Table of Contents

Letter to Student ............................................................................................................................ p 2
Letter to Instructor .......................................................................................................................... p 4

Unit 1
Introduction to Unit 1 ....................................................................................................................... p 7
1.1: Simplifying Expressions and Solving Equations ................................................................... p 8
1.2: Evaluating and Calculating ...................................................................................................... p 17
1.3: Intercepts and Graphs ............................................................................................................. p 22
1.4: Functions and Function Notation ............................................................................................ p 40
1.5: Domain and Range of a Function ............................................................................................ p 51
1.6: Graphs/Tables of Linear, Quadratic and Exponential Functions ........................................ p 61
1.7: Introduction to Formulas of Linear, Exponential, and Quadratic Functions ....................... p 80
Unit 1 Outcome Overview ............................................................................................................. p 92
Unit 1 Practice Test ....................................................................................................................... p 93
Unit 1 Flashcards for Review ........................................................................................................ p 97

Unit 2
Introduction to unit 2 ...................................................................................................................... p 103
2.1: More in-depth Understanding of the Formulas of Linear, Exponential, Quadratic Functions ........................................................................................ p 104
2.2: Solving Graphically with the Graphing Calculator ................................................................. p 113
2.3: Polynomial Vocabulary, Expanding and Factoring, Zero-product Property ....................... p 124
2.4: Square Root Property and the Quadratic Formula ................................................................. p 139
Unit 2 Outcome Overview ............................................................................................................. p 151
Jeopardy Review Game ................................................................................................................ p 152
Unit 2 Practice Test ...................................................................................................................... p 154
Unit 2 Flashcards for Review ........................................................................................................ p 156

Unit 3
Introduction to the unit 3 ................................................................................................................. p 162
3.1a: Applications Basics ............................................................................................................ p 163
3.1b: Applications Continued ...................................................................................................... p 173
3.2a: Exponent Rules Basics ....................................................................................................... p 183
3.2b: Exponent Rules Advanced ................................................................................................ p 197
3.3a: Systems of Equations Basics ............................................................................................... p 205
3.3b: Systems of Equations Applications .................................................................................... p 212
3.4: Introduction to Rational Expressions ..................................................................................... p 218
3.5a: Linear Inequalities Symbolically ......................................................................................... p 228
3.5b: Linear Inequalities Numerical, Graphical, and Applications ........................................... p 231
Unit 3 Outcome Overview ............................................................................................................. p 238
Unit 3 Practice Test ...................................................................................................................... p 239

Solutions to Selected Exercises ................................................................................................... p 247
Letter to the Student

Welcome to the first module of this three credit course. This is a math course for college students that need a bit of algebra review before starting the college level math course. This course is meant to bring student’s skills up to college level in one semester.

Below are the answers to some common questions about this course. Please be sure to ask your instructor further questions if you have them. Your instructor is eager and willing to help you, but in college it is your job to seek out your instructor if you have questions or need assistance!

(For students taking this course as a series of three 1-credit courses) Why is this course broken into three 5-week modules instead of the usual 15-week course?
The content of this course is cumulative. If you understand the content of the first five weeks (if you pass the first module with a C or higher), then you will be able to use that knowledge in the next five weeks. Once you have successfully mastered the content of the first five weeks, you will find that the next five weeks are much easier to understand. If you do not master the content of the first five weeks (if you get a D or an F in the first five weeks), then it is extremely unlikely that you would be successful in the next five weeks. For that reason, students are required to first master the content of the first five weeks and then move ahead to the next five weeks of the course.

We want to help students be successful. With the old course format, we often found students that were unsuccessful in the first five weeks of the course, but then they were stuck in the course for another ten weeks of the semester! Those students ended up wasting ten weeks in confusion, and had to pay for three full credits without even understanding the first credit-worth of material. With this five-week format, the students will move ahead to the second module only when he/she has successfully mastered the content of the first module. This will help the student be successful in subsequent modules!

What is different about College math courses and High School math courses?
- In High School, students attend math class every day and in college students only have class three times a week (sometimes only one or two times a week).
- In High School, math class may cover one section of the book in one week or more. In college, class will generally cover one section or more each class period.
- Students often report that they did little or no homework at all in High School in order to pass their math class. But in college, successful students find that they need to spend SIX TO NINE hours each week working on math in order to pass the math course. MOST OF THE STUDENT’S LEARNING IN COLLEGE WILL TAKE PLACE OUTSIDE OF CLASS TIME. 2/3 of student learning time is out of class learning, and only 1/3 of student learning time is in class learning!
- In High School, extra credit and attendance, and more could help a student pass. In college, all that really matters is content mastery. If the student can pass the tests, then the student can generally pass the college math course. If the student fails the tests, then the student will fail the college math course.
- If a student misses a day of class in High School, then the student is generally allowed to make up any missed work or tests. In college, instructors treat student attendance like attendance at a job. If a person doesn’t show up to a job then he/she could expect to get fired…and if a student doesn’t show up to class in college when there is a test, then the student can expect to get a 0 on the test.
- Much of the day-to-day content in High School is focused on learning how to successfully pass state exams (like the NY State Regents Exam) instead of really focusing on understanding and applying mathematical topics (this is a complicated problem that even your High School teachers are having trouble finding a way out of). In college, the day-to-day class time is focused on learning the mathematics, the reason why the mathematics is true, and how to apply the mathematics in real-world situations...college instructors cringe when students ask “is this going to be on the test?”
What should I do day-to-day in order to be successful in this course?
You need to spend SIX TO NINE HOURS each week working on the course material outside of class time in order to be successful in this course. During that time you should spend most of your time practicing and taking quizzes on MyOpenMath.com, completing Study Plan problems on MyOpenMath.com, re-practicing problems from the book and from lecture, and working on written homework. Remember that you will learn math by doing math (not by simply reading math). Taking 12 credits of college courses means you should be spending 12 hours in class, and an additional 24 or more hours outside of class time on the course material! That’s 36 hours per week spent on college which is a full time job...that’s why 12 credits is considered “full load.” If you find yourself struggling with the content in this class then you need to seek help IMMEDIATELY. The course is only five weeks long, and college math courses move at an extremely fast pace. You should first seek help from your instructor or the free Math Center available on campus. If more help is needed you can seek help from a private tutor (some are also available free of charge on campus). This is a five week course. If you wait 1 week to seek help, then you have wasted 20% of the course time!

Why computer practice AND written practice?
In the past, before personal computers were common, math instructors asked their students to go home and practice the odds from the book and check their answers in the back to ensure accuracy. Unfortunately, if the student got stuck in the middle of the solving process, then there was no help available in the middle of the night at their house! Also, students sometimes neglected to check their work in the back of the book, and so the practice time was spent practicing all the problems incorrectly! The computer will help students by offering similar examples, telling the student when an answer is wrong, giving more chances to practice when a concept needs more practice, offering videos and textbook pages related to the content, and more. The computer practice is mostly “skill-and-drill.” The written practice allows students the opportunity to apply the knowledge, go into more depth with the content, and show work (which can get partial credit that the computer won’t give!). Written work will also give the instructor the opportunity to give you written feedback on your work which can be very helpful in the learning process.

You CAN do this!!
The instructors that have been teaching this course for many years all have the same opinion: students who do everything we recommend (come to class, do all the reading and homework, etc.) generally pass the course, and students who don’t do these things generally fail the course.
Which type of student are you going to be? It’s up to you!
Letter to the Instructor

This course, and this manual, were created with the intention of improving the pass rates in the Intermediate Algebra remedial-level course at Dutchess Community College (DCC) in Poughkeepsie, NY. Like everywhere else in the nation, many students that begin college here at DCC are lacking the math skills necessary to enter a college-level math course. And unfortunately, many of the students that place into a developmental math course never finish the course and therefore never finish a college degree. Across the country there are many wonderful initiatives taking place at 2-year colleges as well as 4-year colleges in an attempt to help improve student success in developmental math courses. After learning about many of those initiatives and engaging in years of discussion, the math faculty at DCC agreed that the following course changes would help improve our student’s chances at success:

1. We have changed the 3-credit fifteen week course into three sequential 1-credit courses (called modules) that are each five weeks in length. Before a student is allowed to move on to the next module, the student must earn a C or higher in the prerequisite module. Each five-week module is offered during each five week period of the semester (at the same time slot) so students are able to repeat a module or move ahead to the next module immediately.
   Why this helps students: This change will help the student because each student can now earn 1 or 2 credits during the semester even if he/she failed one of the modules during the semester. In the original fifteen week course, the students either earned 3 credits or 0 credits...there was no in-between. Now there will be a large proportion of students that will earn either 1 or 2 or 3 credits, and only a very small proportion of students will earn 0 credits.

2. The order of the content of the semester was adjusted to make the modules cumulative instead of disjoint topics. So, for example, we do NOT have five weeks of linear functions, and then five weeks of quadratic functions, and then five weeks of exponential functions. If we had done that, then the student’s chances of success in each module would be largely independent of whether or not the student passed the previous module (understanding quadratic functions in one unit doesn’t really help the student understand exponential functions in the next unit). So, instead, the course is broken up in a way that introduces the basic elements of all three of these types of functions in the first module. Then in the second module the students will get into the more “difficult aspects” of these functions. Then in the last module students will begin to specialize and gain more detailed preparation for his/her college-level math course. This cumulative approach to the topics also helps students understand the material in a deeper way by having students learn about three important function types while at the same time comparing these functions to one another.
   Why this helps students: Content-mastery in one module will help the student’s success in the next module. The first module is an introduction to the concepts. The second module builds on that by gaining more details about each function type. The last module helps students get some specialized preparation for the college level course he/she will take next. One additional benefit of this content-order is that students are less likely to forget entire function types (sometimes in the old course format students would forget all about linear functions by the time they finish learning about quadratic functions!).

3. Day-to-day student learning should follow the steps below:
   1) Read the section from this manual. Watch videos on MyOpenMath.com associated with the section.
   2) Attend class where the section will be covered, questions answered, and more in-depth learning will be discussed.
   3) After class, complete the Practice and Quizzes on MyOpenMath.com, and complete the written homework which has more in-depth learning.
   4) Go back and review old material from the course to keep the content fresh.
   5) Read the next section from this manual and repeat the process.
   Why this helps students: Encouraging students to follow this day-to-day schedule will help them develop regular study habits that help them master the content. Encouraging them to follow these steps can also help them learn exactly how their time should be spent during study time. Many students at this level spend time “reading notes” or “reading the book” and think that’s enough to learn the material. Encouraging this 5-step process every single day will help students develop a regular day-to-day study habit that has a much better chance of improving success.
There will, inevitably, be a few students who “cheat” on the out-of-class MyOpenMath.com work by having a friend do it, or have a friend help so much that it was essentially completed by the friend. MyOpenMath.com should be enough of the overall grade that students don’t want to skip it, but it should also be a small enough part of the grade that it can’t “get the student through” the course even when the student fails the in class exams! Students should regularly be reminded that the out-of-class work is for their own learning benefit. And having a friend do his/her homework will not help him/her master the material and pass the final exam!!

It is highly recommended that the first day, or two days, or three days of class time be spent in the computer lab where the instructor can do a very quick review of old material, and then the remainder of class time can be spent with the students practicing on MyOpenMath.com. This gives the students the opportunity to get comfortable with the features of MyOpenMath.com while the instructor is there to help and answer questions.

In order to encourage students to use this book as a resource, instructors are encouraged to use the examples from the book during class time, and also encouraged to have students complete some of the written homework problems together in groups during class time. This gets students used to using the book and understanding how to approach the written practice. In turn, we hope it will help them feel more comfortable reading and practicing more outside of class time!
Introduction to Unit 1

This is the first unit of the Intermediate Algebra course. During the first unit, students should master the following learning outcomes which will be used throughout the remainder of the course:

- Evaluating expressions and calculating expressions with the calculator.
- Solving linear equations including literal linear equations.
- Finding points on graphs including finding the intercepts of linear functions.
- Understand how to enter a function in the graphing calculator, and have an introductory-level of understanding of viewing windows and tables on the calculator.
- Identifying functions and using function notation.
- Identifying linear functions, exponential functions, and quadratic functions when given a formula.
- Identifying linear functions and exponential functions when given a table of values.
- Identifying linear functions, exponential functions, and quadratic functions when given a graph.
- Identifying the slope, y-intercept, and formula for a linear function when given a graph or table or information about the function.
- Identifying the factor, percent rate, and y-intercept for an exponential function when given the graph or table or information about the function.
- Identifying the y-intercept, vertex, and if the function is open up or open down when given a formula/graph of a quadratic function.

Recommended Schedule for Unit 1:

1.1: Simplify and solve (including literal equations). 2 days.
1.2: Evaluating and using calculator to perform calculations. 1 day.
1.3: Intercepts and Graphs. 2 days.
1.4: Functions and function notation. 1 day.
1.5: Domain and range. 1 day.
1.6: Graphs and Tables of linear, quadratic, exponential functions. 3 days.
1.7: Equations of linear, quadratic, exponential functions. 3 days.

Review and testing. 2 days.
Section 1.1
Simplifying Expressions and Solving Equations

**Simplifying Expressions**: The following steps will help maintain accuracy when simplifying expressions.

1. **Distribute to clear all parenthesis**
2. **Identify like terms and circle/square/triangle them**
3. **Gather and combine like terms to simplify expression**

**Example 1**: Simplify the expression \(3x - 5 + x - 1 + 8x - 10x\)

**Solution**:

\[
\begin{align*}
3x &- 5 + x - 1 + 8x - 10x \\
\text{Circle like terms so you know to combine them} &
\end{align*}
\]

\(2x - 6\)

Combine like terms together (3+1+8-10 = 2 and -5 – 1 = -6)

**Example 2**: Simplify the expression \(2 - (3b + 4) - 3(5b - 9) + 6b\)

**Solution**:

\[
\begin{align*}
2 &- 3b - 4 - 15b + 27 + 6b \\
\text{Distribute to clear away all parenthesis.} &
\end{align*}
\]

\(2 - 3b - 4 - 15b + 27 + 6b\)

\(2 - 3b - 4 - 15b + 27 + 6b\)

\(-12b + 25\)

Combine like terms together (-3 – 15 + 6 = -12 and 2 – 4 + 27 = 25)

**Example 3**: Simplify the expression \(\frac{x}{3} - \frac{1}{2} + 5x - 9 - \frac{1}{5}x\)

**Solution**:

\[
\begin{align*}
\frac{x}{3} &- \frac{1}{2} + 5x - 9 - \frac{1}{5}x \\
\text{Circle like terms so you know to combine them} &
\end{align*}
\]

\(\frac{77}{15}x - \frac{19}{2}\)

Remember that \(\frac{x}{3}\) is the same as \(\frac{1}{3}x\)

Combine like terms together (1/3 + 5 – 1/5 = 77/15 and 1/2 - 9 = -19/2)

Remember that you can quickly add fractions on your calculator using MATH, ENTER, ENTER.
**Solving 1-Variable Linear Equations:** The following steps will help maintain accuracy when solving 1-variable linear equations.

1. Clear all parenthesis on both sides by distributing
2. Clear all fractions by multiplying both sides by a common factor of the denominators
3. Do additions/subtractions on both sides to get all variable-terms on one side, and all constant terms on the other side
4. Do multiplications/divisions on both sides to solve for the variable
5. Check your solution by plugging it into the original equation

**Example 4:** Solve the equation $-\frac{x}{5} = 13$. Give the EXACT answer.

**Solution:** The left side of the equation can be re-written as $-1/5x$.

$$-\frac{x}{5} = 13$$

$$-\frac{1}{5}x = 13$$

Re-write the left-side if you like. You can skip this step if you like.

$$-5 \times \left( -\frac{1}{5}x \right) = (13) \times -5$$

Multiply both sides of the equation by $-5$.

$$-5 \times \left( -\frac{1}{5}x \right) = -65$$

The -5 cancels with the -1/5 on the left side.

$$x = -65$$

The solution to the equation is $x = -65$

$$-\frac{(-65)}{5} = 13 \text{ Correct!}$$

Plug the value $x = -65$ into the original equation to make sure it works.
**Example 5:** Solve the equation \(3 = -\frac{5}{7}a\). Give the EXACT answer.

Solution:

\[
3 = -\frac{5}{7}a
\]

\[
-\frac{7}{5} \times (3) = \left(-\frac{5}{7}a\right) \times -\frac{7}{5}
\]

Multiply both sides of the equation by \(-7/5\)

\[
-\frac{21}{5} = \left(-\frac{5}{7}a\right) \times -\frac{2}{5}
\]

The \(-5/7\) cancels with the \(-7/5\) on the right side.

\[
a = -\frac{21}{5}
\]

The solution to the equation is \(a=-\frac{21}{5}\)

\[
3 = -\frac{5}{7} \left(-\frac{21}{5}\right) \text{Correct!}
\]

Plug the value \(x = -\frac{65}{15}\) into the original equation to make sure it works.

**Example 6:** Solve the equation \(7x - 3 = -5 + 13x\). Give the EXACT answer.

Solution:

\[
7x - 3 = -5 + 13x
\]

\[
7x - 3 - 7x + 5 = -5 + 13x - 7x + 5
\]

Subtract \(7x\) on both sides, and add \(5\) on both sides to bring the \(x\)-terms to the right side, and bring the constant terms to the left side.

\[
2x - 3 = 7x + 5 = -5 + 13x - 7x + 5
\]

The \(7x\)'s cancel on the left. The \(5\)'s cancel on the right.

\[
-3 + 5 = 13x - 7x
\]

\[
2 = 6x
\]

Simplify both sides of the equation

\[
\frac{2}{6} = \frac{6x}{6}
\]

Divide both sides of the equation by \(6\)

\[
\frac{1}{3} = \frac{6x}{6}
\]

The \(6\)'s cancel on the right.

\[
x = 1/3
\]

The solution to the equation is \(x=1/3\)

\[
7\left(\frac{1}{3}\right) - 3 = -5 + 13\left(\frac{1}{3}\right)
\]

\[
-\frac{2}{3} = -\frac{2}{3} \text{Correct!}
\]

Plug the value \(x = 1/3\) into the original equation to make sure it works. Pull out the calculator to quickly check that the left side is the same as the right side.
Example 7: Solve the equation \( \frac{1}{3}(7 - 5x) = \frac{2}{5}x + 8 \). Give the EXACT answer.

Solution:

\[
\frac{7}{3} - \frac{5}{3}x = \frac{2}{5}x + 8
\]

Distribute to clear parenthesis

\[
15 \times \left( \frac{7}{3} - \frac{5}{3}x \right) = \left( \frac{2}{5}x + 8 \right) \times 15
\]

The least common denominator for all terms in the equation is 15, and so we multiply both sides of the equation by 15

\[
35 - 25x = 6x + 120
\]

Distribute the 15 to each term.

\[
35 - 25x + 25x - 120 = 6x + 120 + 25x - 120
\]

Bring x-terms to the right, constant terms to the left

\[
35 - 25x + 25x - 120 = 6x + 120 + 25x - 120
\]

Terms cancelling

\[
-85 = 31x
\]

Simplify on the left and right side

\[
\frac{-85}{31} = \frac{31x}{31}
\]

Divide both sides by 31

\[
x = -\frac{85}{31}
\]

The solution to the equation is \( x = -\frac{85}{31} \)

\[
\frac{1}{3}(7 - 5 \times -\frac{85}{31}) = \frac{2}{5}( -\frac{85}{31}) + 8
\]

Plug \( x = -\frac{85}{31} \) into the original equation to check.

\[
\frac{214}{31} = \frac{214}{31}
\]

Correct!
Review of Solving Equations Literal Equations

The following steps will help maintain accuracy when solving multi variable linear equations.

Make clear to yourself the goal...get WHICH variable alone??

Figure out how the other letters are ATTACHED to your variable to decide how to move them.
Example: If they are they ADDED, then you need to subtract them away
if they are they MULTIPLIED, then you need to divide them away

Do additions and subtractions FIRST. Then multiplications and divisions.

Example 8: Solve the equation \( T = 3fh \) solve for \( f \).

Solution: We want to solve for \( f \). We see that the 3 and the \( h \) are MULTIPLIED to the \( f \). So we need to divide both sides of the equation by \( 3h \) in order to isolate \( f \).

\[
\frac{T}{3h} = \frac{3fh}{3h} \quad \text{Divide both sides by } 3h \text{ to isolate } f
\]

\[
\frac{T}{3h} = \frac{4f}{3h} \quad \text{The } 3 \text{ and the } h \text{ cancel on the right}
\]

\[
\frac{T}{3h} = f \quad \text{The solution is } f = \frac{T}{3h}
\]

Example 9: Solve the equation \( A = \frac{1}{2}bh \) solve for \( b \).

Solution: We want to solve for \( b \). We will first clear fractions by multiplying both sides by 2.

\[
2 \ast (A) = 2 \ast \left(\frac{1}{2}bh\right) \quad \text{Clear fractions by multiplying both sides by } 2
\]

\[2A = bh\quad \text{Simplify}\]

\[
\frac{2A}{h} = \frac{bh}{h} \quad \text{Divide both sides by } h \text{ in order to isolate } b
\]

\[
\frac{2A}{h} = b \quad \text{The solution is } b = \frac{2A}{h}
\]
Example 10: Solve the equation $8x - 3y = 15$ solve for $y$.

Solution: We want to solve for $y$. We'll need to subtract away the $8x$ and divide away the $-3$. We always do additions/subtractions first.

\[
8x - 3y - 8x = 15 - 8x
\]
Subtract $8x$ on both sides to get it away from the $y$-term

\[-3y = -8x + 15
\]
Simplify

\[
\frac{-3y}{-3} = \frac{-8x + 15}{-3}
\]
Divide both sides by $-3$ in order to isolate $y$

\[
y = \frac{8}{3}x + \frac{15}{-3}
\]
Divide both terms on the right by $-3$

\[
y = -\frac{8}{3}x - 5
\]
Simplify. The solution is $y = -\frac{8}{3}x - 5$

Example 11: Solve the equation $mx + b = 7$ solve for $x$.

Solution: We want to solve for $x$. We'll need to subtract away the $b$ and divide away the $m$. We always do additions/subtractions first.

\[
mx + b - b = 7 - b
\]
Subtract $b$ on both sides to get it away from the $x$-term

\[
mx = 7 - b
\]
Simplify

\[
\frac{mx}{m} = \frac{7 - b}{m}
\]
Divide both sides by $m$ in order to isolate $x$

\[
x = \frac{7}{m} - \frac{b}{m}
\]
Divide both terms on the right by $m$. The solution is $x = \frac{7}{m} - \frac{b}{m}$

Example 12: Solve the equation $L = \frac{2}{3}ar^2m$ solve for $a$.

Solution: We want to solve for $a$. We will first clear fractions by multiplying both sides by $3$.

\[
3 \times (L) = \left(\frac{2}{3}ar^2m\right) \times 3
\]
Clear fractions by multiplying both sides by $3$

\[
3L = 2ar^2m
\]
Simplify

\[
\frac{3L}{2r^2m} = \frac{2ar^2m}{2r^2m}
\]
Divide both sides by $2r^2m$ in order to isolate $a$

\[
\frac{3L}{2r^2m} = a
\]
The solution is $a = \frac{3L}{2r^2m}$
Section 1.1 Written Practice and Reflection:
Simplifying Expressions and Solving Equations

1) Simplify the expression $5 + 3x - x - 4 - 9x$.

2) Simplify the expression $\frac{10}{7}k + \frac{10}{7} - 2k + \frac{2}{7} + \frac{2}{7}k - 6$

3) Simplify the expression $4y^2 - 5y^3 - 7y^2 + 7y^3$

4) Simplify the expression $100[0.02(m + 2)]$

5) Simplify the expression $-4(t - 2) - 6$

6) Simplify the expression $-5(3y - 8) + 4(2y + 8)$

7) Simplify the expression $2(3x - 6) - (5x + 3)$

8) Simplify the expression $-2(-2y + 3) - (3y - 5) + 5y + 4$

9) Simplify the expression $-5.5(3x + 6) - 1.4(2x - 4)$

10) Find the exact solution. Show all steps and work. $6x + 10 = 28$

11) Solve the following equations. Show all work. Leave your answer in exact form and check your solutions!
   a) $15x - 130 = 7 - 85x$
   b) $13(y - 4) - (5 + 7y) = 8 - (2y + 5)$
   c) $\frac{2}{3}b - 7 = 5b + \frac{1}{6}$

12) Find the exact solution. Show all steps and work. $7x - 25 = 2x$

13) Find the exact solution. Show all steps and work. $-2x + 5 = 23$

14) Find the exact solution. Show all steps and work. $3 - (5x + 4) = 13$

15) Find the exact solution. Show all steps and work. $7x + 52 = 2x - 13$

16) Find the exact solution. Show all steps and work. $8x + 4 = 5x + 8$

17) Find the exact solution. Show all steps and work. $8x - 9 = 11x + 12$

18) Find the exact solution. Show all steps and work. $6 - 4x = 7x - 9x + 10$

19) Find the exact solution. Show all steps and work. $7 + 10p - 4 = 7p + 39 - 3p$

20) Find the exact solution. Show all steps and work. $3(2t - 5) + 2t = 30 - t$
21) Find the exact solution. Show all steps and work. \(-2p + 8 = 5 - (7p + 3)\)

22) Find the exact solution. Show all steps and work. \(2(6x + 24) = 2(7x + 14)\)

23) Find the exact solution. Show all steps and work. \(-(8y + 5) - (-7y - 8) = -3\)

24) Find the exact solution. Show all steps and work. \(8(2x - 6) = 2(8x + 4)\)

25) Find the exact solution. Show all steps and work. \(4(3x - 15) = 12(x - 5)\)

26) Find the exact solution. Show all steps and work. \(11x - 5(x - 2) = 6x + 5\)

27) Find the exact solution. Show all steps and work. \(\frac{5}{2}x + \frac{1}{4}x = \frac{5}{4} + x\)

28) Find the exact solution. Show all steps and work. \(\frac{4}{5}x - \frac{1}{4}x + 6 = \frac{7}{10}x\)

29) Find the exact solution. Show all steps and work. \(\frac{1}{3}(3x + 5) - \frac{1}{5}(x + 7) = 7\)

30) Find the exact solution. Show all steps and work. \(-\frac{1}{4}(x - 8) + \frac{1}{2}(x + 2) = x + 1\)

31) Find the exact solution. Show all steps and work. \(0.12(6 + x) + 0.02x = 0.04(24 + x)\)

32) Find the exact solution. Show all steps and work. \(0.60x + 0.05(14 - x) = 0.10(62)\)

33) Find the exact solution. Show all steps and work. \(0.19(10,000) - 0.06y = 0.04(y + 10,000)\)

34) Solve the equation \(4M - T = W\) for \(M\).

35) Solve the following equations for the indicated variable. Show all work.
   a) \(V = \frac{4}{3}\pi r^3\) Solve for \(\pi\)

   b) \(7x + 2y = 19\) Solve for \(y\)

   c) \(12x - 5y = 20\) Solve for \(y\)

   d) \(y - \frac{2}{3} = \frac{7}{8}(x - 16)\) Solve for \(y\)

   e) \(y - \frac{2}{3} = \frac{7}{8}(x - 16)\) Solve for \(x\)

36) Suppose \(D = jm\). Solve for \(j\). Show all steps and work.

37) Suppose \(C = tr\). Solve for \(t\). Show all steps and work.

38) Suppose \(a = 9b\). Solve for \(b\). Show all steps and work.
39) Suppose \( a = gmb \). Solve for \( b \). Show all steps and work.

40) Suppose \( f = \frac{1}{2}hr \). Solve for \( h \). Show all steps and work.

41) Suppose \( F = \frac{1}{6}\pi r^2 p \). Solve for \( p \). Show all steps and work.

42) Suppose \( P = a + b + 4c \). Solve for \( a \). Show all steps and work.

43) Suppose \( p = 2m + 2w \). Solve for \( m \). Show all steps and work.

44) Suppose \( A = q + qvd \). Solve for \( v \). Show all steps and work.

45) Suppose \( Ax + Hy = L \). Solve for \( x \). Show all steps and work.

46) Suppose \( A = \frac{5}{4}(h - 47) \). Solve for \( h \). Show all steps and work.

47) Suppose \( U = F(a + R) \). Solve for \( a \). Show all steps and work.
Section 1.2
Evaluating and Calculating

Here are some basic things to always remember when doing calculations with the graphing calculator:

- **Messy problems**: If the problem is messy, take it in a few steps if needed! Otherwise, you need to use parenthesis carefully.

- **Entering Fractions in Calculator**: To enter the fraction $\frac{1}{2}$ into the calculator just type $1 \div 2$.

- **Turn answers into fractions**: To change decimals into fractions, type MATH and then press ENTER ENTER (press enter twice).

- **Negative vs. Subtraction**: There is a difference in your calculator between the negative button (next to the decimal button) and the subtraction button (next to the addition button). Be sure to use them correctly!

- **Scientific Notation**: $3.75E - 4$ is the way your calculator gives scientific notation. It means $3.75 \times 10^{-4}$ which is the decimal number 0.000375. On the calculator $4.956E7$ means $4.956 \times 10^7$ which is the number 49,560,000.

- **0 in the numerator**: 0/8 means “take 0 things and divide them into 8 pieces” which would give 0 things in each group. Therefore $0/8 = 0$.

- **0 in the denominator**: 8/0 means “take 8 things and divide them into 0 pieces” which isn’t possible. Therefore $8/0 = \text{undefined}$. Your calculator will say “Err: divide by 0” when you divide by 0 on the calculator. But the correct way to state the answer is by writing “undefined.”

- **Exponents**: to create exponents on the calculator, you can use the $^\text{^}$ button. So $7^3$ would be $7^3$ in the calculator.

- **Square roots**: The square root button on your calculator is found by hitting 2nd and then the $\sqrt{x}$ button.

**Example 1**: Calculate the expression $\frac{-3-7}{7+1}$. Give your answer as a simplified fraction.

**Solution**: We need to divide the entire numerator by the entire denominator. So we need to wrap parenthesis around the numerator, and parenthesis around the denominator. We also note that the negative button should be pressed to write -3, and we will use the subtraction button to subtract 7.

Notice that the negative symbol is smaller and slightly raised compared to the subtraction sign. The calculator will always return the decimal answer. But, we have been asked to give the answer as a simplified fraction.

So, we need to type MATH and then press ENTER and then ENTER again in order to convert that answer to a simplified fraction.

So the answer is $-5/4$. 


Example 2: Calculate the value of $b^2 - 4ac$ when $a = 2$ and $b = -5$ and $c = -3$.

Solution: First we remove the variables and replace them with empty parenthesis. Then we plug in the values. Then we calculate.

$$b^2 - 4ac = ( )^2 - 4( ) ( ) = (-5)^2 - 4(2)(-3) = 49$$

Example 3: Calculate the expression $\frac{-(-5) + \sqrt{(-5)^2 - 4(3)(-3)}}{2(3)}$. Give your answer rounded to four decimal places of accuracy.

Solution: This is messy enough that we should probably break this up into a couple steps.

$$\frac{-(-5) + \sqrt{61}}{2(3)}$$

first calculate the value under the radical which is 61

$$\frac{-(-5) + \sqrt{61}}{2(3)}$$

simplify the $-(-5)$ and the $2(3)$

$$= \frac{5 + \sqrt{61}}{6}$$

type the numerator in the calculator, press enter, divide by 6, press enter

And so the value of the expression, rounded to four decimal places, is 2.1350. Notice that we type the numerator in the calculator and press ENTER. Then we divide that entire value by the denominator to get the answer.

Example 4: Calculate the value of $\frac{-5.4 \times 10^{14}}{10.8 \times 10^{20}}$. Give the answer in scientific notation, and also in decimal form.

Solution: In the calculator we need to remember to wrap parenthesis around the entire numerator and the entire denominator.

This tells us that the answer in scientific notation is $-5 \times 10^{-7}$.

To convert to decimal notation we need to divide the number $-5$ by 10 seven times in a row. And so the decimal answer is $-0.0000005$. 

Example 5: Calculate the value of $\frac{-b}{2a}$ when $a = 5$ and $b = -6$.

Solution: First we remove the variables and replace them with empty parenthesis. Then we plug in the values. Then we calculate.

$$\frac{-b}{2a} = \frac{-(-6)}{2(5)} = \frac{6}{2(5)} = \frac{6}{10}$$

NOW BE CAREFUL IF YOU ARE USING THE CALCULATOR!!! The entire value of 2*5 needs to be in the denominator!!!

Example 6: Calculate the value of $\frac{y_2 - y_1}{x_2 - x_1}$ when $x_1 = 7$ and $y_1 = -4$ and $x_2 = -12$ and $y_2 = 15$.

Solution: First we remove the variables and replace them with empty parenthesis. Then we plug in the values. Be sure to calculate the numerator and denominator separately before dividing!

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(15) - (-4)}{(-12) - (7)} = \frac{19}{-19} = -1$$

Example 7: Calculate the value of $\frac{4}{3} \pi r^3$ when $r = 17$. Give the answer to 2 decimal places of accuracy.

Solution: On the calculator, we can find $\pi$ by pressing $2^{nd}$ and then the $^\wedge$ button. Here’s how it would be entered into the calculator to get an answer of 20,579.53.
Section 1.2 Written Practice and Reflection:
Evaluating and Calculating

1) Use the calculator to calculate the value of the expression.

   a) \( \frac{3}{5} \left( \frac{-2}{3} - \frac{1}{11} \right) = \quad \) (give decimal rounded to the hundredth)

   b) \( \frac{3}{5} \left( \frac{-2}{3} - \frac{1}{11} \right) = \quad \) (give exact fractional answer)

   c) \( \frac{(3500)(-76540)}{0.01} = \quad \) (give scientific notation)

   d) \( \frac{(3500)(-76540)}{0.01} = \quad \) (give answer as a decimal)

   e) \( \frac{-4+\sqrt{2500-4(3)(-15)}}{2(7)} = \quad \) (give decimal rounded to 4 decimal places)

   f) \( \frac{\frac{1}{2} \left( \frac{-2}{3} \right)}{1 \frac{1}{4}} = \quad \) (give exact fractional answer)

   g) \( \frac{\frac{1}{2} \left( \frac{-2}{3} \right)}{1 \frac{1}{4}} = \quad \) (give answer as a decimal rounded to 4 decimal places)

   h) \( \frac{7}{0} = \quad \)

   i) \( 0 \div 2 = \quad \)

   j) \( \frac{72}{16} = \quad \) (give answer a simplified improper fraction)

2) Calculate \( a - b + c \) when \( a = -7 \) and \( b = -2 \) and \( c = -3 \). Show the substitution step, and then use your calculator to get the correct answer.

3) Calculate \( b^2 - 4ac \) when \( a = -8 \) and \( b = -5 \) and \( c = 20 \). Show the substitution step, and then use your calculator to get the correct answer.

4) Calculate \( \frac{b}{2a} \) when \( a = -7 \) and \( b = -2 \). Show the substitution step, and then use your calculator to get the correct answer.

5) Calculate \( \frac{y_2 - y_1}{x_2 - x_1} \) when \( x_1 = -7 \) and \( y_1 = 2 \) and \( x_2 = -3 \) and \( y_2 = -5 \). Show the substitution step, and then use your calculator to get the correct answer.

6) Evaluate \( a^2 - d^4 \) if \( a = 2 \) and \( d = -2 \).
7) Evaluate the formula \( C = \frac{5}{9} (F - 32) \) to find the value of \( C \) if \( F = -40 \).

8) Evaluate \( r^2 - s^2 \) if \( r = -4 \) and \( s = -5 \).

9) Evaluate \( \frac{a-c}{b-c} \) if \( a = -6 \) and \( b = 8 \) and \( c = 9 \).

10) The area \( A \) of a triangle with base \( b \) and height \( h \) is given by \( A = \frac{1}{2}bh \). Find the area when \( b = 17 \) meters and \( h = 36 \) meters.

11) Evaluate the expression \( b^2 - 4ac \) when \( a = 13 \) and \( b = -25 \) and \( c = -1 \).

12) Evaluate the expression \( x - y \) when \( x = -4 \) and \( y = -3 \).

13) Evaluate the expression \( a - b - c \) when \( a = -19 \) and \( b = -14 \) and \( c = -6 \).

14) Evaluate \( -x^2 - x + 3 \) when \( x = -18 \).

15) Suppose that \( b = 15 \) and \( a = 141 \). Find \( x = \frac{-b}{2a} \). Give exact fractional answer. Also give decimal answer rounded to 2 decimal places.

16) Evaluate \( x^2 - x + 3 \) when \( x = -19 \).

17) Evaluate the expression \( \frac{-b+\sqrt{b^2-4ac}}{2a} \) when \( a = 2 \) and \( b = -506 \) and \( c = 5 \). Round to 4 decimal places.

18) Evaluate the expression \( \frac{y_2-y_1}{x_2-x_1} \) when \( y_2 = -6 \) and \( y_1 = 5 \) and \( x_2 = -15 \) and \( x_1 = -5 \). Give your answer as an exact reduced fraction, and then give your answer as a decimal.

19) Suppose the value \( 9.06E - 5 \) was given on the graphing calculator as an answer. Write the value in standard (decimal) notation.

20) Suppose the value \( 9.86E3 \) was given on the graphing calculator as an answer. Write the value in standard (decimal) notation.

21) Use the calculator to approximate the value of \( \frac{-b+\sqrt{b^2-4ac}}{2a} \) when \( a = -5 \) and \( b = -13 \) and \( c = -7 \). Round to 4 decimal places of accuracy.

23) Evaluate each of the following expressions at the given value.
   
   a) \( \frac{7}{9}x + 14 \) when \( x = -5 \)
   
   b) \( \frac{2}{3}x^2 - 5x + \frac{7}{11} \) when \( x = -8 \)
   
   c) \( \frac{3}{5}y^4 - y - \frac{2}{9} \) when \( y = -\frac{1}{6} \)
Section 1.3

Intercepts and Graphs

Learning Outcomes:
- Find x-intercepts and y-intercepts of linear equations
- Enter equations into the Y= menu and create a viewing window that allows for viewing the complete graph
- Set up the calculator to display an automatically filled table of values, and then view that table
- Set up the calculator to display a manually filled table of values, and then view that table

Vocabulary:
- Complete graph
- X-intercept
- Y-intercept
- Xmin, XMax, XScI, YMin, YMax, YScI
- Standard Window
- TblStart
- ΔTbl

Two-Variable Equations:
In a two variable equation we refer to the input variable as the independent variable, and we refer to the output variable as the dependent variable. The numbers and letters in the equation that are fixed and unchanging are called constants or parameters. A solution to a two variable equation is an ordered pair (x, y), called a point, where we list the independent value first, and the dependent value second. Two variable equations have an infinite number of solutions (there are an infinite number of points that are solutions to any two variable equation).

Independent variable
- Input variable
- Plotted along the horizontal axis on a graph
- We can select any independent values we want to input into the equation (from the set of acceptable input-values).
- In a formula, the independent variable is usually NOT solved for. If a formula is solved for one letter, then that letter would NOT be considered the independent variable.
- In the point (x, y) we know that x is the independent value.

Dependent variable
- Output variable
- Plotted along the vertical axis on a graph
- The value of the dependent variable depends on the value we select for the independent variable
- In a formula, we usually SOLVE for the dependent variable. If a formula is solved for one letter, then that letter would be considered the dependent variable.
- In the point (x, y) we know that y is the dependent value.

Evaluate: Evaluating means you are finding the value of the dependent variable when you are given the value of the independent variable (finding the output if you know the input)

Solve: Solving means you are finding the value of the independent variable when you are given the value of the dependent variable (finding the input if you know the output)
When trying to find a viewing window that is appropriate for an equation on the graphing calculator, we will generally want a window that allows us to see a **complete graph** of the function: a graph that shows all the “important” points includes the y-intercept, x-intercepts, local maximums and minimums, inflection points, and more.

**X-Intercept of a Function**
- A point on a graph is an x-intercept if and only if it is a point where the graph intersects the x-axis.
- A point on a graph is an x-intercept if and only if the output y-value of the graph is $y=0$ at that point.
- Graphs can have multiple x-intercepts.
- If you have a 2-variable equation, then you can find the x-intercept(s) of that equation by setting $y=0$ in the equation and solving for the x-value(s).
- The x-value(s) that make the output equal 0 are called **ROOT(s)** of the function, or they are called **ZERO(s)** of the function. The x-value in an x-intercept is always a root/zero of the function.

**Y-Intercept of a Function**
- A point on a graph is a y-intercept if and only if it is a point where the graph intersects the y-axis.
- A point on a graph is a y-intercept if and only if the input x-value of the graph is $x=0$ at that point.
- If you have a 2-variable equation, then you can find the y-intercept of the equation by setting $x=0$ and evaluating for the y-value.
1. Enter the equation in the Y= menu. The button \( Y_0 \) will produce the X variable.

2. Set up the table by hitting \( 2^{\text{nd}} \) and then WINDOW.

   Enter the x-value you want the table to START at.

   Enter the amount you want the x-values to increment by.

   Set Indpnt and Depend to AUTO by pressing ENTER when the cursor is on Auto. This will automatically fill in the table for you when you view it.

3. View the table by pressing \( 2^{\text{nd}} \) and then GRAPH.
1. Enter the equation in the Y= menu. The button will produce the X variable.

2. Set up the table by hitting 2nd and then WINDOW.

   It doesn’t matter what you put in the TblStart or the ΔTbl area.

   Set Indpnt to Ask by placing the cursor on Ask and pressing ENTER and leave the Depend as AUTO.

   This will allow you to PICK x-values and it will automatically fill in the y-values.

3. View the table by pressing 2nd and then GRAPH. Type an x-value. Press ENTER.
1. Enter the equation in the Y= menu. The button will produce the X variable.

2. To select a window that allows you to see the parts of the graph you would like to view, click the button.

3. Leave Xres set at 1 at all times. You can manually change the other values to create the window you want understanding the following:

   - **Xmin**: the LOWEST XVALUE that will be displayed on the graph
   - **Xmax**: the HIGHEST XVALUE that will be displayed on the graph
   - **Xscl**: the distance between the tick-marks on the x-axis that will be displayed on the graph
   - **Ymin**: the LOWEST YVALUE that will be displayed on the graph
   - **Ymax**: the HIGHEST YVALUE that will be displayed on the graph
   - **Yscl**: the distance between the tick-marks on the y-axis that will be displayed on the graph

Example: The window below produces the following graph of $Y=9.4X - 0.6$

A **Standard Viewing Window** is a window set as follows:

   - XMin: $-10$
   - XMax: 10
   - Xscl: 1
   - YMin: $-10$
   - YMax: 10
   - Yscl: 1
**Example 1**: The formula $P = 300(1.028)^t$ approximates the population of the U.S. where $P$ is the number of people in millions, and $t$ is the number of years after October 26th, 2006.

(a) What are the variables in this equation? Which is the independent and which is the dependent?

**Solution**: The variables in the equation are $t$ and $P$. Since the equation is solved for $P$, we would consider $P$ the dependent variable, and we would consider $t$ the independent variable. So, if we made a graph, the $t$ would be along the horizontal axis, and the $P$ would be along the vertical axis. All points on the graph would have the form $(t, P)$.

(b) What are the constants in this equation?

**Solution**: The values 300 and 1.028 are the constants (or parameters) in the equation. They do not change/vary.

(c) Give three solutions to this equation. Interpret the real-world meaning of each point.

**Solution**: A solution to the equation $P = 300(1.028)^t$ is a point (an ordered pair) that solves the equation (makes the left side equal the right side). In this case, each solution will have the form $(t, P)$. We can select any value we want to input for the variable $t$ and then find the value of the variable $P$. We should select values that make sense in the real-world context (population of the U.S. as time goes by). We will select $t = 0$, and $t=2.5$, and $t = 5$.

If $t=0$, then $P = 300(1.028)^0$ and so $P = 300$. So a solution to the equation is $(0, 300)$. This tells us that, according to this model, the population of the U.S. was 300 million on October 26, 2006.

If $t=2.5$, then $P = 300(1.028)^{2.5}$ and so $P \approx 321.443$. So another solution to the equation is $(2.5, 321.443)$. This tells us that, according to this model, the population was about 321,443,000 at the end of April, 2009.

If $t=5$, then $P = 300(1.028)^5$ and so $P \approx 344.419$. So another solution to the equation is $(5, 344.419)$. This tells us that, according to this model, the population was about 344,419,000 on October 26, 2011.
**Example 2:** Consider the equation that relates Fahrenheit temperature, \( F \), to Celsius temperature, \( C \). The equation can be written as

\[
F = \frac{9}{5}C + 32.
\]

(a) Identify the independent and dependent variable in the equation \( F = \frac{9}{5}C + 32 \).

**Solution:** As is, the equation has independent variable \( C \) and dependent variable \( F \). On a graph, \( C \) is the horizontal and \( F \) is the vertical. All solutions to the equation have the form \((C, F)\).

(b) Solve the equation \( F = \frac{9}{5}C + 32 \) for \( C \).

**Solution:** To solve for \( C \) means we need to get \( C \) isolated on one side of the equation, and we need to get all other values onto the opposite side of the equation.

\[
\begin{align*}
F &= \frac{9}{5}C + 32 \\
5 \cdot F &= 5 \cdot \left( \frac{9}{5}C + 32 \right) & \text{Multiply both sides by 5 to clear fractions} \\
5F &= 9C + 160 & \text{Distribute and simplify both sides of equation} \\
5F - 160 &= 9C + 160 - 160 & \text{Subtract 160 on both sides} \\
5F - 160 &= 9C & \text{Simplify both sides of equation} \\
\frac{5F - 160}{9} &= \frac{9C}{9} & \text{Divide both sides of equation by 9 to isolate \( C \)} \\
\frac{5}{9}F - \frac{160}{9} &= C & \text{Simplify both sides}
\end{align*}
\]

(c) Determine which variable is independent and which variable is dependent in your new equation \( C = \frac{5}{9}F - \frac{160}{9} \).

**Solution:** Since the equation is solved for \( C \), we would now consider \( C \) the dependent variable, and we would consider \( F \) the independent variable. On a graph, \( F \) is the horizontal and \( C \) is the vertical. All solutions to the equation have the form \((F, C)\).

(d) What is the Fahrenheit temperature when Celsius is 0 degrees? What is the Celsius temperature when Fahrenheit is 85 degrees?

**Solution:** We can use EITHER equation to find the answers. But it is always easier to evaluate an equation instead of being forced to solve an equation.

If \( C = 0 \) then since we are given \( C = 0 \) and asked to find \( F \), we will use the equation that has \( C \) as the input. So we are evaluating the equation \( F = \frac{9}{5}C + 32 \) when \( C = 0 \) which means \( F = \frac{9}{5}(0) + 32 \) which means \( F = 32 \). This means that when it is 0 degrees Celsius outside, it will be 32 degrees Fahrenheit outside.

If \( F = 85 \) then since we are given \( F = 85 \) and asked to find \( C \), we will use the equation that has \( F \) as the input. So we are evaluating the equation \( C = \frac{5}{9}F - \frac{160}{9} \) when \( F = 85 \) which means \( C = \frac{5}{9}(85) - \frac{160}{9} \) which means \( C = 29\frac{\pi}{9} \approx 29.4 \). This means that when it is 85 degrees Fahrenheit outside, it will be about 29.4 degrees Celsius outside.
Example 3: Suppose $15 = 3x - 5y$.

(a) Solve the equation for $y$.
Solution: In order to solve the equation for $y$ we need to isolate $y$ on one side of the equation.

$$15 = 3x - 5y$$

$$15 - 3x = 3x - 5y - 3x$$ \hspace{1cm} \text{Bring all the x-terms to the left}

$$15 - 3x = -5y$$ \hspace{1cm} \text{Simplify both sides of equation}

$$\frac{15-3x}{-5} = \frac{-5y}{-5}$$ \hspace{1cm} \text{Divide both sides by $-5$ to isolate $y$}

$$-3 + \frac{3}{5}x = y$$ \hspace{1cm} \text{Simplify both sides of equation}

(b) Complete a table of values for $x=0$, $x=5$, $x=10$ and for $y=-6$, $y=-9$, $y=-18$.
Solution: The equation can be written $15 = 3x - 5y$ or can be written as $-3 + \frac{3}{5}x = y$. We can use either equation to find solutions.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>-5</td>
<td>-6</td>
</tr>
<tr>
<td>-10</td>
<td>-9</td>
</tr>
<tr>
<td>-25</td>
<td>-18</td>
</tr>
</tbody>
</table>

If $x=0$ then $y = -3 + \frac{3}{5}(0)$ and so $y = -3$.

If $x=5$ then $y = -3 + \frac{3}{5}(5)$ and so $y = 0$.

If $x=10$ then $y = -3 + \frac{3}{5}(10)$ and so $y = 3$.

If $y = -6$ then $15 = 3x - 5(-6)$ and so $15 = 3x + 30$ and so $-15 = 3x$ and so $x = -5$.

If $y = -9$ then $15 = 3x - 5(-9)$ and so $15 = 3x + 45$ and so $-30 = 3x$ and so $x = -10$.

If $y = -18$ then $15 = 3x - 5(-18)$ and so $15 = 3x + 90$ and so $-75 = 3x$ and so $x = -25$.

(c) Plot the points from the table and sketch the general graph.
Example 4: Complete the table of values with the help of your graphing calculator.

\[ y = -2x + 5 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
1. First we put the equation in the Y= menu by letting \( Y_1 = -2x + 5 \).
2. Then we hit 2nd WINDOW to access the TBL SETUP menu.
3. Since the x-values of this table start at \( x = -4 \) and increment by units of 2, we let
   \( \text{TBLStart} = -4 \)
   \( \Delta Tbl = 2 \)
   Make sure Auto is selected for Indpnt
4. We view the table by hitting 2nd GRAPH to access the Table.

Example 5: Complete the table of values with the help of your graphing calculator.

\[ y = -x^2 + 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
1. First we put the equation in the Y= menu by letting \( Y_1 = -x^2 + 1 \)
2. Then we hit 2nd WINDOW to access the TBL SETUP menu.
3. Since the x-values of this table start at \( x = -5 \) and increment by units of 3, we let
   \( \text{TBLStart} = -5 \)
   \( \Delta Tbl = 3 \)
   Make sure Auto is selected for Indpnt
4. We view the table by hitting 2nd GRAPH to access the Table.
Example 6: Complete the table of values with the help of your graphing calculator.

\[ y = -2x + 5 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-85</td>
<td></td>
</tr>
<tr>
<td>-9.4</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td></td>
</tr>
<tr>
<td>6300</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
1. First we put the equation in the Y= menu by letting Y1= -2x+5.
2. Then we hit 2nd WINDOW to access the TBL SETUP menu. Make sure Indpnt is set to Ask.
3. We view the table by hitting 2nd GRAPH to access the Table. Type in -85 press ENTER. Type in -9.4 press ENTER. Continue to input the x-values and press enter.

Example 7: Complete the table of values with the help of your graphing calculator.

\[ y = -x^2 + 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>145</td>
<td></td>
</tr>
<tr>
<td>35.58</td>
<td></td>
</tr>
<tr>
<td>-17</td>
<td></td>
</tr>
<tr>
<td>41/3</td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>2.69</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
1. First we put the equation in the Y= menu by letting Y1= -x^2+1.
2. Then we hit 2nd WINDOW to access the TBL SETUP menu. Make sure IndPnt is set to Ask.
3. We view the table by hitting 2nd GRAPH to access the Table. Type in 145 press ENTER. Type in 35.58 press ENTER. Continue to input the x-values and press enter.
Example 8: Suppose that \( V = -75M + 2500 \) where \( V \) is the volume of water in a tank (in gallons) and \( M \) is the number of minutes that have passed.

(a) Use the formula to find \( V \) when \( M=18 \). Then write a sentence that interprets the practical meaning of the point.
Solution: Using the formula we see that \( V = -75(18) + 2500 \) and so \( V = 1150 \). So the point \((18, 1150)\) is on the graph. This tells us that after 18 minutes have passed, the tank has a total of 1150 gallons of water in it.

(b) Use the formula to find \( M \) when \( V=18 \). Then write a sentence that interprets the practical meaning of the point.
Solution: Using the formula we need to solve the \( 18 = -75M + 2500 \).

\[
18 = -75M + 2500 \\
-2500 \quad -2500 \\
\hline \\
-2482 = -75M \\
\frac{-2482}{-75} = \frac{-75M}{-75} \\
\frac{2482}{75} = M \quad 33.093333 \approx M
\]

So the approximate point \((33.1, 18)\) is on the graph. This tells us that after approximately 33.1 minutes have passed, the tank has a total of 18 gallons of water in it.

(c) Use the formula to find \( V \) when \( M=0 \). Then write a sentence that interprets the practical meaning of the point.
Solution: Using the formula we see that \( V = -75(0) + 2500 \) and so \( V = 2500 \). So the point \((0, 2500)\) is on the graph. This tells us that after 0 minutes have passed (when we start), the tank has a total of 2500 gallons of water in it.

(d) Use the formula to find \( M \) when \( V=0 \). Then write a sentence that interprets the practical meaning of the point.
Solution: Using the formula we need to solve the \( 0 = -75M + 2500 \).

\[
0 = -75M + 2500 \\
-2500 \quad -2500 \\
\hline \\
-2500 = -75M \\
\frac{-2500}{-75} = \frac{-75M}{-75} \\
\frac{2500}{75} = M \quad 33.333333 \approx M
\]

So the approximate point \((33.3, 0)\) is on the graph. This tells us that after approximately 33.3 minutes have passed, the tank has a total of 0 gallons of water in it. In other words, after about 33.3 minutes, the tank is completely empty.
(e) Use the table of values on your calculator to check the accuracy of your 4 points that you calculated.

Solution: First we put the function into the Y= menu.

Next we select 2nd WINDOW in order to set up the table. We want to manually input specific x-values. And so we let Indpnt be set to Ask.

Lastly, we select 2nd GRAPH in order to view the table. We input each x-value we want to see, and press enter after each.

These points verify that our calculations above were correct.

(f) Plot the 4 points you have found on a well-labeled graph that you create BY HAND. Use the 4 points to help you fill in the shape of the graph in general.

Solution: We create a graph that allows us to view the points (0,2500) and (18,1150) and (33.1, 18) and (33.3,0). Notice that we do not need to include input or output values that are negative since we can’t have a negative volume of water, and we can’t have a negative number of minutes. When we plot the points we find that they follow the pattern of a line.

(g) Put the formula in the calculator in Y=. Set an appropriate viewing window that allows you to see the graph that you created by hand. What window are you using?

Solution: We already have the function in the Y= menu. Now we set a viewing window that allows us to see the graph above.
Section 1.3 Written Practice and Reflection
Intercepts and Graphs

1) For each table given below, do the following:
   a) Complete the table of values. Show all work to find the values in the table.
   b) Plot the points (with clear labels) and create the general graph for this two-variable equation.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>−5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>1000</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>−7</td>
<td>14</td>
</tr>
</tbody>
</table>

2) Below are three formulas. Answer each question below for each formula.

   Formula 1: \( C = 18.5Q + 12,500 \) where \( Q \) represents the number of items a company produces, and \( C \) represents the total production costs the company has when producing that many items.

   Formula 2: \( H = −16(t − 5)^2 + 800 \) where \( t \) represents the number of seconds after a ball has been launched in the air, and \( H \) represents the height of the ball above ground (in feet).

   Formula 3: \( N = 450,000(0.9)^t \) where \( t \) represents the number of years after 1980, and \( N \) represents the number of people in the town of Mathville.

   a) Identify the INDEPENDENT variable. Identify the DEPENDENT variable.
   b) Solve for the dependent value when the independent value is 0, and then 3, and then 10. Show all work.
   c) For each point you found in part b, write a complete sentence that interprets the real-world meaning of the point.

3) Suppose that \(-8A − 7B = 53\).

   a) Solve the equation for \( B \). In your equation, which variable would be graphed along the vertical axis? Which variable would be graphed along the horizontal axis?

   b) Find the point on the graph where \( A=0 \). Show all work.

   c) Find the point on the graph where \( B=0 \). Show all work.

   d) Find the point on the graph where \( A=5 \). Show all work.

   e) Find the point on the graph where \( B=5 \). Show all work.

   f) Plot the four points you have found on the graph. Use those four points to help you create the general graph of this equation.
4) Complete the given ordered pairs for the equation.
   \[y = 3x + 7\]
   \[(6, \quad ) \quad (0, \quad ) \quad (-3, \quad )\]

5) Complete the ordered pair for the equation
   \[y = 3x + 6.\]
   \[( , -9)\]

6) Complete the table of values for the equation \(7x + 9y = 63\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

7) Complete the table of values. Then plot the ordered pairs on a graph. \(x - 2y = 8\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>

8) Complete the table of values. Then plot the ordered pairs on a graph. \(3x - 5y = 15\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-5</td>
<td>-5</td>
</tr>
</tbody>
</table>

9) The function \(y = \frac{2}{3}x - 12\) is graphed below.

   (a) FIRST use the formula to find the x-intercept and the y-intercept of the function. Find these points, and label them on the graph. This will help you understand the SCALE of this graph!!

   (b) Label all the tick marks on both axis.

   (c) The viewing window that will create the graph above:

   XMin: __________  YMin: __________
   XMax: __________  YMax: __________
   XScI: __________  YScI: __________
10) The function $y = -250x + 30$ is graphed below.

(a) FIRST use the formula to find the x-intercept and the y-intercept of the function. Find these points, and label them on the graph. This will help you understand the SCALE of this graph!!

(b) Label all the tick marks on both axis.

(c) The viewing window that will create the graph above:

<table>
<thead>
<tr>
<th>XMin:</th>
<th>XMax:</th>
<th>XSc1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YMin:</th>
<th>YMax:</th>
<th>YSc1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11) Below is a graph of $y = \frac{5}{3}x + 3000$.

(a) Find and label two points on this graph by finding both intercepts using the formula.

(b) Use the two points you have just labeled on the graph to help you determine the value of the other tick marks on the graph. Label all the tick marks in the entire window.

(c) Enter the equation in your graphing calculator in the Y= menu. Then complete the viewing window settings in order to create the exact same graph on your graphing calculator.

<table>
<thead>
<tr>
<th>XMin:</th>
<th>XMax:</th>
<th>XSc1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YMin:</th>
<th>YMax:</th>
<th>YSc1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12) Below is a graph of $y = 0.1 + 2.78 \times 2.7^{-0.37t}$

(a) The graph below has an XMax of 25 and it has a YMax of 4. Label the tick marks and values along both axis.

(b) Enter the equation in your graphing calculator in the Y= menu. Then complete the viewing window settings in order to create the exact same graph on your graphing calculator.

<table>
<thead>
<tr>
<th>XMin:</th>
<th>XMax:</th>
<th>XSc1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YMin:</th>
<th>YMax:</th>
<th>YSc1:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
13) The equation \( N = 2003 + 80(T - 2001) \) gives the number of birds in a zoo’s aviary in the year \( T \). The population of birds was introduced to the zoo in the year 1990.

(a) If we draw a graph of the equation, then...
Which variable would be along the horizontal axis?
What is the real-world meaning of variable along horizontal axis?

(b) If we draw a graph of the equation, then...
Which variable would be along the vertical axis?
What is the real-world meaning of variable along vertical axis?

(c) Suppose that \( T = 2001 \). What point does this reference on the graph of the equation?

(d) Write a complete sentence that explains the practical, real-world meaning (in terms of birds and years) of the point you found in part (c). Be sure to include all units in your sentence.

14) Below is a graph of \( f(x) = 100(x - 0.3)^2 + 1.4 \)

(a) Use the two labeled points on the graph to help you determine the value of the other tick marks on the graph. Label all the tick marks in the entire window.

(b) Enter the function in your graphing calculator in the Y= menu. Then complete the viewing window settings in order to create the exact same graph on your graphing calculator.

<table>
<thead>
<tr>
<th>XMin:</th>
<th>YMin:</th>
</tr>
</thead>
<tbody>
<tr>
<td>XMax:</td>
<td>YMax:</td>
</tr>
<tr>
<td>XSc:</td>
<td>YSc:</td>
</tr>
</tbody>
</table>

15) Suppose that \( D = -45H + 450 \) where \( D \) is the distance a car is from home (in miles) after \( H \) hours have passed.

(a) Use the formula to find \( D \) when \( H=5 \). Then write a sentence that interprets the practical meaning of the point.

(b) Use the formula to find \( H \) when \( D=5 \). Then write a sentence that interprets the practical meaning of the point.

(c) Use the formula to find \( D \) when \( H=0 \). Then write a sentence that interprets the practical meaning of the point.

(d) Use the formula to find \( H \) when \( D=0 \). Then write a sentence that interprets the practical meaning of the point.

(e) Use the table of values on your calculator to check the accuracy of your 4 points that you calculated.

(f) Plot the 4 points you have found on a well-labeled graph that you create BY HAND. Use the 4 points to help you fill in the shape of the graph in general.

(g) Put the formula in the calculator in Y=. Set an appropriate viewing window that allows you to see the graph that you created by hand. What window are you using?
16) Suppose that \( P = 3.75Q - 35,200 \) where \( P \) is the profit a company makes (in dollars) when selling a total of \( Q \) items.

(a) Use the formula to find \( P \) when \( Q=1000 \). Then write a sentence that interprets the practical meaning of the point.

(b) Use the formula to find \( Q \) when \( P=1000 \). Then write a sentence that interprets the practical meaning of the point.

(c) Use the formula to find the vertical intercept (the \( y \)-intercept). Then write a sentence that interprets the practical meaning of the point.

(d) Use the formula to find the horizontal intercept (the \( x \)-intercept). Then write a sentence that interprets the practical meaning of the point.

(e) Use the table of values on your calculator to check the accuracy of your 4 points that you calculated.

(f) Plot the 4 points you have found on a well-labeled graph that you create BY HAND. Use the 4 points to help you fill in the shape of the graph in general.

(g) Put the formula in the calculator in \( Y= \). Set an appropriate viewing window that allows you to see the graph that you created by hand. What window are you using?

17) Find the \( x \)-intercept and \( y \)-intercept of the following graph. Write your answers as ordered pairs (points).

18) Find the \( x \)-intercept and \( y \)-intercept of the following graph. Write your answers as ordered pairs (points).

19) Suppose that \( x - y = 6 \).

(a) Find the \( x \)-intercept. Write your answer as an ordered pair (a point).

(b) Find the \( y \)-intercept. Write your answer as an ordered pair (a point).

(c) Re-write the equation so that it is solved for \( y \).

20) Suppose that \( 3x - 7y = 63 \).

(a) Find the \( x \)-intercept. Write your answer as an ordered pair (a point).

(b) Find the \( y \)-intercept. Write your answer as an ordered pair (a point).

(c) Re-write the equation so that it is solved for \( y \).

21) Find the \( y \)-intercept of the function \( y = (x - 3)^2 - 9 \). Write your answer as an ordered pair.

22) Suppose the graph of \( y \) passes through the points \((-1,0)\) and \((0,-3)\) and \((3,0)\). Identify the \( x \)-intercept(s).
23) A table of \( y = ax^2 + bx + c \) is given. Use this table to solve \( ax^2 + bx + c = 0 \).

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) What is the left-most x-intercept?
(b) What is the right-most x-intercept?
(c) What are the solutions to this equation?

24) Graph \( y = 2x^2 - 6 \) using a standard viewing window. Sketch a well-labeled graph of the result.

25) Graph \( y = x^3 + 4x^2 \) using a standard viewing window. Sketch a well-labeled graph of the result.

26) Graph \( y = \frac{8}{x^2+2} \) using a standard viewing window. Sketch a well-labeled graph of the result.

27) Find an appropriate viewing window for \( y = x^2 + 40 \) for x-values between -2 and 2.

28) Find an appropriate viewing window for \( y = x^3 + 3x^2 - 105x \) for x-values between -15 and 15.

29) Use the intercepts of \( 5x + 4y = 20 \) and then use them to graph the equation.

30) Use the intercepts of \( 3x - 9 = 3y \) and then use them to graph the equation.
Section 1.4
Functions and Function Notation

Learning Outcomes:
- Identify if an input-output table represents a function or not
- Identify if a graph represents a function or not
- Identify if a formula represents a function or not
- Use function notation to evaluate and solve from a formula, table, or graph

Vocabulary:
- Function
- Function notation

What is a Function and what do we mean by Function Notation

In the previous section we reviewed two variable equations. Recall that in any two-variable equation we will consider one of the variables to be the INPUT variable (the independent variable), and we consider the other variable to be the OUTPUT variable (the dependent variable). If \( x \) is the independent variable and \( y \) is the dependent variable, then all solutions to the two variable equation will have the form \((x, y)\). Now we introduce the term FUNCTION.

- **Y is a function of \( x \)**, and we say \( y = f(x) \), if each \( x \) input results in ONLY ONE \( y \)-output value.

- **Y is NOT a function of \( x \)** if there exists at least one \( x \)-input value that results in TWO OR MORE \( y \)-output values.

- An input-output table is a function if each input maps to only one output. An input-output table is NOT a function if there exists at least one input-value that maps to multiple output values.

- A graph is a function if it passes the vertical line test (any vertical line will only intersect the function at one point at a time). A graph is NOT a function if it fails the vertical line test (a vertical line could intersect the function at more than one point at a time).

- An equation/formula is a function if it can be uniquely solved for the dependent variable. An equation/formula is NOT a function if the output variable equals multiple expressions in terms of the input.
**Example 1**: The table below gives some solutions to a two-variable equation with independent variable x and dependent variable y.

Solution:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>-30</td>
</tr>
<tr>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>-5</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>75</td>
</tr>
</tbody>
</table>

In this example, **y is a function of x** because every x-value is mapped to ONLY ONE y-output value.

Since it is a function, we can say that \( y = f(x) \).

In this example, if we input \( x = -5 \), then we will output \( y = 4 \).
So we can say that \( f(-5) = 4 \).

In this example, if we input \( x = 0 \), then we will output \( y = 18 \).
So we can say that \( f(0) = 18 \).

In this example, if we input \( x = 9 \), then we will output \( y = -30 \).
So we can say that \( f(9) = -30 \).

In this example, if we input \( x = 7 \), then we will output \( y = 75 \).
So we can say that \( f(7) = 75 \).

The other rows of the table are just repeating the same points.
So, there is no need to re-state them.

**Example 2**: The table below gives some solutions to a two-variable equation with independent variable x and dependent variable y.

Solution:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>9</td>
<td>-30</td>
</tr>
<tr>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>-5</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>-30</td>
</tr>
</tbody>
</table>

In this example, **y is NOT a function of x** because the x-value \( x = -5 \) is mapped to TWO OUTPUT VALUES.

Notice that none of the other points in the table pose a problem for y to be a function of x. Only the \( x = -5 \) poses a problem since \( x = -5 \) maps to two different output values.

**Example 3**: The graph below gives a graph of a two variable equation with independent variable x and dependent variable y.

In this example, **y is NOT a function of x** since this graph fails the vertical line test. There exists an x-value that maps to TWO OUTPUT VALUES.

Therefore, y is not a function of x.

This vertical can be moved anywhere and it will only intersect the graph in one place at a time.
Therefore it passes the vertical line test and so y is a function of x.
Example 4: The graph below gives the graph of a two variable equation with independent variable $x$ and dependent variable $y$.

In this example, $y$ is a function of $x$ since this graph passes the vertical line test. There exists no $x$-value that maps to TWO OUTPUT VALUES. Therefore, $y$ is a function of $x$.

Example 5: Each equation below is a function where $y = f(x)$. We know this because each equation can be solved uniquely for $y$.

- $y = 4(x - 3) + x^2 - 9$
  - In this case $y$ is already solved for $y$ uniquely and we can write $f(x) = 4(x - 3) + x^2 - 9$

- $3x - 7y = 9$
  - In this case we can solve for $y$ to write the equation as $y = \frac{3}{7}x - \frac{9}{7}$ and we can write $f(x) = \frac{3}{7}x - \frac{9}{7}$

- $\frac{y}{4} = 1.03^x$
  - In this case we can solve for $y$ to write the equation as $y = 4(1.03^x)$ and we can write $f(x) = 4(1.03)^x$

- $3x^2 - 5y + 2 = 3y + 4x - 8$
  - In this case we can solve for $y$ to write the equation as $y = \frac{3}{8}x^2 - \frac{1}{2}x + \frac{5}{4}$ and we can write $f(x) = \frac{3}{8}x^2 - \frac{1}{2}x + \frac{5}{4}$

Example 6: Each equation below can NOT be solved uniquely for $y$. Therefore, in each equation below we know that $y$ is NOT a function of $x$.

- $y^2 = x$
  - In this case if we solve for $y$ we will have TWO possibilities. We can say that $y = \sqrt{x}$ AND we can say that $y = -\sqrt{x}$. For example, if we input $x=9$, then the $y$ output-value could be 3 or -3. Therefore, when we input a value for $x$, we can get TWO POSSIBLE outputs for $y$. So $y$ is not a function of $x$.

- $|y| = x$
  - In this case if we solve for $y$ we will have TWO possibilities. For example, if we input $x=5$, then the $y$ output-value could be 5 or $y$ could be -5. Therefore, when we input a value for $x$, we can get TWO POSSIBLE outputs for $y$. So $y$ is not a function of $x$.

- $x = 3 + y^4$
  - In this case if we solve for $y$ we will have TWO possibilities. We can say that $y = \sqrt[4]{x - 3}$ and $y = -\sqrt[4]{x - 3}$. For example, if we input $x=19$, then $y$ output-value could be 2 or $y$ could be -2. Therefore, when we input a value for $x$, we can get TWO POSSIBLE outputs for $y$. So $y$ is not a function of $x$.
**Why do we use the notation f(x) instead of using the usual y?**

The first time you use function notation it may seem confusing and unnecessary. However, as you progress in your mathematical education, you will find that function notation not only makes the mathematics more understandable, but also you will find that function notation will be absolutely essential in upper-level mathematics. For now, at least observe that function notation allows us to easily relate the input and the output of a solution (whereas the non-function notation method did not allow for this). See example 7 below to see this comparison.

**Example 7:** Consider the side-by-side comparison of evaluating a two-variable equation with function notation and the same problem without function notation. Suppose that \( f(x) = 7x - 4 \). Evaluate the function when \( x=3 \).

**Solution with function notation:**

\[
f(3) = 7(3) - 4
\]

\[
f(3) = 17
\]

Glancing at the statement \( f(3) = 17 \) we can see that when we input \( x=3 \) we will output \( y=17 \). This tells me that \( (3,17) \) is a solution to the equation, and the point \( (3, 17) \) is a point on the graph. This is one bonus we get from using function notation. We can easily see the input and the output in the final answer.

I can clearly differentiate between the function \( f(x) \) and the value \( f(3) \). When we say \( f(x) \) we are referring to the entire function, and when we say \( f(3) \) we are referring only to the \( y \)-value of the function when \( x=3 \). The function notation allows us to clearly differentiate between specific \( y \)-values that occur on the function and the set of all \( y \)-values that the function produces. This is another bonus of using function notation.

**Solution without function notation:**

\[
y = 7(3) - 4
\]

\[
y = 17
\]

Glancing at the statement \( y = 17 \) ONLY tells me that \( y=17 \). This statement does not tell me anything about the \( x \)-value that produces this height on the graph. This is one difficulty with not using function notation.

The statement \( y = 17 \) can be confusing because at the beginning of the problem we stated that \( y = 7x - 4 \). To say that \( y = 7x - 4 \) means that \( y \) is a variable that changes depending on the value of \( x \). At the end of the problem we state that \( y = 17 \) which seems to imply that \( y \) is a constant value of 17. Which is it?? Is \( y = 7x - 4 \) or is \( y = 17 \)?? Is \( y \) a variable or is \( y \) a constant?? This is another difficulty with not using function notation.

**Example 8:** It is important to clearly differentiate between evaluating vs. solving when using function notation. In this example suppose that \( f(x) = 3x - 5 \). Solve or evaluate each of the following.

\[
f(-8)
\]

We know that \( y = f(x) \) we see that we are GIVEN \( x = -8 \) and we are FINDING \( y = f(-8) \).

So in the formula we replace \( x \) with \(-8\...\)

\[
f(x) = 3x - 5
\]

\[
f(-8) = 3(-8) - 5
\]

\[
f(-8) = -29
\]

This tells us that the point \((-8, -29)\) is on the graph.

\[
f(x) = -8
\]

We know that \( y = f(x) \) we see that we are GIVEN \( y = -8 \) we are FINDING \( x \).

So in the formula we replace \( y=f(x) \) with \(-8...\)

\[
f(x) = 3x - 5
\]

\[
-8 = 3x - 5
\]

add 5 on both sides

\[
-3 = 3x
\]

divide both sides by 3

\[
-1 = x
\]

This tells us that the point \((-1, -8)\) is on the graph.
Why do we Care about Functions?

Not all equations/graphs/tables are functions. But most of your algebra education will involve the study of functions while almost completely ignoring other types of equations/graphs/tables. Why are functions so important? The reason we focus so much time and energy on functions is that function can be used for modeling. **We can use functions to make predictions.**

**Example 9:** Below are two possible graphs that relate the input $T$ to the output $P$. In both graphs $T$ represents the number of years after opening, and $P$ represents the profit that Acme Widget company earns that year. In both graphs $T$ is the independent variable and $P$ is the dependent variable.

**Graph A:**

![Graph A](image)

**Graph B:**

![Graph B](image)

(a) Determine if graph A or graph B or both are functions or not. Explain.

**Solution:**
Graph A is not a function because it fails the vertical line test. Graph B is a function because it passes the vertical line test.

(b) Approximate the profit that Acme makes after 15 years using both graphs. Discuss which graph makes sense to use as a profit model and which does not make sense to use as a profit model. Explain.

**Solution:**
In Graph A, if we input $T=15$ we find TWO POSSIBLE OUTPUTS. One solution is approximately (15, 400) and another solution is approximately (15,600). In real-world terms, this would literally mean that in the 15th year Acme’s profit was $400 AND in the 15th year Acme’s profit was $600. This doesn’t make sense in the real world. You can’t have two different profits at the same time! This model would be completely useless for the company to use if it wanted to make predictions about the profit in the future, or understand what the profit was in the past.

In Graph B, if we input $T=15$ we find that the solution is approximately (15, 100). In real-world terms this would mean that in the 15th year Acme had a profit of $100. Notice that Graph B would be very useful to the Acme company since they could use it to make predictions about the profit at any time.

This example helps us see the reason that a function is useful for real-world models. We can use functions to help us make predictions. We would NOT want to use something that is not a function because then we have the possibility of getting TWO OR MORE predictions instead of just one prediction like we want. Acme doesn’t want a graph that tells them two different possibilities!
Section 1.4 Written Practice and Reflection

Functions and Function Notation

1. For each table of values, determine if \( y \) could be a function of \( x \) or not. If it is not a function, then change ONE THING in the table to make it a function.

<table>
<thead>
<tr>
<th>Table A</th>
<th>Table B</th>
<th>Table C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>( y )</td>
<td>( x )</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>-15</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>-118</td>
<td>7</td>
</tr>
</tbody>
</table>

2. The table below lists some solutions to the function \( f(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Use the table to find \( f(4) \).

(b) Use the table to solve the equation \( f(x) = 4 \)

3. If \( g(x) \) is a function and we know that \( g(2) = 7 \), then what point do you know is on the graph of \( g(x) \)?

4. Suppose \( f(x) = 2x - 8 \)

(a) Find \( f(3) \). Show your work.

(b) Solve the equation \( f(x) = 3 \). Show your work.

5. Evaluate/solve each. Show all work.

a) \( f(x) = 7x - 3 \) \quad f(-2) = \underline{\phantom{000}} \quad f(x) = \frac{1}{9} \underline{\phantom{000}}

b) \( g(x) = 4(x - 7) + \frac{1}{3} \) \quad g(12) = \underline{\phantom{000}} \quad g(x) = 1 \underline{\phantom{000}}

c) \( h(x) = \frac{2}{7}x - 4 \) \quad h(16) = \underline{\phantom{000}} \quad h(x) = 13 \underline{\phantom{000}}
6. Below is a graph of the function \( g(x) \).

(a) Use the graph to find \( g(0) \).

(b) Use the graph to solve the equation \( g(x) = 0 \).

7. Below is the graph of the function \( j(x) \).

(a) Use the graph to find \( j(0) \).

(b) Use the graph to solve the equation \( j(x) = 0 \).

(c) Use the graph to find \( j(67.5) \).

(d) Use the graph to solve the equation \( j(x) = -7.5 \).

8. The table below gives the profit, \( P \), of a company (in dollars) when the company sells \( Q \) items.

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-15,000</td>
</tr>
<tr>
<td>200</td>
<td>-7,500</td>
</tr>
<tr>
<td>400</td>
<td>0</td>
</tr>
<tr>
<td>600</td>
<td>7,500</td>
</tr>
<tr>
<td>800</td>
<td>15,000</td>
</tr>
</tbody>
</table>

\( P(Q) = 0 \) (solve or evaluate and then write a complete sentence that gives the real world meaning of the calculation)

\( P(0) = \ldots \) (solve or evaluate and then write a complete sentence that gives the real world meaning of the calculation)

9. The weight of a man, \( W \), can be approximated by the function \( W(H) = 5(H - 69) + 150 \) where \( H \) is the man’s height in inches, and \( W \) is the man’s weight in pounds.

\( W(72) = \ldots \) (solve or evaluate and then write a complete sentence that gives the real world meaning of the calculation)

\( W(H) = 175 \) (solve or evaluate and then write a complete sentence that gives the real world meaning of the calculation)
10. The height of an object, H, in feet is given in the graph after t second have passed. In the graph the dependent variable is the height of the object in feet, and the independent variable is the number of seconds after it has been launched in the air.

\[ H(0) = \underline{\hspace{2cm}} \] (solve or evaluate and then write a complete sentence that gives the real world meaning of the calculation)

\[ H(t) = 0 \] (solve or evaluate and then write a complete sentence that gives the real world meaning of the calculation)

11. Use the calculator to calculate each. **Show the substitution step**, and then use your calculator to get the correct answer.

   (a) Find \( f \left( \frac{1}{4} \right) \) if \( f(x) = x^2 - 7x - 8 \). Give answer as an exact fraction, and then as a decimal rounded to 4 decimal places.

   (b) Find \( g(1500) \) if \( g(t) = \frac{3-\sqrt{t^2+2}}{t-7} \). Round your answer to four decimal places.

   (c) Find \( h(-5) \) if \( h(w) = -w^4 - w^2 - w + 8 \).

12. Suppose the function \( H(t) = -16t^2 + 100t + 250 \) gives the height of an object above ground as time passes in seconds. In this function t represents the number of seconds since the object has been launched, and H represents the height of the object above ground in feet.

   (a) Find \( H(0) \) and then write a complete sentence that interprets the real world meaning of what you have found.

   (b) Find \( H(3) \) and then write a complete sentence that interprets the real world meaning of what you have found.

   (c) Find \( H(5) \) and then write a complete sentence that interprets the real world meaning of what you have found.

13. Suppose the function \( C(Q) = \frac{3Q+15000}{Q} \) relates the average cost of production to the number of items produced. In this function C represents the average cost per item (dollars per item), and Q represents the number of items the company is producing.

   (a) Find \( C(1) \) and then write a complete sentence that interprets the real world meaning of what you have found.

   (b) Find \( C(2000) \) and then write a complete sentence that interprets the real world meaning of what you have found.

   (c) Find \( C(10,000) \) and then write a complete sentence that interprets the real world meaning of what you have found.
14. Suppose the function \( P(t) = \frac{14}{1+13e^{-0.0121t}} \) relates the population of the world, in billions of people, to the number of years since 1800. In this function \( P \) represents the population of the world in billions of people, and \( t \) represents the number of years since 1800 (so \( t=0 \) means the year 1800, and \( t=1 \) means the year 1801, etc.).

(a) Find \( P(0) \) and then write a complete sentence that interprets the real world meaning of what you have found.

(b) Find \( P(150) \) and then write a complete sentence that interprets the real world meaning of what you have found.

(c) Find the population of the world in 2011 according to this model. Give the function notation for this calculation, and then write a complete sentence that interprets the real world meaning of what you have found.

15. Write a complete sentence or two that explains how a person can look at a graph and decide if the graph is a function or not a function.

16. Does the table describe \( y \) as a function of \( x \)? Explain.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-6</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-6</td>
</tr>
</tbody>
</table>

17. Determine whether the following rule defines \( y \) as a function of \( x \). Explain.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>7</th>
<th>4</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>-4</th>
<th>-7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>49</td>
<td>16</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>16</td>
<td>49</td>
</tr>
</tbody>
</table>

18. Evaluate the function \( r(t) = -t^3 - 3t^2 + t + 8 \) at the indicated values.
   (a) \( r(1) \)
   (b) \( r(-2) \)

19. Given the function \( f \) described by \( f(x) = \frac{x-4}{9x-13} \), find the function value \( f(10) \).

20. Evaluate the function \( s(t) = \sqrt{t} + 2 \) at the indicated values.
   (a) \( s(-2) \)
   (b) \( s(23) \)

21. Suppose the graph of \( f(x) \) passes through the points \((2,4)\) and \((3,4)\) and \((7,2)\) and \((4,4)\). Then find each of the following.
   (a) \( f(2) \)
   (b) \( f(4) \)
22. Use the graph of \( f \) to evaluate the expressions.

\[(a) \ f(-1) \quad (b) \ f(1)\]

23. Use the graph of \( f(x) \) to find all values of \( x \) such that \( f(x) = 1 \).

24. Use the equation of \( g(x) \) to find all values of \( x \) such that \( g(x) = 2 \).

\[ g(x) = \frac{2}{3}x - 15 \]

25. Given that \( g(x) = \frac{x-1}{x+7} \), find each of the following.

\[(a) \ g(2) \quad (b) \ g(1) \quad (c) \ g(-7) \quad (d) \ g(-14.25)\]

26. Suppose that \( g(x) = \frac{x}{\sqrt{9-x^2}} \). Find each of the following.

\[(a) \ g(0) \quad (b) \ g(-3) \quad (c) \ g(5) \quad (d) \ g\left(\frac{2}{3}\right)\]

27. Let \( g(x) = -3x - 2 \). Find \( x \) if \( g(x) = 1 \).

28. Let \( g \) be a function defined by \( g(x) = 9x - 5 \). Determine the value of \( x \) for which \( g(x) = 13 \).

29. Let \( g(x) = 2x + 5 \). Find \( x \) if \( g(x) = 3 \).
### 30. Use the table below to find the indicated function values.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>0</td>
</tr>
<tr>
<td>-4</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>

(a) \( f(-8) \)  
(b) \( f(8) \)  
(c) For what value of \( x \) is \( f(x) = 10 \)?

### 31. Use the table below to find the indicated function values.

<table>
<thead>
<tr>
<th>x</th>
<th>h(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>10</td>
</tr>
<tr>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) \( h(-5) \)  
(b) \( h(5) \)  
(c) For what value of \( x \) is \( h(x) = 5 \)?
Section 1.5
Domain and Range of a Function

Learning Outcomes:
- Write sets of real numbers using interval notation and inequality notation
- Identify the domain of a function from the graph
- Identify the domain of a function from a formula
- Identify the range of a function from the graph

Vocabulary:
- Domain of a function $f(x)$
- Range of a function $f(x)$

❖ Interval Notation and Inequality Notation

There are two types of notation commonly used for describing sets of real numbers.

- **Inequality notation** describes the set of values using the symbols $<$ and $>$ and $\leq$ and $\geq$.
  - $x < a$ represents “all $x$-values that are less than $a$”
  - $x > a$ represents “all $x$-values that are more than $a$”
  - $x \leq a$ represents “all $x$-values that are less than or equal to $a$”
  - $x \geq a$ represents “all $x$-values that are greater than or equal to $a$”

- **Interval notation** describes the set of values using brackets and parenthesis. Brackets are used when set includes the end-point value, and parenthesis are used for strict inequality.
  
  $(-\infty, a)$ represents “all values that are less than $a$”
  $(-\infty, a]$ represents “all values that are less than or equal to $a$”
  
  $(a, \infty)$ represents “all values that are greater than $a$”
  $[a, \infty)$ represents “all values that are greater than or equal to $a$”
  
  $[a, b]$ represents “all values that are greater than or equal to $a$, and also less than or equal to $b$”
  $[a, b)$ represents “all values that are greater than or equal to $a$, and also less than $b$”
  
  $(a, b]$ represents “all values that are greater than $a$, and also less than or equal to $b$”
  $(a, b)$ represents “all values that are greater than $a$, and also less than $b$”

**When writing interval notation, use a parenthesis for strict inequality, and use a bracket for $\leq$ and $\geq$.

**When writing interval notation, write the values as LOW, HIGH surrounded by either parenthesis or brackets.
Example 1 (using interval and inequality notation):
Describe the bolded set of x-values on the number line below using interval notation, inequality notation, and in words.

In words:
The bolded set of x-values is the set of numbers that are less than 27.

Inequality Notation: \( x < 27 \)
Interval Notation: \((-\infty, 27)\)

Example 2 (using interval and inequality notation):
Describe the bolded set of x-values on the number line below using interval notation, inequality notation, and in words.

In words:
The bolded set of x-values is the set of numbers that are greater than 27 and also less than or equal to 32.

Inequality Notation: \( 27 < x \leq 32 \)
Interval Notation: \((27, 32]\)

Example 3 (using interval and inequality notation):
Describe the bolded set of x-values on the number line below using interval notation, inequality notation, and in words.

In words:
The bolded set of x-values is the set of numbers that are greater than or equal to 8.

Inequality Notation: \( 8 \leq x \) or \( x \geq 8 \)
Interval Notation: \([8, \infty)\)
**Example 4 (using interval and inequality notation):**
Describe the bolded set of y-values on the number line below using interval notation, inequality notation, and in words.

In words:
The bolded set of y-values is the set of numbers that are greater than or equal to 14.

Inequality Notation:
\[ y \geq 14 \]

Interval Notation:
\([14, \infty)\)

**Example 5 (using interval and inequality notation):**
Describe the bolded set of y-values on the number line below using interval notation, inequality notation, and in words.

In words:
The bolded set of y-values is the set of numbers that are greater than 97 and less than or equal to 138.

Inequality Notation:
\[ 97 < y \leq 138 \]

Interval Notation:
\((97,138]\)
The **domain** of a function is the ENTIRE SET of numbers that you can INPUT into the function that will produce defined real numbers. Some functions can have any number at all “plugged” into them. But some functions will not be defined when you “plug” in certain numbers. The domain of a function in a real-life contextual problem can be restricted even more because in the real-world you may not be able to input every value (because not every value makes sense in the real world for all problems).

**Example 6 (domain from graph):**
Consider the function pictured in the graph. If the entire graph is shown, then we see that the graph only occurs between x=−5 and x=4. This means that when we consider values outside of the span from x=−5 to x=4, we will NOT find the graph! For example, we can’t find any graph when x=−10, and we can’t find any graph when x=6. That means that the function is not producing real-valued outputs when x=−10 or when x=6 (otherwise there would be graph at those values!). So, the only input values for which the function produces real-valued outputs is from x=−5 to x=4. This means the domain of this function is all real numbers between x=−5 and x=4. The domain is −5 ≤ x ≤ 4. This can also be written as [−5,4].

**Example 7 (domain from graph):**
Consider the function pictured in the graph to the left. If we assume that the graph continues the pattern that we see forever, then we see that the graph extends forever to the left, and forever to the right. So, no matter which value of X we input, we will find the graph at that x-value! This means the domain of this function is all real numbers. The domain is −∞ < x < ∞. This can also be written as (−∞,∞).
**Example 8 (domain from graph):**
Consider the function pictured in the graph to the right. If we assume that the graph continues the pattern that we see forever, then we see that the graph extends forever to the left, and forever to the right. So, no matter which value of $X$ we input, we will find the graph at that $x$-value! This means the domain of this function is all real numbers. The domain is $-\infty < x < \infty$. This can also be written as $(-\infty, \infty)$.

**Example 9 (range from graph):**
Consider the function pictured in the graph. If the entire graph is shown, then we see that the graph only occurs between $y=-1.5$ and $y=3.4$. This means that when we consider values outside of the span from $y=-1.5$ to $y=3.4$, we will NOT find the graph! For example, we can’t find any graph when $y=-10$, and we can’t find any graph when $y=6$. That means that the function is not producing real-valued outputs when $y=-10$ or when $y=6$ (otherwise there would be graph at those values!). So, the only output values for which the function produces real-valued outputs is from $y=-1.5$ to $y=3.4$. This means the range of this function is all real numbers between $y=-1.5$ and $y=3.4$. The range is $-1.5 \leq y \leq 3.4$. 

The range of a function is the ENTIRE SET of numbers that the function will produce (OUTPUT) when you plug in numbers from the domain. Some functions can output any number. But some functions will only output a certain set of numbers.
The function above exists up-and-down from $-1$ up to $1$.

**Example 10 (range from graph):**
Consider the function pictured in the graph to the left. If we assume that the graph continues the pattern that we see forever, then we see that the height of the graph bounces back and forth between a height of $-1$ and a height of $1$. This means the range of this function is $-1 < y < 1$.

**Example 11 (range from graph):**
Consider the function pictured in the graph. If we assume that the graph continues the pattern that we see forever, then we see that the graph extends forever upward. But the lowest the graph gets is $y=5$. This means the range of this function is $y \geq 5$.

The function above exists up-and-down from $5$ up to $\infty$. 
Example 12 (domain from equation):

Consider the function $f(x) = \frac{3}{x-5}$. To find the domain of $f(x)$ we need to find all the input values that will produce defined real numbers out of the function. For example, I could plug in $x=6$ and the output would be $f(6) = \frac{3}{6-5} = \frac{3}{1} = 3$. So, when we plug in $x=6$ we get an output of $f(6) = 3$. Since $y=3$ is a real number output, it was fine to plug in $x=6$. But, if we plugged in $x=5$ a different situation occurs: $f(5) = \frac{3}{5-5} = \frac{3}{0} = \text{undefined}$. Since division by 0 is undefined, we can’t plug in $x=5$ into this function. When we plug in $x=5$ the output is not a real number! This means that $x=5$ is NOT in the domain. But, if you plug in any other number (numbers bigger than 5 or smaller than 5), then you will get real number outputs. So, every number other than 5 is valid to input. This means that the domain of the function $f(x) = \frac{3}{x-5}$ is all real $x$-values except 5. The domain is $x \neq 5$.

Example 13 (domain from equation): Consider the function $f(x) = \sqrt{x}$. To find the domain we need to find all the input values that will produce defined real numbers out of the function. For example, I could plug in $x=9$ and the output would be $f(9) = \sqrt{9} = 3$. So, when we plug in $x=9$ we get an output of $f(9) = 3$. Since $y=3$ is a real number output, it was fine to plug in $x=9$. But, if we plugged in $x= -4$ a different situation occurs: $f(-4) = \sqrt{-4}$ which is not a real number (the square root of a negative number is not a real number). Since the square root of negative numbers are not real number outputs, we can’t plug $x= -4$ into this function. When we plug in $x= -4$ the output is not a real number! This means that $x= -4$ is NOT in the domain. In fact, any negative number would have produced this same issue. Since we can’t take the square root of ANY negative number, we wouldn’t be able to “plug” in ANY negative numbers into this function! So, all numbers other than negative numbers are valid to plug in to the function. This means that the domain of the function $f(x) = \sqrt{x}$ is all real numbers that are not negative (all non-negative real numbers). The domain is $x \geq 0$.

Example 14 (domain from equation): Consider the function $f(x) = 3x + 1$. No matter what number you select for $x$, you can plug it in to the function, and then you can calculate the output. The output will always be three times $x$ and then add 1. There is no $x$ value that would not produce real valued outputs. There will never be an $x$-value that produces division by 0 or square roots of negatives. So, the domain is all real numbers.
Example 15 (domain in context)

Consider the function $V(h)$ that gives the vertical height of a pendulum that swings back and forth. Suppose the pendulum swings 10 cm to the left, and 10 cm to the right, and then continues to swing back and forth until it stops. $h$ is the horizontal distance that the pendulum is from the center position, and $V$ is the vertical height that the pendulum is from the ground. In this case, negative $h$-values represent the pendulum swinging to the left of center, and positive $h$-values represent the pendulum swinging to the right of center (and $h=0$ means the pendulum is at the center). Then the only input values we could consider would be the values between $h = -10$ up to $h = 10$ because we can only input horizontal distances from $-10$ cm up to $10$ cm. So the domain is $-10 \leq h \leq 10$.

Example 16 (domain in context): Consider the function $V(h)$ that gives the vertical height of a pendulum that swings back and forth. Suppose the pendulum swings 10 cm to the left, and 10 cm to the right, and then continues to swing back and forth until it stops. $h$ is the horizontal distance that the pendulum is from the center position, and $V$ is the vertical height that the pendulum is from the ground. In this case, negative $h$-values represent the pendulum swinging to the left of center, and positive $h$-values represent the pendulum swinging to the right of center (and $h=0$ means the pendulum is at the center). Then the only input values we could consider would be the values between $h = -10$ up to $h = 10$ because we can only input horizontal distances from $-10$ cm up to $10$ cm. So the domain is $-10 \leq h \leq 10$.

Example 17 (domain in context): Consider the function $P(Q)$ that gives the profit of a company when selling $Q$ items in a month. Suppose this company is only able to produce at most 150,000 items in any given month due to production limitations. Then the function $P(Q)$ would have a domain of $0 \leq Q \leq 150,000$ because the MOST the company can produce is 150,000 items and the LEAST the company can produce is 0 items. So we can only consider input-values ranging from 0 items up to 150,000 items.

Example 18 (domain in context): Consider the function $P(t)$ that gives the population of a country $t$ years after today. Then the input-values, $t$, represent the number of years after today’s date. Presumably this country also had a population last year, and the year before, and so on. Therefore, we may want to input values such as $t = -1$ which would represent 1 year before today, and $t = -2$ which would represent 2 years before today. We may also want to determine the population of the country in the future by inputting values such as $t = 1$ and $t = 2$ which would represent 1 year from today and 2 years from today respectively. Therefore, we may want to consider both negative and positive input-values, and so the domain may include both negative and positive numbers. How low we should consider, and how high we should consider may depend upon the given country. For example, it would not make sense to consider $t = -2000$ if we were talking about the United States because the United States did not exist 2000 years ago! And, it would probably not make sense to consider values like $t = 500$ because we could not possibly know all the factors that will influence the population between now and 500 years from now! So, using this function $P(t)$ should probably be restricted to years NEAR today. We would try to restrict our input values only to values where the function $P(t)$ can still be used as a good model for the population.
Section 1.5 Written Practice and Reflection
Domain and Range of a Function

1) Write each set in interval notation. Also sketch and shade a real number line to graphically display the given set.
   (a) \(7 < x \leq 12\)   (b) \(-9 \leq x\)   (c) \(x < 5\)   (d) \(-4 \leq x < -1\)

2) Write each set in inequality notation. Also sketch and shade a real number line to graphically display the given set.
   (a) \((-2, 1)\)   (b) \((−∞, 4]\)   (c) \([5, 9)\)   (d) \([−15, ∞)\)

3) Write each set of bolded x-values in both interval notation and inequality notation.
   (a) \(x = 34\)   (b) \(x = -13\)   (c) \(x = 1\)   (d) \(x = 11\)

4) Write each set of bolded y-values in both interval notation and inequality notation.
   (a) \(y = 17\)   (b) \(y = 135\)   (c) \(y = -8\)   (d) \(y = -2\)

5) In your own words, accurately explain the meaning of the **domain** of a function \(y = f(x)\).

6) In your own words, accurately explain the meaning of the **range** of a function \(y = f(x)\).

7) Suppose that \(g(t) = \frac{t^2 + 8}{t-5}\)
   (a) Find \(g(-4)\).
   (b) What is the domain of \(g(t)\)?

8) Suppose that \(f(x) = 3x^2 - 5x + 1\).
   (a) Find \(f(-4)\).
   (b) What is the domain of \(f(x)\)?

9) Suppose that \(h(x) = \sqrt{x + 5}\).
   (a) Find \(h(-4)\).
   (b) What is the domain of \(h(x)\)?

10) Suppose that \(P(Q) = 3.5Q - 150\).
    (a) Find \(P(-4)\).
    (b) What is the domain of \(P(Q)\)?
11) What is the domain and range of the function in the graph below?

![Graph](image)

12) What is the domain and range of the function in the graph below? Assume the graph gets closer and closer to the x-axis on the right, but it never actually hits the x-axis.

![Graph](image)

13) Identify the domain and identify the range of each function graphed below.

![Graphs](image)

(a)  
(b)  
(c)  

14) Find the domain of the function \( p(x) = 2x - 9 \).

15) Find the domain of the function \( f(x) = x^2 - 2 \).

16) Find the domain of the function \( f(x) = \frac{3}{x+1} \).

17) Find the domain of the function \( f(x) = \frac{x}{x^2+4} \).

18) Find the domain of the function \( f(x) = \sqrt{6 - x} \).
A function maps each input value from the domain to one specific output value in the range. Functions, therefore, are defining a specific relationship between \( x \) and \( y = f(x) \). The remainder of this book will focus on a few of the simplest patterns that can exist between variables: the linear pattern, the exponential pattern, and the quadratic pattern.

- **Linear functions** have a constant amount added to the \( y \)-value every time \( x \) increases by one unit.

- **Exponential functions** have a constant amount multiplied to the \( y \)-value every time \( x \) increases by one unit.

- **Quadratic functions** either increase to a maximum point and then decrease, or they decrease to a minimum point and then increase. There are more details to quadratics that you will learn as you proceed further in algebra.

Continue reading for more details on each function.
As $x$ increases by 1 unit we ADD a fixed, constant amount to the $y$-value.

The graph of a linear function is a straight line (not a vertical line because that would not be a function). Linear functions could have a graph that is increasing or decreasing, or is horizontal.

Increasing,  
$slope$ is positive  
Decreasing,  
$slope$ is negative  
Horizontal,  
slope is 0

A linear function has a CONSTANT RATE OF CHANGE which we call the constant $slope$. The slope of a linear function is the change in the $y$-value divided by the change in the $x$-value. In a linear function, this change in $y$-value divided by change in $x$-value will always be a constant, fixed value. In mathematical notation we write “change in $y$-value divided by change in $x$-value is the slope” as follows:

$$\frac{\Delta y}{\Delta x} = \text{change in y value} \div \text{change in x value}$$

Positive slope means the function is increasing. Negative slope means the function is decreasing. Remember to always travel from left-to-right while asking if the graph is going up or if the graph is going down.

If the line is horizontal the slope is 0 because

$$\frac{\Delta y}{\Delta x} = \frac{\text{travel up 0 units}}{\text{travel over 1 unit}} = 0.$$ 

If the line is vertical, then it is not a function and it has an undefined slope because

$$\frac{\Delta y}{\Delta x} = \frac{\text{travel up 1 unit}}{\text{travel over 0 units}} = \text{undefined}.$$ 

A linear function will have exactly one $y$-intercept which is the point on the graph where $x=0$. A linear function may have one $x$-intercept which is the point on the graph where $y=0$. Horizontal lines will not have an $x$-intercept (except the function $f(x)=0$ which is the $x$-axis).
Note: We will consider ONLY exponential functions that have a positive y-intercept.

**Quick Review of Percentages**

<table>
<thead>
<tr>
<th>Changing percentages into decimals:</th>
<th>Percent MORE than:</th>
<th>Percent LESS than:</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5% = 0.035</td>
<td>3.5% MORE than 75</td>
<td>3.5% LESS than 75</td>
</tr>
<tr>
<td>103.5% = 1.035</td>
<td>= 100% of 75 + 3.5% of 75</td>
<td>=100% of 75 – 3.5% of 75</td>
</tr>
<tr>
<td>96.5% = 0.965</td>
<td>= 103.5% of 75</td>
<td>=96.5% of 75</td>
</tr>
<tr>
<td></td>
<td>= 1.035× 75</td>
<td>=0.965× 75</td>
</tr>
</tbody>
</table>

- In an exponential function, as x increases by 1 unit, we MULTIPLY a constant amount to the y-value (called the **factor**).

\[
\text{factor} = \frac{f(x+1)}{f(x)} = \frac{\text{y-value on the right}}{\text{y-value on the left}}
\]

for any two points that are exactly 1 x-unit away from each other

- In an exponential function, as x increases by 1 unit, the y-value increases or decreases by a fixed percentage which we call the **PERCENTAGE RATE**.

\[
decimal \text{ rate} = decimal \text{ factor} - 1 \quad \text{and} \quad decimal \text{ factor} = decimal \text{ rate} + 1
\]

- Factor greater than 1 ↔ function is increasing. Percentage rate is positive ↔ function is increasing.
- Factor between 0 and 1 ↔ function is decreasing. Percentage rate is negative ↔ function is decreasing.

- The graph of an exponential function is increasing and concave up (y-values increase larger and larger amounts at higher x-values) or it is decreasing and concave up (y-values decrease smaller and smaller amounts at higher x-values). Exponential functions have one y-intercept which is the point on the graph where x=0.

- The graph of the exponential function will never reach a height of y=0 and it will never become negative. In other words, the exponential function is always positive in the output (the height on the graph is always above the x-axis).

- Note that the y-values are always positive because, in this course, we only consider exponential functions with a positive y-intercept. In later courses you will expand this definition of exponential function to consider a wider range of exponential functions such as exponential functions that start below the x-axis.
• The graph of a quadratic function is called a **parabola**. It is either open up, or open down.

  ![Parabola open up](image1)

  ![Parabola open down](image2)

• If the parabola is open up, then the graph will decrease down to a **minimum point called the vertex**, and then increase from there.

• If the parabola is open down, then the graph will increase up to a **maximum point called the vertex**, and then decrease from there.

• Quadratic functions have one y-intercept which occurs where the x-value is 0.

• Quadratic functions can have zero, or one, or two x-intercepts which is point(s) on the graph where y=0.

• Quadratic functions are symmetric about the vertical line that passes through the vertex. We call this vertical line the **axis of symmetry**. If the vertex is the point \((a,b)\), then the vertical line of symmetry is the line \(x=a\).
For examples 1-6 do the following:

a) Decide if the graph is linear, exponential, or quadratic.

b) Identify the important features of the function:
   - If it is linear, identify if it is increasing or decreasing, the slope and y-intercept and x-intercept.
   - If it is exponential, identify if it is increasing or decreasing, the y-intercept and factor and percent rate of growth or decay. Label the y-intercept on the graph with an ordered pair.
   - If it is quadratic, identify if it is open up or open down, the y-intercept, the vertex, the vertical line of symmetry, and any x-intercept(s).

Example 1:

Solution:
This function has the shape of a quadratic function that is open down. The y-intercept is the point (0,5/3) which is approximately (0,1.67).

There are two x-intercepts. They are \((-1,0)\) and \((5,0)\).

The vertex of this quadratic function is the MAXIMUM point on the graph. It is the point \((2,3)\). Therefore, the vertical line of symmetry is the line \(x=2\).

Example 2:

Solution:
This function has the shape of a linear function that is increasing. That means the slope will be positive. The y-intercept is the point \((0, -3)\).

The x-intercept is the point \((2,0)\).

The slope of this function is \(\frac{\Delta y}{\Delta x} = \frac{\text{increase 3 units on } y}{\text{increase 2 units on } x} = \frac{3}{2} = \frac{1.5}{1} = 1.5\). We can use ANY two points on the graph to travel between in order to determine the slope. We found the points \((0, -3)\) and \((2,0)\) already, and so they were easiest to use in this case.

This tells us that every time we increase the x-value by 1 unit, we add 1.5 to the y-value.
Example 3:

Solution:
This function has the shape of an exponential function that is increasing. That means the factor will be greater than 1. In other words, every time $x$ increases by 1 unit, we will multiply the $y$-value by a number greater than 1. This also means that the percent rate will be positive. In other words, every time $x$ increases by 1 unit, the $y$-value will increase by a fixed percentage.

The $y$-intercept is the point $(0, 3)$. There is no $x$-intercept since exponential functions do not have $x$-intercepts.

The factor of this function is found by first finding two points on the graph that are one $x$-unit away from each other. We will use the points $(0,3)$ and $(1,4.5)$ since they are clear and easily found on this graph.

$$factor = \frac{f(1)}{f(0)} = \frac{4.5}{3} = 1.5$$

This tells us that every time the $x$-value increases by 1 unit, we multiply the $y$-value by 1.5.

The percent rate of this function is found using the factor.

$$decimal \ rate = decimal \ factor - 1 = 1.5 - 1 = 0.5$$
$$percentage \ rate = 50\%$$

This tells us that every time the $x$-value increases by 1 unit, we increase the $y$-value by 50%.
Example 4:

Solution:
This function has the shape of an exponential function that is decreasing.
That means the factor will be between 0 and 1, and the percent rate will be negative.

The y-intercept is the point (0, 100). There is no x-intercept since exponential functions do not have x-intercepts.

The factor of this function is found by finding two points on the graph that are one x-unit away from each other. We will use the points (0,100) and (1,90):

\[
\text{factor} = \frac{f(1)}{f(0)} = \frac{90}{100} = 0.9 = 90\%
\]

This tells us that every time the x-value increases by 1 unit, we multiply the y-value by 0.9. In other words, every time we increase the x-value by 1 unit, we multiply the y-value by 90%.

The percent rate of this function is found using the factor.

\[
\text{rate} = \text{factor} - 1 = 0.9 - 1 = -0.1 = -10\%
\]

This tells us that every time the x-value increases by 1 unit, we decrease the y-value by 10%.

Notice that decreasing y by 10% is the same as multiplying y by 90%.
Example 5: The function, \( Q = f(x) \), below relates the number of items that a company sells, \( Q \), to the price per item, \( x \).

(a) What is the \( y \)-intercept of the function? Give a real-world interpretation of the meaning of that point.

Solution: The \( y \)-intercept is the point \( (0, 100000) \). All points on this function have the form \((\text{price per item, number sold})\). In real-world terms this tells us that when each item costs \( \$0 \) (when they are free), they will sell 100,000 items.

(b) What is the \( x \)-intercept of the function? Give a real-world interpretation of the meaning of that point.

Solution: The \( x \)-intercept is the point \( (6.67, 0) \). In real-world terms this tells us that when each items costs \( \$6.67 \), then they will sell no items at all.

(c) Find \( f(4) \). Then interpret the real-world meaning of the point you have found.

Solution: On the graph we see that \( f(4) = 40,000 \). In real-world terms this tells us that when each item costs \( \$4.00 \), then they will sell 40,000 items.

(d) What is the slope of the line? Give a real-world interpretation of the meaning of the slope.

Solution: To find the slope, we can use any two points on the line. We will use the point \( (0, 100000) \) and the point \( (4, 40000) \).

The slope of the line is \( \frac{\Delta \text{output}}{\Delta \text{input}} = \frac{\text{down 60,000}}{\text{over 4}} = \frac{-60,000}{4} = -15,000 \).

On the graph, every time the \( x \)-value increases by 1, then the \( y \)-value decreases by 15,000.

In real-world terms this tells us that every time the price of the items increases by \( \$1.00 \), then they will sell 15,000 fewer items.

Example 6: The function, \( D = f(x) \), below relates the distance that two planes are from each other (in miles), where \( x \) is the number of minutes that have passed since one of the planes took off.

(a) What is the \( y \)-intercept of the function? Give a real-world interpretation of the meaning of that point.

Solution: The \( y \)-intercept is the point \( (0, 30) \). All points on this function have the form \((\text{minutes, miles apart})\). Therefore, this point tells us that the two planes are 30 miles apart to start (when 0 minutes have passed).

(b) What is the vertex of the function? Give a real-world interpretation of the meaning of that point.

Solution: The vertex of the function is the point \( (11, 7) \). The vertex of this graph is the \( \text{MINIMUM} \) point on the graph. It tells us that after 11 minutes, the two planes are 7 miles apart. And this is the \( \text{CLOSEST} \) (the minimum distance) that the two planes will get to one another.

(c) The function doesn’t have any \( x \)-intercepts. Explain what it would mean in real-world terms if the function did have an \( x \)-intercept.

Solution: If there were an \( x \)-intercept, then that would be a point where the \( y \)-value is 0. If the \( y \)-value is 0 in this function, that would tell us that the two planes are 0 miles apart...which means the two planes crashed into each other!
For examples 7-11 do the following:

a) Decide if the function in the table is linear, exponential, or neither.

b) Identify the important features of the function:
   - If it is linear, identify if it is increasing or decreasing, the slope and y-intercept and x-intercept.
   - If it is exponential, identify if it is increasing or decreasing, the y-intercept and factor and the rate as a percent. Label the intercept on the graph with an ordered pair.

**Example 7:**

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-7.5</td>
</tr>
<tr>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>-4.5</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-1.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

**Solution:**

This function is linear because there is a constant rate of change.

Every time x increases by 1 unit, the y-value increases by 1.5 units.

The y-intercept is the point (0, -3). They x-intercept is the point (2, 0).

Your plotted graph should look similar to this.

**Example 8:**

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>.88888888889</td>
</tr>
<tr>
<td>-2</td>
<td>1.3333333333</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4.5</td>
</tr>
<tr>
<td>2</td>
<td>6.75</td>
</tr>
<tr>
<td>3</td>
<td>10.125</td>
</tr>
</tbody>
</table>

**Solution**

This is an exponential function because every time x increases by 1 we multiply the y-value by the number 1.5.

1.33333333333/0.88888888888 = 1.5
2/1.33333333333 = 1.5
3/2 = 1.5
4.5/3 = 1.5
6.75/4.5 = 1.5
10.125/6.75 = 1.5

The factor is 1.5. This tells us that every time the x-value increases by 1 unit, we multiply the y-value by 150%.

The y-intercept is the point (0, 3). There is no x-intercept since exponential functions do not have x-intercepts.

The percent rate of this function is found using the factor.

\[ \text{rate} = \text{factor} - 1 = 1.5 - 1 = 0.5 = 50\% \]

This tells us that every time the x-value increases by 1 unit, we increase the y-value by 50%.

Your plotted graph should look similar to this.
Example 9:

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>137.1742112</td>
</tr>
<tr>
<td>-2</td>
<td>123.45679</td>
</tr>
<tr>
<td>-1</td>
<td>111.1111111</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>81</td>
</tr>
<tr>
<td>3</td>
<td>72.9</td>
</tr>
<tr>
<td>4</td>
<td>65.61</td>
</tr>
</tbody>
</table>

Solution

This is an exponential function because every time x increases by 1 we multiply the y-value by the number 0.9.

\[
\frac{123.45679}{137.1742112} = 0.9 \\
\frac{111.1111111}{123.45679} = 0.9 \\
\frac{100}{111.1111111} = 0.9 \\
\frac{90}{100} = 0.9 \\
\frac{81}{90} = 0.9 \\
\frac{72.9}{81} = 0.9 \\
\frac{65.61}{72.9} = 0.9
\]

The factor is 0.9. This tells us that every time the x-value increases by 1 unit, we multiply the y-value by 90%.

The y-intercept is the point (0, 100). There is no x-intercept since exponential functions do not have x-intercepts.

The percent rate of this function is found using the factor.

\[
rate = factor - 1 = 0.9 - 1 = -0.1 = -10\%
\]

This tells us that every time the x-value increases by 1 unit, we decrease the y-value by 10%.

Note that decreasing y by 10% is the same as multiplying y by 90%.

Your plotted graph should look similar to this.
Example 10: Consider the function in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,000</td>
</tr>
<tr>
<td>2</td>
<td>70,000</td>
</tr>
<tr>
<td>5</td>
<td>25,000</td>
</tr>
<tr>
<td>6.5</td>
<td>2,500</td>
</tr>
<tr>
<td>6.66</td>
<td>100</td>
</tr>
<tr>
<td>6.67</td>
<td>-50</td>
</tr>
</tbody>
</table>

(a) What is the y-intercept of the function? Give a real-world interpretation of the meaning of that point.

Solution:
The y-intercept is the point (0,100000). In real-world terms this tells us that when each item costs $0 (when they are free), they will sell 100,000 items.

(b) Is this a linear function? If so, what is the slope?

Solution:
To be a linear function means that there must be a constant rate of change between any two points on the function.

So, to determine if it is linear we will calculate the rate of change between each successive point in the table:

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{70,000 - 100,000}{2 - 0} = \frac{down 30,000}{over 2} = \frac{-30,000}{2} = -15,000
\]

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{25,000 - 70,000}{5 - 2} = \frac{down 45,000}{over 3} = \frac{-45,000}{3} = -15,000
\]

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{2,500 - 25,000}{6.5 - 5} = \frac{down 22,500}{over 1.5} = \frac{-22,500}{1.5} = -15,000
\]

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{100 - 2500}{6.66 - 6.5} = \frac{down 2400}{over .16} = \frac{-2,400}{2} = -15,000
\]

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{-50 - 100}{6.67 - 6.66} = \frac{down 150}{over .01} = \frac{-150}{.01} = -15,000
\]

On the function, we see that the rate of change remains a constant value of $-15,000 between each point.

Therefore this is a linear function with a slope of $-15,000$. Every time the x-value increases by 1, then the y-value decreases by 15,000.
**Example 11:** Is the function below linear, exponential, or neither? If it’s linear, then find the slope. If it’s exponential, then find the factor and the percent rate.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>29</td>
</tr>
<tr>
<td>8</td>
<td>39</td>
</tr>
<tr>
<td>9</td>
<td>51</td>
</tr>
<tr>
<td>10</td>
<td>65</td>
</tr>
<tr>
<td>11</td>
<td>81</td>
</tr>
<tr>
<td>12</td>
<td>99</td>
</tr>
</tbody>
</table>

**Solution:** If the function is linear, then there will be a constant (unchanging) rate of change between each successive point in the table. We will test to see if that happens:

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{39 - 29}{8 - 7} = \frac{10}{1} = -10
\]

\[
\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{51 - 39}{9 - 8} = \frac{12}{1} = -12
\]

We can see that the rate of change is not constant since the rate of change between the first two points in the table is -10, but the rate of change between the second two points in the table is -12. Therefore, we can conclude that this function is NOT linear.

If the function is exponential, then there will be a constant multiple that is multiplied to the y-value every time we increase x by one unit. We will test to see if that happens:

\[
\frac{39}{29} \approx 1.3448
\]

\[
\frac{51}{39} \approx 1.3077
\]

We can see that the number that we multiply by as x increases by one unit is not constant. Therefore, we can conclude that this function is NOT exponential.

This function is not linear, and this function is not exponential. It is some other sort of function entirely.
# Quick Summary of Linear, Exponential, Quadratic

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>Exponential</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Looks like</strong></td>
<td><img src="#" alt="Sloping Upward" /></td>
<td><img src="#" alt="Increasing and concave up" /></td>
<td><img src="#" alt="Parabola open up" /></td>
</tr>
<tr>
<td></td>
<td><img src="#" alt="Sloping Downward" /></td>
<td><img src="#" alt="Decreasing and concave up" /></td>
<td><img src="#" alt="Parabola open down" /></td>
</tr>
<tr>
<td><strong>Vertical lines are not functions.</strong></td>
<td><img src="#" alt="Horizontal" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Domain and Range</strong></td>
<td>Domain all real numbers</td>
<td>Domain all real numbers</td>
<td>Domain all real numbers</td>
</tr>
<tr>
<td></td>
<td>Range all real numbers (constant functions have a range of only one y-value)</td>
<td>Range $y &gt; 0$</td>
<td>Range when open up: $y \geq (y \text{- value of vertex})$</td>
</tr>
<tr>
<td></td>
<td><img src="#" alt="Domain all real numbers" /></td>
<td><img src="#" alt="Range all real numbers" /></td>
<td><img src="#" alt="Range when open down: $y \leq (y \text{- value of vertex})$" /></td>
</tr>
<tr>
<td><strong>Identifying Feature</strong></td>
<td>Constant amount ADDED to the y-value every time x changes by 1 unit.</td>
<td>Constant amount MULTIPLIED to the y-value every time x changes by 1 unit.</td>
<td>Increases to a maximum point and then decreases, OR decreases to a minimum point and then increases.</td>
</tr>
<tr>
<td><strong>Special Characteristics</strong></td>
<td>SLOPE is constant rate of change.</td>
<td>Every time x increases by 1, we multiply the y-value by the FACTOR which is $\frac{f(x+1)}{f(x)}$.</td>
<td>If parabola opens up, then the lowest point on the graph is the vertex.</td>
</tr>
<tr>
<td></td>
<td>$\Delta y = \frac{y_2 - y_1}{x_2 - x_1}$</td>
<td>RATE is the fixed percentage increase or decrease.</td>
<td>If parabola opens down, then the highest point on the graph is the vertex.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Factor = 1 + (decimal rate)</td>
<td>vertex occurs at $x = \frac{-b}{2a}$ evaluate function at this x-value in order to find the y-value of the vertex</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(decimal rate) = Factor – 1</td>
<td></td>
</tr>
<tr>
<td><strong>Increasing/Decreasing</strong></td>
<td>Slope positive ↔ Increasing</td>
<td>Factor $&gt; 1$ ↔ Rate positive ↔ Increasing</td>
<td>Open up: decreases down to vertex, then turns and starts increasing after vertex.</td>
</tr>
<tr>
<td></td>
<td>Slope negative ↔ Decreasing</td>
<td>Factor $&lt; 1$ ↔ Rate negative ↔ Decreasing</td>
<td>Open down: increases up to vertex, then turns and starts decreasing after vertex.</td>
</tr>
<tr>
<td><strong>x-intercept(s)</strong></td>
<td>Will have one x-intercept (assuming it is not a horizontal line). Horizontal lines will have no x-intercept if above/below the x-axis.</td>
<td>We will only consider exponential functions of the form $f(x) = a(b)^x$ with a positive y-intercept. These type of exponential function have NO X-INTERCEPT.</td>
<td>Parabolas can have 0, or 1, or 2 x-intercepts.</td>
</tr>
<tr>
<td><strong>Equation Form</strong></td>
<td>Can be written in the form $f(x) = mx + b$ OR $f(x) = m(x - x_1) + y_1$</td>
<td>Can be written in the form $f(x) = a(b)^x$</td>
<td>Can be written in the form $f(x) = ax^2 + bx + c$</td>
</tr>
</tbody>
</table>
Section 1.6 Written Practice and Reflection
Graphs of Linear, Exponential, and Quadratic Functions

For each function in PROBLEMS 1 THRU 6 below, do the following:

a) Decide if the graph is linear, exponential, or quadratic.

b) Identify the important features of the function:
   - If it is linear, identify if it is increasing or decreasing, the slope and y-intercept and x-intercept.
   - If it is exponential, identify if it is increasing or decreasing, the y-intercept and factor and rate as a percent. Label the intercept on the graph with an ordered pair.
   - If it is quadratic, identify if it is open up or open down, the y-intercept, the vertex, the vertical line of symmetry, and any x-intercept(s).

1) A graph together with seven points on the graph are given below.

   \[ f(-3)=16 \]
   \[ f(-2)=17 \]
   \[ f(-1)=18 \]
   \[ f(0)=19 \]
   \[ f(1)=20 \]
   \[ f(2)=21 \]
   \[ f(3)=22 \]

2) A graph together with seven points on the graph are given below.

   \[ f(-3)= -90 \]
   \[ f(-2)= -95.2 \]
   \[ f(-1)= -100.4 \]
   \[ f(0)= -105.6 \]
   \[ f(1)= -110.8 \]
   \[ f(2)= -116 \]
   \[ f(3)= -121.2 \]

3) A graph together with seven points on the graph are given below.

   \[ f(-3)= 0.375 \]
   \[ f(-2)= 0.75 \]
   \[ f(-1)= 1.5 \]
   \[ f(0)= 3 \]
   \[ f(1)= 6 \]
   \[ f(2)= 12 \]
   \[ f(3)= 24 \]
4) A graph together with six points on the graph are given below.

5) A graph together with a few points on the graph are given below.

6) A graph with clear labels is given below.

7) The function below relates the profit of Acme (in dollars) to the number items that Acme sells. In the graph the independent variable is the number of items sold, and the dependent variable is the profit earned.

(a) What is the y-intercept of the function? Interpret the real-world meaning of this point.
(b) What is the x-intercept of the function? Interpret the real-world meaning of this point.
(c) Calculate the slope of the line. Interpret the real-world meaning of the slope.
(d) Identify the viewing window you need to use to re-create the exact same graph on your graphing calculator.

\[
\begin{align*}
f(-3) &= 480 \\
f(-2) &= 336 \\
f(-1) &= 235.2 \\
f(0) &= 164.64 \\
f(1) &= 115.248 \\
f(2) &= 80.6736 \\
\end{align*}
\]
8) The function given in the graph below relates the amount spent on advertising each day (in dollars per day) to the total profit earned that month. In this function the independent variable is the dollars per day spent on advertising, and the dependent variable is the profit for the month in dollars.

(a) What is the y-intercept of the function? Interpret the real-world meaning of this point.
(b) What are the x-intercept(s) of the function? Interpret the real-world meaning of these points. Be sure to explain if they don’t make sense in the real-world!
(c) Is this parabola open up or down? Describe what happens to profit as the amount spent on advertising changes from $0 per day up to $38.21 dollars per day.
(d) Write a sentence that clearly explains the practical significance of the point (17, 450000) on this function.

(e) Identify the viewing window you need to use to re-create the exact same graph on your graphing calculator.

   Xmin: __________
   Xmax: __________
   Xscl: __________
   Ymin: __________
   Ymax: __________
   Yscl: __________

9) The function below relates the price of a certain item (in dollars) to the number of years that have passed. In this function the independent variable is the number of years that have gone by, and the dependent variable is price of that particular item (in dollars).

(a) On this function, \( f(0) = 1 \). Interpret the real-world meaning of this point.
(b) On this function, \( f(1) = 1.035 \). Interpret the real-world meaning of this point.
(c) Identify the factor, and identify the percent rate of this exponential function. Interpret the real-world meaning of the rate (as a percent).
(d) Identify the viewing window you need to use to re-create the exact same graph on your graphing calculator.

   Xmin: __________
   Xmax: __________
   Xscl: __________
   Ymin: __________
   Ymax: __________
   Yscl: __________

10) Domain and range of linear, quadratic, and exponential functions.
(a) What is the domain of a linear function?
(b) What is the range of a linear function that has a positive slope or a negative slope?
(c) What is the domain of a quadratic function?
(d) How would you find the range of a quadratic function that opens up? Sketch a well-labeled example and give the range of the function in your sketch.
(e) How would you find the range of a quadratic function that opens down? Sketch a well-labeled example and give the range of the function in your sketch.
(f) What is the domain of an exponential function?
(g) What is the range of an exponential function?
11) Answer the questions regarding each function below.
(a) Is the graph linear, quadratic, exponential, or neither?
(b) Identify the vertex. Give your answer as an ordered pair.
(c) Does the graph open upward or downward?
(d) Identify where the function is increasing.
(e) Identify where the function is decreasing.

12) Sketch a graph that shows the height of a ball as the output variable, and the number of seconds after being thrown as the input variable.

13) For each exponential function below, determine the value of the y-intercept and the factor.

14) For each linear function below, answer the questions.
(a) Is the slope positive, negative or zero?
(b) Is the y-value of the y-intercept positive, negative, or zero?

15) Draw a line with a slope of 7 and a y-intercept of (0, −3).
16) Draw a line with a slope of $-\frac{5}{7}$ and a y-intercept of (0, 6).

17) Draw a line with a slope of $\frac{4}{3}$ that passes through (2, $-3$).

18) Identify the slope of each the line below.

(a) ![Graph](image-a)

(b) ![Graph](image-b)

(c) ![Graph](image-c)

(d) ![Graph](image-d)

19) For tables A, B, C, D, and E below do the following:

   a) Decide if the table is linear, or exponential, or neither.

   b) Identify the important features of the function:

      • If it is linear, identify if it is increasing or decreasing, the slope and y-intercept and x-intercept.

      • If it is exponential, identify if it is increasing or decreasing, the y-intercept, and factor and rate as a percent.

   c) Sketch a graph of the function that includes the points that were given.

   d) Identify a viewing window that would allow you to see the graph.

### Table A

<table>
<thead>
<tr>
<th>x</th>
<th>A(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>13,210</td>
</tr>
<tr>
<td>0</td>
<td>3,963</td>
</tr>
<tr>
<td>1</td>
<td>1,188.9</td>
</tr>
<tr>
<td>2</td>
<td>356.67</td>
</tr>
<tr>
<td>3</td>
<td>107.001</td>
</tr>
</tbody>
</table>

### Table B

<table>
<thead>
<tr>
<th>x</th>
<th>A(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>13,210</td>
</tr>
<tr>
<td>0</td>
<td>3,963</td>
</tr>
<tr>
<td>1</td>
<td>-5284</td>
</tr>
<tr>
<td>2</td>
<td>-14,531</td>
</tr>
<tr>
<td>3</td>
<td>-23,778</td>
</tr>
</tbody>
</table>

### Table C

<table>
<thead>
<tr>
<th>x</th>
<th>A(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>32</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table D

<table>
<thead>
<tr>
<th>x</th>
<th>A(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-125</td>
</tr>
<tr>
<td>-3</td>
<td>-99</td>
</tr>
<tr>
<td>0</td>
<td>-60</td>
</tr>
<tr>
<td>15</td>
<td>135</td>
</tr>
<tr>
<td>100</td>
<td>1240</td>
</tr>
</tbody>
</table>

### Table E

<table>
<thead>
<tr>
<th>x</th>
<th>A(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>17.5</td>
</tr>
<tr>
<td>2</td>
<td>30.625</td>
</tr>
<tr>
<td>3</td>
<td>53.59375</td>
</tr>
</tbody>
</table>
20) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>( \frac{1}{9} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>81</td>
</tr>
<tr>
<td>3</td>
<td>729</td>
</tr>
</tbody>
</table>

21) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

22) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

23) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-7</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

24) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>

25) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3125</td>
</tr>
<tr>
<td>1</td>
<td>625</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

26) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>64</td>
</tr>
<tr>
<td>-2</td>
<td>32</td>
</tr>
<tr>
<td>-1</td>
<td>16</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

27) Determine whether the function given below is linear or exponential or neither. If it is linear, then identify the slope and \( y \)-intercept. If it is exponential, then identify the factor and \( y \)-intercept.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>5.12</td>
</tr>
<tr>
<td>-1</td>
<td>6.4</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>12.5</td>
</tr>
</tbody>
</table>
Section 1.7

Introduction to Formulas of Linear, Exponential, and Quadratic Functions

<table>
<thead>
<tr>
<th>Learning Outcomes:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Given a formula, identify if it is the formula of a linear or quadratic or exponential function (or neither).</td>
</tr>
<tr>
<td>• Given a formula of a linear function, identify the slope and y-intercept of the line.</td>
</tr>
<tr>
<td>• Given a formula of a quadratic function, identify the y-intercept, and identify if it is open up/down.</td>
</tr>
<tr>
<td>• Given a formula of an exponential function, identify the y-intercept, identify the factor, identify the percent rate of increase/decrease.</td>
</tr>
<tr>
<td>• Given slope and y-intercept, identify the equation of the linear function.</td>
</tr>
<tr>
<td>• Given the factor and y-intercept, identify the equation of the exponential function</td>
</tr>
</tbody>
</table>

❖ Slope-Intercept Form of a Linear Function

In general, the **slope-intercept form** of a linear function is the form \( f(x) = mx + b \) where

- \( m \) is the **slope**
- \( b \) is the **y-intercept**

The form \( f(x) = mx + b \) is called the slope-intercept form of a line because you can clearly see the slope and the y-intercept in the equation.

Remember that the y-intercept of a function is the point where the x-value is 0. In the function \( f(x) = mx + b \), the y-intercept is the point \((0,b)\).

All linear functions CAN be written in slope-intercept form. But, be careful! Sometimes linear functions are given in other forms, and you may need to re-write them in order to view the slope-intercept form of the line!

❖ Point-Slope Form of a Linear Function

In general, the **point-slope form** of a linear function is the form \( f(x) = m(x - x_1) + y_1 \) where

- \( m \) is the **slope**
- \((x_1, y_1)\) is a **point on the line**

The form \( f(x) = m(x - x_1) + y_1 \) is called the point-slope form of a line because we can clearly see the slope of the line and a point on the line in the equation.

This form of the line is sometimes more useful than the slope-intercept form because we sometimes know a point or several points on the line that ARE NOT the y-intercept of the equation.
Example 1: Below is a table that identifies the slope and y-intercept of various linear functions. Notice that FIRST re-writing the equation in the form Y=mX+b is helpful because then we can just glance at the equation and identify the slope and y-intercept!

<table>
<thead>
<tr>
<th>Equation</th>
<th>Written in slope-intercept form</th>
<th>Slope of line</th>
<th>Y-intercept of line</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3x + 9 )</td>
<td>( f(x) = 3x + 9 )</td>
<td>3</td>
<td>(0,9)</td>
</tr>
<tr>
<td>( y = 5x - 2 )</td>
<td>( y = 5x - 2 )</td>
<td>5</td>
<td>(0, -2)</td>
</tr>
<tr>
<td>( g(t) = 7 + 2t )</td>
<td>( g(t) = 2t + 7 )</td>
<td>2</td>
<td>(0,7)</td>
</tr>
<tr>
<td>( f(x) = x - 10 )</td>
<td>( f(x) = 1x - 10 )</td>
<td>1</td>
<td>(0, -10)</td>
</tr>
<tr>
<td>( y = 8 - x )</td>
<td>( y = -1x + 8 )</td>
<td>-1</td>
<td>(0,8)</td>
</tr>
<tr>
<td>( f(x) = 4x )</td>
<td>( f(x) = 4x + 0 )</td>
<td>4</td>
<td>(0,0)</td>
</tr>
<tr>
<td>( f(x) = 6 )</td>
<td>( f(x) = 0x + 6 )</td>
<td>0</td>
<td>(0,6)</td>
</tr>
<tr>
<td>( y + 2 = 4x )</td>
<td>( y = 4x - 2 )</td>
<td>4</td>
<td>(0, -2)</td>
</tr>
<tr>
<td>( y - 6x = 3 )</td>
<td>( y = 6x + 3 )</td>
<td>6</td>
<td>(0,3)</td>
</tr>
<tr>
<td>( 2y + 6x = 8 )</td>
<td>( y = -3x + 4 )</td>
<td>-3</td>
<td>(0,4)</td>
</tr>
</tbody>
</table>

Example 2: Below is a table that identifies the slope and one point on the line of various linear functions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation written in slope-intercept form</th>
<th>Slope of line</th>
<th>One point on this line that we can quickly see using the formula...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3(x - 9) + 2 )</td>
<td>( f(x) = 3(x - 9) + 2 )</td>
<td>3</td>
<td>(9,2)</td>
</tr>
<tr>
<td>( y = 5(x - 2) - 4 )</td>
<td>( y = 5(x - 2) - 4 )</td>
<td>5</td>
<td>(2, -4)</td>
</tr>
<tr>
<td>( g(t) = \frac{1}{2}(x - 8) - 3 )</td>
<td>( g(t) = \frac{1}{2}(x - 8) - 3 )</td>
<td>( \frac{1}{2} )</td>
<td>(8, -3)</td>
</tr>
<tr>
<td>( f(x) = -8(x + 1) + 7 )</td>
<td>( f(x) = -8(x + 1) + 7 )</td>
<td>-8</td>
<td>(-1,7)</td>
</tr>
<tr>
<td>( y = 2(x + 15) - 1 )</td>
<td>( y = 2(x + 15) - 1 )</td>
<td>2</td>
<td>(-15, -1)</td>
</tr>
<tr>
<td>( g(t) = \frac{2}{3}(x + 12) - 100 )</td>
<td>( g(t) = \frac{2}{3}(x + 12) - 100 )</td>
<td>( \frac{2}{3} )</td>
<td>(-12, -100)</td>
</tr>
</tbody>
</table>

Example 3: Find the equation of the line with slope 3 and y-intercept (0, 5).

Solution:
The equation of a line can be written as \( f(x) = mx + b \) where \( m \) is the slope and \( (0,b) \) is the y-intercept. We have been told that \( m=3 \) and \( b=5 \). So, the equation of the line can be written as \( f(x) = 3x + 5 \).

Example 4: Find the equation of the line that passes through (0,4) and (7, 2).

Solution:
We will need to the slope of the line. We know that slope is \( \frac{\Delta y}{\Delta x} = \frac{2-4}{7-0} = \frac{-2}{7} \). So the slope is -2/7.
The y-intercept was given as (0, 4). So the equation of the line can be written as \( f(x) = -\frac{2}{7}x + 4 \).
**Example 5**: Find the equation of the line in the graph below.

![Graph with points (0,5), (5,3), (10,1), (15,-1), (20,-3)]

**Solution**:
This is a linear function, and we see that the slope is 
\[
\Delta y \over \Delta x = {3-5 \over 5-(0)} = {\frac{-2}{5}}.
\]

Next, we see that the y-intercept of this line is the point (0,5).

So the slope-intercept form of this line is 
\[
f(x) = \frac{-2}{5}x + 5.
\]

**Example 6**: Find the equation of the line in the table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>100</td>
</tr>
<tr>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>12</td>
<td>49</td>
</tr>
<tr>
<td>37</td>
<td>-26</td>
</tr>
<tr>
<td>150</td>
<td>-365</td>
</tr>
</tbody>
</table>

**Solution**:
We first need to verify that this is a linear function by finding the slope (the constant rate of change).

\[
\text{slope first two points} = \Delta y \over \Delta x = {85 - 100 \over 0 - (-5)} = -15 \over 5 = -3
\]

\[
\text{slope of points three and four} = \Delta y \over \Delta x = {49 - (-26) \over 37 - (12)} = {75 \over 25} = -3
\]

\[
\text{slope points two and three} = \Delta y \over \Delta x = {49 - 85 \over 12 - (0)} = -36 \over 12 = -3
\]

\[
\text{slope of points four and five} = \Delta y \over \Delta x = {365 - (-26) \over 150 - (37)} = -339 \over 113 = -3
\]

We have now identified that this is a linear function because it has a constant rate of change. We have also identified that the slope of this line is -3. Every time $x$ increases by 1 unit, we add -3 to the $y$-value.

We also see that the $y$-intercept is the point (0, 85).

Therefore, the slope-intercept form of this linear function is 
\[
f(x) = -3x + 85.
\]
In general, exponential functions can be written in the form $f(x) = a(b)^x$ where

- **$(0, a)$ is the y-intercept.** In this class we will only consider exponential functions with a positive y-intercept.
- **$b$ represents the factor.** As $x$ increases by 1, the y-value is multiplied by $b$. Because of this, we can conclude the following:
  - **The function is increasing if the factor is** $> 1$.
    
    For example, if the factor is 1.083, then every time $x$ increases by 1, we multiply the y-value by 108.3% which makes the y-value increase (since 108.3% of something is 100% of it PLUS 8.3% more of it).
  - **The function is decreasing if the factor is** $0 < b < 1$.
    
    For example, if the factor is 0.86, then every time $x$ increases by 1, we multiply the y-value by 86% which makes the y-value decrease (since 86% of something is 100% of it MINUS 14% of it).

- **Decimal Rate = Decimal Factor − 1**
  
  Because of this we can conclude the following:
  - **A negative percent rate means the function is decreasing.**
    
    For example, if the factor is 0.86, then the function is decreasing at a rate of 14% because the rate is $0.86 - 1 = -0.14$ which is a decreasing rate of 14%.
  - **A positive percent rate means the function is increasing.**
    
    For example, if the factor is 1.083, then the function is increasing at a rate of 8.3% because the rate is $1.083 - 1 = 0.083$ which is an increasing rate of 8.3%.

**Example 7:** Below is a table that identifies the y-intercept and growth/decay factor of various exponential functions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Growth or decay?</th>
<th>Factor of the function</th>
<th>Y-intercept of function</th>
<th>Percent Rate of growth/decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 4(0.95^x)$</td>
<td>Decay</td>
<td>0.95</td>
<td>(0,4)</td>
<td>Decay rate of 5%</td>
</tr>
<tr>
<td>$f(x) = 7(1.02^x)$</td>
<td>Growth</td>
<td>1.02</td>
<td>(0,7)</td>
<td>Growth rate of 2%</td>
</tr>
<tr>
<td>$f(x) = (0.05^x)$</td>
<td>Decay</td>
<td>0.05</td>
<td>(0,1)</td>
<td>Decay rate of 95%</td>
</tr>
<tr>
<td>$f(x) = (0.01^x)$</td>
<td>decay</td>
<td>0.01</td>
<td>(0,1)</td>
<td>Decay rate of 99%</td>
</tr>
<tr>
<td>$f(x) = 3(9^x)$</td>
<td>Growth</td>
<td>9</td>
<td>(0,3)</td>
<td>Growth rate of 800%</td>
</tr>
<tr>
<td>$f(x) = 5(2^x)$</td>
<td>Growth</td>
<td>2</td>
<td>(0,5)</td>
<td>Growth rate of 100%</td>
</tr>
<tr>
<td>$f(x) = 13(0.8)^x$</td>
<td>Decay</td>
<td>0.8</td>
<td>(0,13)</td>
<td>Decay rate of 20%</td>
</tr>
</tbody>
</table>
Example 8: Find the equation of the exponential function that passes through (0,50) and has growth factor 1.07.
Solution: 
The function has the form \( f(x) = a(b)^x \) where \( a \) is the y-intercept and \( b \) is the factor. Therefore, we can say that the formula is \( f(x) = 50(1.07)^x \).

Example 9: Find the equation of the exponential function that passes through (0,3500) and has decay factor 0.97.
Solution: 
The function has the form \( f(x) = a(b)^x \) where \( a \) is the y-intercept and \( b \) is the factor. Therefore, we can say that the formula is \( f(x) = 3500(0.97)^x \).

Example 10: Find the equation of the exponential function with points in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>160</td>
<td>136</td>
<td>115.6</td>
<td>98.26</td>
</tr>
</tbody>
</table>

Solution: 
We first test to make sure there is a constant factor between each successive point in the table.
\[
\frac{136}{160} = 0.85 \quad \frac{115.6}{136} = 0.85 \quad \frac{98.26}{115.6} = 0.85 \quad \text{So we see there is a constant factor of 0.85.}
\]

The factor of this exponential is 0.85, and we also see that the y-intercept is (0, 160). Therefore, the formula of this function is \( f(x) = 160(0.85)^x \).

Formula of Quadratic Functions

Quadradic functions have equations that can be written in the form \( f(x) = ax^2 + bx + c \) where \( a \neq 0 \).

We call this the standard form of a quadratic function, or the expanded form of a quadratic function.

In the next unit of the course and future courses, you will explore various ways of re-writing quadratic functions including the factored form and the vertex form. But you will only be asked to work with the standard form of the quadratic function in this unit.

The quadratic function \( f(x) = ax^2 + bx + c \) where \( a \neq 0 \) has the following properties:

- The graph of \( f(x) = ax^2 + bx + c \) is a parabola that is open up when \( a > 0 \), and it is a parabola that is open down when \( a < 0 \).
- The y-intercept of \( f(x) \) is the point \( (0, c) \).
- The vertex of the parabola and the x-intercepts of the parabola can also be found using the formula of the quadratic function. But, we will work on those concepts during unit 2.
**Example 11:** Use the formula of the function \( f(x) = 2x^2 - 8x + 7 \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

\[
\text{Solution:}
\]

The function \( f(x) = 2x^2 - 8x + 7 \) is a quadratic function with \( a = 2 \) and \( b = -8 \) and \( c = 7 \). Since \( a \) is greater than 0, we know that the graph of \( f(x) \) will be a parabola that is open up.

The y-intercept is \((0, c)\). So, the y-intercept of this function is \((0, 7)\).

We will learn how to find the vertex in the next unit.

**Example 12:** Use the formula of the function \( f(x) = -3x^2 + 15x \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

\[
\text{Solution:}
\]

The function \( f(x) = -3x^2 + 15x \) is a quadratic function with \( a = -3 \) and \( b = 15 \) and \( c = 0 \).

- Since \( a \) is less than 0, we know that the graph of \( f(x) \) will be a parabola that is open down.
- The y-intercept is \((0, c)\). So, the y-intercept of this function is \((0, 0)\). Notice that this is also an x-intercept since the y-value is 0.

We will learn how to find the vertex in the next unit.
Example 13: Use the formula of the function \( f(x) = -5x^2 + 4 \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

Solution: The function \( f(x) = -5x^2 + 4 \) is a quadratic function with \( a = -5 \) and \( b = 0 \) and \( c = 4 \).

- Since \( a \) is less than 0, we know that the graph of \( f(x) \) will be a parabola that is open down.
- The y-intercept is \((0, c)\). So, the y-intercept of this function is \((0, 4)\).
- We will learn how to find the vertex in the next unit.
<table>
<thead>
<tr>
<th>Looks like</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sloping Upward</td>
<td>Vertical lines are not functions.</td>
<td>Sloping Downward</td>
</tr>
<tr>
<td>Horizontal</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domain and Range</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain all real numbers</td>
<td>Domain all real numbers</td>
<td>Domain all real numbers</td>
</tr>
<tr>
<td>Range all real numbers (constant functions have a range of only one y-value)</td>
<td>Range y &gt; 0</td>
<td>Range when open up: ( y \geq (y - \text{value of vertex}) )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Range when open down: ( y \leq (y - \text{value of vertex}) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Identifying Feature</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant amount ADDED to the y-value every time x changes by 1 unit.</td>
<td>Constant amount MULTIPLIED to the y-value every time x changes by 1 unit.</td>
<td>Increases to a maximum point and then decreases, OR decreases to a minimum point and then increases.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Special Characteristics</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SLOPE is constant rate of change.</td>
<td>Every time x increases by 1, we multiply the y-value by the FACTOR which is ( \frac{f(x+1)}{f(x)} ).</td>
<td>If parabola opens up, then the lowest point on the graph is the vertex.</td>
</tr>
<tr>
<td>[ \text{Slope is } \frac{\Delta y}{\Delta x} = \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}} = \frac{y_2 - y_1}{x_2 - x_1} ]</td>
<td>RATE is the fixed percentage increase or decrease.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor = 1 + (decimal rate)</td>
<td>If parabola opens down, then the highest point on the graph is the vertex.</td>
</tr>
<tr>
<td></td>
<td>(decimal rate) = Factor – 1</td>
<td>vertex occurs at ( x = \frac{-b}{2a} ) evaluate function at this x-value in order to find the y-value of the vertex</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Increasing/Decreasing</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope positive ↔ Increasing</td>
<td>Factor&gt;1 ↔ Rate positive ↔ Increasing</td>
<td>Open up: decreases down to vertex, then turns and starts increasing after vertex.</td>
</tr>
<tr>
<td>Slope negative ↔ Decreasing</td>
<td>0&lt;Factor&lt;1 ↔ Rate negative ↔ Decreasing</td>
<td>Open down: increases up to vertex, then turns and starts decreasing after vertex.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x-intercept(s)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Will have one x-intercept (assuming it is not a horizontal line).</td>
<td>We will only consider exponential functions of the form ( f(x) = a(b)^x ) with a positive y-intercept. These type of exponential function have NO X-INTERCEPT.</td>
<td>Parabolas can have 0, or 1, or 2 x-intercepts.</td>
</tr>
<tr>
<td>Horizontal lines will have no x-intercept if above/below the x-axis.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation Form</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Can be written in the form ( f(x) = mx + b ) OR ( f(x) = m(x - x_1) + y_1 )</td>
<td>Can be written in the form ( f(x) = a(b)^x )</td>
<td>Can be written in the form ( f(x) = ax^2 + bx + c )</td>
</tr>
</tbody>
</table>
Section 1.7 Written Practice and Reflection
Introduction to Formulas of Linear, Exponential, and Quadratic Functions

1) Identify slope and y-intercept of each linear function.

(a) \( Y = -2X + 3 \)
   - slope: _________________
   - Y-intercept: ( , )

(b) \( Y = 7X \)
   - slope: _________________
   - Y-intercept: ( , )

(c) \( Y = 10 \)
   - slope: _________________
   - Y-intercept: ( , )

(d) \( Y = 4(x - 2) + 1 \)
   - slope: _________________
   - Y-intercept: ( , )

(e) \( 2X + Y = 6 \)
   - slope: _________________
   - Y-intercept: ( , )

2) Complete the following table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Factor for this function?</th>
<th>Is this growth or decay?</th>
<th>Rate of growth or decay (as a PERCENT)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3(1.09)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 75(0.68)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = (1.6)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 158(.98)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3) Complete the following table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 6x - 8 )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( f(x) = -2x^2 + 5 )</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>( f(x) = 1 + 7x - x^2 )</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

4) Use the formula of the function \( f(x) = -10x^2 + x - 4 \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

5) Use the formula of the function \( f(x) = 3x^2 - 5x + 9 \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

6) Use the formula of the function \( f(x) = 6x^2 + 7 \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

7) Use the formula of the function \( f(x) = x^2 - 4x - 10 \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

8) Use the formula of the function \( f(x) = -2x^2 + 7x \) to identify the y-intercept and the general shape of the function. Sketch and label the graph with the help of your graphing calculator.

9) Find the equation of the line that passes through \((0, 15)\) and has slope \(-2\).
10) Find the equation of the line that has slope $-\dfrac{4}{9}$ and passes through $(0, 3)$.

11) Find the equation of the exponential function that passes through $(0, 200)$ and has factor $1.42$.

12) Find the equation of the exponential function that passes through $(0, 13)$ and has factor $0.81$.

13) Use graph of the function $f(x)$ given below to answer the questions.

- **a)** Identify the $y$-intercept of the function.
- **b)** Identify the slope of the function.
- **c)** Find the equation of the line.
- **d)** Use the equation of the line to solve for the $x$-intercept of the function. Show all work.
- **e)** Use the equation of the function to find/solve $f(x) = \dfrac{8}{9}$. Show all work.
- **f)** Use the equation of the function to find/solve $f\left(\dfrac{5}{11}\right)$. Show all work.
- **g)** What viewing window should be used to re-create the given graph? Check your answer by graphing your equation on that viewing window on your calculator.

14) Use graph of the function $f(x)$ given below to answer the questions.

- **a)** Identify the $y$-intercept of the function.
- **b)** Identify the slope of the function.
- **c)** Find the equation of the line.
- **d)** Use the equation of the line to solve for the $x$-intercept of the function. Show all work.
- **e)** Use the equation of the function to find/solve $f\left(-\dfrac{7}{11}\right)$. Show all work.
- **f)** Use the equation of the function to find/solve $f(x) = \dfrac{2}{3}$. Show all work.
- **g)** What viewing window should be used to re-create the given graph? Check your answer by graphing your equation on that viewing window on your calculator.
15) Use graph of the exponential function \( f(x) \) given below to answer the questions.

![Graph of exponential function](image)

(a) Identify the y-intercept of the function.
(b) Identify the factor of the function. Show all work.
(c) Is this function increasing or decreasing? Make sure the factor you calculated in part (b) matches up with your answer!
(d) Find the equation of the function.
(e) Find the rate of the function as a percentage. Show all work.
(f) Use the equation of the function to find/solve \( f(4.3) \). Show all work. Round to 4 decimal places.
(g) What viewing window should be used to re-create the given graph? Check your answer by graphing your equation on that viewing window on your calculator.

16) Use graph of the function \( f(x) \) given below to answer the questions.

![Graph of exponential function](image)

(a) Identify the y-intercept of the function.
(b) Identify the factor of the function. Show all work.
(c) Is this function increasing or decreasing? Make sure the factor you calculated in part (b) matches up with your answer!
(d) Find the equation of the function.
(e) Find the rate of the function as a percentage. Show all work.
(f) Use the equation of the function to find/solve \( f(-1.8) \). Show all work. Round to 4 decimal places.
(g) What viewing window should be used to re-create the given graph? Check your answer by graphing your equation on that viewing window on your calculator.
17) Use the given table of values of the function $f(x)$ given below to answer the questions.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-62.2</td>
</tr>
<tr>
<td>0</td>
<td>-25</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>10</td>
<td>99</td>
</tr>
</tbody>
</table>

a) Identify the y-intercept of the function.
b) Show the work to demonstrate that $f(x)$ is a linear function. Identify the slope of the function.
c) Find the equation of the line.
d) Use the equation of the line to solve for the x-intercept of the function.
e) Use the equation of the function to find/solve $f\left(\frac{3}{6}\right)$. Show all work.
f) Use the equation of the function to find/solve $f\left(-\frac{7}{2}\right)$. Show all work.

18) Use the given table of values of the function $f(x)$ given below to answer the questions.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>7.25</td>
</tr>
<tr>
<td>7</td>
<td>-5.75</td>
</tr>
</tbody>
</table>

a) Identify the y-intercept of the function.
b) Show the work to demonstrate that $f(x)$ is a linear function. Identify the slope of the function.
c) Find the equation of the line.
d) Use the equation of the line to solve for the x-intercept of the function.
e) Use the equation of the function to find/solve $f(14)$. Show all work.
f) Use the equation of the function to find/solve $f(x) = 11$. Show all work.

19) Use the given table of values of the function $f(x)$ given below to answer the questions.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1350</td>
</tr>
<tr>
<td>1</td>
<td>918</td>
</tr>
<tr>
<td>2</td>
<td>624.24</td>
</tr>
<tr>
<td>3</td>
<td>424.4832</td>
</tr>
</tbody>
</table>

a) Identify the y-intercept of the function.
b) Show the work to demonstrate that $f(x)$ is an exponential function. Identify the factor of the function.
c) Find the equation of the function.
d) Find the rate of the function as a percentage.
e) Use the equation of the function to find/solve $f(13)$. Show all work. Round to 4 decimal places.

20) Use the given table of values of the function $f(x)$ given below to answer the questions.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>420</td>
</tr>
<tr>
<td>1</td>
<td>562.8</td>
</tr>
<tr>
<td>2</td>
<td>754.152</td>
</tr>
<tr>
<td>3</td>
<td>1010.56368</td>
</tr>
</tbody>
</table>

a) Identify the y-intercept of the function.
b) Show the work to demonstrate that $f(x)$ is an exponential function. Identify the factor of the function.
c) Find the equation of the function.
d) Find the rate of the function as a percentage.
e) Use the equation of the function to find/solve $f(-2)$. Show all work. Round to 4 decimal places.
Unit Outcome Overview

Section 1.1 Simplify and Solve
- Simplify expressions by distributing and then gathering/combining like terms.
- Solve 1-variable linear equations and then check the solution in the original equation.
- Solve linear equations for a variable when there are multiple letters in the equation.
- Re-write a two-variable equation for either variable, depending on which is dependent.

Section 1.2 Evaluate and Calculate
- Use parenthesis correctly when evaluating.
- Use the TI graphing calculator to effectively calculate the value of expressions.

Section 1.3 Intercepts and Viewing Windows
- Identify independent and dependent variable in an equation that is solved for one variable or given on a graph.
- Complete an input-output table by using the formula.
- Plot the graph of a two-variable equation by using an input/output table and/or formula.
- Enter functions into the Y= menu.
- Set up the calculator to display an automatically filled table of values, and then view that table.
- Set up the calculator to display a manually filled table of values, and then view that table.
- Set up the viewing window of a calculator after understanding what the graph should look like first.

Section 1.4 Functions and Function Notation
- Identify if an input-output table represents a function or not.
- Identify if a graph represents a function or not.
- Identify if a formula represents a function or not.
- Use function notation to evaluate and solve from a formula, table, or graph.

Section 1.5 Domain and Range
- Identify the domain of a function from the graph.
- Identify the domain of a function from a formula.
- Identify the range of a function from the graph.

Section 1.6 Graphs and Tables of Linear, Exponential, and Quadratic Functions
- Given a graph, identify if it could be a linear function, or an exponential function, or a quadratic function, or neither
- If a graph is linear, identify the slope and y-intercept (and x-intercept if it is on the graph)
- If a graph is exponential, identify the factor and percent rate and y-intercept
- If a graph is quadratic, identify the y-intercept, and vertex, if it is open up or down, if it has a maximum or minimum, x-intercept(s)
- Given a graph, identify where it is increasing or decreasing
- Use a table of values to identify if a function is linear or exponential or neither
- If a table of values is linear, then identify the slope of the line and the y-intercept of the line
- If a table of values is exponential, then identify the factor and percent rate and y-intercept

Section 1.7 Introduction to Formulas of Linear, Exponential, and Quadratic Functions
- Given a formula, identify if it is the formula of a linear or quadratic or exponential function (or neither).
- Given a formula of a linear function, identify the slope and y-intercept of the line.
- Given a formula of a quadratic function, identify the y-intercept, and identify if it is open up/down.
- Given a formula of an exponential function, identify the y-intercept, identify the factor, identify the percent rate of increase/decrease.
- Given y-intercept and slope, identify the equation of the linear function.
- Given y-intercept and factor, identify the equation of the exponential function.
1) Consider the table below.

(a) In the table, is \( y \) a function of \( x \)? (circle one answer)
   - Yes, \( y \) is a function of \( x \)
   - No, \( y \) is not a function of \( x \)

(b) Explain, using complete sentences, your answer to part (a).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

2) Consider the graph below.

(a) In the graph, is \( y \) a function of \( x \)? (circle one answer)
   - Yes, \( y \) is a function of \( x \)
   - No, \( y \) is not a function of \( x \)

(b) Explain, using complete sentences, your answer to part (a).

3) Suppose that \( f(x) = 8x - 15 \). Evaluate or solve the following. Show all work.

(a) \( f(x) = 30 \)

(b) \( f(7) \)

4) Consider the graph of the function \( g(x) \) given below.

(a) Circle ALL the statements that are true. \( g(2) = 0 \)  \( g(2) = -2 \)  \( g(4) = 2 \)

(b) Assuming that \( g(x) \) is a quadratic function, the domain of \( g(x) \) is ____________________________.

(c) Assuming that \( g(x) \) is a quadratic function, the range of \( g(x) \) is ____________________________.
5) Identify the domain of each of the following functions.

(a) The domain of \( f(x) = \frac{18 + 5}{x - 3} \) is ________________________________.

(b) The domain of \( f(x) = 7x^2 - 8x + 4 \) is ________________________________.

(c) The domain of \( f(x) = \sqrt{x - 10} \) is ________________________________.

(d) The domain of \( f(x) = 15x + 2 \) is ________________________________.

(e) The domain of \( f(x) = 10 \) is ________________________________.

6) Evaluate each of the following expressions at the given value. Show the substitution step and then you can use the calculator to calculate the value of the answer.

(a) \( x^2 - x + 3 \) when \( x = -87 \)

(b) \( \frac{y_2 - y_1}{x_2 - x_1} \) when \( x_1 = -8 \) and \( y_1 = 17 \) and \( x_2 = -9 \) and \( y_2 = 4 \)

(c) \( -\frac{b}{2a} \) when \( a = 400 \) and \( b = -6000 \)

(d) \( b^2 - 4ac \) when \( a = 15 \) and \( b = -35 \) and \( c = -27 \)

(e) \( -\frac{-b + \sqrt{b^2 - 4ac}}{2a} \) when \( a = -25 \) and \( b = -587 \) and \( c = 75 \) Round to 4 decimal places.

7) Solve each equation for the exact value of the identified variable. Show all work.

(a) \( 3 - (7x + 4) = 15x - 4(3 - 2x) \) Solve for \( x \).

(b) \( \frac{1}{3}x - 8 = \frac{2}{3} + 7x \) Solve for \( x \).

(c) \( A = \frac{1}{2}bh \) Solve for \( b \).

(d) \( 7x - 4y = 8 \) Solve for \( y \).

8) Identify the exact viewing window in the graph below.

\[
\begin{array}{c}
\text{XMin} = \underline{\phantom{0000}} \\
\text{XMax} = \underline{\phantom{0000}} \\
\text{XScI} = \underline{\phantom{0000}} \\
\text{YMin} = \underline{\phantom{0000}} \\
\text{YMax} = \underline{\phantom{0000}} \\
\text{YScI} = \underline{\phantom{0000}} \\
\end{array}
\]
9) Suppose that $3x + 5y = 15$. Show all work.
   (a) The y-intercept of $3x + 5y = 15$ is the point $(            ,            )$.
   (b) The x-intercept of $3x + 5y = 15$ is the point $(            ,            )$.
   (c) Draw a sketch of $3x + 5y = 15$ that clearly shows both the x-intercept and the y-intercept. Be sure to label your x-axis and your y-axis with a scale and tick-marks that appropriate for this equation.

10) Identify slope and y-intercept of each linear function.
   a) $Y = 8X - 5$  slope: _________________  Y-intercept: (          ,          )
   b) $Y = 6X$  slope: _________________  Y-intercept: (          ,          )
   c) $Y = \frac{2}{3}$  slope: _________________  Y-intercept: (          ,          )
   d) $Y = -3(x - 5) + 1$  slope: _________________  Y-intercept: (          ,          )
   e) $5x + Y = 7$  slope: _________________  Y-intercept: (          ,          )

11) Complete the following table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Factor for this function?</th>
<th>Is this growth or decay?</th>
<th>Rate of growth or decay (as a PERCENT)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 9(1.02)^x$</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 85(0.95)^x$</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = (1.4)^x$</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 75(0.75)^x$</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

12) Complete the following table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2 - 2x + 3$</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>$f(x) = 5x^2 - 8$</td>
<td>( , )</td>
<td></td>
</tr>
<tr>
<td>$f(x) = 1 + 4x - 2x^2$</td>
<td>( , )</td>
<td></td>
</tr>
</tbody>
</table>

13) Answer the questions about the exponential function in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8500</td>
</tr>
<tr>
<td>1</td>
<td>7650</td>
</tr>
<tr>
<td>2</td>
<td>6885</td>
</tr>
<tr>
<td>3</td>
<td>6196.5</td>
</tr>
<tr>
<td>4</td>
<td>5576.85</td>
</tr>
<tr>
<td>5</td>
<td>5019.165</td>
</tr>
<tr>
<td>6</td>
<td>4517.2485</td>
</tr>
</tbody>
</table>

(a) Identify if the exponential function is increasing or decreasing.
(b) Identify the factor of the exponential function.
(c) Identify the percentage rate for the exponential function.
(d) Identify the equation for the exponential function.
14) \( f(x) = -0.5x^2 + 8x - 1.2 \)

a) Circle what TYPE of function is shown.
- function is linear
- function is exponential
- function is quadratic
- function is neither

b) Identify the important features of the function:
- If it is linear:
  - Is it increasing or decreasing?
  - Identify the slope
  - Identify the y-intercept
  - Identify the x-intercept
- If it is exponential:
  - Is it increasing or decreasing?
  - Identify the y-intercept
  - Identify the factor
  - Identify the rate as a percent.
- If it is quadratic:
  - Is it open up or open down?
  - Identify the y-intercept
  - Identify the vertex

15) What is an appropriate viewing window for this function that will show the complete graph?

XMin = _________  
XMax = _________  
XSec = _________  
YMin = _________  
YMax = _________  
YSec = _________

16) Find the equation of the line that has slope 17 and y-intercept (0, 3).

17) Find the equation of the exponential function that has y-intercept (0, 1.7) and factor 1.3.
Instructions: On the following pages you will find flashcards for Unit 1. Cut along the lines to create your own flashcards to review basic concepts from Unit 1.
<table>
<thead>
<tr>
<th>domain of a function?</th>
<th>formula for factor of exponential equation of exponential function?</th>
</tr>
</thead>
<tbody>
<tr>
<td>equation of exponential function?</td>
<td>Formula to relate factor and rate</td>
</tr>
<tr>
<td>equation of quadratic function?</td>
<td>How do we find a y-intercept?</td>
</tr>
<tr>
<td>( f(7) ) means ( x=7 ) or ( y=7 )?</td>
<td>How do we find an x-intercept?</td>
</tr>
<tr>
<td>( f(x)=7 ) means ( x=7 ) or ( y=7 )?</td>
<td>point-slope form of a line?</td>
</tr>
<tr>
<td>Factor = (2nd y) / (1st y)</td>
<td>the set of all x-values that produce outputs for the function</td>
</tr>
<tr>
<td>---------------------------</td>
<td>--------------------------------------------------------------</td>
</tr>
<tr>
<td>Factor = 1+rate</td>
<td>y=a(b)^x</td>
</tr>
<tr>
<td>let x=0 to find y-intercept</td>
<td>y=ax^{2}+bx+c</td>
</tr>
<tr>
<td>let y=0 to find x-intercept</td>
<td>f(7) means x=7</td>
</tr>
<tr>
<td>y=m(x-x1)+y1</td>
<td>f(x)=7 means y=7.</td>
</tr>
<tr>
<td>range of a function?</td>
<td>3E5 on the calculator?</td>
</tr>
<tr>
<td>---------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>slope formula</td>
<td>y=3(x-5)+1 what is the y-intercept?</td>
</tr>
<tr>
<td>slope-intercept</td>
<td>y=3x^2+5x-4 what is the y-intercept?</td>
</tr>
<tr>
<td>form of a line?</td>
<td>y=45(1.03)^x what is the rate of growth?</td>
</tr>
<tr>
<td>Standard Viewing</td>
<td>y=62(0.92)^x what is the rate of decrease?</td>
</tr>
<tr>
<td>Window</td>
<td></td>
</tr>
<tr>
<td>300,000</td>
<td>the set of all y-values that the function produces</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>the y-intercept is (0, -14)</td>
<td>(y_2-y_1) / (x_2-x_1)</td>
</tr>
<tr>
<td>the y-intercept is (0, -4)</td>
<td>y = mx + b</td>
</tr>
<tr>
<td>growing at rate of 3%</td>
<td>Press ZOOM 6 [-10,10] x [-10,10]</td>
</tr>
<tr>
<td>decreasing at a rate of 8%</td>
<td>.0002</td>
</tr>
</tbody>
</table>
Unit 2
**Introduction to Unit 2**

This is the second unit (the second five weeks) of the Intermediate Algebra course. During the first unit, students should have mastered the following learning outcomes which will be used throughout this module:

- Review concepts such as evaluating expressions, solving simple linear equations, and finding points on graphs.
- Using function notation
- Identifying linear functions, exponential functions, and quadratic functions when given a formula.
- Identifying linear functions and exponential functions when given a table of values.
- Identifying linear functions, exponential functions, and quadratic functions when given a graph.
- Identifying the slope, y-intercept, and formula for a linear function when given a graph or table or information about the function.
- Identifying the factor, percent rate, and y-intercept for an exponential function when given the graph or table or information about the function.
- Identifying the y-intercept, vertex, and if the function is open up or open down when given a formula/graph of a quadratic function.
- Understand how to enter a function in the graphing calculator, and have an introductory-level of understanding of viewing windows.
- Understand the real-world meaning of points on a graph when a function is a model for a real-world situation.

During this unit, students will further develop understanding of these three types of functions and continue to learn about the graphing calculator as well. Some learning outcomes of this module include:

- Find formulas of linear and exponential functions when given information about these functions.
- Understand the real-world meaning of points on a graph when a function is a model for a real-world situation.
- Use the graphing calculator to evaluate for the output-values of functions graphically.
- Use the graphing calculator to solve for the input-value of functions graphically.
- Use the graphing calculator to find local maximum and local minimum on the calculator.
- Use the graphing calculator to solve equations graphically (and approximate numerically).
- Re-write quadratics functions in expanded and factored form, and understand some polynomial vocabulary.
- Use factoring and the zero-product property to solve some quadratic equations.
- Use the zero-product property to solve some quadratic equations.
- Use the quadratic formula to solve any quadratic equation.

Note: many algebra textbooks include a section on “Exponent Rules” before discussion expanding and factoring. This book reserves the topic of Exponent Rules until Module 3. It is not necessary to first master Exponent Rules in order to expand basic quadratics.

**Recommended Schedule for Unit 2:**

2.1: More in depth understanding of Linear, exponential, quadratic functions. **2 days.**

2.2: Solving graphically with graphing calculator. **2 days.**

2.3: Polynomial vocabulary, expanding/factoring, zero-product property. **4 days.**

2.4: Quadratic formula. **4 days.**

Review and testing. **3 days.**
Section 2.1
More in-depth Understanding of the Formulas of Linear, Exponential, Quadratic Functions

Learning Outcomes:

- Find the formula of a linear function using slope-intercept form and/or point-slope form when given a graph or appropriate information about the line.
- Find the formula of an exponential function when given a graph or appropriate information about the function.
- Find the vertex of a quadratic function using the formula of the quadratic function.

Formula of Linear Functions

- Linear functions can be written in slope-intercept form as \( f(x) = mx + b \) where \( m \) is the slope of the line and \((0, b)\) is the \( y \)-intercept of the line.

- Linear functions can be written in point-slope form as \( f(x) = m(x - x_1) + y_1 \) where \( m \) is the slope of the line and \((x_1, y_1)\) is a point on the line.

Example 1: Find the equation of the line with slope 8 and \( y \)-intercept (0, 7).

Solution:
The equation of a line can be written as \( f(x) = mx + b \) where \( m \) is the slope and \((0, b)\) is the \( y \)-intercept.
We have been told that \( m=8 \) and \( b=7 \).
So, the equation of the line can be written as \( f(x) = 8x + 7 \).

Example 2: Find the equation of the line that passes through \((0,15)\) and \((12, 9)\).

Solution:
We will need to the slope of the line. We know that slope is \( \frac{\Delta y}{\Delta x} = \frac{9-15}{12-0} = \frac{-6}{12} = -\frac{1}{2} \). So the slope is \(-1/2\).
The \( y \)-intercept was given as \((0, 15)\).
So the equation of the line can be written as \( f(x) = -\frac{1}{2}x + 15 \). In this equation we used the slope-intercept form of the line since we have the \( y \)-intercept.

Notice that an alternative way to write the equation would be to write \( f(x) = -\frac{1}{2}(x - 12) + 9 \) by using the slope and the point \((12, 9)\). In this equation we have used the point-slope form of the line. Notice that you could distribute the \(-1/2\) and simplify which would leave the equation in slope-intercept form.
Example 3: Find the equation of the line in the graph below.

\[ f(x) = -\frac{2}{5}(x - 5) + 3 \]

or

\[ f(x) = -\frac{2}{5}(x - 10) + 1 \]

or

\[ f(x) = -\frac{2}{5}(x - 15) - 1 \]

or

\[ f(x) = -\frac{2}{5}(x - 20) - 3 \]

Example 4: Find the equation of the line in the table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>100</td>
</tr>
<tr>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>12</td>
<td>49</td>
</tr>
<tr>
<td>37</td>
<td>-26</td>
</tr>
<tr>
<td>150</td>
<td>-365</td>
</tr>
</tbody>
</table>

Solution:

We first need to verify that this is a linear function by finding the slope (the constant rate of change).

\[ \text{slope first two points} = \frac{\Delta y}{\Delta x} = \frac{85 - 100}{0 - (-5)} = \frac{-15}{5} = -3 \]

\[ \text{slope of points three and four} = \frac{\Delta y}{\Delta x} = \frac{-26 - 49}{37 - (12)} = \frac{-75}{25} = -3 \]

\[ \text{slope of points two and three} = \frac{\Delta y}{\Delta x} = \frac{49 - 85}{12 - (0)} = \frac{-36}{12} = -3 \]

\[ \text{slope of points four and five} = \frac{\Delta y}{\Delta x} = \frac{-365 - (-26)}{150 - (37)} = \frac{-339}{113} = -3 \]

We have now identified that this is a linear function because it has a constant rate of change. We have also identified that the slope of this line is -3. Every time x increases by 1 unit, we add -3 to the y-value.

We also see that the y-intercept is the point (0, 85).

Therefore, the slope-intercept form of this linear function is \( f(x) = -3x + 85 \).
**Example 5:** Find the equation of the line with slope 8 that passes through the point (7, 5).

**Solution:** We have been given the slope of the line, but this time we have NOT been given the y-intercept of the line. Instead, we have been given another point on the line which is not the y-intercept. It would be easiest, in this case, to start with the point-slope form of the line (since we have a point and the slope).

We know the equation of the line can be written in the form \( y = m(x - x_1) + y_1 \).

We know that the slope is \( m = 8 \) and we know that a point on the line is \((x_1, y_1) = (7,5)\).

And so, substituting the values into the point-slope formula of a line we get the following equation of the line:

\[
y = 8(x - 7) + 5
\]

We can re-write the equation in slope-intercept form by distributing the 8 and simplifying:

\[
y = 8x - 56 + 5 = 8x - 51
\]

And so the equation of the line is \( y = 8x - 51 \).

This shows that it is an increasing line with a slope of 8 and a y-intercept of \((0, -51)\).

**Example 6:** Find the equation of the line in the table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>49</td>
</tr>
<tr>
<td>37</td>
<td>-26</td>
</tr>
<tr>
<td>150</td>
<td>-365</td>
</tr>
</tbody>
</table>

**Solution:** We first need to verify that this is a linear function by finding the slope (the constant rate of change).

\[
slope \ of \ first \ two \ points = \frac{\Delta y}{\Delta x} = \frac{-26 - 49}{37 - 12} = \frac{-75}{25} = -3
\]

\[
slope \ of \ points \ two \ and \ three = \frac{\Delta y}{\Delta x} = \frac{-365 - (-26)}{150 - 37} = \frac{-339}{113} = -3
\]

We have now identified that this is a linear function because it has a constant rate of change. We have also identified that the slope of this line is -3. Every time \( x \) increases by 1 unit, we add -3 to the \( y \)-value.

We have not been given the y-intercept of this equation, so it would simplest to use the point-slope form of the linear function.

We have three different points on this line, and ANY ONE OF THESE POINTS would be valid to use.

The point-slope form of a linear function is not unique. This means that we could write the point-slope form of a line in more than one way. All of the following equations represent THE SAME linear function, \( f(x) \), even though they each are written in different forms!

In this example the formula of this linear function could be represented as

\[
f(x) = -3(x - 12) + 49 \quad \text{or} \quad f(x) = -3(x - 37) - 26 \quad \text{or} \quad f(x) = -3(x - 150) - 365
\]

Observe that each of these three forms of the function could be re-written in slope-intercept form as \( f(x) = -3x + 85 \). The slope-intercept form of a function is unique because there is only one slope and one y-intercept for any given function.
Example 7: Find the equation of the line that passes through the points \((-7, 4)\) and \((-2,14)\).

Solution: We first find the slope of the line.

\[
\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(14) - (4)}{(-2) - (-7)} = \frac{10}{5} = 2
\]

Next we can use the point slope form in order to write the equation of the line. We can use EITHER point when we write the equations.

\[f(x) = 2(x + 7) + 4 \quad \text{or} \quad f(x) = 2(x + 2) + 14\]

Notice that both of these equations could be simplified to the slope-intercept form of this line which is \(f(x) = 2x + 18\).

Alternatively, we could use the slope-intercept form of the line in order to write the equation. If we want to skip using the point-slope form and directly use the slope-intercept form, then we could do the following:

\[
y = mx + b
\]

\[y = 2x + b\] 

\[4 = 2(-7) + b\]
\[4 = -14 + b\]
\[18 = b\]
And therefore \(y = 2x + 18\).

Please observe that this method of finding the equation does involve more work and solving. It works, but it may involve more work on your part (especially if there are fractions involved). Instead, it is usually preferable to use the point-slope form of a line.

Formula of Exponential Functions

- Exponential functions can be written in the form \(f(x) = a(b)^x\) where \((0, a)\) is the y-intercept of the exponential function, and \(b\) is the factor of the exponential function.

Example 8: Find the equation of the exponential function that passes through \((0,3)\) and has growth factor 1.25.

Solution:
The function has the form \(f(x) = a(b)^x\) where \(a\) is the y-intercept and \(b\) is the factor.
Therefore, we can say that the formula is \(f(x) = 3(1.25^x)\).

Example 9: Find the equation of the exponential function that passes through \((0,7)\) and has decay factor 0.89.

Solution: Exponential functions can be written in the form \(y = a(b)^x\) where \(a\) is the y-intercept and \(b\) is the factor.
Therefore, we can say that the formula is \(f(x) = 7(0.89^x)\).

Example 10: Find the equation of the exponential function that passes through \((0,15)\) and increases at a rate of 7.08%.

Solution: Exponential functions can be written in the form \(y = a(b)^x\) where \(a\) is the y-intercept and \(b\) is the factor.
We have been told that the percent rate is 7.08% = +0.0708. Recall that \(\text{Factor} = 1 + \text{(decimal rate)}\).
This means the factor in the function is \(1+0.0708=1.0708\).
Therefore the formula is \(f(x) = 15(1.0708^x)\).
Example 11: Find the equation of the exponential function that passes through (0,13) and decreases at a rate of 2.6%.

Solution: Exponential functions can be written in the form $y = a(b)^x$ where $a$ is the y-intercept and $b$ is the factor. The function is decreasing at a rate of 2.6%. So, this means the rate is $-2.6\%$.

As a decimal, we can write the rate as $-0.026$. Recall that Factor = $1 + \text{decimal rate}$.

This means the factor in the function is $1 + (-0.026) = 1 - 0.026 = 0.974$.

Therefore, the formula is $f(x) = 13(0.974^x)$.

Example 12: Find the equation of the exponential function with points in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>160</td>
<td>136</td>
<td>115.6</td>
<td>98.26</td>
</tr>
</tbody>
</table>

Solution: Exponential functions can be written in the form $y = a(b)^x$ where $a$ is the y-intercept and $b$ is the factor.

Recall that the factor of an exponential

$$f(x+1) = \frac{y-value \ on \ the \ right}{y-value \ on \ the \ left}$$

for any two points that are exactly 1 x-unit away from each other.

We first test to make sure there is a constant factor between each successive point in the table.

$$\frac{136}{160} = 0.85 \quad \frac{115.6}{136} = 0.85 \quad \frac{98.26}{115.6} = 0.85$$

So we see there is a constant factor of 0.85.

The factor of this exponential is 0.85, and we also see that the y-intercept is (0, 160). Therefore, the formula of this function is $f(x) = 160(0.85)^x$.

Formula of Quadratic Functions

Recall that quadratic functions have equations that can be written in the form $f(x) = ax^2 + bx + c$ where $a \neq 0$. We call this the standard form of a quadratic function, or the expanded form of a quadratic function.

The quadratic function $f(x) = ax^2 + bx + c$ where $a \neq 0$ has the following properties:

- The graph of $f(x) = ax^2 + bx + c$ is a parabola that is open up when $a > 0$, and it is a parabola that is open down when $a < 0$.

- The y-intercept of $f(x)$ is the point $(0, c)$.

- The vertex of the parabola occurs at $x = -\frac{b}{2a}$. Once you have found the x-value of the vertex, you can find the y-value of the vertex by evaluating the function!
  
  - If the parabola is open up, that means the vertex is the MINIMUM point on the function.
  
  - If the parabola is open down, that means the vertex is the MAXIMUM point on the function.

- The x-intercepts of the parabola can also be found using the formula of the quadratic function. But, we will work on that concept in later sections.
Example 13: Use the formula of the function $f(x) = 2x^2 - 8x + 3$ to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph with the help of your graphing calculator.

Solution:
The function $f(x) = 2x^2 - 8x + 3$ is a quadratic function with $a = 2$ and $b = -8$ and $c = 3$.

Since $a$ is greater than 0, we know that the graph of $f(x)$ will be a parabola that is open up.

The $y$-intercept is $(0, c)$. So, the $y$-intercept of this function is $(0, 3)$.

The vertex occurs at $x = \frac{-b}{2a}$.

The $x$-value of the vertex is $x = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$.

The $y$-value of the vertex can be found by evaluating the function when $x=2$.

We find that $f(2) = 2(2)^2 - 8(2) + 3 = -5$.

Therefore, the vertex of this parabola is $(2, -5)$. This function is open up, and so the vertex is a MINIMUM.

Example 14: Use the formula of the function $f(x) = -3x^2 + 15x$ to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph with the help of your graphing calculator.

Solution: The function $f(x) = -3x^2 + 15x$ is a quadratic function with $a = -3$ and $b = 15$ and $c = 0$.

- Since $a$ is less than 0, we know that the graph of $f(x)$ will be a parabola that is open down.

- The $y$-intercept is $(0, c)$. So, the $y$-intercept of this function is $(0, 0)$. Notice that this is also an $x$-intercept since the $y$-value is 0.

- The vertex occurs at $x = \frac{-b}{2a}$. The $x$-value of the vertex is $x = \frac{-15}{2(-3)} = \frac{-15}{-6} = 2.5$.

The $y$-value of the vertex can be found by evaluating the function when $x=2.5$.

We find that $f(2.5) = -3(2.5)^2 + 15(2.5) = 18.75$. Therefore, the vertex of this parabola is $(2.5, 18.75)$. This function is open down, and so the vertex is a MAXIMUM.
Example 15: Use the formula of the function $f(x) = -5x^2 + 4$ to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph with the help of your graphing calculator.

Solution: The function $f(x) = -5x^2 + 4$ is a quadratic function with $a = -5$ and $b = 0$ and $c = 4$.

- Since $a$ is less than 0, we know that the graph of $f(x)$ will be a parabola that is open down.
- The y-intercept is $(0, c)$. So, the y-intercept of this function is $(0, 4)$.
- The vertex occurs at $x = \frac{-b}{2a}$. The x-value of the vertex is $x = \frac{-b}{2a} = \frac{-0}{2(-5)} = \frac{0}{-10} = 0$.

The y-value of the vertex can be found by evaluating the function when $x=0$.
We find that $f(0) = -5(0)^2 + 4 = 4$. Therefore, the vertex of this parabola is $(0, 4)$. This function is open down, and so the vertex is a MAXIMUM.

y-intercept and vertex at (0,4) (maximum)
Section 2.1 Written Practice and Reflection

More in-depth Understanding of the Formulas of Linear, Exponential, Quadratic Functions

1) Find an equation of the line that passes through (0, -4) and has slope 2. Use slope-intercept form.

2) Find an equation of the line with slope 2 that passes through the point (7, 3). You can use slope-intercept form OR point-slope form.

3) Find an equation of the line with slope 6 that passes through the point (-3, 4). You can use slope-intercept form OR point-slope form.

4) Find an equation of the line that passes through (0, 2) and (8, -10). Show your work. You can use slope-intercept form OR point-slope form.

5) Find an equation of the line that passes through (0, -9) and (-7, -1). Show your work. You can use slope-intercept form OR point-slope form.

6) Find an equation of the line that passes through the point (7, 3) and (11, 11). You can use slope-intercept form OR point-slope form.

7) Find an equation of the line that passes through the point (-3, 4) and (1, -8). You can use slope-intercept form OR point-slope form.

8) Find the equation of the exponential function that passes through (0, 4) and has growth factor 1.03.

9) Find the equation of the exponential function that passes through (0, 5) and has decay factor 0.93.

10) Find the equation of the exponential function that passes through (0, 4) and has growth rate 6.5%.

11) Find the equation of the exponential function that passes through (0, 30) and grows at a rate of 10%.

12) Find the equation of the exponential function that passes through (0, 7) and decreases at a rate of 3.2%.

13) Find the equation of the exponential function that passes through (0, 12) and decays at a rate of 13%.

14) Find an equation of the line in the graph below. Show your work.

15) Find an equation of the line in the graph below. Show your work.
16) Find an equation of the line in the graph below. Show all your work.

\begin{align*}
\text{Graph with points:} & \quad (75, 20), (125, 50), (180, 83) \\
\end{align*}

17) Find an equation of the line in the graph below. Show all your work.

\begin{align*}
\text{Graph with points:} & \quad (-25, 65), (-5, -35) \\
\end{align*}

18) Find the equation of the function with points given in the table below.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>1</td>
<td>450</td>
</tr>
<tr>
<td>2</td>
<td>405</td>
</tr>
<tr>
<td>3</td>
<td>364.5</td>
</tr>
</tbody>
</table>

19) Complete the following table.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
<th>Vertex?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x) = x^2 - 6x - 8)</td>
<td>( , , )</td>
<td>( , , )</td>
<td></td>
</tr>
<tr>
<td>(f(x) = -2x^2 + 5)</td>
<td>( , , )</td>
<td>( , , )</td>
<td></td>
</tr>
<tr>
<td>(f(x) = 1 + 7x - x^2)</td>
<td>( , , )</td>
<td>( , , )</td>
<td></td>
</tr>
</tbody>
</table>

20) Use the formula of the function \(f(x) = -10x^2 + x - 4\) to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph and then check using your graphing calculator.

21) Use the formula of the function \(f(x) = 3x^2 - 5x + 9\) to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph and then check using your graphing calculator.

22) Use the formula of the function \(f(x) = -2x^2 + 7x\) to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph and then check using your graphing calculator.

23) Use the formula of the function \(f(x) = 6x^2 + 7\) to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph and then check using your graphing calculator.

24) Use the formula of the function \(f(x) = x^2 + 4x - 10\) to identify the y-intercept, vertex, and general shape of the function. Sketch and label the graph and then check using your graphing calculator.
Section 2.2
Solving Graphically with the Graphing Calculator

Learning Outcomes:

- Use TI graphing calculator to evaluate functions (numerically and graphically)
- Use TI graphing calculator to solve for the input-value using Calc Intersect
- Use TI graphing calculator to solve for maximum and minimum points

Vocabulary:

- Calc Intersect
- Calc Value
- Calc Maximum
- Calc Minimum
- Set up an appropriate viewing window for the real-world situation
- Evaluate graphically, and interpret real-world meaning
- Solve for input value graphically, and interpret real-world meaning
- Solve for maximum or minimum value graphically, and interpret real-world meaning
- Solve equations by using graphical/numerical method instead of solving by hand

Recall from unit 1

**Evaluating** means we are given the input value of a function and asked to identify the value of the output.

**Solving** means that we are given the value of the output, and asked to solve for the value of the input.

In the first unit we used formulas and tables to evaluate and solve functions.

In this section we will create a graph on the calculator, and then use the graph to evaluate and solve.

Once a graph has been created on the graphing calculator, it is a very useful tool for solving problems and answering questions. To solve graphically means that we use the graph to find the answer (rather than using symbolic algebra or a table of values). We will solve graphically to answer three different questions:

- How do you find the y-value graphically when you have been given the x-value?
- How do you find the x-value graphically when you have been given the y-value?
- How do you find the local maximum or local minimum points on a graph (such as the vertex of a quadratic graphically)?
**Graphically Evaluate to find Y-value**

On the graphing calculator, if you have been given the x-value and asked to find the y-value on a function:

1. Create a graph on a viewing window that allows you to see the point on the graph with the given x-value.
2. Press TRACE.
3. Type the given x-value.
4. Press ENTER.
5. The calculator will put the cursor on that point on the graph, and the bottom of the calculator will list the x-value and the y-value of the point. The y-value of that point is the answer you were seeking!

**Graphically Solve to find X-value**

On the graphing calculator, if you have been given the y-value and asked to find the x-value on the function:

1. Create a graph on a viewing window that allows you to see the point(s) on the graph with the given y-value (make a graph that you can see to that height). Remember that there may be more than one answer!
2. In Y2 enter the given y-value. This will draw a horizontal line at that height. We need to find where Y1 = Y2 because that will tell us when the function’s output reaches a height of Y2.
3. Press 2nd TRACE to access the calculate menu.
4. Select Option 5: intersect in order to find the place(s) on the graph where the Y2 horizontal line intersects the function.
5. The calculator asks you to select “first curve” and you should just press enter.
6. The calculator jumps the cursor to the other equation and asks you to select “second curve” and you should again press ENTER.
7. The calculator asks you to make a “guess” about where the point of intersection occurs. You need to move your cursor near the point of intersection (by using the left and right arrow buttons) and then press ENTER.
8. The calculator will put the cursor on the point of intersection and list the x-value and y-value on the bottom of the screen.
9. The x-value of that point is your answer!

If there is another point of intersection, then you need to repeat the process again (starting at part 3).
Example 1 (finding y-value graphically): Suppose \( f(x) = x^2 - 7x - 8 \). Graphically find \( f(15) \).

Solution: We first create the graph on the calculator on a viewing window that allows us to see the point where \( x=15 \). This means we need to include \( x=15 \) in the viewing window. We will let \( \text{XMin} = -10 \) and let \( \text{XMax} = 20 \). Then we press Zoom 0 (Zoom Fit) to let the calculator automatically set the YMin and YMax. When we press Zoom Fit the calculator graphs the function, and if you go back to window, you see that the calculator automatically selected \( \text{YMin} = -20.24... \) and \( \text{YMax} = 252 \).

Next we press TRACE, type 15, then press ENTER. The calculator puts the cursor on the point (15, 112) and we can see the x-value and y-value of that point listed on the bottom of the screen. So the answer to the question is that \( f(15) = 112 \).

Example 2 (finding the x-value graphically): Find where the function \( f(x) = -5x^2 - 10x + 15 \) has a height of \( -4 \).

Solution: To say that the height is \( -4 \) means that the y-value is \( -4 \). We enter the function in the calculator and create a viewing window that allows us to see this function at a height of \( -4 \). We can start with a Standard viewing window.

Next we need to add \( Y2 = -4 \) in the Y= menu and see this horizontal line intersect the function in two different places. The two points of intersection tell us that there will be two different x-values where the function has a y-value of \( -4 \).

To find the two solutions we press 2nd TRACE and select option 5: Intersect.

We will first find the point of intersection on the left side. Then we start over and find the point of intersection on the right side. Press ENTER when it says “first curve,” and then ENTER when it says “second curve,” and then move the cursor on top of the point of intersection and press ENTER again.

So this function has a y-value of -4 when \( x \) is approximately \(-3.19089\) and also when \( x \) is approximately 1.1908902.

So we have learned that \( f(-3.19089) \approx -4 \) and \( f(1.1908902) \approx -4 \).
Example 3 (finding the x-value graphically): Find where the function \( f(x) = 850(0.72)^x \) reaches a height of 500. Round your answer to 4 decimal places.

Solution: We start by entering the function \( f(x) \) into Y1, and since we want to find where the function reaches a y-value of 500, we also enter 500 into Y2. We need to find the point(s) where the function Y1 intersects with Y2 since that will be the point(s) when the function \( f(x) \) has a height of Y2.

Now we need to set a viewing window that will allow us to view the two functions intersecting. Y1 is an exponential function that has a y-intercept of (0,850) and then decreases from there. We know that it will decrease to a height of 500 at some point...but we don’t know the x-value where that height will be reached. So, we should create a window that has a YMax that is higher than 850 and we need to make a best guess about the x-values for the WINDOW.

We will start with the window below:

We can see from this graph that the exponential function reaches a height of 500 around an x-value of x=2. To solve for the x-value to 4 decimal places of accuracy we will need to use Calc Intersect.

Press 2nd Trace and select Option 5: Intersect.

The cursor will automatically be placed on Y1. Press ENTER to indicate that this is the first function you want to intersect.

The cursor will automatically jump to Y2. Press ENTER to indicate that this is the second function you want to intersect.

The calculator will ask for a “guess” about where the point of intersection occurs. Your cursor will already be sitting close enough to the point of intersection. So you can simply press ENTER again.

This tells us that the function \( f(x) = 850(0.72)^x \) reaches a height of 500 when \( x \approx 1.6153 \).
To find a point on the graph that is a PEAK point or a VALLEY point, follow the steps below:

1. Create a graph on a viewing window that allows you to see the point. It is helpful if the point you are trying to find is not right on the edge of the screen (since you may end up with writing covering the point of interest).

2. Press 2nd TRACE to access the Calculate menu.

3. Select either Maximum or Minimum depending on if the point you are finding is a high-point or a low-point.

4. When the cursor asks for a “left-bound,” move the cursor to the left side of the point and press enter.

5. When the cursor asks for a “right-bound,” move the cursor to the right side of the point and press enter.

6. When the cursor asks for a “guess” move your cursor on top of the point (approximately) and press enter.

7. The calculator will put the cursor on the max or min point you are trying to find, and the bottom of the calculator will list the x-value and the y-value of that point.

8. **NOTE that the x-value of the calculator’s answer USUALLY HAS ERROR in it when finding max or min!!** You need to give your answer as the correct, exact value. For example, if the calculator says it is x=.99999999, then you need to write x=1.

**Example 4 (finding the vertex graphically):** Graphically find the vertex of the function $f(x) = x^2 - 6x + 10$.

**Solution:** We first graph the function on a window that allows us to view the vertex. We will start with a Standard viewing window by pressing Zoom 6. This turns out to be a good window because we can clearly see the vertex.

![Graphical View of Vertex](image)

Next we press 2nd TRACE and select option 3: Minimum because the vertex is a low-point on this parabola (since the parabola is open up we know the vertex is a minimum!).

The calculator asks for a left-bound, so we move the cursor to the left-side of the vertex and press enter. The calculator asks for a right-bound, so we move the cursor to the right-side of the vertex and press enter. The calculator asks for a guess, so we put the cursor on the vertex and press enter.

![Calculator Steps](image)

This calculator gives the following answer for the vertex:

![Calculator Answer](image)

Note that **your calculator will probably give something a bit different.** The point is that the x-value given by the calculator usually has a bit of error in it. The correct vertex is the point (3,1). The calculator will give values NEAR x=3, but often will give some error in that answer.
Example 5 (finding a vertex graphically): Graphically find the vertex of the function $f(x) = -3x^2 + 15x + 42$.

Solution: We will start with a Standard viewing window by pressing Zoom 6. This turns out to be a bad window because it doesn't go high enough to see the point we are interested in finding.

We can adjust the window by making the YMax a higher value. Now it is set to YMax = 10. We know the shape of the graph is a parabola that opens downward because the (leading coefficient of the function is negative). Somewhere above this screen the parabola reaches the maximum height at the vertex. We need to estimate a new YMax-value that will be high enough to allow us to see that vertex. We will change YMax to 75. That's high enough for us to clearly see the vertex on the graph. You make select a YMax that is different from this, but 75 is an appropriate value.

Next we press 2nd TRACE and select option 4: Maximum because the vertex is a high-point on this parabola (since the parabola is open down we know the vertex is a maximum!).

The calculator asks for a left-bound, so we move the cursor to the left-side of the vertex and press enter. The calculator asks for a right-bound, so we move the cursor to the right-side of the vertex and press enter. The calculator asks for a guess, so we put the cursor on the vertex and press enter.

Note that your calculator will probably give something a bit different. The point is that the x-value given by the calculator usually has a bit of error in it. The correct vertex is the point (2.5, 60.75). The calculator will give values NEAR x=2.5, but often will give some error in that answer.
In order to solve an equation in 1-variable, we have three following three options:

1) Solve the equation using symbolic methods. This means that we take the symbolic equation and use algebraic techniques to isolate the value of the variable.

2) Solve the equation using numeric methods. This means that we use a table of values in order to identify the value(s) of the input variable that solves the equation. To solve the equation means that we need to identify the value(s) of the input-variable that will make the output of the two functions equal.

3) Solve the equation using graphical methods. This means that we view the left-side of the equation as one function, and view the right-side of the equation as a second function. To solve the equation means that we need to identify the value(s) of the input-variable that will make the output of the two functions equal. This means that we need to find the point(s) of intersection of the two functions. The x-value of the intersection point(s) will solve the equation.

Example 6: Solve the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) using three different techniques (numerical, graphical, symbolic).

(a) First approximate the solution using numerical techniques.

**Solution:** To solve “numerically” means we want to use numbers/tables to solve. In order to do this, observe the following:

Solving the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) means that we want to find the x-value(s) that make

\[
\text{the OUTPUT of } \left(-\frac{2}{3}x + 7\right) = \text{ the OUTPUT of } \left(2x - \frac{13}{7}\right)
\]

This means that we can view the left side of the equation as one function: the function \(Y_1 = -\frac{2}{3}x + 7\).

We can view the right side of the equation as a second function: the function \(Y_2 = 2x - \frac{13}{7}\).

Solving the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) means that we are finding the x-value that makes \(Y_1 = Y_2\).

So we will put the two functions in the calculator, view the table of values, and try to find the x-value(s) where the two functions are equal.

We know that \(Y1\) is a line that “starts” at a height of 7 and goes down from there. We know that \(Y2\) is a line that “starts” at a height of \(-\frac{13}{7} \approx -1.9\) and goes up from there. So, the two functions will have an equal output value for some positive x-value that we don’t yet know.

So we conclude that the solution to the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) must be an x-value that is between x=3 and x=4. This is an approximation of the answer rather than an exact answer. But, it’s a good start!
(b) Solve for the solution graphically with the help of the graphing calculator.

**Solution:** Now we will try to find the solution to the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) by solving graphically. This means that we want to find the x-value(s) that make the OUTPUT of Y1 equal to the OUTPUT of Y2 on the graph.

A point where the two functions intersect will be a point where they have the same y-value when inputting the same x-value. The point of intersection(s) will tell us the x-value where the two functions have the same output.

So we need to find where Y1 and Y2 intersect. The x-value of that point will be the solution of this equation.

We have the two functions in the Y= menu.

We select a viewing window that will allow us to see the point of intersection. We saw in part (a) that the point of intersection will occur somewhere between x=3 and x=4, and somewhere between y=4.14 and y=6.14.

We see that the solution to the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) is \(x \approx 3.3214\).

(c) Solve the equation by hand. Show all work.

**Solution:** To solve the equation \(-\frac{2}{3}x + 7 = 2x - \frac{13}{7}\) by hand, we start by multiplying both sides by the least common multiple which is 21.

\[
21 \left(-\frac{2}{3}x + 7\right) = 21 \left(2x - \frac{13}{7}\right)
\]

Now we distribute the 21 to each term to simplify both sides.

\[-14x + 147 = 42x - 39\]

We add 14x to both sides in order to gather the x-terms on the right.

\[147 = 56x - 39\]

We add 39 on both sides in order to bring the constant terms to the left.

\[186 = 56x\]

We divide by 56 on both sides in order to isolate x.

\[
\frac{186}{56} = x
\]

Simplify the fraction to get \(x = \frac{93}{28}\) which is \(x \approx 3.3214\).
**Example 7:** Solve the equation \( \frac{2}{3}(x - 7) + 9 = x^2 \) numerically and graphically.

(a) Find and approximate solution **numerically**.

**Solution:** To solve numerically means we will use the table of values to help find the solution. We let the left-side of the equation be \( Y_1 \), and we let the right-side of the equation be \( Y_2 \). Then we look at the table of values to approximate the \( x \)-value that will make \( Y_1 = Y_2 \).

We know that \( Y_1 \) is a line that passes through the point (7,9) and it is an increasing function. We know that \( Y_2 \) is a parabola that opens up, and it has a y-intercept of (0,0). Since it is a line intersecting a parabola, there are potentially TWO points where these two functions could intersect.

And so we conclude that the equation \( \frac{2}{3}(x - 7) + 9 = x^2 \) has one solution between \( x = -2 \) and \( x = -1 \), and it has another solution somewhere between \( x = 2 \) and \( x = 3 \).

(b) Solve **graphically** with the help of the graphing calculator.

**Solution:** To solve graphically means we will use the graph in order to find the point of intersection.

Using Calc Intersect twice we find the two points of intersection. This tells us that the solutions to the equation \( \frac{2}{3}(x - 7) + 9 = x^2 \) are \( x \approx -1.7749 \) and \( x \approx 2.4415 \).
Section 2.2 Written Practice and Reflection
Solving Graphically with the Graphing Calculator

1) \( f(x) = -0.2x^2 + 7x - 1 \)
   a) What kind of function is this? Explain how you know the type of function it is from looking at the formula.
   b) What is the y-intercept of the graph? How can you tell by looking at the formula?
   c) Graph the function from \( X = -10 \) to \( X = 40 \). Make sure you set a window that allows you to see the complete shape of the graph. Report the window that you used.
   d) Find \( f(3) \) graphically. Write down a brief description of how you used the calculator to solve the problem.
   e) Solve \( f(x) = 15 \) graphically. Write down a brief description of how you used the calculator to solve the problem.
   f) Find the vertex of the parabola graphically. Write down a brief description of how you used the calculator to solve the problem.
   g) Find both x-intercepts of the function by solving graphically on the calculator. Round your answers to 4 decimal places of accuracy.

2) \( f(x) = 15x^2 + 2x - 3 \)
   a) Will the parabola open up or down? How can you tell by looking at the formula?
   b) What is the y-intercept of the graph? How can you tell by looking at the formula?
   c) Graph the function from \( X = -3 \) to \( X = 3 \). Make sure you set a window that allows you to see the complete shape of the graph. Report the window that you used.
   d) Find \( f(-1) \) graphically. Write down a brief description of how you used the calculator to solve the problem.
   e) Solve \( f(x) = 50 \) graphically. Write down a brief description of how you used the calculator to solve the problem.
   f) Find the vertex of the parabola graphically. Write down a brief description of how you used the calculator to solve the problem.
   g) Find both x-intercepts of the function by solving graphically on the calculator. Round your answers to 4 decimal places of accuracy.

3) \( f(x) = 350(0.97)^x \)
   a) What kind of function is this? Explain how you know the type of function it is from looking at the formula.
   b) Will the graph be increasing or decreasing? How can you tell by looking at the formula?
   c) What is the y-intercept of the graph? How can you tell by looking at the formula?
   d) What is the factor? What is the percent rate? Every time \( x \) increases by 1 unit, what happens to the y-value of the function?
   e) Graph the function from \( X = -5 \) to \( X = 20 \). Make sure you set a window that allows you to see the complete shape of the graph. Report the window that you used.
   f) Find \( f(12) \) graphically. Write down a brief description of how you used the calculator to solve the problem.
   g) Solve \( f(x) = 75 \) graphically. Write down a brief description of how you used the calculator to solve the problem.

4) \( f(x) = 23(1.17)^x \)
   a) What kind of function is this? Explain how you know the type of function it is from looking at the formula.
   b) Will the graph be increasing or decreasing? How can you tell by looking at the formula?
   c) What is the y-intercept of the graph? How can you tell by looking at the formula?
   d) What is the factor? What is the percent rate? Every time \( x \) increases by 1 unit, what happens to the y-value of the function?
   e) Graph the function from \( X = -5 \) to \( X = 20 \). Make sure you set a window that allows you to see the complete shape of the graph. Report the window that you used.
   f) Find \( f(4) \) graphically. Write down a brief description of how you used the calculator to solve the problem.
   g) Solve \( f(x) = 75 \) graphically. Write down a brief description of how you used the calculator to solve the problem.
5) Use your graphing calculator to find where \( f(x) = 8(1.93)^x \) has a height of 3. Round your answer(s) to 4 decimal places of accuracy. To set a viewing window, be sure to think about what you already know about the features and shape of the graph of \( f(x) \).

6) Use your graphing calculator to find where \( f(x) = -3x^2 + 16x + 2 \) has a height of \(-6\). Round your answer(s) to 4 decimal places of accuracy. To set a viewing window, be sure to think about what you already know about the features and shape of the graph of \( f(x) \).

7) Suppose that \( f(x) = -0.96(x + 3)(x - 4)(x - 8) \). Set your viewing window so that \( \text{XMin} = -10 \) and \( \text{XMax} = 10 \). Then use Zoom 0 to set the Y-Window. Find the local maximum and local minimum points that occur on the function. Round your answer(s) to 4 decimal places of accuracy.

8) Use a calculator to find \( S(t) \) for the value of \( t \) given in the following table.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( S(t) = 6.9t - 14.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td></td>
</tr>
</tbody>
</table>

9) Find an equation of the line, \( f(x) \), that passes through \((-3, 2)\) and \((5, 18)\). Then find an equation of the function \( g(x) \) that passes through \((0,3)\) and increases at a rate of 5.1%. Finally, use your graphing calculator to find when the two functions intersect. Round to 4 decimal places.

10) Find an equation of the line, \( f(x) \), that has slope \(-3\) and passes through the point \((4, 7)\). Then find an equation of the function, \( g(x) \), that passes through \((0,5)\) and increases at a rate of 2.8%. Finally, use your graphing calculator to find when the two functions intersect. Round to 4 decimal places.

11) A function has y-intercept \((0, 15)\) and every time \( x \) increases by 1 unit the y-value is 95% of the previous y-value. Find the function.

12) Find an equation of the function \( f(x) \) that has y-intercept \((0, 35)\) and decreases at a rate of 8.2%. Then use the graphing calculator to find when the function reaches a height of 17. Round to 4 decimal places.

13) A function has y-intercept \((0, 34)\) and ever y time \( x \) increases by 1 unit the y-value increases by 2.4%. Find the function.

14) A function has y-intercept \((0, 215)\) and every time \( x \) increases by 1 unit the y-value decreases by 4.8%. Find the function.

15) Solve the equations below by solving graphically using Calc Intersect (with accuracy to 4 decimal places).

\[
\begin{align*}
(a) & \quad -\frac{3}{2}(x + 2) - 4 = 2(x - 1) + 5 \\
(b) & \quad -2x^2 + 5x + 3 = 3x - 4 \\
(c) & \quad x^2 - 3x - 7 = -3x^2 + 5x + 1
\end{align*}
\]
### Section 2.3
Polynomial Vocabulary, Expanding and Factoring, Zero-product Property

#### Learning Outcomes:
- Polynomial vocabulary
- Expand/Multiply expressions
- Factor expressions by factoring out the greatest common factor
- Factor trinomial expressions
- Understand what the zero-product property is and how/when to use it
- Use zero-product property to solve equations and solve for intercepts of functions
- Factor and then use zero-product property

#### Vocabulary:
- Zero-product property
- Factored form
- Expanded form
- Factorable vs. not factorable

---

#### Polynomial Vocabulary

**Monomial:** an algebraic expression of the form $ax^n$ where $a$ is any real number, and $n$ is a natural number (0, 1, 2, 3, ...).

<table>
<thead>
<tr>
<th>Examples of algebraic expressions that ARE monomials: (have the form $ax^n$ with $n$ a whole number)</th>
<th>Examples of algebraic expressions that ARE NOT monomials: (do not have the form $ax^n$ with $n$ a whole number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x^3$</td>
<td>$5x^{-3}$ (can’t have negative exponents)</td>
</tr>
<tr>
<td>$-\frac{1}{2}x^8$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$-0.006x^{48}$</td>
<td>$-0.006x^{1/48}$ (can’t have non-integer exponents)</td>
</tr>
<tr>
<td>$x^2$ (same as $1x^2$)</td>
<td>$\sin(x^2)$ (not allowed to be a trig function like sin, cos, tan)</td>
</tr>
<tr>
<td>$3x$ (same as $3x^1$)</td>
<td>$\sqrt{x}$ (can’t have roots of the variable, $x$)</td>
</tr>
<tr>
<td>$-4$ (same as $-4x^0$)</td>
<td>$-4^x$ (can’t have $x$ in the exponent)</td>
</tr>
</tbody>
</table>

**Binomial:** A binomial is exactly two monomials added together.
For example, $3x^5 + 8x^2$ is a binomial, and $2x^2 - 5x$ is a binomial.

**Trinomial:** A trinomial is exactly three monomials added together.
For example, $3x^3 + 8x^2 - 7x^0$ is a trinomial, and $2x^2 - 5x - 2$ is a trinomial.
Polynomial: an expression that involves added/subtracted monomials. We also refer to monomials as polynomials (even though there is only one of them).

For example, \(2x^2 - 5x + 6\) and \(7x + 8x^5 - \frac{1}{4}x^2 + 250x^3\) are polynomials.

Note: The expression \(2x^2 + \frac{5}{x} - 8\) and the expression \(3x^4 - 3x + 7x^5\) are not polynomials because they involve additions/subtractions of non-monomials!

Degree of a monomial: The degree of \(ax^n\) is \(n\).

For example, the degree of \(-7x^3\) is 3, and the degree of \(\frac{2}{3}x\) is 1 (since it can be written as \(\frac{2}{3}x^1\)), and the degree of 6 is 0 (since it can be written as \(6x^0\)).

Degree of polynomial: The degree of a polynomial is if the degree of the highest-degree monomial within the polynomial.

For example, the degree of \(2x^5 - 17x + 3x^8 + 7\) is 8, and the degree of \(9x + 6\) is 1, and the degree of \(3x^2 - 5x + 2 - x^4\) is 4.

Coefficient of \(ax^n\): The coefficient of \(ax^n\) is \(a\).

For example, the coefficient of \(-7x^3\) is \(-7\), and the coefficient of \(\frac{2}{3}x\) is \(\frac{2}{3}\), and the coefficient of 6 is 6.

Leading coefficient: The leading coefficient of a polynomial is the coefficient of the term of highest degree. To find the leading coefficient, find the term with the HIGHEST DEGREE and the coefficient of that term is the leading coefficient of the polynomial.

For example, the leading coefficient of \(2x^5 - 17x + 3x^8 + 7\) is 3, and the leading coefficient of \(9x + 6\) is 9, and the leading coefficient of \(3x^2 - 5x + 2 - x^4\) is \(-1\).

Constant term: The constant term of a polynomial is the term with no \(x\) in it. It is the coefficient of \(x^0\).

For example, the constant term of \(2x^5 - 17x + 3x^8 + 7\) is 7, and the constant term of \(9x + 6\) is 6, and the constant term of \(3x^2 - 5x - x^4\) is 0 (since it is not there!).

Factored form of a polynomial: A polynomial is written in factored form if it is written as a PRODUCT of algebraic expressions. We will explore how to write some polynomials in this form during this section.

For example, \((2x - 3)(5 + 7x)\) is a polynomial written in factored form, and \(3x(x - 8)(3x + 9)\) is a polynomial written in factored form.

Expanded/Standard form of a polynomial: The form of a polynomial that is multiplied out and simplified to gather and combine all like terms, and then write each term in descending degree order.

For example, \(5x^3 - 8x + 1\) is a polynomial written in expanded form, and \(-18x^{10} + 4x^6 + 9x^5 - 3x\) is a polynomial written in expanded form.

Factorable quadratic function: A quadratic function is factorable over the rational numbers (we usually just say “factorable”) when it can be written in the form \(f(x) = a(x - x_1)(x - x_2)\) where \((x_1, 0)\) and \((x_2, 0)\) are the \(x\)-intercepts of the quadratic function where both \(x\)-intercepts are rational numbers, and \(a\) is a real number.

Non-factorable Quadratic Function: A quadratic function is not factorable over the rational numbers (we usually just say “not factorable”) when it cannot be written in factored form. We will see that most quadratic functions are not factorable because most quadratic functions either do not have \(x\)-intercepts, or have \(x\)-intercepts that are irrational numbers.
It will sometimes be useful to have a polynomial written in factored form, and other times it is more useful to have the polynomial written in expanded form. For this reason, algebra students should be able to convert polynomials back and forth between the expanded/standard form and the factored form. We will learn that many polynomials are not factorable over the rational numbers, but if they are factorable, it will be useful to write them in the factored form.

**Multiplying a Monomial and a Polynomial:** To multiply a monomial times a polynomial, you need to distribute the monomial, and then use rules of exponents to simplify.

**Example 1:** Expand and simplify the expression $-5x^3(2x^2 - 4x + 6)$.

**Solution:**
First we distribute the $-5x^3$ to each of the three terms inside of the parenthesis to get

$(-5x^3)(2x^2) - (5x^3)(-4x) - (5x^3)(6)$

Then we simplify each of the three terms to get $-10x^5 + 20x^4 - 30x^3$.

**Multiplying Binomial times Binomial:** To multiply a binomial times a binomial there are several techniques available. Here we will demonstrate the FOIL method, the VERTICAL method, and also the BOX method for multiplying two binomials.

**Example 2:** Expand and simplify the expression $(5x - 2)(4x + 3)$.

**Solution using FOIL:** We will expand the polynomial $(5x - 2)(4x + 3)$ using the FOIL method. FOIL stands for First, Outer, Inner, Last, and it reminds you of all the quantities that need to be multiplied together...then add all the quantities together and simplify.

- **First:** The first term in the $(5x - 2)$ is $5x$. Multiply that to the first term in the $(4x + 3)$ which is $4x$. So we get $(5x)(4x) = 20x^2$
- **Outer:** The outer terms in the polynomial $(5x - 2)(4x + 3)$ are $5x$ and $3$ since they are on the “outside” of the expression. We multiply them to get $(5x)(3) = 15x$.
- **Inner:** The inner terms in the polynomial $(5x - 2)(4x + 3)$ are $-2$ and $4x$ since they are on the “inside” of the expression. We multiply them to get $(-2)(4x) = -8x$.
- **Last:** The last term in $(5x - 2)$ is $-2$ and the last term in $(4x + 3)$ is $3$. We multiply them together to get $(-2)(3) = -6$.

Now we add all of these 4 values together to get $20x^2 + 15x - 8x - 6$ which simplifies as $20x^2 + 7x - 6$. 

126
Solution using vertical method: We will expand the polynomial $(5x - 2)(4x + 3)$ using the vertical method. This is just like multiplying two numbers together. Stack the values on top of one another, and multiply just like you multiply two digit numbers.

\[
\begin{array}{c}
5x - 2 \\
-2 \\
4x + 3 \\
3 \\
\hline
15x - 6 \\
20x^2 - 8x \\
20x^2 + 7x - 6 \\
\end{array}
\]

Solution using box method: We will expand the polynomial $(5x - 2)(4x + 3)$ using the box method. This method creates a box that is segmented into 4 equal parts. We write one factor across the top, and the other factor down the side. Then multiply the respective terms. Finally, we “add up each part” inside the box to find the expanded terms. See below.

\[
\begin{array}{c|c|c}
5x & -2 \\
4x & 20x^2 & -8x \\
3 & 15x & -6 \\
\end{array}
\]

Now we add up all terms in the box to get the expanded form: $20x^2 + 15x - 8x - 6$ which simplifies as $20x^2 + 7x - 6$.

Multiplying Polynomial times Polynomial: To multiply two polynomials that have more than two terms, it is easiest to use the vertical method. This method is just like multiplying 2 digit and 3 digit numbers (or more) back in elementary school. Stack them up, and multiply terms.

Example 3: Multiply/Expand $(2x^2 + 3x - 5)(3x^2 - 7x + 1)$.

Solution: We stack the polynomials on top of one another and multiply just like multiplying numbers (the vertical method).
Factoring a Polynomial

Recall that writing an expression in factored form means that we are writing the expression as a product of two or more expressions. We will find that having an expression written in factored form can greatly simplify the task of solving equations. So, being able to factor an expression will be very helpful in certain situations.

**Factoring out the Greatest Common Factor (GCF):** To “factor out the greatest common factor” of an expression means that we need to
- find the gcf of each term in the expression
- divide each term by the gcf
- write the original expression as a product using that gcf

**Example 4:** Factor the expression $14x^4 - 7x^3 + 21x^2 + 35x$ by factoring out the greatest common factor.

**Solution:** To do this, we first find the gcf of each of the four terms. We see that each of the four terms have a $7x$ as a factor. So, we factor the $7x$ out of each term (divide each term by $7x$), which leaves the following:

$$14x^4 - 7x^3 + 21x^2 + 35x = 7x(2x^3 - 1x^2 + 3x + 5)$$

**Expanded form**

**Factored form**
How to Factor Quadratic Functions of the Form $ax^2 + bx + c$: (note: proof of why this method work given after examples)

**STEP 1:** Factor out the GCF from the expression.
If the leading coefficient is negative, be sure to factor out the negative!
If the GCF is 1, then you can skip this step.

**STEP 2:** Find ac for your new quadratic that has removed the GCF, and then find 2 numbers that multiply to ac while at the same time add to b.

**STEP 3:** Create a 2X2 table.
Put the leading term $ax^2$ in both top boxes.
Put the two values you selected in the bottom two boxes. Both terms in the bottom are x-terms.

**STEP 4:** Simplify fractions on the left-side of the table.
Simplify fractions on the right-side of the table.

**STEP 5:** The left side of the table can be used to determine one factor, and the right side of the table can be used to determine the other factor. Don’t forget to write the GCF out from the factored expression!

**Example 5:** Write the function $f(x) = -40x^2 + 34x - 6$ in factored form.

**Solution:**

Step 1: $f(x) = -2(20x^2 - 17x + 3)$  
Factor out the GCF which is -2.

Step 2: ac = $20 \times 3 = 60$
Two numbers: -12 and -5
Find a*c=60 and then find two numbers that multiply to ac while at the same time add to -17.

Step 3:

<table>
<thead>
<tr>
<th>$20x^2$</th>
<th>$20x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12x</td>
<td>-5x</td>
</tr>
</tbody>
</table>

Step 4:

<table>
<thead>
<tr>
<th>$20x^2$</th>
<th>$20x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x</td>
<td>4x</td>
</tr>
<tr>
<td>-3</td>
<td>-1</td>
</tr>
</tbody>
</table>

Step 5: $f(x) = -2(5x - 3)(4x - 1)$  
Write the factored form. Don’t forget to include that GCF!

Note: In order to find the two numbers that multiply to ac in step 2, you can work methodically up from 1. Write down all the ways to multiply to 60 with two integers. Focus only on positives. Then select the set of numbers that you can get to add to the middle term, b, if you select negatives correctly.

Example:

| 60 = 1 * 60  | 60 = 4 * 15  |
| 60 = 2 * 30  | 60 = 5 * 12  |
| 60 = 3 * 20  | 60 = 6 * 10  |
Example 6: Factor the function \( f(x) = 120x^2 + 130x + 25 \).

Solution:
Step 1: \( f(x) = 5(24x^2 + 26x + 5) \)

Step 2: \( ac = 24 \times 5 = 120 \)
Two numbers: 20 and 6

Step 3:

\[
\begin{array}{c|c|c}
24x^2 & 24x^2 \\
-20x & 6x \\
\end{array}
\]

Step 4:

\[
\begin{array}{c|c|c}
24x^2 = 6x & 24x^2 = 4x \\
-20x & 6x \\
\end{array}
\]

Step 5: \( f(x) = 5(6x + 5)(4x + 1) \)

Make a 2X2 table. Put the leading term \( 24x^2 \) in both top boxes. Put the two values you found in the bottom two spots. Both of these are \( x \)-terms.

Simplify on the left and simplify on the right. These are your factors.

Write the factored form. Don’t forget to include that GCF!

Example 7: Factor the function \( f(x) = -36x^2 - 48x + 105 \).

Solution:
Step 1: \( f(x) = -3(12x^2 + 16x - 35) \)

Step 2: \( ac = 12 \times -35 = -420 \)
Two numbers: 30 and -14

Step 3:

\[
\begin{array}{c|c|c}
12x^2 & 12x^2 \\
-30x & -14x \\
\end{array}
\]

Step 4:

\[
\begin{array}{c|c|c}
12x^2 = 2x & 12x^2 = 6x \\
-30x & -14x \\
\end{array}
\]

Step 5: \( f(x) = -3(2x + 5)(6x - 7) \)

Make a 2X2 table. Put the leading term \( 12x^2 \) in both top boxes. Put the two values you found in the bottom two spots. Both of these are \( x \)-terms.

Simplify on the left and simplify on the right. These are used to write factors.

Write the factored form. Don’t forget to include that GCF!
Example 8: Write the function \( f(x) = x^2 - 25 \) in factored form.

Solution: First note that the function can be written in the form \( f(x) = x^2 + 0x - 25 \)

Step 1: \( f(x) = x^2 + 0x - 25 \)

Step 2: \( ac = 1 \cdot -25 = -25 \)
Two numbers: 5 and -5

Step 3:

\[
\begin{array}{c|c}
1x^2 & 1x^2 \\
5x & -5x \\
\end{array}
\]

Step 4:

\[
\begin{array}{c|c}
\frac{1x^2}{5x} & \frac{1x^2}{-5x} \\
\frac{x}{5} & \frac{-x}{-5} \\
\end{array}
\]

Step 5: \( f(x) = (x + 5)(x - 5) \)

The GCF is 1, and so we don’t worry about this step.

Find \( a \cdot c = -25 \) and then find two numbers that multiply to \( ac \) while at the same time add to 0.

Make a 2X2 table. Put the leading term \( x^2 \) in both top boxes.
Put the two values you found in the bottom two spots. Both of these are \( x \)-terms.

Simplify on the left and simplify on the right. These are used to write factors.

Write the factored form. This time the GCF was 1, so we don’t write it.

Example 9: Factor the function \( f(x) = x^2 - 15x + 56 \).

Solution:

Step 1: \( f(x) = x^2 - 15x + 56 \)

Step 2: \( ac = 1 \cdot 56 = 56 \)
Two numbers: -7 and -8

Step 3:

\[
\begin{array}{c|c}
x^2 & x^2 \\
-7x & -8x \\
\end{array}
\]

Step 4:

\[
\begin{array}{c|c}
\frac{1x^2}{-7x} & \frac{x^2}{-8x} \\
\frac{x}{-7} & \frac{x}{-8} \\
\end{array}
\]

Step 5: \( f(x) = (x - 7)(x - 8) \)

The GCF is 1, and so we don’t worry about this step.

Find \( a \cdot c = 56 \) and then find two numbers that multiply to \( ac \) while at the same time add to -15.

Make a 2X2 table. Put the leading term \( x^2 \) in both top boxes.
Put the two values you found in the bottom two spots. Both of these are \( x \)-terms.

Simplify on the left and simplify on the right. These are used to write factors.

Write the factored form. This time the GCF was 1, so we don’t write it.
Why does this factoring method work?

Suppose we have a factorable quadratic function \((ax + b)(cx + d) = (ac)x^2 + (ad + bc)x + (bd) = Ax^2 + Bx + C\) where we the GCF of \(Ax^2 + Bx + C\) is 1. Assume also that A is positive (we can factor out negative one first if necessary).

Then, according to this method, we start by finding AC:

\[
AC = acbd
\]

Next we find two numbers that multiply to the value \(AC = acbd\) while at the same time adding to the value \(B = ad + bc\). Those two numbers would be \((ad)\) and \((bc)\).

Now we create the table as follows:

<table>
<thead>
<tr>
<th>(Ax^2)</th>
<th>(Ax^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((ad)x)</td>
<td>((bc)x)</td>
</tr>
</tbody>
</table>

which can be written as

<table>
<thead>
<tr>
<th>(\frac{(ac)x^2}{(ad)x})</th>
<th>(\frac{(ac)x^2}{(bc)x})</th>
</tr>
</thead>
</table>

Now we simplify the left and simplify the right to get values in the table to help us identify the factors:

\[
\frac{(ac)x^2}{(ad)x} = \frac{(c)x}{d} \quad \text{and} \quad \frac{(ae)x^2}{(be)x} = \frac{ax}{b}
\]

And so we see that the factored form would be \((cx + d)(ax + b)\) or \((ax + b)(cx + d)\) as we started with originally.

---

**Zero-Product Property**

If we multiply two numbers together and get an answer of 0, then we know that one (or both) of the two numbers must have been 0. In fact, the only way to get an answer of 0 when multiplying numbers is if one of the numbers being multiplied is 0. This leads us to the following property:

**Zero-Product Property:** If \(M \cdot N = 0\) then either \(M = 0\) or \(N = 0\) or both equal 0.

This is a useful property because we can use it to solve factored expressions set equal to 0.

**Example 10:** Solve the equation \((x - 5)(x + 13) = 0\).

**Solution:** Notice that this equation has the form \(M \cdot N = 0\) where \(M\) is \((x - 5)\) and \(N\) is \((x + 13)\). The only way to multiply these two values together and get an answer of 0 is if \((x - 5) = 0\) or if \((x + 13) = 0\) or if both are 0.

Now we can solve both of these simple linear equations to determine what the value of \(x\) would need to be in order to make each expression equal to 0.

We know that \(x - 5 = 0\) when \(x = 5\), and we know that \(x + 13 = 0\) when \(x = -13\).

So the solutions to the equation \((x - 5)(x + 13) = 0\) are the numbers \(x = 5\) and \(x = -13\).

We can check these solutions to make sure they are correct: (see next page)
If \( x = 5 \) is substituted into the original equation, then we get
\[
(5 - 5)(5 + 13) = 0 \\
(0)(18) = 0 \\
0 = 0 \quad \text{So } 0 \text{ works as a solution to this equation.}
\]

If \( x = -13 \) is substituted into the original equation, then we get
\[
(-13 - 5)(-13 + 13) = 0 \\
(-18)(0) = 0 \\
0 = 0 \quad \text{So } 13 \text{ works as a solution to this equation as well.}
\]

So we can now conclude that there are two solutions to the equation \( (x - 5)(x + 13) = 0 \) which are \( x = 5 \) and \( x = -13 \).

Example 11: Solve the equation \( 0 = x(3x - 15)(5 + 23x) \).

Solution: Notice that this equation has the form \( M \cdot N \cdot P = 0 \) where \( M \) is \( (x) \) and \( N \) is \( (3x - 15) \) and \( P \) is \( (5 + 23x) \). The only way to multiply these three values together and get an answer of 0 is if one or more of these expressions is equal to 0.

Now we can solve each of these simple linear equations to determine what the value of \( x \) would need to be in order to make each expression equal to 0.

We know that \( x = 0 \) when \( x = 0 \), and we know that \( 3x - 15 = 0 \) when \( x = 5 \) and we know \( 5 + 23x = 0 \) when \( x = -\frac{5}{23} \).

So the solutions to the equation \( x(3x - 15)(5 + 23x) = 0 \) are the numbers \( x = 0 \) and \( x = 5 \) and \( x = -\frac{5}{23} \).

We can check these solutions to make sure they are correct:

If \( x = 0 \) is substituted into the original equation, then we get
\[
(0)(3(0) - 15)(5 + 23(0)) = 0 \\
(0)(-15)(5) = 0 \\
0 = 0 \\
\text{So } 0 \text{ works as a solution to this equation.}
\]

If \( x = 5 \) is substituted into the original equation, then we get
\[
(5)(3(5) - 15)(5 + 23(5)) = 0 \\
(5)(0)(120) = 0 \\
0 = 0 \\
\text{So } 5 \text{ works as a solution to this equation as well.}
\]

If \( x = -\frac{5}{23} \) is substituted into the original equation, then we get
\[
\left(-\frac{5}{23}\right)\left(3\left(-\frac{5}{23}\right) - 15\right)\left(5 + 23\left(-\frac{5}{23}\right)\right) = 0 \\
\left(-\frac{5}{23}\right)\left(-\frac{360}{23}\right)(0) = 0 \\
0 = 0 \\
\text{So } -\frac{5}{23} \text{ works as a solution to this equation as well.}
\]

So we can now conclude that there are three solutions to the equation \( x(3x - 15)(5 + 23x) = 0 \) which are \( x = 0 \) and \( x = 5 \) and \( x = -\frac{5}{23} \).
Example 12: Find the x-intercepts of the function \( f(x) = x^2 - 64 \) by symbolically solving. Then support your answer graphically.

Solution: We know that x-intercepts occur where the y-value equals 0. So, we let \( y=0 \) in the equation and solve for the unknown \( x \)-values.

<table>
<thead>
<tr>
<th>( 0 = x^2 - 64 )</th>
<th>Let the y-value equal 0 to solve for the unknown x-values.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 = (x - 8)(x + 8) )</td>
<td>Factor the right-side of the equation.</td>
</tr>
<tr>
<td>( x - 8 = 0 ) or ( x + 8 = 0 )</td>
<td>Use the zero-product property to solve for ( x ).</td>
</tr>
<tr>
<td>( x = 8 ) and ( x = -8 ) are the two solutions</td>
<td></td>
</tr>
<tr>
<td>The x-intercepts are the points ( (8,0) ) and ( (-8,0) )</td>
<td>Answer the initial question.</td>
</tr>
</tbody>
</table>

To support the answer graphically, we will graph the function and check to see that the x-intercepts occur at these two points. We set \( Y_2=0 \) and find the two points where the function intersects a height of \( Y_2=0 \). The graphs below show support of our symbolic solutions.

Example 13: Symbolically determine the x-intercepts of the function \( -3x^2 - 9x + 84 = f(x) \).

Solution:

<table>
<thead>
<tr>
<th>(-3x^2 - 9x + 84 = 0)</th>
<th>To find x-intercepts means we need to let the output value equal 0 and then solve for the input value.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3(x^2 + 3x - 28))</td>
<td>Factor out (-3) on the left side.</td>
</tr>
<tr>
<td>(-3(x + 7)(x - 4) = 0)</td>
<td>Factor the left side completely.</td>
</tr>
<tr>
<td>( x + 7 = 0 ) or ( x - 4 = 0 )</td>
<td>Use the zero-product property to solve for ( x ).</td>
</tr>
<tr>
<td>( x = -7 ) and ( x = 4 ) are the two solutions</td>
<td></td>
</tr>
<tr>
<td>The x-intercepts are ((-7,0)) and ((4,0)).</td>
<td>Answer the initial question.</td>
</tr>
</tbody>
</table>

To support the answer graphically, we will graph the function and check to see that the x-intercepts occur at these two points. We set \( Y_2=0 \) and find the two points where the function intersects a height of \( Y_2=0 \). The graphs below show support of our symbolic solutions.
Example 14: Symbolically determine the x-intercepts of the function \( f(x) = 6x^2 - 1x - 40 \).

Solution:

\[
\begin{array}{c|c|c}
 ac & -240 & \text{The GCF is 1. So we proceed to finding AC which is -240. The two numbers are -16 and 15 since } -16 \times 15 = -240 \text{ and } -16 + 15 = -1 \\
\hline
6x^2 & 6x^2 & \text{Create a 2x2 table. Put } 6x^2 \text{ in the two top spots. Put } -16x \text{ and } 15x \text{ in the bottom two spots.} \\
-16x & 15x & \\
\hline
3x & 2x & \text{Simplify the left and simplify the right.} \\
-8 & 5 & \\
\end{array}
\]

\[
0 = (3x - 8)(2x + 5)
\]

Write the original equation in factored form.

\[
x = \frac{8}{3} \text{ or } 2x + 5 = 0
\]

\[
x = \frac{8}{3} \text{ and } x = -5/2 \text{ are the two solutions}
\]

Answer the initial question.

To support the answer graphically, we will graph the function and check to see that the x-intercepts occur at these two points. We set \( Y_2 = 0 \) and find the two points where the function intersects a height of \( Y_2 = 0 \). The graphs below show support of our symbolic solutions.

Example 15: Symbolically solve the equation \( x^3 - 3x^2 - 28x = 0 \).

Solution:

\[
0 = x^3 - 3x^2 - 28x
\]

First observe that we can factor out a common factor of \( x \) from each term.

\[
0 = x(x^2 - 3x - 28)
\]

Factor the trinomial and re-write the original equation in factored form.

\[
x = 0 \text{ or } x - 7 = 0 \text{ or } x + 4 = 0
\]

\[
x = 0 \text{ and } x = 7 \text{ and } x = -4 \text{ are the three solutions}
\]

Answer the initial question.

To support the answer graphically, we will graph the function and check to see that the x-intercepts occur at these two points. We set \( Y_2 = 0 \) and find the two points where the function intersects a height of \( Y_2 = 0 \). The graphs below show support of our symbolic solutions.
**Example 16**: Symbolically determine where the function \( f(x) = x^2 - 9x - 21 \) reaches a height (a y-value) of 15.

**Solution**: We want to find where the function has a y-value of 15. So we let \( y=15 \) in the function, and symbolically solve for the unknown \( x \)-values.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 15 = x^2 - 9x - 21 )</td>
<td>Let the y-value equal 15 and we need to solve for the unknown x-values.</td>
</tr>
<tr>
<td>( 0 = x^2 - 9x - 36 )</td>
<td>Re-write the equation so it is a quadratic set equal to 0. This will allow us to use the zero product property. Subtract 15 on both sides.</td>
</tr>
<tr>
<td>( 0 = (x - 12)(x + 3) )</td>
<td>Factor the right side of the equation.</td>
</tr>
<tr>
<td>( x - 12 = 0 ) or ( x + 3 = 0 )</td>
<td>Use the zero-product property to solve for ( x ).</td>
</tr>
<tr>
<td>( x = 12 ) and ( x = -3 ) are the two solutions</td>
<td>The function will have a height (a y-value) of 15 when the ( x )-value is 12 and when the ( x )-value is (-3).</td>
</tr>
</tbody>
</table>

To support the answer graphically, we will graph the left-side and right-side of the original equation and check to see that the solutions occur at these two points of intersection. The graphs below show support of our symbolic solutions.
2.3 Written Practice and Reflection
Polynomial Vocabulary, Expanding and Factoring, and the Zero-product Property

1) Complete the following table in order to practice the vocabulary of polynomials.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Degree?</th>
<th>Leading coefficient?</th>
<th>Constant term?</th>
<th>Coefficient of $x^2$?</th>
<th>Coefficient of $x$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x^2 + 3x - 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3x - 7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x - 8x^2 + 7x^4 + 9 - 2x^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$15x^8 + 9x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Expand and simplify $-2x^2(3x^5 - 5x + 7)$. Show all work.

3) Expand and simplify $(2x - 1)(3x + 5)$. Show all work.

4) Expand and simplify $(4x^2 + 2x - 4)(2x^2 - 5x + 3)$. Show all work.

5) Solve each of the following equations by factoring and then using the zero-product property. Be sure to show all steps in the process!
   (a) $x^2 - 12x + 27 = 0$
   (b) $5x^2 - 18x - 8 = 0$
   (c) $6x^2 + x - 2 = 0$

6) The height of a ball, in feet, $t$ seconds after it has been thrown is given by the function $H(t) = -16t^2 + 64t + 80$.
   (a) How high is the ball initially? Be sure to use units in your answer!
   (b) SYMBOLICALLY find how long it takes until the ball hits the ground by factoring and solving. Show all your work. Write a complete sentence to answer the question. Check your work graphically!
   (c) SYMBOLICALLY find the highest that the ball gets, and when does it reach that height? Show all your work. Write a complete sentence to answer the question. Check your work graphically!
   (d) What would be an appropriate REAL WORLD domain to consider for this problem? Explain.
   (e) What would be an appropriate REAL WORLD range to consider for this problem? Explain.

7) Consider the $3^{rd}$ degree polynomial $f(x) = 2x^3 - 22x^2 + 36x$.
   (a) Find the $x$-intercepts of the function $f(x)$ by factoring and using the zero-product property. Show your work. Check your work graphically!
   (b) What is the $y$-intercept of this function? Write your answer as an ordered pair (a point).
   (c) Graphically find all local maximums and local minimums (all the peak points and valley points) on the graph of $f(x)$. Give each of the answers as ordered pairs rounded to 4 decimal places.
   (d) Sketch a well-labeled complete graph of $f(x)$ that labels all intercepts, maximums, and minimums.
8) Complete the table.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree?</th>
<th>Leading coefficient?</th>
<th>Coefficient of $x^2$?</th>
<th>Constant term?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = -8x^9 + 12 - 7x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 11 + 2x^{10} - 6x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 18 + 17x^7$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 16x - 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 10x^4 + 8 - 5x^2 - x^{14}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 7x^3 - 3x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9) Expand/Multiply each of the following and then combine like-terms in order to simplify completely.

(a) $-7x(x - 5)$
(b) $(x + 8)9x$
(c) $(x + 2)(x + 7)$
(d) $(x - 3)(x - 5)$
(e) $(4a - 3)(5a + 5)$
(f) $(r - 5)(r + 7)$
(g) $(6z - 1)(5 - 8z)$
(h) $(-5x - 3)(x - 5)$
(i) $(x^2 + x + 7)(x - 7)$

10) Factor each of the following polynomials if possible.

(a) $s^2 + 6s + 5$
(b) $r^2 + 4r + 3$
(c) $v^2 - 15v + 56$
(d) $v^2 - 4v - 12$
(e) $6x^2 - 7x - 3$
(f) $3a^2 - 14a + 8$
(g) $8y^2 + 19y + 11$
(h) $15x^2 - x - 6$
(i) $20v^2 + 19v + 3$
(j) $2v^2 - 24v + 54$
(k) $8s^2 - 30s + 18$
(l) $7n^3 - 21n^2 - 126n$
(m) $4r^3 + 12r^2 - 72r$
(n) $20w^3 - 41w^2 + 20w$
(o) $8x^3 + 28x^2 + 24x$
(p) $3x^3 - 15x^2 + 18x$
(q) $60x^4 + 8x^3 - 4x^2$
(r) $25x^4 - 160x^3 + 60x^2$

11) Solve each of the following equations using the zero-product property.

(a) $8x(x - 2) = 0$
(b) $(s + 9)(s - 12) = 0$
(c) $(10a + 7)(3a - 9) = 0$
(d) $t^2 - 4t = 0$
(e) $10x^4 + 2x^3 = 0$
(f) $8x^2 - 5x = 0$
(g) $-3x^2 = 21x$
(h) $5x^2 = 3x$
(i) $4t^4 + 21t^3 = 0$
(j) $w^2 + 3w - 28 = 0$
(k) $5x^2 + 6x - 63 = 0$
(l) $5x^2 - 11x = 12$
(m) $8x^2 - 12x - 56 = 0$

12) Find an equation of the line, $f(x)$, that has slope 2 and passes through the point $(7, 3)$. Then BY HAND find when the line $f(x)$ intersects the quadratic function $g(x) = x^2 + 3x - 67$.

13) Find an equation of the line, $f(x)$, that has slope $-3$ and passes through the point $(2, 5)$. Then BY HAND find when the line $f(x)$ intersects the quadratic function $g(x) = x^2 - 4x - 19$. 
Suppose we have a quadratic function set equal to zero and we want to solve for the x-values. So far we have been able to solve the equation by hand by first factoring and then using the zero-product property. Unfortunately, factoring and then using the zero-product property is only useful if the quadratic function has a very specific form. You can only factor and solve if the quadratic function you are working with is factorable!

**Note:** The zero-product property can only be used to solve \(0 = ax^2 + bx + c\) when the quadratic \(ax^2 + bx + c\) is factorable. Unfortunately, most quadratics are not factorable!!! So, most quadratics cannot be solved with the zero-product property!!!

**Example 1:** Try to solve the following equations by factoring and then using the zero-product property.

(a) \(0 = x^2 + 2x - 1\)  
(b) \(0 = x^2 + 2\)  
(c) \(0 = x^2 - 2\)

**Solution:**

(a) To solve the equation \(0 = x^2 + 2x - 1\) we try to factor. We need two numbers that multiply to -1 and at the same time add to 2. The only way to multiply to -1 with integers is to multiply 1*-1. Unfortunately, when we add -1 + 1 we get 0 and not the value 2. Therefore, we conclude that we cannot factor in order to solve this equation! We can say that the function \(f(x) = x^2 + 2x - 1\) is not factorable.

(b) To solve the equation \(0 = x^2 + 2\) we try to factor. Remember that we can rewrite the equation as \(0 = x^2 + 0x + 2\). We need two numbers that multiply to 2 and at the same time add to 0. The only way to multiply to 2 with integers is to multiply 1*2 or to multiply -1*-2. Unfortunately, when we add 1 + 2 we get 3 and when we add -1 + -2 we get -3. Neither option gives us 0 which we needed. Therefore, we conclude that we cannot factor in order to solve this equation! We can say that the function \(f(x) = x^2 + 2\) is not factorable.

(c) To solve the equation \(0 = x^2 - 2\) we try to factor. Remember that we can rewrite the equation as \(0 = x^2 + 0x - 2\). We need two numbers that multiply to -2 and at the same time add to 0. The only way to multiply to -2 with integers is to multiply 1*-2 or to multiply -1*2. Unfortunately, when we add 1 + -2 we get -1 and when we add -1 + 2 we get 1. Neither option gives us 0 which we needed. Therefore, we conclude that we cannot factor in order to solve this equation! We can say that the function \(f(x) = x^2 - 2\) is not factorable.
So what do we do if we want to solve a quadratic equation that isn’t factorable?

- If the quadratic equation has the form \((\text{linear expression})^2 = \text{Non-Negative number}\), then we can solve for \(x\) using the square root property.

- Any quadratic equation, no matter the form, can be solved using the quadratic formula.

Both the Square Root Property and the Quadratic Formula will require that we make use of square roots. Therefore, we will begin with some discussion of Square Roots.

**Understanding Square Roots:**

The square root is the inverse operation of squaring a value:

\[ \sqrt{M} = A \] tells us that \(M = A^2\)

**Example 2:**

\[ \sqrt{4} = 2 \] tells us that \(4 = 2^2\)

\[ \sqrt{9} = 3 \] tells us that \(9 = 3^2\)

\[ \sqrt{729} = 27 \] tells us that \(729 = 27^2\)

\[ \sqrt{453} \approx 21.2838 \] tells us that \(453 \approx 21.2838^2\)

**Square Root Property**

Many students forget that equations such as \(x^2 = 4\) have TWO solutions rather than only one solution. In fact, all equations of the form \(x^2 = (\text{positive number})\) have two real solutions. This leads us to the following property:

**Square Root Property:** If \(G^2 = a\) where \(a\) is any positive number, then \(G = \pm\sqrt{a}\).

In other words, the two solutions to the equation are \(G = \sqrt{a}\) and \(G = -\sqrt{a}\).

**Example 3:** Solve the equation \(x^2 = 9\).

**Solution:** The equation has the form \(G^2 = a\), and so we can use the square root property. We take the square root of both sides.

\[ \sqrt{x^2} = \pm\sqrt{9} \]

The square root property tells us that there are TWO solutions.

\[ x = 3 \text{ and } x = -3 \]

Check: It is true that \((3)^2 = 9\) and it is true that \((-3)^2 = 9\). So both \(x\)-values are solutions to the original equation.
Example 4: Find the exact solutions of the equation $3(x)^2 = 5$. Then approximate the solutions to 3 decimal places.

Solution:

$3(x)^2 = 5$

$(x)^2 = \frac{5}{3}$

Divide both sides by 3.

$\sqrt{(x)^2} = \pm \sqrt{\frac{5}{3}}$

Take the square root of both sides to “un-do” the square on the $x$.

$x = \pm \sqrt{\frac{5}{3}}$

Remember that there are TWO square roots, so there will be two answers.

$x = \sqrt{\frac{5}{3}}$ and $x = -\sqrt{\frac{5}{3}}$

These are the exact solutions.

$x \approx 1.291$ and $x \approx -1.291$

These are the approximate solutions.

Example 5: Find the exact solutions of the equation $(x + 2)^2 = 6$. Then approximate the solutions to 4 decimal places.

Solution:

$(x + 2)^2 = 6$

$\sqrt{(x + 2)^2} = \pm \sqrt{6}$

Take the square root of both sides to “un-do” the square.

$x + 2 = \pm \sqrt{6}$

Remember there are TWO solutions.

$x = -2 \pm \sqrt{6}$

Subtract 2 on both sides to isolate $x$. These are the exact solutions.

$x \approx 0.4495$ and $x \approx -4.4495$

Approximate these values on the calculator.

Example 6: Find where the function $f(x) = (2x - 7)^2$ has a height (a y-value) of 10. Find the exact solutions. Then approximate the solutions to 4 decimal places.

Solution:

$(2x - 7)^2 = 10$

We first let the y-value of the function be 10.

$\sqrt{(2x - 7)^2} = \sqrt{10}$

Take the square root of both sides to “un-do” the square.

$2x - 7 = \pm \sqrt{10}$

Remember there are TWO solutions.

Continued on next page
\[ 2x = 7 \pm \sqrt{10} \quad \text{ADD 7 on both sides to get closer to isolating x.} \]

\[ \frac{2x}{2} = \frac{7 \pm \sqrt{10}}{2} \quad \text{Divide left side and right side by 2 to isolate x. Remember that EVERYTHING needs to be divided by 2.} \]

\[ x = \frac{7 + \sqrt{10}}{2} \quad \text{and} \quad x = \frac{7 - \sqrt{10}}{2} \quad \text{These are the exact solutions.} \]

\[ x \approx 1.9189 \quad \text{and} \quad x \approx 5.0811 \quad \text{Approximate these values on the calculator.} \]

**Example 7:** Find the x-intercepts of the function \( f(x) = 2(2x - 3)^2 - 1 \). Approximate the solutions to 4 decimal places.

**Solution:**

\[ 0 = 2(2x - 3)^2 - 1 \quad \text{x-intercepts occur where the y-value is 0, so we let y=0.} \]

\[ 1 = 2(2x - 3)^2 \quad \text{Add 1 on both sides of equation.} \]

\[ \frac{1}{2} = (2x - 3)^2 \quad \text{Re-write the equation in the form } a = G^2 \text{ so that we can use the square root property.} \]

\[ \sqrt{\frac{1}{2}} = \sqrt{(2x - 3)^2} \quad \text{Take the square root of both sides to “un-do” the square.} \]

\[ \pm \sqrt{\frac{1}{2}} = 2x - 3 \quad \text{Remember there are TWO solutions} \]

\[ \frac{3 \pm \sqrt{\frac{1}{2}}}{2} = x \quad \text{ADD 3 on both sides and then divide everything by 2 to isolate x. These are the EXACT solutions.} \]

\[ x \approx 1.8536 \quad \text{and} \quad x \approx 1.1464 \quad \text{Approximate these values on the calculator.} \]

The x-intercepts are \((1.8536,0)\) and \((1.1464,0)\)

The square root property and the zero product property allow us to solve quadratic equations that have very specific forms. But, the quadratic formula will allow us to solve quadratic equations in ANY form. The quadratic formula is the most useful thing to know if you are interested in solving quadratic equations by hand.
If \( ax^2 + bx + c = 0 \) where \( a \neq 0 \) then 
\[
X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Note:** When you use the quadratic formula, you can get no solutions, or one solution, or two solutions. The solution(s) you get will solve the equation \( ax^2 + bx + c = 0 \). You should check your solution(s) by plugging them back into the original equation!

**Note:** The quadratic formula can be used to solve ANY quadratic equation. The quadratic doesn’t have to be in a specific form since you could always re-write any quadratic function in the form \( ax^2 + bx + c \). Because of this, if you want to solve a quadratic equation by hand, the quadratic formula is the best thing to know!!

- Factoring and the zero-product property won’t always work (since most quadratics aren’t factorable). But factoring and solving is quick and easy if it is factorable.
- Square root property won’t always work (since most quadratic equations can’t easily be written in the form \( G^2 = a \)). But solving with the square root property is quick and easy if the equation is able to be solved that way.
- You can always take any quadratic function and write it in the form \( f(x) = ax^2 + bx + c \) where \( a \neq 0 \). Therefore, ANY quadratic equation can be solved using the quadratic formula!

**Discriminant of a quadratic function:** The discriminant of the quadratic function \( f(x) = ax^2 + bx + c \) is the value \( b^2 - 4ac \). The discriminant is the number underneath the radical when you use the quadratic formula. The value of the discriminant will give information about the x-intercepts of the function. If the discriminant is negative, then there are no x-intercepts (since there will be no real solutions to the quadratic formula). If the discriminant is 0, then there is exactly one x-intercept (since the quadratic formula will only give one solution). If the discriminant is positive, then there are two x-intercepts (since the quadratic formula will give two solutions).

**How to use the quadratic formula to solve quadratic equations:**

1. Make sure the equation satisfies the conditions of the quadratic formula. That is, make sure you have an equation of the form \( ax^2 + bx + c = 0 \).
2. Identify \( a \) and \( b \) and \( c \) in the function.
3. Write the quadratic formula down on your paper.
4. Substitute the values for \( a \), \( b \), \( c \) into the quadratic formula. Remember that \( b^2 \) will ALWAYS be a positive number!
5. Use your calculator to find the discriminate which is the value UNDER the square root (do NOT take the square root yet). Determine if there will be no solutions, 1 solution, or 2 solutions using the discriminate.
6. Simplify to find the two EXACT answers. You can use your calculator to find the approximate solutions if necessary.
**Example 8:** Find the x-intercepts of the function \( f(x) = x^2 + 9x + 18 \) by hand.

**Solution:** Notice, first, that this could be solved by factoring and using the zero-product property. But for this example we will use the quadratic formula. Either method will yield the same results.

To find the x-intercepts we need to solve the equation
\[
0 = x^2 + 9x + 18
\]
\( a = 1 \) and \( b = 9 \) and \( c = 18 \) in this function

The equation is written in the form
\[
a x^2 + bx + c = 0.
\]
So we can use the quadratic formula to solve it.

Identify \( a \) and \( b \) and \( c \) in the function.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Write the quadratic formula down on your paper.

\[
x = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(18)}}{2(1)}
\]

Substitute the values for \( a, b, c \). Remember that \( b^2 \) will ALWAYS be a positive number!

\[
x = \frac{-9 \pm \sqrt{9}}{2(1)}
\]

Use your calculator to find the value UNDER the square root (do NOT take the square root yet).

Since it is a positive number, we know there will be two x-intercepts.

\[
x = \frac{-9 + \sqrt{9}}{2} \quad \text{and} \quad x = \frac{-9 - \sqrt{9}}{2}
\]

Simplify to find the two EXACT answers.

\[
x = -3 \quad \text{and} \quad x = -6
\]

This means the x-intercepts are the points \((-3,0)\) and \((-6,0)\).

Answer the initial question.

The graphing calculator helps us ensure that our symbolic calculations were correct:

![Graph](image-url)
Example 9: Find the x-intercepts of the function \( f(x) = x^2 - 10x + 25 \) by hand.

Solution: Notice, first, that this could be solved by factoring and using the zero-product property. But for this example we will use the quadratic formula. Either method will yield the same results.

We need to solve the equation

\[
0 = x^2 - 10x + 25
\]

\( a = 1 \) and \( b = -10 \) and \( c = 25 \) in this function

The equation is written in the form

\[
a x^2 + bx + c = 0
\]

So we can use the quadratic formula to solve it.

Identify \( a \) and \( b \) and \( c \) in the function.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Write the quadratic formula down on your paper.

\[
x = \frac{-(10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}
\]

Substitute the values for \( a, b, c \). Remember that \( b^2 \) will ALWAYS be a positive number!

\[
x = \frac{10 \pm 0}{2}
\]

Use your calculator to find the value UNDER the square root (do NOT take the square root yet). Since it is 0, we know there will be exactly 1 x-intercept.

\[
x = \frac{10}{2} \text{ and } x = \frac{10}{2}
\]

Simplify to find the two EXACT answers.

\[
x = 5 \text{ and } x = 5 \text{ (only one root)}
\]

This means there is only one x-intercept at the point \((5,0)\).

Answer the initial question.

The graphing calculator helps us ensure that our symbolic calculations were correct:
**Example 10**: Solve the equation $3x^2 + 10x = 9$ by hand.

**Solution**: 

Solve the equation $3x^2 + 10x = 9$. First re-write the equation by subtracting $3x^2$ and $10x$ from both sides.

Now we need to solve the equation 

$0 = -3x^2 - 10x + 9$

$a = -3$ and $b = -10$ and $c = 9$ in this function

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

First re-write the equation so that it is written in the form $ax^2 + bx + c = 0$. In this case, we bring all terms to the right so that we can use the quadratic formula to solve it.

Identify $a$ and $b$ and $c$ in the function.

Write the quadratic formula down on your paper

Substitute the values for $a$, $b$, $c$. Remember that $b^2$ will ALWAYS be a positive number!

Use your calculator to find the value UNDER the square root (do NOT take the square root yet). Since it is positive we know there will be two $x$-intercepts.

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(-3)(9)}}{2(-3)}$

These are the two EXACT answers.

$x = \frac{-(-10) \pm \sqrt{208}}{2(-3)}$

Simplified form of these values: 

$x = \frac{5 + 2\sqrt{13}}{-3}$ and $x = \frac{5 - 2\sqrt{13}}{-3}$

$x \approx -4.0704$ and $x \approx 0.7370$ (these are the approximations)

$x \approx -4.0704$ and $x \approx 0.7370$ (these are the approximations)

This means the solutions are $x \approx -4.0704$ and $x \approx 0.7370$

Now we can approximate the solutions since the values are not rational numbers.

Answer the initial question.

The graphing calculator helps us ensure that our symbolic calculations were correct:
**Example 11**: Find the x-intercepts of the function \( f(x) = 2(x^2 + 2) + 3x \) by hand.

**Solution:**

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribute the 2 in ( f(x) = 2(x^2 + 2) + 3x ) to get ( f(x) = 2x^2 + 4 + 3x ). And then re-order the terms to get ( f(x) = 2x^2 + 3x + 4 ).</td>
<td>First re-write the equation so that it is written in the form ( ax^2 + bx + c = 0 ). In this case, we bring all terms to the right so that we can use the quadratic formula to solve it.</td>
</tr>
<tr>
<td>We need to solve ( 0 = 2x^2 + 3x + 4 ). ( a = 2 ) and ( b = 3 ) and ( c = 4 ) in this function.</td>
<td>Identify ( a ) and ( b ) and ( c ) in the function.</td>
</tr>
<tr>
<td>( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} )</td>
<td>Write the quadratic formula down on your paper.</td>
</tr>
<tr>
<td>( x = \frac{-3 \pm \sqrt{3^2 - 4(2)(4)}}{2(2)} )</td>
<td>Substitute the values for ( a ), ( b ), ( c ). Remember that ( b^2 ) will ALWAYS be a positive number!</td>
</tr>
<tr>
<td>( x = \frac{-3 \pm \sqrt{-23}}{2(2)} )</td>
<td>Use your calculator to find the value UNDER the square root (do NOT take the square root yet). Since it is negative we know there are no x-intercepts.</td>
</tr>
<tr>
<td>( x = \frac{-3 + \sqrt{-23}}{4} ) and ( x = \frac{-3 - \sqrt{-23}}{4} ) (observe that these are complex numbers)</td>
<td>Simplify to find the two EXACT answers. Observe that the square root of a negative number is a COMPLEX NUMBER (it is not a real number that can be found on the real number line).</td>
</tr>
</tbody>
</table>

The solutions to this equation are complex numbers. There are no x-intercepts because these solutions cannot be found on the real number line. | Answer the initial question. |

The graphing calculator helps us ensure that our symbolic calculations were correct:
Section 2.4 Written Practice and Reflection
Square Root Property and Quadratic Formula

1) Answer the following questions about quadratic function using complete sentences that completely and accurately answer the questions posed.

(a) Explain what you know about the graph of a quadratic function \( f(x) \) when the discriminant of \( f(x) \) is negative. Notice that this means that solving the equation \( 0 = ax^2 + bx + c \) results in the solution(s) \( X = \frac{-b \pm \sqrt{NegativeNumber}}{2a} \).

(b) Explain what you know about the graph of a quadratic function \( f(x) \) when the discriminant of \( f(x) \) is zero. Notice that this means that solving the equation \( 0 = ax^2 + bx + c \) results in the solution(s) \( X = \frac{-b \pm \sqrt{zero}}{2a} \).

(c) Explain what you know about the graph of a quadratic function \( f(x) \) when the discriminant of \( f(x) \) is positive. Notice that this means that solving the equation \( 0 = ax^2 + bx + c \) results in the solution(s) \( X = \frac{-b \pm \sqrt{PositiveNumber}}{2a} \).

(d) What is the number under the radical called?

2) Aircraft A is travelling due East at a constant speed, and Aircraft B is travelling due North at a constant speed. The function \( D(t) = 0.2074t^2 - 4.57t + 30.40 \) gives the distance that two planes are from each other (in miles) as the minutes pass.

(a) Solve the equation \( 0 = .2074t^2 - 4.57t + 30.40 \) by hand. Show all your work and if the answers are complex, please say so.

(b) Write a clear and complete explanation that explains why your answer to part (a) makes sense in the context of this problem. Explain what it would mean in real-world terms if your function had an x-intercept.

3) Suppose that \( f(x) = (45x - 7)^2 - 8 \).

(a) Use the square root property to find the x-intercepts of the function. HINT: first re-write it in the form \( \text{(expression)}^2 = \text{(positive number)} \) and then solve. Show your work.

(b) Find the y-intercept of \( f(x) \) by hand. Show your work.

(c) Write \( f(x) \) in the form \( f(x) = ax^2 + bx + c \) by expanding and simplifying. Show your work.

(d) Find the vertex of \( f(x) \).

(e) Solve for the x-intercepts by using the quadratic formula and your expanded form. Make sure you get the same answer you got in part a.
4) An object is launched in the air. The height of the object, \( H \), above ground in feet, after \( t \) seconds have passed is given by the function \( H(t) = -16t^2 + 85t + 7 \).

   (a) How high is the object to start?

   (b) When does the object reach its highest height?

   (c) What is the highest the object get?

   (d) When does the object hit the ground? Solve by hand and show all work.

   (e) What's an appropriate real-world domain for this function? Explain.

   (f) What's an appropriate real-world range for this function? Explain.

5) Use the square root property to solve each equation.

   (a) \( x^2 = 196 \) 
   (b) \( 6x^2 - 11 = 0 \) 
   (c) \( (x + 5)^2 = 17 \)

6) Use the square root property to solve each equation.

   (a) \( (x - 9)^2 = 49 \) 
   (b) \( (5k - 2)^2 = 8 \) 
   (c) \( 2(x + 8)^2 = 2 \)

7) Use the quadratic formula to solve the equation.

   (a) \( 4x^2 + 9x + 5 = 0 \) 
   (b) \( -x^2 - 14x - 49 = 0 \) 
   (c) \( x^2 - 9x - 52 = 0 \) 
   (d) \( 5x^2 - 7x - 3 = 0 \)

8) Use the quadratic formula to solve the equation.

   (a) \( -3x^2 = -4x + 6 \)
   (b) \( 2k(k + 11) = 11 \)
   (c) \( 3x^2 - 5x - 7 = 0 \)
   (d) \( x^2 - 6x - 4 = 0 \)

9) Find the x-intercepts (if they exist) using the quadratic formula.

   (a) \( y = x^2 - 6x - 4 \)
   (b) \( y = -5x^2 - 13x - 6 \)
   (c) \( y = x^2 + 2x + 7 \)

10) Find the x-intercepts (if they exist) using the quadratic formula.

    (a) \( y = x^2 + 8 \)
    (b) \( y = 4x^2 + 6x - 2 \)
    (c) \( y = 5x^2 + 5x - 5 \)

11) The graph of \( y = ax^2 + bx + c \) is given. Use it to answer the questions.

    (a) Determine whether \( a > 0 \) or \( a < 0 \) in the function \( y = ax^2 + bx + c \).

    (b) Identify the solution(s), if they exist, to \( ax^2 + bx + c = 0 \).

    (c) Is the discriminant of \( y = ax^2 + bx + c \) positive, negative, or zero?
12) The graph of $y = ax^2 + bx + c$ is given. Use it to answer the questions.

(a) Determine whether $a>0$ or $a<0$ in the function $y = ax^2 + bx + c$.

(b) Identify the solution(s), if they exist, to $ax^2 + bx + c = 0$.

(c) Is the discriminant of $y = ax^2 + bx + c$ positive, negative, or zero?

13) The graph of $y = ax^2 + bx + c$ is given. Use it to answer the questions.

(a) Determine whether $a>0$ or $a<0$ in the function $y = ax^2 + bx + c$.

(b) Identify the solution(s), if they exist, to $ax^2 + bx + c = 0$.

(c) Is the discriminant of $y = ax^2 + bx + c$ positive, negative, or zero?

14) Challenge problem! Solve the following equations.

(a) Solve by hand using the quadratic formula. Show all your work and steps. Check your work graphically! Hint: Clear away the fraction by multiplying both sides by the denominator!

\[ \frac{3}{x + 5} = x - 7 \]

(b) Solve by hand. Show all your work and steps. Check your work graphically! Hint: Start by making the substitution $x^2 = w$ which means that $x^4 = w^2$.

\[ x^4 - 15x^2 + 54 = 0 \]

15) For each equation, state the value of the discriminant, and then state what that tells you about the number of solutions to the given equation.

(a) $12x^2 + 3x - 13 = 0$

(b) $x^2 - 14x + 49 = 0$

(c) $x(x + 9) = 4$
Unit 2 Outcome Overview

Section 2.1
- Find the formula of a linear function using slope-intercept form and/or point-slope form when given a graph or appropriate information about the line.
- Find the formula of an exponential function when given a graph or appropriate information about the function.
- Find the vertex of a quadratic function using the formula of the quadratic function.

Section 2.2
- Use TI graphing calculator to evaluate functions (numerically and graphically)
- Use TI graphing calculator to solve for the input-value using Calc Intersect
- Use TI graphing calculator to solve for maximum and minimum points
- Set up an appropriate viewing window for the real-world situation
- Evaluate graphically, and interpret real-world meaning
- Solve for input value graphically, and interpret real-world meaning
- Solve for maximum or minimum value graphically, and interpret real-world meaning
- Solve equations by using graphical/numerical method instead of solving by hand

Section 2.3
- Polynomial vocabulary
- Expand/Multiply expressions
- Factor expressions by factoring out the greatest common factor
- Factor trinomial expressions
- Understand what the zero-product property is and how/when to use it
- Use zero-product property to solve equations and solve for intercepts of functions
- Factor and then use zero-product property

Section 2.4
- Use the square root property to solve equations of the form \( G^2 = a \) where a is a real number.
- Memorize quadratic formula
- Understand when and how to use the quadratic formula
- Understand what the quadratic formula tells you (what are you solving for)
- Understand what the discriminant tells us about the graph of a quadratic function
## Jeopardy Review Game

### Module 2

<table>
<thead>
<tr>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section 2.1: Equations of linears, quadratics, exponentials</td>
<td>Section 2.2: Solving graphically on calculator</td>
<td>Section 2.3: Expanding and factoring</td>
<td>Section 2.3: Zero-product property</td>
<td>Section 2.4: Solve with quadratic formula</td>
</tr>
</tbody>
</table>
Section 2.1
100: Find the equation of the line that has slope 17 and passes through the point (4, 9).
200: Find the equation of the function that has a y-intercept of (0, 15) and increases at a rate of 7.5%.
300: Find the EXACT vertex of the function \( f(x) = 3x^2 - 5x + 2 \). Give answer as an ORDERED PAIR.
400: Find the equation of the function that has a y-intercept of (0, 24) and decreases at a rate of 4.6%.
500: Find the equation of the line that passes through the points (15, −70) and (40, −20).

Section 2.2
100: Graph \( f(x) = 0.1(x + 2)(x − 3)(x − 7) \) on a standard viewing window (-10 to 10, -10 to 10). Find the local maximum point. Round the values to 4 decimal places of accuracy.
200: Graph \( f(x) = 0.1(x + 2)(x − 3)(x − 7) \) on a standard viewing window (-10 to 10, -10 to 10). Find the local minimum point. Round the values to 4 decimal places of accuracy.
300: Use graphing calculator to graphically solve for where \( f(x) = 3(1.05)^x \) reaches a height of 7. Round the answer accurately to 4 decimal places. Briefly explain how you used the calculator to solve the problem.
400: Graph \( f(x) = x^3 - 2x + 7 \) on a standard viewing window (-10 to 10, -10 to 10). Find the x-intercept. Round the values to 4 decimal places of accuracy.
500: Graph \( f(x) = x^3 - 5x - 1 \) on a standard viewing window (-10 to 10, -10 to 10). Find the x-intercepts. Round the values to 4 decimal places of accuracy.

Section 2.3 expand/factor
100: Expand/Multiply and then simplify. Give your answer in descending degree order. \((3x - 7)(2x + 5)\)
200: Expand/Multiply and then simplify. Give your answer in descending degree order. \((x - 4)^2\)
300: Factor. \( 8x^2 + 10x - 3 \)
400: Factor. \( 8x^2 + 10x + 3 \)
500: Factor. \( 12x^3 - 10x^2 - 12x \)

Section 2.3 Zero-product property
100: Solve \((2x - 5)(x + 80) = 0\) using the zero-product property.
200: Solve \(5x - 30x^2 = 0\) using the zero-product property.
300: Solve \(x^2 + 5x = 36\) using the zero-product property.
400: Solve \(8x^2 = 15 - 14x\) using the zero-product property.
500: Solve \(20x^3 + 38x^2 + 12x = 0\) using the zero-product property.

Section 2.4 Solve with Quadratic Formula
100: Find the exact solution(s) to the equation \( 8x^2 - 5x - 7 = 0 \). Then give the approximate solution(s) to 4 decimal places of accuracy.
200: Find the exact solution(s) to the equation \( 3x - 17 = -4x^2 \). Then give the approximate solution(s) to 4 decimal places of accuracy.
300: Find the x-intercepts of the function \( f(x) = -7x^2 - 84x - 252 \). Sketch the graph labeling the x-intercept(s) and y-intercept.
400: Find the x-intercepts of the function \( f(x) = 2x^2 + 3x + 7 \). Sketch the graph labeling the vertex and y-intercept.
500: Find the x-intercepts of the function \( f(x) = -3x^2 + 4x + 8 \). Sketch the graph labeling the vertex, the x-intercepts, and y-intercept.

Section 2.4 Understand Quadratic Formula
100: Identify the discriminate of \( f(x) = ax^2 + bx + c \).
200: If a quadratic function has a discriminate that is negative, then what do you know about the x-intercepts of that quadratic function?
300: If a quadratic function has 2 x-intercepts, then what do you know about the discriminate of that quadratic function?
400: Find the discriminate of \( f(x) = 7x^2 - 4x + 2 \). What does the discriminate tell you about the x-intercepts of the function?
500: Find the discriminate of \( f(x) = -7x^2 - 2 \). What does the discriminate tell you about the x-intercepts of the function?
Practice Exam 2

Instructions: Answer the following questions. Check and correct your work when you’re done!

1) Find the equation of each function.

(a) Find the equation of the line with slope 8 that passes through (0, 7).

(b) Find the equation of the line with slope $-4$ that passes through (15,3).

(c) Find the equation of the line that passes through (17,4) and (22,1).

(d) Find the equation of the line that passes through (−3, −5) and (−7, −10).

(e) Find the equation of the exponential function with y-intercept (0, 85) and factor 1.35.

(f) Find the equation of the exponential function with y-intercept (0, 17) that increases at a rate of 4.1%.

(g) Find the equation of the exponential function with y-intercept (0, 560) that decreases at a rate of 3.6%.

(h) Find the equation of the exponential function that passes through (0,500) and (1, 425).

2) Use the graphing calculator to GRAPHICALLY solve each of the following problems. To “show work” be sure to explain what you entered in Y=, what you used for a viewing window, and what calculator feature you used to solve the problem. Round to 4 decimal places if necessary, but be sure to give exact values if you can.

(a) $f(x) = 5(1.03)^x$ Find where $f(x) = 7$.

(b) $g(x) = -3(x - 4)^2 + 6$ Find $g(-2)$.

(c) $k(t) = \frac{2}{3}(t - 17) + 50$ Find where $k(t) = 20$.

(d) $m(t) = 0.5(t - 2)(t - 4)(t - 6)$ Find the local maximum and local minimum points (the peak and valley points) of the function $m(t)$. Use t-values from $t = 0$ up to $t = 8$.

(e) Graphically solve the equation $\frac{2}{3}x - 5 = 7x + \frac{5}{6}$.

3) An object is launched in the air. Its height above ground (in feet) after t seconds have passed is given by the function $D(t) = -16t^2 + 96t + 15$. You can solve the following problems using any method you prefer. Be sure to give your answers using CORRECT UNITS!!

(a) How high was the object initially?

(b) What is the highest the object gets?

(c) When does the object reach the peak height?

(d) When does the object hit the ground?

4) The population of a country is $P(t) = \frac{3,565,000(0.964)^t}{t}$ where $t$ is the number of years after 2000, and $P(t)$ is the population of the country.

(a) What was the population of the country in the year 2000?

(b) What was the population of the country in 2005? Round to the nearest person.

(c) At what percent rate is the population increasing or decreasing? Write a complete sentence to answer this question.

(d) When will the population of this country be 3 million people? Give your answer to 4 decimal places of accuracy.
5) Complete the following table to demonstrate your understanding of polynomial vocabulary.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Degree?</th>
<th>Leading coefficient?</th>
<th>Constant term?</th>
<th>Coefficient of $x^2$?</th>
<th>Coefficient of $x$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x^2 - 4x^3 + 8 - 9x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6 - \frac{1}{2}x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$9$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-15x + x^3 - x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6) Re-write the following functions in expanded form (be sure to simplify the expanded form as well).

   (a) $f(x) = 7x^2(3x^4 - 5x + 2x^2 + 9)$
   (b) $g(x) = (x - 5)(x + 7)$
   (c) $h(t) = (3t + 5)(5t - 1)$
   (d) $P(Q) = (Q + 5)(2Q^2 - 3Q + 4)$

7) Re-write each of the following functions in factored form. If the function is not factorable, say so.

   (a) $f(x) = x^2 - 10x + 21$
   (b) $g(t) = x^2 + 5$
   (c) $h(k) = 6k^2 + 7k - 5$
   (d) $k(t) = 6t^2 - t - 5$
   (e) $m(s) = s^2 - 81$
   (f) $f(x) = 4x^2 + 20x + 25$

8) Find the x-intercept(s) of each of the following functions USING THE ZERO-PRODUCT PROPERTY. Show your work.

   (a) $f(x) = (x - 8)(x + 3)$
   (b) $g(x) = x^2 - 3x - 40$
   (c) $h(x) = 8x^2 + 2x - 3$
   (d) $L(x) = 12x^3 + 15x^2 - 18x$

9) Solve the equation $17 = (3x - 5)^2$ USING THE SQUARE ROOT PROPERTY. Give your answer in exact form, and then give your answer as an approximated decimal rounded to 4 decimal places. Show your work.

10) For each of the functions below, do the following:

   - Identify if the function is open up or open down and explain how you can determine this from the given formula
   - Identify the $y$-intercept of the function and the vertex of the function (write both as ordered pairs!)
   - Identify the value of the discriminant. Explain what the discriminant tells you about the $x$-intercepts.
   - Use the quadratic formula to find the $x$-intercept(s) of the function. Give the answers EXACTLY and then give the answers as decimals rounded to 4 decimal places if necessary
   - Sketch a graph. Label the $y$-intercept, vertex, and any $x$-intercept(s).

   (a) $g(t) = -3t^2 + 8t + 10$
   (b) $f(x) = 4x^2 + 56x + 196$
   (c) $h(t) = -0.65(t - 7.1)^2 + 4.3$
   (d) $m(t) = 10t^2 + 550$
Flash Cards for Unit 2

Instructions: On the following pages you will find flashcards for Unit 2. Cut along the lines to create your own flashcards to review basic concepts from Unit 2.
| **f(x)=2x-5**  
Find/solve f(10). | **Formula for finding vertex of quadratic?** |
| --- | --- |
| **f(x)=2x-5**  
Find/solve f(x)=7. | **Formula for slope of a line?** |
| **Find the equation of the line with slope 3 that passes through (4, 2).** | **If I know y, how do I graphically solve for x?** |
| **Find vertex of**  
y= -7x^2 +35  
on calculator. What feature? | **If there are 2 x-intercepts, then what do you know about the discriminant?** |
| **Find vertex of**  
y= 7x^2-35  
on calculator. What feature? | **If there are no x-intercepts, then what do you know about the discriminant?** |
<table>
<thead>
<tr>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = \frac{-b}{2a}$</td>
</tr>
<tr>
<td>$f(10) = 2(10) - 5$</td>
</tr>
<tr>
<td>$f(10) = 15$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$</td>
</tr>
<tr>
<td>$7 = 2x - 5$</td>
</tr>
<tr>
<td>$x = 6$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Put $y$ in $y_2$. Calc Intersect.</td>
</tr>
<tr>
<td>$y = 3(x-4) + 2$</td>
</tr>
<tr>
<td>or $y = 3x - 10$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discriminant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discriminant is positive</td>
</tr>
<tr>
<td>Calc Maximum.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discriminant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discriminant is negative</td>
</tr>
<tr>
<td>Calc Minimum.</td>
</tr>
<tr>
<td>Question</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>If there is only 1 x-intercept, then what do you know about the</td>
</tr>
<tr>
<td>discriminant?</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Point-slope form of a line?</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Quadratic formula?</td>
</tr>
<tr>
<td>Solve ((x-3)(x+7)=0) with zero-product property.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>We use the quadratic formula when we are trying to solve what</td>
</tr>
<tr>
<td>equation?</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>-4</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>-7</td>
</tr>
</tbody>
</table>
Unit 3
**Introduction to Unit 3**

This is the final unit of the Intermediate Algebra course. After successfully completing this unit, students will proceed to the college-level courses such as College Algebra, Algebra and Trigonometry for Pre-Calculus, and Math for Elementary School Teachers.

This final unit is designed to allow the instructors some flexibility in selecting which topics to cover. Students who will be going on to College Algebra for programs such as Business will need more skills related to interpreting real-world applications of mathematics, and be allowed a bit more flexibility in the method of solving problems. Students going to the Calculus track will need to have a solid understanding of the rules of exponents, and achieve a more solid understanding of algebraic manipulation.

The following timeline provides the recommended path to follow for students on different tracks.

<table>
<thead>
<tr>
<th>Days</th>
<th>MAT110 and MAT107</th>
<th>MAT184 track</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Section 3.1a and Section 3.1 b (applications basics and applications continued)</td>
<td>Section 3.1a (applications basics)</td>
</tr>
<tr>
<td>2</td>
<td>Section 3.2a (exponent rules basics)</td>
<td>Section 3.2a and Section 3.2b (exponent rules basics and exponent rules advanced)</td>
</tr>
<tr>
<td>3</td>
<td>Section 3.3a and Section 3.3b (systems of equations basics and systems of equations applications)</td>
<td>Section 3.3a (systems of equations basics)</td>
</tr>
<tr>
<td>4</td>
<td>Section 3.5a and Section 3.5b (inequalities)</td>
<td>Section 4 (intro to rational expressions)</td>
</tr>
<tr>
<td>5</td>
<td>Review and Cumulative Final Exam</td>
<td>Section 3.5a (inequalities symbolically)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Review and Cumulative Final Exam</td>
</tr>
</tbody>
</table>

**NOTES:** Each of the section assignment contains cumulative review as well as new content practice in order to help students begin to prepare for the cumulative final exam.
Section 3.1a
Applications Basics

Learning Outcomes:
- Given the equation of a linear or quadratic or exponential function in a real-world, contextual problem, find features of the function and interpret the real-world, contextual meaning of each feature.
- Given points on a linear function or points on an exponential function in a real-world, contextual problem, find the equation of the function and interpret the real-world, contextual meaning of the features.
- Given real-world contextual situations, use algebraic or graphical methods to solve problems and interpret the real-world, contextual meaning of each.

In this section we will cover examples of applications of the concepts covered in unit 1 and unit 2. There are no new mathematical concepts in this section - only applications of the concepts from unit 1 and unit 2.

Section 3.1b will cover further examples of applications in order to give the students who are planning to take MAT110 the opportunity to spend additional time on this important topic.

Example 1: The function \( P(q) = 25.75q - 6500 \) relates the number of items sold, \( q \), to the profit earned (\( P \) in dollars) by a certain company.

(a) Explain how you know that \( P(q) \) is a linear function from the formula of the function.

Solution: We know that \( P(q) \) is linear because it has the form \( P(q) = mq + b \), which is the same as \( f(x) = mx + b \) only it uses different letters for the variables. In this case, \( m = 25.75 \) and \( b = -6500 \).

(b) Identify the slope of the line, and write a complete sentence that interprets the practical meaning of the slope.

Solution: The slope of the line is 25.75. This means that every time \( q \) increases by 1, then \( P \) increases by 25.75. \( q \) represents the number of items sold, and \( P \) represents the profit earned. So this tells us that every time they sell one additional item, they will increase the profit by $25.75.

(c) Identify the \( y \)-intercept (vertical intercept) of the function, and write a complete sentence that interprets the practical meaning of that point.

Solution: The \( y \)-intercept is the point \((0, -6500)\). The 0 is the number of items, and the \(-6500\) is the profit in dollars. So this point tells us that when they sell 0 items the profit will be in the red by $6500. In other words, they are in debt by $6500 before selling any items (this is the fixed costs).

(d) Identify the \( x \)-intercept (horizontal intercept) of the function, and write a complete sentence that interprets the practical meaning of that point.

Solution: The horizontal intercept occurs where the height of the graph is 0. So, we let \( P = 0 \) and solve for the value of \( q \).

\[
0 = 25.75q - 6500 \\
+6500 \quad + \quad + 6500 \\
------------------- \\
6500 = 25.75q \\
\div 25.75 \quad \div 25.75 \\
252 \approx q
\]
The horizontal intercept is approximately the point (252, 0). This tells us that when they sell 252 items they will make a profit of $0. This is the break-even point. They break even by selling 252 items. If they sell less than 252 items they will still have negative profit. If they sell more than 252 items they will make a positive profit.

(e) Sketch a complete graph of the function \( P(q) \) for items ranging from 0 items up to 500 items sold. Label the graph completely. Identify the viewing window that would create such a graph.

Solution: The items are along the horizontal axis. The profit in dollars is along the vertical axis. There is a y-intercept of (0, -6500) and an x-intercept of (252, 0). The graph is a line through these points. We extend the horizontal axis from 0 up to 500 (as the problem specified to do). This will create a low P-value of -6500 at the point (0, -6500) and this will create a high P-value of 6375 at the point (500, 6375).

Example 2: The function \( H(t) = -16t^2 + 144t + 576 \) gives the height, \( H \) in feet, of an object where \( t \) is the number of seconds after the object was launched.

(a) Identify the vertical intercept (y-intercept) and write a complete sentence that interprets the practical meaning of this point.

Solution: The vertical intercept (y-intercept) occurs when \( t = 0 \). It is the point (0, 576). The input represents the number of seconds after the object was launched, and the output represents the height of the object in feet. The point (0, 576) tells us that the object was 576 feet above ground at the moment when it was launched (when 0 seconds had passed).

(b) Is this parabola open up or open down? Explain how you know by looking at the formula.

Solution: The parabola is open down. We know this because the value of \( a \) in the expression \( ax^2 + bx + c \) is negative.

(c) Identify the vertex of the parabola. Is the vertex a maximum or is it a minimum?

Solution: In this quadratic function, \( a = -16 \) and \( b = 144 \) and \( c = 576 \). The vertex occurs at \( x = \frac{-b}{2a} \)

\[
x = \frac{-(144)}{2(-16)} = \frac{144}{32} = 4.5
\]

The H-value (the y-value) of the vertex occurs at \( H(4.5) = -16(4.5)^2 + 144(4.5) + 576 = 900 \).

Therefore, the vertex is the point (4.5, 900). Since the parabola is open down, we know that this vertex will be the maximum (highest) point on this function.

(d) Write a complete sentence that interprets the practical meaning of the vertex. Make sure to include the meaning of it being a maximum or minimum of the parabola.

Solution: The point (4.5, 900) is the highest point on this graph. In practical terms, this tells us that the object will reach its highest altitude of 900 feet exactly 4.5 seconds after it was launched.
Example 3: The function \( P(t) = 300(1.025)^t \) gives the population of a town \( P \), in thousands of people, where \( t \) is the number of years after 1980.

(a) Explain how you know, from the formula, that \( P(t) \) is an exponential function.

Solution: \( P(t) \) is an exponential function because it has the form \( P(t) = a(b)^t \) where \( a \) is 300 and \( b \) is 1.025.

(b) Identify the \( y \)-intercept of the function, and write a complete sentence that interprets the practical meaning of the point.

Solution: The function has a \( y \)-intercept of \((0, 300)\). The 0 is the number of years after 1980 (0 years after 1980) and the 300 is the population of the town in thousands (300 thousand people). So, this means that the town had a population of 300,000 people in the year 1980.

(c) Identify the factor of the function, and identify the rate of increase of the function as a percent. Write a complete sentence that interprets the practical meaning of the rate (as a percent).

Solution: The factor of the function is 1.025.

\[
(\text{decimal rate}) = \text{Factor} - 1 = 1.025 - 1 = 0.025.
\]

This means the percent rate is 2.5%.

So as \( t \) increases by 1 unit, the \( P \) increases by 2.5%. In other words, the population increases by 2.5% every year.

(d) Find \( P(25) \) and write a complete sentence that interprets the practical meaning of your finding.

Solution: \( P(25) = 300(1.025)^{25} \approx 556.183 \). This tells us that 25 years after 1980 (in the year 2005) the population of the town was about 556,183 people.

(e) Create a graph of the function for the years 1980 through 2015. Label the graph and determine the viewing window that will create the same graph.

Solution: The horizontal axis represents the number of years since 1980. So, the horizontal axis will run from \( t=0 \) up to \( t=35 \). The vertical axis represents the population of the town in thousands. The low \( P \)-value of the graph is 300 which occurs at the point \((0, 300)\), and the high \( P \)-value of the graph is 711.962 which occurs at \((35, 711.962)\).
Example 4: The tables below give the amount of money (in dollars) in Bill’s account and Paige’s account over the months.

<table>
<thead>
<tr>
<th>t months</th>
<th>B(t)= amount in Bill’s account</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$5,000</td>
</tr>
<tr>
<td>1</td>
<td>$5,140</td>
</tr>
<tr>
<td>2</td>
<td>$5,283.92</td>
</tr>
<tr>
<td>3</td>
<td>$5,431.87</td>
</tr>
<tr>
<td>4</td>
<td>$5,583.96</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t months</th>
<th>P(t)= amount in Paige’s account</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$23,000</td>
</tr>
<tr>
<td>6</td>
<td>$21,000</td>
</tr>
<tr>
<td>12</td>
<td>$17,000</td>
</tr>
<tr>
<td>15</td>
<td>$15,000</td>
</tr>
<tr>
<td>24</td>
<td>$9,000</td>
</tr>
</tbody>
</table>

(a) Find a formula for B(t), the amount of money in Bill’s account after t months. Give a brief description of what is happening with Bill’s account.
Solution: First we need to determine the type of function B(t) is. Linear is the “easiest” type of function, and so we can first check the table to see if B(t) has a constant rate of change. We see that from t=0 to t=1 there is a slope of 140 dollars per month. And from t=1 to t=2 there is a slope of 143.92 dollars per month. Since that slope is not constant, we can conclude that the function B(t) is not a linear function.

Next we check to see if B(t) is an exponential function. By calculating ratio’s between each successive point we see that there is a constant factor. Therefore, B(t) is an exponential function.

\[
\frac{5140}{5000} = 1.028 \quad \text{and} \quad \frac{5283.92}{5140} = 1.028 \quad \text{and} \quad \frac{5431.87}{5283.92} \approx 1.028 \quad \text{and} \quad \frac{5583.96}{5431.87} \approx 1.028
\]

B(t) is an exponential function with factor 1.028. This tells us that every time x increases by 1 unit, the y-value is multiplied by 1.028. In other words, B(t) is increasing at a rate of 2.8%. Every month, Bill’s account balance increases by 2.8%. The y-intercept is (0,5000) and so the formula for B(t) is \( B(t) = 5000(1.028)^t \).

(b) Find a formula for P(t), the amount of money in Paige’s account after t months. Give a brief description of what is happening with Paige’s account.
Solution: First we need to determine the type of function P(t) is. Linear is the “easiest” type of function, and so we can first check the table to see if P(t) has a constant rate of change. By calculating average rates of change between successive points in the table we see that P(t) has a constant rate of change, and therefore we know it is linear.

Average rate = \( \frac{21000−23000}{6−3} = \frac{-2000}{3} \approx -666.67 \)

Average rate = \( \frac{15000−17000}{15−12} = \frac{-2000}{3} \approx -666.67 \)

Average rate = \( \frac{17000−21000}{12−6} = \frac{-4000}{6} = \frac{-2000}{3} \approx -666.67 \)

Average rate = \( \frac{9000−15000}{24−15} = \frac{-6000}{9} = \frac{-2000}{3} \approx -666.67 \)

So P(t) is a linear function with slope \(-2000/3\) and if we use the first point in the table, we can say that a formula for P(t) is

\[ P(t) = -\frac{2000}{3}(t-3) + 23000 \]

In slope-intercept form this is written as \( P(t) = -\frac{2000}{3}t + 25000 \). This tells us that Paige started with $25,000, and then every three months Paige removes $2000 from the account. Or, every month she removes about $666.67 from the account.

(c) Solve for when Paige and Bill have the same amount of money GRAPHICALLY.
Solution: We put B(t) in Y1, and we put P(t) in Y2. Then we set up a window that allows us to see the point of intersection.

This tells us that after about 23.25 months Paige and Bill will have the same amount of money in their accounts. They will both have $9501.36 at that time.
Example 5: A new housing community digs and fills a lake for the community. They stock the lake with fish, but a disease strikes and the number of living fish in the lake decreases at a constant rate as the weeks pass. 12 weeks after the lake is filled with fish, there are a total of 5,560 fish in the lake, 20 weeks after the lake is filled with fish, there are a total of 3,000 fish in the lake.

(a) Find an equation of the function that gives the number of fish in the lake, \( F \), when \( W \) weeks have passed since the lake was stocked with fish. Show your work.

Solution:

We are first asked to find an equation for the function that gives the number of fish in the lake, \( F \), when \( W \) weeks have passed. Before finding the equation, we should first clearly identify the meaning of the independent and dependent variable.

\( W \): the number of weeks that have passed since the lake was first stocked with fish (INPUT variable, INDEPENDENT)

\( F(W) \): the number of fish in the lake (OUTPUT variable, DEPENDENT)

Next, we need to clearly identify how the information in the problem relates to the mathematics of the problem. We look at the information to identify points, intercepts, information about the type of function, etc:

- The statement “the number of living fish in the lake decreases at a constant rate as the weeks pass” tells us that this is a LINEAR function (since it has a constant rate of change). Also, since it is DECREASING, we know the SLOPE WILL BE NEGATIVE.
- The statement “12 weeks after the lake is filled with fish, there are a total of 5,560 fish in the lake” tells us that when \( W=12 \), then \( F=5560 \). In other words, \( F(12)=5560 \). Or, we know that the point \( (12,5560) \) is on the graph of the function.
- The statement “20 weeks after the lake is filled with fish, there are a total of 3,000 fish in the lake” tells us that when \( W=10 \), then \( F=3000 \). In other words, \( F(20)=3000 \). Or, we know that the point \( (20,3000) \) is on the graph of the function.

So, we know that we are finding the equation of a linear function with a negative slope. And we know that the function passes through the points \((12,5560)\) and \((20,3000)\). This is enough information to find the equation.

\[
\text{slope } = \frac{\Delta y}{\Delta x} = \frac{3000 - 5560}{20 - 12} = \frac{-2560}{8} = -320
\]

Using the point-slope form of a line we can write the equation as

\[
F(W) = -320(W - 12) + 5560 \quad \text{or} \quad F(W) = -320(W - 20) + 3000
\]

Or using the slope-intercept form of the line we find that the equation is \( F(W) = -320W + 9400 \).

(b) Write complete sentences to interpret the real-world, contextual meaning of the y-intercept and the slope.

Solution:

The slope of the line is \(-320\). As a fraction we can write this as \(-\frac{320}{1}\text{ fish}\text{ per week}\). Slope is a number that tells us how the line is CHANGING. This means that every week the lake loses another 320 fish.

The y-intercept is \((0, 9400)\) which tells us that TO START (when 0 weeks have passed), there were 9400 fish in the lake.
**Example 6:** Alex invests $500 in Bank A and his money increases at a rate of 5.1% per year. That same day Bob deposits $1000 in Bank B which results in his money increasing at a rate of 3.2% per year.

(a) Find a function $A(t)$ that gives the amount of money in Alex's account after $t$ years.

**Solution:** $A(t)$ must be an exponential function since every time $t$ increases by 1 year, we need to increase the output-value, $A$, by a fixed percentage.

Recall that exponential functions have the form $f(x) = a \cdot b^x$ where $a$ is the y-intercept and $b$ is the factor.

This exponential function has y-intercept $(0, 500)$ and it has rate 5.1% = 0.051. Remember that factor = 1+(decimal rate). So the factor is 1.051. Therefore, the function $A(t)$ can be written as $A(t) = 500(1.051)^t$.

(b) Find a function $B(t)$ that gives the amount of money in Bob's account after $t$ years.

**Solution:** $B(t)$ must be an exponential function since every time $t$ increases by 1 year, we need to increase the output-value, $B$, by a fixed percentage. This exponential function has y-intercept $(0, 1000)$ and it has rate 3.2%. So the factor is 1.032. Therefore, the function $B(t)$ can be written as $B(t) = 1000(1.032)^t$.

(c) Graph both functions on your calculator at the same time. Assume that both Alex and Bob leave their money in the accounts for 50 years. Be sure to use an appropriate viewing window in practical terms. Report the window you have used.

**Solution:** Since the starting time is $t=0$ and the last time to consider is $t=50$, we will let XMin=0 and XMax=50. Then we press ZOOM 0 to automatically set the YMin and YMax. The resulting window is given below.

(d) Determine the amount of money in Alex's account after 12 years. Determine the amount of money in Bob's account after 12 years.

**Solution:** We have been given that $t=12$, and we are asked to find $A(12)$ and to find $B(12)$. So, that means that we need to let the input-value be 12 and find the output value on the graph.

On the graph screen we press TRACE. The cursor will automatically put the cursor on the function Y1 which is $A(t)$. Type 12, and then press enter. We see that $A(12) \approx 908.24$. This tells us that Alex will have $908.24 in his account after 12 years.

On the graph screen we press TRACE. THEN PRESS THE DOWN ARROW ONCE. This will place the cursor on the function Y2 which is $B(t)$. Type 12, and then press enter. We see that $B(12) \approx 1459.34$. This tells us that Bob will have $1,459.34 in his account after 12 years.
Graphically determine the amount of time until there is $4500 in Alex’s account. Graphically determine the amount of time until there is $4500 in Bob’s account. Briefly explain your calculator steps.

Solution: We have been given that the output value is 4500, and we want to solve for the value of the input-value on each of the two functions. We will let Y3=4500 in order to produce a horizontal line at a height of 4500. Then we will use Calc Intersect in order to identify the value of t when functions reach a height of 4500.

To find the intersection of Y1 and Y3, select Calc Intersect. Press Enter on First curve, making sure that the cursor is on Y1. The calculator will then automatically jump the cursor to Y2. But, we are interested in finding the intersection of Y1 and Y3. So, you need to press the DOWN arrow in order to jump the cursor to Y3, and then press ENTER. This will find the intersection of Y1 and Y3 at \(x \approx 44.17234\). This tells us that Alex will have $4500 in the account after approximately 44.17234 years.

To find the intersection of Y2 and Y3, select Calc Intersect. The calculator will automatically place the cursor to Y1. But, we are interested in finding the intersection of Y2 and Y3. So, you need to press the DOWN arrow in order to jump the cursor to Y2, and then press ENTER. Then continue from there. This will find the intersection of Y2 and Y3 at \(x \approx 47.7505\). This tells us that Bob will have $4500 in the account after approximately 47.7505 years.

(f) Graphically find the intersection of A(t) and B(t). Write a sentence or two that explains the meaning of the point of intersection in this context.

Solution: We see on the graph that there is one point of intersection between A(t) and B(t) that occurs at approximately 38 years. We will use Calc Intersect in order to find the point of intersection between Y1 and Y2.

This tells us that Alex and Bob will have the same amount in their accounts after 37.9944 years. At that time they will both have $3309.42 in their accounts.
Warm-up/Review problems:

1) 
   (a) Find the intercepts and slope of the function. Then sketch a graph of \( f(x) = 4x - 7 \) that labels the y-intercept and at least one other point.
   (b) Find the intercepts and slope of the function. Then sketch a graph of \( f(x) = -2(x - 3) + 5 \) that labels at least two points.
   (c) Solve the equation \( 4x - 7 = -2(x - 3) + 5 \) by hand.
   (d) Solve the equation \( 4x - 7 = -2(x - 3) + 5 \) graphically with the help of the graphing calculator.
   (e) Make sure your answers to part (c) and (d) match.

2) 
   (a) Find the intercepts, vertex and general shape of the graph. Then sketch a graph of \( f(x) = 3x^2 - 5x - 4 \) that labels the y-intercept and the vertex and any x-intercepts.
   (b) Find the intercepts, vertex and general shape of the graph. Then sketch a graph of \( f(x) = -7x^2 + 2x + 1 \) that labels the y-intercept and the vertex and any x-intercepts.
   (c) Solve the equation \( 3x^2 - 5x - 4 = -7x^2 + 2x + 1 \) by hand.
   (d) Solve the equation \( 3x^2 - 5x - 4 = -7x^2 + 2x + 1 \) graphically.
   (e) Make sure your answers to parts (c) and (d) match.

3) 
   (a) Solve the equation \( 12x^2 + 13x - 14 = 0 \) by hand using factoring.
   (b) Solve the equation \( 12x^2 + 13x - 14 = 0 \) by hand using the quadratic formula.
   (c) Solve the equation \( 12x^2 + 13x - 14 = 0 \) graphically.
   (d) Make sure your answers to parts (a), (b), and (c) match.

4) Solve the following equations by hand. Check your answer by graphically solving on the calculator. Show all work.
   (a) \( 7x - (2 - x) = 0 \)
   (b) \( 7x(2 - x) = 0 \)
   (c) \( -5(x - 4) + 7 = 2x + 9 \)
   (d) \( -5(x^2 - 4) + 7 = 2x + 9 \)
   (e) \( \frac{3}{5}(x - 4)^2 + 1 = 0 \)
   (f) \( \frac{3}{5}(x - 4) + 1 = 0 \)

Applications/Contextual Problems:

5) The table below gives the amount of money in Dante’s account, in dollars, \( t \) years after he originally invested the money.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( D(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15,000</td>
</tr>
<tr>
<td>1</td>
<td>15,510</td>
</tr>
<tr>
<td>2</td>
<td>16,037.34</td>
</tr>
<tr>
<td>3</td>
<td>16,582.61</td>
</tr>
<tr>
<td>4</td>
<td>17,146.42</td>
</tr>
</tbody>
</table>

   (a) Find a formula for \( D(t) \). Show all work.
   (b) What interest rate is Dante getting each year on this account?
   (c) Graph the function from year 0 to year 30.
   (d) Find when Dante will have double the amount of money that he originally invested.

6) A new housing community digs and fills a lake for the community. They stock the lake with fish, but a disease strikes and the number of living fish in the lake decreases by 45 fish each week. After 10 weeks the number of fish in the lake is down to a total of 750 fish. Find an equation of the function that gives the number of fish in the lake, \( F \), when \( X \) weeks have passed since the lake was stocked with fish.
7) Cooperative Industries International makes 256 additional dollars in profit for every widget that they sell, and when they sell exactly 250 widgets, they make a total profit of $4,000. Find an equation of the function that gives the company profit, $P$, when they sell $X$ widgets.

8) Country A has a population of $A(t) = 75(0.83)^t$ where $t$ is the number of years after 2000 and $A(t)$ is the population of country A in millions of people. Country B has a population of $B(t) = 20(1.2)^t$ where $t$ is the number of years after 2000 and $B(t)$ is the population of country B in millions of people.

(a) Graph both functions on the same axis for the years 2000 through 2010.
(b) What was the population of each country in 2000?
(c) Write complete sentences to describe exactly how the population of each country is changing.
(d) Graphically find $A(2)$ and $B(2)$. Then write a sentence that interprets the real world meaning of what you have found.
(e) Graphically solve $A(t)=60$ and $B(t)=60$. Then write a sentence that interprets the real world meaning of what you have found.
(f) Graphically solve $A(t)=B(t)$. Then write a sentence that interprets the real world meaning of what you have found.

9) Collaborative Collectables sells items that all cost the same amount. They make a total profit of $6,450 when they sell exactly 800 items, but when they sell 1000 items they make a profit of $86,450. Find an equation of the function that gives the company profit, $P$, when they sell $Q$ items. Show your work.

10) Every year Sally earns 105% of her previous year’s salary. When she was hired, she started out at a salary of $45,000 per year. Write an equation that gives Sally’s salary, $S$, when $X$ years have passed after being hired. Use your equation to find how much Sally will be earning after 10 years with the company.

11) Bob writes a popular novel and earns $1,000,000 during its premier month. The next month, and every month thereafter, Bob earns 65% of his previous month’s earnings on the book. Write an equation that gives Bob’s monthly earnings, $Y$, after $X$ months have passed since the book premier. How much will Bob make 1 year after the book premieres?

12) The average price of a gallon of milk was about $3.50 in 2007. If the U.S. has an annual inflation rate of 3.5%, then find an equation that gives the average price of a gallon of milk $T$ years after 2007. How much will a gallon of milk cost when you are 50 years old?

13) Afghanistan had about 25.8 million people in 1999 and their population is growing at a rate of 3.95% per year. Find an equation that gives the population of Afghanistan $t$ years after 1999. According to this equation, what is the population of Afghanistan in the year 2010?

14) A store owner prices an item at $150 when he first stocks the item in the store. Every week that the item doesn’t sell, the owner reduces the price of the item another 3%. Find an equation that gives the price of the item, $P$, after $X$ weeks of not selling the item.

15) A population of trees is dying off due to disease. If there are 4500 trees to start, and the trees are dying off at a rate of 17% per month, find an equation that gives the total number of live trees, $T$, after $X$ months have passed.
16) The function \( P(x) = -1000x^2 + 12,700x - 12,200 \) gives the profit a company earns from selling a certain item. In this case, \( x \) is the dollar price they charge for each item, and \( P(x) \) is the dollar profit they earn when selling them at that price.
   a) Find the y-intercept.
   b) Find the x-intercept(s).
   c) Find the vertex.
   d) Write a complete sentence to interpret the real-world, contextual meaning of the y-intercept.
   e) Write a complete sentence to interpret the real-world, contextual meaning of the x-intercept(s).
   f) How much should this company charge for each item in order to earn the most profit? How much profit will they make when they charge that price?

17) The function \( M(t) = -45t + 900 \) gives the distance that Mary is north of home. In this function \( t \) is the number of hours that have passed since Mary started driving, and \( M(t) \) is her distance north of home in miles.
   a) Find the y-intercept. Write a complete sentence to interpret the real-world, contextual meaning of the point.
   b) Find the x-intercept. Write a complete sentence to interpret the real-world, contextual meaning of the point.
   c) Find the slope. Write a complete sentence to interpret the real-world, contextual meaning of the slope.
   d) Find/solve \( M(10) \). Write a complete sentence to interpret the real-world, contextual meaning of the point.
   e) Find/solve \( M(t) = 10 \). Write a complete sentence to interpret the real-world, contextual meaning of the point.
Section 3.1b

Applications Continued

Learning Outcomes:
- Given the equation of a linear or quadratic or exponential function in a real-world, contextual problem, find points and slope and factor and percent rate and interpret the real-world, contextual meaning of each.
- Given points on a linear function or points on an exponential function in a real-world, contextual problem, find the equation of the function and interpret the meaning of points and slope and factor and percent rate in real-world, contextual meaning of each.
- Given real-world contextual situations, use algebraic or graphical methods to solve problems and interpret the real-world, contextual meaning of each.

In this section we will cover additional examples of applications of the concepts covered in unit 1 and unit 2. This section provides the students who are planning to take MAT110 the opportunity to spend additional time on this important topic.

Example 1: The function, $Q=f(x)$, below relates the number of items that a company sells, $Q$, to the price per item, $x$.

<table>
<thead>
<tr>
<th>x dollars per item</th>
<th>Q=f(x) items sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100,000</td>
</tr>
<tr>
<td>2</td>
<td>70,000</td>
</tr>
<tr>
<td>5</td>
<td>25,000</td>
</tr>
<tr>
<td>6.5</td>
<td>2,500</td>
</tr>
<tr>
<td>6.66</td>
<td>100</td>
</tr>
<tr>
<td>6.67</td>
<td>-50</td>
</tr>
</tbody>
</table>

(a) What is the y-intercept of the function? Give a real-world interpretation of the meaning of that point.

Solution:
The y-intercept is the point $(0,100000)$. In real-world terms this tells us that when each item costs $0 (when they are free), they will sell 100,000 items.

(b) Is this a linear function? If so, what is the slope? Give a real-world interpretation of the meaning of that point.

Solution:
To be a linear function means that there must be a constant rate of change between any two points on the function. So, to determine if it is linear we will calculate the rate of change between each successive point in the table:

$$\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{70,000 - 100,000}{2 - 0} = \frac{-30,000 \text{ items}}{2 \text{ dollars}} = -15,000 \text{ items per dollar}$$

$$\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{25,000 - 70,000}{5 - 2} = \frac{-45,000 \text{ items}}{3 \text{ dollars}} = -15,000 \text{ items per dollar}$$

$$\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{2,500 - 25,000}{6.5 - 5} = \frac{-22,500 \text{ items}}{1.5 \text{ dollars}} = -15,000 \text{ items per dollar}$$

$$\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{100 - 25,000}{6.66 - 6.5} = \frac{-2,400 \text{ items}}{2 \text{ dollars}} = -15,000 \text{ items per dollar}$$

$$\frac{\Delta \text{output}}{\Delta \text{input}} = \frac{-50 - 100}{6.67 - 6.66} = \frac{-150 \text{ items}}{.01 \text{ dollars}} = -15,000 \text{ items per dollar}$$

On the function, we see that the rate of change remains a constant value of $-15,000$ between each point.

Therefore this is a linear function with a slope of $-15,000$. Every time the $x$-value increases by 1, then the $y$-value decreases by 15,000.

In real-world terms this tells us that every time the price of the items increases by $1.00, then they will sell 15,000 fewer items.
Example 2: The table below lists the profit, \( P \), that Bamboozle Bracelets earned in dollars when selling \( Q \) Bracelets.

<table>
<thead>
<tr>
<th>Q Bracelets sold</th>
<th>P(Q) dollars of profit earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>-5,562.5</td>
</tr>
<tr>
<td>380</td>
<td>-2,007</td>
</tr>
<tr>
<td>460</td>
<td>181</td>
</tr>
<tr>
<td>2500</td>
<td>55,975</td>
</tr>
<tr>
<td>10,000</td>
<td>261,100</td>
</tr>
</tbody>
</table>

(a) Find a formula for the function \( P(Q) \). Show your work/process.

(b) Find the y-intercept of your function. Write a sentence that interprets the real-world meaning of the y-intercept you found.

(c) Find the x-intercept(s) of your function. Write a sentence that interprets the real-world meaning of the x-intercept(s) you found.

(d) How much additional profit does Bamboozle make for every additional item it sells? Explain.

Solution:

(a) In order to find a formula for \( P(Q) \) we should first decide what type of function it is. We will test to see if it is linear first because the linear pattern is simple to test for...we need to determine if the function has a constant slope.

Rate between (250, -5563) and (380, -2007):
\[
rate = \frac{-2007 - (-5563.5)}{380 - 250} = \frac{3555.5}{130} = \frac{27.35}{item}
\]

Rate between (380, -2007) and (460, 181):
\[
rate = \frac{181 - (-2007)}{460 - 380} = \frac{2188}{80} = \frac{27.35}{item}
\]

Rate between (460, 181) and (2500, 55975):
\[
rate = \frac{55975 - 181}{2500 - 460} = \frac{55794}{2040} = \frac{27.35}{item}
\]

Since the average rate of change between each point is exactly the same value, we conclude that this is a linear function with a slope of 27.35. The y-intercept is not given, and so we will use the point-slope form to write the formula. If we use the first point in the table, we can write the formula of the function as \( P(Q) = 27.35(Q - 250) - 5,562.5 \). If we distribute the 27.35 and simplify, we can write the formula in slope-intercept form as \( P(Q) = 27.35Q - 12,400 \).

(b) Since we have the slope intercept form of the line as \( P(Q) = 27.35Q - 12,400 \), we can say that the y-intercept of the function is the point \((0, -12400)\). Points on this function have the form (quantity of bracelets sold, profit earned). So, the y-intercept tells us that when they sell 0 items, they will have a negative profit...the company will be in the red by $12,400. This means that they have fixed costs of $12,400 (operating expenses that they have before selling any items).

(c) To find the x-intercept of the function \( P(Q) = 27.35Q - 12,400 \) we need to set the output-value to 0 and solve for the input value. This is a linear function, and so it will be simple to solve by hand:
\[
0 = 27.35Q - 12400
+12400 +12400
\]
\[
12400 = 27.35Q
\]
\[
\div 27.35 \div 27.35
\]
\[
453 \approx Q
\]

And so we can conclude that the x-intercept is approximately the point \((453,0)\). In real-world terms this tells us that when the company sells 453 bracelets, then they will have a profit of exactly $0. So, they will no longer be in debt, but they will not be earning a positive profit yet. This is called the break-even point. They need to sell 453 bracelets to break even.

(d) The slope of this function is 27.35 dollars per item. Every time the company sells one more item, the profit will increase by $27.35.
Example 3: The number of people shopping in a popular downtown neighborhood on a certain day in December is approximated by the function \( N(t) = -10t^2 + 120t + 15 \). In this function, \( t \) represents the number of hours after 6 am on that day, and \( N(t) \) represents the number of people in the area. This function works as a model for \( 0 \leq t \leq 12 \).

(a) Explain the real-world meaning of saying that “this function works as a model for \( 0 \leq t \leq 12 \).”

Solution: The input-values represent the number of hours after 6 am. So, \( t=0 \) represents 6 am, and \( t=12 \) represents 6 pm. So, when we say that the domain is \( 0 \leq t \leq 12 \) we are saying that the model only makes sense for predicting the number of people between the hours of 6 am and 6 pm.

(b) Solve the equation \( N(t) = 100 \). Then explain the real-world meaning of any solutions.

Solution: We have been asked to solve where this quadratic function has an output value of 100.

\[
100 = -10t^2 + 120t + 15
\]

\[
0 = -10t^2 + 120t - 85 \quad \text{subtract 100 on both sides to set equation equal to 0}
\]

\[
t = \frac{-120 \pm \sqrt{120^2 - 4(-10)(-85)}}{2(-10)} \quad \text{Use quadratic formula to solve for } t
\]

\[
t \approx 0.76 \quad \text{and} \quad t \approx 11.24
\]

We can check these two solutions by plugging them back into the original equation:

\(-10(.76)^2 + 120(.76) + 15 \approx 100\) and \(-10(.76)^2 + 120(.76) + 15 \approx 100\)

This tells us that there will be approximately 100 people shopping 11.24 hours after 6 am (about 5:15 pm), and there will be approximately 100 people shopping 0.76 hours after 6 am (about 6:45 am).

(c) Find the vertex of the function and then write a sentence that interprets the real-world meaning of the vertex.

Solution: The vertex of the parabola \( N(t) = -10t^2 + 120t + 15 \) occurs at \( t = \frac{-120}{2(-10)} = 6 \). The height of the vertex is \( N(6) = -10(6)^2 + 120(6) + 15 = 375 \)

And so the vertex is the point \((6, 375)\). This tells us that 6 hours after 6am (at noon), there will be 375 people shopping. The parabola is open down, and so the vertex is the maximum point on the function. This tells us that the highest number of people on the street will be 375 people, and that occurs at noon.

(d) During what time of day is the number of people shopping in the neighborhood increasing?

Solution: The domain of the function is \( 0 \leq t \leq 12 \), and the vertex occurs at \((6, 375)\) which is a maximum. So, we know that from \( t=0 \) to \( t=6 \) the function is increasing. This tells us that the number of people shopping in the neighborhood increases between 6am and noon.
Example 4: A square cabin with width $W$, in feet, has total construction cost, in dollars, given by the formula

$$C(W) = 18W^2 + 145W + 510.$$

(a) Clearly identify the real-world meaning of the INPUT variable (the variable along the horizontal axis). Clearly identify the real-world meaning of the OUTPUT variable (the variable along the vertical axis).

Solution: $W$ is the input variable (the horizontal variable), and it represents the length of the side of the square cabin in feet.

The output variable (the vertical variable) is $C$ which represents the total construction cost (in dollars) for building a square cabin that is $W$ feet in length. In the graphing calculator we will see $X$ representing the width, and we will see $Y$ representing the cost.

(b) Suppose that the largest square cabin that the construction company is willing to build is a cabin with a width of 50 feet. You want to graph this window on a viewing window that makes sense in practical terms. Find appropriate values for $X_{\text{Min}}$ and $Y_{\text{Min}}$.

Solution: We will use $X_{\text{Min}}$ of 0 because the cabin could be any length higher than 0 feet. We will use $X_{\text{Max}}$ of 50 because the largest possible length of the cabin wall is 50 feet according to the given information.

(c) Use ZOOM FIT (ZOOM 0) in order to have the calculator automatically set the $Y_{\text{Min}}$ and $Y_{\text{Max}}$. Tell what values you have for $Y_{\text{Min}}$ and $Y_{\text{Max}}$ after pressing Zoom Fit.

Solution: We put the formula in the Y= menu. Then we let $X_{\text{Min}}=0$ and we let $X_{\text{Max}}=50$. Then we press Zoom 0. The calculator graphs the function.

We can then go back to WINDOW and see that the $Y_{\text{Min}}$ has been automatically been set to a value of 510 and $Y_{\text{Max}}$ has been automatically set to a value of 52,760. This means that the lowest possible construction cost is $510, and the highest possible construction cost is $52,760.

(d) Graphically find the $C$-value when $W=12$. Then write a complete sentence that explains the real-world meaning of what you have found.

Solution: Since $W$ is the input-variable, we know that the input value is 12 and we need to find the output value on the graph. On the graph screen we type TRACE and then type 12 and then press ENTER.

We see that when $W=12$, then $C=4842$. In other words, $C(12)=4842$. This tells us that when the square cabin has a side length of 12 feet, then the construction cost of the cabin is $4,842.

(e) Graphically find the $W$-value when $C=10,000$. Then write a complete sentence that explains the real-world meaning of what you have found.

Solution: Since $C$ is the output-variable, we know that the output value is 10,000 and we need to find the input value on the graph. We add the line $Y_2=10,000$ to the graph, and find the point of intersection.
We see that when $C=10,000$ that means that $W \approx 19.2841$. So, if the cabin costs $10,000 to construct, then it had a width of about 19.2841 feet.

(f) What is the $y$-intercept of this function? Explain the real-world meaning of the $y$-intercept.

Solution: The vertical intercept is the point $(0, 510)$ which is the constant term of the function. So, if the cabin has walls of length 0 feet (so no cabin is built at all), then the construction costs are $510. This tells us that the fixed costs of constructing the cabin are $510.

Example 5: The height of a ball above the ground (in feet) $t$ seconds after it has been launched is given by the function $H(t) = -16t^2 + 144t + 8$. Graph the function on your calculator. Use the viewing window $X_{\text{Min}} = -2$ and $X_{\text{Max}} = 10$ and $Y_{\text{Min}} = -50$ and $Y_{\text{Max}} = 400$.

(a) Find the vertical intercept and explain the meaning of that point in context.

Solution: The vertical intercept of the quadratic function $H(t) = -16t^2 + 144t + 8$ is the point $(0,8)$.

0 is the $t$-value, and 8 is the $H$-value. So this tells us that 0 seconds after the ball has launched it will be 8 feet above ground. In other words, the ball starts out 8 feet above ground at the time of launch.

(b) Graphically find when the ball hits the ground. Round your answer to 4 decimal places. Briefly explain your calculator steps.

Solution: To hit the ground means that the height of the ball is 0 feet.

So, we know that the output-value is 0, and we need to find the value(s) of the input. We put the function $H(t)$ into $Y_1$, and we let $Y_2=0$ in order to produce a horizontal line at a height of 0. The horizontal line will lay right on top of the x-axis, and so we won’t see it on the graph.

We are finding where the function $H(t)$ intersects the x-axis since this will be the point(s) where the graph has a height of 0. We see that the graph intersects the x-axis in two different points. We will solve for both.
This point of intersection on the left side tells us that the ball hits the ground approximately 0.0552 seconds BEFORE it was launched. That makes no sense! We wouldn’t consider t-values that occur before t=0 when the ball was launched. So, although this is an x-intercept of the parabola, this x-intercept doesn’t make sense in the context of this problem.

The point of intersection on the right side tells us that the ball hits the ground approximately 9.06 seconds after it was launched. So we conclude that the ball started at 8 feet above ground, traveled upward to a peak, then fell down and hit the ground at about 9.06 seconds.

(c) Graphically find the highest the ball gets and find the length of time until it reaches that height. Briefly explain your calculator steps.

Solution: The highest the ball gets would be the point on the graph with the highest H-value. That means we are finding the point on the graph with the highest output-value. That is the vertex of the parabola.

We could solve the problem by using the vertex formula, \( x = -\frac{b}{2a} \). But, we have been asked to solve the problem graphically. So, instead we will use the Calc Maximum feature on the calculator.

Remember that your own calculator will probably give a slightly different answer for the x-value. The maximum and minimum feature on the calculator will usually report the x-value with a little bit of error.

The actual point that is the maximum is the point (4.5, 332). This tells us that the ball reaches the highest point after 4.5 seconds. The highest the ball gets is 332 feet high.

(d) What is the domain of this function in practical terms? Explain.

Solution: Recall that the domain of a function is the set of all inputs that can be plugged into the function in order to produce real-valued outputs. The domain of a quadratic function is all real numbers.

However, this quadratic function is being used to model a real-world situation where the input-values represent the number of seconds after a ball was launched. We can’t plug in any value for t. For example, we can’t plug in negative t-values because that would represent times before the ball was launched. And we can’t plug in values of t above 9.06 seconds because the ball hits the ground (and presumably stops) at 9.06 seconds.

So, for this model, we would say that the domain is t-values from 0 seconds up to 9.06 seconds. In other words, the domain is \( 0 \leq t \leq 9.06 \).

(e) What is the range of this function in practical terms? Explain.

Solution: Recall that the domain of a function is the set of all inputs that can be plugged into the function in order to produce real-valued outputs. The range of this quadratic function is \( y \leq 332 \) since the parabola has output values from a height of y=332 and below.

But, again, we need to consider the real-world meaning of these y-values. In this model, the output-values represent the height of a ball above ground. Since we would assume the ball stopped when it hit the ground, we would assume that the y-values can’t actually be negative (that would mean the ball went below ground!).

So, we would say that the range for this model is heights from 0 feet up to 332 feet. In other words the range is \( 0 \leq H \leq 332 \).
1) Suppose you have one piece of white paper, and suppose the piece of paper is 0.004 inches in thickness. You fold the paper in half over and over again.

<table>
<thead>
<tr>
<th>Number of folds, $x$</th>
<th>Height of paper (in inches) $H(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table of values.
(b) Explain why the height of this stack of paper is an EXPONENTIAL function of the number of folds made to the paper. Use complete sentences.
(c) What is the factor of the exponential function? What is the y-intercept of the exponential function? What is the percent rate of the exponential function (as a percent)? Write sentences that interpret the real-world meaning of the y-intercept and percent rate.
(d) Find the equation of the function. Use the equation to find and interpret $H(20)$.

2) Suppose you are 20 feet away from a big bag of money and to get the money you simply need to walk to the money. But you can only take steps that are exactly 1.1 feet long at a time.

<table>
<thead>
<tr>
<th>Number of steps taken, $x$</th>
<th>REMAINING distance to money (in feet) $D(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table of values.
(b) Explain why the remaining distance to the money is a LINEAR function of the number of steps you have taken.
(c) What is the slope of the linear function? What is the y-intercept of the linear function? Write sentences that interpret the real-world meaning of the slope and y-intercept.
(d) Find the equation of the function. Use the equation to find and interpret $D(x)=0$.

3) Suppose a big field contains nine fuzzy rabbits. Every month two more fuzzy rabbits are added to the field.

<table>
<thead>
<tr>
<th>Number of months, $x$</th>
<th>Total number of rabbits in field $R(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table of values.
(b) Explain why the number of rabbits in the field is a linear function of the number of months that have passed.
(c) What is the slope of the linear function? What is the y-intercept of the linear function? Write sentences to interpret the real-world meaning of the slope and y-intercept.
(d) Find the formula of the function. Find and interpret $R(20)$. 
4) Suppose you have a lot of white paper, and suppose that each piece of paper is 0.004 inches in thickness. You create a stack of paper by adding more and more pieces of paper to the pile.

<table>
<thead>
<tr>
<th>Number of pieces of paper, x</th>
<th>Height of stack (in inches) H(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table of values.
(b) Explain why the height of this stack of paper is a LINEAR function of the number of pieces of paper in the pile. Use complete sentences.
(c) What is the slope of the linear function? What is the y-intercept of the linear function? Write sentences that interpret the real-world meaning of the slope and y-intercept.
(d) Find the equation of the function, H(x). Use the equation to find and interpret H(1000).

5) The function below gives the number of trees in a forest. In this table, w represents the number of weeks after a lumber company started removing trees, and T represents the number of trees in the forest.

<table>
<thead>
<tr>
<th>w</th>
<th>T(w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>825</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>450</td>
</tr>
<tr>
<td>9</td>
<td>150</td>
</tr>
</tbody>
</table>

(a) Show that T(w) is a linear function by finding the constant slope between each successive point in the table.
(b) What is the slope? What is the y-intercept? Write sentences to interpret the practical meaning of each.
(c) Write the equation for T(w).
(d) How many trees were there after 1 week, 2 weeks, 4 weeks, 10 weeks, 11 weeks?

6) The function below gives the number of people in the town of Mathburg. In this table, t represents the number of years after 2000 and P represents the number of people in the town.

<table>
<thead>
<tr>
<th>t</th>
<th>P(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>45,300</td>
</tr>
<tr>
<td>1</td>
<td>46,206</td>
</tr>
<tr>
<td>2</td>
<td>47,130</td>
</tr>
<tr>
<td>3</td>
<td>48,073</td>
</tr>
<tr>
<td>4</td>
<td>49,034</td>
</tr>
<tr>
<td>5</td>
<td>50,015</td>
</tr>
</tbody>
</table>

(a) Show that P(t) is an exponential function by finding the constant factor between each successive point in the table. Round to the hundredths place.
(b) What is the rate (as a percent)? What is the y-intercept? Write sentences that interpret the practical meaning of each.
(c) Write the equation for P(t).
(d) If this pattern continued, then what would the population of Mathburg be in 2006, 2007, and 2008?

7) Bob has a rich uncle that gives him a job right out of High School. His uncle gives him $50,000 upon being hired, and then Bob earns an additional $30,000 at the end of each day he works!

(a) Create table of values that gives the total amount Bob has earned, T(x), after x days of working for his uncle. The table should include the values for 0 days, 1 day, 2 days, 3 days, 4 days, and 5 days. Be sure to label the meaning of each column in your table.
(b) Would this be a linear function or an exponential function? Explain.
(c) If it is linear, then identify the slope and y-intercept and interpret both. If it is exponential, then find the factor and percent rate and y-intercept and interpret each.
(d) Find a formula for the function. Call it F(x). Use the formula to find and interpret F(x)=1,000,000.
8) Two planes are flying in the air. The distance that the two planes are from one another, in miles, after \( t \) minutes have passed is given by the function \( D(t) = 0.19t^2 - 4.18t + 29.99 \).

(a) Graph the function on the window \( \text{XMin} = -5, \text{XMax} = 30, \text{YMin} = -10, \text{YMax} = 50 \). Sketch a well-labeled graph that includes labeling the real-world meaning of both axis.

(b) Identify the \( y \)-intercept and write a sentence that interprets the real-world meaning of the \( y \)-intercept.

(c) Graphically solve \( D(t) = 13 \). Write sentence(s) that interpret the real-world meaning of your answer(s).

(d) Graphically solve for the vertex of the parabola. Write a sentence that interprets the real-world meaning of your answer.

9) Tina is offered a job where she will be paid a salary of $0.01 to start working, $0.02 at the end of her first day, $0.04 at the end of her second day, $0.08 at the end of her third day, etc. This pattern will then continue as long as she stays with the company.

(a) Create table of values that gives the amount earned on the \( x \)th day of working (NOT the total amount earned). Let \( F(x) \) be the amount made that day, and let \( x \) be the number of days of working for this company. The table should include the values for 0 days, 1 day, 2 days, 3 days, 4 days, and 5 days. Be sure to label the meaning of each column in your table!

(b) Would this be a linear function or an exponential function? Explain.

(c) If it is linear, then identify the slope and \( y \)-intercept and interpret both. If it is exponential, then find the factor and percent rate and \( y \)-intercept and interpret each.

(d) Find a formula for the function. Use the formula to find and interpret \( F(30) \) and interpret.

10) According to one model, the world’s population (in millions of people) can be given by the function \( P(t) = 100(1.008)^t \) where \( t \) is the number of years since 1500.

(a) According to this model, calculate \( P(0) \) and then write a sentence to interpret the real-world meaning of your finding.

(b) In actuality, the population of the world in 1500 was approximately 500 million people. The model is not accurate...how many people difference is the model prediction vs. the actual population value?

(c) According to this model, calculate \( P(100), P(200), P(300), P(400), P(500) \). Write sentences to interpret the real-world meaning of your findings.

(d) Calculate the amount of error between your calculations and the actual number of people in the world at those times. Then calculate the PERCENT difference between the model and the actual number of people. Use the information below:

<table>
<thead>
<tr>
<th>Year</th>
<th>Approximate number of people in the world</th>
</tr>
</thead>
<tbody>
<tr>
<td>1600</td>
<td>580 million</td>
</tr>
<tr>
<td>1700</td>
<td>682 million</td>
</tr>
<tr>
<td>1800</td>
<td>978 million</td>
</tr>
<tr>
<td>1900</td>
<td>1750 million = 1.75 billion</td>
</tr>
<tr>
<td>2000</td>
<td>6000 million = 6 billion</td>
</tr>
</tbody>
</table>

(e) Put the model in the graphing calculator in \( Y1 \). Graph it using a window \( \text{XMin} = -100, \text{XMax} = 700, \text{YMin} = -500, \text{YMax} = 8000 \). Then use the calculator to graphically determine the year in which the model predicts we will have a world population of 7 billion people. (The world actually reached a population of 7 billion in October of 2011).

11) It’s 65 degrees Fahrenheit outside and a cold front moves in. The temperature drops 3 degrees Fahrenheit every hour!

(a) Create table of values that gives the temperature outside, \( T(x) \), after \( x \) hours have passed. Your table should include the temperature after 0 hours, 1 hours, 2 hours, 3 hours, 4 hours, and 5 hours. Be sure to label the meaning of each of the columns in your table!

(b) Would this be a linear function or an exponential function? Explain.

(c) If it is linear, then identify the slope and \( y \)-intercept and interpret both. If it is exponential, then find the factor and percent rate and \( y \)-intercept and interpret each.

(d) Find a formula for the function. Call it \( F(T) \). Use the formula to find and interpret \( F(30) \) and interpret. Does this value make sense in real-world terms? Explain.
12) A certain movie opens and earns $1.8 million dollars during its opening week. A week later it earns 20% less at the theaters. A week after that it earns 20% less than the previous week. Every week the movie continues to earn less and less money in the theaters in this same manner.
(a) Write a function that outputs the amount of money the movie makes in the theaters X weeks after it opens.
(b) Graph the function for the first year (there are 52 weeks in a year). Report the window you used. Sketch a well-labeled graph that includes the real-world meaning of each axis.
(c) Graphically solve for when the movie earns only $10,000 in the theaters.

13) You bake a cake in the oven and then place the cake in the refrigerator to cool. When you place the cake in the refrigerator it is 250 degrees Fahrenheit. Every minute it cools 10%.
(a) Create table of values that gives the temperature of the cake, F(T), after T minutes have passed. The table should include the values for 0 minutes, 1 minute, 2 minutes, 3 minutes, 4 minutes, and 5 minutes. Be sure to label the meaning of each column in your table!
(b) Would this be a linear function or an exponential function? Explain.
(c) If it is linear, then identify the slope and y-intercept and interpret both. If it is exponential, then find the factor and percent rate and y-intercept and interpret each.
(d) Find a formula for the function. Call it F(t). Use the formula to find and interpret F(30) and interpret. Does this value make sense in real-world terms? Explain.

14) Suppose a company has total costs of $C = 2,500 + 10Q$ when they sell Q items. Suppose, also, that when they sell Q items, then the price of each item is $P$ dollars where $P = 300 - 5Q$.
(a) REVENUE is found by taking (price per item)*(number of items sold). Find the function, $R(Q)$, that gives the company revenue, R, when selling Q items. Simplify your equation.
(b) PROFIT is found by calculating (revenue) – (cost). Find the function, $P(Q)$, that gives the company profit, P, when selling Q items. Simplify your equation.
(c) How many items should the company sell to break even? Solve graphically. Briefly explain calculator steps and write a complete sentence to answer the question.
(d) How many items should the company sell to maximize profit? Solve graphically. Briefly explain calculator steps and write a complete sentence to answer the question.

15) The height of a ball above the ground (in feet) t seconds after it has been launched is given by the function
$$H(t) = -16t^2 + 115t + 13$$
Graph the function on your calculator. Use the viewing window XMin = -2 and XMax = 10 and YMin = -50 and YMax = 300.
(a) Find the vertical intercept and explain the meaning of that point in context.
(b) Graphically find when the ball hits the ground. Round your answer to 4 decimal places. Briefly explain your calculator steps.
(c) Graphically find the highest the ball gets and find the length of time until it reaches that height. Briefly explain your calculator steps.
(d) What is the domain of this function in practical terms? Explain.
(e) What is the range of this function in practical terms? Explain.
In this section there will be several “rules” of exponents given, and students will be expected to use those rules to simplify and re-write expressions.

**A poor approach to this section would be to do the following:**
- memorize the rules
- practice a few problems to use the rules
- move on to the next section (and begin the process of forgetting the rules)

Unfortunately, most of the students that take this approach end up forgetting the rules by the next semester, and a lot of students even forget these rules before the unit test!

**A better approach to this section is to focus on trying to understand why the general rules are what they are:**
- practice coming up with each general rule (not by repeating a memorized rule, but by logically figuring out how the rule is what it is)
- practice explaining why the rule makes sense (practice by explaining it to a friend and/or practice by writing down a few simple examples of why the rule works)

If a student truly understands why the exponent rules are true, then the student won’t need to memorize rules at all...the student can create the rule when he/she needs it, because the rule makes logical sense to the student!

**Example 1:** When asked to simplify $a^{30}a^4$ many students incorrectly say that the answer is $a^{120}$ because they AREN’T THINKING ABOUT WHAT THE EXPRESSION REPRESENTS, but are instead incorrectly using a rule that has been forgotten. If the student instead views the problem in terms of what $a^{30}a^4$ represents, then the student would work out the fact that

$$a^{30}a^4 = \left( \underbrace{a \cdot a \cdots a}_{30 \text{ times}} \right) \left( \underbrace{a \cdot a \cdot a \cdot a}_{4 \text{ times}} \right) = \underbrace{a \cdot a \cdots a}_{34 \text{ times}} = a^{34}$$

Rather than approaching this section as a set of rules that students need to “memorize and then regurgitate,” students should instead try to understand why the rules work!!
**Exponential Expression**: The expression $a^m$ has BASE $a$ and EXPONENT $m$. When $m$ is a positive integer, then $a^m$ represents multiplying $a$ times itself $m$ times in a row.

**Example 2**: Explain the difference between $(2t)^4$ and $2t^4$.

**Solution**:

$(2t)^4$ means $(2t)(2t)(2t)(2t)$ which, by the commutative property is $(2 \cdot 2 \cdot 2 \cdot 2)(t \cdot t \cdot t \cdot t)$, which can be simplified as $16t^4$.

But $2t^4$ means $2(t)(t)(t)(t)$ which can be simplified as $2t^4$.

Notice that $(2t)^4$ has base $(2t)$, and $2t^4$ means $2$ times the quantity $t^4$ which has base $t$. So, the expressions $(2t)^4$ and $2t^4$ are different.

**Example 3**: Explain the difference between $(-3)^4$ and $-3^4$.

**Solution**:

$(-3)^4$ means $(-3)(-3)(-3)(-3) = 81$.


Notice that the base in $(-3)^4$ is $(-3)$ which means $(-3)$ is being multiplied to itself four times in a row.

But, in the expression $-3^4$ there is a negative out front, and it is only the $3$ that is being multiplied to itself four times in a row.

The negative in $-3^4$ is not being raised to the fourth power!
Example 4: Simplify the expression \((7^4)(7^2)\).

Solution: First we think about the meaning of each expression within the product.

\[
7^4 \text{ means } 7 \cdot 7 \cdot 7 \cdot 7 \quad \text{and} \quad 7^2 \text{ means } 7 \cdot 7
\]

If we multiply \(7^4\) and \(7^2\), then the result is \((7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7) = 7^6\).

Notice that it is much simpler to remember the rule of adding the exponents than to write out all the 7's! But, if you forget the rule, making up a quick example like this one on your paper can help remind you that there would be six of the 7's all multiplied together, and so the example would help remind you that you need to add the exponents in order to get the correct number of 7's.

Example 5: Simplify the expression \((-5k)^{15}(-5k)^8\) using the product rule.

Solution: We see that the two exponential expressions that we are multiplying have the same base. So, we have 15 of the \((-5k)\) multiplied together, and then we would have 8 more of the \((-5k)\) multiplied after that. This means we should have a total of 23 \((-5k)\) multiplied in a row.

\[
(-5k)^{15}(-5k)^8 = (-5k)^{15+8} = (-5k)^{23}
\]

Example 6: Simplify the expression \((-2x)^{30}(-2x)^{-20}\) using the product rule.

Solution: We see that the two exponential expressions that we are multiplying have the same base. Therefore we can use the product rule. In the answer, we keep the same base, and add the exponents in order to simplify the expression.

\[
(-2x)^{30}(-2x)^{-20} = (-2x)^{30+(-20)} = (-2x)^{10}
\]

Note: We will discuss the meaning of negative exponents later in this section.
Example 7: Simplify the expression \( \frac{5^7}{5^3} \).

Solution: First we think about the meaning of the expression.

\[ \frac{5^7}{5^3} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5} = \frac{5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5 \cdot 5 = 5^4 \]

We see that three of the 5's canceled out leaving four remaining 5's in the numerator.

Notice that it is much simpler to remember the rule of subtracting the exponents than to write out all the 5's! But, if you forget the rule, making up a quick example like this one on your paper can help remind you that you need to subtract the exponents (since 5's cancel out) in order to get the correct number of 5's.

Example 8: Simplify the expression \( \frac{(12mr^2)^5}{(12mr^2)^{-7}} \).

Solution: We see that the two exponential expressions that we are dividing have the same base. Therefore, we can use the quotient rule. According to the quotient rule, we keep the same base, and subtract the numerator’s exponent minus the denominator’s exponent in order to simplify the expression.

\[ \frac{(12mr^2)^5}{(12mr^2)^{-7}} = (12mr^2)^{5-(-7)} = (12mr^2)^{12} \]

Note: We will discuss the meaning of negative exponents later in this section.

Example 9: Simplify the expression \( \frac{12x^6}{3x^{-4}} \).

Solution: First note that the coefficient 12 in the numerator and the coefficient 3 in the denominator are not being raised to a power. So, we can simplify them separately.

Next we see that the two exponential expressions that we are dividing have the same base. Therefore, we can keep the same base, and subtract the numerator’s exponent minus the denominator’s exponent in order to simplify the expression.

\[ \frac{12x^6}{3x^{-4}} = \frac{12 \cdot x^6}{3 \cdot x^{-4}} = 4x^6(-4) = 4x^{10} \]

Note: We will discuss the meaning of negative exponents later in this section.
Example 10: Simplify the expression $(5^4)^3$.

Solution: First we think about the meaning of the expression. Inside the parenthesis we have $5^4 = 5 \cdot 5 \cdot 5 \cdot 5$. We need to multiply the $(5 \cdot 5 \cdot 5 \cdot 5)$ times itself three times in a row which gives

$$(5^4)^3 = (5 \cdot 5 \cdot 5 \cdot 5)^3$$
$$= (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$$
$$= 5^{12}$$

Notice that it is much simpler to remember the rule of multiplying the exponents than to write out all the 5’s! But, if you forget the rule, making up a quick example like this one on your paper can help remind you that you need to multiply the exponents (since there would be four 5’s multiplied three times) in order to get the correct number of 5’s.

Example 11: Simplify the expression $5(x^2)^4$.

Solution: First note that the coefficient 5 is not being raised to a power. So, we re-write the coefficient 5 and then simplify the rest of the expression separately.

$$5(x^2)^4 = 5x^{8}$$

Example 12: Simplify the expression $(x^{-8})^{-2}$.

Solution: We see that the exponential expression is being raised to a power. Therefore, we can use the power rule. According to the rule, we need to keep the same base, and multiply the exponents in order to simplify the expression.

$$(x^{-8})^{-2} = x^{16}$$

Note: We will discuss the meaning of negative exponents later in this section.
Example 13: Simplify the expression \((5a)^3\).

Solution: First we begin by thinking about the meaning of the expression.

The base of this exponential is \(5a\). And so \((5a)^3 = (5a)(5a)(5a)\).

And by the commutative property this is the same as \((5a)^3 = 5^3a^3\).

Notice that it is much simpler to remember this rule of exponents than to write out all the 5’s and all the a’s! But, if you forget the rule, making up a quick example like this one on your paper can help remind you that there will be three 5’s and there will be 3 a’s, and so both values must be raised to the 3\(^{rd}\) power.

Example 14: Simplify the expression \((3x)^5\).

Solution: We see that we have two values multiplied together, and the entire expression is being raised to a power. This means that we can use this rule of exponents.

According to this rule,

\[
(3x)^5 = 3^5x^5 = 243x^5
\]

Example 15: Simplify the expression \((-x^3)^2\).

Solution: The base of this expression can be thought of as \(-1 \cdot x^3\). This is a good way to think of it so that we don’t forget to raise the \(-1\) to the power of 2. It is a very common error to forget to raise the negative to the power!!

According to this rule,

\[
(-x^3)^2 = (-1)^2(x^3)^2 = 1x^6 = x^6
\]

Don’t forget that the negative in the base acts like the number -1. Many students re-write the expression \(-x^3\) as the expression \(-1x^3\) in the first step to help ensure that the negative isn’t forgotten. That negative in the base must also be raised to the power!
To simplify an expression with an exponent of 0, remember that $a^0 = 1$ for all real numbers $a$ (except $0^0$ is undefined).

One explanation of why this makes sense:
Suppose that $a \neq 0$.

Then $\frac{a^n}{a^n} = 1$ since this is just a number divided by itself (which is always 1).

But we know that $\frac{a^n}{a^n} = a^{n-n} = a^0$ according to the quotient rule.

So we know that $\frac{a^n}{a^n} = 1$ and we know that $\frac{a^n}{a^n} = a^0$.

Therefore we can conclude that $1 = \frac{a^n}{a^n} = a^0$.

Therefore, we can conclude that $1 = a^0$.

Example 16: Simplify the expression $5^0$.

Solution: According to the rule, we can conclude that $5^0 = 1$.

If we forget the rule, we can try to work it out as follows:

$5^0 = 5^{n-n} = \frac{5^n}{5^n}$ according to the quotient rule.

And we know

$\frac{5^n}{5^n} = \frac{5\cdot5\cdot\cdots\cdot5}{5\cdot5\cdot\cdots\cdot5} = 1$ since it is a number divided by itself.

So we can see that $5^0 = \frac{5^n}{5^n} = 1$, and so we know that $5^0 = 1$.

Example 17: Simplify the expression $(175x)^0$ assuming that $x \neq 0$.

Solution: According to the rule, we know that raising this expression to the zero power will give an answer of 1.

$(175x)^0 = 1$

Example 18: Simplify the expression $\left(\frac{4(3xy^6)^3}{15y^9}\right)^0$ assuming that $x \neq 0$ and $y \neq 0$.

Solution: According to the rule, we know that raising this expression to the zero power will give an answer of 1.

$\left(\frac{4(3xy^6)^3}{15y^9}\right)^0 = 1$

Example 19: Simplify the expression $-7(x^3)^0$ assuming that $x \neq 0$.

Solution: First observe that the exponent 0 is only applied to the base ($x^3$). The -7 out front is not being raised to the 0 power.

So we know that $(x^3)^0 = 1$ according to the rule. And the -7 is multiplied to that after we apply exponents. And so we have

$-7(x^3)^0 = -7(1) = -7$. 

In other words, when \( n \) is a positive number, then \( a^{-n} \) represents DIVIDING by \( a \) \( n \) times in a row.

<table>
<thead>
<tr>
<th>Negative Exponents</th>
<th>( a^{-n} = \frac{1}{a^n} ) and ( \frac{1}{a^{-n}} = a^n )</th>
</tr>
</thead>
</table>

Note: Many students think of negative exponents as "flipping" the expression.

- If the expression is in the numerator raised to a negative exponent, then you can "flip it" to the denominator with a positive exponent. So, in order to rewrite \( a^{-n} = \frac{a^{-n}}{1} = \frac{1}{a^n} \), we “flip” the original base down to the denominator with a positive exponent.

- If the expression is in the denominator raised to a negative exponent, then you can “flip it” to the numerator with a positive exponent. So, in order to rewrite \( \frac{1}{a^{-n}} = \frac{1}{(1/a^n)} = a^n \), we “flip” the original base up to the numerator with a positive exponent.

Why this makes sense:

Suppose that \( a \neq 0 \).

Then \( \frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n} \) because \( 1 = a^0 \) by the quotient rule.

Therefore \( \frac{1}{a^n} = a^{-n} \).

We can also see that if \( a \neq 0 \),

Then \( \frac{1}{a^{-n}} = \frac{1}{(1/a^n)} = 1 \div \frac{1}{a^n} = 1 \times a^n = a^n \) because division of fraction is the same as multiplication of reciprocals.

Therefore \( \frac{1}{a^{-n}} = a^n \).
Example 20: Consider the expression $\frac{3^4}{3^6}$.

(a) Simplify the expression and write the answer as an exponential expression with negative exponents.

Solution to (a): According to the quotient rule, $\frac{3^4}{3^6} = 3^{4-6} = 3^{-2}$.

(b) Simplify the expression and write the answer as an exponential expression with no negative exponents.

Solution to (b): Another way to view the problem is by noticing that $\frac{3^4}{3^6} = \frac{3\cdot3\cdot3\cdot3}{3\cdot3\cdot3\cdot3\cdot3\cdot3} = \frac{1}{3\cdot3} = \frac{1}{3^2}$.

And so another way to write the answer is $\frac{1}{3^2}$.

Notice that we can write the answer as $3^{-2}$ or we can write the answer as $\frac{1}{3^2}$.

According to this rule of negative exponents, they are the same. $\frac{1}{3^2} = 3^{-2}$. Writing $3^{-2}$ is the same as saying that we are dividing by $3^2$.

Example 21: Re-write the expression $7^{-5}$ as an exponential expression with no negative exponents.

Solution: According to this rule of negative exponents, writing $7^{-5}$ is the same as saying that we need to divide by $7^5$. And so,

$7^{-5} = \frac{1}{7^5}$

Example 22: Simplify the expression $\frac{1}{(-13\text{term})^{-3}}$ as an exponential expression with no negative exponents.

Solution: According to this rule of exponents, the expression can be re-written as $\frac{1}{(-13\text{term})^{-3}} = (-13\text{term})^3$

Example 23: Simplify the expression $\frac{1}{(-50x)^{-4}} = (-50x)^4$.

Solution: According to this rule of exponents, the expression can be re-written as $\frac{1}{(-50x)^{-4}} = (-50x)^4$

Example 24: Re-write each expression using negative exponents in order to eliminate all fractions.

(a) $\frac{7}{x} \quad (b) \frac{3x^2-4x+7}{(8x-3)} \quad (c) \frac{x}{(1.05)^{3x}} \quad (d) \frac{(185+x)^3}{(x-20)^2}$

Solution: According to this rule of exponents, if we have an expression in the denominator, then we can get an equivalent expression if we “flip the denominator” up to the numerator with an exponent of the opposite sign.

(a) $\frac{7}{x} = \frac{7}{x^1} = 7x^{-1}$

(c) $\frac{x}{(1.05)^{3x}} = x(1.05)^{-3x}$

(b) $\frac{3x^2-4x+7}{(8x-3)} = \frac{(3x^2-4x+7)}{(8x-3)^1}$

$= (3x^2 - 4x + 7)(8x - 3)^{-1}$

(d) $\frac{(185+x)^3}{(x-20)^2} = (185 + x)^3(x - 20)^{-2}$
Scientific Notation: To write a number in scientific notation is to write the number in the following form:

\[ A \times 10^N \]

where

- \( A \) is a real number such that \( 1 < |A| < 10 \)
- \( N \) is an integer which are numbers of the form \(-3, -2, -1, 0, +1, +2, +3 \) ...

Example 25:

<table>
<thead>
<tr>
<th>Number Written in Scientific Notation</th>
<th>Number Written in Decimal Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5.687 \times 10^7 )</td>
<td>56,870,000</td>
</tr>
<tr>
<td>(-9.09543 \times 10^{14} )</td>
<td>(-909,543,000,000,000 )</td>
</tr>
<tr>
<td>( 2.4 \times 10^{-8} )</td>
<td>0.0000000024</td>
</tr>
<tr>
<td>(-3.00084 \times 10^{-6} )</td>
<td>(-0.00000300084 )</td>
</tr>
<tr>
<td>( 6.089 \times 10^0 )</td>
<td>6.089</td>
</tr>
</tbody>
</table>

Scientific notation is often used to write extremely big numbers (numbers in the billions, trillions, and higher), and also used to write little tiny decimals (numbers in the billionths, trillionths, and smaller).

Example 26: Write each number in scientific notation.

(a) \( 17,534,000,000,000 \)

(b) \( 0.000000000000894 \)

Solution to (a): In order to write the number \( 17,534,000,000,000 \) in scientific notation, we need to write it in the form

\[(\text{number between 1 and 10}) \times (\text{multiple of 10})\]

This would mean that we would place the decimal point between the 1 and the 7 (to make the number 1.7534 which is a number between 1 and 10), and then multiply that number by 10 thirteen times in a row.

And so, in scientific notation, \( 17,534,000,000,000 = 1.7534 \times 10^{13} \).

Solution to (b): In order to write the number \( 0.000000000000894 \) in scientific notation, we need to write it in the form

\[(\text{number between 1 and 10}) \times (\text{multiple of 10})\]

This would mean that we would place the decimal point between the 8 and the 9 (to make the number 8.94 which is a number between 1 and 10), and then divide that number by 10 thirteen times in a row. That is the same thing as multiplying by \( 10^{-13} \).

And so, in scientific notation, \( 0.000000000000894 = 8.94 \times 10^{-13} \).
Example 27: Write each number in decimal form.

(a) \(6.114 \times 10^{-8}\)  

(b) \(-1.4905 \times 10^5\)

Solution:

(a) To take the number \(6.114\) and multiply it by \(10^{-8}\) is the same as dividing the number \(6.114\) by \(10\) eight times in a row. Therefore, we can take the number \(6.114\) and move the decimal point eight positions to the left.

\[
6.114 \times 10^{-8} = \frac{6.114}{10^8} = 0.0000006114
\]

(b) To take the number \(-1.4905\) and multiply by \(10^5\) is the means we take the number \(-1.4905\) and move the decimal point 5 places to the right.

\[
-1.4905 \times 10^5 = -149,050
\]

Example 28: The Sun is approximately \(1.58 \times 10^{-5}\) light years away from Earth. A light year is a unit of length where 1 light year is approximately \(5.9 \times 10^{12}\) miles. Use this information to determine the distance to the Sun in miles.

Solution: The distance to the Sun is

\[
(1.58 \times 10^{-5} \text{ light years}) \times \left(\frac{5.9 \times 10^{12} \text{ miles}}{1 \text{ light year}}\right)
\]

\[
= (1.58 \times 10^{-5}) \times (5.9 \times 10^{12}) \text{ miles}
\]

\[
= (1.58 \times 5.9) \times (10^{-5} \times 10^{12}) \text{ miles} \quad \text{by using the commutative property}
\]

\[
= (9.322) \times (10^7) \text{ miles} \quad \text{by using the product rule}
\]

And so, in scientific notation, the distance to the Sun is approximately \(9.322 \times 10^7\) miles. We can write this distance as a decimal if we want. The distance to the Sun is approximately \(93,220,000\) miles.

Example 29: The U.S. National Debt on 1/21/2012 was approximately \(15.26 \times 10^{12}\). At that time, there were approximately \(312,000,000\) people in the U.S. If every man, woman, and child in the U.S. were to contribute an equal amount to paying off the National Debt, then how much would every man, woman, and child need to contribute?

Solution: We need to find the amount of money per person. So, we need to divide the total amount of money by the total number of people.

\[
\frac{15.26 \times 10^{12}}{312,000,000} \text{ dollars}
\]

\[
= \frac{15.26 \times 10^{12}}{3.12 \times 10^8} \text{ dollars}
\]

Write the denominator in scientific notation so we can easily simplify

\[
= \left(\frac{15.26}{3.12}\right) \times \left(\frac{10^{12}}{10^8}\right) \text{ dollars per person}
\]

Divide the decimal values, and the multiples of 10 as separate pieces

\[
\approx (4.89) \times (10^4) \text{ dollars per person}
\]

Simplify using the quotient rule

And so, to pay off the National Debt, each man, woman, and child would need to contribute \(4.89 \times 10^4\) dollars. We can write that as a decimal if we want. To pay off the National Debt, each man, woman, and child would need to contribute \(48,900\).
Example 30: A red blood cell is approximately 8 micrometers in diameter. A micrometer is one millionth of a meter. A millimeter is one thousandth of a meter. How big is a red blood cell in meters? How big is a red blood cell in millimeters?

Solution: We have been told that 1 micrometer = 10^{-6} meters. Therefore the red blood cell diameter is

\[ 8 \text{ micrometers} \times \left( \frac{10^{-6} \text{ meters}}{1 \text{ micrometer}} \right) = (8 \times 10^{-6}) \text{ meters} \]

And so a red blood cell is 8 \times 10^{-6} meters in diameter. As a decimal, we would say that a red blood cell is 0.000008 meters in diameter.

We have been told that 1 millimeter = 10^{-3} meters. Therefore the red blood cell diameter is

\[ 0.000008 \text{ meters} \times \left( \frac{1 \text{ millimeter}}{10^{-3} \text{ meter}} \right) = (0.000008 \times 10^{3}) \text{ mm} = 0.008 \text{ mm} \]

And so a red blood cell is 0.008 mm in diameter.
Section 3.2a Written Practice and Reflection
Exponent Rules Basics

1) Simplify each function completely. Remove all parenthesis from the expression. Write your final answer using only POSITIVE exponents.

(a) \( f(x) = 7(-5x)^4 \)  
(b) \( g(x) = (-8x)^{19}(-8x)^{30} \)  
(c) \( h(w) = \frac{18w^{20}}{3w^5} \)  
(d) \( p(b) = 8(b^6)^3 \)  
(e) \( f(t) = 5(2t^3)^4 \)  
(f) \( P(x) = 12(4x)^0 \)  
(g) \( k(x) = 8x^{-4} \)  
(h) \( L(m) = \frac{3}{(4m)^{-5}} \)

2) Simplify each expression completely. Remove all parenthesis from the expression. Write your final answer using only POSITIVE exponents.

(a) \( k(t) = 3t^{-5} \)  
(b) \( f(w) = 7(4w^2)^3 \)  
(c) \( P(b) = 150(3b^2)^0 \)  
(d) \( L(a) = \frac{8}{(3a)^{-2}} \)  
(e) \( y(x) = (-3x)^7(9x)^4 \)  
(f) \( r(v) = \frac{4v^{12}}{20v^4} \)  
(g) \( f(x) = 500(-8x)^2 \)  
(h) \( p(t) = 3(t^4)^8 \)

3) Re-write each expression/function in simplified form with NO FRACTIONS (so you may need to use negative exponents).

(a) \( \frac{740}{x^{-8}} \)  
(b) \( \frac{825x^{-80}y^6}{x^{-20}y^{18}} \)  
(b) \( \left( \frac{400a^{15}}{2b^7} \right)^3 \)  
(d) \( \frac{x^7y^{-3}}{a^3b^{-5}} \)  
(e) \( f(x) = \frac{6}{x} \)  
(f) \( g(x) = \frac{7}{(1-x)} \)  
(g) \( g(x) = \frac{7}{(x-3)^6} \)  
(h) \( k(x) = \frac{1950}{2^x} \)
4) Re-write each expression/function in simplified form with NO FRACTIONS (so you may need to use negative exponents).

(a) \( \frac{12x}{(x-4)^{-3}} \)

(b) \( \frac{75x^{-30}m^{10}}{x^{-15}m^{50}} \)

(c) \( \left( \frac{8a^{-9}}{5b^4} \right)^{-3} \)

(d) \( \frac{7m^{-7}x^{10}}{8p^{13}y^{-15}} \)

(e) \( f(t) = \frac{150}{t} \)

(f) \( W(m) = \frac{7m+3}{m-4} \)

(g) \( g(x) = \frac{100x}{(x+15)^6} \)

(h) \( k(t) = \frac{75}{1500(1.04)^t} \)

5) Write each of the following numbers in scientific notation.

(a) 0.0000000000054112

(b) 74,852,000,000,000

(c) −850,000,000,000,000

(d) −0.0000000000000113

6) Write each of the following numbers in scientific notation.

(a) 62,009,000,000

(b) 0.00000008907

(c) −0.00056

(d) −10,009,000

7) The chart below gives the approximate volume of the planets.

<table>
<thead>
<tr>
<th>Celestial Object</th>
<th>Approximate Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>( 1.41 \times 10^{18} ) cubic km</td>
</tr>
<tr>
<td>Jupiter</td>
<td>( 1.52 \times 10^{15} ) cubic km</td>
</tr>
<tr>
<td>Earth</td>
<td>( 1.09 \times 10^{12} ) cubic km</td>
</tr>
</tbody>
</table>

(a) How many Earth’s could fit inside the Sun?

(b) How many Earth’s could fit inside Jupiter?

(c) How many Jupiter’s could fit inside the Sun?

8) The Earth is 150 million km away from the Sun on average. Neptune is 4.5 billion km away from the Sun on average.

(a) Write Earth’s average distance from the Sun in scientific notation AND write Earth’s average distance from the Sun as a decimal (with all the 0’s written).

(b) Write Neptune’s average distance from the Sun in scientific notation AND write Neptune’s average distance from the Sun as a decimal (with all the 0’s written).

(b) How many km further from the Sun is Neptune?

9) It turns out that 18 grams of water contains approximately \( 6.022 \times 10^{23} \) molecules of water. Use this information to determine approximately how much ONE mole of water weighs in grams (give your answer in scientific notation).

10) 180 grams of sugar contains approximately \( 6.022 \times 10^{23} \) molecules of water. Use this information to determine approximately how much ONE molecule of sugar weighs in grams (give your answer in scientific notation).

11) In 2011, one estimate put the Japanese National Debt at approximately \( 9.63 \times 10^{14} \) yen.

(a) Assuming that 1 Japanese yen is equivalent to 0.0129 American dollars, find the Japanese National Debt in American dollars. Write your answer in scientific notation, and also write your answer in decimal form (with all of the zeros).

(b) There are approximately 128 million people in Japan. How much would each citizen need to pay if they each contributed an equal amount to pay off the National debt? Give your answer in American dollars. Also give your answer in Yen. Note: the average household income in Japan is approximately 5,475,000 yen.
Section 3.2b
Exponent Rules Advanced

Learning Outcomes:
- Re-write radical expressions using rational exponents
- Re-write rational exponent expressions using radicals
- Simplify square roots
- Simplify exponential expressions involving multiple steps

Fractional Exponents \( a^{1/n} = \sqrt[n]{a} \) and \( a^{b/n} = \sqrt[n]{a^b} = (\sqrt[n]{a})^b \)

We can re-write expressions with radicals by writing them with fractional exponents using the fact that \( a^{1/n} = \sqrt[n]{a} \).

One explanation of why this makes sense:

As an example, notice that we know that \( \sqrt{a} \cdot \sqrt{a} = a \), but we also know from the product rule that \( a^{1/2} \cdot a^{1/2} = a \). So breaking \( a \) down into a multiple of two equivalent things can be done in two different ways.

This must mean that \( \sqrt{a} \cdot \sqrt{a} = a^{1/2} \cdot a^{1/2} \) and so \( \sqrt{a} = a^{1/2} \).

Another example, notice that we know that \( \sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a \), but the product rule tells us that \( a^{1/3} \cdot a^{1/3} \cdot a^{1/3} = a \). So breaking \( a \) down into a multiple of three equivalent things can be done in two different ways.

This must mean that \( \sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a^{1/3} \cdot a^{1/3} \cdot a^{1/3} \) and so \( \sqrt[3]{a} = a^{1/3} \).

In general, we can say that \( \sqrt[n]{a} \cdot \sqrt[n]{a} \cdot \sqrt[n]{a} = a \), and we can say that \( a^{1/n} \cdot a^{1/n} \cdot \ldots \cdot a^{1/n} = a \). And so \( \sqrt[n]{a} = a^{1/n} \).

Example 1: Re-write the expression \( \sqrt{5} \) in exponential notation.

Solution: \( \sqrt{5} \) can be re-written as \( 2^{\sqrt{5}} \). According to this rule, we can re-write this as \( 5^{1/2} \). Therefore, \( \sqrt{5} = 5^{1/2} \). We can check the value on the calculator.
Example 2: Calculate the value of $8^{4/3}$ by hand without a calculator. Then check using the calculator.

Solution: $8^{4/3}$ is the same as $(8^{1/3})^4$ according to the power rule.

Now observe that inside the parenthesis we have $8^{1/3}$ which is the same as $\sqrt[3]{8}$. We know that $\sqrt[3]{8} = 2$.

Therefore, we can now say that $8^{4/3} = (8^{1/3})^4 = (\sqrt[3]{8})^4 = (2)^4 = 16$.

We check the value on the calculator.

Note: It would be equivalent to re-write $8^{4/3}$ as $(8^4)^{1/3}$ which is $(4096)^{1/3} = 16$. But this would probably not be the approach to take if you were doing the problem without a calculator since we don't know the cube root of 4096 off the top of our heads!

Example 3: Calculate the value of $(-32)^{7/5}$.

Solution: $(-32)^{7/5}$ is the same as $((-32)^{1/5})^7$.

Now observe that inside the parenthesis we have $(-32)^{1/5}$ which is the same as $\sqrt[5]{-32}$. We know that $\sqrt[5]{-32} = -2$.

Therefore, we can now say that $(-32)^{7/5} = ((-32)^{1/5})^7 = (\sqrt[5]{-32})^7 = (-2)^7 = -128$.

We check the value on the calculator.

Note: It is equivalent to apply the exponents in the other order as $(-32)^{7/5} = ((-32)^7)^{1/5}$. But -32 raised to the 7th power is so big that this would be a difficult approach to the problem!

Example 4: Write the expression $(-4x)^{-10}$ as an exponential expression with no negative exponents.

Solution: According to this rule of negative exponents, the expression $(-4x)^{-10}$ means that we are dividing by $(-4x)^{10}$. And so we can write

$$(-4x)^{-10} = \frac{1}{(-4x)^{10}}$$

We could re-write the expression further by applying the exponent, 10, to the (-4) in the base and also applying the exponent, 10, to the (x) in the base.

$$(-4x)^{-10} = \frac{1}{(-4)^{10}x^{10}} = \frac{1}{1,048,576x^{10}}$$
Example 5: Rewrite each expression as an expression with radicals instead of fractional exponents.

(a) \(17(x - 30)^{2/3}\)

Solution:
(a) First we can re-write the expression using the power rule.
\[17(x - 30)^{2/3} = 17((x - 30)^{1/3})^2\]
Lastly, we can re-write the fractional exponent as a radical expression.
\[= 17\left(\sqrt[3]{x - 30}\right)^2\]
Note: This is equivalent to writing the expression as \(17\sqrt[3]{(x - 30)^2}\). The order of applying the exponents does not matter.

(b) \(\frac{450x^{1/4}}{75x^{1/3}}\)

First we observe that the fraction \(450/75\) simplifies to 6. Next, we can apply the quotient rule which means we need to subtract the exponents \(1/4 - 1/3 = -1/12\).
\[\frac{450x^{1/4}}{75x^{1/3}} = 6x^{-1/12}\]
Next we re-write the expression using only positive exponents.
\[= \frac{6}{x^{1/12}}\]
Lastly, we can re-write the fractional exponent using a radical.
\[= \frac{6}{\sqrt[12]{x}}\]

Example 6: Re-write the expression \(\frac{2x^3y^{-4}}{5a^{-7}b^8}\) as an expression with no negative exponents.

Solution: First notice that the 2 and the 5 do not have exponents at all, and so we only need to re-write them.
Also notice that the \(x^3\) and the \(b^8\) do not have negative exponents, and so they will remain unchanged.
Only the \(y^{-4}\) and the \(a^{-7}\) need to be altered.
According to this rule of exponents we know that \(y^{-4}\) is the same as \(\frac{1}{y^4}\). And we know that \(\frac{1}{a^{-7}}\) is the same as \(a^7\).
Therefore, the \(a^{-7}\) flips up to the numerator to become \(a^7\), and \(y^{-4}\) flips down to the denominator to become \(y^4\).
\[\frac{2x^3y^{-4}}{5a^{-7}b^8} = \frac{2x^3a^7}{5b^8y^4}\]
Example 7: Simplify each expression, and then write the answer using only exponents (no radicals).

(a) \(5\sqrt{x - 3}\)

(b) \(\frac{5x^2}{\sqrt{x} \cdot \sqrt{x}}\)

Solution to (a): To simplify the expression \(5\sqrt{x - 3}\) we observe that the radical can be re-written using an exponent of \((1/2)\). The 5 is not under the radical, and so it will remain unchanged. Therefore, we can conclude that \(5\sqrt{x - 3} = 5(x - 3)^{1/2}\).

Solution to (b): To simplify the expression \(\frac{5x^2}{\sqrt{x} \cdot \sqrt{x}}\) we can first re-write the square root as an exponent of \((1/2)\), and we can re-write the cube root as an exponent of \((1/3)\).

\[
\frac{5x^2}{\sqrt{x} \cdot \sqrt{x}} = \frac{5x^2}{x^{1/2} \cdot x^{1/3}}
\]

Next we can simplify the denominator using the product rule. We add the exponents \(1/2 + 1/3\) to get \(5/6\).

\[
= \frac{5x^2}{x^{5/6}}
\]

Lastly, we can simplify the expression using the quotient rule. We subtract the exponents \(2 - 5/6\) to get \(7/6\).

\[
= 5x^{7/6}
\]

Therefore, we can conclude that \(\frac{5x^2}{\sqrt{x} \cdot \sqrt{x}} = 5x^{7/6}\).

If you would like to check your answer, you can select ANY \(x\)-value and evaluate the original expression as well as the simplified expression. Since the two expressions should be the same, we should get the same value for the original and the simplified expression no matter which \(x\)-value you select to evaluate with. For example, we can evaluate the original and the simplified version at an \(x\)-value of \(x = 17.4\).

\[
\begin{align*}
(5\sqrt{17.4^2}) & / (\sqrt{17.4} \cdot \sqrt{17.4}) \\
5(17.4) & / (17.4) \\
140.481772 & / 140.481772
\end{align*}
\]

Note: The “Square Root of a Product” is actually just repeating the rule \((a \cdot b)^m = a^m \cdot b^m\) which we will discuss in unit 3. This is true since this “Square Root of a Product” rule can be written equivalently as \((M \cdot N)^{1/2} = M^{1/2} \cdot N^{1/2}\).

Example 8: Below are a few examples to demonstrate how we can re-write/simplify some expressions using the “Square Root of a Product Rule.”

\[
\begin{align*}
\sqrt{12} &= \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3} \\
\sqrt{32} &= \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2} \\
\sqrt{45} &= \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5} \\
\sqrt{700} &= \sqrt{100 \cdot 7} = \sqrt{100} \cdot \sqrt{7} = 10\sqrt{7}
\end{align*}
\]
1. Re-write the number under the radical as a product of (perfect square)*(other number). Use the largest perfect square value possible.
   Example: re-write $\sqrt{50}$ as $\sqrt{25 \cdot 2}$ since 25 is a perfect square.

2. Use the “Square Root of a Product” rule to re-write the radical expression as a product of TWO radical expressions.
   Example: rewrite $\sqrt{25 \cdot 2}$ as $\sqrt{25} \cdot \sqrt{2}$

3. Simplify the radical expression involving the perfect square in order to get rid of one of the radicals
   Example: simplify the $\sqrt{25}$ and re-write it as 5.

So, putting it all together, we have the following: $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5 \cdot \sqrt{2}$

**Example 9**: Simplify the expression $3 - \sqrt{750}$.

**Solution**: The expression $\sqrt{750}$ can be simplified since 750 can be written as the product $25 \cdot 30$.

$$3 - \sqrt{750} = 3 - \sqrt{25 \cdot 30}$$

$$= 3 - \sqrt{25} \cdot \sqrt{30}$$

$$= 3 - 5\sqrt{30}$$

**Example 10**: Simplify the expression $\frac{10 + \sqrt{72}}{6}$.

**Solution**: The expression $\sqrt{72}$ can be simplified since 72 can be written as the product $36 \cdot 2$.

$$\frac{10 + \sqrt{72}}{6} = \frac{10 + \sqrt{36 \cdot 2}}{6} = \frac{10 + \sqrt{36} \cdot \sqrt{2}}{6} = \frac{10 + 6\sqrt{2}}{6}$$

$$= \frac{2(5 + 3\sqrt{2})}{2(3)}$$ factor out the 2 in the numerator/denominator

$$= \frac{2(5 + 3\sqrt{2})}{2(3)}$$ cancel the 2/2 to reduce fractions

$$= \frac{5 + 3\sqrt{2}}{3}$$

Be sure to check on the calculator to make sure that the original value of $\frac{10 + \sqrt{72}}{6}$ is the same as the simplified value as $\frac{5 + 3\sqrt{2}}{3}$. 
Example 11: Simplify the expression \( \frac{-15 + \sqrt{126}}{12} \).

Solution: The expression \( \sqrt{126} \) can be simplified since 126 can be written as the product 9 \cdot 14.

\[
\frac{-15 + \sqrt{126}}{12} = \frac{-15 + \sqrt{9 \cdot 14}}{12} = \frac{-15 + 3 \cdot \sqrt{14}}{12}
\]

\[
= \frac{3(-5 + \sqrt{14})}{3(4)} \quad \text{factor out the 3 in the numerator/denominator}
\]

\[
= \frac{-5 + \sqrt{14}}{4} \quad \text{cancel the 3/3 to reduce fractions}
\]

Be sure to check on the calculator to make sure that the original value of \( \frac{-15 + \sqrt{126}}{12} \) is the same as the simplified value as \( \frac{-5 + \sqrt{14}}{4} \).
Section 3.2b Written Practice and Reflection

Exponent Rules Advanced

1) Re-write each radical expression in exponential form.

(a) \( \sqrt[3]{7} \)  
(b) \( \sqrt[7]{w} \)  
(c) \( \sqrt{x} \)  
(d) \( \sqrt[3]{x - 7} \)  
(e) \( \sqrt[4]{13^5} \)  
(f) \( \sqrt[6]{y^5} \)  
(g) \( \sqrt[n]{8^5} \)  
(h) \( \sqrt[4]{(x + 10)^3} \)

2) Re-write each radical expression in exponential form.

(a) \( \sqrt[7]{6} \)  
(b) \( \sqrt[4]{t} \)  
(c) \( \sqrt{m} \)  
(d) \( \sqrt[6]{x - 2} \)  
(e) \( \sqrt[5]{7^6} \)  
(f) \( \sqrt{x^3} \)  
(g) \( \sqrt[9]{(3x)^4} \)  
(h) \( \sqrt[5]{(7 + y)^4} \)

3) Re-write each exponential expression using radicals.

(a) \( 19^{1/8} \)  
(b) \( t^{1/4} \)  
(c) \( (x + 9)^{1/3} \)  
(d) \( \frac{1}{x^{1/3}} \)  
(e) \( 7^{3/4} \)  
(f) \( m^{8/9} \)  
(g) \( (5y + x)^{3/4} \)  
(h) \( \frac{1}{(7x + 3)^{2/5}} \)

4) Re-write each exponential expression using radicals.

(a) \( 3^{1/5} \)  
(b) \( y^{1/4} \)  
(c) \( (7 - t)^{1/2} \)  
(d) \( \frac{1}{p^{1/8}} \)  
(e) \( 9^{7/9} \)  
(f) \( x^{2/3} \)  
(g) \( (4 - 8x)^{8/11} \)  
(h) \( \frac{1}{(6x - 1)^{3/4}} \)
5) Simplify and then re-write each expression with either fractional exponents or with radicals.

<table>
<thead>
<tr>
<th>Radical Expression</th>
<th>Expression with Fractional Exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>$17\sqrt{x} \sqrt{y}$</td>
<td>$20x^{2/3}$ $\frac{1}{m^{1/4}}$</td>
</tr>
<tr>
<td>$\frac{3\sqrt{a} - 4}{15\sqrt{a} - 4}$</td>
<td>$\frac{85x^{-1/2}}{\sqrt{x}}$</td>
</tr>
</tbody>
</table>

6) Re-write and completely simplify each function **using negative exponents so that it no longer contains fractions or radicals**.

(a) $f(t) = \frac{15t\sqrt{t}}{3t^{-7}}$

(b) $f(x) = \frac{8x^7(4x)^2}{2x^5}$

(c) $g(t) = \frac{t}{\sqrt{t}}$

(d) $p(q) = \frac{\sqrt{q-20}}{(q-20)^{3/4}}$

(e) $m(x) = \left(\frac{8x^7}{16x^{-4}}\right)^{-2}$

(f) $f(x) = \frac{30x^{-3}(x-5)^{70}}{5(x-5)^{80}(x+1)^{-10}}$

7) Simplify each expression completely.

(a) $\sqrt{24}$

(b) $\frac{2-\sqrt{8}}{2}$

(c) $\frac{5+\sqrt{50}}{10}$

(d) $\frac{12-\sqrt{32}}{4}$

8) Find the exact solutions to the equation $0 = 15x^2 - 4x - 8$. Write the exact answers in simplified form including simplifying the radical.

9) Find the exact solutions to the equation $0 = -6x^2 + 2x + 5$. Write the exact answers in simplified form including simplifying the radical.
Section 3.3a
Systems of Two Linear Equations Basics

Learning Outcomes:
- Understand what a system of two linear equations is, and understand what it means to be a solution to a system of equations
- Solve a system of two linear equations graphically
- Solve a system of two linear equations by hand using substitution
- Solve a system of two linear equations by hand using elimination

A system of two linear equations is two linear equations in the same variables given simultaneously. This situation occurs in real-world problems because there are situations where two variables may have a relationship with each other in more than one way. Section 3.3b will cover applications of systems of linear equations.

Solution to a System of Equations

The solution to a system of two linear equations, if it exists, is a point or points that will solve BOTH equations. A solution of the system, if it exists, is a point of intersection of the two lines. Note: A solution is a POINT (not just an x-value or just a y-value).

- No solution to the system: Note that when the two lines are parallel and have a different y-intercept, then the system does not have any solutions because there would be no point of intersection of the two lines. In a real world problem, this would tell us that it is IMPOSSIBLE to satisfy both conditions at the same time.
- Every point on the line is a solution to the system: Note that when the two lines lay atop one another, then the points of intersection are every single point on the lines. So, if the two lines lay atop one another, then every point on the line is a solution to the system of equations. In a real world problem, this would tell us that EVERY POINT on one equation also satisfies the conditions of the other equation.
- One point is a solution to the system: Most systems of two linear equations that we consider will have exactly one solution (one point of intersection). We could graphically find the point of intersection of the two lines, or we could symbolically solve the system of equations using either the elimination method or the substitution method. In a real world problem, this point of intersection is the ONLY POINT that will satisfy both conditions at the same time.

Solving a System Graphically

1. Solve both equations for the dependent variable.
2. Graph both equations. Put one equation in Y1, and the other in Y2. Set a window that allows you to see the intersection of the two lines.
3. Use Calc Intersect to solve for the point of intersection of the two lines.

Solving a System using Substitution Method

1. Solve one of the equations for one of the variables (doesn’t matter which equation, and doesn’t matter which variable).
2. Substitute your solved expression into the other equation in order to re-write the other equation as a linear equation with only 1 variable.
3. Solve your equation for the value of the variable.
4. Now that you know the value of one of the variables, substitute this value back into either equation in order to solve for the value of the other variable. It doesn’t matter if you use the first equation or the second equation, because this is a point that lies on both lines.
5. Check your solution in both equations.
1. Multiply one or both equations with the intent that a variable will cancel upon adding. The value(s) you multiply by should result in one term in your new equation being the additive inverse of a term in the other equation.

2. Add the two equations together. Because of the value that was multiplied in the first step, when you add the two equations, you will have one variable cancel out in the resulting equation.

3. Solve the new equation for the value of the variable.

4. Plug your solved value back into either equation in order to solve for the value of the other variable.

5. Check your solution in both equations.

Example 1: Solve the system of equations GRAPHICALLY

\[\begin{align*}
4x - 3y &= 7 \\
5x + 2y &= 1
\end{align*}\]

Solution: To solve the system of equations means that we need to find the point(s) of intersection of the two lines. We can do this on the graphing calculator by graphing one equation in Y1 and the other equation in Y2. So, we will start by solving both equations for Y.

\[\begin{align*}
4x - 3y &= 7 \\
+3y &= +3y \\
\hline
4x &= 3y + 7 \\
-7 &= -7 \\
\hline
4x - 7 &= 3y \\
\div 3 &= \div 3 \\
\hline
\frac{4x - 7}{3} &= y
\end{align*}\]

And so we put \(Y1 = \frac{4x - 7}{3}\) into the calculator to represent the first equation in the system.

\[\begin{align*}
5x + 2y &= 1 \\
-5x &= -5x \\
\hline
2y &= -5x + 1 \\
\div 2 &= \div 2 \\
\hline
y &= \frac{-5x + 1}{2}
\end{align*}\]

And so we put \(Y2 = \frac{-5x + 1}{2}\) into the calculator to represent the second equation in the system.

Since \(Y1\) is a line with y-intercept \((0, -7/3)\) that slopes up with slope 4/3, and \(Y2\) is a line with y-intercept \((0, 1/2)\) that slopes down with slope -5/2, we can guess that the intersection of the two lines would occur pretty quickly (probably before \(x=2\)). So we set a standard viewing window in order to see the point of intersection.

The intersection point is approximately \((0.7391, -1.3478)\). This tells us that the solution to the system of equation is approximately \((0.7391, -1.3478)\).
So \( x = \frac{17}{23} \).

So \( y = -\frac{31}{23} \).

So the exact solution to the system is \( \left( \frac{17}{23}, -\frac{31}{23} \right) \).

**Example 2:** Solve the system of equations using SUBSTITUTION.

\[
\begin{align*}
5x &= y + 4 \\
y &= -2x + 1
\end{align*}
\]

**Solution:**

<table>
<thead>
<tr>
<th>Equation</th>
<th>First SOLVE ONE OF THE EQUATIONS for one of the variables.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -2x + 1 )</td>
<td>( x = \frac{5}{7} ) (correct)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
<th>Second, SUBSTITUTE INTO THE OTHER EQUATION the value you solved for.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x = y + 4 )</td>
<td>( 5x = (-2x + 1) + 4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
<th>Next SOLVE THIS EQUATION for the variable in the equation. In this case, we are solving for the value of ( x ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x = y + 4 )</td>
<td>( 5x = (-2x + 1) + 4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
<th>Lastly, SUBSTITUTE THE KNOWN VALUE into one of the original equations and solve for the other value. In this case, since we know the value of ( x ), we plug that into one of the equations in order to find the value of ( y ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x = y + 4 )</td>
<td>( y = -2 \left( \frac{5}{7} \right) + 1 )</td>
</tr>
</tbody>
</table>

Check your answer by plugging the point into both equations and checking.

Note: If we want to convert those decimal approximations into exact values, you can
1. Hit 2nd MODE (in order to quit and return to the home screen).
2. Hit the button \( \boxed{\text{Frac}} \) and press ENTER. The calculator will repeat the \( x \)-value you found in the point of intersection.
3. Hit MATH ENTER ENTER in order to convert that \( x \)-value to a fraction.
4. Now that you have the exact fractional answer for \( x \), you can evaluate either equation in the system in order to find the exact value of \( y \).
Example 3: Solve the system of equations using SUBSTITUTION. Show all work.

\[ 2x - 7y = 4 \]
\[ x - 8y = 2 \]

Solution:

\[
\begin{align*}
x - 8y = 2 \\
x = 8y + 2
\end{align*}
\]

First SOLVE ONE OF THE EQUATIONS for one of the variables. In this case, the second equation is already solved for \( y \).

\[
\begin{align*}
2x - 7y &= 4 \\
2(8y + 2) - 7y &= 4
\end{align*}
\]

Second, SUBSTITUTE INTO THE OTHER EQUATION the value you solved for.

\[
\begin{align*}
2(8y + 2) - 7y &= 4 \\
16y + 4 - 7y &= 4 \\
9y + 4 &= 4 \\
9y &= 0 \\
y &= 0
\end{align*}
\]

Next SOLVE THIS EQUATION for the variable in the equation. In this case, we solve for the value of \( y \).

\[
\begin{align*}
x - 8y &= 2 \\
x - 8(0) &= 2 \\
x &= 2
\end{align*}
\]

Lastly, SUBSTITUTE THE KNOWN VALUE into one of the original equations and solve for the other value.

\[
\begin{align*}
2x - 7y &= 4 \quad \text{and} \quad 2(2) - 7(0) = 4 \\
x - 8y &= 2 \quad \text{and} \quad 2(2) - 8(0) = 2
\end{align*}
\]

Both are correct.

Check your answer by plugging the point into both equations and checking.
**Example 4**: Solve the system of equations using ELIMINATION. Show all work.

\[
\begin{align*}
5y + 2x &= 7 \\
4y - 8x &= 2
\end{align*}
\]

**Solution:**

<table>
<thead>
<tr>
<th>Re-written first equation by multiplying by 4:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4 \cdot (5y + 2x) = (7) \cdot 4)</td>
</tr>
<tr>
<td>(4y - 8x = 2)</td>
</tr>
</tbody>
</table>

\[\downarrow\]

Simplified system of equations:

\[
\begin{align*}
20y + 8x &= 28 \\
4y - 8x &= 2
\end{align*}
\]

\[
24y = 30
\]

\[
y = \frac{30}{24} = \frac{5}{4}
\]

First **MULTIPLY ONE OR BOTH EQUATIONS** by a fixed number so that one of the variables will cancel out when the equations are added. In this case we have (+2x) in the first equation and (-8x) in the second equation. If the first equation were multiplied by 4, then it would become (+8x) in the first equation...that would mean the two values would cancel each other out when added. Then, **ADD THE TWO EQUATIONS** together in order to be left with an equation with only one variable. Next **SOLVE THIS EQUATION** for the variable in the equation. Lastly, **SUBSTITUTE THE KNOWN VALUE** into one of the original equations and solve for the other value. Check your answer by plugging the point into both equations and checking.

\[
5y + 2x = 7 \text{ and so } 5(\frac{5}{4}) + 2\left(\frac{3}{8}\right) = 7 \text{ which works}
\]

\[
4y - 8x = 2 \text{ and so } 4(\frac{5}{4}) - 8\left(\frac{3}{8}\right) = 2 \text{ which works}
\]

And so the point works as a solution.

**Example 5**: Solve the system of equations using ELIMINATION. Show all work.

\[
\begin{align*}
3y + 5x &= 8 \\
4y - 2x &= 9
\end{align*}
\]

**Solution:**

<table>
<thead>
<tr>
<th>Re-written first and second equation by multiplying:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4 \cdot (3y + 5x) = (8) \cdot 4)</td>
</tr>
<tr>
<td>(-3 \cdot (4y - 2x) = (9) \cdot -3)</td>
</tr>
</tbody>
</table>

\[\downarrow\]

Simplified system of equations:

\[
\begin{align*}
12y + 20x &= 32 \\
-12y + 6x &= -27
\end{align*}
\]

\[
26x = 5
\]

\[
x = \frac{5}{26}
\]

First **MULTIPLY ONE OR BOTH EQUATIONS** by a fixed number so that one of the variables will cancel out when the equations are added. In this case we have (+3y) in the first equation and (+4y) in the second equation. If the first equation were multiplied by 4, and the second equation were multiplied by (-3), then the two would cancel each other out when added together. Then, **ADD THE TWO EQUATIONS** together in order to be left with an equation with only one variable. Next **SOLVE THIS EQUATION** for the variable in the equation. Lastly, **SUBSTITUTE THE KNOWN VALUE** into one of the original equations and solve for the other value. Check your answer by plugging the point into both equations and checking.

\[
3y + 5x = 8 \text{ and so } 3(\frac{5}{26}) + 5(\frac{3}{8}) = 8 \text{ which works.}
\]

\[
4y - 2x = 9 \text{ and so } 4(\frac{5}{26}) - 2(\frac{3}{8}) = 9 \text{ which also works.}
\]

And so the point works as a solution.
Section 3.3a Written Practice and Reflection
Systems of Two Linear Equations Basics

1) Solve the following system of equations using SUBSTITUTION. Be sure to clearly show your work on each step, and remember that a solution to a system of equations is BOTH an x-value and a y-value. Then solve graphically to make sure it is correct!

\[
\begin{align*}
2x + 5y &= 10 \\
y &= -4x + 1
\end{align*}
\]

2) Solve the following system of equations using SUBSTITUTION. Be sure to clearly show your work on each step, and remember that a solution to a system of equations is BOTH an x-value and a y-value. Then solve graphically to make sure it is correct!

\[
\begin{align*}
-3x + 2y &= 4 \\
x &= y - 5
\end{align*}
\]

3) Solve the following system of equations using SUBSTITUTION. Be sure to clearly show your work on each step, and remember that a solution to a system of equations is BOTH an x-value and a y-value. Then solve graphically to make sure it is correct!

\[
\begin{align*}
2x &= 6 - 10y \\
3y &= 9x + 15
\end{align*}
\]

4) Solve the following system of equations using ELIMINATION. Be sure to clearly show your work on each step, and remember that a solution to a system of equations is BOTH an x-value and a y-value. Then solve graphically to make sure it is correct!

\[
\begin{align*}
2x + 5y &= 10 \\
x - 5y &= 4
\end{align*}
\]

5) Solve the following system of equations using ELIMINATION. Be sure to clearly show your work on each step, and remember that a solution to a system of equations is BOTH an x-value and a y-value. Then solve graphically to make sure it is correct!

\[
\begin{align*}
4x + 2y &= 5 \\
2x - 5y &= 6
\end{align*}
\]

6) Solve the following system of equations using ELIMINATION. Be sure to clearly show your work on each step, and remember that a solution to a system of equations is BOTH an x-value and a y-value. Then solve graphically to make sure it is correct!

\[
\begin{align*}
5x + 3y &= 1 \\
2x - 4y &= 6
\end{align*}
\]

7) Use the calculator to solve graphically.

\[
\begin{align*}
\frac{1}{2}x + \frac{1}{8}y &= \frac{19}{40} \\
\frac{1}{6}x - y &= -\frac{7}{15}
\end{align*}
\]

8) Use the calculator to solve graphically.

\[
\begin{align*}
0.3x + 0.1y &= 85 \\
0.4x - 0.5y &= -45
\end{align*}
\]

9) Use the calculator to solve graphically.

\[
\begin{align*}
x - 3y &= 9 \\
-2x + 6y &= -8
\end{align*}
\]
10) Use substitution to solve the system.
5x + y = 18
y = 4x

11) Use substitution to solve the system.
x = 5y - 4
x + 5y = 8

12) Use substitution to solve the system.
2x - 5y = 9
4x + y = -15

13) Use elimination to solve the system.
x + 6y = 51
-x + 3y = 12

14) Use elimination to solve the system.
3x - 4y = 1
9x + 8y = 8

15) Use elimination to solve the system.
2x + 3y = 2
4x + 6y = 4
Section 3.3b
Systems of Two Linear Equations Applications

Learning Outcomes:
• Set up and solve systems of linear equations in real-world applications. Interpret the answer.

Example 1: Suppose Bob takes a trip in his car. He drives an average speed of 60 mph for part of the trip, and he drives an average speed of 50 mph for the rest of the trip. His trip takes a total of 5.9 hours, and he goes a total distance of 337 miles. We want to determine the amount of time he drove at each speed.

In order to determine the amount of time he traveled at each speed, we can set up a system of linear equations and solve the system. In the end, we want the solution of the system to be a POINT that will tell us the amount of time traveled at each speed. Therefore, we will define the variable X as the number of hours spent driving at 60mph, and the variable Y as the number of hours spent driving at 50mph. In the end, we will have a point (X, Y) that tells us X: the hours spent travelling at 60 mph, and Y: the hours spent travelling at 50 mph.

In this problem, we have been given two separate "types" of information of the relationship between X and Y. We've been told some information about the total miles traveled in relation to X and Y, and we've also been told some information about the total hours traveled in relation to X and Y. We will use this information to create two separate equations that both contain X and Y:

NOTE: To write the equations, it is often helpful to first write the equations in WORDS.

Total Distance Traveled Relationship between X and Y:

\[(\text{total miles traveled at a speed of 60mph}) + (\text{total miles traveled at a speed of 50mph}) = (337\text{miles})\]

\[\left(\frac{60 \text{ miles}}{1 \text{ hour}}\right)(X \text{ hours}) + \left(\frac{50 \text{ miles}}{1 \text{ hour}}\right)(Y \text{ hours}) = (337\text{miles})\]

and so the equation is \(60X + 50Y = 337\)

Total Time Traveled Relationship between X and Y:

\[(\text{total hours traveling at a speed of 60 mph}) + (\text{total hours traveling at a speed of 50 mph}) = (5.9 \text{ hours})\]

\[(X \text{ hours}) + (Y \text{ hours}) = (5.9\text{hours})\]

and so the equation is \(X + Y = 5.9\)

And so the system of equations would be

\[\begin{cases} 
60X + 50Y = 337 \\
X + Y = 5.9 
\end{cases}\]

Notice that in this example the X-variable represents the same thing (hours spent traveling at 50 mph) in both equations. And the Y-variable represents the same thing (hours spent traveling at 50 mph) in both equations. We could graph both equations on the same set of axis because they are both relationships between the same variables.

Notice also that if a point happens to land on BOTH EQUATIONS at the same time, then that point would satisfy the distance traveled relationship, and it would also satisfy the time traveled relationship. Any point that lies on both equations would satisfy both conditions simultaneously.
We can solve the system of equations using either substitution, elimination, or graphically. For this example, we will solve the system graphically.

The system of equations is
\[
\begin{align*}
60X + 50Y &= 337 \\
X + Y &= 5.9
\end{align*}
\]

To solve graphically with the calculator, we first need to re-write equation so that they are solved for \( y \).

\[
\begin{align*}
60X + 50Y &= 337 \\
50Y &= 337 - 60X \\
Y &= \frac{337 - 60X}{50} \\
X + Y &= 5.9 \\
Y &= 5.9 - X
\end{align*}
\]

Now we can put the equations into Y1 and Y2 in the calculator. Next we need to set a viewing window that allows us to see the intersection of the two graphs. Since this is a contextual problem, we should create the viewing window by considering the real-world meaning of the variables. We should only view \( X \) and \( Y \) values that make sense in real-world terms.

**NOTE: To find a viewing window you need to stop and think about the MEANING of the variables. What would be the lowest value that makes sense, and what would be the highest value that makes sense in REALITY?**

\( X \) represents the total hours spent travelling at 60 mph. So, the smallest \( X \) could be is 0 hours travelling at that speed. The total trip only lasted 5.9 hours. So, the largest \( X \) could be is 5.9 hours travelling at that speed. Therefore

\[
X_{\text{Min}} = 0 \quad \text{XMax} = 5.9
\]

\( Y \) represents the total hours spent travelling at 50 mph. So, the smallest \( Y \) could be is 0 hours travelling at that speed. The total trip only lasted 5.9 hours. So, the largest \( Y \) could be is 5.9 hours travelling at that speed. Therefore

\[
Y_{\text{Min}} = 0 \quad \text{YMax} = 5.9
\]

In the graph we have created, the horizontal axis represents the number of hours spent travelling at 60 mph, and the vertical axis represents the number of hours spent travelling at 50 mph.

All points on the graph of Y1 have the meaning \((X, Y)\): (hours travelling at 60 mph, hours travelling at 50 mph) where the points each satisfy the hours necessary in order to travel a total of 337 miles on the trip.

All points on the graph of Y2 have the meaning \((X, Y)\): (hours travelling at 60 mph, hours travelling at 50 mph) where the points each satisfy the hours necessary in order to travel a total of 5.9 hours for the trip.

The point \((4.2, 1.7)\) is the solution to the system of equations. It is the one point that is on BOTH graphs. When the car travels 4.2 hours at 60 mph and travels 1.7 hours at 50 mph, then the car will travel exactly 337 miles total, and the trip will take a total of 5.9 hours.
**Example 2**: Below is the nutritional information from www.nutritiondata.self.com for a McDonald’s Big Mac and also for a glass of Skim Milk.

<table>
<thead>
<tr>
<th><strong>Big Mac</strong></th>
<th><strong>Skim Milk</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nutrition Facts</strong></td>
<td><strong>Nutrition Facts</strong></td>
</tr>
<tr>
<td>Serving Size: 210 g</td>
<td>Serving Size: 217 g</td>
</tr>
<tr>
<td><strong>Amount Per Serving</strong></td>
<td><strong>Amount Per Serving</strong></td>
</tr>
<tr>
<td>Calories: 590</td>
<td>Calories: 86</td>
</tr>
<tr>
<td>% Daily Value</td>
<td>% Daily Value</td>
</tr>
<tr>
<td>Total Fat: 11 g</td>
<td>Total Fat: 0 g</td>
</tr>
<tr>
<td>Saturated Fat: 2 g</td>
<td>Saturated Fat: 0 g</td>
</tr>
<tr>
<td>Trans Fat: 0 g</td>
<td>Trans Fat: 0 g</td>
</tr>
<tr>
<td>Cholesterol: 55 mg</td>
<td>Cholesterol: 0 mg</td>
</tr>
<tr>
<td>Sodium: 520 mg</td>
<td>Sodium: 2 mg</td>
</tr>
<tr>
<td>Total Carbohydrate: 42 g</td>
<td>Total Carbohydrate: 1 g</td>
</tr>
<tr>
<td>Dietary Fiber: 3 g</td>
<td>Dietary Fiber: 0 g</td>
</tr>
<tr>
<td>Sugars: 6 g</td>
<td>Sugars: 0 g</td>
</tr>
<tr>
<td>Protein: 34 g</td>
<td>Protein: 5 g</td>
</tr>
<tr>
<td>Vitamin A: 6%</td>
<td>Vitamin A: 29%</td>
</tr>
<tr>
<td>Vitamin C: 0%</td>
<td>Vitamin C: 4%</td>
</tr>
<tr>
<td>Calcium: 3%</td>
<td>Calcium: 5%</td>
</tr>
<tr>
<td>Iron: 2%</td>
<td>Iron: 1%</td>
</tr>
<tr>
<td><em>Percent Daily Values are based on a 2000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.</em></td>
<td><em>Percent Daily Values are based on a 2000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.</em></td>
</tr>
</tbody>
</table>

(a) Write an equation that relates the total servings of Big Macs, $B$, and total servings of Skim Milk, $S$, that she can drink in order to exactly meet her daily CALORIE requirements.

**Solution**: Since $B$ represents the servings of Big Macs, and $S$ represents the servings of Skim Milk, we can say that

\[
(\text{calories from Big Macs}) + (\text{Calories from Skim Milk}) = (\text{Total Calories})
\]

\[
\left(\frac{590 \text{ calories}}{1 \text{ Big Mac}}\right)(B) + \left(\frac{86 \text{ calories}}{1 \text{ Skim Milk}}\right)(S) = 1700 \text{ total calories}
\]

And so the equation can be written as $590B + 86S = 1700$.

(b) Write an equation that relates the total servings of Big Macs, $B$, and total servings of Skim Milk, $M$, that she can drink in order to exactly meet her daily protein requirements.

**Solution**: Since $B$ represents the servings of Big Macs, and $S$ represents the servings of Skim Milk, we can say that

\[
(\text{protein from Big Macs}) + (\text{protein from Skim Milk}) = (\text{Total protein})
\]

\[
\left(\frac{24 \text{ g protein}}{1 \text{ Big Mac}}\right)(B) + \left(\frac{8 \text{ g protein}}{1 \text{ Skim Milk}}\right)(S) = 45 \text{ g protein}
\]

And so the equation can be written as $24B + 8S = 45$.

(c) How many servings of Big Mac, and how much Skim Milk can Marcia consume in order to exactly meet BOTH her calorie requirements and her fat requirements for the day?

**Solution**: Since we have two equations that give relationships between the same two variables, we can now say that we have a system of equations. The system of equations is

\[
\begin{align*}
590B + 86S &= 1700 \\
24B + 8S &= 45
\end{align*}
\]

Where $B$ represents the total servings of Big Macs that Marcia eats, and $S$ represents the total servings of Skim Milk that Marcia drinks.

- Points on the first equation will tell us how many Big Macs, and how much Skim Milk that Marcia can consume in order to exactly meet her calorie requirements.
Points on the second equation will tell us how many Big Macs, and how much Skim Milk that Marcia can consume in order to exactly meet her protein requirements.

Points on BOTH equations will tell us how many Big Macs and how much Skim Milk Marcia needs to consume in order to meet BOTH her calorie and her protein requirements.

In order to find point(s) that are on both equations, we need to solve the system of equations. For this problem, we will solve graphically. We will call B the dependent variable (to be graphed on the vertical axis), and we will call S the independent variable (to be graphed on the horizontal axis). It doesn’t matter if you do it the other-way-around…as long as you pick one to be dependent and then stick with it.

Since B is the dependent variable, we will solve both equations for B:

\[ 590B + 86S = 1700 \]
\[ 24B + 8S = 45 \]

Subtract 86S on both sides
Divide both sides by 590
\[ B = (-86S + 1700)/590 \]

Subtract 8S on both sides
Divide both sides by 34
\[ B = (-8S + 45)/24 \]

This is the equation we can enter into Y1.
This is the equation we can enter into Y2.

We set a viewing window that makes sense in the problem:

X represents the number of glasses of milk she consumes, and Y represents the number of Big Macs she eats. So, X and Y must be positive. And, X and Y probably can’t get very high since no one can eat a huge number of Big Macs or drink a huge number of glasses of milk. We will try to max it out at 5 Big Macs and 5 glasses of milk.

We see that these two graphs are not going to intersect in the first quadrant (which is the only quadrant that makes sense to consider since X and Y need to be positive in this situation). In fact, it looks like they are going to intersect somewhere in quadrant II.

If we change the viewing window and find the point of intersection, we see that the two lines intersect at about (-5.4, 3.7).

This would literally mean that she needs to eat NEGATIVE 5.4 glasses of milk, and also eat 3.7 Big Macs in order to meet both her calorie and protein requirements. That makes no sense!!!

So, even though this system of equations has a solution in the “land of theoretical math,” we would ultimately conclude that it is not possible for Marcia to only eat Big Macs and drink skim milk in order to meet both her calorie and protein requirements.
It’s just not possible!
Example 3: Bob has a total of $55,300 that he wants to invest. Bob puts some of the money in Bank X that earns 2.8% interest each year. Bob puts the rest of the money in Bank Y that earns 3.4% interest each year. At the end of the year, Bob has earned a total of $1,750.60 in interest from the two accounts all together. Let \( X \) represent the amount of money Bob invested in bank X, and let \( Y \) represent the amount of money Bob invested in Bank Y. Write a system of equations, and then solve the system in order to determine the amount that Bob invested in each bank.

Solution: We start by writing two equations that both involve the variables \( X \) and \( Y \).

\[
\begin{align*}
(amount \text{ invested in Bank } X) \, + \, (amount \text{ invested in Bank } Y) &= \text{ $55,300 total} \\
X + Y &= 55300
\end{align*}
\]

\[
\begin{align*}
(interest \text{ earned from Bank } X) \, + \, (interest \text{ earned from Bank } Y) &= \text{ $1,750.60 total} \\
(2.8\% \text{ of money invested}) \, + \, (3.4\% \text{ of money invested}) &= \text{ $1,750.60 total} \\
0.028X + 0.034Y &= 1750.60
\end{align*}
\]

And so the system of equation is

\[
\begin{align*}
X + Y &= 55300 \\
0.028X + 0.034Y &= 1750.60
\end{align*}
\]

Now we will solve the system of equations graphically. In order to enter the equations in the calculator, we first need to solve both equations for \( Y \).

\[
\begin{align*}
X + Y &= 55300 \\
Y &= 55300 - X
\end{align*}
\]

\[
\begin{align*}
0.028X + 0.034Y &= 1750.60 \\
0.034Y &= 1750.60 - 0.028X \\
Y &= \frac{1750.60 - 0.028X}{0.034}
\end{align*}
\]

Now we enter the functions in the calculator in Y1 and Y2 and set a viewing window so that we can see the point of intersection.

To create the viewing window, we think about the meaning of each variable. \( X \) represents the amount of money invested in Bank X. The smallest amount would be $0. The largest amount would be $55,300 since that is the total amount invested in the two banks all together. The same logic holds for \( Y \).

So we set XMin = 0, XMax = 55300, and we set YMin = 0 and YMax = 55300.

We find the point of intersection is \((21600, 33700)\). This tells us that we invested $21,600 in Bank X, and we invested $33,700 in bank Y. Doing so results in a total investment of $55,300 in the banks, and results in total interest of $1750.60 earned.
Section 3.3b Written Practice and Reflection
Systems of Two Linear Equations Applications

1) I invest a total of $2400 into bank A and bank B. Bank A gives 5% interest over the year, and bank B gives 6% interest over the year. I get a total of $126 in interest at the end of the year. Let A represent the amount I invest in bank A and let B represent the amount I invest in bank B.

(a) Write a system of equations that relates the amount I invest in bank A and the amount I invest in bank B.
(b) Solve your system of equations from part (a). Show your work and/or explain calculator steps.
(c) How much do I invest in bank A? How much do I invest in bank B?

2) I invest a total of $15,000 into two banks. Bank A gives me 3% interest over the course of one year, and bank B gives me 4% interest over the course of one year. At the end of the year I have earned a total of $585 in interest. Let A stand for the amount of money I invested in bank A and let B stand for the amount of money I invested in bank B.

(a) Write a system of equations that relates the amount of money I invested in bank A and the amount of money I invested in bank B.
(b) Solve the system of equations. Show your work and/or explain any calculator steps.
(c) How much money did I invest in bank A and how much money did I invest in bank B?

3) I’m on a special diet where I only eat two types of food: food X and food Y. Each serving of food X contains 9 g of protein and 350 mg of potassium. Each serving of food Y contains 3.5 g of protein and 400 mg of potassium. I am supposed to eat exactly 30 g of protein and 2000 mg of potassium every day. Let X represent the number of servings of food X that I eat in a day, and let Y represent the number of servings of food Y that I eat in a day.

(a) Write a system of equations that relates the total number of servings of food X and the total number of servings of food Y I can eat in one day.
(b) Solve the system of equations. Show your work and/or explain any calculator steps.
(c) How many servings of each food should I eat to exactly meet my dietary requirements?

4) Bob needs 2200 calories each day, and he needs 60 grams of fat each day. Bob wants to eat only food A and food B. Food A contains 140 calories in each serving, and contains 35 grams of fat in each serving. Food B contains 305 calories in each serving, and contains 3 grams of fat in each serving.

(a) Write a system of equations that relates the total number of servings of food X and the total number of servings of food Y I can eat in one day.
(b) Solve the system of equations. Show your work and/or explain any calculator steps.
(c) How many servings of each food should I eat to exactly meet my dietary requirements?

5) Tina drives 60mph straight north for X hours, and drives 45mph straight north for Y hours. She drives for a total of 22 hours, and her total distance traveled is 1186.5 miles.

(a) Write a system of equations that relates the number of hours she travels at 60 mph to the number of hours she travels at 45mph.
(b) Solve the system of equations. Show your work and/or explain any calculator steps.
(c) How long did she travel at 60mph, and how long at 45mph?

6) I car travels at 30 mph for X hours, and drives 40 mph for Y hours. The trip takes a total of 10.8 minutes (which is 0.18 hours). The car travels a total of 6.9 miles.

(a) Write a system of equations that relates the number of hours she travels at 30 mph to the number of hours she travels at 40mph.
(b) Solve the system of equations. Show your work and/or explain any calculator steps.
(c) How long did the car travel at 30 mph, and how long at 40 mph?
Section 3.4

Introduction to rational expressions

Learning Outcomes:
- Identify the domain of rational expressions
- Re-write rational expressions with the numerator and denominator in factored form.
- Simplify rational expressions.
- Multiply, divide, add, subtraction fractions by hand (with no calculator).
- Multiply, add, subtract rational expressions.

A rational expression is a fraction where the numerator and denominator are both polynomials, and the denominator cannot be zero since that would make the rational expression undefined.

\[
\text{rational expression} = \frac{\text{polynomial}}{\text{polynomial}}
\]

When working with rational expressions, we must always carefully keep track of the domain of the expression in order to always understand the input-values that are acceptable to use and not use.

Domain of a rational expression

The domain of a rational expression is all input values that produce a non-zero denominator in the rational expression.

Important note about the domain restrictions

The simplified expression needs to be EQUIVALENT to the original expression. Therefore the domain of the simplified expression must be the same as the domain of the original expression. If you neglect to include the domain restrictions when writing your simplified expression, then you have not necessarily identified a simplified expression that is equivalent to the original expression! Don't forget to include the original domain restrictions with your expression even if the factors in the original expression cancel.

Simplifying Rational Expressions

To simplify a fraction means that we have cancelled all common factors greater than 1 in the numerator and denominator. To simplify a fraction (including a rational expression), you should follow these steps:

1) Factor both the numerator and denominator completely.

2) Identify the domain BEFORE cancelling any common factors. Make sure you keep track of the domain restrictions, as they will continue to be the domain restrictions even if factors cancel in the expression.

3) Cancel all common factors from the numerator and denominator. Re-state the original domain! Even if factors have cancelled, the original domain restrictions are still in effect!
Example 1: Identify the domain of the rational expression \( \frac{(30x-5)(x+19)}{(x-8)(x+1)} \)

Solution: The domain of this rational expression is all x-values that make the denominator non-zero. To find the domain, we need to find the x-value(s) that make the denominator zero and then exclude those values from the domain.

\( (x - 8)(x + 1) = 0 \)
\( x - 8 = 0 \) and \( x + 1 = 0 \) using the zero – product property

\( x = 8 \) and \( x = -1 \)

This tells us that when \( x = -1 \) and when \( x = 8 \), this rational expression will be undefined. But, every other x-value would produce real-valued, non-zero denominator values.

So, we can conclude that the domain of this rational expression is all real numbers except \(-1 \) and \( 8 \).
This is usually written as \( x \neq -1 \) and \( x \neq 8 \).

Example 2: Identify the domain of the rational expression \( \frac{x-9}{3x^3+36x^2+96x} \)

Solution: The domain of this rational expression is all x-values that make the denominator non-zero. To find the domain, we need to find the x-value(s) that make the denominator zero and then exclude those values from the domain.

\( 3x(x^2 + 12x + 32) = 0 \)
\( 3x(x + 8)(x + 4) = 0 \)
\( 3x = 0 \) and \( x + 8 = 0 \) and \( x + 4 = 0 \) using the zero – product property
\( x = 0 \) and \( x = -8 \) and \( x = -4 \)

This tells us that when \( x = 0 \) and when \( x = -8 \) and when \( x = -4 \), this rational expression will be undefined. But, every other x-value would produce real-valued, non-zero denominator values.

So, we can conclude that the domain of this rational expression is all real numbers except \( 0 \) and \( -8 \) and \( -4 \).
This is usually written as \( x \neq 0 \) and \( x \neq -8 \) and \( x \neq -4 \).
Example 3: Simplify the rational expression \( \frac{x^2 + 11x + 28}{(x + 7)(x - 3)} \).

Solution: We first need to factor the numerator and denominator. The numerator factors as \( x^2 + 11x + 28 = (x + 7)(x + 4) \). The denominator is already factored. Now we can rewrite the rational expression with the numerator and denominator in factored form as follows:

\[
\frac{x^2 + 11x + 28}{(x + 7)(x - 3)} = \frac{(x + 7)(x + 4)}{(x + 7)(x - 3)}
\]

Next we identify the domain of the rational expression: the domain is \( x \neq -7 \) and \( x \neq 3 \) because those are the x-values that make the denominator equal zero.

Finally, we cancel all common factors in the expression. Note how we can easily keep track of the original domain restrictions by continually writing them at each stage of the simplification process:

\[
\frac{x^2 + 11x + 28}{(x + 7)(x - 3)} = \frac{(x + 7)(x + 4)}{(x + 7)(x - 3)} = \frac{(x + 7)}{(x - 3)} \quad x \neq -7 \text{ and } x \neq 3
\]

And so the rational expression, in simplified form, is \( \frac{x + 4}{x - 3} \) where \( x \neq -7 \) and \( x \neq 3 \).

Example 4: Simplify the rational expression \( \frac{6x^2 - 17x - 14}{2x^2 - 5x - 7} \).

Solution: We first need to factor the numerator and denominator. The numerator factors as \( 6x^2 - 17x - 14 = (2x - 7)(3x + 2) \). The denominator factors as \( 2x^2 - 5x - 7 = (x + 1)(2x - 7) \). Now we can rewrite the rational expression with the numerator and denominator in factored form as follows:

\[
\frac{6x^2 - 17x - 14}{2x^2 - 5x - 7} = \frac{(2x - 7)(3x + 2)}{(x + 1)(2x - 7)}
\]

Next we identify the domain of the rational expression: the domain is \( x \neq -1 \) and \( x \neq 7/2 \) because those are the x-values that make the denominator equal zero.

Finally, we cancel all common factors in the expression. Note how we can easily keep track of the original domain restrictions by continually writing them at each stage of the simplification process:

\[
\frac{6x^2 - 17x - 14}{2x^2 - 5x - 7} = \frac{(2x - 7)(3x + 2)}{(x + 1)(2x - 7)} = \frac{(3x + 2)}{(x + 1)} \quad x \neq -1 \text{ and } x \neq 7/2
\]

And so the rational expression, in simplified form, is \( \frac{3x + 2}{x + 1} \) where \( x \neq -1 \) and \( x \neq 7/2 \).
Multiplying numerical fractions

To multiply the fractions \( \frac{a}{b} \times \frac{c}{d} \), we multiply straight across the numerator, and multiply straight across the denominator. Remember that it is easiest if you FIRST cancel common factors from the numerator and denominator.

\[
\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}
\]

Example 5: Calculate the value of \( \frac{7}{20} \times \frac{15}{28} \) without using a calculator.

Solution: To multiply the fractions, we multiply straight across the numerator, and straight across the denominator.

\[
\frac{7}{20} \times \frac{15}{28} = \frac{7 \times 15}{20 \times 28}
\]

Remember that we can cancel common factors in the numerator and denominator. So, below, the numbers have been written in factored form so that we can easily see the factors which cancel.

\[
\frac{7}{20} \times \frac{15}{28} = \frac{(7)(5 \times 3)}{(5 \times 4)(7 \times 4)} = \frac{(2)(4 \times 3)}{(4 \times 4)} = \frac{3}{16}
\]

Multiplying Rational Expressions

Multiplying rational expressions is done with a very similar procedure used to multiply numerical fractions. To multiply rational expressions you should follow these steps:

1) For each expression, factor the numerator completely, and factor the denominator completely.

2) Identify the domain of the original expressions. The combined restrictions will be the overall domain restrictions. Make sure you keep track of the domain restrictions, as they will continue to be the domain restrictions even if factors cancel in the expression.

3) Multiply straight across the numerator, and multiply straight across the denominator in order to create one fraction. Do NOT expand the expressions. Leave everything in factored form! Re-state the original domain.

4) Cancel all common factors from the numerator and denominator. Re-state the original domain! Even if factors have cancelled, the original domain restrictions are still in effect!

Example 6: Calculate and completely simplify \( \frac{3x - 15}{(x + 4)(x + 2)} \times \frac{x^2 + 10x + 16}{7x - 25} \). Be sure to identify the domain!

Solution: To multiply, we first rewrite each rational expression with a factored numerator and a factored denominator, and identify the domain of the original expression.

\[
\frac{3x - 15}{(x + 4)(x + 2)} \times \frac{x^2 + 10x + 16}{7x - 25} = \frac{3(x - 5)(x + 2)}{(x + 4)(7)(x - 5)} \text{ domain: } x \neq -4, -2, 5
\]

Now we multiply straight across the numerator, and straight across the denominator:

\[
= \frac{3(x - 5)(x + 8)(x + 2)}{(x + 4)(x + 2)(7)(x - 5)} \quad x \neq -4, -2, 5
\]

Now we cancel the common factors and keep the original domain restrictions:

\[
= \frac{3(x - 5)(x + 8)}{(x + 4)(7)(x - 5)} = \frac{3(x + 8)}{(x + 4)(7)} \quad x \neq -4, -2, 5
\]

And so the simplified answer is \( \frac{3(x + 8)}{7(x + 4)} \) where \( x \neq -4, -2, 5 \).
To add or subtract fractions, we first need to rewrite the fractions with a common denominator. We then add/subtract the numerators and keep the common denominator.

**Example 7:** Calculate and completely simplify \( \frac{1}{2} + \frac{2}{5} \).

**Solution:** We first rewrite both fractions using a common denominator of 10. We selected 10 since it is a common multiple of the denominators 2 and 5. We now rewrite each fraction as an equivalent fraction that has a denominator of 10.

So, in this case, we will multiply the fraction \( \frac{1}{2} \) by \( \frac{5}{5} \) in order to make the denominator of the equivalent fraction 10.

\[
\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}
\]

We will multiply the fraction \( \frac{2}{5} \) by \( \frac{2}{2} \) in order to make the denominator of the equivalent fraction 10.

\[
\frac{2}{5} \times \frac{2}{2} = \frac{4}{10}
\]

Now we can add the fractions because they have the same denominator.

\[
\frac{5}{10} + \frac{4}{10} = \frac{9}{10}
\]

And so \( \frac{1}{2} + \frac{2}{5} = \frac{9}{10} \).

**Adding Rational Expressions**

Adding/subtracting rational expressions is done with a very similar procedure used to add/subtract numerical fractions. To add or subtract rational expressions you should follow these steps:

1) For each expression, factor the numerator completely, and factor the denominator completely.

2) Identify the domain of the original expressions. The combined domain restrictions will be the overall domain restrictions. Make sure you keep track of the domain restrictions, as they will continue to be the domain restrictions even if factors cancel in the expression.

3) Find the common denominator for the expressions, and rewrite both expressions using that common denominator. This common denominator must be a common factor of both denominators. Leave everything in factored form! Re-state the original domain.

4) Add across the numerators and keep the common denominator. Re-state the original domain.

5) Cancel all common factors from the numerator and denominator. Re-state the original domain! Even if factors have cancelled, the original domain restrictions are still in effect!
**Example 8:** Calculate and completely simplify \( \frac{8}{x} + \frac{3}{5x} \). Identify the domain.

**Solution:** There is no factoring to be completed in these expressions, and so we can immediately proceed to finding the domain. We see that the domain is \( x \neq 0 \) since that is the only \( x \)-value that would make the denominators zero.

The common denominator is \( 5x \) because \( 5x \) is a common multiple of both \( x \) and \( 5x \). We now need to rewrite both rational expressions with \( 5x \) as the denominator.

In order to rewrite the expression \( \frac{8}{x} \) with a denominator of \( 5x \) we need to multiply \( \frac{8}{x} \) by \( \frac{5}{5} \).

\[
\frac{8}{x} \times \frac{5}{5} = \frac{40}{5x}
\]

The expression \( \frac{3}{5x} \) is already written using the common denominator.

Now that we can write the rational expressions with the same denominator, we can add across the numerator, and keep the denominator:

\[
\frac{8}{x} + \frac{3}{5x} = \frac{40}{5x} + \frac{3}{5x} = \frac{43}{5x}
\]

And so the answer is \( \frac{43}{5x} \) where \( x \neq 0 \).
Example 9: Calculate and completely simplify \( \frac{x}{x+5} + \frac{4}{9x} \). Identify the domain.

Solution: There is no factoring to be completed in these expressions, and so we can immediately proceed to finding the domain. We see that the domain is \( x \neq -5 \) and \( x \neq 0 \) since those are the x-values that would make the denominators zero.

The common denominator is \((9x)(x + 5)\) because \((9x)(x + 5)\) is a common multiple of both denominators. We now need to rewrite both rational expressions with \((9x)(x + 5)\) as the denominator.

In order to rewrite the expression \( \frac{x}{x+5} \) with a denominator of \((9x)(x + 5)\) we need to multiply by \(\frac{(9x)}{(9x)}\):

\[
\frac{x}{(x+5)}\times\frac{(9x)}{(9x)} = \frac{9x^2}{(9x)(x+5)}
\]

In order to rewrite the expression \( \frac{4}{9x} \) with a denominator of \((9x)(x + 5)\) we need to multiply by \(\frac{(x+5)}{(x+5)}\):

\[
\frac{4}{9x}\times\frac{(x+5)}{(x+5)} = \frac{4(x+5)}{(9x)(x+5)}
\]

Now that we can write the rational expressions with the same denominator, we can add across the numerator, and keep the denominator:

\[
\frac{x}{x+5} + \frac{4}{9x} \quad x \neq 0, -5
\]

\[
= \frac{9x^2}{9x(x+5)} + \frac{4(x+5)}{9x(x+5)} \quad x \neq 0, -5
\]

\[
= \frac{9x^2 + 4(x+5)}{9x(x+5)} \quad x \neq 0, -5
\]

We can now simplify the expression by multiplying the numerator:

\[
\frac{9x^2 + 4x + 20}{9x(x+5)} \quad x \neq 0, -5
\]

Since the numerator is not factorable, we cannot simplify this expression and further.

And so the answer is \( \frac{9x^2 + 4x + 20}{9x(x+5)} \) where \( x \neq 0, -5 \).
Section 3.4 Written Practice
Introduction to Rational Expressions

1) For each rational expression, completely factor the numerator and denominator, and identify the domain of the expression.

   a) \[ \frac{28x^2 + 12x}{x - 9} \]

   b) \[ \frac{x^2 + x - 20}{3(x + 4)} \]

   c) \[ \frac{2x^2 - 5x - 12}{5x^2 + 2x - 16} \]

   d) \[ \frac{(3x + 2)(2x + 1)}{(3x - 2)(2x - 1)} \]

   e) \[ \frac{12x^3 - 14x^2 - 10x}{3x^2 - 30x + 48} \]

2) For each rational expression, completely factor the numerator and denominator, and identify the domain of the expression.

   a) \[ \frac{7x + 15}{x^2 - 16} \]

   b) \[ \frac{x^2 - 10x + 9}{3x^2 - 22x - 24} \]

   c) \[ \frac{4x^2 - 25x - 21}{3x + 1} \]

   d) \[ \frac{4x^2 - 3x - 10}{4x^2 + 3x - 10} \]

   e) \[ \frac{18x^3 + 33x^2 + 9x}{x^2 - 6x + 8} \]
3) For each rational expression, perform the following steps in order:
   1. Completely factor the numerators and denominators.
   2. Identify the domain of the expression overall.
   3. Multiply the expressions and identify the domain overall.
   4. Simplify the expression and identify the domain overall.

   a) \[
   \frac{3(x-5)}{x+2} \cdot \frac{5(x+2)}{x-1}
   \]

   b) \[
   \frac{x^2+6x-16}{x-3} \cdot \frac{x^2+5x-24}{5x+2}
   \]

   c) \[
   \frac{x^2-49}{x-1} \cdot \frac{x^2-1}{2x-14}
   \]

   d) \[
   \frac{10x^2-3x-1}{(2x-1)(x+4)} \cdot \frac{x+4}{10x+2}
   \]

   e) \[
   \frac{x+1}{8x^2+10x-3} \cdot \frac{8x^2-14x+3}{x-3}
   \]

4) For each rational expression, perform the following steps in order:
   1. Completely factor the numerators and denominators.
   2. Identify the domain of the expression overall.
   3. Multiply the expressions and identify the domain overall.
   4. Simplify the expression and identify the domain overall.

   a) \[
   \frac{6(x-1)}{x-4} \cdot \frac{3x-12}{12x+6}
   \]

   b) \[
   \frac{x^2+7x+12}{7x+12} \cdot \frac{x^2-9}{(x+3)(x-8)}
   \]

   c) \[
   \frac{8x^2+2x-1}{6x+5} \cdot \frac{7x-3}{4x-1}
   \]

   d) \[
   \frac{x-2}{x+8} \cdot \frac{x^2-64}{6x^2-11x-2}
   \]

   e) \[
   \frac{x+3}{8x^2+10x+3} \cdot \frac{8x^2-10x+3}{x-9}
   \]
5) For each rational expression, perform the following steps in order:
   1. Completely factor the numerators and denominators.
   2. Identify the domain of the expression overall.
   3. Add/subtract the expressions and identify the domain overall.
   4. Simplify the expression and identify the domain overall.

   a) \( \frac{7}{x} + \frac{5}{13x} \)

   b) \( \frac{8x}{x+3} + \frac{x-1}{x+2} \)

   c) \( \frac{7}{x-4} - \frac{5x}{x+2} \)

   d) \( \frac{x-3}{(x-1)(x+5)} + \frac{5}{x-2} \)

   e) \( \frac{x+2}{x^2+7x+6} + \frac{5}{(x-1)} \)

6) For each rational expression, perform the following steps in order:
   1. Completely factor the numerators and denominators.
   2. Identify the domain of the expression overall.
   3. Add/subtract the expressions and identify the domain overall.
   4. Simplify the expression and identify the domain overall.

   a) \( \frac{2}{5x} + \frac{3}{x} \)

   b) \( \frac{x}{x+7} + \frac{x-4}{x+1} \)

   c) \( \frac{5}{x-2} - \frac{x+2}{x-9} \)

   d) \( \frac{x-4}{x+7} + \frac{2x}{(x-1)(x+3)} \)

   e) \( \frac{2}{(x-3)} - \frac{x+1}{x^2+10x+16} \)
Section 3.5a
Linear Inequalities Symbolically

Learning Outcomes:
- Understand what a linear inequality is, and what the solution to a linear inequality is
- Solve linear inequalities by hand

An algebraic equation is two algebraic expressions that are equal to one another. Equations have an equal sign, and we can solve equations using the addition rule (adding and subtracting the same quantity on both sides of the equation) and the multiplication rule (multiplying and dividing the same non-zero quantity on both sides of the equations).

Algebraic inequalities are very similar to algebraic equations. Algebraic inequalities have an inequality symbol rather than an equal sign. Algebraic inequalities can have the following symbols:
- \(<\) which means “less than”
- \(\geq\) which means “greater than or equal to”

The solution to an algebraic inequality is a set of values that satisfy the inequality.

Example 1: The solution to the inequality \(x + 1 \geq 8\) is all of the x-values that satisfy the inequality. Some examples of x-values that satisfy the inequality are x = 9, x = 50.95, x = 7.3, and x = 7. Notice that each of these x-values make the inequality true, and therefore they are each solutions:

\[(9) + 1 \geq 8\] and so x=9 is a solution to the inequality

\[(50.95) + 1 \geq 8\] and so x=50.95 is a solution to the inequality

\[(7.3) + 1 \geq 8\] and so x=7.3 is a solution to the inequality

\[(7) + 1 \geq 8\] and so x=7 is a solution to the inequality. Notice that 7+1 is equal to 8…so it is true to say that 7+1 is “greater than or equal to 8”

So we have listed 4 different solutions to this inequality. But notice that there are an infinite set of x-values that we could pick that would be solutions to this inequality! Every number that is bigger than or equal to x=7 would be a solution to this inequality! So the SOLUTION SET is the values \(x \geq 7\).

How to Solve a Linear Inequality Symbolically (by hand)

We solve inequalities symbolically just like solving linear equations by hand with only one exception. The only difference is the following: If you multiply or divide both sides by a negative number, then you need to reverse the inequality. Otherwise, linear inequalities are solved using exactly the same process as solving linear equations.

Notice that this rule makes sense because the solution set for \(-1x > 0\) is all x-values that are negative. So, when we divide both sides by \(-1\) we need to get a solution set \(x < 0\). Therefore, it makes logical sense to switch the inequality when we divide by a negative.
Example 2: Solve the inequality $5x - (3 + 2x) + 1 \leq -7x + 1x - 5(2 + 4x)$ SYMBOLICALLY. Show all work.

egin{align*}
5x - 3 - 2x + 1 & \leq -7x + 1x - 10 - 20x \\
3x - 2 & \leq -26x - 10 \\
3x - 2 + 26x & \leq -26x - 10 + 26x \\
29x - 2 & \leq -26x - 10 + 26x \\
29x - 2 + 2 & \leq -10 + 2 \\
29x - 2 + 2 & \leq -8 \\
29x & \leq -8 \\
\frac{29x}{29} & \leq \frac{-8}{29} \\
x & \leq -\frac{8}{29}
\end{align*}

So the solution to the inequality is the set of real numbers less than or equal to $-\frac{8}{29}$. The solution is written $x \leq -\frac{8}{29}$. Notice that we solved the inequality using the same steps that we would use to solve an equation. The only difference was that we wrote the inequality symbol $\leq$ rather than writing an equal sign.

In the next example we will see a very similar process with only one exception. When we divide both sides by a negative number, we reverse the inequality.

Example 3: Solve the inequality $-5(x + 3) < 7$ SYMBOLICALLY. Show all work.

egin{align*}
-5x - 15 & < 7 \\
-5x - 15 + 15 & < 7 + 15 \\
-5x & < 22 \\
\frac{-5x}{-5} & > \frac{22}{-5} \\
x & > -\frac{22}{5}
\end{align*}

So the solution to the inequality is the set of real numbers greater than $-\frac{22}{5}$. The solution is to the inequality is written $x > -\frac{22}{5}$. 
Section 3.5a Written Practice and Reflection

Inequalities Symbolically

1) Solve the inequality symbolically. \(3x + \frac{4}{3} > \frac{2}{5}x - 3\)

2) Solve the inequality symbolically. \(\frac{6}{5}(2x - 3) < 5 - 2x\)

3) Solve the inequality symbolically. \(\frac{1}{2}(x - 5) < \frac{1}{3}(2x - 1)\)

4) Solve the inequality symbolically. \(3 - 4x \leq \frac{-2}{3}\)

5) Solve the inequality symbolically. \(2 - (3x - 5) > 5(4 - 8x)\)

6) Solve the inequality symbolically. \(\frac{2}{3}x - \frac{1}{7} > 5 + 8x\)

7) Solve the inequality symbolically. \(\frac{4}{7} - 9x < 3 + \frac{5}{6}x\)

8) Solve the inequality symbolically. \(6(5x - 3) \leq 4 - (2x + 9)\)

9) Solve the inequality symbolically. \(\frac{5}{6} \geq \frac{-7}{9}x\)

10) Solve the inequality symbolically. \(\frac{-2}{3}x \leq \frac{-18}{9}\)

11) Consider the function \(f(x) = -7x^2 + 21x + 8\).
   a) Is the function open up, or is it open down?
   b) Identify the vertex of the function.
   c) Identify the y-intercept.
   d) Identify the x-intercept(s) if they exist.
   e) Identify the domain of the function. Write the domain in interval notation and also in inequality notation.
   f) Identify the range of the function. Write the range in interval notation and also in inequality notation.

12) Consider the function \(f(x) = 45(1.03)^x\).
   a) Is the function increasing or decreasing?
   b) Identify the percentage rate of growth or decay.
   c) Identify the y-intercept.
   d) Identify the domain of the function. Write the domain in interval notation and also in inequality notation.
   e) Identify the range of the function. Write the range in interval notation and also in inequality notation.

13) Consider the function \(f(x) = \frac{2}{3}(x - 15) + 8\).
   a) Is the function increasing or decreasing?
   b) Identify the slope.
   c) Identify the x-intercept.
   d) Identify the y-intercept.
   e) Identify the domain of the function. Write the domain in interval notation and also in inequality notation.
   f) Identify the range of the function. Write the range in interval notation and also in inequality notation.
Section 3.5b
Linear Inequalities Numerical, Graphical, and Applications

Learning Outcomes:
- Solve inequalities numerically
- Solve inequalities graphically
- Solve inequality application problems

Example 1: Consider the table below that gives several points on the lines $Y_1$ and $Y_2$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30</td>
<td>-2.5</td>
<td>-4.9</td>
</tr>
<tr>
<td>-25</td>
<td>-3.0</td>
<td>-4.8</td>
</tr>
<tr>
<td>-20</td>
<td>-3.5</td>
<td>-4.7</td>
</tr>
<tr>
<td>-15</td>
<td>-4.0</td>
<td>-4.6</td>
</tr>
<tr>
<td>-10</td>
<td>-4.5</td>
<td>-4.5</td>
</tr>
<tr>
<td>-5</td>
<td>-5.0</td>
<td>-4.4</td>
</tr>
<tr>
<td>0</td>
<td>-5.5</td>
<td>-4.3</td>
</tr>
<tr>
<td>5</td>
<td>-6.0</td>
<td>-4.2</td>
</tr>
</tbody>
</table>

(a) Use the table to solve where $Y_1 \leq Y_2$.

(b) Find a formula for $Y_1$.

(c) Find a formula for $Y_2$.

(d) Use the formulas you found in part (b) and part (c) to solve the inequality $Y_1 \leq Y_2$ by hand. Show your work and make sure you get the same answer that you got in part (a)!

Solution:

(a) Notice in the table of values that $Y_1$ is SMALLER than $Y_2$ in the table when $x = -5$ and after. We see that $Y_1$ is EQUAL TO $Y_2$ in the table when $x = -10$. If we make the assumption that the pattern we see in the table continues, then we would have to assume that for $x$-values smaller than -10 we will find $Y_1$ to be GREATER than $Y_2$, and for $x$-values bigger than -10 we will find $Y_1$ to be SMALLER than $Y_2$. Therefore, the set of $x$-values that make $Y_1 \leq Y_2$ are the $x$-values greater than or equal to -10. So the solution to the inequality is $x \geq -10$.

(b) To find the formula for $Y_1$, we first note that $Y_1$ is a linear function because it has a constant slope. The slope of the line is:

$$m = \frac{\text{output}}{\text{input}} = \frac{\text{down 0.5}}{\text{over 5}} = \frac{-0.5}{5} = -\frac{1}{10} = -0.1$$

So it is a line of the form $Y_1 = mx + b$ where we know that $m = -0.1$ and we see in the table that $b = -5.5$ since that is the $y$-intercept. Therefore, the equation of the line is $Y_1 = -0.1x - 5.5$.

(c) To find the formula for $Y_2$, we first note that $Y_2$ is a linear function because it has a constant slope. The slope of the line is:

$$m = \frac{\text{output}}{\text{input}} = \frac{\text{up 0.1}}{\text{over 5}} = \frac{0.1}{5} = \frac{1}{50} = 0.02$$

So it is a line of the form $Y_2 = mx + b$ where we know that $m = 0.02$ and we see in the table that $b = -4.3$ since that is the $y$-intercept. Therefore, the equation of the line is $Y_2 = 0.02x - 4.3$.

(d) Now we solve the inequality $Y_1 \leq Y_2$. This means we are solving the inequality $-0.1x - 5.5 \leq 0.02x - 4.3$. We start by adding 5.5 to both sides of the inequality to get:
Now we subtract 0.02x on both sides to get
\[-0.1x \leq 0.02x + 1.2\]
Now we divide both sides by $-0.12$. Since we divided by a negative we need to reverse the inequality which gives
\[x \geq -10\]
And so our symbolic solution is the same as our original numerical solution!

**Example 2:** Consider the graph below. Solve the inequality $Y_1 > Y_2$.

![Graph](image)

**Solution:**
Notice in the graph that $Y_1$ is BIGGER/HIGHER than $Y_2$ in the second quadrant and for all x-values up to x=.375.

If we make the assumption that the graph we see continues the same pattern, then we would have to assume that

for x-values smaller than x=.375 we will find $Y_1$ to be GREATER than $Y_2$,

and for x-values bigger than x=.375 we will find $Y_1$ to be SMALLER than $Y_2$.

Therefore, the set of x-values that make $Y_1 > Y_2$ are the x-values less than .375. So the solution to the inequality is $x < .375$.

**Example 3:** Acme makes and sells widgets. Acme’s total cost per month, C, when producing W widgets is given by the function $C(W) = 7.50W + 55,000$. Acme’s revenue when selling W widgets is given by the function $R(W) = 12W$. Solve the inequality $C < R$ and explain the meaning of your answer in practical terms.

**Solution:** We are solving the inequality $C < R$. We know that $C=7.5W+55000$ and we know that $R=12W$.

So to say that $C<R$ is the same as saying $7.5W+55000<12W$. Now we can solve the inequality.

\[
7.5W + 55000 < 12W \\
55000 < 4.5W \\
12,222 < W
\]

This means that if they sell MORE THAN 12,222 widgets, their revenue will be greater than their costs ($C<R$).

So selling more than 12,222 widgets will produce a positive profit.
Section 3.5b Written Practice and Reflection
Linear Inequalities Numerical, Graphical, and Applications

1) Consider the table below that gives several points on the lines \( Y_1 \) and \( Y_2 \). Find where \( Y_1 \leq Y_2 \).

<table>
<thead>
<tr>
<th>X</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>83</td>
<td>69</td>
</tr>
<tr>
<td>9.5</td>
<td>81</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>79</td>
<td>79</td>
</tr>
<tr>
<td>10.5</td>
<td>77</td>
<td>84</td>
</tr>
<tr>
<td>11</td>
<td>75</td>
<td>89</td>
</tr>
<tr>
<td>11.5</td>
<td>73</td>
<td>94</td>
</tr>
</tbody>
</table>

(a) Circle the X-values below that ARE solutions to the inequality \( Y_1 \leq Y_2 \).
\( X = 9 \quad X = 9.5 \quad X = 10 \quad X = 10.5 \quad X = 11 \quad X = 11.5 \)

(b) Circle the X-values below that ARE solutions to the inequality \( Y_1 \leq Y_2 \).
\( X = 9.6 \quad X = 9.999 \quad X = 10 \quad X = 10.001 \quad X = 10.3 \)

(c) Complete the following sentence: The solution to the inequality \( Y_1 \leq Y_2 \) is all the X-values that are _______ or equal to __________.

(d) What is the solution to the inequality \( Y_1 \leq Y_2 \) ?

2) Consider the table below that gives several points on the lines \( Y_1 \) and \( Y_2 \). Find where \( Y_1 < Y_2 \).

<table>
<thead>
<tr>
<th>X</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-300</td>
<td>83</td>
<td>69</td>
</tr>
<tr>
<td>-290</td>
<td>81</td>
<td>74</td>
</tr>
<tr>
<td>-280</td>
<td>79</td>
<td>79</td>
</tr>
<tr>
<td>-270</td>
<td>77</td>
<td>84</td>
</tr>
<tr>
<td>-260</td>
<td>75</td>
<td>89</td>
</tr>
<tr>
<td>-250</td>
<td>73</td>
<td>94</td>
</tr>
</tbody>
</table>

(a) Circle the X-values below that ARE solutions to the inequality \( Y_1 < Y_2 \).
\( X = -300 \quad X = -290 \quad X = -280 \quad X = -270 \quad X = -260 \quad X = -250 \)

(b) Circle the X-values below that ARE solutions to the inequality \( Y_1 < Y_2 \).
\( X = -281 \quad X = -280.4 \quad X = -280 \quad X = -279.9 \quad X = -275 \quad X = -271 \)

(c) Complete the following sentence: The solution to the inequality \( Y_1 < Y_2 \) is all the X-values that are __________.

(d) What is the solution to the inequality \( Y_1 < Y_2 \) ?

3) Consider the table below that gives several points on the lines \( Y_1 \) and \( Y_2 \). Find where \( Y_1 \leq Y_2 \).

<table>
<thead>
<tr>
<th>X</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50</td>
<td>15</td>
<td>16.2</td>
</tr>
<tr>
<td>-49</td>
<td>15.5</td>
<td>16.3</td>
</tr>
<tr>
<td>-48</td>
<td>16</td>
<td>16.4</td>
</tr>
<tr>
<td>-47</td>
<td>16.5</td>
<td>16.5</td>
</tr>
<tr>
<td>-46</td>
<td>17</td>
<td>16.6</td>
</tr>
<tr>
<td>-45</td>
<td>17.5</td>
<td>16.7</td>
</tr>
</tbody>
</table>

(a) Circle the X-values below that ARE solutions to the inequality \( Y_1 \leq Y_2 \).
\( X = -49 \quad X = -48 \quad X = -47 \quad X = -46 \quad X = -45 \)

(b) Circle the X-values below that ARE solutions to the inequality \( Y_1 \leq Y_2 \).
\( X = -47.5 \quad X = -47.1 \quad X = -47.001 \quad X = -46.999 \quad X = -46.5 \)

(c) Complete the following sentence: The solution to the inequality \( Y_1 \leq Y_2 \) is all the X-values that are __________ or equal to __________.

(d) What is the solution to the inequality \( Y_1 \leq Y_2 \) ?
4) Consider the table below that gives several points on the lines $Y_1$ and $Y_2$. Find where $Y_2 < Y_1$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>130</td>
<td>85</td>
</tr>
<tr>
<td>15</td>
<td>110</td>
<td>80</td>
</tr>
<tr>
<td>20</td>
<td>90</td>
<td>75</td>
</tr>
<tr>
<td>25</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
<td>65</td>
</tr>
</tbody>
</table>

(a) Circle the X-values below that ARE solutions to the inequality $Y_2 < Y_1$.

$X = 15 \quad X = 20 \quad X = 25 \quad X = 30 \quad X = 35$

(b) Circle the X-values below that ARE solutions to the inequality $Y_2 < Y_1$.

$X = 23 \quad X = 24 \quad X = 24.9 \quad X = 25.3 \quad X = 26 \quad X = 27$

(c) Complete the following sentence: The solution to the inequality $Y_2 < Y_1$ is all the $X$-values that are ______________.

(d) What is the solution to the inequality $Y_2 < Y_1$?

5) Consider the table below that gives several points on the lines $Y_1$ and $Y_2$. Find where $Y_1 \leq Y_2$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>17</td>
<td>11</td>
</tr>
<tr>
<td>44</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>45</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>46</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>47</td>
<td>1</td>
<td>19</td>
</tr>
<tr>
<td>48</td>
<td>-3</td>
<td>21</td>
</tr>
</tbody>
</table>

6) Consider the table below that gives several points on the lines $Y_1$ and $Y_2$. Find where $Y_1 < Y_2$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>-12</td>
<td>-6</td>
</tr>
<tr>
<td>107</td>
<td>-10</td>
<td>-7</td>
</tr>
<tr>
<td>110</td>
<td>-8</td>
<td>-8</td>
</tr>
<tr>
<td>113</td>
<td>-6</td>
<td>-9</td>
</tr>
<tr>
<td>116</td>
<td>-4</td>
<td>-10</td>
</tr>
<tr>
<td>119</td>
<td>-2</td>
<td>-11</td>
</tr>
</tbody>
</table>

7) Consider the table below that gives several points on the lines $Y_1$ and $Y_2$. Find where $Y_2 < Y_1$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>14</td>
<td>-6</td>
</tr>
<tr>
<td>-15</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>-10</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>-5</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>-6</td>
<td>24</td>
</tr>
</tbody>
</table>
8) Aiden weighs 34 pounds today and gains 2 pounds per month at a constant rate. Chase weighs 30 pounds today and gains weight at a constant rate of 3 pounds per month.

(a) Complete each table of values.

<table>
<thead>
<tr>
<th>T=months after today</th>
<th>A=Aiden's weight in pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Find an equation that gives Aiden's weight, A, after T months. Find an equation that gives Chase's weight, C, after T months. Show all work.

(c) Solve the inequality A<C using the table of values. Write a sentence that explains the meaning of your answer.

(d) Now solve the same inequality SYMBOLICALLY to check your answer. Show all your steps/work.

9) Judy leaves her house and drives straight north to a camp ground travelling at 15 mph. Judy’s husband, Bob, is at the mall, which is at the camp ground which is 160 miles away. He gets in his car and drives straight south towards home travelling 25 mph.

(a) Complete the table of values that relates Judy’s distance from home after T hours, and complete the table of values that relates Bob’s distance from home after T hours.

<table>
<thead>
<tr>
<th>T hours</th>
<th>J miles from home</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

(b) Solve the inequality J>B using the table of values, and then write a sentence that interprets the meaning of the answer.

(c) Write a function that gives Judy’s distance from home, J, after she has traveled for T hours. Write a function that gives Bob’s distance from home, B, after he has traveled for T hours.

(d) Solve the inequality J>B by hand using the formulas you created in part (c). Show all steps/work.

10) Acme makes widgets, but due to production restraints is only able to produce at most 2,000 widgets in any given month. Each widget costs $45 to produce, and the company also has fixed costs of $18,650 every month. This means that Acme’s total cost per month, C, when producing W widgets is given by the function \( C(W) = 45W + 18,650 \). Acme sells each widget for $70. Therefore, Acme’s revenue when selling W widgets is given by the function \( R(W) = 70W \).

(a) Complete the table of values for the total cost, and complete the table of values for the total revenue.

<table>
<thead>
<tr>
<th>W widgets</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Solve the inequality \( C < R \) by using the table of values and explain the meaning of your answer in practical terms.

(c) Solve the inequality \( C < R \) by hand using the formulas. Show your work.
11) Consider the graph below that displays the lines $Y_1 = 3x - 4$ and $Y_2 = -1x + 6$ on a standard viewing window. Suppose that the point of intersection of the two graphs is the point (2.5, 3.5). Solve the inequality $Y_1 \leq Y_2$.

12) Consider the graph below that displays the lines $Y_1 = -3x + 6$ and $Y_2 = -0.5x + 2$ on a standard viewing window. Suppose that the point of intersection of the two graphs is the point (1.6, 1.2). Solve the inequality $Y_1 < Y_2$.

13) Consider the graph below that displays the lines $Y_1 = 2X + 5$ and $Y_2 = 3x + 10$. Suppose that the point of intersection of the two graphs is the point (-5, -5). Solve the inequality $Y_1 > Y_2$.

14) Consider the graph below that displays the lines $Y_1 = \frac{2}{3}X + 7$ and $Y_2 = 8$. Suppose that the point of intersection of the two graphs is the point (1.5, 8). Solve the inequality $Y_2 < Y_1$.

15) Consider the graph below that displays the lines $Y_1 = 0.5X + 1$ and $Y_2 = -4X + 1$ on a standard viewing window. Suppose that the point of intersection of the two graphs is the point (0,1). Solve the inequality $Y_2 < Y_1$.

16) Consider the graph below that displays the lines $Y_1 = -4$ and $Y_2 = 2X + 8$ on a standard viewing window. Suppose that the point of intersection of the two graphs is the point (-6, -4). Solve the inequality $Y_1 \geq Y_2$.

17) Consider the graph below that displays the lines $Y_1 = -1X + 2$ and $Y_2 = -3X + 6$ on a standard viewing window. Suppose that the point of intersection of the two graphs is the point (2, 0). Solve the inequality $Y_2 > Y_1$. 


18) Bob starts a job where he earns $35,000 per year, but each year he gets a $5,000 raise. Jim starts a job where he earns $60,000 per year, but each year he gets a $1,000 raise.
   (a) Write an equation that gives Bob’s salary, \( B \), after he has been working the job for \( T \) years.
   (b) Write an equation that gives Jim’s salary, \( J \), after he has been working the job for \( T \) years.
   (c) Graph the two functions on the same set of axis. Graph for the first 10 years of working. Label each graph clearly.
   (d) Graphically solve the inequality \( J > B \). Briefly explain your calculator steps. Write a sentence that interprets the meaning of your answer.

19) Acme Rubber Factory produces rubber sandals and rubber ducks. Each pair of sandals requires the use of 1.5 grams of rubber and each rubber duck requires the use of 0.8 grams of rubber. The company has exactly 50,000 grams of rubber to use. So, if they make \( R \) rubber ducks and \( S \) pairs of sandals, then we know that \( 0.8R + 1.5S = 50,000 \).
   (a) Solve the equation \( 0.8R + 1.5S = 50,000 \) for \( S \). Show your work.
   (b) Sketch a graph of your equation from part (a) for values up to 65,000 rubber ducks. Label both axis with the meaning of the axis, and the scale on the axis.
   (c) Solve the inequality \( S > 20,000 \). Write a sentence that explains the meaning of your answer. Show all work/calculator steps.
Unit 3 Outcome Overview

Section 3.1a and 3.1b:
- Given the equation of a linear or quadratic or exponential function in a real-world, contextual problem, find features of the function and interpret the real-world, contextual meaning of each feature.
- Given points on a linear function or points on an exponential function in a real-world, contextual problem, find the equation of the function and interpret the real-world, contextual meaning of the features.
- Given real-world contextual situations, use algebraic or graphical methods to solve problems and interpret the real-world, contextual meaning of each.

Section 3.2a:
- Understand when and how to use Product Rule
- Understand when and how to use Quotient Rule
- Understand when and how to use Power Rule
- Understand how to simplify when 0 is in the exponent
- Understand how to re-write expressions with negative exponents
- Properly use scientific notation

Section 3.2b:
- Re-write radical expressions using rational exponents
- Re-write rational exponent expressions using radicals
- Simplify square roots
- Simplify exponential expressions involving multiple steps

Section 3.3a:
- Understand what a system of two linear equations is, and understand what it means to be a solution to a system of equations
- Solve a system of two linear equations graphically
- Solve a system of two linear equations by hand using substitution
- Solve a system of two linear equations by hand using elimination

Section 3.3b
- Set up and solve systems of linear equations in real-world applications. Interpret the answer.

Section 3.4:
- Re-write the numerator and denominator of a rational expression in factored form
- Identify the domain of a rational expression
- Simplify a rational expression and identify the domain
- Multiply a rational expression and identify the domain
- Add/subtract a rational expression and identify the domain

Section 3.5a:
- Understand what a linear inequality is, and what the solution to a linear inequality is
- Solve linear inequalities symbolically

Section 3.5b
- Solve inequalities numerically
- Solve inequalities graphically
Practice Exam 3 (cumulative exam for all three units)

1) Solve each of the following equations. Show all work.

(a) \[12x - 2(2 + 4x) = 4 - (2x + 5)\] Solve for \(x\) by hand. Show all work.

(b) \[\frac{1}{2} + 3x = \frac{2}{3}x - \frac{1}{3}\] Solve for \(x\) by hand. Show all work.

(c) \[2x - 3y = 15\] Solve for \(y\).

(d) \[V = \pi r^2 h\] Solve for \(h\).

(e) \[5x^2 - 7x - 23 = 0\] Solve for \(x\) by hand.

(f) \[12x(3x - 7)(x + 50) = 0\] Solve for \(x\) by hand.

2) Simplify/re-write using rules of exponents.

(a) \[(75^{8})(75^{20}) = \] _______________

(b) \[\frac{(36)^{12}}{(36)^{4}} = \] _______________

(c) \[750^{-86} = \] _______________ re-write using only positive exponents

(d) \[\frac{3x^5}{7y^{-8}} = \] _______________ re-write using only positive exponents

(e) \[3050^0 = \] _______________

(f) \[(2x^3)^5 = \] _______________

(g) \[\sqrt{x + 3} = \] _______________ re-write using exponential notation

(h) \[0.00004867 = \] _______________ re-write using scientific notation

(i) \[\frac{7x^5}{y^-8} = \] _______________ re-write as an expression that contains no fractions

(j) \[\frac{8x^{-4}}{m^{-3}w^5} = \] _______________ re-write using only positive exponents
3) Expand and simplify the following polynomials.

(a) \((x - 8)(x + 2) = \) _____________________

(b) \((2x + 5)(4x - 1) = \) _____________________

(c) \((x + 4)^2 = \) ______________________________

(d) \((x - 5)(x + 5) = \) _____________________

(e) \((2x^2 - 3x + 5)(x + 7) = \) ______________

4) Solve the equation by factoring and using the zero-product property. Show your work.

(a) \(x^2 + 2x - 15 = 0\)

(b) \(3x(x - 4)(7x + 2) = 0\)

(c) \(8x^2 - 18x - 5 = 0\)

(d) \(8x^2 + 3x - 5 = 0\)

5) Solve the equation using the quadratic formula. Show your work. If there are no real solutions, then say so.

(a) \(-3x^2 - 5x + 17 = 0\)

(b) \(2x^2 + 3x + 4 = 0\)

6) For each function, identify if the function is LINEAR, QUADRATIC, EXPONENTIAL, or NEITHER.

(a) \(f(x) = x^2 \) ________________________________

(b) \(f(x) = 2x \) ________________________________

(c) \(f(x) = (x + 2)(x - 3) \) ________________________________

(d) \(f(x) = 3^x \) ________________________________

(e) \(f(x) = x^3 \) ________________________________
7) Is the function in the table linear, exponential, or neither? Explain how you know.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>36</td>
</tr>
<tr>
<td>1</td>
<td>108</td>
</tr>
<tr>
<td>2</td>
<td>324</td>
</tr>
<tr>
<td>3</td>
<td>972</td>
</tr>
</tbody>
</table>

8) For each table, determine if y is a function of x.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>22</td>
<td>9</td>
</tr>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

For each table, determine if y is a function of x.

- **Table A** is (circle one) a function of x
- **Table B** is (circle one) a function of x
- **Table C** is (circle one) a function of x

9) (a) What is the domain of the function in the graph?

(b) What is the range of the function in the graph?

(c) Find/solve \( f(2) \)

(d) Find/solve \( x = 2 \)
10) The height of a ball, in feet, $t$ seconds after it has been thrown is given by the function

$$H(t) = -16t^2 + 112t + 128$$

(a) Find the vertical intercept (the y-intercept): $(\phantom{0}, \phantom{0})$

(b) Find the horizontal intercepts (the x-intercepts) of the function. $(\phantom{0}, \phantom{0})$ and $(\phantom{0}, \phantom{0})$

(c) Find the vertex of the function. Write the vertex as an ordered pair (a point). $(\phantom{0}, \phantom{0})$

(d) How high is the ball initially? ______________________ FEET

(e) When does the ball hit the ground? ______________________ SECONDS

(f) What is the highest the ball gets? ______________________ FEET

(g) When does the ball reach its highest height? ______________________ SECONDS

11) Consider the graph of the quadratic function, $f(x)$, given in the graph below.

(a) Is the leading coefficient of $f(x)$ positive or negative?

(b) Identify the y-intercept(s).

(c) Identify the x-intercept(s).

(d) Identify the vertex.

(e) Is the discriminate of $f(x)$ positive, negative, or zero?
12) Identify slope and y-intercept of each linear function.

(a) \( Y = 3 - 2X \)  
slope:  \( \)  
Y-intercept:  \( ( , ) \)

(b) \( Y = 7X \)  
slope:  \( \)  
Y-intercept:  \( ( , ) \)

(c) \( Y = 10 \)  
slope:  \( \)  
Y-intercept:  \( ( , ) \)

(d) \( 3X + Y = 4 \)  
slope:  \( \)  
Y-intercept:  \( ( , ) \)

(e) \( 2X + 3Y = 5 \)  
slope:  \( \)  
Y-intercept:  \( ( , ) \)

13) Identify the y-intercept, factor, if the function is growth or decay, and the percent rate of growth or decay (as a percent) for each exponential function.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Factor for this function?</th>
<th>Is this growth or decay?</th>
<th>Rate of growth or decay (as a PERCENT)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3(1.09)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 75(0.68)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = (0.98)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 158(1.35)^x )</td>
<td>( , )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

14) Identify the y-intercept, if the graph will open up or open down, and the vertex.

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 6x - 8 )</td>
<td>( , )</td>
<td></td>
<td>( , )</td>
</tr>
<tr>
<td>( f(x) = x^2 + 5 )</td>
<td>( , )</td>
<td></td>
<td>( , )</td>
</tr>
<tr>
<td>( f(x) = 54 + 7x - x^2 )</td>
<td>( , )</td>
<td></td>
<td>( , )</td>
</tr>
</tbody>
</table>
15) Find the equation of the line with slope 5 that passes through (0, 7).

16) Find the equation of the line with slope 8 that passes through (2, -9).

17) Find the equation of the line that passes through (-5, -13) and (-12, -69). Show your work.

18) Find the equation of the exponential function with y-intercept (0, 52) that has factor 0.59.

19) Find the equation of the exponential function with y-intercept (0, 560) that grows at a rate of 75%.

20) Find the equation of the exponential function with y-intercept (0, 8) that decays at a rate of 63%.

21) Suppose \((x) = 150x - 0.75\).

   a) What type of function is this (linear, exponential, quadratic, or neither)?

   b) What is the y-intercept of \(f(x) = 150x - 0.75\)?

      The y-intercept is the point \((\quad, \quad)\)

   c) Calculate the x-intercept of \(f(x) = 150x - 0.75\) BY HAND. Show your work and DO NOT ROUND.

      The x-intercept is the point \((\quad, \quad)\)

   d) Label the graph below with both intercepts of \(f(x) = 150x - 0.75\).

   ![Graph]

   e) Find the exact viewing window that was used to create the graph above. Check using your calculator!

      XMin: \_________
      YMin: \_________
      XMax: \_________
      YMax: \_________
      XScl: \_________
      YScl: \_________

22) The number of people in a country (in millions) \(T\) years after today is \(N(T) = 125(0.95)^T\). Graphically find the number of years until there are 70 million people in the country. Then briefly explain your calculator steps to solving graphically. Round to 4 decimal places.

   Number of years until there are 70 million people (Round to 4 decimal places): \______________

   Briefly explain calculator steps: \________________________________________
   \________________________________________
23) Use your graphing calculator to graphically solve the following equation. Round the answer(s) to 4 decimal places. Use a standard viewing window to graph.

\[ 0.05(x + 8)(x + 3)(x - 4) = -2 \]

Solution(s) to 4 decimal places: ____________________________________________________

24) Bob writes a popular novel and earns $1,000,000 during its premier month. The next month, and every month thereafter, Bob earns 65% of his previous month’s earnings on the book. Write an equation that gives Bob’s monthly earnings, Y, after X months have passed since the book premier.

25) Collaborative Collectables sells items that all cost the same amount. The profit increases $400 for every additional item they sell, and when they sell 1000 items they make a profit of $86,450. Find an equation of the function that gives the company profit, P, when they sell Q items.

26) A town starts with 140,000 people and increases by 1.8% each year. Find an equation that gives the population of the town, P, after T years have passed.

27) A car starts out many miles away from home and then travels towards home at a constant speed. After 3 hours of driving the car is 534 miles north of home. After 8 hours of driving the car is 234 miles away from home. Find an equation that gives the distance the car is from home, D, where T is the number of hours since the car started driving.

28) Solve the inequality \( Y_1 \leq Y_2 \).

Solution to \( Y_1 \leq Y_2 \) is ________________

29) Solve the inequality \( 8x - 17 \leq 50 + 7x \) by hand. Show all work.

30) Solve the inequality \( Y_1 > Y_2 \) using the table below. Assume the patterns in the table continue forever.

<table>
<thead>
<tr>
<th>x</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>50</td>
<td>34</td>
</tr>
<tr>
<td>0</td>
<td>48</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>10</td>
<td>44</td>
<td>52</td>
</tr>
<tr>
<td>15</td>
<td>42</td>
<td>58</td>
</tr>
</tbody>
</table>

31) Solve the system of equations. Show all work.

\[
\begin{align*}
5y + 2x &= 7 \\
5y - 8x &= -2
\end{align*}
\]
32) Anna has a total of $1800 to invest, and she decides to split the money into two pieces to invest in two different banks. Anna invests X dollars in a bank that gives 3.5% interest each year. She invests the remaining Y dollars in another bank that offers 2.8% interest each year. At the end of a year, Anna has earned a total of $56.35 in interest. How much money did she invest in each bank? Show all work to write the two equations and then solve the system of equations.

33) Re-write the expression so that the numerator and denominator are both in factored form. Simplify. Then identify the domain of the expression.
\[
\frac{x^2-81}{3x^2-25x-18}
\]

34) Perform the calculation and write the answer in simplified form. Identify the domain of the expression.
\[
\frac{9x}{x-3} - \frac{7}{x+2}
\]

35) Re-write the numerators and denominators in factored form. Perform the calculation and write the answer in simplified form. Identify the domain of the expression.
\[
\frac{6x^2+11x+3}{(x+2)(x+5)} \div \frac{(2x+3)(x-7)}{3x^2+7x+2}
\]
Solutions to Selected Exercises

Section 1.1
1) \(-7x + 1\)
3) \(2y^3 - 3y^2\)
5) \(-4t + 2\)
7) \(1x - 15\)
9) \(-19.3x - 27.4\)
11a) \(x = 1.37\) or \(x = 137/100\)
11b) \(x = 7.5\) or \(x = 15/2\)
11c) \(x = -43/26\)
13) \(x = -9\)
15) \(x = -13\)
17) \(x = -7\)
19) \(p = 6\)
21) \(p = -1.2\) or \(p = -6/5\)
23) \(y = 6\)
25) all real numbers
27) \(x = 5/7\)
29) \(x = 101/12\)
31) \(x = 2.4\)
33) \(x = 15,000\)

35a) \(\frac{3V}{4r^3} = \pi\)
35b) \(y = \frac{19-2x}{2}\) or \(y = \frac{-7}{2}x + \frac{19}{2}\)
35c) \(y = \frac{20-12x}{-5}\) or \(y = \frac{12}{5}x - 4\)
35d) \(y = \frac{7}{8}(x - 16) + 2/3\)
35e) \(\frac{8}{7}y + \frac{320}{21} = x\) or \(\frac{8}{7}(y - \frac{2}{3}) + 16 = x\)
37) \(\frac{c}{r} = t\)
39) \(\frac{a}{gm} = b\)
41) \(\frac{6F}{\pi r^2} = p\)
43) \(\frac{p-2w}{2} = m\) or \(\frac{1}{2}p - w = m\)
45) \(x = \frac{L-Hy}{A}\) or \(x = \frac{L}{A} - \frac{Hy}{A}\)
47) \(\frac{U}{F} - R = a\) or \(\frac{U-FR}{F} = a\)

Section 1.2
1a) \(-0.45\)
1b) \(-5/11\)
1c) \(-2.6789 \times 10^{10}\)
1d) \(-26,789,000,000\)
1e) \(3.4121\)
1f) \(14/11\)
1g) \(1.2727\)
1h) undefined
1i) 0
1j) \(\frac{9}{2}\)
3) 665
5) \(-1.75\)

7) \(-40\)
9) 15
11) 677
13) 1
15) \(-\frac{5}{94} \approx -0.05\)
17) 252.9901
19) 0.0000906
21) \(-1.8385\)
23a) 91/9
23b) 2749/33
23c) 3418/9
Section 1.3

1)

<table>
<thead>
<tr>
<th>$y = -3x + 15$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
</tr>
<tr>
<td>-2</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>$7/3 \approx 2.33$</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>$20/3 \approx 6.67$</td>
</tr>
</tbody>
</table>

$P = 3.75Q - 12,500$

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-12,500</td>
</tr>
<tr>
<td>1000</td>
<td>-8,750</td>
</tr>
<tr>
<td>$10,000/3 \approx 3333.33$</td>
<td>0</td>
</tr>
<tr>
<td>3600</td>
<td>1000</td>
</tr>
<tr>
<td>$22,000/3 \approx 7333.33$</td>
<td>15,000</td>
</tr>
</tbody>
</table>

$4x - 7y = 21$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>5.25</td>
<td>0</td>
</tr>
<tr>
<td>-7</td>
<td>-7</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

3a) $-\frac{8}{7}A - \frac{53}{7} = B$
3b) $A = 0, B = -\frac{53}{7} \approx -7.57$
3c) $A = -6.625, B = 0$
3d) $A = 5, B = -\frac{93}{7} \approx -13.29$
3e) $A = -11, B = 5$
3f)
5) \((-5, -9)\)

7) \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-4 & -6 \\
2 & -3 \\
-2 & -5 \\
0 & -4 \\
\hline
\end{array}
\]

9) x-intercept is (18,0). Y-intercept is (0, -12).
The viewing window is
- Xmin: -2
- XMax: 22
- Xscl: 2
- YMin: -15
- Ymax: 3
- Yscl: 3

11) x-intercept is (1800, 0). Y-intercept is (0, 3000).
The viewing window is
- Xmin: -900
- XMax: 2700
- Xscl: 900
- YMin: -3000
- Ymax: 6000
- Yscl: 1000

13a) The horizontal axis is T: the year.
13b) The vertical axis is N: the number of birds in the aviary.
13d) In the year 2001 there are 2003 birds in the aviary.

15a) D=225. After 5 hours pass, the car is 225 miles from home.
15b) D=5 means H≈ 9.9. The car is 5 miles from home after 9.9 hours have passed.
15c) D=450. To start, when no time has passed, the car is 450 miles from home.
15d) D=0 means H=10. The car is home (0 miles away from home) after 10 hours have passed.
15f)
15g) Make sure your window allows you to clearly see both intercepts!
17a) x-intercept is (4,0)
17b) y-intercept is (0,−4)
17c) range is \( y \geq -4 \)
19a) x-intercept is (6,0)
19b) y-intercept is (0,−6)
19c) \( y = x - 6 \)
21) y-intercept is (0, 0)
23a) The left-most x-intercept is (-1, 0)
23b) The right-most x-intercept is (2,0)
23c) The solutions to the equation \( ax^2 + bx + c = 0 \) are the x-values that make the output equal 0. So, the solutions to the equation are \( x = -1 \) and \( x = 2 \).
25) Set your window as follows:
   - Xmin: -10
   - Xmax: 10
   - Ymin: -10
   - Ymax: 10
27) Put function in Y= menu. Set Xmin=-2 and set xmax = 2. Then hit Zoom 0. This will automatically set the Ymin equal to 40, and ymax equal to 44. This would allow you to clearly see the points of this function for \( x=-2 \) up to \( x=2 \). But, you may change to a different y-window that is wider than this if you want.
29) The x-intercept is (4, 0) and the y-intercept is (0, 5).

Section 1.4

1) Table A: function because each x-value maps to only one unique y-value
   Table B: not a function because x=5 is mapped to two different y-values
   Table C: function because each x-value maps to only one unique y-value
3) (2,7) is on the graph
5a) \( f(-2) = -17 \), and then \( f(x) = \frac{1}{9} \) when \( x = \frac{4}{9} \)
5b) \( g(12) = \frac{61}{3} \), and then \( g(x) = 1 \) when \( x = \frac{43}{6} \)
5c) \( h(16) = \frac{4}{7} \), and then \( h(x) = 13 \) when \( x = 59.5 = \frac{119}{2} \)
7a) \( f(0) = -15 \)
7b) \( x = 45 \)
7c) \( f(67.5) = 7.5 \)
7d) \( x = 22.5 \)
9) \( W(72) = 165 \) which means that a 72 inch tall man is expected to weigh 165 pounds.
   \( W(H) = 175 \) means that a man that is 175 pounds is expected to be 74 inches tall.
11a) \( f \left( \frac{1}{6} \right) = -\frac{329}{36} \approx -9.1389 \)
11b) \( g(1500) \approx -0.0239 \)
11c) \( h(-5) = -637 \)
13a) \( C(1) = 15,003 \)  
   If the company is only producing 1 item, then it costs them $15,003 per item to produce each item.
13b) \( C(2000) = 10.5 \)  
   If the company is producing 2,000 items, then it costs them $10.50 per item to produce each item.
13c) \( C(10,000) = 4.5 \)  
   If the company is producing 10,000 items, then it costs them $4.50 per item to produce each item.
15) A graph is a function if it passes the vertical line test: you can pass a vertical line through the graph at ANY point and it will only intersect the graph at one point at a time.
17) yes. This is a function because each x-value maps to only one unique y-value.
19) \( f(10) = 6/77 \)
21a) \( f(2) = 4 \)
21b) \( f(4) = 4 \)
23) two solutions: \( x = -1 \) and \( x = 5 \)
25a) \( g(2) = 1/9 \)
25b) \( g(1) = 0 \)
25c) \( g(-7) = undefined \)
25d) \( g(-14.25) = 61/29 \)
27) \( x = -1 \)
29) \( x = -1 \)
31a) \( h(-5) = 5 \)
31b) \( h(5) = 5 \)
31c) \( x = -5 \) and \( x = 5 \)

Section 1.5
1a) \( (7,12] \)
1b) \([−9, ∞) \)
1c) \((-∞,5) \)
1d) \([−4,−1) \)
3a) \([34,40) \) or \( 34 \leq x < 40 \)
3b) \((-13,∞) \) or \( −13 < x < ∞ \)
3c) \((1,7] \) or \( 1 < x \leq 7 \)
3d) \((-∞,11) \) or \( x < 11 \)
5) \( answers \ will \ vary \)
7a) \( g(-4) = -8/3 \)
7b) The domain is all real numbers except \( x=5 \). The domain is \( x \neq 5 \).
9a) \( h(-4) = 1 \)
9b) The domain is all real numbers greater than or equal to \( x = -5 \). The domain is \( x \geq -5 \).
11) The domain is all real numbers.
13a) The domain is \( 1 \leq x \leq 7 \). The range is \(-7 \leq y \leq -4 \).
13b) The domain is \( 1 < x \leq 4 \). The range is \( 2 < y \leq 4 \).
13c) The domain is all real numbers. The range is \( y \geq -4 \).
13d) The domain is all real numbers. The range is \( y \geq 3 \).
15) The domain is all real numbers.
17) The domain is all real numbers.

Section 1.6

1) The function is linear. The function is increasing. The slope is 1. The y-intercept is (0,19). The x-intercept is (-19,0) (found by following the slope backwards until y=0).
3) The function is exponential. The function is increasing. The factor is 2. The rate is 100%. The y-intercept is (0,3).
5) The function is quadratic. It is open down. The y-intercept is (0, -4). The vertex is (2.5, 2.25). There are two x-intercepts: (1,0) and (4,0). The vertical line of symmetry is x=2.5.
7a) The y-intercept is (0, -14500). If they sell nothing, then the profit will be in the red by $14,500.
7b) The x-intercept is (5800, 0). If they sell 5800 they will break even (have a profit of exactly $0).
7c) The slope is 2.5. Every time they sell one more item, they will make an additional $2.50 of profit.
7d) XMin = -1000, XMax = 9000, YMin = -20,000, YMax = 10,000
9a) To start (when no time has passed), the item costs $1.
9b) After 1 year, the item now costs $1.035 (essentially it is $1.04).
9c) The factor is 1.035. The rate is 3.5% increase. Each year the price of the item increases by 3.5%.
9d) XMin = -10, XMax = 100, XScl = 10, YMin = -5, YMax = 35, YScl = 35
11 i a) quadratic
11 i b) vertex is (-1, -1)
11 i c) opens up
11 i d) increasing for $x \geq -1$
11 i e) decreasing for $x \leq -1$
11 ii a) quadratic
11 ii b) vertex is (-3, 1)
11 ii c) opens down
11 ii d) increasing for $x \leq -3$
11 ii e) decreasing for $x \geq -3$
13 i) the y-intercept is (0,5). The factor is 2.
13 ii) The y-intercept is (0,6). The factor is 1/3.
19 table A a) exponential
19 table A b) decreasing function, y-intercept of (0, 3963), factor of 0.3, decreasing at rate of 70%.
19 table A d) XMin=-1m XMax = 3, YMin= -1000, YMax = 15,000
19 table C a) neither
19 table C b) not linear and not exponential, increasing function but it does not have a constant slope or constant factor
19 table C d) XMin = 0 XMax = 40 YMin = -3 YMax = 3
21) linear. Slope of 6. Y-intercept of (0, -2)
23) neither. No constant slope. No constant factor.
25) exponential. Factor is 0.2. Y-intercept is (0, 3125).
27) exponential. Factor is 1.25. Y-intercept is (0, 8).

Section 1.7

1a) slope -2, y-intercept (0, 3)
1c) slope 0, y-intercept (0, 10)
1d) slope 4, y-intercept (0, -7)
3)
<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 6x - 8 )</td>
<td>(0, -8)</td>
<td>up</td>
<td>(3, -17)</td>
</tr>
<tr>
<td>( f(x) = -2x^2 + 5 )</td>
<td>(0, 5)</td>
<td>down</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>( f(x) = 1 + 7x - x^2 )</td>
<td>(0, 1)</td>
<td>down</td>
<td>(3.5, 13.25)</td>
</tr>
</tbody>
</table>

5) y-intercept is (0,9). Parabola that opens up.

7) y-intercept is (0, -10). Parabola that opens up.

9) \( f(x) = -2x + 15 \)

11) \( f(x) = 200(1.42)^x \)

13a) (0,6)

13b) slope is -3/7

13c) Equation of line is \( f(x) = -\frac{3}{7}x + 6 \)

13d) Set \( y=0 \) and solve for \( x \). The x-intercept is (14,0).

13e) Set \( y=8/9 \) and solve for \( x \). The solution is \( x=322/27 \)

13f) \( f\left(\frac{5}{11}\right) = \frac{447}{77} \)

13g) XMin: -5 and XMax: 20 and YMin: -2 and YMax: 8

15a) (0, 180)

15b) factor is 1.25

15c) increasing function means the factor should be more than 1. It is.

15d) \( f(x) = 180(1.25)^x \)

15e) rate is 25%.

15f) \( f(4.3) \approx 469.8785 \)

15g) XMin: -5 and XMax: 5 and YMin: -100 and YMax: 1000

17a) (0, -25)

17b) \( \frac{-25-(-62.2)}{0-(-3)} = \frac{37.2}{3} = 12.4 \)

17c) \( f(x) = 12.4x - 25 \)

17d) Let \( y=0 \) and solve for \( x \). The x-intercept is \( \left(\frac{125}{62}, 0\right) \).

17e) Let \( y=3/8 \) and solve for \( x \). The solution is \( x = \frac{1015}{496} \).

17f) \( f\left(\frac{-7}{9}\right) = -\frac{1559}{45} \)

19a) (0, 1350)

19b) \( \frac{918}{1350} = 0.68 \) and \( \frac{624.24}{918} = 0.68 \) and \( \frac{424.4832}{624.24} = 0.68 \). The factor is 0.68.

19c) \( f(x) = 1350(0.68)^x \)

19d) Rate is -32%.

19e) \( f(13) \approx 8.9732 \)
Unit 1 Practice Exam

1) not a function because x=5 maps to two different y-values.

2) Yes this is a function because it passes the vertical line test

3a) \( x = 5.625 \)
3b) \( f(7) = 41 \)

4a) true statements are \( g(2) = -2 \) and \( g(4) = 2 \)
4b) domain is all real numbers
4c) range is \( y \leq 2 \)

5a) \( x \neq 3 \)
5b) all real numbers
5c) \( x \geq 10 \)
5d) all real numbers
5e) all real numbers

6a) 7659
6b) 13
6c) 7.5
6d) 2845
6e) \(-23.6071\)

7a) \( x = \frac{11}{30} \)
7b) \( x = -1.3 \)
7c) \( \frac{2A}{h} = b \)
7d) \( y = \frac{8-7x}{-4} \) or \( y = \frac{7}{4}x - 2 \)

8) Xmin: -100, XMax: 200 XScl: 50 YMin: -30 YMax: 45 YScl: 15

9a) (0, 3)
9b) (5,0)

10a) slope 8, y-intercept (0, -5)
10b) slope 6, y-intercept (0, 0)
10c) slope 0, y-intercept (0, 2/3)
10d) slope -3, y-intercept (0, 16)
10e) slope \(-5\), y-intercept (0, 7)
11)  
<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Factor for this function</th>
<th>Is this growth or decay?</th>
<th>Rate of growth or decay (as a PERCENT)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 9(1.02)^x )</td>
<td>(0, 9)</td>
<td>1.02</td>
<td>growth</td>
<td>2% growth</td>
</tr>
<tr>
<td>( f(x) = 85(0.95)^x )</td>
<td>(0, 85)</td>
<td>0.95</td>
<td>decay</td>
<td>5% decay</td>
</tr>
<tr>
<td>( f(x) = (1.4)^x )</td>
<td>(0, 1)</td>
<td>1.4</td>
<td>growth</td>
<td>40% growth</td>
</tr>
<tr>
<td>( f(x) = 75(0.75)^x )</td>
<td>(0, 75)</td>
<td>0.75</td>
<td>decay</td>
<td>25% decay</td>
</tr>
</tbody>
</table>

12)  
<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 2x + 3 )</td>
<td>(0, 3)</td>
<td>up</td>
</tr>
<tr>
<td>( f(x) = 5x^2 - 8 )</td>
<td>(0, -8)</td>
<td>up</td>
</tr>
<tr>
<td>( f(x) = 1 + 4x - 2x^2 )</td>
<td>(0, 1)</td>
<td>down</td>
</tr>
</tbody>
</table>

13a) exponential  
13b) Decreasing. Y-intercept is (0, 8500). Factor is 0.9. Rate is −10%.

14a) quadratic  
14b) open down, y-intercept (0, −1.2)  
14c) Xmin: -10 Xmax: 20 XScl: 5 YMin: -10 YMax: 35 YScl: 5

15a) linear  
15b) Increasing. Slope is 1/10. Y-intercept is (0, -7). X-intercept is (70,0).  
15c) \( f(x) = \frac{1}{10}x - 7 \)

16) \( y = 17x + 3 \)

17) \( f(x) = 1.7(1.3)^x \)

**Section 2.1**

1) \( f(x) = 2x - 4 \)
3) \( f(x) = 6(x + 3) + 4 \) which can be written as \( f(x) = 6x + 22 \)
5) \( f(x) = -\frac{9}{7}x - 9 \)
7) \( f(x) = -3(x + 3) + 4 \) or \( f(x) = -3(x - 1) - 8 \) or \( f(x) = -3x - 5 \)
9) \( f(x) = 5(0.93)^x \)
11) \( f(x) = 30(1.1)^x \)
13) \( f(x) = 12(0.87)^x \)
15) \( f(x) = -0.005x + 1.4 \)
17) \( f(x) = -5(x + 25) + 65 \) or \( f(x) = -5(x + 5) - 35 \) or \( f(x) = -5x - 60 \)
19) 

<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 6x - 8 )</td>
<td>(0, -8)</td>
<td>up</td>
<td>(3, -17)</td>
</tr>
<tr>
<td>( f(x) = -2x^2 + 5 )</td>
<td>(0, 5)</td>
<td>down</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>( f(x) = 1 + 7x - x^2 )</td>
<td>(0, 1)</td>
<td>down</td>
<td>(3.5, 13.25)</td>
</tr>
</tbody>
</table>

21) y-intercept is (0,9). Vertex is \((5/6, 83/12)\). Parabola that opens up.
23) y-intercept is (0,7). Vertex is (0,7). Parabola that opens up.

**Section 2.2**

1a) quadratic function because it has the form \( y = ax^2 + bx + c \)
1b) The y-intercept is (0, -1)
1d) Type TRACE, 3, ENTER. \( f(3) = 18.2 \)
1e) Let \( Y2 = 15 \). Use calc intersect. \( x \approx 2.4584 \) and \( x \approx 32.5416 \)
1f) vertex at \((17.5, 60.25)\)
1g) Let \( Y2 = 0 \) and use calc intersect to find \( x \approx 0.1434 \) and \( x \approx 34.8566 \)

3a) exponential function because it has the form \( y = ab^x \)
3b) decreasing because the factor is less than 1
3c) y-intercept is \((0, 350)\)
3d) factor is 0.97. Rate is \(-3\%\). Every time \( x \) increases by 1 unit, \( y \) decreases by 3%.
3f) Typed TRACE, 12, ENTER. \( f(12) \approx 242.8448 \)
3g) Let \( Y2 = 75 \) and use calc intersect to find \( x \approx 50.5740 \)

5) \( x \approx -1.4917 \) using calc intersect with \( y2=3 \)
7) min at \((-0.2146, -92.5767)\) and max at \((6.2145, 34.9767)\)
9) \( f(x) = 2x + 8 \) and \( g(x) = 3(1.051)^x \). Intersection at \((-2.6877, 2.6246)\).
11) \( f(x) = 15(0.95)^x \)
13) \( f(x) = 34(1.024)^x \)
15) (a) \( x \approx -3.2353 \) (b) \( x \approx -1.4365 \) and \( x \approx 2.4365 \) (c) \( x \approx -0.7321 \) and \( x \approx 2.7341 \)
Section 2.3

1) | Expression | Deg? | Leading coefficient? | Constant term? | Coefficient of $x^2$? | Coefficient of $x$? |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x^2 + 3x - 4$</td>
<td>2</td>
<td>5</td>
<td>-4</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>$3x - 7$</td>
<td>1</td>
<td>3</td>
<td>-7</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$4x - 8x^2 + 7x^4 + 9 - 2x^3$</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>-8</td>
<td>4</td>
</tr>
<tr>
<td>$15x^8 + 9x$</td>
<td>8</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>$4$</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3) $6x^2 + 7x - 5$

5a) $(x - 9)(x - 3) = 0$
and so $x = 9$ and $x = 3$

5b) $(5x + 2)(x - 4) = 0$
and so $x = -2/5$ and $x = 4$

5c) $(3x + 2)(2x - 1) = 0$
and so $x = -2/3$ and $x = 1/2$

7a) $(0,0)$ and $(2,0)$ and $(9,0)$
7b) $(0,0)$
7c) maximum at $(0.9382, 16.0620)$,
minimum at $(6.3951, -146.4324)$

9a) $-7x^2 + 35x$
9b) $9x^2 + 72x$
9c) $x^2 + 9x + 14$
9d) $x^2 - 8x + 15$
9e) $20a^2 + 5a - 15$
9f) $r^2 + 2r - 35$
9g) $-48z^2 + 38z - 5$
9h) $-5x^2 + 22x + 15$
9i) $x^3 - 6x^2 - 49$

11a) $x = 0$ and $x = 2$
11b) $s = -9$ and $s = 12$
11c) $a = -7/10$ and $a = 3$
11d) $t(t - 4) = 0$ and so $t = 0$ and $t = 4$
11e) $2x^3(5x + 1) = 0$ and so $x = 0$ and $x = -1/5$
11f) $x(8x - 5) = 0$ and so $x = 0$ and $x = 5/8$
11g) \(3x(x + 7) = 0\) and so \(x = 0\) and \(x = -7\)
11h) \(x(5x - 3) = 0\) and so \(x = 0\) and \(x = 3/5\)
11i) \(t^3(4t + 21) = 0\) and so \(t = 0\) and \(t = -21/4\)
11j) \((w + 7)(w - 4) = 0\) and so \(w = -7\) and \(w = 4\)
11k) \((5x + 21)(x - 3) = 0\) and so \(x = -21/5\) and \(x = 3\)
11l) \((5x + 4)(x - 3) = 0\) and so \(x = -4/5\) and \(x = 3\)
11m) \(4(x + 2)(2x - 7) = 0\) and so \(x = -2\) and \(x = 7/2\)

13) \(f(x) = -3(x - 2) + 5\) and they are equal when \(x = -2\) and when \(x = 3\).

Section 2.4

1a) The graph will have NO x-intercepts because there will be no real solutions to the equation \(0 = ax^2 + bx + c\).
1b) The graph will have EXACTLY ONE x-intercept because there will be exactly one solution to the equation \(0 = ax^2 + bx + c\).
1c) The graph will have EXACTLY TWO x-intercepts because there will be exactly two solutions to the equation \(0 = ax^2 + bx + c\).
1d) discriminant

3a) \(x = \frac{7 \pm \sqrt{8}}{45}\) which may also be written as \(x = \frac{7 \pm 2\sqrt{2}}{45}\)
3b) (0, 41)
3c) \(f(x) = 2025x^2 - 630x + 41\)
3d) \(\left(\frac{7}{45}, -8\right)\)
3e) same answers as part a.
5a) \(x = \sqrt{196}\) and \(x = -\sqrt{196}\)
5b) \(x = \frac{11}{\sqrt{6}}\) and \(x = -\frac{11}{\sqrt{6}}\)
5c) \(x = -5 + \sqrt{17}\) and \(x = -5 - \sqrt{17}\)
7a) \(x = -1\) and \(x = -5/4\)
7b) \(x = -7\)
7c) \(x = -4\) and \(x = 13\)
7d) \(x = \frac{7 \pm \sqrt{109}}{10}\)
9a) \(\left(\frac{6 + \sqrt{52}}{2}, 0\right)\) and \(\left(\frac{6 - \sqrt{52}}{2}, 0\right)\)
9b) \((-2, 0)\) and \((-0.6, 0)\)
9c) no x - intercepts because the discriminate is negative
11a) \(a > 0\)
11b) \(x = -2\) and \(x = -5\)
11c) positive because there are two x intercepts
13a) \(a < 0\)
13b) \(x = -3\)
13c) zero because there is one x intercept
15a) discriminant is 633. Two solutions
15b) discriminant is 0. One solution
15c) discriminant is 97. Two solutions

Solutions to Jeopardy Review Game

Section 2.1 Jeopardy Game

100: Find the equation of the line that has slope 17 and passes through the point (4, 9).
Solution: \(y = 17(x - 4) = 9\) or \(y = 17x - 59\)
200: Find the equation of the function that has a y-intercept of (0, 15) and increases at a rate of 7.5%.
Solution: \(y = 15(1.075)^x\)
300: Find the EXACT vertex of the function \(f(x) = 3x^2 - 5x + 2\). Give answer as an ORDERED PAIR.
Solution: the vertex is \(\left(\frac{5}{6}, -\frac{1}{12}\right)\)
400: Find the equation of the function that has a y-intercept of (0,24) and decreases at a rate of 4.6%.
Solution: $y = 24(0.954)^x$  
**500**: Find the equation of the line that passes through the points $(15, -70)$ and $(40, -20)$.  
Solution: $y = 2(x - 15) - 70$ or $y = 2(x - 40) - 20$ or $y = 2x - 100$  
**Section 2.2 Jeopardy Game**  
**100**: Graph $f(x) = 0.1(x + 2)(x - 3)(x - 7)$ on a standard viewing window (-10 to 10, -10 to 10). Find the local maximum point. Round the values to 4 decimal places of accuracy.  
Solution: $(0.0632, 4.2031)$  
**200**: Graph $f(x) = 0.1(x + 2)(x - 3)(x - 7)$ on a standard viewing window (-10 to 10, -10 to 10). Find the local minimum point. Round the values to 4 decimal places of accuracy.  
Solution: minimum at $(5.2701, -2.8550)$  
**300**: Use graphing calculator to graphically solve for where $f(x) = 3(1.05)^x$ reaches a height of 7. Round the answer accurately to 4 decimal places. Briefly explain how you used the calculator to solve the problem.  
Solution: when $x$ is approximately 17.3662  
**400**: Graph $f(x) = x^3 - 2x + 7$ on a standard viewing window (-10 to 10, -10 to 10). Find the $x$-intercept. Round the values to 4 decimal places of accuracy.  
Solution: $(-2.2583, 0)$  
**500**: Graph $f(x) = x^3 - 5x - 1$ on a standard viewing window (-10 to 10, -10 to 10). Find the $x$-intercepts. Round the values to 4 decimal places of accuracy.  
Solution: $(-2.1284, 0)$ and $(-0.2016, 0)$ and $(2.3301, 0)$  
**Section 2.3 expand/factor Jeopardy Game**  
**100**: Expand/Multiply and then simplify. Give your answer in descending degree order. $(3x - 7)(2x + 5)$  
Solution: $6x^2 + x - 35$  
**200**: Expand/Multiply and then simplify. Give your answer in descending degree order. $(x - 4)^2$  
Solution: $x^2 - 8x + 16$  
**300**: Factor. $8x^2 + 10x - 3$  
Solution: $(2x + 3)(4x - 1)$  
**400**: Factor. $8x^2 + 10x + 3$  
Solution: $(4x + 3)(2x + 1)$  
**500**: Factor. $12x^3 - 10x^2 - 12x$  
Solution: $2x(2x-3)(3x+2)$  
**Section 2.3 Zero-product property Jeopardy Game**  
**100**: Solve $(2x - 5)(x + 80) = 0$ using the zero-product property.  
Solution: $x = 5/2$ and $x = -80$  
**200**: Solve $5x - 30x^2 = 0$ using the zero-product property.  
Solution: $x = 0$ and $x = 1/6$  
**300**: Solve $x^2 + 5x = 36$ using the zero-product property.  
Solution: $x = -9$ and $x = 4$  
**400**: Solve $8x^2 = 15 - 14x$ using the zero-product property.  
Solution: $x = -5/2$ and $x = 3/4$  
**500**: Solve $20x^3 + 38x^2 + 12x = 0$ using the zero-product property.  
Solution: $x = 0$ and $x = -3/2$ and $x = -2/5$  
**Section 2.4 Solve with Quadratic Formula Jeopardy Game**  
**100**: Find the exact solution(s) to the equation $8x^2 - 5x - 7 = 0$. Then given the approximate solution(s) to 4 decimal places of accuracy.
Solution: \( x = \frac{5 + \sqrt{249}}{16} \approx 1.2989 \) and \( x \approx -0.6737 \)

200: Find the exact solution(s) to the equation \( 3x - 17 = -4x^2 \). Then given the approximate solution(s) to 4 decimal places of accuracy.

Solution: \( x = \frac{-3 + \sqrt{201}}{8} \approx 1.7204 \) and \( x \approx -2.4704 \)

300: Find the x-intercepts of the function \( f(x) = -7x^2 - 84x - 252 \). Sketch the graph labeling the x-intercept(s) and y-intercept.

Solution: \( x = -6 \)

400: Find the x-intercepts of the function \( f(x) = 2x^2 + 3x + 7 \). Sketch the graph labeling the vertex and y-intercept.

Solution: no x-intercepts

500: Find the x-intercepts of the function \( f(x) = -3x^2 + 4x + 8 \). Sketch the graph labeling the vertex, the x-intercepts, and y-intercept.

Solution: \( x = \frac{-4 + \sqrt{104}}{-6} \approx -1.0902 \) and \( x \approx 2.4305 \)

Section 2.4 Understand Quadratic Formula Jeopardy Game

100: Identify the discriminate of \( f(x) = ax^2 + bx + c \).

Solution: \( b^2 - 4ac \)

200: If a quadratic function has a discriminate that is negative, then what do you know about the x-intercepts of that quadratic function?

Solution: negative discriminate means no x intercepts

300: If a quadratic function has 2 x-intercepts, then what do you know about the discriminate of that quadratic function?

Solution: 2 x-intercepts means the discriminate is positive

400: Find the discriminate of \( f(x) = 7x^2 - 4x + 2 \). What does the discriminate tell you about the x-intercepts of the function?

Solution: discriminate is -40. Since it is negative, there are no x-intercepts.

500: Find the discriminate of \( f(x) = -7x^2 - 2 \). What does the discriminate tell you about the x-intercepts of the function?

Solution: discriminate is -56. There are no x-intercepts since the discriminate is negative.
1a) \( y = 8x + 7 \)
1b) \( y = -4(x-15)+3 \)
1c) \( y = -\frac{3}{5}(x-17)+4 \) or \( y = -\frac{3}{5}(x-22)+1 \) or \( y = -\frac{3}{5}x+14.2 \)
1d) \( y = \frac{5}{4}(x+3) - 5 \) or \( y = \frac{5}{4}(x+7)-10 \) or \( y = \frac{5}{4}x-1.25 \)
1e) \( y = 85(1.35)^x \)
1f) \( y = 17(1.041)^x \)
1g) \( y = 560(0.964)^x \)
1h) \( y = 500(0.85)^x \)

2a) \( x \approx 11.3831 \) by letting \( Y_2 = 7 \) and using calc intersect
2b) \( g(-2) = -102 \) use TRACE -2 ENTER
2c) \( x = -28 \) by letting \( Y_2 = 20 \) and using calc intersect
2d) local maximum at \((2.8453, 1.5396)\). local minimum at \((5.1547, -1.5396)\).

3a) It is initially 15 feet high.
3b) The highest it gets is 159 feet high.
3c) It reaches the highest height at 3 seconds.
3d) The object hits the ground at about 6.15 seconds.

4a) 3,565,000 people (this is the y-intercept...when \( t=0 \))
4b) 2,967,869 people in 2005 (let \( t=5 \) to find value of \( P \))
4c) The population is decreasing at a rate of 3.6% per year. (factor is .964 and so rate is -.036)
4d) 4.7063 years after 2000 there will be a population of 3 million. (let \( y_2 = 3 \) and use calc intersect)

5)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Degree?</th>
<th>Leading coefficient?</th>
<th>Constant term?</th>
<th>Coefficient of ( x^2 )?</th>
<th>Coefficient of ( x )?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 7x^2 - 4x^3 + 8 - 9x )</td>
<td>3</td>
<td>-4</td>
<td>8</td>
<td>7</td>
<td>-9</td>
</tr>
<tr>
<td>( 6 - \frac{1}{2}x )</td>
<td>1</td>
<td>-1/2</td>
<td>6</td>
<td>0</td>
<td>-1/2</td>
</tr>
<tr>
<td>( 9 )</td>
<td>0</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( -15x + x^3 - x^2 )</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-15</td>
</tr>
</tbody>
</table>

6a) \( f(x) = 21x^6 - 35x^3 + 14x^4 + 63x^2 \)
6b) \( g(x) = x^2 + 2x - 35 \)
6c) \( h(t) = 15t^2 + 22t - 5 \)
6d) \( P(Q) = 2Q^3 + 7Q^2 - 11Q + 20 \)

7a) \( (x-7)(x-3) \)
7b) not factorable
7c) \( (3k + 5)(2k - 1) \)
7d) \( (t-1)(6t + 5) \)
7e) \( (s-9)(s+9) \)
7f) \((2x + 5)(2x + 5)\) or \((2x + 5)^2\)
8a) \(x = 8\) and \(x = -3\)
8b) \(x = 8\) and \(x = -5\)
8c) \(x = 1/2\) and \(x = -3/4\)
8d) \(x = 0\) and \(x = -2\) and \(x = 3/4\)

9) \(x = \frac{5\pm\sqrt{17}}{3}\) \(x \approx 3.0410\) and \(x \approx 0.2923\)

10a) open down since leading coefficient is negative, y-intercept of \((0,10)\), vertex of \((4/3, \frac{46}{3})\), discriminant is 184, discriminant is positive which tells us the function has two x-intercepts, x-intercepts are \(x = \frac{4\pm\sqrt{46}}{3}\) which is approximately \((3.5941,0)\) and \((-0.9274,0)\).
10b) open up since leading coefficient is positive, y-intercept is \((0,196)\), vertex is \((-7,0)\), discriminant is 0, only one x-intercept at \((-7,0)\).
10c) open down since leading coefficient is negative, y-intercept of \((0,-28.4665)\), vertex of \((7.1, 4.3)\), discriminant is 11.18, discriminant is positive which tells us the function has two x-intercepts, x-intercepts are \(x = \frac{-9.23\pm\sqrt{11.18}}{-1.3}\) which is approximately \((9.6720,0)\) and \((4.5280,0)\).
10d) open up since leading coefficient is 0, y-intercept is \((0,550)\), vertex is \((0,550)\), discriminate is -22000, no x-intercepts.

Section 3.1a

1c and 1d) \(x=3\)
3a and 3b and 3c) \(x = -7/4\) and \(x = 2/3\)
5a) \(D(t) = 15,000(1.034)^t\)
5b) 3.4% interest
5d) \(Y_2 = 30000\), Calc Intersect. They intersect at \((20.7313, 30000)\). The amount of money in the account will be double (at $30,000) after 20.7 years.
7) \(P(x) = 256(x - 250) + 4000\) or \(P(x) = 256x - 60,000\)
9) \(P(Q) = 400(Q - 800) + 6450\)
or \(P(Q) = 400(Q - 1000) + 86450\)
or \(P(Q) = 400Q - 313,550\)
11) \(Y = 1,000,000(0.65)^x\). He’ll make $5688.01 in the month that is 1 year after the book came out.
13) \(P(t) = 25,800,000(1.0395)^t\). They’ll have a population of about 39,508,372 in 2010 (11 years after 1999).
15) \(T(x) = 4500(0.83)^x\)
17a) The y-intercept is \((0, 900)\). To start, Mary is 900 miles north of home.
17b) The x-intercept is \((20, 0)\). After 20 hours of driving, Mary is 0 miles from home (she is home).
17c) The slope is \(-45\). Every hour, Mary gets 45 miles closer to home. This means she is driving at 45mph towards home.
17d) \(M(10) = 450\). After 10 hours of driving Mary is 450 miles north of home.
17e) Solution is \(t \approx 19.8\). After about 19.8 hours of driving Mary is 10 miles north of home.
Section 3.1b

1a)  

<table>
<thead>
<tr>
<th>Number of folds</th>
<th>Height of paper (in inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.004 inches</td>
</tr>
<tr>
<td>1</td>
<td>0.008</td>
</tr>
<tr>
<td>2</td>
<td>0.016</td>
</tr>
<tr>
<td>3</td>
<td>0.032</td>
</tr>
<tr>
<td>4</td>
<td>0.064</td>
</tr>
<tr>
<td>5</td>
<td>0.128</td>
</tr>
</tbody>
</table>

1b) It is exponential because every time x increases by 1 unit (every time we fold it one more time), H(x) is multiplied by 2 (the height of the stack doubles).

c) Factor = 2, rate of growth = 1= 100% growth, y-intercept = (0, .004).

d) Equation: \( H(x) = .004(2)^x \).

\( H(20) = 4194.304 \). After 20 folds, it is 4,194.304 inches tall (this is approximately 350 feet high).

3a)  

<table>
<thead>
<tr>
<th>Number of months</th>
<th>Total number of rabbits in field</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

3b) Linear because every time one more month passes, the are 2 more rabbits added to the field (Every time x increases by 1, we add 2 to the y-value).

3c) Slope = 2. Y-intercept= (0, 9). Meanings given in the initial information.

3d) \( R(x) = 2x + 9 \)

5a)  

\[
\begin{align*}
(600-825)/(3-0) &= -75 \\
(450-600)/(5-3) &= -75 \\
(150-450)/(9-5) &= -75
\end{align*}
\]

5b) The slope is -75. This means that every week, there are 75 fewer trees in the forest.
The y-intercept is (0,825). This means that there were 825 trees to start.

5c) \( T(w) = -75w + 825 \)

5d) \( T(1) = 750. \) After 1 week, 750 trees remain

\( T(2) = 675. \) After 2 weeks, 675 trees remain

\( T(4) = 525. \) After 4 weeks, 525 trees remain

\( T(10) = 75. \) After 10 weeks, 75 trees remain

\( T(11) = 0. \) After 11 weeks, 0 trees remain

7a) \( F(0)=50,000. \) \( F(1)=80,000. \) \( F(2)=110,000. \) \( F(3)=140,000. \) \( F(4)=170,000. \) \( F(5)=200,000. \)

7b) Linear because there is a constant rate of change of 30,000 dollars per day.

7c) Slope is 30,000. Y-intercept is (0,50000).

7d) \( F(x) = 30,000x + 50,000. \) \( F(x)=1,000,000 \) when \( x \approx 31.7 \). This would mean that Bob earns a total of \$1,000,000 after about 31.7 days of working.
9a) \( F(0) = 0.01 \) and \( F(1) = 0.02 \) and \( F(2) = 0.04 \) and \( F(3) = 0.08 \) and \( F(4) = 0.16 \) and \( F(5) = 0.32 \)

9b) Exponential function because we are multiplying by 2 each day

9c) Factor is 2 and y-intercept is \((0, 0.01)\)

9d) \( F(x) = 0.01(2)^x \). \( F(30) = 10,737,418 \). She will earn $10,737,418 on her 30th day. That is just how much she would earn ON THAT DAY (not the total amount she earned for the whole month).

11a) 

<table>
<thead>
<tr>
<th>Number of hours</th>
<th>F(T) Temp in Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>65</td>
</tr>
<tr>
<td>1</td>
<td>62</td>
</tr>
<tr>
<td>2</td>
<td>59</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

11b) linear. Slope of -3. Y-intercept of \((0, 65)\). Every hour the temperature decreases by 3 degrees. The temp started at 65 degrees.

11c) \( F(T) = -3T + 65 \)

\( F(24) = -7 \). This would mean that it is -7 degrees F outside after 1 day (this makes no sense! The temperature wouldn't continue to drop at 3 degrees per hour for this long!)

11d) \( F(T) = -3T + 65 \)

\( F(30) = -35 \). This would mean that it is NEGATIVE 35 degrees Fahrenheit outside after 30 hours! That is ridiculous!

13a) 

<table>
<thead>
<tr>
<th>Number of minutes</th>
<th>F(T) temperature of cake</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>250</td>
</tr>
<tr>
<td>1</td>
<td>225</td>
</tr>
<tr>
<td>2</td>
<td>202.5</td>
</tr>
<tr>
<td>3</td>
<td>182.25</td>
</tr>
<tr>
<td>4</td>
<td>164.025</td>
</tr>
<tr>
<td>5</td>
<td>147.6225</td>
</tr>
</tbody>
</table>

13b) This is exponential because we are multiplying by .9 every time x increases by 1.

13c) The factor is 0.9. Rate is \(-10\%\). Y-intercept is \((0, 250)\). The temperature of the cake starts at 250 degrees and decreases by 10% each minute. It starts at 250 degrees and each minute it is 90% of the temperature of the previous minute.

13d) \( F(t) = 250(0.9)^t \), \( F(30) = 10.5978 \) which means that the cake is 10.5978 degrees after 30 minutes of cooling. This is ridiculous unless your kitchen happened to be 10 degrees!

15a) \((0, 13)\). The ball is 13 feet above ground to start.

15b) The ball hits the ground after 7.2988 seconds. Let Y2 = 0 and use Calc intersect. Ignore the solution on the left since that is a time that occurs before the ball was thrown.

15c) The ball reaches it’s highest point at 3.5938 seconds when it reaches a height of 219.6406 feet above ground. Use Calc Maximum on the calculator.

15d) The domain in practical terms is from 0 seconds (when the ball is thrown) to 7.2988 seconds (when it hits the ground).

15e) The range in practical terms is from 0 feet (the lowest it gets) to 219.6406 feet (the highest it gets).
Section 3.2a

1a) \( f(x) = 4375x^4 \)
1b) \( g(x) = (-8)^{49}x^{49} = -8^{49}x^{49} \)
1c) \( h(w) = 6w^{15} \)
1d) \( p(b) = 8b^{18} \)
1e) \( f(t) = 80t^{12} \)
1f) \( P(x) = 12 \)
1g) \( k(x) = \frac{8}{x^4} \)
1h) \( L(m) = 3072m^5 \)

3a) \( 740x^8 \)
3b) \( 825x^{-60}y^{-12} \)
3c) \( 8,000,000a^{45}b^{-21} \)
3d) \( x^7y^{-3}a^{-3}b^5 \)
3e) \( 6x^{-1} \)
3f) \( 7(1-x)^{-1} \)
3g) \( 7(x-3)^{-6} \)
3h) \( 1950(2^{-x}) \)

5a) \( 5.4112 \times 10^{-12} \)
5b) \( 7.4852 \times 10^{13} \)
5c) \( -8.5 \times 10^{17} \)
5d) \( -1.13 \times 10^{-14} \)

7a) Approximately 1,293,578 Earth’s could fit inside the Sun
7b) Approximately 1,394 Earth’s could fit inside Jupiter
7c) Approximately 928 Jupiter’s could fit inside the Sun

9) \( \frac{18 \text{ grams}}{(6.022 \times 10^{23}) \text{ molecules}} = 2.989 \times 10^{-23} \text{ grams per molecule} \)
One molecule of water weighs approximately \( 2.989 \times 10^{-23} \) grams

11a) \( (1.24227 \times 10^{13}) \) dollars which is \$12,422,700,000,000.
11b) Each citizen would need to pay $97,052.34 which is 7,523,437.5 yen.
Section 3.2b

1a) \(7^{1/3}\)
1b) \(w^{1/7}\)
1c) \(x^{1/2}\)
1d) \((x - 7)^{1/3}\)
1e) \((13)^{4/5}\)
1f) \(y^{5/6}\)
1g) \(n^{6/5}\)
1h) \((x + 10)^{3/4}\)

3a) \(\sqrt[18]{19}\)
3b) \(\sqrt[3]{t}\)
3c) \(\sqrt[3]{x + 9}\)
3d) \(\frac{1}{\sqrt{x}}\)
3e) \(\sqrt[3]{7}\)
3f) \(\sqrt[8]{m^9}\)
3g) \(\sqrt[3]{(5y + x)^3}\)
3h) \(\sqrt[5]{(7x + 3)^2}\)

5) | Radical Expression | Expression with Fractional Exponents |
--- | --- |
\(17\sqrt[5]{x} \sqrt[5]{y}\) | \(17x^{5/2} y^{1/2}\) |
\(20\sqrt[3]{x^2} \sqrt[3]{m}\) | \(\frac{20x^{2/3}}{m^{-1/4}}\) |
\(\frac{3\sqrt{a - 4}}{15\sqrt{a - 4}}\) | \(\frac{1}{5}(a - 4)^{1/3} = \frac{1}{5} (a - 4)^{1/6}\) |
\(\frac{85}{\sqrt{x}}\) | \(85x^{-1/2}\) |

7a) \(\sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \sqrt{6} = 2\sqrt{6}\)

7b) \(\frac{2 - \sqrt{11}}{2} = \frac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2}\)

7c) \(\frac{5 + \sqrt{55}}{10} = \frac{5 + \sqrt{5 \times 11}}{10} = \frac{5 + 5\sqrt{11}}{10} = \frac{1 + \sqrt{11}}{2}\)

7d) \(\frac{12 - \sqrt{37}}{4} = \frac{12 - \sqrt{16 \times 2}}{4} = \frac{12 - 4\sqrt{2}}{4} = 3 - \sqrt{2}\)

9) \(x = \frac{-2 + \sqrt{124}}{-12} = \frac{-2 + \sqrt{4 \times 31}}{-12} = \frac{-2 + 2\sqrt{31}}{-12} = \frac{-1 + \sqrt{31}}{-6} = \frac{1 - \sqrt{31}}{6}\)
and
\(x = \frac{-2 - \sqrt{124}}{-12} = \frac{-2 - \sqrt{4 \times 31}}{-12} = \frac{-2 - 2\sqrt{31}}{-12} = \frac{-1 - \sqrt{31}}{-6} = \frac{1 + \sqrt{31}}{6}\)
Section 3.3a

1) \((- \frac{5}{18}, \frac{19}{9})\)

3) \((-1.375, 0.875)\)

5) \((\frac{37}{24}, -\frac{7}{12})\)

7) \((0.8, 0.6)\)

9) No solution. These are parallel lines that never intersect.

11) \((2, 1.2)\)

13) \((9, 7)\)

15) All points on the line are solutions to the system of equations (they are the same line).
   The solution is \(\{(x, y) | 2x + 3y = 2\}\).

Section 3.3b

1a) \(A + B = 2400\) and \(0.05A + 0.06B = 126\)

1b) \((1800, 600)\) where A is the independent variable, and B is the dependent variable

1c) $1800 invested in bank A, $600 invested in bank B.

3a) \(350X + 400Y = 2000\) and \(9X + 3.5Y = 30\)

3b) Solution is approximately \((2.1, 3.2)\)

3c) Need to eat 2.1 servings of food X and 3.2 servings of food Y.

5a) \(X + Y = 22\) and \(60X + 45Y = 1186.5\)

5b) \((13.1, 8.9)\)

5c) The car travels 13.1 hours at 60mph, and travels 89 hours at 45 mph.

Section 3.4

1a) \(\frac{4x(7x+3)}{(x-9)}\) domain: \(x \neq 9\)

1b) \(\frac{(x+5)(x-4)}{3(x+4)}\) domain \(x \neq -4\)

1c) \(\frac{(2x+3)(x-4)}{(5x-8)(x+2)}\) domain \(x \neq \frac{8}{5}, x \neq -2\)

1d) \(\frac{(3x+2)(2x+1)}{(3x-2)(2x-1)}\) domain \(x \neq -\frac{2}{3}, x \neq \frac{1}{2}\)

1e) \(\frac{x(3x-5)(2x+1)}{3(x-2)(x-8)}\) domain \(x \neq 2, x \neq 8\)

3a) \(\frac{15(x-5)}{(x-1)}\) domain \(x \neq 2, x \neq 1\)

3b) \(\frac{(x+8)(x-2)(x+8)(x-3)}{(x-8)(5x+2)}\) domain \(x \neq 8, x \neq -\frac{2}{5}\)

3c) \(\frac{(x-7)(x+7)(x-1)(x+1)}{(x-1)+2(x-7)} = \frac{(x+7)(x+1)}{2}\) domain \(x \neq 1, x \neq 7\)
\[ \frac{(2x-1)(5x+1)(x+4)}{(2x-1)(x+4)+2(5x+1)} = \frac{1}{2} \text{ domain } x \neq \frac{1}{2}, x \neq -4, x \neq -\frac{1}{5} \]

\[ \frac{(x+1)(5x+1)(2x-3)(4x-1)}{(4x-1)(2x+3)(x-3)} = \frac{(x+1)(5x+1)(2x-3)}{(2x+3)(x-3)} \text{ domain } x \neq \frac{1}{4}, x \neq -\frac{3}{2}, x \neq 3 \]

\[ \frac{91x}{13x} + \frac{5}{13x} = \frac{9x+5}{13x} \text{ domain } x \neq 0 \]

\[ \frac{8x(x+2)}{(x+3)(x+2)} + \frac{(x-1)(x+3)}{(x+2)(x+3)} = \frac{9x^2+18x-3}{(x+3)(x+2)} \text{ domain } x \neq -3, x \neq -2 \]

\[ \frac{7(x+2)}{(x-4)(x+2)} + \frac{5(x-4)}{(x-4)(x+2)} = \frac{12x^2-20x+14}{(x-4)(x+2)} \text{ domain } x \neq 4, x \neq -2 \]

\[ \frac{(x-3)(x-2)}{(x-1)(x+5)(x-2)} + \frac{5(x-1)(x+5)}{(x-2)(x-1)(x+5)} = \frac{6x^2+15x-19}{(x-1)(x+5)(x-2)} \text{ domain } x \neq 1, x \neq 2, x \neq -5 \]

\[ \frac{(x+2)(x-1)}{(x+6)(x+1)(x-1)} + \frac{5(x+6)(x+1)}{(x-1)(x+6)(x+1)} = \frac{6x^2+36x+28}{(x-1)(x+6)(x+1)} \text{ domain } x \neq 1, x \neq -6, x \neq -1 \]

**Section 3.5a**

1) \( x > -\frac{5}{3} \)
3) \( x > -13 \)
5) \( x > \frac{13}{37} \)
7) \( x > -\frac{102}{413} \)
9) \( x \geq -\frac{1}{14} \)
11a) down
11b) (1.5, 23.75)
11c) (0, 8)
11d) \( x = \frac{-21\pm\sqrt{665}}{-14} \) which is approximately (-0.342, 0) and (3.342, 0)
11e) Domain is all real numbers. That is \((-\infty, \infty)\) or \(-\infty < x < \infty\)
11f) Range is \( y \leq 23.75 \) or \((-\infty, 23.75]\).
13a) Increasing because the slope is positive.
13b) Slope is 2/3. Over 3, up 2.
13c) x-intercept is (3, 0).
13d) y-intercept is (0, -2)
13e) Domain is all real numbers. That is \((-\infty, \infty)\) or \(-\infty < x < \infty\)
13f) Range is all real numbers. That is \((-\infty, \infty)\) or \(-\infty < y < \infty\)

**Section 3.5b**

1a) 10, 10.5, 11, 11.5
1b) 10, 10.001, 10.3
1c) greater than or equal to 10
1d) \( x \geq 10 \)
3a) -49, -48, -47
3b) \(-47.5, -47.1, -47.0001\)
3c) less than or equal to -47
3d) \(x \leq -47\)
5) \(x \geq 44\)
7) \(x < -10\)
9b) \(T > 4\). After 4 hours Judy is further away from home than Bob.
9c) \(J(T) = 15T\) and \(B(T) = -25T + 160\)
9d) \(T > 4\).
11) \(x \leq 2.5\)
13) \(x < -5\)
15) \(x > 0\)
17) \(x < 2\)
19a) \(S = (50,000 - 0.8R)/1.5\)
19c) \(R < 25000\). When they are making less than 25,000 rubber ducks, then they will be making more than 20,000 rubber sandals.

### Unit 3 Practice Exam

1a) \(x = 1/2\)
1b) \(x = -5/14\)
1c) \(y = (15 - 2x)/-3\) or \(y = \frac{2}{3}x - 5\)
1d) \(h = \frac{V}{\pi r^2}\)
1e) \(x = \frac{7\pm\sqrt{509}}{10}\)
1f) \(x = 0, x = \frac{7}{3}, x = -50\)

2a) \((75)^{28}\)
2b) \(36^8\)
2c) \(\frac{1}{750^{86}}\)
2d) \(\frac{3x^5y^8}{7}\)
2e) 1
2f) \(2^5x^{15} = 32x^{15}\)
2g) \((x + 3)^{1/2}\)
2h) \(4.86710^{-5}\)
2i) \(7x^5y^{-8}\)
2j) \(8x^{-4}m^3w^{-5}\)

3a) \(x^2 - 6x - 16\)
3b) \(8x^2 + 18x - 5\)
3c) \(x^2 + 8x + 16\)
3d) \(x^2 - 25\)
3e) \(2x^3 + 11x^2 - 16x + 35\)

4a) \(x = 3\) and \(x = -5\)
4b) \(x = 0\) and \(x = 4\) and \(x = -2/7\)
4c) \( x = -1/4 \) and \( x = 5/2 \)
4d) \( x = 5/8 \) and \( x = -1 \)

5a) \( x = \frac{5 \pm \sqrt{229}}{6} \)
5b) \( x = \frac{-3 \pm \sqrt{-23}}{4} \) no real solutions since the discriminate is negative

6a) quadratic
6b) linear
6c) quadratic
6d) exponential
6e) neither

7) exponential because every time \( x \) increases by 1 unit, the y-value is multiplied by the constant factor of 3.

8) Table A is a function of \( x \), Table B is a function of \( x \), Table C is NOT a function of \( x \)

9a) \( x \geq -2 \)
9b) \( y \geq 1 \)
9c) \( f(2) = 3 \)
9d) \( f(x) = 2 \) when \( x = -1 \)

10a) (0, 128)
10b) (-1, 0) and (8, 0)
10c) (3.5, 324)
10d) The ball is initially 128 feet high.
10e) The ball hits the ground at 8 seconds.
10f) The highest the ball gets is 324 feet high.
10g) The ball reaches the highest point at 3.5 seconds.

11a) negative because it opens down
11b) (0, 3)
11c) (-2,0) and (6,0)
11d) (2, 4)
11e) positive since there are two x-intercepts

12a) slope is -2, y-intercept is (0, 3)
12b) slope is 7, y-intercept is (0, 0)
12c) slope is 0, y-intercept is (0, 10)
12d) slope is -3, y-intercept is (0, 4)
12e) slope is -2/3, y-intercept is (0, 5/3)
13)  
<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Factor for this function?</th>
<th>Is this growth or decay?</th>
<th>Rate of growth or decay (as a PERCENT)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 3(1.09)^x )</td>
<td>(0, 3)</td>
<td>1.09</td>
<td>growth</td>
<td>+9%</td>
</tr>
<tr>
<td>( f(x) = 75(0.68)^x )</td>
<td>(0, 75)</td>
<td>0.68</td>
<td>decay</td>
<td>−32%</td>
</tr>
<tr>
<td>( f(x) = (0.98)^x )</td>
<td>(0, 1)</td>
<td>0.98</td>
<td>decay</td>
<td>−2%</td>
</tr>
<tr>
<td>( f(x) = 158(1.35)^x )</td>
<td>(0, 158)</td>
<td>1.35</td>
<td>growth</td>
<td>+35%</td>
</tr>
</tbody>
</table>

14)  
<table>
<thead>
<tr>
<th>Function</th>
<th>Y-intercept of function</th>
<th>Open up or Open down?</th>
<th>Vertex</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = x^2 - 6x - 8 )</td>
<td>(0, -8)</td>
<td>up</td>
<td>(3, -17)</td>
</tr>
<tr>
<td>( f(x) = x^2 + 5 )</td>
<td>(0, 5)</td>
<td>up</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>( f(x) = 54 + 7x - x^2)</td>
<td>(0, 54)</td>
<td>down</td>
<td>(3.5, 69.25)</td>
</tr>
</tbody>
</table>

15) \( y = 5x + 7 \)

16) \( y = 8(x - 2) - 9 \) or \( y = 8x - 25 \)

17) \( f(x) = 8x + 27 \)

18) \( y = 52(0.59)^x \)

19) \( f(x) = 560(1.75)^x \)

20) \( y = 8(0.37)^x \)

21a) linear  
21b) (0, −.75)  
21c) (.005, 0)  
21e) xmin: -.01, xmax .01, xscl: .005, ymin: -1, ymax: 1, yscl: .25
22) Let \( Y_2 = 70 \), Calc Intersect. Intersection occurs at about \( x = 11.30398 \).

23) Let \( Y_1 \) be the left side of the equation, and let \( y_2 = -2 \). Use Calc Intersect. The solutions are approximately \( x = -1.8885 \) and \( x = -8.5711 \) and \( x = 3.4596 \).

24) \( Y = 1,000,000(0.65)^x \)

25) \( P = 400(Q-1000)+86450 \) or \( P = 400 - 313550 \)

26) \( P = 140,000(1.018)^T \)

27) \( D = -60(T - 3) + 534 \) or \( D = -60(T - 8) + 234 \) or \( D = -60T + 714 \)

28) \( x \leq -2 \)

29) \( x \leq 67 \)

30) \( x < 5 \)

31) \((0.9, 1.04)\)

32) System is
\[
1800 = X + Y \\
.035X + .028Y = 56.35
\]
The solution is \((850, 950)\).
She invested $850 into account \( X \), and she invested $950 into account \( Y \).

33) \[
\frac{(x-9)(x+9)}{(3x-2)(x+9)} = \frac{x-9}{3x-2} \text{ domain is } x \neq \frac{2}{3} \text{ and } x \neq -9
\]

34) \[
\frac{9x(x+2)}{(x-3)(x+2)} - \frac{7(x-3)}{(x+2)(x-3)} = \frac{9x^2 + 18x - 7x + 21}{(x-3)(x+2)} = \frac{9x^2 + 11x - 21}{(x-3)(x+2)} \text{ domain is } x \neq 3 \text{ and } x \neq -2
\]

35) \[
\frac{(2x + 3)(3x + 1)}{(x + 2)(x + 5)} - \frac{(2x + 3)(x - 7)}{(x + 2)(3x + 1)}
\]
\[
= \frac{(2x + 3)(3x + 1)}{(x + 2)(x + 5)} \times \frac{(x + 2)(3x + 1)}{(2x + 3)(x - 7)}
\]
\[
= \frac{(3x + 1)^2}{(x + 5)(x - 7)}
\]

\( domain \ x \neq -2, x \neq -5, x \neq -\frac{1}{3}, x \neq 7, x \neq -3/2 \)