activity

Building an Analog Clock

Introduction to BlocksCAD

This activity is going to introduce you to block programming through a website called BlocksCAD. The block hides the written code to make this a nice introduction to something a computer scientist might do.



On the left is the bank on blocks available to you.

Your workplace is the middle section. You drag and drop blocks from the bank and they can connect together if you want them to be part of the same group.

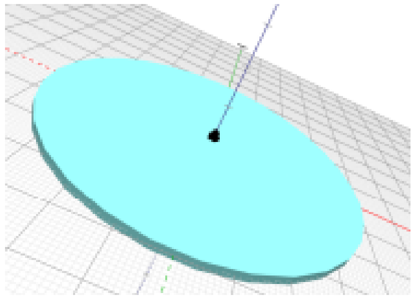
If you hover over a block, it gives you a description of what it does.

On the right it displays what you have done. To refresh the display, hit the render button.

Task: Your task is to build an analog clock that adjusts its hands based on the input values

Part 1: Clock Body

We are going to experiment with the cylinder, union, and color blocks to build our clock body.



We have to use 3D blocks for this project. So, to create the face of the clock we are going to use a cylinder block with a radius of 40 and a height of 2. Make sure to center the cylinder. You can choose the color if you would like.

What is the circumference and the area of the face of the cylinder?

$C = 2\pi r$ $\quad$ $A = \pi r^2$

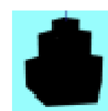
$C = 2\pi 40$ $\quad$ $A = \pi(40)^2$

$C = 80\pi \approx 251.33$ $\quad$ $A = 1600\pi \approx 5,026.55$

Now we need to add the nobs in the center of the face for the hands to turn on. For this create three cylinders that are not centered.

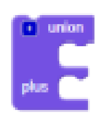
Cylinder 1: r = 1.5, h= 2 Cylinder 2: r = 1, h = 3 Cylinder 3: r = 0.5, h = 3.5

What is the circumference and area of each of the faces of the cylinders?



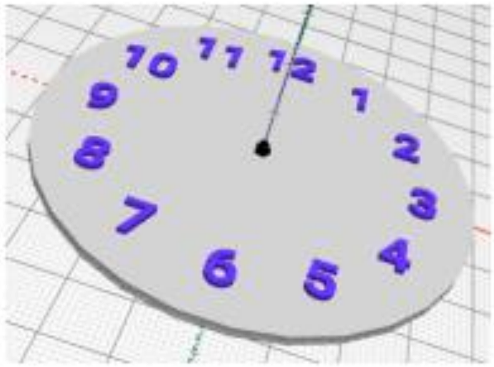
C1 $r=1.5$	C2 $r=1$	C3 $r=0.5$
$C = 2\pi(1.5)$	$C = 2\pi(1)$	$C = 2\pi(0.5)$
$C = 3\pi$	$C = 2\pi$	$C = \pi$
$A = \pi(1.5)^2$	$A = \pi(1)^2$	$A = \pi(0.5)^2$
$A = 2.25\pi$	$A = \pi$	$A = 0.25\pi$

The union block combines 2 or more objects. Use the block to combine all of the cylinders to create the clock body.



Part 2: Numbers

We are going to add the numbers to our clock.



It would take a while and would be repetitive to do each number separately. There is a way to make things simpler by using a block called a loop block or count with/do block.

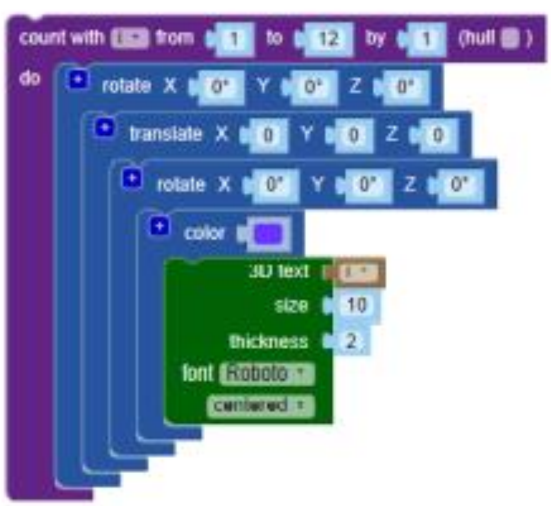
Watch these videos to learn more

Loops Part 1: https://www.youtube.com/watch?v=OQeyRl3j3FU

Loops Part 2: https://www.youtube.com/watch?v=LSPY_KG8hSQ

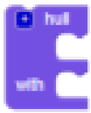
Start with the 3Dtext block and the count with I block to get your numbers. You will then need to rotate, translate, and rotate the numbers to position them correctly. Use the math blocks to help you with the spacing.

You will rotate in the Z direction. You should translate in the Y direction to move the numbers to the outer edge.



Part 3: Creating the Hour & Minute Hand

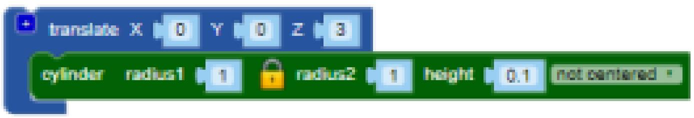
We are going to create the hour and minute hands. There is no 3D line in BlocksCAD, so we have to create one by using the hull block to combine the cylinders. The hull block combines blocks by wrapping them together.

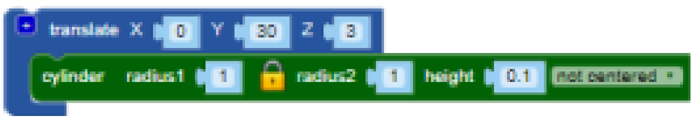


Minute Hand

Start with 2 cylinder blocks with radius = 1 and height = 0.1 not centered.

You need one at the center and one to be near the numbers. You can do this by translating the cylinders





To connect the cylinders, use a hull block.

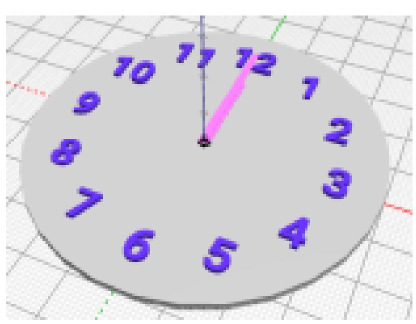
Hour Hand

Duplicate the minute hand blocks by right clicking on the hull block.

Change the radius of the cylinders to 1.5.

Change the translate Z values to 2.

Change the translate Y value from 30 to 20.



Part 4: Animating the Clock Hands

The final step is to make the clock hands rotate.

To start we are going to create hour and minute variables so that we don't have to adjust the code in multiple locations when we want to change the time.

Watch this short video to learn more about variables in computer science.

https://www.youtube.com/watch?v=I3OeE52zIns

Creating the Variables

Create a minute variable and hour variable, and place a math block in it so you are able to change the input.

Minute Hand

To make the minute hand rotate, you just need to add a rotate block outside of the hull block and change the Z value.

What would the angle measure be for 1 minute (Knowing that there are 360 degrees in a circle and 60 minutes in one complete circle)?

$\frac{360°}{60min}$ $\quad\quad$ $minute \times \frac{360}{60}$

Use the math block in the Z value to help you.

Hour Hand

The hour hand is a little tricky because it is determined by both the hour and minute.

We are going to start with just the hour part. What would the angle measure be for 1 hour?

Eq1 = $\quad hour \times \frac{360}{12}$

Now we need to add the minute effect.

We know that $hour \times \frac{360}{12}$ $\quad\quad$ We also know $hour = \frac{minutes}{60}$

Eq2 = Plug the second equation in the first equation to get an equation for the movement of the hour hand with respect to the minute value.

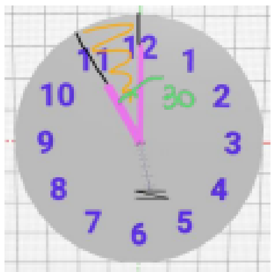
So, our final equation for the Z value would be $Eq1 + Eq2$

$$\left(hour \times \frac{360}{12}\right) + \left(\frac{minutes}{60} \times \frac{360}{12}\right)$$

My radius = 40 (from clock face)

Practice finding arc length & sector area

1. Find the arc length and sector area of the section between the hands when the time is 11:00.



minute ∡ = 0 → 360

hour ∡ = $11\left(\frac{360}{12}\right) + 0 = 330$

$360 - 330 = 30$

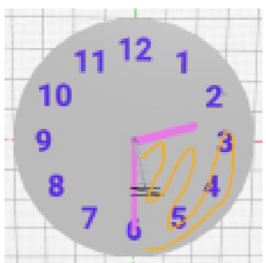
Arc = $\frac{30}{360} \times 2\pi(40)$

Arc = 20.94

Area = $\frac{30}{360} \times \pi(40)^2$

Area = 418.88

2. Find the arc length and sector area of the section between the hands when the time is 2:30.



minute ∡ = $30 \times \frac{360}{60} = 180$

hour ∡ = $2\left(\frac{360}{12}\right) + \left(\frac{30}{60} \times \frac{360}{12}\right) = 75$

$180 - 75 = 105°$

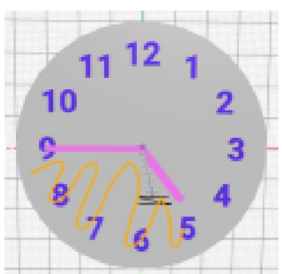
Arc = $\frac{105}{360} \times 2\pi(40)$

Arc = 73.30

Area = $\frac{105}{360} \times \pi(40)^2$

Area = 1466.08

3. Find the arc length and sector area of the section between the hands when the time is 4:45.



minute ∡ = $45 \times \frac{360}{60} = 270$

hour ∡ = $4\left(\frac{360}{12}\right) + \left(\frac{45}{60} \times \frac{360}{12}\right) = 142.5$

$270 - 142.5 = 127.5$

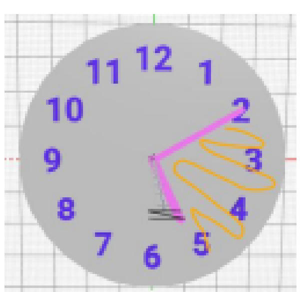
Arc = $\frac{127.5}{360} \times 2\pi(40)$

Arc = 89.01

Area = $\frac{127.5}{360} \times \pi(40)^2$

Area = 1780.24

4. Find the arc length and sector area of the section between the hands when the time is 7:10.



minute ∡ = $10 \times \frac{360}{60} = 60$

hour ∡ = $7\left(\frac{360}{12}\right) + \left(\frac{10}{60} \times \frac{360}{12}\right) = 215$

$215 - 60 = 155$

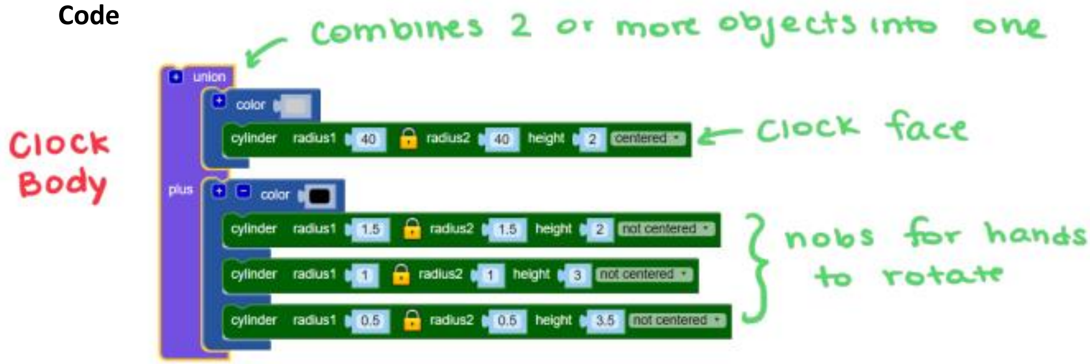
Arc = $\frac{155}{360} \times 2\pi(40)$

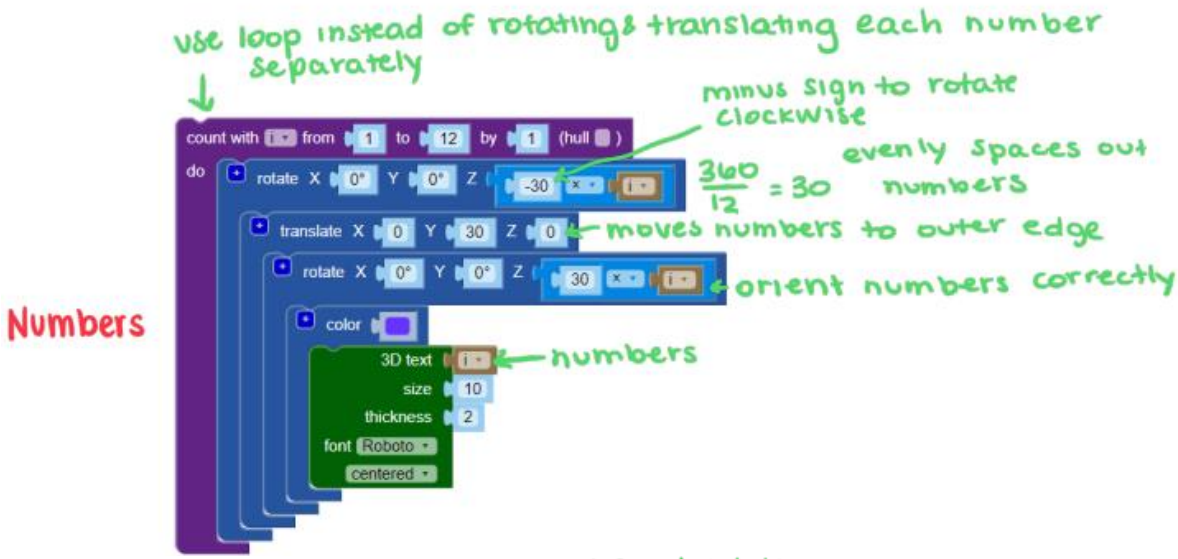
Arc = 108.21

Area = $\frac{155}{360} \times \pi(40)^2$

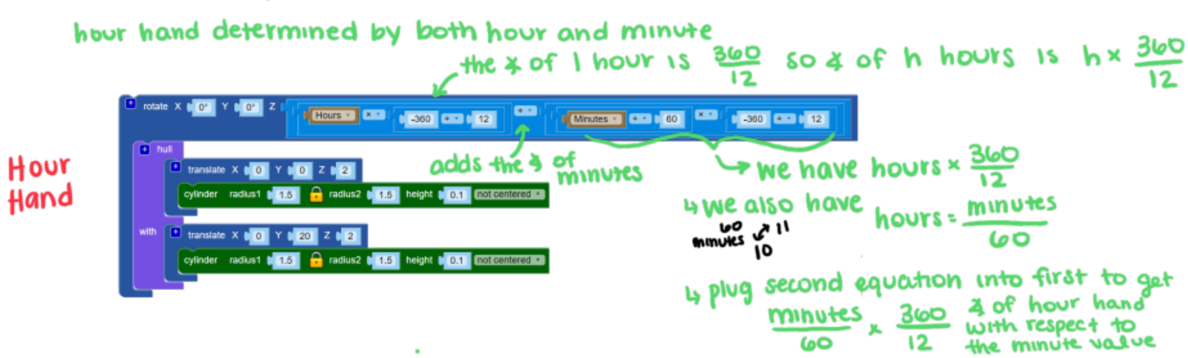
Area = 2164.21

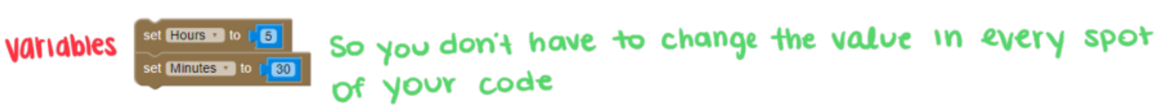
Code


Clock Body
union
color
cylinder radius1 40 radius2 40 height 2 centered
plus
color
cylinder radius1 1.5 radius2 1.5 height 2 not centered
cylinder radius1 1 radius2 1 height 3 not centered
cylinder radius1 0.5 radius2 0.5 height 3.5 not centered
combines 2 or more objects into one
clock face
nobs for hands to rotate


Numbers
use loop instead of rotating & translating each number separately
count with i from 1 to 12 by 1 (hull)
do
rotate X 0° Y 0° Z -30 × i
translate X 0 Y 30 Z 0
rotate X 0° Y 0° Z 30 × i
color
3D text i
size 10
thickness 2
font Roboto
centered
minus sign to rotate clockwise
360/12 = 30
evenly spaces out numbers
moves numbers to outer edge
orient numbers correctly
numbers


Minute Hand
rotate X 0° Y 0° Z Minutes × -360 ÷ 60
hull
translate X 0 Y 0 Z 3
cylinder radius1 1 radius2 1 height 0.1 not centered
with
translate X 0 Y 30 Z 3
cylinder radius1 1 radius2 1 height 0.1 not centered
minus sign to rotate the correct direction
combines by wrapping objects
the ∡ of 1 minute is 360°/60 min
so ∡ of m minutes m × 360/60
there is no 3D line so we connect 2 cylinders instead


Hour Hand
hour hand determined by both hour and minute
the ∡ of 1 hour is 360/12 so ∡ of h hours is h × 360/12
rotate X 0° Y 0° Z Hours × -360 ÷ 12 + Minutes ÷ 60 × -360 ÷ 12
hull
translate X 0 Y 0 Z 2
cylinder radius1 1.5 radius2 1.5 height 0.1 not centered
with
translate X 0 Y 20 Z 2
cylinder radius1 1.5 radius2 1.5 height 0.1 not centered
adds the ∡ of minutes
we have hours × 360/12
we also have hours = minutes/60
plug second equation into first to get minutes/60 × 360/12 ∡ of hour hand with respect to the minute value


Variables
set Hours to 5
set Minutes to 30
So you don't have to change the value in every spot of your code