SPEEDY 3-D: A STUDY OF THE EFFECTS OF SPATIAL ABILITY AMONG COLLEGE STUDENTS

By

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State University of New York at Fredonia
Department of Mathematical Sciences

CERTIFICATE OF PROJECT WORK

We, the undersigned, certify that this project entitled SPEEDY 3-D: A STUDY OF THE EFFECTS OF SPATIAL ABILITY AMONG COLLEGE STUDENTS by Sara A. Maiorana, candidate for the degree of Master of Science in Education, Mathematics Education (7-12), is acceptable in form and content and demonstrates a satisfactory knowledge of the field covered by this project.

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Abstract

This research investigates how college students' spatial skills vary by age, gender, college major and additional factors. Specifically, it explores students' abilities to visualize two-dimensional air nets corresponding to two-dimensional illustrations of three-dimensional cubes. Also, this study examines how the use of a tangible air net manipulative affects performance. During this study, students answered a five-problem quiz involving matching and creating two-dimensional air nets for a given cube and vice versa. The results of the assessment were compared to those from a survey on the students' age, gender, college major, ethnicity, and students' perceptions of which problems were the most difficult and least difficult. It was hypothesized that male mathematics majors with access to a manipulative would perform best on the given spatial skills problems. The results of this study indicated that gender and college major had no statistical significance in spatial ability test score. Additional results revealed that there was a significant difference in test score by class, particularly with the use of a manipulative, and that the most difficult problem and least difficult problem on the assessment were both of the unfolding-type spatial ability task. These findings have noteworthy implications for in-service and pre-service mathematics teachers, particularly at the secondary level, regarding lesson planning and implementation when teaching spatial reasoning.

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Introduction

This research investigates how college students' spatial skills vary by age, gender, college major and additional factors. Specifically, it explores students' abilities to visualize two-dimensional air nets corresponding to two-dimensional illustrations of three-dimensional cubes. Also, this study examines how the use of a tangible air net manipulative affects performance.

Spatial reasoning and visualization is a crucial skill that must be mastered in the mathematics curriculum, particularly in geometry. Having taught two sections of a Regents geometry class during my student teaching placement in a low income rural school district, I noticed that students often struggle to visualize a locus of points or geometric figures. After analysis of the Common Core curriculum and state assessments, it is evident that spatial visualization has become increasingly prominent. Therefore, it is important for mathematics educators to understand how to teach these geometry topics while incorporating spatial visualization tasks.

Spatial ability plays a major role in other fields such as engineering and the arts. Many people would consider Science, Technology, Engineering, and Mathematics (STEM) fields and the arts at opposite ends of the spectrum, yet spatial skills are the common overlap. In fact, I believe that art majors specializing in drawing and sculpting do not realize that they are using this mathematical tool in their work. My study focuses on determining the specific aspects of spatial ability that are the most challenging for students.

> *It is hypothesized that factors including age, gender, and college major will influence students' spatial ability in visually matching two-dimensional air nets with two-dimensional illustrations of three-dimensional geometric figures. It is also hypothesized that students with access to a hands-on manipulative will perform better on these types of*

problems than students without access to a hands-on manipulative. Furthermore, it is hypothesized that male college students majoring in STEM fields will be able to successfully match these figures more accurately than their female counterparts.

The hypothesis was tested through administering two versions of a spatial visualization test to a sample of students from the population. An example of these spatial ability problems is shown in Figure 1.

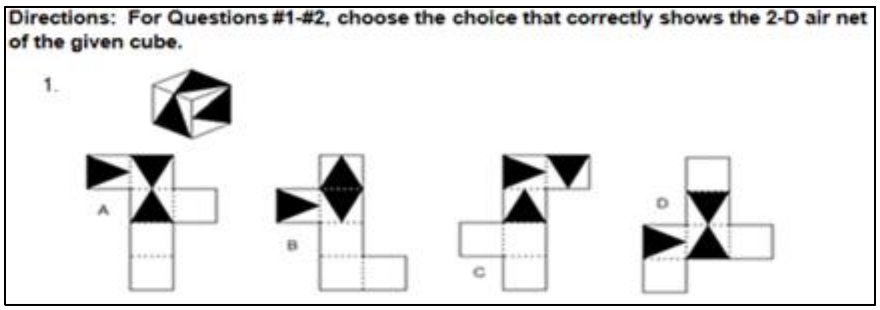


Figure 1. Sample Question from the Spatial Ability Test

One test was administered as a paper-and-pencil test alone, while the second test was administered with a paper manipulative of an air net that could be drawn on, folded, etc. Following the assessment, students were asked to respond to a survey that indicated various factors that may impact spatial ability as well as a brief analysis of the specific problems and topics that were the most difficult. A variety of related research studies and results are provided in the next section.

The following literature review discusses the role of spatial ability in education, different strategies that students use in spatial reasoning, and successful techniques for improving students' spatial skills. Also, there is an analysis of how contextual factors, specifically age, gender, and course of study, impact the ability to complete spatial ability tasks. Finally, this literature review provides sample problems and geometric puzzles of how spatial ability has previously been measured.

Literature Review

The purpose of this literature review is to examine the existing research regarding how factors such as gender, age, and course of study influence spatial ability. This literature review is divided into four categories: Defining Spatial Ability, Teaching Spatial Ability, Assessing Spatial Ability, and Factors that Influence Spatial Ability. The first section introduces the concept of spatial ability and its relation to mathematics. Next, a discussion of how students learn and improve their spatial skills is given with descriptions of two intervention strategies that have been shown to be effective. The following section features a summary of relevant research evaluating how spatial ability varies by gender, age, and educational background. Finally, the literature review ends with examples of various forms of assessments used in recent studies that measured spatial ability.

Defining Spatial Ability and its Importance to Education

Spatial ability is absolutely essential to living in a three-dimensional world. According to Patkin and Dayan (2012), spatial reasoning skills affect our ability to "navigate from place to place, detect an object moving towards us, estimate quantities, understand drawings and charts, and compose various items" (p. 179-180). Spatial ability is listed as one of Howard Gardner's multiple intelligences, focusing on non-verbal cognitive abilities (Gardner, 2006). Spatial intelligence covers a wide variety of right-brain thinking and processes including visual, motor, analytic, and behavioral skills (Meadmore, Dror, & Bucks, 2009; Patkin & Dayan, 2012). Generally, spatial ability can be divided into three categories. These categories are mental rotation (MR), spatial visualization (SV), and spatial orientation (SO).

The three categories of spatial ability play a major role in geometric thinking and understanding (Unal, Jakubowski, & Corey, 2009). Educational research regarding spatial ability dates back to the 1940s, but behavioral analysis of spatial ability has been noted earlier by research conducted in the field of psychology. Spatial reasoning skills are absolutely essential for improving mathematical comprehension and mathematical pedagogy for mathematics educators. In fact, the National Council of Teachers of Mathematics (NCTM) identified spatial ability as a central goal in high school mathematics, specifically in geometry courses (The National Council of Teachers of Mathematics, 2013). The role of spatial ability is recognized in various other subjects including chemistry, geosciences, physics, and engineering. Patkin and Dayan (2012) suggest that one's spatial ability influences and can determine one's level of achievement in mathematics and other related fields.

Learning and Teaching Spatial Ability

Mathematics educators Dina van Hiele-Geldof and Pierre M. van Hiele developed a model to represent an individual's level of geometric thought and development that has been used for over 20 years (van Hiele & van Hiele, 1959). The van Heile model features five levels of geometric cognitive development: (0) Visualization, (1) Analysis, (2) Abstraction, (3) Deduction, and (4) Rigour, as described in Figure 2 below.

- *Level 0 (Visualization):* The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual consideration of the concept as a whole without explicit regard to properties of its components.
- *Level 1 (Analysis):* The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established.
- *Level 2 (Abstraction):* The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of set of properties in determining a concept.
- *Level 3 (Deduction):* The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions and theorems.
- *Level 4 (Rigour):* The student can compare systems based on different axioms and can study various geometries in the absence of concrete models.

Figure 2. Description of the Levels of van Hiele's Model of Geometric Development

Although spatial ability and the van Hiele model of geometric thinking have previously been studied separately, Unal et al. (2009) linked one's level of geometric development to spatial ability. Generally, students who are at lower levels of geometric thinking also have lower spatial abilities. In a secondary level geometry setting, students must have a level of understanding equivalent to van Hiele's level (3) or higher in order to be successful. To assist learners with low spatial ability, research recommends including appropriate instructional tasks such as hands-on paper-folding and tracing, or alternative implementation strategies.

Recent studies show that computer-based instruction can dramatically improve spatial ability (Hannafin, Truxaw, Vermillion, & Liu, 2008; Patkin & Dayan, 2012). A study conducted by Hannafin, et al. (2008) used Geometer's sketchpad as a form of computer-based instruction to teach spatial skills. After students' existing spatial skills were evaluated via a pre-test, students were divided into a control group and an experimental group. The control group received a tutorial booklet to learn about spatial reasoning, while the experimental group completed an online lesson using Geometer's sketchpad. The researchers concluded that the students in the experimental group expressed more interest in the computer-based instruction. Post-test results also indicated that, aside from being well-liked, the Geometer's sketchpad program proved to be an effective method of spatial ability instruction.

Another successful instructional method supported the idea of computer-based instruction, as well. Patkin and Dayan's 2012 study followed a similar outline consisting of a pre-test, instructional period, and post-test. The intervention unit was divided into three phases. The first phase, called the challenge phase, introduced high school students to Op-Art, a program that uses shading and artistic illusions to create depth and movement in geometric figures. The second phase of the intervention used computer-based instruction to represent and build 3-D models. These spatial ability tasks were performed in pairs. Finally, the last phase of the intervention unit featured a class discussion where students summarized and formalized what they had learned. Post-test scores provided evidence of substantial improvement in spatial ability in the experimental group when compared to students in the control group. This result has significant implications for mathematics educators regarding spatial ability topics, particularly in accounting for demographic factors that may contribute to student difficulties.

Factors that Influence Spatial Ability

Several studies analyzing spatial skills aimed to compare abilities between genders, various age groups, and other influencing factors. The purpose was to determine not only *if* these factors play a role in spatial skills, but also *how* these factors influence one's ability. This subsection specifically discusses gender, age, and educational background as potential influential factors.

Presence of a Gender Difference

Much research exists debating whether or not a gender gap exists for spatial ability and/or general mathematics performance. Some argue that there is no gender difference in spatial reasoning skills (Felix, Parker, Lee, & Gabriel, 2011; Rahman, Bakare, & Serinsue, 2011), but the majority of research provides evidence of a male advantage (Geiser, Lehmann, & Eid 2008; Jansen & Heil, 2010; Li & O'Boyle, 2011; Neubauer, Bergner, & Schatz, 2010; Quaiser-Pohl & Lehmann, 2002; Robert & Cheverier, 2003). Specifically, these studies indicated a male advantage according to mental rotation test scores.

Research conducted by Geiser, et al. (2008) found that males scored higher on mental rotation tests that analyzed spatial visualization of three-dimensional objects. Similar mental rotation tests provided the same results (Jansen & Heil, 2010; Li & O'Boyle, 2011). Regardless of additional factors such as age, Jansen and Heil (2010) concluded that males generally provided more correct answers than females. In a 2011 study, Li and O'Boyle evaluated the presence of a gender difference while subjects concurrently utilized verbal memory and spatial memory. While this dual-task assessment provided evidence of improved spatial ability for both males and females, males continued to have significantly higher scores on Mental Rotation Tests (MRT). Aside from test scores and performance, Robert and Cheverier (2003) indicated that men also had faster response times on such assessments, which suggested that right-hemisphere spatial processing speed is dominant in males.

Additional research aimed to compare spatial skill performance by gender on multiple types of mental rotations tests (Neubauer, et al., 2010). The authors used two different forms of mental rotation tests: one utilizes two-dimensional illustrations of three-dimensional objects, the second projects the objects in three-dimensional space. While male participants outperformed female participants in the two-dimensional projections, there was no evidence of a significant gender difference in the three-dimensional rotation tests. The format of the spatial skills assessment may also affect spatial ability across genders. Jansen and Heil (2010) found that there was a large discrepancy when they compared the performance of participants male and female on a paper-and-pencil MRT among young adults (ages 20-30). However, when participants were assessed via computer simulations, the comparison of scores by gender showed no substantial difference. The implementation of time as a factor also affected the presence of a gender difference (Jansen & Heil, 2010). One study conducted by Jansen and Heil (2010) found that, while males outperformed females in a timed MRT, a comparison of the scores in an untimed condition eliminated any evidence of a gender gap in performance.

Causes and Impact of the Gender Difference

A difference in cognitive abilities among men and women may be to blame for the gender gap in spatial reasoning. In fact, Estes and Felker (2012) noted that spatial ability tasks including mental rotation of three-dimensional abstract objects have the most prevalent gender gap of all cognitive sex differences. Keith, et al. (2011) found that females tend to excel in left-hemisphere verbal working memory and processing speed, while cognition in males shows advantages in right-hemisphere visuospatial memory. As a result, spatial skills can be used to predict mathematical ability later in adulthood (Ganley & Vasilyeva, 2011).

Further research investigated whether or not confidence had a causal relationship in spatial ability, specifically in terms of mental rotation (Estes & Felker, 2012). In the four experiments Estes and Felker conducted, the researchers manipulated subjects' confidence with these visuospatial concepts to determine its effect on MRT scores. Results supported the hypothesis that confidence facilitates the sex difference in MR abilities, implying that the gender difference lies solely in MRT performance as opposed to actual spatial ability. Similarly, the results of surveys conducted by Quaiser-Pohl & Lehmann (2002) indicate that women have higher anxiety levels and negative attitudes towards mathematics and related fields. Consequently, these feelings affected their choice of educational and occupational paths and contributed to the underrepresentation of women in STEM fields (Benbow, et al., 2000; Quaiser-Pohl & Lehmann, 2002). However, gender differences are not the only important topic of study in terms of comparing spatial skills.

Spatial Ability across Age Groups

Cognitive development varies not only by gender, but also by age. In fact, Jansen & Heil (2010) imply that age may contribute to the gender gap in spatial ability for certain age groups.

In a study of people ranging in age from nine to 23 years old, mental rotation test scores improved with age (Geiser, et. al., 2008). Jansen & Heil (2010) also proposed the idea that mental rotation test scores peak in young adulthood. Specifically, participants between 20 and 30 years of age performed better than participants who were middle-aged or older. MR test scores for older adults were significantly worse and showed a decrease in spatial ability. In fact, the gender difference in test scores was minimized as subjects aged past about 40 years of age (Jansen & Heil, 2010; Mammarella, et al., 2013).

Cognitive studies infer that spatial ability follows a parabolic path for both males and females as visuospatial processes eventually decline with age. Meadmore, et al. (2009) explained that the right hemisphere of the brain responsible for such reasoning and thinking is more susceptible to age-related deterioration than the left hemisphere, which confirms the reduction of the gender gap in older adults when compared to their younger counterparts. Other research proposed that poor spatial ability in elderly adults is linked to their approach to mental rotation tasks. Studies have shown that older adults tend to use a more holistic strategy by attempting to rotate the entire figure, as opposed to more of a piece-by-piece approach often used by younger adults and adolescents (Jansen & Heil, 2010). Aside from a cognitive perspective, an individual's personal experiences relative to his age can influence how he learns and processes spatial tasks. For example, the time period in which one was raised, the educational system at that period in time, and additional background information are suggested to have played a role in spatial skills.

Educational Background and Spatial Ability

The gender difference in spatial ability was evaluated by Hoffman, et al. (2011) in terms of living environment and other outside factors. He found a significant variance in the presence of a gender difference between matriarchal and patriarchal societies, thus proposing that nurture plays a role in the development of one's spatial and visualization skills. These societies differ in values and education, offering the idea that background education may play a role.

The effect of educational background on spatial ability has been an important topic of study. The outcomes of Brownlow, et al.'s (2003) study demonstrated that a background in college level science affects spatial ability. A more thorough network of science knowledge led to improved spatial ability performance. A comparison of scores by gender yielded the smallest gender gap for students with an extensive background in science. Physical science majors also showed better mental rotation ability compared to college students studying social sciences, arts, or humanities (Li & O'Boyle, 2011; Quaiser-Pohl & Lehmann, 2002). Educational background, along with the other factors mentioned above, should all be considered in order to create spatial reasoning tests to assess these skills.

Assessing Spatial Ability

Spatial visualization or spatial orientation tasks are often measured in standard paper-and-pencil tests, including state curriculum assessments and the DAS-II Cognitive Tests (Ganley & Vasilyeva, 2011; Keith, Reynolds, Roberts, Winter, & Austin 2011). These forms of spatial reasoning often involve spatial working memory as well. Thus, simple storage tasks like maintaining patterns are used to evaluate spatial (object location) memory and overall spatial skills (Mammarella, Borella, Pastore, & Pazzaglia, 2013). Rahman, Bakare, and Serinsu, (2011) also assessed spatial location memory by requiring participants to recall abstract geometric designs.

In several recent studies, a wide variety of mental rotation tests are used to evaluate spatial ability, including the Purdue Visual Rotations Test, the Peters, et al. version, and the

Vandenberg and Kuse Mental Rotations Test (Brownlow, McPheron, & Acks, 2003; Geiser, Lehmann, & Eid, 2008; Quaiser-Pohl & Lehmann, 2002). Several variations of these mental rotation tests exist. One way the mental rotation tests are varied is by altering the format in which the test is administered. For example, Jansen and Heil (2010) studied the difference in performance on a paper-and-pencil visual rotation test compared to a computer-simulated mental rotation test. Another distinction to consider is the number of dimensions in the test. Several authors investigated how performance varies in MRTs using two-dimensional illustrations of three-dimensional objects versus MRTs using actual three-dimensional projections (Felix, Parker, Lee, & Gabriel, 2011; Neubauer, Bergner, & Schatz, 2010; Robert & Cheverier, 2003). A sample of a typical MRT problem is given below in Figure 3. A standard figure is given along with four choices. Participants were asked to choose which option(s) were a rotated version of the standard figure and which option(s) were distracters. Specifically, this problem comes from a study which evaluated the role of confidence in mental rotation ability (Estes & Felker, 2010).

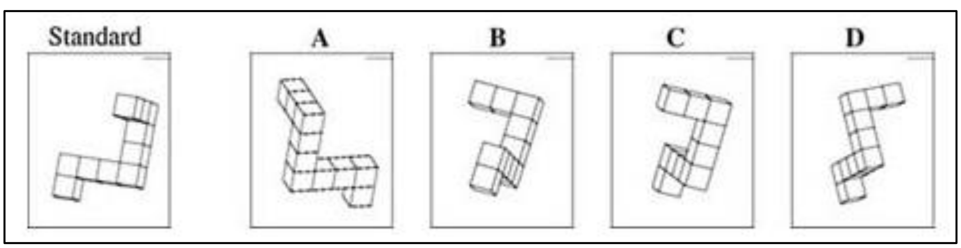


Figure 3. Sample MRT problem used by Estes & Felker (2010)

Similar examples were used in a study conducted by Li & O'Boyle (2011). However, they also examined the effect of mental rotation performance with segmented figures compared to intact figures. Participants were shown an original intact figure that was then "exploded", or segmented, into portions, some of which were rotated. The task was to then select which new intact figure resulted from connecting the disjoint pieces at the middle. A sample MRT problem is illustrated in Figure 4 below.

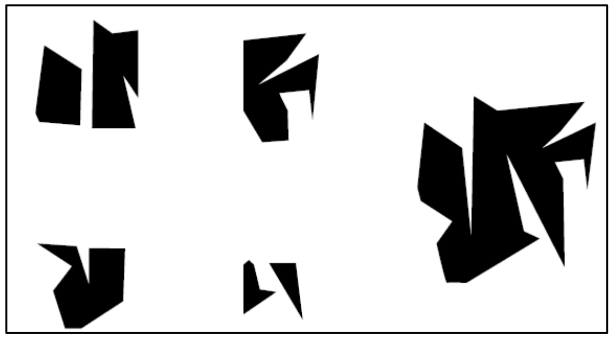


Figure 4. Sample MRT problem used by Li & O'Boyle (2011).

Spatial skills can also be assessed through processing speed as opposed to test performance. A two-dimensional, four-piece jigsaw puzzle was used by Hoffman, Gneezy, and List (2011) to measure spatial ability as a function of the time taken for participants to complete the puzzle. To better assess spatial ability, three-dimensional gate puzzles were also used. Alt, Bodlaender, van Kreveld, Rote, and Tel (2009) provide examples of gate puzzles, and two-layer puzzles that may be beneficial to measure spatial reasoning skills. Gate puzzles consist of pieces of two (or sometimes three) vertical legs connected by a horizontal rod that must be placed in specific spaces on the board based on leg length (see Figure 5). Research finds that the average time to complete gate puzzles ranges from 30 minutes to one hour.

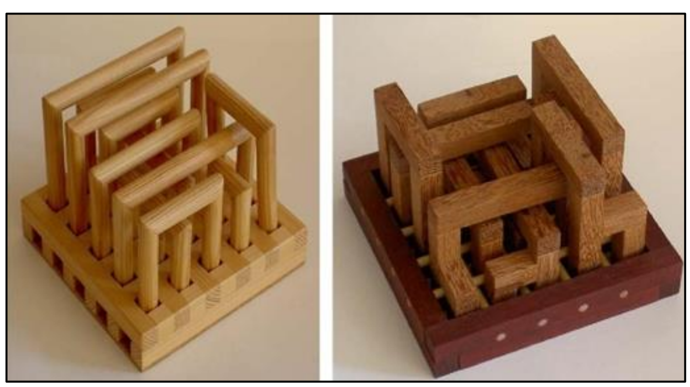


Figure 5. Gate puzzles with two legs (left) and three legs (right) used by Hoffman et al. (2011).

Two-layer puzzles are almost like a three-dimensional jigsaw puzzle. The pieces must be arranged in two layers so that opposite pieces connect appropriately. Examples of two-layer puzzles are given in Figure 6.

Figure 6. Example of a two-layer puzzle with solution used by Hoffman et al. (2011).

These puzzles can be easily adapted to have more than two layers, or to use slanted tops to create varying orientations. Presently, spatial skills are more commonly assessed in traditional mathematics assessments.

The current New York State mathematics curriculum has placed an increased emphasis on spatial ability with the recent implementation of the Common Core State Standards (CCSS). As early as Kindergarten, teachers are asked to focus mathematics instruction time on "describing shapes and space" (p. 12). Specifically, at the adolescent level, students are required to understand rotations of two-dimensional objects and visualize cross sections of three-dimensional figures (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). A CCSS for Geometric Measurement and Dimension is illustrated in Figure 7.

Geometric Measurement and Dimension (G-GMD):

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Figure 7. CCSS addressing spatial ability (high school)

The evidence from earlier research supports the idea of a male advantage in spatial ability, mental rotation performance, and timing/accuracy of completing geometric puzzles. Also, it demonstrates an age gap in spatial reasoning skills. This study used a paper-and-pencil assessment converting between two-dimensional and three-dimensional visualizations to evaluate how these forms of measuring spatial ability vary by gender, age, course of study, and additional factors.

Experimental Design

This experiment tested the hypothesis that female non-mathematics majors would have the most difficulty with spatial visualization of two-dimensional air nets and their corresponding illustrations of cubes when compared to a population of male and female students in varying majors. During this study, students answered a five-problem test that contained different types of problems involving matching and creating two-dimensional air nets for a given cube and vice versa. Four of the five problems were of multiple-choice format, while the fifth problem required students to draw in the faces of a cube into a sample air net. The results of the test were compared to a survey that students answered regarding specific demographic factors and their perceptions of which problems were the most difficult and least difficult.

Participants

This study was conducted in a comprehensive, selective, public, residential, liberal arts university located in the Northeast. The college has a population of about 5,400 students consisting of 5,100 undergraduate students and 300 graduate students. The majority of these students are from the county or surrounding counties. The participants include students with various majors enrolled in *University Precalculus* (referred to as Course A) as well as mathematics, mathematics education, and related majors enrolled in *History of Mathematics*

(Course B) and *Reading and Writing Mathematics* (Course C). A total of 43 students participated in this study (nine students from Course A, eighteen students from Course B, and sixteen students from Course C).

Course A (*University Precalculus*) focuses on topics such as algebraic and transcendental functions, trigonometry, analytic geometry, applications, and computational technology to prepare students for *University Calculus I*. Students enrolled in Course A are non-mathematics majors. Specifically, the participants enrolled in this course majored in general science, social science, or arts/humanities. Course B (*History of Mathematics*) is intended for mathematics majors and mathematics education majors. The purpose of this course is to provide pre-service teachers with a historical perspective for topics taught in secondary mathematics while they experience and solve historically significant problems. Course C (*Reading & Writing Mathematics*) uses a problem-solving approach to improve pre-service teachers' ability to write and communicate mathematics. Particularly, childhood mathematics education majors and secondary mathematics education majors study how reading and writing mathematics can help with teaching and learning mathematics.

Additional demographic information on the university population is given in Figure 8a.

Population Breakdown:

- Freshmen (21.9%)
- Transfers: (8.5%)
- Female: 55.2% undergraduate; 77.5% graduate
- Male: 44.8% undergraduate; 22.5% graduate

Reported Ethnicity of Undergraduates:

- White: 81%
- Unknown: 3.8%
- Hispanic: 4.7%
- Black, Non-Hispanic: 4.2%
- Multiple Races: 2.4%
- Asian: 3.5%
- American Indian: 0.3%
- Native Hawaiian, Pacific Islander: 0.1%

Figure 8a. Demographic Information on University Population

Demographic information of the study's participants are summarized in Figure 8b.

Gender	# of Students
Male	13
Female	30

Age	# of Students
18-19	8
20-21	30
22-23	5

Race/Ethnicity	# of Students
Caucasian	42
African American	1

Figure 8b. Demographics of Student Participants

Signed consent forms were obtained from each student prior to the beginning of the study (see Appendix A).

Design

This experiment was designed to determine whether age, gender, and college major would impact performance on a spatial visualization test involving converting between two-dimensional figures and their three-dimensional equivalents. Problems were chosen based on the type of visualization strategy it required (either two-dimensional to three-dimensional or three-dimensional to two-dimensional). Also, test problems were considered to be of varying difficulty levels based on the figures' patterns and orientations.

In order to provide a quality instrument, first the assessment was administered to a pilot group. The researcher discussed the requirements for participation with the pilot group which consisted of college freshmen non-mathematics majors enrolled in a *Survey of Precalculus* course. The principal researcher emphasized that participation in the piloting of the assessment had no impact on their grade and that the data and test scores collected were anonymous and confidential. Students were given approximately 20 minutes to complete the pilot assessment during the scheduled class time. After some initial feedback, the assessment was altered to include a section on the types of strategies that the students used when completing the spatial ability problems. Additionally, the survey was edited to include a question about students' thoughts on the use of the manipulative.

Prior to the beginning of the study, the researcher reviewed the requirements for participation in the study and the policy for dropping out of the study. The participants were made aware that there was no penalty for choosing not to participate, for dropping out of the study at any time, and that this study had no impact on their grade. Informed consent was obtained by the researcher for each participant in the study. The researcher allotted 20 minutes during the scheduled class time to administer the assessment. Participants were asked to complete a five-problem spatial ability test followed by a brief survey. The test consisted of problems that required students to visually match a two-dimensional illustration of a three-dimensional object with its corresponding two-dimensional air net. The five-question post-assessment survey sought information about the participants' age, gender, college major, ethnicity, and GPA. The survey also asked students to explain which problems they felt were the easiest and most difficult, what kinds of strategies they used to solve these problems, if any additional factors may have impacted their performance, and their thoughts on the use of a manipulative for the assessment regardless of their access to the manipulative itself.

Instrument Items and Justification

The same instrument was administered to two groups of students. One group of students had access to a paper air net of a cube as a hands-on manipulative, while the other group of students did not. Each problem required students to visualize three-dimensional figures from the two-dimensional figures given on the assessment.

Students were instructed to match cubes to their corresponding air nets (see Figure 9a) and to match air nets to their corresponding cubes (see Figure 9b).

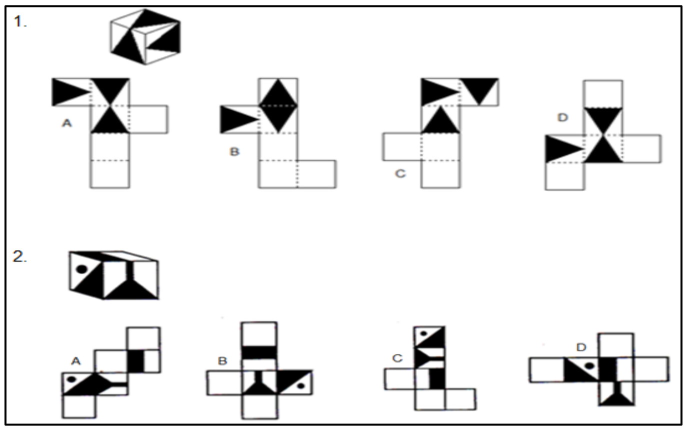


Figure 9a. Instrument Problem: Unfolding Task

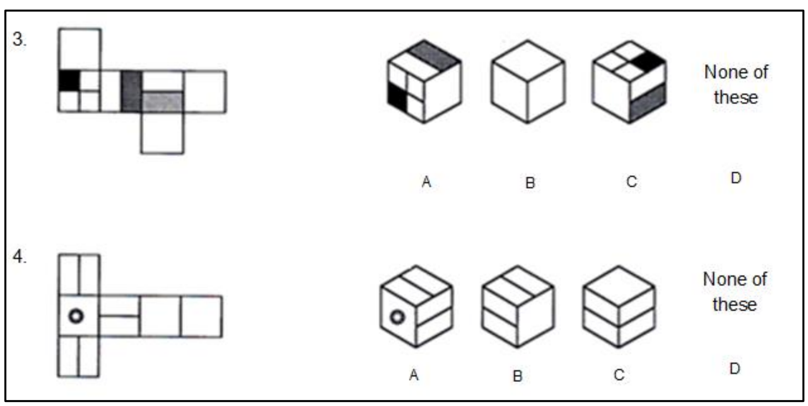


Figure 9b. Instrument Problem: Folding Task

The two types of questions helped to determine if visualizing from three dimensions to two dimensions is less difficult, more difficult, or at the same level of difficulty as visualizing from two dimensions to three dimensions.

The final problem on the instrument took on a different approach (see Figure 9c). Instead of matching an air net to its cube, students were asked to fill in the faces of a given cube into a blank air net. This type of problem required students to actually construct their own air net and was designed to eliminate the possibility of simply guessing on the earlier multiple choice problems.

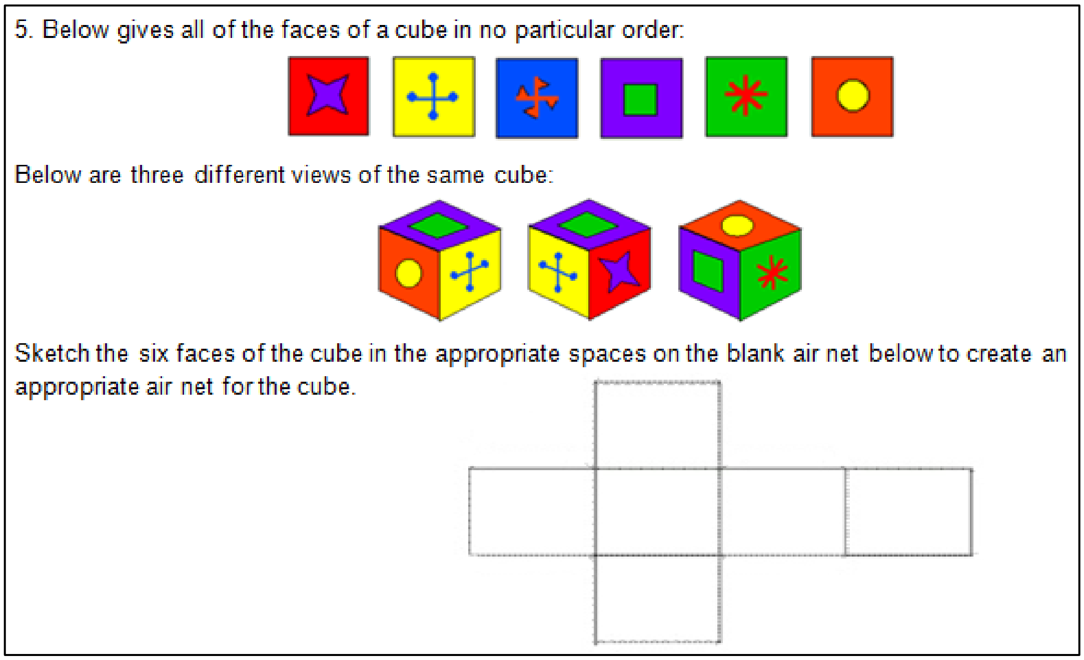


Figure 9c. Instrument Problem: Creating a Cube's Air Net

Each of the five problems asked students to show their work and/or explain the strategy or method that they used to arrive at their answer.

The purpose of the follow-up survey questions was to determine what factors, if any, influence spatial ability as well as which aspects of spatial visualization are the most difficult for students. The survey questions are listed in Figure 10.

Post-Assessment Survey

1. Please fill in the following information.

Age: Gender: College Major: Race/Ethnicity: GPA:

2. Which problem was the most difficult and why?

3. Which problem was the least difficult and why?

4. Do you think any other factors may have influenced your ability to complete these problems? If so, explain.

5. Do you think this assessment would have been easier with the use of a hands-on manipulative? Explain. **OR**
Do you think this assessment would have been more difficult without the use of a hands-on manipulative? Explain.

Figure 10. Follow-up Survey Questions

The aforementioned test problems and survey questions were used for data collection and analysis as described in the following section.

Methods of Data Analysis

Data Collection

The data for this experiment was generated in the beginning of the Spring 2014 semester at the university. All data was collected within three weeks. Participants were administered both the assessment and post-assessment survey on the same day as discussed in previous sections. Participants were given 20 minutes to complete the assessment. The assessment was graded as follows: For problems one through four, a correct response received one point and an incorrect response received zero points. There could not be a partially correct response due to the multiple-choice format. For problem five, the air net was checked to determine if it correctly produced each of the three given cubes. If only two or fewer of the faces were present, zero points were awarded. If all three of the faces were present but transposed, one point was awarded. If all three faces were present and in the correct position, two points were awarded. Each given cube had the possibility of 0, 1 or 2 points which gave a total possible score of six points for this problem. Partial credit was allowed for this problem. Figure 11 summarizes the way the assessment was graded.

Problem #	Points Possible
1	0 or 1
2	0 or 1
3	0 or 1
4	0 or 1
5	0 - 6

Figure 11. Summary of Scoring Rubric

The surveys were evaluated as follows: The responses for questions one and four were tallied and used for statistical analysis to determine if these demographic factors affected performance on spatial tasks. The analysis for question five aimed to determine whether access to a hands-on manipulative would be unnecessary or helpful for these types of spatial ability tasks. The responses for questions two and three regarding which problems were the least and most difficult were divided into three categories: Matching a cube to its air net (unfolding), matching an air net to its cube (folding), and creating a cube's air net (open-response). The methods or explanations that students provided were divided into five categories: trial-and-error, pattern recognition, drawing a picture, mental visualization, or no strategy mentioned. The prominent statistic that the researcher was seeking was the type of spatial visualization task the students viewed as the most difficult and the types of strategies students used to solve these problems.

Data Analysis

The scores on the five-problem test were analyzed for qualitative purposes. Students' numerical scores (out of 10) were then compared by question-type (i.e. folding, unfolding, and open-response). Qualitative data was collected from student explanations of their solutions as well as responses to the survey questions. The strategies that students mentioned were evaluated to determine which methods led to the most successful and least successful outcomes. Responses to the survey were compared based on the problems students found easy and which problems students found difficult. A comparison of scores was also conducted across gender, age, major, and course. Finally, a qualitative analysis in regard to the use of a manipulative was conducted. Statistical software was used to test the hypothesis relative to the data collected.

Results

Five primary results were evident following the collection and analysis of the quiz scores. These results allowed for the hypothesis to be rejected. Therefore, factors such as age, gender, and college major did not have an impact on spatial skills. Additionally, the use of a hands-on manipulative in the form of a paper air net did not influence scores for all participants in the study. An analysis of variance (ANOVA) was utilized to determine the significance of the treatment at the 0.05 level with regard to the following results.

Result #1- There was no significant difference in spatial ability test scores by gender (p = .104).

Upon analysis of the total scores on the assessment, it was determined that no substantial difference existed between the scores of male participants and the scores of female participants. Mean scores for the assessment were out of a maximum of 10 points. Figure 12a and Figure 12b compare the results for the entire population ($N = 43$).

Gender	N	Mean Score (out of 10)
Male	13	7.8
Female	30	6.4

Figure 12a. Comparison of Mean Scores by Gender

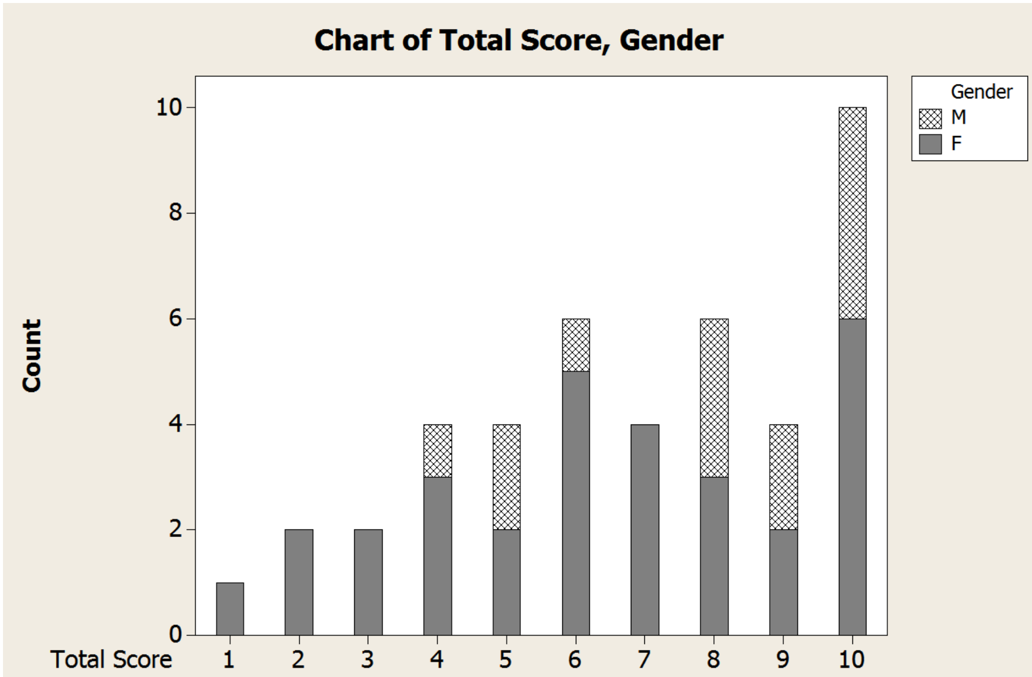


Figure 12b. Bar Graph of Total Assessment Score by Gender

The resulting p-value of the ANOVA test ($p = 0.104$) shows no statistical significance by gender, thus refuting previous findings that, contrarily, support the presence of a gender gap in spatial ability tasks. However, the chart illustrates that male participants scored four points or higher, whereas only females were in the one to three point range. There was a higher frequency of females scoring in the four to seven point range than males. For higher scores (eight to 10 points), there were more scores by males than by females. The discrepancy between the chart in Figure 12b and the ANOVA test is probably due to the heavily biased sample. Since the population of study participants were mostly female, the ANOVA test may have been skewed.

Result #2- There was no significant difference in spatial ability test scores by major (p = .068).

Upon analysis of the total scores on the assessment, it was determined that no substantial difference existed between test scores and major. The list of majors was divided into eight categories: Mathematical Sciences, Adolescent Education (Mathematics), Childhood Education (Mathematics), General Sciences, Mathematics Education/Sociology, Social Sciences, Arts/Humanities, and Other. Figure 13a and Figure 13b compare the results for the entire population ($N = 43$). Again, mean scores are given out of a maximum of 10 points.

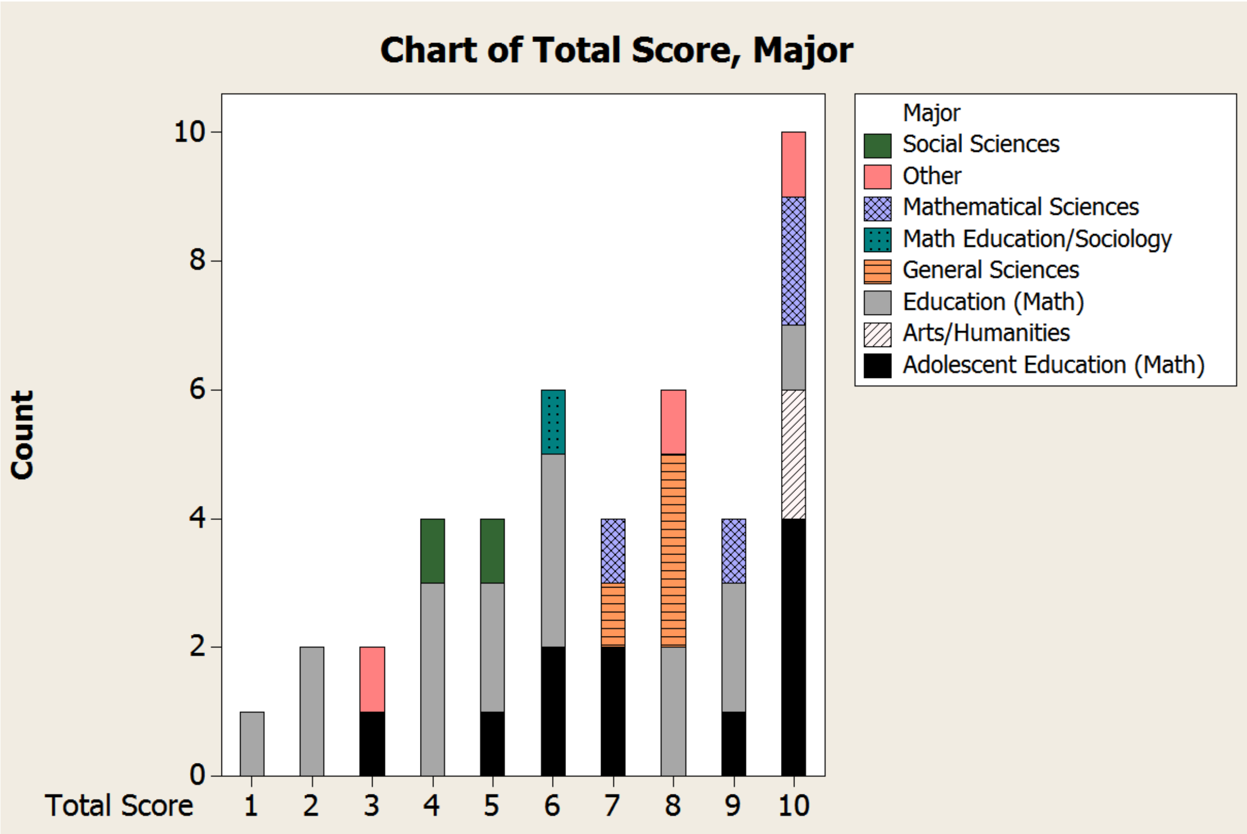


Figure 13a. Bar Graph of Total Assessment Score by Major

This chart shows that childhood mathematics education majors were more likely to score in the zero to six point range. In fact, the students who scored one or two points on the assessment were all childhood mathematics education majors. All students studying some form of general science scored a seven or eight on the assessment. The majority of students who received perfect scores had some type of mathematics-based major (either mathematics education or applied mathematics). Figure 13b below provides the mean scores for each category of college major.

Major	N	Mean Score (out of 10)
Mathematical Sciences	4	9.0
Math Adolescent Education	11	7.5
Math Childhood Education	16	5.6
General Sciences	4	7.8
Math Education/Sociology	1	6.0
Social Sciences	2	4.5
Arts/Humanities	2	10.0
Other	3	7.0
Total	43	6.9

Figure 13b. Comparison of Mean Scores by Major

The fact that the mean score for arts/humanities majors was a 10 out of 10 was unexpected; however, the sample of participants is heavily biased towards mathematics and STEM majors, so there was a very small population of these students which may account for this finding.

Grouping information using the Tukey Method and 95% confidence intervals showed that there was no significant difference in scores by college major. Hence, the resulting p-value of the ANOVA test ($p = 0.068$) disproves the claim that mathematics or science majors would perform better on spatial ability assessments.

Result #3- There was a significant difference in test scores between the use of a manipulative and course ($p = 0$).

Upon analysis of the total scores on the assessment, it was determined that no substantial difference existed between the scores (out of 10 points) of participants with access to the manipulative and the scores of participants without access to the manipulative. Figure 14a and Figure 14b compare the results for the entire population ($N = 43$).

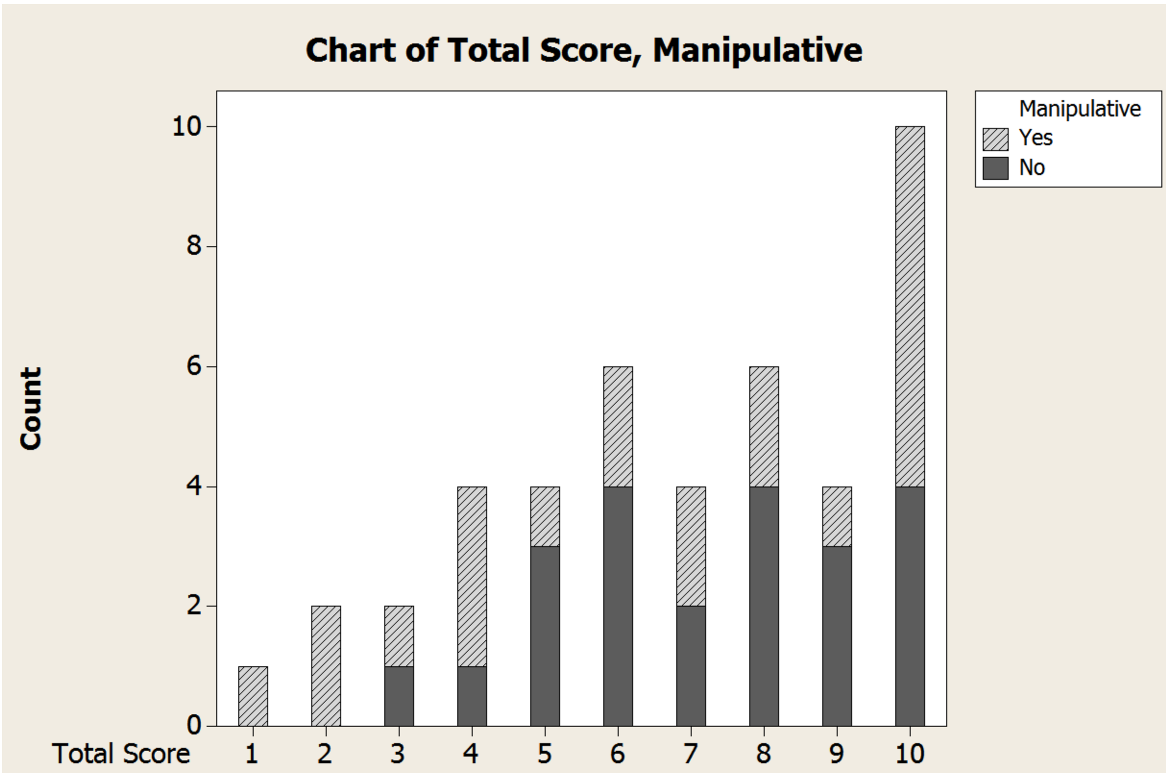


Figure 14a. Bar Graph of Total Assessment Score by Use of Manipulative

Manipulative	N	Mean Score
Yes	21	6.5
No	22	7.2

Figure 14b. Comparison of Mean Scores by Use of Manipulative

The chart in Figure 14a shows that students without the manipulative scored three points or higher, while students with access to the manipulative scored across the board (one through 10 points). The mean scores compared by access to a manipulative show that students without the manipulative scored minimally higher than students with access to a manipulative. There was no statistical significance by use of a manipulative which contradicts the researcher's original assertion that students with access to a manipulative would have higher scores. However, when compared by course, a significant difference in scores by the use of a manipulative was present ($p = 0$). Figure 15 shows an interaction plot for the total score using fitted means.

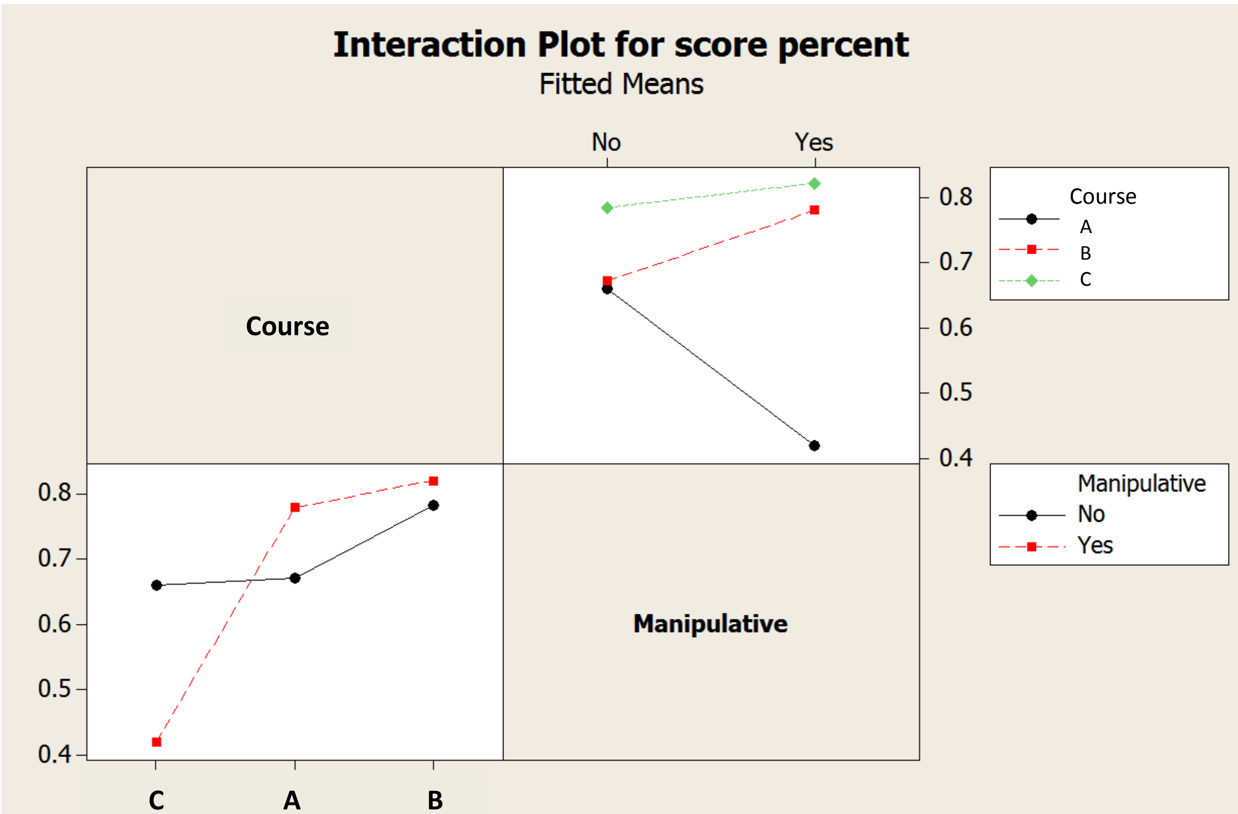


Figure 15. Interaction Plot of Scores and Use of Manipulative by Course

Notice that for both Course A and Course B, the use of a manipulative produced a higher mean score on the assessment which follows the claim cited in the hypothesis statement, namely that students with access to a manipulative would perform better on the spatial skills test. Conversely, students in Course C who had the manipulative performed significantly worse on the spatial ability problems than students in the same course who did not have the manipulative.

Result #4- There was a significant difference in spatial ability test scores by course (p = .042).

Assessment results indicated that the mean scores by course were significantly different. Grouping information using the Tukey Method and 95% confidence intervals showed that a significant difference existed in scores by class. Recall that Course A consisted of mostly general science majors and included no mathematics majors; Course B consisted mostly of mathematics adolescent education majors and applied mathematics majors; Course C was comprised of mostly childhood education majors whose concentration was mathematics, with some mathematics adolescent education majors. A main effects plot for total score by course is shown in Figure 16.

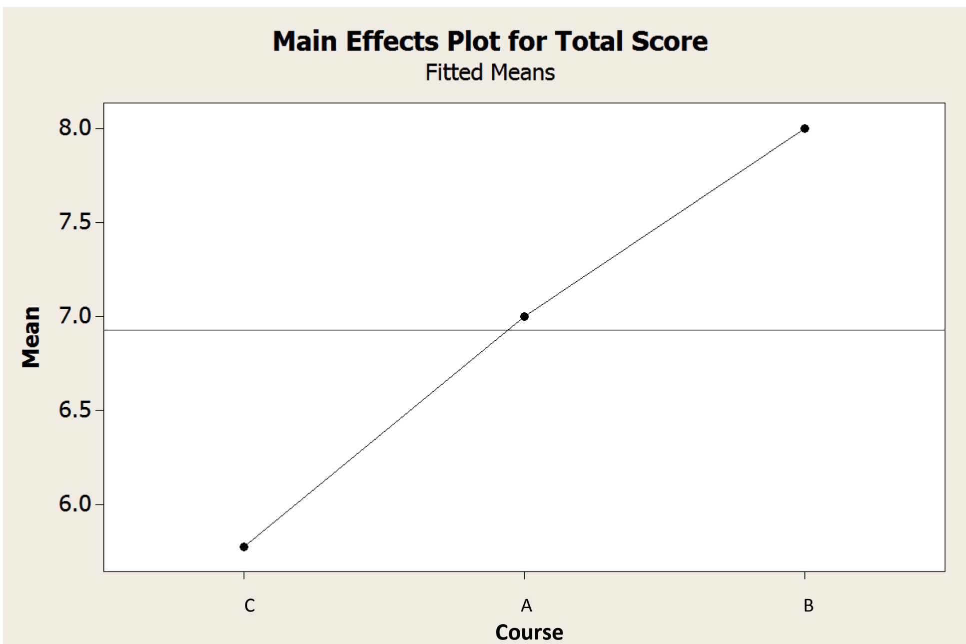


Figure 16. Main Effects Plot for Total Score by Course

Hence, the resulting p-value of the ANOVA test ($p = 0.042$) demonstrates that mean scores for students in Course C were significantly lower than those in Course A or Course B. Perhaps the childhood mathematics education majors did not experience these types of spatial ability tasks in their course of study while the upper level mathematics and STEM majors are more familiar with this content. Despite the fact that spatial reasoning is becoming increasingly prominent throughout the mathematics curriculum at all grade levels, the pre-service teacher courses may not have adjusted their program to reflect this change.

Result #5- There was a significant difference in scores between Problem #1 and Problem #2, which are of the same type (p = 0).

The analysis of the assessment aimed to determine which spatial ability task was viewed as more difficult by adolescents: either visualizing from a three-dimensional cube to a two-dimensional air net (unfolding), or visualizing from a two-dimensional air net to a three-dimensional cube (folding). However, test scores showed a significant difference in scores between Problems #1 and #2, which are of the same type ($p = 0$). Figure 17a shows problems 1 and 2.

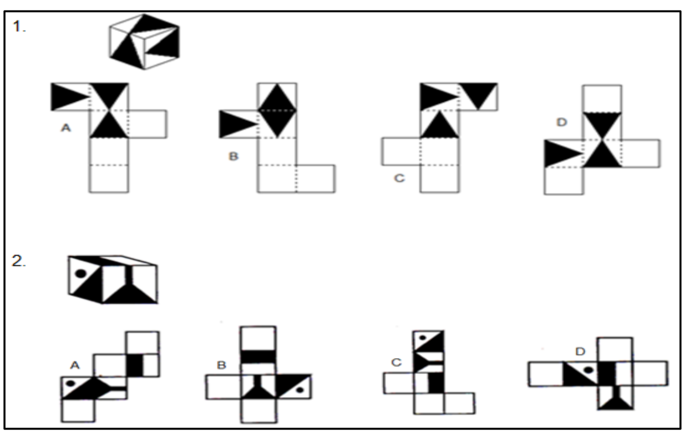


Figure 17a. Assessment Problems #1 and #2

Figure 17b provides the comparative means for each problem. Specifically, the line plot in the figure helps to illustrate the significant difference between problem 1 and problem 2.

Problem #	Mean (out of 1)
1	0.5470
2	0.8028
3	0.6633
4	0.6865
5	0.7136

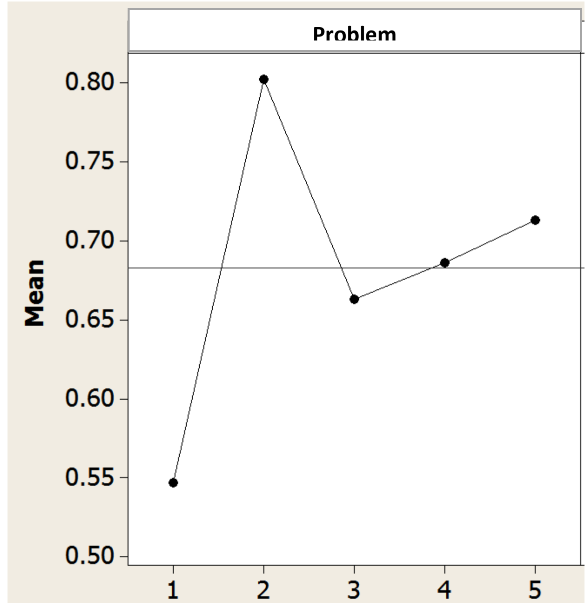


Figure 17b. Comparison of Means by Problem

Since the purpose of the test was to identify which type of spatial task was the most difficult for college students, it was unexpected to have such a dramatic difference between problems of the same type. Again, the test problems were designed to have a variety of patterns and orientations which may explain the inconsistency that students had for the unfolding-type problems.

The researcher hypothesized that demographic factors would influence performance on a five-problem spatial skills test. Specifically, the claims were that males would perform better than females, STEM majors would score higher than non-STEM majors, and having access to a hands-on manipulative during the test would improve performance. Although the sample used in this study demonstrated no significant difference by gender or major, results did indicate that scores were significantly different by course. Also, this research suggested that the manipulative did influence student performance, but opposite to the researcher's original claim.

Analysis of Assessment Problems

In order to discern which types of spatial ability problems are the hardest for students and to determine the aspects of these problems that are causing difficulties, each problem was individually examined. Figure 18 lists each problem, the type of spatial ability task that it is testing, and the percentage of correct and incorrect responses. To remain consistent with the all-or-nothing nature of the multiple choice questions, scores from 0 to 5 points were considered incorrect for problem #5 and a score of 6 points was considered correct.

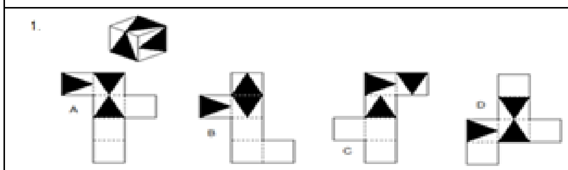
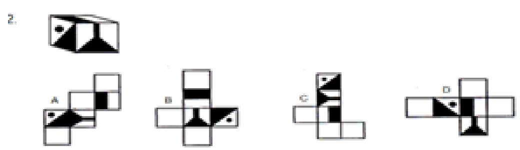
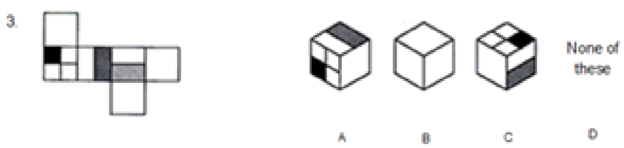
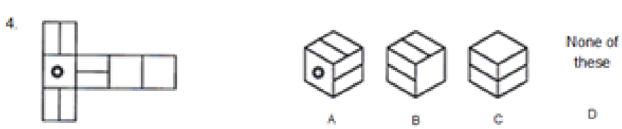
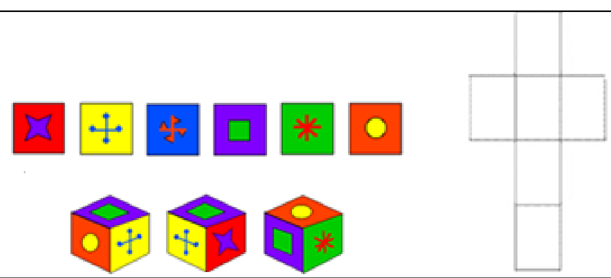
Problem	Problem Type	Percent Correct	Percent Incorrect
	Unfolding	53.5%	46.5%
	Unfolding	79.1%	20.9%
	Folding	65.1%	34.9%
	Folding	67.4%	32.6%
	Combination	44.2%	55.8%

Figure 18. Analysis of Individual Problems

The percentage of correct responses for both folding problems were very close which suggests that students were consistent on this type of spatial ability task. However, the percentage of correct responses for the unfolding problems were inconsistent which supports the significant difference in scores between problem #1 and problem #2 found in Result #5. Based on the percentages given in Figure 18, it is clear that problem #1 proved to be the least successful while students had the most success with problem #2. Although the chart states that 55.8% answered problem #5 incorrectly, there were several students who still received partial credit. Only one student received zero points for this problem, which agrees with the fact that problem #1 was the least successful.

Survey Analysis

Following the completion of the five-problem test, students were asked to complete a free-response survey. The purpose of the survey was to further expand on the factors that may influence spatial skills as well as to gain insight on which problems were easiest and most difficult for the students and why. Recall from Figure 10 (shown again below) that the survey asked the following questions:

Post-Assessment Survey

1. Please fill in the following information.

Age: Gender: College Major: Race/Ethnicity: GPA:

2. Which problem was the most difficult and why?

3. Which problem was the least difficult and why?

4. Do you think any other factors may have influenced your ability to complete these problems? If so, explain.

5. Do you think this assessment would have been easier with the use of a hands-on manipulative? Explain. **OR**
Do you think this assessment would have been more difficult without the use of a hands-on manipulative? Explain.

Figure 10. Follow-up Survey Questions

The majority of students identified Problems #1 and #5 as the most difficult. The difficulty with Problem #1 supports the findings of Result #5, namely that there was a significant difference in scores comparing problem #1 and problem #2. One student explained that Problem #1 was the most difficult because "it was hard to build the four choices in my head, also because it was the first one and I didn't really have a method yet." Another challenge with Problem #1 that multiple students expressed was that the patterns on the faces of the cube in Problem #1 were "too simple" since they were all triangles. In these cases, several students tried to draw a picture to differentiate the orientation of the triangles. Figure 19 shows a sample of one student's work for Problem #1. This student is a 21-year-old male enrolled in *University Precalculus* majoring in the arts/humanities.

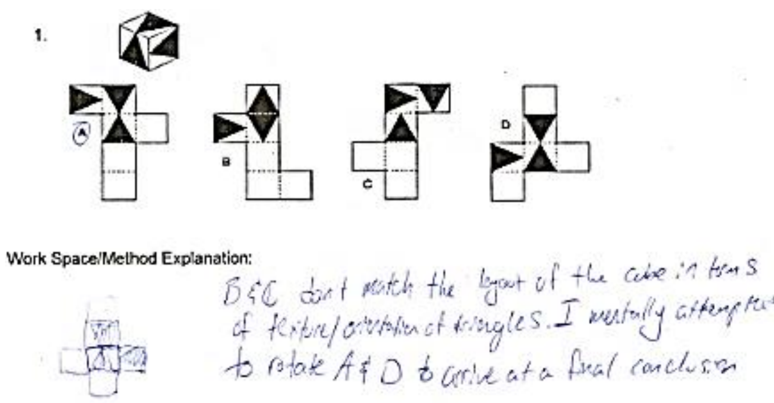


Figure 19. Sample Solution and Method for Problem 1

The strategy mentioned in Figure 19 was categorized as 'pattern recognition'. This particular student used the orientation of the triangles on the cube and air net choices to eliminate choices B and C. Then, some mental rotation and visualization led the student to correctly select choice A as the final answer. The student writes, "B & C don't match the layout of the cube in terms of texture/orientation of triangles. I mentally attempted to rotate A & D to arrive at a final conclusion."

The issue with Problem #5 was that students felt that it was “too complex” or they “could not manipulate it.” Generally, students recognized which sides needed to be adjacent to each other to fold into the given cubes. Figure 20 and Figure 21show some methods that students used to complete Problem #5. The student work shown in Figure 20 is a 21-year-old female enrolled in *History of Mathematics* with a double major in mathematics and mathematics education. Her strategy was categorized as drawing a picture. Specifically, this student drew portions of the air net that would create the three given cubes and then tried to combine them into the air net template. The student’s attempt to arrange the three portions of the air net was not fully correct as portion two did not properly fit into the given template based on her other two arrangements. One point was deducted since the student’s solution did not produce the second cube (the square face was missing). According to the survey responses, this participant identified this problem as the most difficult.

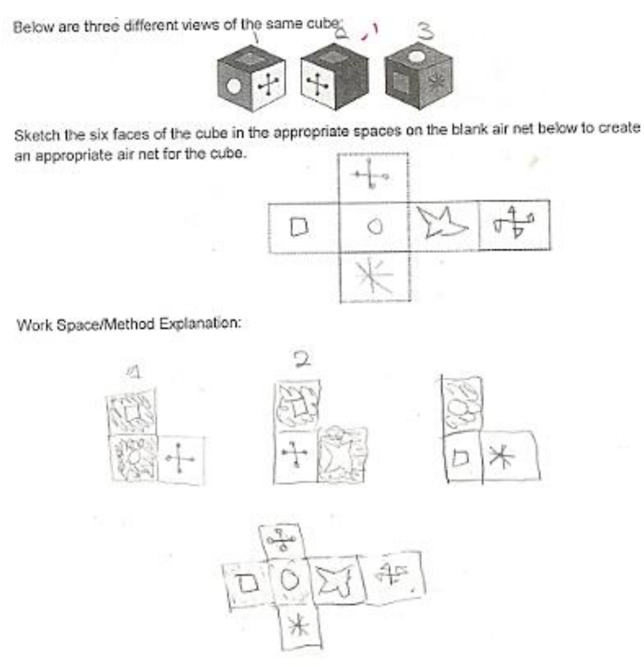


Figure 20. Sample Solution and Method for Problem #5

The second sample solution for problem #5 in Figure 21 is from a 19-year-old female enrolled in *Reading & Writing Mathematics* with a childhood mathematics education major. She also ranked this problem as the most difficult. Although the student describes her method as a process of elimination, her strategy was categorized as mental visualization. Specifically, this student used one cube for reference points by labeling the top, left, and right faces. Then, the student used the remaining cubes and mental visualization to determine which face must be on the bottom, front, and back. Unfortunately, the student did not accurately arrange the top, left, and right faces into the air net template which threw off the rest of the solution.

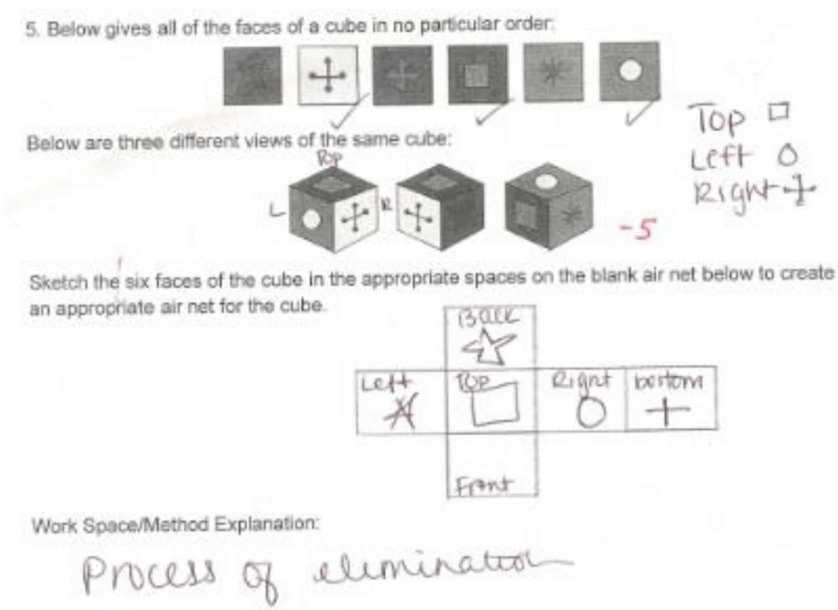


Figure 21. Sample Solution and Method for Problem #5

According to the survey results, 12 students found Problem #2 to be easy. The fact that many students found Problem #2 to be easy supports the findings of Result #5 which was that there was a significant difference in scores between problem #1 and problem #2. One student stated that the least difficult problem was #2 “because the shapes were more prominent and looked less confusing." Several other students noted that they used the orientation of the shapes and lines as a guide. A sample of student work for this problem is shown in Figure 22. This male student is a 21-year-old physics major enrolled in *History of Mathematics*. His strategy was categorized as 'pattern recognition'. He explains how he used the orientation of the patterns on the faces of the cube to eliminate one choice and then checked the alignment and orientation with the remaining choices. In fact, this student listed problem #2 as the easiest problem in his follow-up survey because, "it was easy to infer what side is next to what based on the orientation of the images."

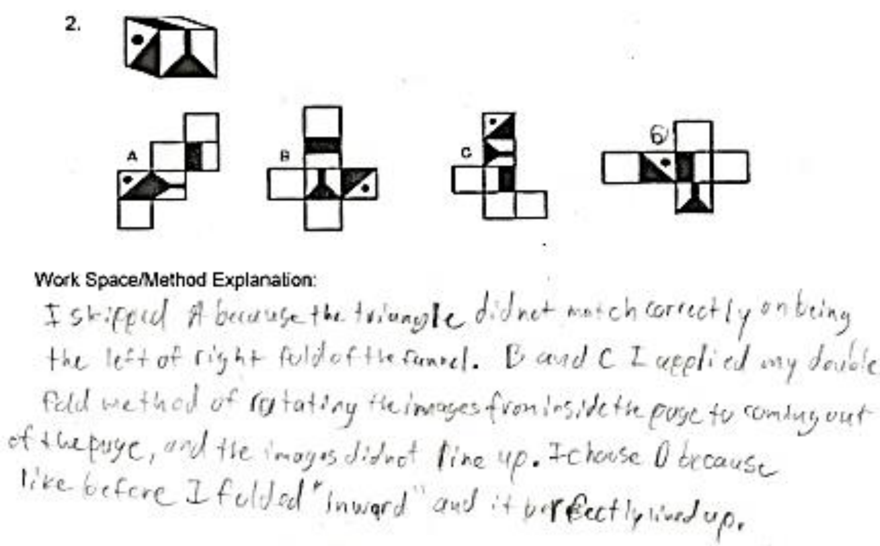


Figure 22. Sample Solution and Method for Problem #2

However, the most students (15 out of 43) ranked Problem #4 as the easiest question based on survey responses. Similar to Problem #2, a pattern recognition strategy was used. Figure 23 shows one student's explanation for their answer choice. This student is a 21-year-old female enrolled in *Reading & Writing Mathematics* studying mathematics education and sociology. She used a pattern recognition strategy, noting that the orientation of the lines after they are folded must be parallel, which led this student to the correct answer. In fact, she noted on the survey that she found this problem to be the easiest.

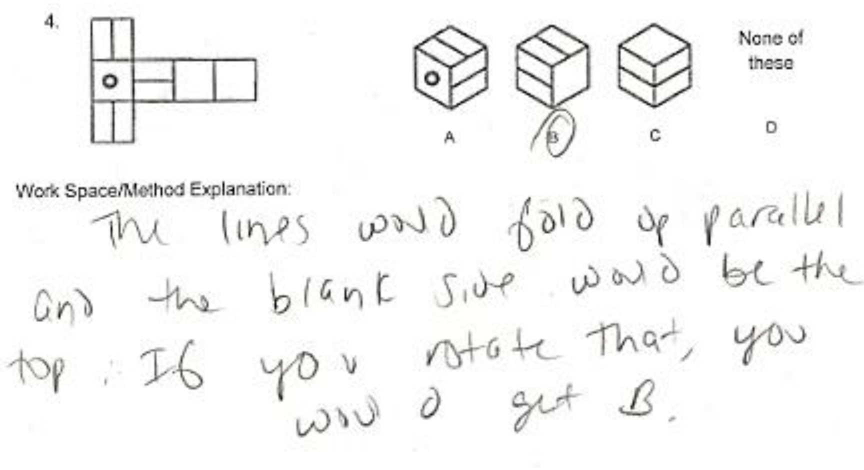


Figure 23. Sample Solution and Method for Problem #4

The problems that were viewed as the most difficult consisted of an unfolding problem and an open-response problem that used a combination of both folding and unfolding techniques. Ironically, the problems that were viewed as the easiest consisted of an unfolding question and a folding question. The survey responses about the easiest and most difficult problems are consistent with the actual test and individual problem scores. However, the survey does not yield any conclusive results about which type of spatial ability task is the most difficult for adolescents. One could speculate, though, that college students find folding problems to be easier than unfolding problems. Students were more consistent in their performance for this problem type. Also, more students got both folding problems correct as opposed to the unfolding problems where several students only were correct for one of the unfolding type questions. Even though problem #5 used a combination of folding and unfolding to reach a conclusion, it still follows the format of problems #1 and #2 (unfolding problems) which transition from a three-dimensional cube to a two-dimensional air net. Since more than half of the students ranked this problem and problem #1 as the most difficult, it may be inferred that unfolding is considered to be the most difficult spatial ability task for college students. The results and conclusions drawn from the descriptive and inferential statistics can be used to enhance mathematics education, particularly in the area of spatial reasoning.

Implications for Teaching

The hypothesis tested which factors, if any, influence spatial ability as well as which types of spatial ability tasks are viewed as the most difficult by college students. Based on student performance on the instrument, along with their comments on the post-assessment survey, four areas of improvement for classroom instruction and student scores emerged.

Implication #1- Mathematics Education courses for pre-service teachers should put more emphasis on spatial ability tasks.

Results #3 and #4 showed that students in Course C scored significantly lower on the spatial ability test than the other two classes. Again, *Reading & Writing Mathematics* is intended for early childhood through adolescent mathematics education majors. The results suggest that future mathematics teachers struggle with spatial visualization. Pre-service mathematics teachers must reach certain level of mastery with a specific topic before they can eventually teach it themselves. In this case, it is evident that mastery of spatial skills is lacking for this group of students. A particularly noteworthy find was that Course C students, consisting of mostly elementary education majors, did even worse with the use of a manipulative when compared to Course C students who did not have access to the manipulative. Since pre-service teacher training at the elementary level is so manipulative-oriented, the decrease in scores was quite unexpected. Perhaps a specific unit or course focusing on spatial reasoning would be beneficial to improve this skill in pre-service elementary mathematics teachers.

Implication #2- Educators should teach specific strategies for solving spatial visualization problems.

Regardless of the type of problem, a large number of students found the entire spatial ability test to be challenging. In fact, when asked which problems were the most difficult, some students wrote, “all of them.” In order to make a mathematical task less complicated, it is important to tackle the problem with a strategy. The process of folding and unfolding cubes/air nets is a very unique task that is often not seen on traditional mathematics assessments; however, these core skills are used in other areas of mathematics. Drawing a picture proved to be the most effective strategy. Pattern recognition and mental visualization also demonstrated a relatively high success rate. Figure 24 shows an analysis of test scores by strategy.

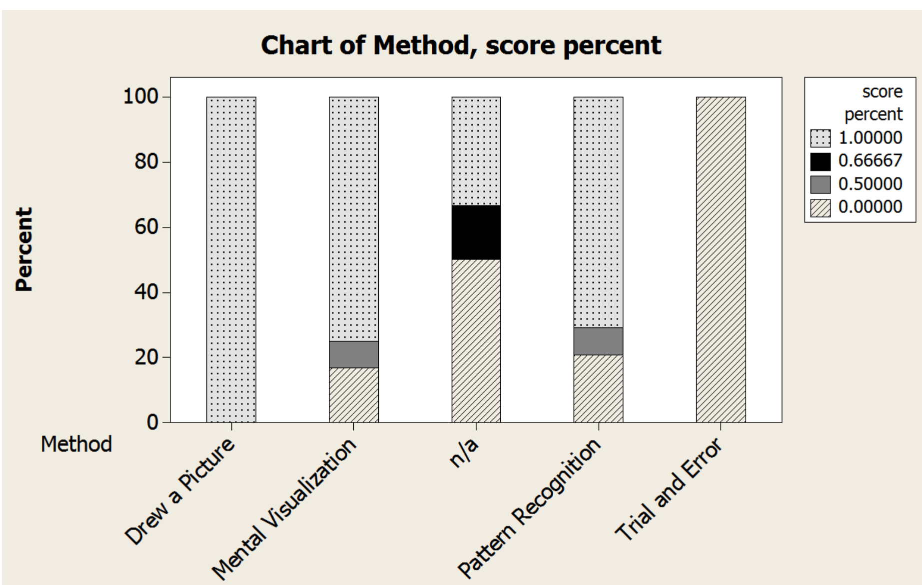


Figure 24. Student Scores by Method

It is clear that when students had a specific strategy to solve the problem they were more successful than students who simply guessed. The survey indicated that some students listed prerequisite knowledge or learning style as a factor that may have influenced their ability to complete these problems. Therefore, it is crucial that teachers teach more than one specific method using different approaches to spatial ability tasks in order to address all learning styles in their classrooms.

Although spatial reasoning is a heavily visual topic, educators must adapt their lessons to aid non-visual learners. Specifically, some helpful instructional techniques are to use manipulatives, encourage mathematical solutions through diagrams, use visual aids and technology. Spatial-visual learning is the ability to solve problems with pictures and charts as opposed to words. Thus, mathematics teachers should incorporate pictorial brain teasers and three-dimensional block puzzles to encourage visual thinking daily. Similarly, educators should also welcome and support alternative solutions through diagrams and illustrations. Alternative assignments such as building mathematical models or presenting information in a diorama will also hone in on students' visual-spatial skills. For students who struggle with mental visualization, visual aids are imperative in the classroom. Visual aids can be as simple as pictures drawn on the board, supplementary handouts/charts, or three-dimensional objects used for demonstration. Mathematical technology and educational technology such as interactive whiteboards, calculators, and computer software can offer a wide variety of advanced visual aids to use in the mathematics classroom for teaching spatial skills, as well. These technological aids will be discussed in more detail in Implication #4.

Implication #3- The quality of example problems used in the mathematics classroom is important.

The results revealed a significant difference in scores between Problems #1 and #2 which was unexpected since they were the same type of spatial ability task. Although Problem #1 and Problem #2 were both unfolding-type problems, the orientation of the cube and air net as well as the patterns and shapes given on the faces of the cube were different. Mathematics teachers often use multiple examples to practice a specific mathematics concept. This research suggests that the quality of the problems used in mathematics instruction is important to student understanding. Two similar-looking problems may be testing the same skill, but they can differ by a variety of factors producing dramatically different results. Also, mathematics problems come in varying levels of difficulty. When students are tested on the same geometry or spatial skill but are at different levels of cognition in the van Hiele model of geometric thinking, the outcomes and responses obtained will be diverse. Thus, it is imperative for teachers to anticipate these possible challenges and choose example problems, homework problems, and assessment problems appropriately.

Implication #4- Technology is a critical learning tool in the mathematics classroom.

Although the instrument used in this assessment was strictly paper-and-pencil, a computer simulation of these spatial ability tasks could provide a different perspective for students. Several studies and related research found evidence supporting computer-based instruction for spatial ability tasks, as was mentioned in the Literature Review. Along the lines of addressing different learning styles, computer programs could be helpful for visual learners. At the secondary level of mathematics instruction, the use of manipulatives or other diverse strategies tend to get "lost." Teachers often draw three-dimensional figures on the two-dimensional whiteboards and often assume that middle/high school students can readily make sense of these illustrations, but of course, that is not always the case. The use of shading and ability to click/rotate three-dimensional figures on a computer screen or SMARTBoard could improve the instruction and overall student understanding of three-dimensional geometry and related spatial ability tasks.

One specific technological tool to assist with teaching three-dimensional geometry is called *Cabri 3D.* This computer software program brings difficult mathematics topics to life in a visual manner. Students can interact with three dimensional models and manipulatives to solve problems as well as creating visual models themselves. For example, Figure 25 shows an illustration of how a simple triangle can be copied, rotated, translated, etc. to form the net of a three-dimensional tetrahedron.

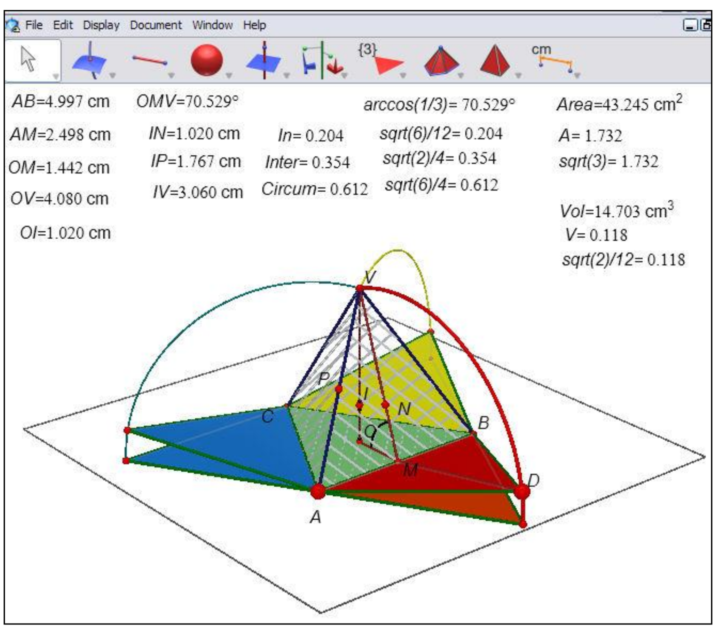


Figure 25. Illustration of a 2-D and 3-D Tetrahedron from Cabri 3D

This study revealed that two-dimensional and three-dimensional representations of cubes were difficult to mentally visualize for college students, so access to tools to ease the visualization struggle for more advanced shapes will yield great promise in middle and high school mathematics classrooms.

Visualization skills and three-dimensional geometry have proven to be a difficult concept for both students and mathematics educators due to their own limited spatial awareness. Through differentiated instructional techniques and materials, teachers can develop an appropriate pedagogical knowledge base to create a challenging spatial skill curriculum for their students.

Suggestions for Future Research

Although this research did not yield conclusive results in regard to factors that influence spatial ability, further research in this area may generate stimulating results. The age group used in this study was solely adolescents, so a sample of younger, school-aged children and/or a sample of middle-aged adults would be valuable for comparison. Similarly, there was virtually no diversity in ethnicity of the participants involved in this specific study. Generally, the sample of participants in this study was heavily weighted with females and students with mathematics-based majors, so a random sample could be a more efficient way to obtain participants. Also, one might consider administering two tests to each participant, one with a manipulative and one without, in order to better determine the effect that a manipulative has on performance by individual. Conducting a similar study with a more diverse sample may yield a more conclusive set of results.

Concluding Remarks

The motivation behind this study was to determine if certain demographic factors influence spatial visualization. Based on the results of this study, it appears as though, given this particular implementation model, that factors including gender, age, ethnicity, and college major do not in fact make a substantial difference in performance on spatial ability tasks. It is clear that the use of a manipulative and the courses taken throughout one's program of study can influence scores on these types of tests. However, it remains unclear which type of spatial visualization task is the most difficult for college students, and therefore, is worthy of further study.

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Appendix A

I would appreciate your collaboration in this very important project. Please sign below to indicate your agreement to participate in this study. You may retain a copy of this letter for your own files. Thank you for giving this request your full consideration.

STUDENT CONSENT FORM

SUNY Fredonia

Voluntary Consent: I have read this memo and I am fully aware of all that this study involves. My signature below indicates that I freely agree to participate in this study. If I withdraw from the study, I understand there will be no penalty assessed to me. I understand that my confidentiality will be maintained. I understand that if I have any questions about the study, I may reach Sara Maiorana by e-mail at: maiorana@fredonia.edu

Please return this original, completed consent form as soon as possible. Thank you for your cooperation.

Student Name (please print): ____

Student Signature: ____

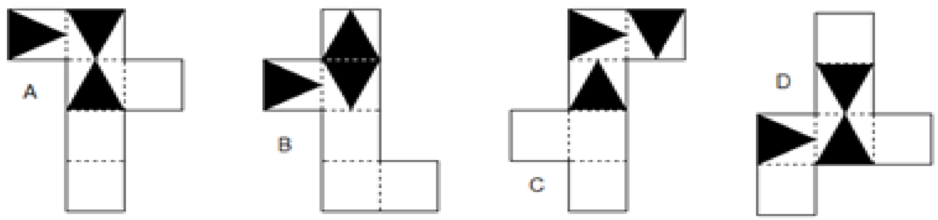
Date: ____________________________

Appendix B

Measuring Spatial Ability Using Air Nets

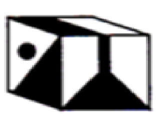
Directions: For Questions #1-#2, choose the choice that correctly shows the 2-D air net of the given cube. Use the space below to show your work and explain how you reached your final answer.

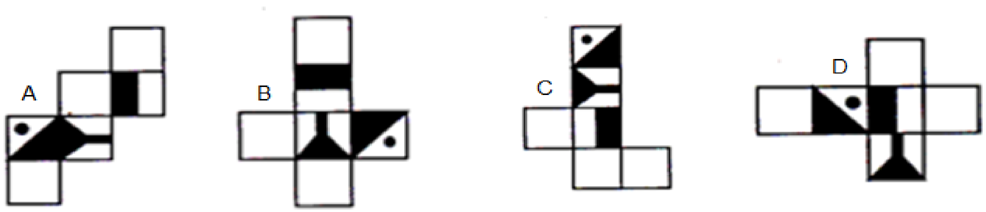
1.



Work Space/Method Explanation:

2.

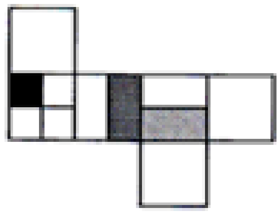


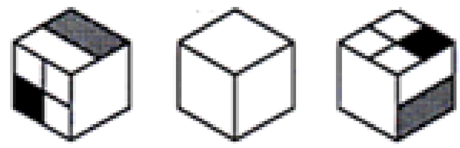


Work Space/Method Explanation:

Directions: For Questions #3-#4, choose the choice that correctly shows the 3-D cube corresponding to the given 2-D air net. Use the space below to show your work and explain how you reached your final answer.

3.



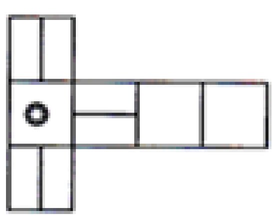


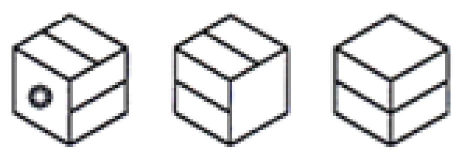
None of these

A B C D

Work Space/Method Explanation:

4.





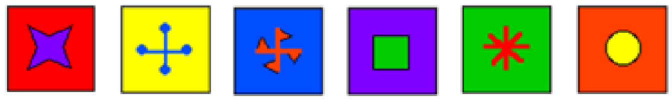
None of these

A B C D

Work Space/Method Explanation:

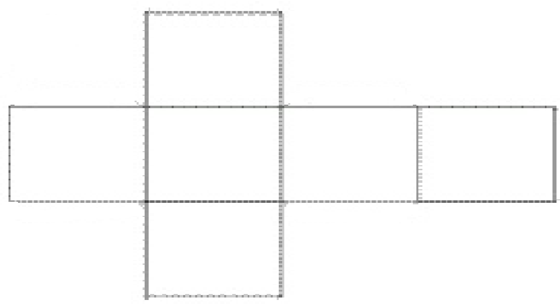
Directions: For Question #5, draw an appropriate air net for the given cube. You may wish to use the last half of this page to show your work. Please explain how you arrived at your final answer.

5. Below gives all of the faces of a cube in no particular order:



Below are three different views of the same cube:

Sketch the six faces of the cube in the appropriate spaces on the blank air net below to create an appropriate air net for the cube.



Work Space/Method Explanation:

Post-Assessment Survey

1. Please fill in the following information.

Age:

Gender:

College Major:

Race/Ethnicity:

GPA:

2. Which problem was the most difficult and why?

3. Which problem was the least difficult and why?

4. Do you think any other factors may have influenced your ability to complete these problems? If so, explain.

5. Do you think this assessment would have been easier with the use of a hands-on manipulative? Explain. **OR**
Do you think this assessment would have been more difficult without the use of a hands-on manipulative? Explain.