The Applications of Trigonometry Throughout Human History

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Abstract

This project investigates the ways that trigonometry has been utilized throughout human history. Information provided by present-day math historians on the internet was carefully read to find out how it was used. I took a course titled Origins of Mathematics that focused on many applications of mathematics. The sections of the class material relevant to this project were used. Mathematicians have made trigonometric measurements in different areas of study such as outer space and three-dimensional shapes. The place of origin for the function sine, one of the main trigonometric functions, is revealed. The form of trigonometry that was studied centuries ago mainly consisted of lines, arcs, shapes and handmade tables. Over time, trigonometry evolved, and today's trigonometry looks at those same concepts as well angles and trigonometric functions. The results of this project show that trigonometry had a purpose of evolving knowledge into one that is more mathematical.

CHAPTER 1: GREECE.

Introduction

The evolution of trigonometry that spans over millennia is seen by people who take a close look into what people have used it for. The application of trigonometry in ancient Greece was a complicated practice. People who wanted to practice mathematics in the BC era did not have the technological resources that we have today – such as calculators and computers. Aspiring mathematicians in that era were unable to use any mathematical shortcuts and they were not provided with prewritten references to aid them in their work. Aspiring mathematicians living in that period needed to make intense use of their eyes and brain.

The ancient Greeks had a vision that bared a vast magnitude. With their vision, they were able to set revolutionary mathematical ideas in stone. A figure who has made this type of contribution is the ancient Greek mathematician and astronomer Aristarchus of Samos (310-230 BC). He created a revolutionary astronomical hypothesis: the Sun, not Earth, was the fixed center of the universe, and Earth, along with the other planets, revolved around the Sun. Aristarchus also said that all the stars were distant suns that remained unmoved and that the size of the universe was much larger than his peers believed. He demonstrated the true power of trigonometry. He was able to measure the heavens simply by the means of his mind and mathematics. Thanks to his demonstrations, we know Earth's circumference (≈ 25,000 miles) and diameter (≈ 8,000 miles), the Moon's diameter (≈ 2,000 miles), the Sun's diameter (≈ 800,000 miles), the distance between Earth and the Moon (≈ 250,000 miles), and the distance between the Moon and the Sun (≈ 100,000,000 miles).

Aristarchus completed the calculation of these large quantities with a few easy steps. His first step was to compute the angle subtended by the Moon. One can take a dime and look at it while looking at the moon's place in the sky. Make sure the dime is 90 cm away from your face to perfectly cover the moon. The diameter of a dime is 0.7 cm. Divide 0.7 by 90 to arrive at a value of 0.008 radians. The measurement of 0.008 radians converts to 0.5º.

Before the sixteenth century, most people believed that the universe was illustrated by the geocentric model. The geocentric model describes the theory that Earth is the center of the universe. The hypothesis by Aristarchus that was previously described is what defines the heliocentric model. The theory remained a hypothesis for several centuries and people continued to believe in the geocentric model.

The Greek mathematician whom many regard as the first “true mathematician” is Pythagoras of Samos. He is extremely well known because the Pythagorean Theorem is named after him. The Pythagorean Theorem states for any right-angled triangle, the square of the length of its hypotenuse is equal to the sum of the square of the other two sides. The equation is written as $a^2 + b^2 = c^2$. The theorem has been used since the time before Pythagoras was born. Despite this massive contribution to mathematics, he is claimed to be a controversial figure. He did not leave any mathematical writings and a majority of the information known about him comes from the writings of the Pythagorean philosopher Philolaus. Other known information of Pythagoras came from later Pythagorean scholars. Pythagoras established his academy in the town of Crotone, in what used to be called Magna Graecia, around 530 BC. The thought of the Pythagorean was quite unique. Although the Pythagorean thought was mostly mathematical, it was also quite mystical because of the philosophical implications. The followers of Pythagoras were split into two groups: the “mathematikoi”, who extended and developed the mathematical and scientific work that Pythagoras started, and the “akousmatikoi”, who focused on the more religious aspects from his teachers. Rivalry built up between the groups over time and they eventually dispersed.

The over-riding proclamation of Pythagoras' school was "Everything is number". This proclamation was effective, and therefore the Pythagoreans practiced a kind of number-worship and considered each number to have its own meaning. For example, number one was the generator of all numbers and number three represented harmony. Number ten was a tribute to the intellectual achievements of the Pythagoreans (Mastin, 2020).

Seeing Trigonometry in Eclipses

An example of an extensive practice of trigonometry in the BC era was in the observation of lunar eclipses. Dennis W. Duke discusses this early trigonometry in his chapter "Hipparchus' Eclipse Trios and Early Trigonometry." Duke's work is the fourth chapter of the academic journal *Centaurus* and is the only chapter in the academic journal that focuses on trigonometry. The discussion of the astronomer Hipparchus by the mathematician Ptolemy in Ptolemy's book *Almagest* is also discussed in this chapter. Duke also follows the methods of modern British astronomical historian Gerald J Toomer, who studied Hipparchus' and Ptolemy's calculations decades before Duke. Hipparchus analyzed two sets of lunar eclipses that took place one hundred years apart. These sets of eclipses were called "Eclipse Trios" because it was claimed that the moon was eclipsed three times in each year that Hipparchus made his analyses. One set of eclipses took place in Babylon in 300 BC. Hipparchus used an eccentric model to analyze this set of eclipses. An eccentric model is a model that is based on the geocentric model. The next set of eclipses took place in Egypt in 200 BC with the use of the epicycle model. The epicycle model is based on the heliocentric model.

The eccentric model that Hipparchus used to make calculations from the eclipses in Babylon is part of the Ptolemaic system. This geocentric system is another item that is found in

Ptolemy's *Almagest.* During the time when Ptolemy lived, it was naturally expected by ancient societies that the heavenly bodies of the universe would travel in uniform motion along a circle, which they deemed as the most perfect path possible. However, they incorrectly observed the paths of the heavenly bodies. As we all know today, their paths are not circular. Ptolemy's model explained the "imperfection" of the path by suggesting that the irregular movements were a combination of several regular circular motions seen in perspective from stationary Earth.

Like all models, the eccentric model came with principles. The first principle was the concept of eccentric motion. "A body travelling at uniform speed on a circular path with Earth at its center will sweep out equal angles in equal times from a perspective seen from Earth." This means if somebody is observing the uniform movement of a planet in the sky, they will see the same side of the planet in any part of the sky at any time. But if the path's center is displaced from Earth, then the body will sweep out equal angles at unequal times. The farther the body is from Earth, the slower it will appear to move.

Ptolemy stated in his work that the purpose of Hipparchus' analysis of each set of eclipses was to calculate the size of the first lunar anomaly. The lunar anomaly is defined as the angular distance of the average longitude of the Moon from the average longitude of its perigee. The lunar anomaly can be symbolized with a lowercase L. The Moon's perigee is the point in its orbit that is closest to Earth. Perigee can be symbolized with Gamma, or Γ. In the case of lunar eclipses, the lunar anomaly is presented with two complex ratios: $\frac{e}{R} = \frac{327^{2}/_{3}}{3144}$ and $\frac{r}{R} = \frac{247^{1}/_{2}}{3122^{1}/_{2}}$.

These two ratios are presented in the beginning of Duke's chapter with insufficient context. However, their significance in trigonometry is made clear a few pages into the chapter. Duke does not clarify what 'e,' 'r,' and 'R' symbolize, but based on the subject, it will be assumed that the ratios are presented in this format: angle to distance.

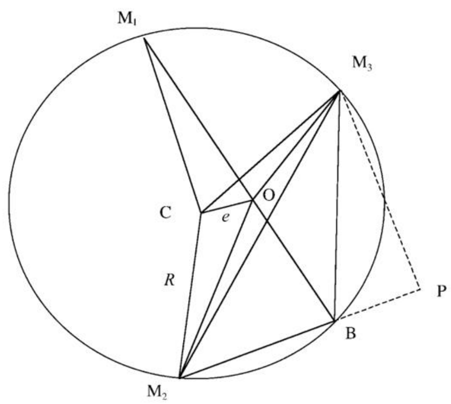

Figure 1. The Lunar Eccentric Model.

Hipparchus used a table of chords to solve the lunar anomaly. Hipparchus and Ptolemy used chord tables to make their trigonometric calculations. A chord is a line segment that connects two points on a curve. These distance scales are different from the ones we use today. The calculations that are being analyzed are given specific numerical values, as presented by Duke. Based on his discoveries, Duke assumes that the chord table is based on entries derived from a circle of circumference 21,600. That is the number of arc minutes in 360°. Some of the entries on the table of chords are $7\frac{1}{2}$ ° = crd 450, 15º = crd 897, and 30º = crd 1780. He also mentions the process of linear interpolation used to compute the chord of an arbitrary angle. Linear interpolation is defined as a method of calculating intermediate data between known values by conceptually drawing a straight line between two adjacent know values, or points. This method can be used in the illustration of graphs. Duke also notes that Ptolemy listed more steps for his calculations than what is expected from a mathematician, per say. Ptolemy went through each of his calculations numerically.

Duke then states that the computation of chord entries can be done to connect crd(180º - α) and crd(α/2). Another note that should be mentioned is that Hipparchus' ratios were originally in sexagesimal form. Sexagesimal is the form of a number with base sixty and it was assumed to only have been practiced by the Babylonians. According to Duke's presentation of data, the complex ratio $\frac{327^2/_3}{3144}$ can be converted to $\frac{6;15,11}{60}$ and $\frac{247^1/_2}{3122^1/_2}$ can be converted to $\frac{4;46,30}{60}$. He discovered a more accurate method of calculating the two complex ratios and his calculations were $\frac{336.07}{3144}$ and $\frac{248.41}{3122^1/_2}$. Unfortunately, Duke does not demonstrate how he converted the ratios from sexagesimal form. His readers simply must trust him. So, I will now demonstrate how to convert a decimal number (base ten) to a number with base sixty.

Let us look at $15{,}208_{10}$. First, we must figure out which 60^n is the largest that can fit in the given number. The largest 60^n that can be used is 60^2, or 3600. That is because 60^3 is 216,000. Divide 15208 by 3600 and we get the quotient 4 and remainder 808. The quotient of 4 is the first digit of the number in sexagesimal form. Now, we divide 808 by 60^1 and get the quotient of 13 with remainder 28. The quotient 13 is the second digit of the number in sexagesimal form. Lastly, because 28 is less than 60, we already found the last digit of the $15{,}208_{10}$ in sexagesimal form. Taking all our values, we write $15{,}208_{10}$ as $4*60^2 + 13*60^1 + 28*60^0$ and then arrive at the answer of $(4,13,28)_{60}$. The presentation of this answer is $(4*60^2, 13*60^1, 28*60^0)_{60}$.

Now let's look at converting from sexagesimal to decimal form. What is $(1,2,5,1,30)_{60}$ in decimal form? First, we see that the first digit 1 is in the place of 60^4, the highest power of 60 in this sexagesimal number. Multiply 1 by 60^4 and get 12,960,000. Second, we see that the second digit 2 is in place of 60^3. Multiply 2 by 60^3 and get 432,000. Third, the third digit 5 is in place of 60^2. Multiply 5 by 60^2 and get 18,000. Fourth, the fourth digit 1 is in the place of 60^1. Multiple 1 by 60^1 and get 60. Fifth, the fifth and last digit 30 is in place of 60^0. Multiply 30 by 60^0 and get 30. Lastly, perform 12,960,000 + 432,000 + 18,000 + 60 + 30. The number in decimal form is 13,410,090.

According to Duke, Hipparchus' calculations were not way off. From his calculations, Duke concluded that Hipparchus must have done a long sequence of number rounding in order to arrive at his answers. He also claims that based on looking at Ptolemy's thorough list of steps, a miscalculation was bound to happen because of failing to be concise. Neither he nor the other Greek mathematicians could not be concise in their work.

Duke discusses how he solved the trigonometric solution of the Babylonian lunar eclipse trio. It is noted in his chapter that there is no direct evidence that Hipparchus used the solution. The evidence is rather found in Ptolemy's *Almagest*. The trigonometric solution is explained in algebraic form and the three eclipses are represented as $M_{1,}$ $M_{2,}$ and M_3. Six given angles are represented in the model. Three are represented by α, and the other three are represented by zeta, or ζ. The symbols C and O are present, and it is assumed that C represents the center point and O represents the observer's point. Each of the six angles can be solved with these formulas: M_1*C *M_2 = α_1, M_2*C *M_3 = α_2, M_1*C *M_3 = α_3, M_1*O*M_2 = ζ_1, M_2*O*M_3 = ζ_2, and M_1*O* M_3 = ζ_3. He uses two different symbols for the six angles instead of naming them $\alpha_{1\text{-}6}$. This condition is created in order to remember what angles associate with C and what angles associate with O. Point B appears to be a random spot on the eclipse trail within the eccentric model and the line segment M_1O intersects the circle at that point. OB = d then becomes the formula that determines the essential and theoretically arbitrary length scale in the eccentric model. Duke continues to replace point C with point B in the angle formulas. The formulas are later transformed to M_1*B *M_2 = $\frac{\alpha 1}{2}$. The other two angles α are split in half as well. The observation that B is the midpoint of eclipses M_2 and M_3 may have caused this. It might appear tricky that we have more than three points to look at one the eclipse trail. But when a conclusion is made, it is possible that this is the strategy of calculating the most accurate solution of the lunar anomaly.

Duke also uses the values from the table of chords in his calculations. One example of this representation is how the sides of M_2M_3B are written: $\frac{\mathrm{M2(B)}}{d} = \frac{\mathrm{crd2}(180º-\zeta 1)}{crd2(O)(M2)B}$. Duke is looking at an eccentric model and dealing with two types of lines. Knowing that one type of line is “on screen” and the second type of line is “off screen”, maintaining consistency might become tricky because one might question that much reliability should be placed on only one model. A correct numerical implementation of the calculation requires the conversion of the intervals in time and longitude into angles in anomaly and longitude. Duke’s presentation of the conversation confuses me – mainly because he brings in new symbols, omega, lowercase delta and lambda. At a point in calculating the solutions for the Babylonian eclipse trio, Duke finds the following angles using Hipparchus’ Babylonian eclipse trio intervals: α_1 = 159;59,14, α_2=153;24,33, α_3=46;36,11, ζ_1=153;5,37, ζ_2=155;23,59, ζ_3=51;30,23 (Duke, 2005).

Duke used Toomer’s methods to guide him through his own calculations. According to Duke, the error that Hipparchus made was that he found 51;19,37 for ζ_3. While doing the “correct method” of calculations, he tried to be as precise as he could. It is possible that the amount of time Duke used to do so was a fraction of the time that Hipparchus used to make his calculations thousands of years ago. The values for R and R’ were found to be not plausible for the radius of a reference circle.

I believe the purpose of Duke’s analysis of the lengthy example of early trigonometry is to show readers the differences between the approach from mathematicians living in ancient times and the approach from today’s mathematicians. Before coming across this chapter on

Hipparchus' analysis of the eclipses, I only watched lunar eclipses with a simple lens; meaning I would look for how much red is projected onto the moon – depending on the Earth's atmosphere. After observing all the mathematics behind eclipses, it appears that astronomy and trigonometry have a big connection with each other.

CHAPTER 2: CHINA.

Professor Jiang-Ping Jeff Chen, a professor at St. Cloud State University, explains the use of trigonometry in China in the 1600s and 1700s. He also cites the works of Chinese mathematics that date back as early as the eleventh century. In the seventeenth and eighteenth century, Chinese mathematicians used trigonometry to measure solids and to calculate measurements in the field of astronomy. The measurement of pyramids is what really sticks out in this history. Their form of trigonometry contained arcs and excluded some of the trigonometric methods that we know today. One of the methods that was applied by Chinese mathematicians was circle division. It involves illustrating right triangles and then applying the Pythagorean theorem to obtain the lengths of the sides of the illustrated triangles. This method is found in the treatise titled "Nine Chapters" by Liu Hui (Chinese mathematician, 220-280 AD). Hui performed this method by beginning with a hexagon inscribed in a circle. For his first step, he bisected one side of the hexagon to form a right triangle with half of the side of the hexagon as its base and the radius of the circle as its hypotenuse. He then used the Pythagorean Theorem to obtain the height of the triangle, or as Chen calls it, the altitude. In the second step, Hui formed a smaller right triangle with a smaller base, smaller height, and smaller hypotenuse. The new base is equal to the difference of the radius and the altitude of the first triangle and the new height is equal to half the side of the hexagon that he started with. Hui repeated these steps and ended up calculating the sides of inscribed 24-sided, 48-sided, and even 96-sided polygons in the same circle. One might think: why did he make polygons with so many sides? It turns out that was the way he found several angles.

Another trigonometric method found in Chinese trigonometry was arc-sagittae. This method describes the procedures for finding the length of one of the following: a chord on a circle, the arc subtending it, the sagitta (the line segment connecting the midpoints of the chord and its arc), and the measure of the area bounded by the chord and its arc. Shen Kuo was a Chinese astronomer and mathematician who used this method. He lived from 1031 to 1095. The procedures can be summarized by this equation: a = c + $\frac{s^2}{r}$, where ‘a’ is the length of the arc, ‘c’ is the chord, ‘s’ the sagitta, and ‘r’ the radius. Years later, in around 1281, Kuo’s arc-chord-sagitta relationship was employed by Wang Xun and Guo Shoujing in the algorithms for converting the position of the sun on the ecliptic to equatorial coordinates. They used this in *Shoushi li,* or “Season Granting Calendar”. In the format described, Wang and Guo used half of the chord, which is dubbed the sine line segment of the half-arc. Therefore, the sagitta of the arc is dubbed the difference of the radius and the cosine line segment of the half-arc in question. Chen describes the connection of Xun and Shoujing’s work to Kuo’s work as the body of work that provides the relations between the length of an arc and the values of sine and cosine of the half-arc.

According to the treatise “Grand Measurement,” trigonometric functions were represented as eight different line segments. Each of them was assigned to an angle or an arc and they were all geometrically portrayed on a quarter of a circle. Chen then explains that knowing how to form families of similar right triangles is key to solving problems in spherical trigonometry, or the study of the relationships between trigonometric functions of the sides and angles of spherical triangles. Spherical triangles can be drawn when looking at diagrams of the planets and stars. Then, in the treatise “Complete Principles of Measurement”, it is revealed how the Jesuits brought their mathematical input to the Chinese. The Italian missionary Giacomo Rho used the following figure to analyze problems involving spherical right triangles. It encompasses four types of problems and generates the results as numerous sets of four *lü* (“related quantities” in English) of trigonometric lines associated with the arcs and angles of the spherical right triangle. The points are represented by Chinese characters. This configuration presented by Chen is fascinating and is an excellent visual representation of an introduction to spherical trigonometry. In addition, he presents a simplified version of the configuration, which resembles a thin slice of the sphere.

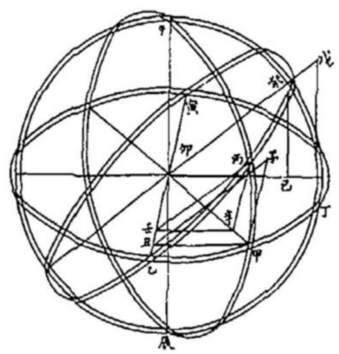

Figure 2. A geometric configuration *in Complete Principles.*

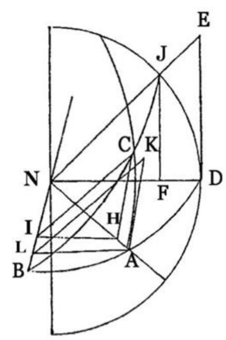

Figure 3: A simplified version of Figure 2, also found in *Complete Principles.*

In figure three, Chen identifies the following plane triangles in the sphere: ALK, HIC, DNE, and FNJ. He explains that they were constructed by the following steps: (1) Construct the tangent line DE and sine FJ for the arc DJ and obtain two right triangles DNE and FNJ. Arc DJ is equal to the angle ABC. From this, we have DE = tan(ABC) and FJ = sin(ABC). (2) Construct the sine segment AL for the arc AB and the tangent segment AK for the arc AC and form the right triangle ALM.

Then, AL = sin(AB). (3) Construct the sine segment CH for the arc AC, draw HI parallel to AL, and form the right triangle HIC so that AK = tan(AC) and CH = sin(AC).

Calculating spherical triangles evolved over time. Wending Mei (1633-1721) used similar right triangles to on the sphere, completely the configuration of solids without utilizing it. But later, he directly applied three-dimensional solids to find families of similar right triangles as well as finding families of similar right triangles on the two-dimensional development of the same solids. In, “Essentials of Spherical Trigonometry”, Mei followed Rho’s practice in “Complete Principles” by directly forming similar right triangles in the sphere. But he added one extra right triangle. From this, a tetrahedron is drawn from the sphere. Chen mentions that Mei did not recognize any solids in his practices. So how was he able to identify families of similar right triangles? His years of experience in experimenting with spherical trigonometry have strengthened his skills (Chen, 2010).

CHAPTER 3: INDIA. LEADING TO THE ISLAMIC WORLD.

A big contribution to trigonometry came from India. In the sexagesimal system, division or multiplication by 120 can be compared to multiplication or division by 20 in the decimal system. With this, a formula by Ptolemy, which is not explicitly stated in the article, can be written as $\frac{c}{120}$ = sinB, where B = $\frac{A}{2}$. The way to look at this equation is that the arc B subtends the half-chord C. This is how the modern sine function used to be represented. Indian astronomer and mathematician Aryabhata I wrote the first table of sines in his work, titled *Aryabhatiya*. He used unique words in his work. The Hindi word *Ardha-jya* was the word for half-chord. Sometimes, Aryabhata would turn this word around – to *jya-ardha* ("chord-half"). In due time, the word was shortened to *jya* or *jiva*. Years later, Muslim scholars got their hands on *Aryabhatiya* to translate it to Arabic. They kept the Hindi word *jiva*, but they did not translate its meaning. In a Semitic language like Arabic, words consist mostly of consonants. Common usage allows for the pronunciation of missing vowels to be understood. Thus, *jiva* can also be pronounced as *jaib* or *jiba*. *Jaib* is Arabic for "fold". Later, the Arabic translation of *Aryabhatiya* was translated to Latin. *Jaib* became *sinus*, the Latin word for bay. This word first appeared in the writings Gherardo of Cremona (1114-1187), an Italian translator who translated many ancient Greek texts into Latin. After more translations, the word *sine* was born. Its well-known symbol of abbreviation, 'sin', was first used in 1624. The notations for the five remaining trigonometric functions were introduced shortly after the introduction of 'sin'.

Arab and Jewish scholars living in Spain, Mesopotamia, and Persia possessed a great knowledge of trigonometry. When Europe entered the Dark Ages in the fifth century, there came a lack of insignificant scientific and cultural advancement. The Dark ages were "lit up" by these scholars who shared their knowledge with everybody. The first table of tangents and cotangents was introduced. It was constructed by Iranian astronomer Habash al-Hasib in 860. A Syrian astronomer named al-Battani gave a rule for finding the elevation of the Sun, θ, above the horizon in terms of length *s* of the shadow cast by a vertical gnomon of height h.[1] His rule can be written as $s = \frac{h sin(90° - \theta)}{sin\theta}$. It is equivalent to the formula $s = hcot\theta$. With his rule, al-Battani was able to construct a "table of shadows", which was a table of cotangents, for each degree from 1° to 90°.

Spherical trigonometry was the main interest of scholars[2] until the sixteenth century. This is a consequence of the predominance of astronomy among the natural sciences. Studying spherical trigonometry means studying spherical triangles. A spherical triangle is a figure that is formed on the surface of a sphere by three great arcs of great circles[3]. The first definition of a spherical triangles is found in Book 1 of the three-book treatise called *Sphaerica,* written by Menelaus of Alexandria. In his work, Menelaus developed the spherical equivalents of Euclid's propositions for planar triangles. The differences between spherical triangles and planar triangles are as follows. Two spherical triangles whose angles are equal in pairs are congruent. Whereas in the case of planar triangles, equal angles means that the triangles are similar. Also, the sum of the angles of a spherical triangle is always greater than 180°. Imagine a middle school trying their very best to maintain the fact that the sum of all angles in a triangle is always 180°; then suddenly a teacher gets a crazy idea and introduces spherical triangles. They would probably feel overwhelmed by the concept.

[1] A gnomon is a device used for timekeeping
[2] Britannica does not specify which scholars looked at spherical trigonometry. So it is assumed that it is referring to European and Middle Eastern Scholars.
[3] A great circle is a circle whose centers coincide with the center of a sphere

Al-Battani and Persian scholar Nasir al-Din al-Tusi continued to develop spherical trigonometry and brought it to its present form. Al-Tusi was the first to write a book on trigonometry solely based on astronomy. However, the first modern book completely devoted to trigonometry first appeared in the Bavarian city of Nürnberg in 1533. The book was titled *On Triangles of Every Kind* and it was written by German mathematician Regiomontanus. The book contains all the theorems needed to solve triangles whether they're spherical or planar. At this period, symbolic algebra had yet to be invented, so all the theorems were expressed in verbal form. Interestingly, the law of sines is essentially stated in the modern way. *On Triangles of Every Kind* was deeply admired by future generations of scientists, including the significant Nicolaus Copernicus. Copernicus studied through the work thoroughly and his annotated copy survives.

The final major development in classical trigonometry was the invention of logarithms by Scottish mathematician John Napier in 1614. His tables of logarithms facilitated the art of numerical computation – including the compilation of trigonometry tables. His contribution was hailed one of the greatest to science (Maor, year unknown).

CHAPTER 4. EGYPT.

The ancient Egyptians strongly made use of trigonometry and the practice is found illustrated on papyri. They strongly demonstrated their roles as scribes. Their practice was illustrated in the form of trigonometric problems that seek the measurements of figures such as rectangles and triangles of a given base and height. One of the problems that were found looked at a rectangle with an area of 12 and a height that is $\frac{3}{4}$ times (written as $\frac{1}{2} + \frac{1}{4}$ times) its base. This problem was found in the Golenishchev papyrus, and a unit of measurement is not given. The process of solving this problem is similar to the process of solving an algebraic equation – except a letter is not used to represent the unknown value. They also had an interesting procedure for finding the area of a circle, which is found in the Rhind papyrus. First, $\frac{1}{9}$ of the circle's diameter is discarded and the result is squared. For example, a circle with diameter 9 has a set area of 64 because $\frac{1}{9}$ of 9 is 1, 9 – 1 is 8, and $8^2 = 64$. The scribe writing this rule recognized that the area of a circle is proportional to the square of the diameter and assumed the value of $\frac{64}{81}$ for the constant of proportionality. The constant of proportionality is defined as: the ratio that relates two given values in what is known as a proportional relationship. It is stated that the approximated value is a good estimate – it is only 0.6% larger than the true value. The true value for the area of a circle with a diameter of 9 is 63.6173. In fact, there is nothing in the papyri indicating that the scribes were aware that this rule was only approximate. This condition later resulted in additional remarkable discoveries.

One of these remarkable discoveries is the rule found in the Golenischev papyrus for the volume of a truncated pyramid. A truncated pyramid is a pyramid with its top portion sliced off, bearing the apex with it. In the problem, the height is set to 6, the base to be a square of side 4, and at the top a square of side 2. First, the height is divided by three, resulting in two. Then, that result is multiplied by twenty-eight, resulting in fifty-six as the volume of the pyramid. An interesting note that is made in the article is that twenty-eight can be computed as (2*2) + (2*4) + (4*4). From this computation, a general formula is formed: $(\frac{h}{3})$ $(a^2 + ab + b^2)$. Apparently, the process of how ancient Egyptian scribes derived the rule is part of an ongoing debate. However, it is assumed that they were aware of rules related to the rule for the volume.

The ancient Egyptians implemented the equivalent of similar triangles to measure distances. They specifically used pyramids to aid their measurements. "The *seked* of a pyramid is stated as the number of palms in the horizontal corresponding to a rise of one cubit (seven palms)." Visually, a seked is a right-triangular slice of a pyramid. For example, if the seked is $\frac{21}{4}$ and the base is 140 cubits, the height becomes $\frac{280}{3}$ cubits, or 93.3333 cubits. A story that has gone around is that Greek philosopher Thales of Miletus had measured the height of pyramids by means of their shadows. However, in the light of seked computations, this report must indicate an aspect of Egyptian surveying that extended back more than a millennium before the time of Thales.

The ancient Egyptian papyri bear witness to a mathematical tradition that is tied to practical accounting and surveying activities of the scribes. After the scribes finished performing more serious computations, they loosened the topic up a bit. For example, they looked at a problem that seeks the total from seven houses, seven cats per house, seven mice per cat, seven ears of wheat per mouse, and seven hekat (ancient Egyptian volume unit) of grain per ear. Going from more serious to less serious problems demonstrates the practicality of ancient Egyptian trigonometry (Berggren, year unknown).

Works Cited

Berggren, John, et al. "Mathematics - Mathematics in Ancient Egypt." *Encyclopædia Britannica,* 2019, www.britannica.com/science/mathematics/Mathematics-in-ancient-Egypt. Accessed November 2021.

Chen, Jiang-Ping. "The Evolution of Transformation Media in Spherical Trigonometry in 17th- and 18th-Century China, and Its Relation to 'Western Learning.'" *Historia Mathematica*, vol. 37, no. 1, Feb. 2010, pp. 62–109, 10.1016/j.hm.2009.06.003. Accessed October 2021.

Duke, Dennis W. "Hipparchus' Eclipse Trios and Early Trigonometry." *Centaurus*, vol. 47, no. 2, Apr. 2005, pp. 163–177. *EBSCOhost*, doi:10.1111/j.1600-0498.2005.470204.x. Accessed October 2021.

Jones, Alexander R. "Ptolemaic System." *Encyclopedia Britannica*, Encyclopedia Britannica, Inc., https://www.britannica.com/science/Ptolemaic-system. Accessed March 2022.

Maor, Eli, and Raymond Barnard. "Trigonometry - India and the Islamic World." *Encyclopedia Britannica,* www.britannica.com/science/trigonometry/India-and-the-Islamic-world. Accessed November 2021.

Mastin, Luke, and William Smith. "Pythagoras of Samos: Famous Mathematician." *The Story of Mathematics - A History of Mathematical Thought from Ancient Times to the Modern Day*, https://www.storyofmathematics.com/greek_pythagoras.html/. Accessed March 2022.

Wikipedia Contributors. “Lunar Theory.” *Wikipedia*, Wikimedia Foundation, 15 May 2022, https://en.wikipedia.org/wiki/Lunar_theory. Accessed November 2021.