INVOLVEMENT OF MATHEMATICS IN ART

by

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Abstract

To numerous people, mathematics may seem mind-numbing, far-off, and even the reverse of art. The two topics are usually separated, denying countless of the knowledge of the strong, still unpredicted associations between mathematics and art. This project brings into focus the different ways that mathematics can help create art both manually and by coding. The art will be displayed on a website that will be coded on Glitch, and where the main programming languages being used are HTML and CSS. Afterward, it will be displayed on the school's student server.

1. Introduction

Growing up, we use to hear a lot about how mathematics is involved in everything we do. How about art? Well, mathematics for a long time has always helped artists invasion ideas for their work; but little do they know is that art has also helped mathematicians figure out how to comprehend some mathematical formulas and theorems.

For my senior project, I decided to create a website, in which I am going to show a piece of art I made from the mathematical formulas. The information about the formulas and theorems will be added to another page after the art is shown. The project will be divided into five sections. Each section will include a formula or a theorem. The following will be the sections shown on the website: The Golden Ratio, Pythagorean Theorem, Prime Numbers, Euclidean Geometry, and Fractals.

The website is created mostly by using HTML and CSS. HTML has been so the website users could go to the different sections of the website; CSS has been used for the website's design, including some of my previous art pieces.

2. Pythagorean Theorem

2.1. What Pythagorean Theorem is about.

In Geometry, the Pythagorean Theorem states that the sum of the squares of the sides of a right triangle equals the square of the hypothenuse, the side opposite of the right angle. It is also known by the familiar algebraic formula "$a^2 + b^2 = c^2$". Additionally, the inverse of the Pythagorean theorem helps figure out if a triangle is a right triangle or not.

2.2. History of Pythagorean Theorem

The famous Pythagorean Theorem was named after the famous mathematician Pythagoras (Ferrao 1). It has been said that the theorem existed way before Pythagoras since some of the earliest forms of the theorem were created before the birth of Pythagoras. Back in time, mathematicians in India would use the Pythagorean Theorem and another theorem called Sulbasutras; those theorems gave the specific rules on how to build altars; for religious reasons in terms of the area of altars as seen in "Indian mathematicians in the ancient times knew the Pythagorean Theorem, they also used something called Sulbasutras that (of which the earliest date from 800-600 B.C.) that discuss the theorem in the context of strict requirements for the orientation, shape, and area of altars for religious purposes" (Ferrao).

"Some ancient clay tablets from Babylonia indicate that the Babylonians in the second millennium B.C., over 1000 years before Pythagoras, had rules for generating Pythagorean triples" (Ferrao) proves that some thoughts saying that Pythagoras was not the one that created the theorem were right. The Babylonians were able to resolve some other inquiries "They could even solve the hypothenuse of an isosceles right-angled triangle, in which they would come up with an approximation of the final value up to five decimal places" (Ferrao).

Aside from the Babylonians, this theorem has been used by the Egyptians to build some of their most famous pyramids, which included the use of angles and the hypotenuse "The Egyptians wanted a perfect 90-degree angle to build the pyramids which were actually two right-angles whose hypotenuse forms the edges of the pyramids" (Ferrao). It has also been used by the Chinese in a different way than the Egyptians "There are some clues that the Chinese had also developed the Pythagorean Theorem using the areas of the sides long before Pythagoras himself" (Ferrao).

However, it has been proven that Pythagoras was the one to grant the final form "Despite these early attempts at the Pythagorean Theorem, many scholars agree that Pythagoras gave the theorem its definitive form" (Barnett).

As seen in "Later in Book VI of *the Elements*, Euclid delivers an even easier demonstration using the proposition that the areas of similar triangles are proportionate to the squares of their corresponding sides" (Britannica); it is visible that the Pythagorean Theorem is the foundation of more than one theorem in the mathematical world "Apparently, Euclid invented the windmill proof so that he could place the Pythagorean Theorem as the capstone to Book I" (Britannica).

The Pythagorean Theorem is known to be used in geometry; little did we know it has been used to create the Mayans' "Long Count calendar" by using one of the Pythagorean triples 3-4-5 "The numbers related to the multiples of the 3-4-5 right triangle appear to reflect the number sets of the Maya long count" (Johnson). In this case, a few modifications have had to be made, such as the replacement of multiples of 2 to multiples of 3, as explained in "Furthermore, it would also appear that when the multiples of the 3-4-5 right triangle undergo relationships of equivalency to the power of three, there appears to exist a modification or extension of the Pythagorean Theorem" (Johnson).

2.3. Process of creating Pythagorean Theorem Art Piece

I used Processing to create the art piece for the Pythagorean Theorem. I first created different classes for the different types of triangles included in the art piece. For the first class, I named it Rotators, since I wanted to make the triangles rotate. I used the integer of type float, which is a floating-point number, meaning it is a decimal. But floats are used when the number needs to be precise. I assigned to the integer the variables c and d to determine the x and y location on the screen. After I assigned to the integer the variable theta, which determines the angle of rotation of the triangle. Then assigned the variable of speed, which determines the speed of the rotation.

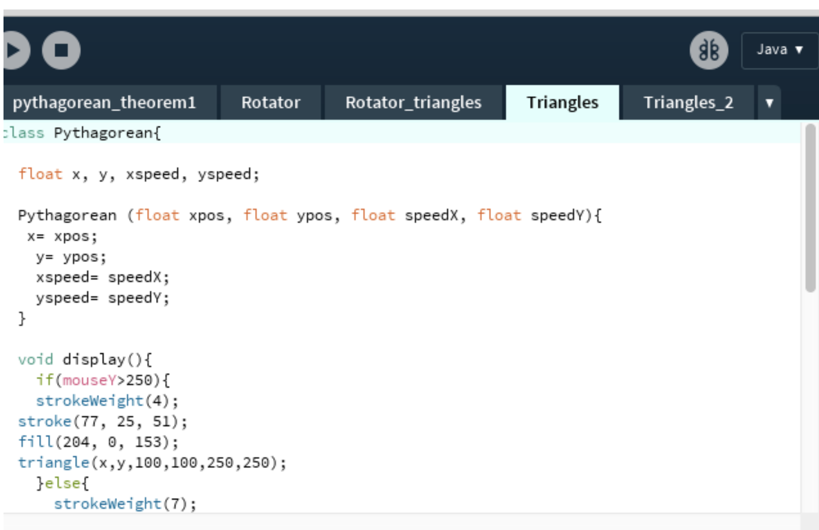

Figure 1: Code for Pythagorean Art

With all the integers being assigned to the float variables, I named them to make it easier while coding in the main class. To make the triangles spin and put the variables theta and to use; I coded the function “void spin()” in which the increment; which increases the value of the integer variable 1 would be included, that is an increase or addition, especially one of a series on a fixed scale “theta += speed”. Void is a keyword used to indicate that a function returns no value; each function must either return a value of a specific data type or return no value. To be able to display the triangles in this class, I coded the function “void display()”. I first started this code with “if(mouse>250)”, so that if the user put the mouse on the bottom of the screen it would change the whole color and size of the triangles. I used “strokeWeigth” to determine the weight of the stroke; then for the color of the stroke, I typed “(255,51,153)” which represents the color being used in the following order: primary, secondary, and tertiary colors. I used the same process to fill the color of the triangles “fill(169,13,153)”. To save the current coordinate system to the stack, the function “pushMatrix()” has been used. Using the function “translate(x, y, z)”, helps specifies an amount of what? to put on view in the display window. The x-parameter states left and right translation, the y-parameter states up and down translation; and the z-parameter specifies translations toward and away from the screen.

Using the function “rotate()”, helps rotates the shape to the amount specified by the angle parameter. To create the triangles, the function “triangle(0,0,50,50,w,w)” and to close that section, the function “popMatrix()” pops the current transformation matrix off the matrix stack and restores the prior coordinate system.

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For the other design in the class “Rotator_triangles”, I had to change the variable names so that they do not clash with the first part of the rotating triangles.

The same goes for the non-rotating triangles, the only different thing is the addition of the “void bounce()”; in which the speed of the x and y variable is based on the width and height of the screen. Like the “Rotator_triangles”, in the class “Triangles_2” the only thing that had to be changed are “void show()” and “void jump()”.

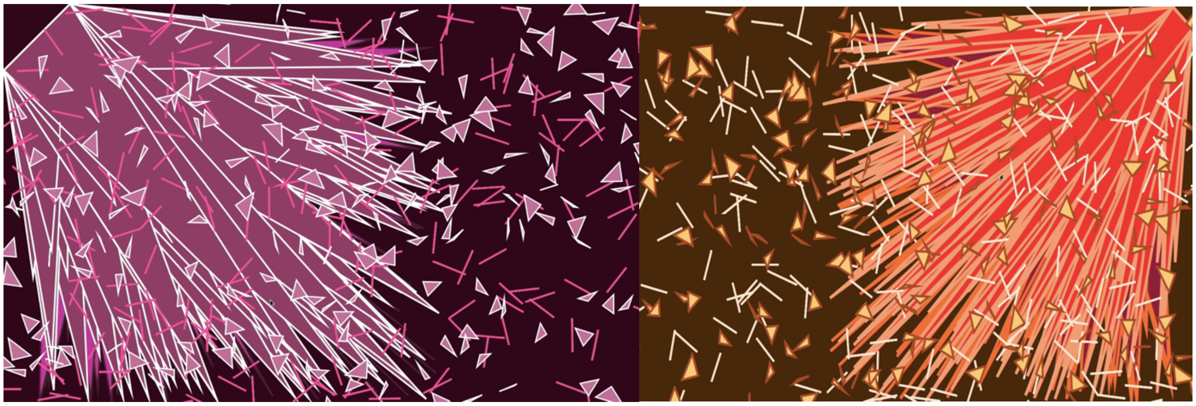

Figure 1: Pythagorean Theorem Image

3. Fractals

3.1. Definition of Fractals

Fractals are infinite patterns and are shown as numerous curves or shapes. They are complex in a way that is self-similar across different scales; meaning if examined, the shapes shown will be the same in a large or smaller part when magnified or reduced to the same size.

3.2. History of Fractals

The first person to introduce the world to the mathematical term fractals was the French-American mathematician, Benoit Mandelbrot in 1967. He established it by describing fractals as captivating objects that created the renowned posters that are hung up by many people in their households. Many think that Mandelbrot was the first mathematician to bring fractals into light, name them; but it has been proved that other mathematicians before him, have been studying this specific field for many years. By presenting a theory in which a group of curves could be put together and get analyzed as a whole “Mandelbrot gave a name to the field; he showed that there was a family of curves that could be grouped together and studied en masse for their properties" (Math Art: Truth, Beauty, and Equations 72).

There are many ways to define fractals, but the original one was the "Hausdorff dimension". The "Hausdorff dimension" is a point that has zero dimension, a line has one, an area has two, and a volume has three. However, it does make us ask, what is in between everything. To understand the correlation, in the book "Math Art: Truth, Beauty, and Equations" written by Stephen Ornes, they explained it by taking a line, and two other shapes: a square and a cube; that have respectively the dimensions 1, 2 and 3. They, then go and divide the line in two which leads them to two segments that join in the middle. By continuing the same process of dividing the sides in half on the other shapes, you wind up with a square divided into four small squares since each side is divided into two ($2^2 = 4$); and a cube divided into eight small cubes since each side is divided in two, $2^3 = 8$. These facts agree with our usual sense of dimension. Fractals, however, have non-integer dimensions.

Mandelbrot also tried to explain Fractals, by studying the length of the coastline in Great Britain as mentioned in "Mandelbrot famously kicked off his inquiry with a simple question: How long is the coastline of Great Britain?" (Math Art: Truth, Beauty, and Equations 73). He simply touched on the fact that by only using a long enough ruler, you would be only at most able to measure the basics of the coastline and not notice the smaller geographical details of its "He pointed out that if you use a long enough ruler, you're not going to measure every bay and outcropping" (Math Art: Truth, Beauty, and Equations 73). The only way to measure the specific details of the coastline would be to use a small ruler "If you use a shorter ruler, you'll catch more of those ins and outs, and you'll end up with a longer measurement" (Math Art: Truth, Beauty, and Equations 76). If we are trying to be more specific, smaller rulers would be helpful since there are so many nooks in a coastline “Use a ruler that’s shorter still, and your total measurement goes up more because now you’ll capture those coves, smaller bays, and so on” (Math Art: Truth, Beauty, and Equations 76). In mid 20^{th} century, an English mathematician, meteorologist, physicist, and psychologist Lewis Fry Richardson was able to figure out an equation that described the connection between the length of the ruler and the measurement of the coastline. Based on Richardson’s explanation of the coastline’s different dimensions “A smooth coastline would have a dimension of one. But if it’s jagged, rocky, and irregular, then its dimension is going to be more than one, but less than two” (Math Art: Truth, Beauty, and Equations 76), Mandelbrot was able to revive that connection by proclaiming that by using Richardson’s equation, you could discuss the roughness of the coastline. Mandelbrot realized that those coastlines were self-similar, which is the trademark, definition of fractals “It means that no matter how far you zoom in or zoom out, you’ll see the same, or almost the same design” (Math Art: Truth, Beauty, and Equations 76). Thanks to his theories of fractals, Mandelbrot inspired multiple artists, making them use his theories to create their art

The term Mandelbrot set is used to refer both to a general class of fractal sets and a particular instance of such a set “The “set” refers to the collection of points that make up its boundaries” (Math Art: Truth, Beauty, and Equations 80). By naming it fractals, Mandelbrot was able to bring the subject to light; but it was not suitable in the mathematical field, especially in the field of geometry. However, with many things changing and different mathematicians proving different theorems; Mandelbrot’s many theorems about fractals could be accepted and used by numerous people.

3.3. Process of creating Fractals Art

To create the fractals art, I coded the section “void setup()” in which you code the desired size and color of the screen by using size() and colorMode(). Along with “rectMode(RADIUS) which draws the image from its center point and uses the third and fourth parameters of rect() to specify half of the rectangle’s width and height.

In the function “void draw()”, I was able to show how the rectangles will change colors when you move the mouse up and down the screen by using “translate()” which specifies an amount to put on view in the display window; “pushMatrix()” saves the current coordinate system to the stack; and “popMatrix()” which pops the current transformation matrix off the matrix stack. Finally, using the functions “void scalingRectangles(float n)” decides the speed and the offset of the recursive rectangles.

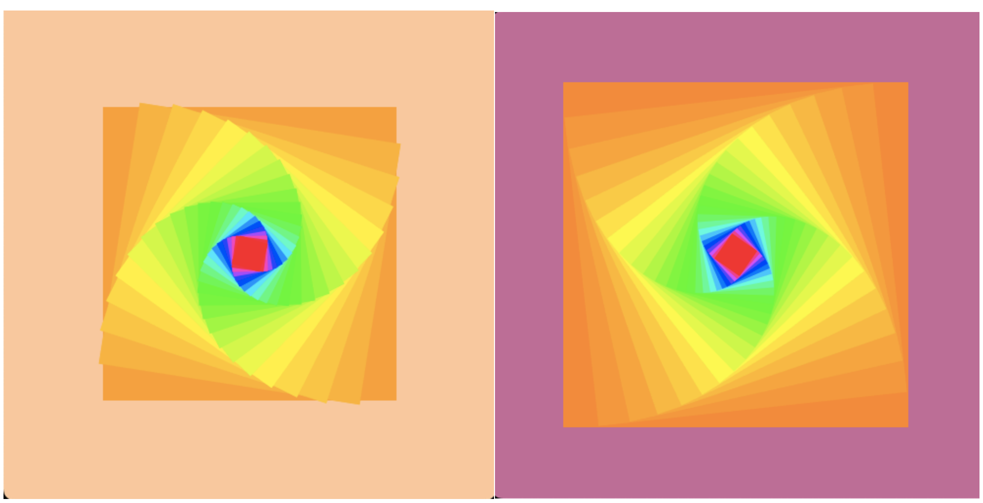

Figure 2: Fractals Image

4. Euclidean Geometry

4.1. Definition of Euclidean Geometry

Euclidean Geometry is the study of geometrical shapes; plane and solid; figures based on different axioms; a statement or proposition which is labeled as being accepted and correct; and theorems. This theorem is the main introduction for studies related to flat and plane surfaces. It is ruled one of the most important theorems in the mathematical world since it allows one to calculate distances and advanced mathematical problems.

4.2. History of Euclidean Geometry

It is possible to say that Euclidean Geometry is a well-known mathematical framework created by the well-known mathematician, Euclid of Alexandria. He was the one, that wrote one of the first and most known methodical arguments about geometry, *Elements*. It became one of the most influential books in the history of mathematics, equally for its method and its mathematical content. The method consists of presuming a small set of intuitively alluring axioms; which are statements or propositions on which an abstractly defined structure is based; and then proving other theorems from those axioms. In the book, *Elements*, 7 axioms are included for plane geometry:

"Axiom 1: Things that are equal to the same thing are equal to one another.

Axiom 2: If equals are added to equals, the wholes are equal.

Axiom 3: If equals are subtracted from equals, the remainders are equal.

Axiom 4: Things that coincide with one another are equal to one another.

Axiom 5: The whole is greater than the part.

Axiom 6 and 7: Things that are double of the same things are equal to one another. Things that are halves of the same things are equal to one another." At the beginning of the book, the five common notions, related to plane geometry, were mentioned:

"1- Any two points can be joined by a straight line.

2- Any straight line segment can be extended indefinitely in a straight line.

3- Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.

4- All right angles are congruent.

5- Parallel postulate. If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

The axioms mentioned refers to 8 concepts: point, straight line segment and line, side of a line, a circle with radius and center, right angle, congruence, inner and right angles, and sum.

The *Elements* started with plane geometry, often taught in school as the first axiomatic system and one of the first formal proofs. With plane geometry, there are multiple theorems included in that section, such as the Congruence of triangles. As we all know, two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion (Britannica), and the congruence theorems state these conditions under which this can happen. The first theorem is "The side-angle-side theorem, also known as the SAS theorem, which states if two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, the triangles are congruent" (Britannica). The earliest and probably the most used theorem deriving from the axioms is the fundamental symmetry property of isosceles triangles; that two sides of a triangle are equal if and only if the angles opposite them are equal. The parallel postulate is one of the founding explanations for the proof of this theorem: that the sum of the angles of a triangle is always 180 degrees.

The two postulates that relate the most to plane geometry are postulates 3 and 5.

The most important difference between plane and solid Euclidean geometry is that human beings can look at the plane from above, whereas three-dimensional space cannot be looked at from outside. Solid Geometry is the study of three-dimensional shapes, meaning that the analysis of angles will be used to see how those shapes have been made.

4.3. Process of creating Euclidean Geometry Art

For the Euclidean Geometry Art, I based it on Solid Geometry, which is the geometry of three-dimensional, Euclidean spaces. I started by drawing a cube. From there, decided to draw around the first one and repeated the same step three more times by enlarging the size and the length of the cubes. Then, I painted the cubes with a mixture of three different colors and outlined them in black to make them more visible to the viewers. The white dotted lines were put in the painting so that we could differentiate the cubes. For the background, I painted purple and orange to try and create an ombre effect so when I added the white dotted lines, it created a volume effect.

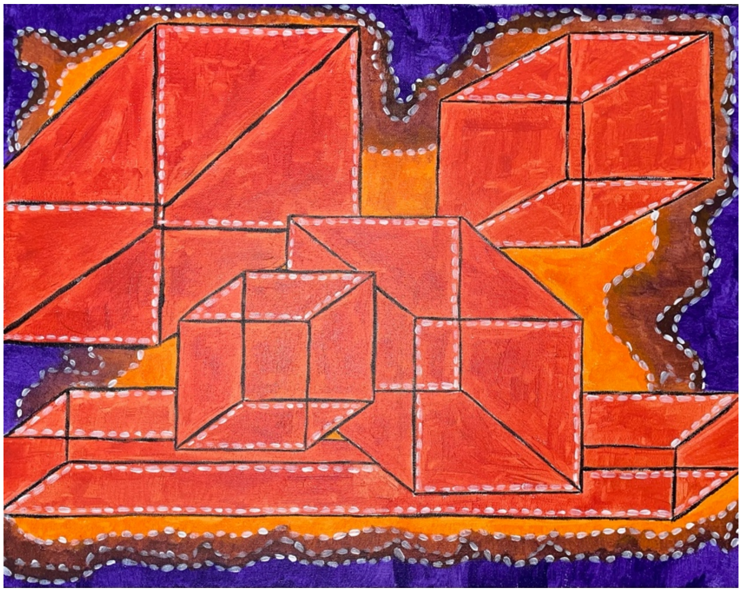

Figure 3: Euclidean Geometry Art

5. Prime Numbers

5.1. Definition of Prime Numbers

Prime Numbers are numbers greater than 1. They only have two factors, 1 and the number itself. This means these numbers cannot be divided by any number than 1 and the number itself without leaving any remainder.

5.2. History of Prime Numbers

It has been said that Egyptians were the ones to discover prime numbers, but the credit was given to Greek mathematicians since they were the ones to officially start the research of those prime numbers. In the ninth book of Euclid's work "*Elements*", the first theorem for prime numbers appeared, explaining how there are endlessly many prime numbers to exist in the world. The theorem states that "there are infinitely many prime numbers", specifically states that within the system of natural numbers the list of prime numbers is endless.

"To be more precise, this theorem claims that if we write a finite list of prime numbers, we will always be able to find another prime number that is not on the list" (de Shalit). To prove this theorem, Euclid utilized another basic theorem that he knew about, stating that "Every natural number can be written as a product of prime numbers".

To test this theorem, composite numbers must be used. A composite number is a whole number that can be written as the product of two smaller numbers. "If you pick a number that is not composite, then that number is prime itself. Otherwise, you can write the number you chose as a product of two smaller numbers. If each of the smaller numbers is prime, you have expressed your number as a product of prime numbers. If not, write the smaller composite numbers as products of still smaller numbers, and so forth" (de Shalit), and as de Shalit mentioned in the article, during the process, you will have to keep replacing any of the composite numbers with products of smaller numbers. Since it is not possible to do it forever, the process must end and all the small numbers you ended up with will not be able to be broken down, making them prime numbers. By using this process, it is now possible to say that using Euclid's proof we will find out that there is an infinitude set of prime numbers.

Many people wonder if it is possible to find out any prime numbers smaller than 100, what method would we use to find our answer, or by checking each number individually to see if it is divisible by smaller numbers? Well, mathematicians used *The Sieve of Eratosthenes*, created by one of the greatest scholars of the Hellenistic period, mathematician, and chief librarian Eratosthenes. This method works well when n is small, making it easier to determine if any natural number less than or equal to n is prime or composite. The following explains the different steps of using *The Sieve of Eratosthenes*:

"-List all integers from 2 to n

-The first integer on the list is,2, and it is prime. Mark out all multiples of 2that are bigger than 2 because they are composite.

-We do not have to compute anything, as we can simply mark out every second number starting at .2.

-The next integer on the list that is not marked out is 3, and it is prime. Mark out all multiples of 3 that are bigger than 3 because they are composite. (Note that some of these, such as 6, will already be marked out).

-We do not have to compute anything, as we can simply mark out every third number starting at *3*.

-The next integer on the list that is not marked out is 5, and it is prime. Mark out all multiples of 5 that are bigger than 5 because they are composite.

-We do not have to compute anything, as we can simply mark out every fifth number starting at 5.

-Continue in this way until there is no next integer on the list that is not marked out." (Pauli).

Different types of Prime Numbers exist in the mathematical world, such as Twin Primes, Cousin Primes, Pythagorean Primes, and many more.

Twin Primes are prime numbers that differ by two. The first known twin prime numbers are (3, 5); (5, 7); (11, 13); (17, 19) …

It is conjectured that there are infinitely many twin primes and while testing suggests that this is true, a theory to support this notion has yet to be discovered. Every twin prime pair except (3, 5) is of the form ($6n - 1$, $6n + 1$). That is the twin prime pair of (17, 19) is of the form (6x3-1, 6x3+1).

Cousin Primes are two odd prime numbers that differ by 4. In other ways, they are presented as pair of odd prime numbers in the form (p, p +4). The first few known cousin primes are: (3, 7), (7, 11), (13, 17), (19, 23), (37, 41), (43, 47), (67, 71), (79, 83), (97, 101), (103, 107), (109, 113), (127, 131), (163, 167).

A Pythagorean prime is a prime number of forms 4n + 1. Pythagorean primes are exactly the odd prime numbers that are the sum of two squares. By the Pythagorean Theorem, they are the odd prime numbers p for which $\sqrt{p}$ is the length of the hypotenuse of a right triangle with integer legs, and they are also the prime numbers p for which p itself is the hypotenuse of a primitive Pythagorean Theorem. The first few prime numbers are: 5, 13, 17, 29, 37, 41, 53, 61, 73, 89, 97, 101, 109, 113.

5.3. Process of creating Prime Numbers Art

I first divided the canvas into four sections; from there used the Pythagorean Theorem to draw triangles within those sections. I kept dividing the triangles, by drawing lines and triangles within those drawn first. Afterward, placed prime numbers and non-prime numbers randomly in the different areas on the canvas. Next, designated the colors orange, yellow, and red to the prime numbers, and the color black to the non-prime numbers. As a result, you get an interesting art piece.

Figure 4: Prime Numbers Art

6. Conclusion

Mathematics governs the universe's physical realm and since art is an expression of human imagination represented in a visual form then one can conclude that mathematics is inevitably involved in art. After all the research and seeing the different ways I could use mathematics while creating these art pieces, it is clear to me now that art is involved in art and the other way around.

- https://students.purchase.edu/melissandre.jones/senior-project/

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