Creating Authentic Meaning for Inferential Statistics in a High School Setting

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Abstract

Creating authentic learning experiences for students to apply statistical concepts in areas that interests them aligns with best practices in the teaching. This four-lesson curriculum project is thus designed for a high school statistics class to support learning of the limited New York State Next Generation Learning Statistics Standards. It can also use statistics standards from other states that offer a full high school statistics class. The project culminates in students performing statistical analysis on real world data they generated from a topic that interests them. Teacher guides and materials for instruction are included with the keys available in the appendix.

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Introduction

Since McGatha et al., (1998) launched a focus of statistics in the middle schools in the late 1980s, statistics has been an increasingly relevant and focused topic in secondary mathematics. As such, more schools are offering elective senior classes focusing on statistics. This curriculum project is designed for such a course in statistics, potentially with dual enrollment with a college. Within this curriculum project, students will learn about the one sample t-test, the two sample t-test, and the equations for determining sample size and margin of error. Then in a culminating project students will investigate data that interests them and perform analysis on data to make statistically valid decisions. In doing this, students will be more likely to be excited about the content because they will be able to apply what they have learned to an area of their interest.

While New York State (NYS) does not have specific statistics standards for a senior level class, there are multiple standards that this curriculum project builds upon. First, in algebra I, students use statistics to determine the shape of the data distribution and compare the measures of the middle and spread. Then, in algebra II, students learn to use statistics and draw conclusions from numerical summaries. In using inferential statistics, such as the z-and t-test, which is not covered in the NYS standards, is where teachers from other states will need to use either their standards, the example standards provided, or guidance from AP Statistics. For example, one of the standards within the South Carolina Framework standards states that students will use inference to describe sampling distributions. The limited NYS Standards state that students will interpret differences in shape, center, and spread in the context of the data. In this project, students will apply this knowledge by completing t-tests and determining if the means are different within context of the problem. One of the assumptions of the t-test is that the data set is normal. If the data is non-normal a t-test is not appropriate. When using a t-test students will build upon this by drawing conclusions on whether the means are significantly different and how to interpret this within context of the problem. This unit of material will follow topics such as displaying distributions, data collection, sampling, the z distribution, hypothesis testing, p values, and the z test. As such, lessons will be done under the assumption that students are familiar with these topics.

Literature Review

In mathematics education, external high stakes tests require teachers to compromise on teaching for learning and instead has allowed teachers to make excuses for not teaching for meaning or for in depth understanding, rather, they try and focus on procedural instruction for problems students are likely to see on the test (McTighe, 2004). This type of instruction can lead to a gap in students relational understanding. Skemp (1976) noted that relational understanding is knowing what to do and why and stressed that this type of understanding is what we typically think of when we consider understanding. Skemp (1976) also presented the idea of instrumental understanding, which is instruction that focuses on rules without reason. An example of this would be students seeking to recall a memorized rule needed to solve a problem, but without understanding, students are not sure why, or specifically how, that rule is applied. For student learning, we need to seek instructional methods that align with relational understanding so students can develop deep learning. So, the conflict of instruction for high stakes test performance is it is in conflict with relational understanding.

One way to increase relational understanding in the mathematics classroom is by placing more emphasis on authentic tasks. Beswick (2010) describes authentic tasks as tasks which convey common contexts that has no ready-made algorithm. While using word problems is a common practice within the mathematics classroom, they are not necessarily authentic because algorithms are often available for them. Additionally, authenticity is dependent upon who is solving the problem, a problem may be authentic for one student but inauthentic for another, thus, creating meaningful, goal directed problems for each student is important (Beswick, 2010). Vos (2018) notes that students are often faced with inauthentic word problems designed to illustrate mathematical concepts but they are not real-world problems. Vos reasons that the inclusion of these problems limits student learning and teaches students that mathematics is for inauthentic situations and that mathematical tasks often contain added artificial complexities when students should be learning that mathematics is useful for solving real life problems through reasoning, creativity, and collaboration. Additionally, just because a task is authentic, the purpose of a problem is important as well. Students will not gain understanding or appreciation for authentic tasks such as determining the perimeter of a marketplace within their hometown if there is little purpose behind these tasks (Vos, 2018). Therefore, creating authentic contexts with purpose are paramount to increasing student relational understanding. These contexts can improve how problems are accessible to students and give them more opportunities to demonstrate their understanding (Beswick, 2010). Vos (2018) notes that to improve and foster deeper student understanding, it is necessary to create authentic problems where students investigate and answer questions the same way that people working in that field would. The most authentic tasks occur when students work on self-selected, challenging, open ended problems that are rooted within a real-life situation (Vos, 2018). The inclusion of such tasks and problems can help to increase student relational understanding of the mathematical concepts within the context.

This curriculum project presents four exemplar lessons designed around relational understanding. Within this curriculum, students are tasked with synthesizing statistical analysis on a topic that they deem important. Since this culminating project is self-selected, challenging and open-ended, it can be ensured that for students, this task is authentic. This authenticity can encourage relational understanding within students instead of instrumental understanding as students will take ownership of presenting statistical analysis on a topic of their choice.

Lesson 1 Lesson Plan

Mathematical Task: Reviewing the uses of the z test and explaining the need for a different test, the t test.
Grade Level, Class: Senior level statistics course
Materials and Sources: Guided Notes Homework sheet
Next Generation Standards **AI-S.ID.2** - Use statistics appropriate to the shape of the data distribution to compare center and spread. **AII-S.IC.3** - Recognize the purposes of and differences among surveys, experiments and observational studies. Explain how randomization relates to each. **AII-S.IC.6a** - Use the tools of statistics to draw conclusions from numerical summaries.
Performance Objectives: - The students will demonstrate the ability to use the four steps of hypothesis testing. - The students will demonstrate the ability to determine the advantages and disadvantages of using the z test. - The students will demonstrate the ability to use a z test to determine whether or not the mean of a sample is different than a hypothesized mean and whether or not 2 means are different from each other. - The students will demonstrate the ability to determine when a z test is not appropriate.
Progressions: Students are already familiar with the z distribution and hypothesis testing. Students will review this distribution by practicing multiple review questions involving the use of a z test. Students will also be able to recognize the limitations of the z test due to the population standard deviation needing to be known to use it or recognize that the z test is inappropriate for sample size under 30. Students will then be introduced to the t test which will be presented as a better alternative to the z test in the following lesson.

<table>
<tr><th>Lesson
Teacher and Student Actions</th><th>Mathematical Questions.</th></tr>
<tr><td>Launch (10-15 minutes)

Prior Knowledge:
- Students know the z distribution as well as hypothesis testing.
- Students know how to perform a hypothesis test using the z distribution to test whether or not a sample mean is significantly different than a hypothesized mean or to determine whether or not two means are different.

Launch:
- Teacher will begin class by presenting the lesson 1 guided notes
- The teacher will begin with a review of hypothesis testing along with the z test for testing a mean.
- Teacher will complete the z test review problem (example 1) with the students, reminding them of the steps along the way.
- Teacher will pose mathematical questions to ensure student understanding of the z test and hypothesis testing.</td><td>● What are the steps in hypothesis testing?

● What is the formula for the z test?

● How do we read a z score table?

● What is the difference between a 1 tailed and a 2 tailed test?</td></tr>
<tr><td>During (15-20 minutes)

- Teacher will assign example 2 for the students to try while the teacher walks around the room and monitors work.
- Teacher will ask guiding mathematical questions as needed to ensure student understanding.
- Teacher will review example 2 with class and ensure students understand the one sample z test.
- After going over example 2 with class, teacher will then discuss the two sample z test through the guided notes and explain when the two sample test is necessary.
- Teacher will go over example 3 with the class to ensure students are able to complete 2 sample z tests.
- Teacher will then explain how the z test is only useful when we know the population standard deviation or when we have sample size greater than 30.
- Teacher will explain the need for the t test when sample size is under 30.</td><td>● How do we read a z table to determine if our z test was significant?

● What is the difference between a one sample z and a two sample z?

● What are the disadvantages of the z test?

● Is it likely that we will know the population standard deviation of our samples?

● Will we always have samples over 30 participants?</td></tr>
<tr><td>After (5-10 minutes)

- Teacher will go over lesson summary to review with students lesson 1.</td><td>● What are problems with the z test that t tests solve?</td></tr>
</table>

- Teacher will assign homework and allow (if time) students to begin working on the assignment to monitor student progress.		
Assessment	The teacher will collect the students' homework to assess the students understanding of the notes and practice problems. The teacher will walk around the room and monitor the students' understanding as they work through homework and guided notes questions. The teacher will assess the students' understanding by their responses to the discussion questions.	

Lesson 1 Guided Notes – Review of the Z test and Cautions about it

In this chapter, you will learn:

- What is a t-test?
- How is the z-test and t-test different and alike?
- How to calculate the t statistic and use it to draw conclusions
- How to find the one sample and two sample t confidence intervals.
- How to apply knowledge of a t-test in a study to make decisions about an area of study that interests you!

Before beginning our new topic of t-tests, let us remind ourselves how to perform a z test as well as how to perform hypothesis testing. Our steps for hypothesis testing were as follows:

1. State the null and alternative hypotheses.
2. State the significance level appropriate for testing as well as the appropriate test to use.
3. Calculate the appropriate test statistic and the corresponding p-value.
4. Draw a conclusion based upon your results from step 3.

The z-statistic was a test statistic we learned to determine whether or not a mean value is different from a hypothesized value using the formula $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$ and then using our z table to determine whether or not the standardized z value was significant or not. Additionally, in order to use a z-test, it is important that our data is assumed to be normal and that n > 30. Let's review with a couple of examples, we will do one together and you will try one!

Example 1: A high school teacher is interested in testing whether his students perform better than the national average on a standardized test, the SAT. He collects data on 100 of his students who have taken the SAT. These 100 students have an average SAT score of 1056. It is known that the national average SAT score is 1030 and have a standard deviation of 51. Do this high school teacher's students perform better than the national average on the SAT? Use a z-test to support your claim.

Remember, we need to follow our hypothesis test steps whenever performing a statistical test.

1. **State the null and alternative hypotheses**: Our null is always a statement of no difference or no change. Since this teacher is interested in determining whether or not his students perform *better* than the national average, this question requires a one-way test and the null and alternative hypothesis will be $H_0: \mu = 1030$ and our alternative will be: $H_a: \mu > 1030$.

2. **State the significance level appropriate for testing and the appropriate test:** Since there is no significance level stated, 95% significance level would be appropriate. We will use the z test since we know the population standard deviation and n > 30.

3. **Calculate the appropriate test statistic and the corresponding p-value:** Since we are interested in testing whether or not a mean is different from a hypothesized mean, we know the population standard deviation and we have n > 30, a z test is appropriate. $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} =$

$\frac{1046-1030}{51/\sqrt{100}} \approx 3.14$. Now, we look at our z-table to find the p-value. Remember, our z table gives us the probability to the left of the curve, since this is a right tailed test, we need to find our z-score, find the corresponding p-value and then calculate 1 – that value. Looking at our z score table for 3.13, we see the corresponding p-value is .9913 as shown below, 1 - .9913 = .0087.

4. **State a conclusion based upon your results from step 3.** Based upon our z-test with p = .0087, we can conclude that this teachers' students SAT scores were significantly better than the national average.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Now, you try!

Example 2: The United States Census Bureau collects data on citizens living in the United States. In a recent study, they found the average number of pets in a US home to be 2.4 with a standard deviation of .9. You are interested in determining whether your town is different than the US national average. You interview 150 people from your town and find the average number of pets to be 2.2. Is this significantly different than the national average? Be sure to list the 4 hypothesis test steps.

We also learned about the 2 sample z test. This test is used when we want to compare the means of two different groups and test if they are significantly different. The two sample test performs very much the same as the one sample test, the only difference is the formula and the fact that we are testing whether or not 2 means are different from each other rather than 1 mean being different than a hypothesized mean. The formula for the two sample z is $z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{\sqrt{n_1}} + \frac{\sigma_2^2}{\sqrt{n_2}}}}$. Let's try an example of a two sample z!

Example 3: A professional sports team is interested in determining what pricing structure in their concessions will drive more sales. One day during a sold out game they tested a pricing structure of having all prices end in .99, on this day, 130 fans spent an average of $11.73 on food with a known standard deviation of 3.24. On another day during a sold out game, the team tested a pricing structure of having even prices, such as $4.00, but advertising them as 20% off regular prices. On this day, 192 fans spent an average of $13.20 on food with a known standard deviation of 3.60. Is there a significant difference in sales between pricing structures? Test at the 95% confidence level.

1. **State the null and alternative hypotheses:** Our null hypothesis is always a statement of no difference or no change, so, $H_0: \mu_1 = \mu_2$. In this case, we are interested in testing whether or not the 2 means are different, so this will be a two tailed test with an alternative of $H_a: \mu_1 \neq \mu_2$.

2. . **State the significance level appropriate for testing and the appropriate test:** This question specifically tells us to test at the 95% confidence level, so that is what we will use. We will use the z statistic because n > 30.

3. Calculate the test statistic and corresponding p-value: Since we are interested in testing whether or not two means are different than each other and in this case we know the standard deviation of both populations, a two sample z test would be appropriate. Since this is a 2-tailed test, if our z score is negative, our p-value will be 2 times the table value, if our z score is positive, our p-value will be 2 times 1 minus the table value.

$$z = \frac{11.73 - 13.20}{\sqrt{\frac{3.24^2}{130} + \frac{3.60^2}{192}}} = -3.82$$

Using our z table, we see that this corresponds to a p value of 2(.00007) = .00014

4. Draw a conclusion based upon step 3. Based upon our z test with $p < .05$, we can conclude that there is a significant difference between the different pricing structures in terms of how much money people spend on average.

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

Overall, the z-test determins if a mean is different than a specified mean or if the two sample means are different. If a z-test is so good, why do we need another test? The z-test is a very good test in 2 specific scenarios, one where we know the population standard deviation and one where we have a large (30 or more) sample size. However, we will not always meet these two assumptions. First, if we knew the population standard deviation then we would likely know everything else about the population in question, it is not often that the population mean is actually known, however, when our sample size gets large, we assume the population standard deviation and the sample standard deviation are one in the same and thus can still use the z test. But, what if we only have 25 participants in a survey? In this case, we have real problems with the z test. Hence, the need for another test, the t test.

Lesson 1 Summary

- The z test is a good method to use to test whether or not a sample mean is different than a hypothesized mean or to test whether or not two means are different from each other when we know the population standard deviation or when the sample size is greater than 30.
- When we perform the z test, we need to follow the 4 steps of hypothesis testing, stating the null and alternative hypotheses, stating the appropriate test to use and the significance level, calculate the test statistic and p value, draw conclusions based upon the test statistic.
- Despite the z test being a strong statistical test, if $n < 30$, the z test is no longer appropriate. If $n > 30$ but we do not know the population standard deviation we can still use the z test with the assumption that the sample standard deviation is the population standard deviation.

Lesson 1 Homework

1. A researcher wants to determine whether a specific study method helps students perform better than the national average on the ACT. He has 22 students use his study method and then records their ACT scores. He thinks that the best way to test whether or not his students do better than the national average is to use a z-test. In this scenario, is a z test appropriate? Why or why not?

2. A car wash is interested in reducing the number of services they offer to streamline operations, they currently offer a soft touch car wash and a touch free car wash. They currently believe that they want to keep the touch free operations because they think people spend more on average per car wash with the touch free versus the soft touch despite both washes being the same price points. On a particular day, the soft touch car washes did 132 washes with an average spend of $9.34 per customer, it is known that the standard deviation is $3.52. The touch free car wash did 146 washes with an average spend of $10.19 per customer, it is known that the standard deviation is $3.96. Is the car wash company's belief that people spend more on average with the touch free car wash correct?

Lesson 2 Lesson Plan

Mathematical Task: Using the t test to determine whether or not there are significant differences between a sample mean and a hypothesized mean or whether there are significant difference between 2 means.
Grade Level, Class: Senior level statistics course
Materials and Sources: Guided Notes Homework sheet
Next Generation Standards **AI-S.ID.2** - Use statistics appropriate to the shape of the data distribution to compare center and spread. **AII-S.IC.3** - Recognize the purposes of and differences among surveys, experiments and observational studies. Explain how randomization relates to each. **AII-S.IC.6a** - Use the tools of statistics to draw conclusions from numerical summaries.
Performance Objectives: - The students will demonstrate the ability to determine the disadvantages of using a z test and why a t test will be appropriate when $n < 30$. - The students will demonstrate the ability to use the four steps of hypothesis testing with a t test. - The students will demonstrate the ability to use a t test and z test to determine whether or not the mean of a sample is different than a hypothesized mean and when each is appropriate. - The students will demonstrate the ability to read a t table to draw conclusions from a t test.
Progressions: Students are already familiar with the z distribution and hypothesis testing. Students will be able to recognize the limitations of the z test due to the population standard deviation needing to be known to use it. Students will then be introduced to the t test which will be presented as an alternative to the z test for when our sample size is under 30. Students will practice with teacher help multiple examples to understand how to perform a t test and draw conclusions from that test, including both one and two sample tests.

Lesson **Teacher and Student Actions**	**Mathematical Questions.**
Launch (10-15 minutes) **Prior Knowledge:** - Students know the z distribution as well as hypothesis testing. - Students know how to perform a hypothesis test using the z distribution to test whether or not a sample mean is significantly different than a hypothesized mean and to determine whether or not two means are different. **Launch:** - Teacher will begin class by introducing the t test as an alternative to the z test when we have sample size under 30. - Teacher will explain that when sample size is above 30, even if we don't know the population standard deviation we will still use the z test because as n gets large, the t test approaches the z test. - The teacher will explain the similarities and differences between the t test and the z test and their formulas.	● What are the steps in hypothesis testing? ● What is the formula for the z test? ● How are the z test and t test similar? How are they different? ● Why should we use a t test instead of a z test? Vice versa?
During (15-20 minutes) - Teacher will begin going over example 1 with students ensuring to ask mathematical questions during example 1 to ensure student understanding. - Teacher will explain the differences between reading the z score table and the t score table and how the t score table can be used to determine whether or not their t test results are significant. - Teacher will assign students example 2 to work thorough and walk around the room and guide students who need help completing the t test or reading the t distribution table. - Teacher will then explain the two sample t test and go over example 3 together with students - Teacher will assign example 4 for students to complete and walk around and guide students as needed.	● What are the disadvantages of the z test? ● Is it likely that we will know the population standard deviation of our samples? ● What is similar between the z and t statistic formulas? ● How is the t table different than the z table? ● How can I tell whether or not my t statistic is significant? ● What conditions do we need to assume to use a two sample t test?
After (5-10 minutes) - The teacher will summarize the lesson, going over the lesson summary with students.	● What are the four steps of hypothesis testing for the t test?

<table>
<tr>
<td colspan="2">

- The teacher will emphasize that the t test takes away the assumption of knowing the population standard deviation that the z test had and is useful when n < 30.
- The teacher will assign lesson 2 homework and allow students to start if time allows.

</td>
<td>

- How do I determine what my critical value is for the t test?
- How can I tell if my t test is one tailed or two tailed?

</td>
</tr>
<tr>
<td>Assessment</td>
<td colspan="2">
The teacher will collect the students' homework to assess the students understanding of the notes and practice problems.

The teacher will walk around the room and monitor the students' understanding as they work through homework and notes questions.

The teacher will assess the students' understanding by their responses to the discussion questions.
</td>
</tr>
</table>

Lesson 2 Guided Notes

As we discussed at the end of lesson 1, there are problems with the z test. The z test either assumes that the population standard deviation is known or when n > 30, we assume that the sample standard deviation is the population standard deviation per the central limit theorem. But, often, we will not have samples of over 30 individuals. In these cases, the z test is not reliable and thus, we need another test, the t test.

The t test is similar to the z test in that it can test whether or not a mean is different than a specified mean and can also help to determine if two sample means are different from each other. The way we use the t table is slightly different than how we use the z table, but the general idea is the same. The t statistic for one sample is calculated with the formula $t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$, note that this looks *extremely* similar to the z score formula with the change being that instead of using σ, we use s and use degrees of freedom n – 1 for determining whether or not our t test is significant or not. For n < 30 the t distribution is flatter than the z distribution but with more probability in the tails. However, as n gets large, the t distribution approaches the z distribution and thus when n > 30, we use the z distribution even if the population standard deviation is not known. When we conduct a t test, we still use our steps of conducting hypothesis testing and follow the same 4 steps, the only change is the statistic we calculate and the table we use. Let's work through one together.

Example 1: Apple advertises that their iPhone battery will last for up to 20 hours of video streaming. A phone repair company feels that this estimate is too high. They randomly select 23 iPhones to test the battery life to determine if Apple's claims are accurate. Of these 23 phones, they stream a 30 hour loop of a movie to test the battery life and find the mean battery life to be

18.2 hours with a standard deviation of 2.9. Is Apple's claim accurate? Use the 4 steps of hypothesis testing to test the claim.

1. State the null and alternative hypotheses: For this, the phone repair company is interested in testing whether or not Apple's claim of battery life is too high, thus we will have a one tailed test. Remember, the null hypothesis is always a statement of no change.

Thus, $H_0: \mu = 20, H_a: \mu < 20$

2. State the significance level appropriate for testing and the appropriate test: For this, we will use the t test since n < 30 and we will test at the 95% level because no significance level is specified.

3. Calculate the appropriate test statistic and corresponding p-value:

$$t = \frac{18.2 - 20}{2.9/\sqrt{22}} = -2.91$$

Now, to check whether or not our test statistic is significant, we need to use our t table. First, in our t-table, we look whether or not we are doing a one tailed test or a two tailed test and then find the appropriate significance level. This test is 95% ($\alpha = .05$), so we find one tail and go to the .05 column. We then find the appropriate degrees of freedom, in this case 21, and find where our .05 column and degrees of freedom row meet as shown below. This cell shows us our t critical value. If our the absolute value of the t value we calculated is larger than this critical value, then our test is significant and p < α. In this case, the t critical value is 1.721 and $|t| = 2.91$.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

4. Draw conclusions based upon step 3: Based upon our t test, with $t = -2.91$ and $p < .05$, we can conclude that Apple's claims of battery life are too high, the batteries of their iPhones do not last 20 hours.

Now, you try!

Example 2: General Motors advertises a truck to get 27 miles per gallon of fuel mileage on the highway. A auto repair shop wants to test this claim and determine whether or not the truck gets different gas mileage than advertised. The auto repair shop conducts a sample of 16 trucks and finds their average fuel mileage to be 26.2 miles with a standard deviation of 4.5. Is there enough evidence to suggest advertisement is incorrect?

In addition to being able to test whether or not a mean is different than a specified mean, much like the two sample z test, we can conduct a two sample t test to determine if two means are different. The two sample t test uses the formula $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ with $n_1 + n_2 - 2$ degrees of freedom. One key assumption in this calculation is that the two populations of study do NOT have the same variance. Let's try an example and then you will try one on your own!

Example 3: A biologist is interested in determining what type of fertilizer helps plants grow better. The biologist suspects that fertilizer A helps plants grow better than fertilizer B. 26 plants were split into 2 separate groups of 13 and grown under the exact same conditions with the exception of the fertilizer they received. The group with fertilizer A had a mean plant height of 26cm with a standard deviation of 4.5, fertilizer B had a mean plant height of 24.2cm with a standard deviation of 3.7. Is there evidence to suggest that fertilizer A is better than fertilizer B?

1. State the null and alternative hypotheses: Since the researcher is interested in testing whether or not fertilizer A is better than fertilizer B, a one tailed test would be appropriate. The null hypothesis is always a statement of no difference or no change.

$$H_0: \mu_{Fertilizer\ A} = \mu_{Fertilizer\ B}, H_a: \mu_{Fertilizer\ A} > \mu_{Fertilizer\ B}$$

2. State the significance level and the appropriate test: In this example, since no significance level is specified, 95% would be appropriate. Since we have n < 30, the t test would be appropriate.

3. Calculate the appropriate statistic and p-value:

$t = \frac{26-24.2}{\sqrt{\frac{4.5^2}{13}+\frac{3.7^2}{13}}} = 1.114$, Since this is a one tailed test with degrees of freedom = 24, we will look at the 95% one tailed column and 24 for df in our t table, doing so yields a critical value of 2.064, since 1.114 < 2.064, p > .05.

4. Draw conclusions based upon step 3:

Since t = 1.114 and p > .05, we can conclude that there is not enough evidence to suggest that fertilizer A is better than fertilizer B.

Now, you try!

Example 4: An education researcher is interested in determining if a particular study method can help students perform better on tests. A group of 20 similar performing students were randomly divided into 2 groups, 10 that don't receive the study method, and 10 who do. All 20 students were then given the same test and their test results were recorded. The group receiving the study method scored an average of 86.24 on the test with a standard deviation of 6.9. The group that didn't receive the study method scored an average of 82.45 with a standard deviation on 8.4. Is there sufficient evidence to suggest that the study method was effective in raising the students scores?

Lesson 2 Summary

- The t test is a good alternative to the z test when we have n < 30. When we have n > 30, the t distribution approaches the z distribution so we will just use z when we have n > 30.
- The t test uses the same formula as the z test with the exception of using s for standard deviation rather than σ.
- Unlike the z test where we can find a direct p value, for the t test, we find a critical t value and then can determine whether or not our t value we calculated is significant or not.

Lesson 2 Homework

1. A bus company is interested in expanding its service to a new neighborhood. However, it will only be profitable for them if the average number of trips a resident in the neighborhood takes is over 4 trips per week. The bus company sends out a survey to 25 random residents within the neighborhood asking them how many times per week they would use the new service. The survey determined that the residents would use the bus service on average 4.4 times per week with a standard deviation of 1.2. Is there enough evidence to suggest that the bus company should expand their service? Be sure to include the 4 hypothesis testing steps and explain what type of test you should use and why.

2. 2 different car manufacturers, Chevy and Ford are advertising that their car gets better gas mileage than the other. A car dealership has 10 of each car and decides to test which car gets better gas mileage. The results for Chevy are an average gas mileage of 29.3 with a standard deviation of 4.7, Ford has an average gas mileage of 28.1 with a standard deviation of 3.2. The car dealership then decides that the Chevy car has better gas mileage than the Ford car. Are they correct?

Lesson 3 Lesson Plan

Mathematical Task: Reviewing the uses of the t test and explaining what a t confidence interval can determine.

Grade Level, Class:
Senior level statistics course

Materials and Sources:
Guided Notes
Homework sheet

Next Generation Standards

AI-S.ID.2
- Use statistics appropriate to the shape of the data distribution to compare center and spread.

AII-S.IC.3
- Recognize the purposes of and differences among surveys, experiments and observational studies. Explain how randomization relates to each.

AII-S.IC.6a
- Use the tools of statistics to draw conclusions from numerical summaries.

Performance Objectives:
- The students will demonstrate the ability to use the t confidence interval formula.
- The students will demonstrate the ability to determine a confidence interval for a mean and confidence intervals for the difference of two means.
- The students will demonstrate the ability to draw conclusions based upon confidence intervals.
- The students will demonstrate the ability to determine when a z test is not appropriate.

Progressions:
Students are already familiar with the t distribution and hypothesis testing using the t test. In this lesson, students will expand upon this knowledge to calculate t confidence intervals. Students additionally will explore the idea that while t tests can tell us whether or not differences are significant, t confidence intervals can tell us how much the differences are.

<table>
<tr><th>Lesson
Teacher and Student Actions</th><th>Mathematical Questions.</th></tr>
<tr><td>Launch (10-15 minutes)

Prior Knowledge:
- Students know the t distribution as well as hypothesis testing with the t test.
- Students know how to perform a hypothesis test using the t distribution to test whether or not a sample mean is significantly different than a hypothesized mean or to determine whether or not two means are different when n < 30.

Launch:
- Teacher will begin class by presenting the lesson 3 guided notes
- The teacher will begin with a review of using t tests for testing whether or not a mean is significantly different than a hypothesized mean or whether or not two means are different.</td><td>● What is the t test used for?

● What can the t test help to determine?

● Can the t test provide us with an estimate for a mean or an estimate for the differences between two means?</td></tr>
<tr><td>During (15-20 minutes)

- Teacher will introduce the idea of the one sample t confidence interval as a way to determine what a population’s mean is based upon the sample.
- Teacher will explain that while the t test is a useful test to determine whether or not a mean is different than a specified mean, it does not tell us an estimate for what that mean is.
- Teacher will go over example 1 with the class and ask mathematical questions to ensure student understanding.
- Teacher will assign example 2 for the students to work on while the teacher walks around and checks student work.
- Teacher will then explain the need for both the t confidence interval and the t test, while both are useful, both don’t provide all of the information that could be useful.
- Teacher will explain the two sample t confidence interval and go over example 3 to show how it can be used to determine the difference between two means.
- Teacher will assign example 4 for the students to work on and walk around and check student work.</td><td>● What are the benefits of completing a t confidence interval?

● What information does the t confidence interval provide that the t test does not?

● What information does the t test provide that the t confidence interval does not?</td></tr>
</table>

After (5-10 minutes) - Teacher will go over lesson summary to review with students lesson 3. - Teacher will assign homework and allow (if time) students to begin working on the assignment to monitor student progress.		● Why are both confidence intervals and t tests necessary?
Assessment	The teacher will collect the students' homework to assess the students understanding of the notes and practice problems. The teacher will walk around the room and monitor the students' understanding as they work through homework and guided notes questions. The teacher will assess the students' understanding by their responses to the discussion questions.	

Lesson 3 Guided Notes

In the last two lessons, we have focused on reviewing z tests and introducing the t test as an alternative to the z test when we have a smaller sample size. While the t tests are valuable resources to determine whether or not a mean is different than a specified mean or whether or not two means are different, they do not provide an estimate of what the population mean actually is or what the difference of the population means are. This is where the one sample t confidence interval and the two sample t confidence interval can be used. Suppose that we have one sample with n < 30 that we are interested in estimating the population mean for, we can use the formula $\bar{x} \pm t^* \frac{s}{\sqrt{n}}, with\ t^*$ being the two tailed critical value for t with a specified confidence level and n – 1 degrees of freedom. This formula provides us with a confidence interval for μ. Let's try using this formula together!

Example 1: In lesson 2 example 1, we determined that Apple's claims that their iPhone battery lasted 20 hours was too high. We now want to calculate an estimate for what the mean battery life is. The phone repair shop found the mean battery life to 18.2 hours with a standard deviation of 2.9 in a sample of 23 phones. Using the one sample t confidence interval formula, determine a 95% confidence interval for the iPhone's battery life.

Since we have df = 22 and want to calculate at the 95% confidence level, we will look at the corresponding portion of our t table to see that $t^* = 2.074$ as shown below.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

We know use our one sample t confidence interval formula and draw a conclusion.

$18.2 \pm 2.074\frac{2.9}{\sqrt{23}} = 16.95 \leq \mu \leq 19.45$. Thus, we can say with 95% confidence that the mean battery life for an iPhone is between 16.95 and 19.45.

Now, you try!

Example 2: You see an advertisement on TV that mentions that high school students on average get 7.2 hours of sleep. You are interested in testing this claim to determine whether students at your school are close to this claim. You conduct a sample of 22 students and find their average sleep time to be 6.4 hours with a standard deviation of 1.1. Complete a 95% confidence interval for the mean sleep time of high school students.

Notice, this confidence interval cannot tell us whether or not high school students have a different mean amount of sleep than 7.2, while it can provide us a confidence interval, a t test would be necessary to determine whether or not the mean is significantly different than 7.2.

In addition to helping us find a interval for the population with a one sample t confidence interval, we can use a two sample t confidence interval to determine a confidence interval for the difference of two means. Suppose we take two samples and are interested in determining the differences in their population means, we can use the two sample t confidence interval with equation $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ where t^* is the t critical value being the two tailed critical value for t with a specified confidence level and the degrees of freedom of the smaller sample. Let's try an example!

Example 3: An education researcher is interested in determining whether chewing gum while studying and then during the test can improve test scores. He chooses a group of 20 students randomly, 10 will be assigned to the gum chewing group, 10 will be assigned to the non gum chewing group and both groups are given an assessment. The gum chewing group scored a mean of 83.4 with a standard deviation of 13.2 while the non gum chewing group scored a mean of 81.2 with a standard deviation of 9.3. Determine a 95% confidence interval for the mean difference between the two groups.

Since both groups are the same size, we can use df = 9. Using our t table we can see the critical t value is 2.262. We now can calculate the confidence interval for the difference in population means between the two groups using the two sample t confidence interval.

$$(83.4 - 81.2) \pm 2.262\sqrt{\frac{13.2^2}{10} + \frac{9.3^2}{10}} = -9.35 \leq \mu_1 - \mu_2 \leq 13.75$$

Thus, we can say with 95% confidence that the gum chewing group performed between 9.35 points worse and 13.75 points better than the non gum chewing group.

Examples like this can be tricky since there is such a wide range of differences possible, this is where a two sample t test would be necessary to determine if there is a difference between the two groups. Now, you try an example!

Example 4: A biologist is comparing two different species of frogs and wants to determine whether one species weighs more than another. The biologist collects 13 frogs of species 1 and 8 of species 2. Species 1 has a mean weight of 56 grams with a standard deviation of 4.8. Species 2 has a mean weight of 61 grams with a standard deviation of 5.3. Construct a 95% confidence interval for the difference in mean weight between species 1 and species 2.

Lesson 3 Summary

- While the t test is a useful test for determining whether or not a mean is different than a specified mean or whether two means are different, it does not provide us with an estimate of these differences.
- The one sample and two sample t confidence intervals can be used to help to estimate a mean and to estimate the differences between two means.

Lesson 3 Homework

1. You are interested in determining whether or not the gas prices near you are the same as the national average. You use a website to look at the prices of gas near you and select at random 15 gas stations and find the average price to be $3.43 with a standard deviation of $.07. Construct a 95% confidence interval for the average price of gas.

2. Walt Disney World is interested in determining whether a promotion helps to increase the amount of money guests spend. 20 transactions in a gift shop were chosen at random, 10 while the promotion wasn't active and 10 while the promotion was active. When the promotion wasn't active, guests spent on average $84.63 with a standard deviation of $11.50. When the promotion was active, guests spent an average of $102.98 with a standard deviation of $13.20. Construct a 95% confidence interval for the difference between means of the two groups.

Lesson 4 Lesson Plan

Mathematical Task: Complete a project of student's choosing that focuses on using t tests and t confidence intervals for data analysis

Grade Level, Class:
Senior level statistics course

Materials and Sources:
Project Worksheet
Google Sheets/Excel

Next Generation Standards

AI-S.ID.2
- Use statistics appropriate to the shape of the data distribution to compare center and spread.

AII-S.IC.3
- Recognize the purposes of and differences among surveys, experiments and observational studies. Explain how randomization relates to each.

AII-S.IC.6a
- Use the tools of statistics to draw conclusions from numerical summaries.

Performance Objectives:
- The students will demonstrate the ability to use both the one sample t test and two sample t test to determine whether means are significantly different.
- The students will demonstrate the ability to use the t confidence interval formula.
- The students will demonstrate the ability to determine a confidence interval for a mean and confidence intervals for the difference of two means.
- The students will demonstrate the ability to draw conclusions based upon confidence intervals.

Progressions:
Students are already familiar with the t distribution and hypothesis testing using the t test as well as t confidence intervals. In this lesson, students will expand upon this knowledge and choose real world data sets to perform data analysis on.

Lesson **Teacher and Student Actions**	**Mathematical Questions.**
Launch (10-15 minutes) **Prior Knowledge:** - Students know the t distribution as well as hypothesis testing with the t test. - Students know how to perform a hypothesis test using the t distribution to test whether or not a sample mean is significantly different than a hypothesized mean or to determine whether or not two means are different when n < 30. - Students know how to complete t confidence intervals **Launch:** - Teacher will begin class by presenting the lesson 4 project.	• What is the t test used for? • What can the t test help to determine? • Can the t test provide us with an estimate for a mean or an estimate for the differences between two means? • What kind of data are you interested in studying? Why? • Is the data you want to choose appropriate for a t test (is it quantitative)? • Are we conducting a 1 sample test or 2 sample test?
During (15-20 minutes) - Teacher will guide student discussion and activity by asking guiding mathematical questions. - Teacher will also walk around the room and help students choose appropriate data sets for their project. - Teacher will walk around the room and assist students in using random number generator for choosing a sampling of a data set.	• What are the benefits of completing a t confidence interval? • What information does the t confidence interval provide that the t test does not? • What information does the t test provide that the t confidence interval does not?

After (5-10 minutes) - Students will continue to work on lesson 4 project.		• Why are both confidence intervals and t tests necessary? • Why do we divide our data into groups of 20 in this project? Why can't we use groups larger than 30?
Assessment	The teacher will collect the students' projects and grade based upon the attached rubric. The teacher will assess the students' understanding by their responses to the discussion questions.	

Lesson 4 Project

Over the past three lessons, we have seen the importance of using a t test as well as constructing t confidence intervals. You now will complete a project with data of your choosing to practice our skills using t tests and t confidence intervals!

Project Requirements

The Data In this project, you can pick from a plethora of available data sources to find a dataset that you are interested in analyzing. Available data can be selected from any dataset in the list of websites below:

- https://www.kaggle.com/datasets - Provides a wide range of datasets from car sales data to data about cancer.
- https://data.gov/ - Provides data focused on agriculture, energy, local governments and health.
- https://data.worldbank.org/ - Provides data focused on economic issues.
- https://data.world/datasets/sports - Provides data focused on sports.

With this data, you will want to ensure that the data is available in .csv format so it can be opened in Excel or Google Sheets. In this project we want to focus on t testing, many of these data sets will have sample sizes above 30. In the case of this, you will check how many pieces of data are in the data set and pick 20 pieces of data at random using this random number generator, putting the minimum number at 1 and the maximum at how many pieces of data you have. You will then use these 20 pieces of data to complete your data analysis. If your data is in different sets of years and you want to perform 2 sample tests, break the years up into 2 separate groups, then select 10 of each set of years using the random number generator. If you have to use the random number generator, write down the 20 data points you used in your data analysis section of your project.

Introduction In this section, in 3-4 sentences you should explain your choice of data set and what you are hoping to learn from the topic you selected.

Hypotheses In this section, you will be stating hypotheses that you will test using a t test in your data set. The hypotheses should be written in words along with mathematical symbols. These hypotheses should be what you think may be true. For example, if I was testing a study method that I think improves test scores, my hypothesis would be that the test scores of the study method are higher than no study method, along with the appropriate mathematical symbols. You can look up information for these – especially for data such as national averages. For example, if I looked at car sales data and wanted to know if the sales at a particular dealership were higher than the national average, I need to know the national average – feel free to look these up. *These hypotheses should be stated before doing any data analysis as to not skew your thinking.*

Data Analysis In this section of your paper, you will include all work along with results (not conclusions) of your data analysis. There is no requirement for the different types of t tests, you can use all 2 sample tests, 1 sample tests, or a mixture of both. *All tests should be conducted at the 95% confidence level.*

Conclusions This section should include conclusions of all stated hypotheses with mathematical proof and explanation from the data analysis completed.

Rubric

	3 – Meets Expectations	2 – Approaching Expectations	1 – Fails to Meet Expectations
Introduction	At least 3-4 sentences are used to explain choice of data set and interest in the data	Fewer than 3 sentences are used to explain choice of data set and interest in the data	Introduction does not explain choice of data set and interest in the data.
Hypotheses	Hypotheses are stated using both words and mathematical notation	Hypotheses are stated but either words or mathematical notation are not used	Hypotheses aren't stated.
Data Analysis	The t test and t confidence intervals are both calculated for each hypothesis and are mathematically correct	The t test and t confidence intervals are both calculated for each hypothesis but there are some mathematical errors.	Incorrect tests are used or the t test and t confidence intervals contain many errors.
Conclusions	Sound conclusions are drawn using data analysis.	Conclusions are drawn but one is incorrect.	Conclusions are drawn but multiple are incorrect.

Sample Project

Below is an example of a completed project to help guide students and help them understand the necessary requirements for meeting expectations for this assignment.

Introduction

I chose to do my project on water quality issues and pollution. Living on a river, I know first hand the importance of having fresh, clean water and the dangers of polluting that source. I found a data set that collected different measurements of the water quality in a particular area and is available at the link https://catalog.data.gov/dataset/water-quality-data-a3c08 .

Hypotheses

For this section, I am interested in determining if the water tested is safe for life. Three measurements they took were salinity (the dissolved salt content of the water in parts per thousand), dissolved oxygen in mg/L and the pH of the water. For each of these, I researched what is considered safe levels and will conduct hypothesis testing to determine if the tested water is safe. The safe salinity level is considered to be around 35 ppt so I will test whether or not our water samples are above 35 ppt, I will test if our sample is below 3mg/L of dissolved oxygen and will test whether or not the water's pH is different than 7.75 (a safe range for pH in fresh water is considered to be between 6.5 and 9). For each, I have listed the hypothesis test we will be conducting below.

Salinity Level

$H_0: \mu = 35, H_a: \mu > 35$

Dissolved Oxygen

$H_0: \mu = 3, H_a: \mu < 3$

pH

$H_0: \mu = 7.75, H_a: \mu \neq 7.75$

Data Analysis:

In my data set, there were 795 samples of the same water source (Bay) done, I used the random number generator to select the following pieces of data: 598, 169, 605, 287, 241, 5, 480, 309, 652, 628, 321, 107, 701, 126, 171, 743, 535, 99, 270, and 356.

Saline

For this data for the saline ppt, we get a result of $\bar{x} = 1.32\ and\ s = 1.44$

Our hypothesis was $H_0: \mu = 35, H_a: \mu > 35$, and calculate the t value of $t = \frac{1.32-35}{1.44/\sqrt{20}} = -104$, for this test, our critical t value is 1.729 thus p > .05. For our confidence interval, the critical t value is 2.093, thus our confidence interval for the salinity is $1.32 \pm 2.093\frac{1.44}{\sqrt{20}} = (.65, 1.99)$

Oxygen

For this data for the oxygen, we get a result of $\bar{x} = 7.65\ and\ s = 2.73$

Our hypothesis was $H_0: \mu = 3, H_a: \mu < 3$ and calculate the t value $t = \frac{7.65-3}{2.73/\sqrt{20}} = 7.62,$ in this test, our critical t value is -1.729, since it is a one tail test to the left, thus p > .05. For our confidence interval, the critical t value is 2.093, thus our confidence interval for the dissolved oxygen is $7.65 \pm 2.093\frac{2.73}{\sqrt{20}} = (6.37, 8.93)$

pH

For this data for the pH, we get a result of $\bar{x} = 7.51\ and\ s = .62$

Our hypothesis was $H_0: \mu = 7.75, H_a: \mu \neq 7.75,$ leading to the t value $t = \frac{7.51-7.75}{.62/\sqrt{20}} = -1.73,$ for this t test, our t critical values are -1.729 and 1.729, thus p < .05. For our confidence interval, the t critical value is 2.093, our confidence interval will be $7.51 \pm 2.093\frac{.62}{\sqrt{20}} = (7.23, 7.80).$

Conclusions

Based upon our t test with t = -104, p > .05 and confidence interval of $(.65, 1.99)$, we can conclude that there is not enough evidence to suggest that the salinity of the water is above 35 parts per thousand. Based upon our t test with t = 7.62, p > .05 and confidence interval of (6.37, 8.93) we can conclude that there is not enough evidence to suggest that the dissolved oxygen in the water is below 3mg/L. Based upon our t test with t = -1.73, p < .05 and confidence interval of (7.23, 7.80) there is enough evidence to suggest that the pH of the water is not 7.75. The goal of our study was to determine if the water was safe for wildlife based on accepted standards. Overall, the results with the exception of pH suggest that the water is safe. However, even though the pH data was significantly different than 7.75, the confidence interval is (7.23, 7.80), the accepted range for pH levels are between 6.5 and 9, since our confidence interval is inside this range, it is likely that the pH levels are acceptable for wildlife.

Classroom Implementation

Unfortunately, due to a change in teaching circumstances and the classes I am now teaching, I was unable to implement this specific curriculum into the classroom. Despite this fact, I have conferenced with other teachers that have taught statistics in the past and one of these teachers was able to implement the project into their teaching. This teacher enjoyed using the project and stated that since students were able to choose their own topic to study, they were more engaged and seemed to care more about the work that they were doing since it was topics that they found important. One issue that this teacher experienced was with the data sources provided in the project. Many of the data sources provide large datasets of unedited data and in a few cases, the teacher had to pare the data down to make it usable for the students. It is their suggestion that when implementing this project that the teacher is careful as to what datasets the students are choosing and double check students chosen data source before they begin attempting to perform analysis using the data set.

Conclusion

This curriculum project provides opportunities for teachers to create authentic learning opportunities within their statistics classrooms. The lessons were designed to help teachers provide the necessary tools for students to complete data analysis and then allow them to take ownership of their work in performing statistical analysis on data sets that are important to them. It is the author’s hope that other teachers are able to use the materials provided within this project, implement them in their own classrooms and for students to make meaningful connections between the tools of statistical analysis and the real-world data they select.

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Appendix

Lesson 1 Example 2

1. $H_0: \mu = 2.4, H_a: \mu \neq 2.4$

2. 95% significance level is appropriate

3. $z = \frac{2.2-2.4}{.9/\sqrt{150}} = -2.72$ p = .00326 * 2 = .00652

4. Since p < .05, we can conclude that the town has significantly different number of pets than the national average.

Lesson 1 HW

1. A z test would not be appropriate in this scenario because there are only 22 students in the sample.

2.

1. $H_0: \mu_{touch\ free} = \mu_{soft\ touch}, H_a: \mu_{touch\ free} > \mu_{soft\ touch}$

2. 95% confidence level would be appropriate

3. $z = \frac{10.19-9.34}{\sqrt{\frac{3.96^2}{146}+\frac{3.52^2}{132}}} = -1.89 - p = (1 - .97062) * 2 = .05876$

4. Since p > .05, there is not enough evidence to suggest that customers of the touch free car wash spend more than the soft touch car wash.

Lesson 2 Example 2

1. $H_0: \mu = 27, H_a: \mu \neq 27.$

2. We will test at the 95% significance level and use a t test since n < 30.

3. $t = \frac{26.2-27}{4.5/\sqrt{16}} = -.711, t\ critical\ value\ of\ 2.131, thus, p >\ .05$

4. With t = -.711, p > .05, there is not enough evidence to suggest that the truck's gas mileage is different from 27.

Lesson 2 Example 4

1. $H_0: \mu_{study} = \mu_{no\ study}, H_a: \mu_{study} > \mu_{no\ study}$

2. We will test at the 95% level using a 2 sample t test.

3. $t = \frac{86.24-82.45}{\sqrt{\frac{6.9^2}{10}+\frac{8.4^2}{10}}} = 1.103, t\ critical\ value\ of 1.729, thus\ p >\ .05$

4. Since t = 1.103 and p > .05, there is not sufficient evidence to suggest that the study method had a significant impact on improving test scores.

Lesson 2 Homework

1.

1. $H_0: \mu = 4, H_a: \mu > 4$

2. We should use a t test since n < 30 and test at the 95% confidence level.

3. $t = \frac{4.4-4}{1.2/\sqrt{25}}$ =1.67, t critical value of 1.711, thus p > .05

4. Based upon our results with t = 1.711 and p > .05, there is not enough evidence to suggest that the bus company should expand their service.

2.

1. $H_0: \mu_{Chevy} = \mu_{Ford}, H_a: \mu_{Chevy} > \mu_{Ford}$

2. We will use a t test at the 95% confidence level since n < 30.

3. $t = \frac{29.3-28.1}{\sqrt{\frac{4.7^2}{10}+\frac{3.2^2}{10}}} = .668, t\ critical\ value\ with\ df = 18\ is\ t = 1.734, p >\ .05$

4. Based upon our t test with t = .668 and p > .05, there is not enough evidence to suggest that Chevy has better gas mileage than Ford.

Lesson 3 Example 2

Since we have a sample of 22 students, we will have 21 for our degrees of freedom. Thus, our critical t value is 2.080. Thus, our confidence interval will be $6.4 \pm 2.080\frac{1.1}{\sqrt{22}} = 5.91 \leq \mu \leq 6.89$. We can say with 95% confidence that the mean sleep time of high school students is between 5.91 and 6.89.

Lesson 3 Example 4

Since species 2 has a sample of 8 we will use df = 7. Our critical t value is 2.365, thus our two sample confidence interval will be $(56 - 61) \pm 2.365\sqrt{\frac{4.8^2}{13} + \frac{5.3^2}{8}} = -10.44 \leq \mu_1 - \mu_2 \leq .44$

Thus, we can say with 95% confidence that species 1 is between 10.44 grams lighter and .44 grams heavier than species 2.

Lesson 3 Homework

1.

We can use df = 14 for this confidence interval, thus, the t critical value is 2.145 and the confidence interval is $3.43 \pm 2.145\frac{.07}{\sqrt{15}} = 3.39 \leq \mu \leq 3.47$. Thus, we can say with 95% confidence that the average gas price is between $3.39 and $3.47

2.

Since both groups have n = 10, we can use df = 9. Using our t table we see the t critical value is 2.262, thus the two sample t confidence interval is

$$(84.63 - 102.98) \pm 2.262\sqrt{\frac{11.50^2}{10} + \frac{13.20^2}{10}} = -30.87 \leq \mu_1 - \mu_2 \leq -5.83$$

Thus, we can say with 95% confidence that when the promotion was active, guests spent between \$5.83 and \$30.87 more than when the promotion wasn't active