Strong Gravitational Lensing Analysis of SDSS J1138+2754 using Lenstool.

Akza Sam$^{1,2}$
Advisor: Dr. Amy Bartholomew$^1$

$^1$Department of Physics and Astronomy
$^2$Honors Program

May 20, 2023
Abstract

Gravitational lensing is a naturally occurring phenomenon that behaves like as a cosmic telescope, magnifying distant background galaxies that are otherwise beyond our reach. Modeling gravitational lenses can probe the mass distribution of the lensing cluster, predict the location of the distant sources that are blocked by the intervening cluster, and provide a foundation for detailed studies of high redshift sources. This project utilizes a parametric modeling software called Lenstool to model the gravitational lens SDSS J1138+2754, detected in the SDSS’s Giant Arcs Survey and imaged using the Hubble Space Telescope. Lenstool produces mass maps describing the mass content within the cluster and the source and image-pair locations.

Keywords: physics; astronomy; gravitational lensing; SDSS J1138+2754; mass maps; galaxy; galaxy cluster
1 Introduction

Strong gravitational lensing is a powerful tool to observe distant galaxies that are otherwise inaccessible to an observer due to its distance, location, and size. The chance alignment between the observer, an intervening mass such as a galaxy cluster, and a distant source will produce lensing effects comparable to effects from optical lenses. The gravitational field of the intervening mass perturbs spacetime and causes light from the source to bend around the mass, creating distortions in the observed images. The strong lensing regime of gravitational lensing is characterized by image distortions such as arcs, high magnification, and multiple images in a given field of view.

In the early 1900s Einstein predicted using general relativity that the gravitational field of a central mass like the Sun produces a deflection two times greater than that predicted by Newtonian deflection (Bovy 2023 in prep). In the Eclipse of 1919, Einstein and Eddington conducted the first test for general relativity by measuring the deflection of a light ray from a background star grazing the surface of the Sun. Their experiment supported general relativity’s prediction of 1.75 arcseconds compared to the Newtonian prediction of 0.875 arcseconds. Studying the gravitational effects of massive bodies on the light from distant stars is now termed microlensing.

It was Fritz Zwicky who suggested in the 1930s that gravitational lensing by galaxy clusters could be used as a tool to measure the unseen, dominant mass in clusters now referred to as dark matter. With incredible technological advancements in space telescopes, galaxy-galaxy lensing is now a popular field with surveys having discovered many instances of strong gravitational lensing.

Aside from strong gravitational lensing and microlensing, there is also weak lensing. Weak gravitational lensing deals with lensing effects on massive galaxies from multiple intervening mass components like the intra-cluster medium (ICM) or diffuse gas between the source and observer. Weak lensing is characterized by stretching distortions on a source galaxy. As a result such minor distortions in the shape of a galaxy can only be detected statistically. Weak lensing is also a very effective tool to probe the mass distribution of galaxies and to measure the dark matter contained within them.

Modeling techniques for strong gravitational lensing allows for accurate mapping of the mass distribution of the lensing cluster. It also allows for the detection of multiple images, positions of sources, and a mass estimate that includes the dark matter content of the cluster. Moreover, strong gravitational lensing allows for the study of distant sources that are otherwise beyond the reach of even the most powerful space telescopes. There is a growing field of astrophysicists who study high redshift sources detected from gravitational lensing. The high magnification allows for the determination of star formation rates and the properties and chemistry of galaxies in the early Universe. Gravitational lensing can be
Figure 1: A simplified version of the lensing geometry with the thin lens approximation (Kneib & Natarajan 2011). The light ray from the source (S) is deflected by an amount $\alpha$ by lens (L), and as a result, the image is observed at (I) at position $\theta_I$ rather than $\theta_S$ in the absence of the lens.

used to measure cosmological parameters such as the Hubble constant. Welch et al. (2022) reported the discovery of a star named Earendel, within a highly magnified lensed image in the Sunrise Arc, which is spectroscopically found to be at a redshift of 6.2 corresponding to a distance fo 12.9 billion light years. Therefore, strong gravitational lensing is a very effective and widely used tool to look further into the history of the Universe and to understand the geometry and properties of the Universe that give rise to such incredible lensing phenomena.

1.1 Deriving the Lensing equation

The theory described in the following sections are directly from Kneib & Natarajan 2011 and are merely brief restatements of the gravitational lensing formalism\textsuperscript{1}. Under the assumption of the “Cosmological Principle” which states that the Universe is homogenous and isotropic, the following metric referred to as the geodesic equation describes the Universe:

$$ds^2 = c^2 dt^2 - a^2(t)(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$  

\textsuperscript{1}For a detailed understanding of gravitational lens theory see: Bovy 2023 in prep
where \(a(t)\) is the scale factor, \(c\) is the speed of light, and \(k\) defines the curvature of the Universe. In the presence of a mass that perturbs spacetime, this metric is:

\[
ds^2 = (1 + \frac{2\Phi}{c^2})c^2dt^2 - (1 - \frac{2\Phi}{c^2})dr^2
\]

(2)

where \(\Phi\) is the 3D gravitational potential for the mass distribution.

Figure 1 shows a simplified version of the lensing geometry for a thin lens approximation. For galaxy-galaxy lensing, the physical extent of the deflector is much smaller than the distances between the observer, lens, and source, and therefore the lens is considered under a thin lens approximation. In the absence of a deflecting potential, the observer (O) sees the distant source at (S) at an angular position of \(\theta_S\). The gravitational effect of the lens at (L) deflects the light from (S) by the amount \(\alpha\), and the observer sees the source at (I) at an angular position of \(\theta_I\). The distances described in Figure 1 between the observer and lens \((D_{OL})\), the lens and source \((D_{LS})\), and the observer and source \((D_{OS})\) are angular diameter distances. The angular diameter distance to a body in space takes into account the physical size of the object, its angular size when viewed from Earth, and the cosmology of the Universe. The geometric equation below relates the source position to the image position, deflection angle, and the angular diameter distances:

\[
\theta_I = \theta_S + \frac{D_{LS}}{D_{OS}} \tilde{\alpha}(\theta_I)
\]

(3)

Light travels along the null geodesic which is defined by \(d\sigma^2 = 0\). The deflection angle can be derived from the null geodesic by solving for the travel time \(t_T(\tilde{\alpha})\) and applying Fermat’s principle which states that light travels the path of least time \(\frac{dt}{dt} = 0\). Here the deflection angle \(\tilde{\alpha}\) is a function of the local Newtonian gravitational potential \(\phi^2_N\) projected in the lens plane.

\[
\tilde{\alpha}(\theta_I) = \frac{2}{c^2} \frac{D_{LS}}{D_{OS}} \nabla_{\theta_I} \phi^2_N(\theta_I)
\]

(4)

Combining equations (3) and (4) yields the lensing equation under the thin lens approximation which predicts the positions of the sources given image locations:

\[
\theta_S = \theta_I - \frac{2\varepsilon}{c^2} \nabla \phi^2_N(\theta_I) = \theta_I - \nabla \varphi(\theta_I)
\]

(5)

Here, \(\varphi\) is the normalized version of the Newtonian projected lensing potential and the distance ratio \(\varepsilon = \frac{D_{LS}}{D_{OS}}\).

Gravitational lensing effects can be modeled mathematically as a transformation from the source plane to the image plane. The source plane is the plane where the source is located and the image plane is the plane where the image
is observed; the two are separated by large distances. For a single lens plane, the Hessian\textsuperscript{2} of the transformation describes the transformation of an element in the image plane \(d\theta_I\) to the source plane \(d\theta_S\) and is called the magnification matrix:

\[
\frac{d\theta_S}{d\theta_I} = A^{-1} = \begin{bmatrix}
1 - \partial_{xx}\phi & -\partial_{xy}\phi \\
-\partial_{xy}\phi & 1 - \partial_{yy}\phi
\end{bmatrix}
\]  

(6)

The magnification matrix can also be expressed in terms of the convergence \(\kappa\) and the shear vector (often denoted as a complex number) \(\gamma = (\gamma_1, \gamma_2)\) as:

\[
A^{-1} = \begin{bmatrix}
1 - \kappa - \gamma_1 & -\gamma_2 \\
-\gamma_2 & 1 - \kappa + \gamma_1
\end{bmatrix}
\]  

(7)

Here, the convergence \(\kappa\) is defined as:

\[
\kappa = \frac{\Delta \phi}{2} = \frac{\Sigma}{\Sigma_{crit}}
\]

(8)

and the shear is given by:

\[
\gamma_1 = \frac{\partial_{xx}\phi - \partial_{yy}\phi}{2} = \partial_{xy}\phi \\
\gamma_2 = \partial_{xy}\phi
\]

(9)

The term \(\Sigma_{crit}\) is the critical lensing surface density and is described as:

\[
\Sigma_{crit} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{LS}D_{OL}} = \frac{cH_0}{4\pi G} \frac{D_{OS}c/H_0}{D_{LS}D_{OL}}
\]

(10)

Multiple imaging of a source corresponds to multiple solutions to equation (5). Multiple images occur when the surface mass density of the cluster core \(\Sigma\) approaches or exceeds the critical surface mass density i.e. \(\Sigma \geq \Sigma_{crit}\).

1.2 Critical and Caustic Lines

Critical lines are the locus of infinite magnification in the image plane represented by two lines that do no intersect. Mathematically, the magnification matrix from equation (7) is real and symmetric and therefore can be diagonalized and expressed in its principal axes:

\[
A^{-1} = \begin{bmatrix}
1 - \kappa - \gamma & 0 \\
0 & 1 - \kappa + \gamma
\end{bmatrix} = (1 - \kappa) \left( \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \frac{\gamma}{1 - \kappa} \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} \right)
\]

(11)

\textsuperscript{2}Hessian is a square matrix that contains second-order partial derivatives of a scalar function.
where $g = \frac{\kappa}{1 - \kappa}$ is the reduced shear and is the quantity that is directly measured from the galaxy shapes in the data. The $(1 - \kappa)$ term describes the isotropic deformation where each point from the source plane is mapped to the same coordinates in the image plane but is magnified. The term involving the shear $\gamma$ describes the anisotropic deformation where points from the source plane are stretched tangentially in the image plane.

The determinant of the magnification matrix, $\mu$, quantifies the magnification and is given by:

$$\mu^{-1} = \text{det}(A^{-1}) = (1 - \kappa)^2 - \gamma^2 = (1 - \kappa)^2(1 - g^2)$$  \hspace{1cm} (12)

Infinite magnification occurs when one of the principal axes in equation (11) is zero and therefore the reduced shear $g = \pm 1$, resulting in two lines referred to as “critical lines.” The external critical line is called the “tangential critical line” along which the deformations are tangential, and the internal critical line is the “radial critical line” along which the deformations are radial. Critical lines describe the magnification and deformations in the image plane, and the corresponding lines that describe the position of the sources in the source plane causing such deformations are called “caustic lines.”

The tangential critical line is also referred to as the Einstein radius $r_E$, and the critical surface mass density $\Sigma_{\text{crit}}$ defined in equation (10) corresponds to the mean surface density enclosed within the Einstein radius. For a given $\Sigma_{\text{crit}}$, the size of the Einstein radius will vary based on the redshift of the lens and source, and the cosmology. Using the location of the tangential critical curve, the total mass enclosed within a circular aperture can be measured and the radial critical curve constrains the gradient of the mass profile near the cluster.

### 2 Data

The image of the lensing field SDSS J1138+2754 was obtained from the Mikulski Archive for Space Telescopes (MAST) maintained by the Space Telescope Science Institute. The field was detected in the Sloan Digital Sky Survey's (SDSS’s) Giant Arcs Survey and the image was obtained under the Hubble Legacy Archive (HLA) mission from the Hubble Space Telescope (HST). The image in Figure 2 was obtained using the F775W filter in the optical wavelength by the WFC3/UVIS instrument on the HST. The exposure time was 2396 seconds, and was overseen by principal investigator Michael D. Gladders (proposal id: 130003) on March 25, 2013. The image is made available to the public via the Hubble Legacy Archive project as a calibration level 3 image which corresponds to combined images or mosaics created by combining data taken during different visits to the target. The target coordinates in (RA,DEC) is (11 38 8.91, +27 54 8.53).
3 Source Extractor

This section describes the steps taken to obtain information from the HST image of the J1138+2754 field that will be used in the modeling process in section 4. The Lenstool modeling software requires inputs via .txt files and does not process images directly. Therefore, it is common practice to extract the necessary positional and photometric information from ccd images using the Source Extractor (also known as SExtractor) software.

Source Extractor accepts .fits images as direct inputs and automatically detects the background pixel level and determines whether pixels belong to the background or are separate objects. SExtractor performs photometry on objects classified as separate objects and writes them to a catalog. The subsections below describe the process of doing photometry using SExtractor.

3.1 Installing Source Extractor

SExtractor was developed on GNU/Linux machines and therefore should compile on POSIX-compliant systems like Apple OS X if the development packages of the ATLAS (v3.6) and FFTw (v3.0) libraries are installed in the system. My personal computer was a Macbook Air running on macOS Big Sur version
11.7.6. The source package of SExtractor v2.25.0 was downloaded from the official GitHub repository.³

Initially, due to my misreading of the manual’s instruction, I attempted to install the entire ATLAS library instead of the development package, which turned out to be extensive and challenging. Therefore, I decided to install Intel’s MKL (Math Kernel Library) along with Intel’s C++ Classic Compiler which were alternative recommendations in the SE manual.

Once installed, I ran the following command⁴ which sets the environment variables to run the software:

```
$ . /opt/intel/oneapi/setvars.sh
```

To compile SExtractor I ran the following commands in the terminal:

```
$ sh autogen.sh
$ ./configure --enable=mkl
$ make -j
$ sudo make install
```

Once compiled the executable ‘sex’ will be created and SExtractor will be ready to run.

### 3.2 SExtractor Inputs

SExtractor is run from the terminal using the following command:

```
$ sex Image --configuration-file
```

The working directory must contain the following files: default.conv, default.nnw, default.psf, default.param, default.sex (configuration file), and the input HST image.

#### 3.2.1 Configuration File: default.sex

The configuration file is an ASCII format file that contains user-defined parameters which instruct SExtractor to process the input Image. SExtractor install comes with a default configuration file called ‘default.sex’ which contains default parameter values that may produce decent photometry for a given image.

In the configuration file found in Appendix 9.1 I have set the detection threshold ‘DETECT.THRESH’ to be 1.5 sigmas, and ‘DETECT.MINAREA’ to be 16 pixels above threshold which seemed to be standard for photometry on galaxy fields.

³MacPorts and Xcode command line tools were installed in my system prior to installing the software mentioned here. MacPorts is a package manager that allows for easy installation of open source software for macOS.
⁴This command needs to be run before each terminal session. This can be automated through the bash profile, but I ran it manually each time.
The 'DEBLEND_NTHRESH' and 'DEBLEND_MINCONT' were set to 32 and 0.005 respectively, again from my research into what is acceptable for the field in consideration from previous studies on similar fields. The 'MAG_ZEROPOINT' was uniquely computed for the HST to be 24.88. And 'MEMORY_PIXSTACK' was set to a large number of 1500000 prior to which I encountered several errors. The rest of the parameters were defaults.

3.2.2 Parameter File: default.param

The 'default.param' file contains a list of hundreds of parameters that SExtractor can obtain from the input image. Values for parameters that are uncommented in the 'default.param' file are written to the catalog name specified in the configuration file.

The parameters I have extracted from SExtractor include but are not limited to, the ID number of each object (NUMBER), x- and y- positions in right ascension (RA) and declination (DEC) (ALPHA_J2000 and DELTA_J2000), semi-major and -minor axes (A_WORLD and B_WORLD), position angle counter-clockwise from the horizontal axis (THETA_IMAGE), ellipticity of the objects (ELLIPTICITY), isophotal magnitude (MAG_ISO) and the star/galaxy classifier (CLASS_STAR).

3.3 SExtractor Output

SExtractor detects all relevant pixels that are above the specified background threshold level in the configuration file. It outputs the values for the uncommented parameters in the 'default.param' file for each of the object it detects. Figure 3 shows an example of all the objects SE detects and does photometry on. SE also detects bad pixel artifacts which can be seen by the overdensity of detections in the four corners of the image and through the middle. I used a software called TOPCAT to project the detections from the SE output catalog onto the HST image. Using TOPCAT, I was able to reduce the initial detections from well above 500 down to a manageable amount by filtering out rows that were above a certain magnitude threshold. With a reduced catalog, I matched the location of the objects of interest from their RA and DEC in DS9 to the ones in the catalog. This allowed me to obtain ID numbers for the galaxies in consideration in order to obtain the corresponding positional and photometric details from the catalog.

4 Lenstool Modeling

Lenstool is a "parametric" modeling software that models the mass distribution of the galaxy cluster that is responsible for lensing distant sources. Since all modeling approaches require the use of parameters, for Lenstool "parametric"
Figure 3: The entire detection of objects within the SExtractor output catalog plotted onto the HST image using the TOPCAT software.

refers to the use of a finite number of mass clumps to represent the matter content of the cluster which are in turn defined by a finite number of parameters depending on the mass profile used (Kneib & Natarajan 2011). These mass clumps describe the galaxy-scale components to represent the galaxies in the cluster, and even fewer large scale mass clumps to represent other sources of mass in a cluster such as dark matter, X-ray gas, and Intra-Cluster Medium.

4.1 Bayesian Modeling

Lenstool's parametric modeling is done via Bayesian MCMC Sampling (Kneib & Natarajan 2011). Under Bayesian modeling the priors are well defined and used as constraints for the model and the unknown parameters are optimized over relevant ranges. Bayesian modeling provides two levels of inference through its exploration of the parameter space (during the burn-in phase) and its model comparison (during the sampling phase). Bayesian statistics computes the conditional probability of unknown parameters based on the given priors that describe the model. Bayes Theorem can be written as:
\[ P(p, D, M) = \frac{P(D|p, M)P(p|M)}{P(D|M)} \] (13)

where the posterior PDF (probability density function) \( P(p|D, M) \) is defined by the product of the likelihood function \( P(D|p, M) \) and the prior PDF \( P(p|M) \) divided by the evidence \( P(D|M) \). The posterior probability function describes the probability of the parameters \( p \) occurring given the observed data \( D \) and the model \( M \). The likelihood function varies with \( p \), and describes the likelihood of the data \( D \) occurring given parameters \( p \) that describe the model \( M \). The prior PDF describes the probability of parameters \( p \) occurring before considering the observed data. The evidence \( P(D|M) \) describes the probability of getting the data \( D \) given the assumed model \( M \). The value of the posterior PDF will be the greatest for the set of parameters \( p \) that result in the best-fit and is consistent with the prior PDF.

4.2 Lenstool inputs

Given that Lenstool models galaxies as mass clumps, inputs to Lenstool must include all the positional information such as the location, size, and shape of these mass clump, and their apparent brightness. Installing Lenstool was fairly straightforward using MacPorts. Lenstool is run via the following command in the terminal:

```
$ lenstool parameterfile.par -a
```

4.2.1 Parameter File

The parameter file (with a .par extension) is the central location with all the user-specified instructions used to model a given lensing field. It is a text file with first and second identifiers which tell the model what to do. Below I briefly describe the first identifiers I have used and the most important parameters necessary for my modeling:\(^5\):

**runmode**: defines what the program will do. The inverse parameter in this section (3 0.05 999) tells the program to use Bayesian approach with a burn-in rate\(^6\) of 0.05 and to sample through 999 models\(^7\).

**grid**: defines the size of the grid, number of potentials, and number of lenses optimized.

\(^5\)For a detailed description of the parameters refer to Kneib 2014

\(^6\)The burn-in rate determines how fast the program explores the parameter space. 0.05 is an acceptable default for a decent model.

\(^7\)Lenstool produces an additional model for some reason.
source: defines characteristics of sources. I have only included one of the source redshifts. Therefore the final model is optimized for a source at redshift of 1.334.

image: defines information about the multiple images or arclets.

potential: explicitly defines information about the mass clump corresponding to the brightest cluster galaxy that causes the most perturbation to the gravitational field.

limit: directly follows the potential identifier and defines the constraints for the same mass clump. Ranges for optimization are specified here.

potfile: defines the parameters for all the galaxy scale mass components. See Section(potfile). These galaxies are optimize together instead of separately in the case of the potential.

cosmology: defines cosmological parameters.

cline: defines parameters that compute the critical and caustic lines for the cluster.

field: defines size of the field used in some calculations.

4.2.2 Potential and Limit

The brightest cluster galaxy (BCG) is modeled as a cluster-scale mass clump taking into account other sources of matter such as dark matter and ICM. Here, the BCG is modeled as a PIEMD (pseudo-isothermal elliptical mass distribution) mass profile. The parameters that define this mass profile are the core radius $r_{\text{core}}$, cut radius $r_{\text{cut}}$, and central velocity dispersion $\sigma_0$, which are scaled to the galaxy luminosity in the following relations (Kneib & Natarajan 2011):

$$\sigma_0 = \sigma_0^* \left( \frac{L}{L^*} \right)^{1/4}$$

$$r_{\text{core}} = r_{\text{core}}^* \left( \frac{L}{L^*} \right)^{1/2}$$

$$r_{\text{cut}} = r_{\text{cut}}^* \left( \frac{L}{L^*} \right)^{\alpha}$$

The total mass of a cluster- or galaxy-scale halo then scales as:

$$M = \left( \frac{\pi}{G} \right) (\sigma_0^*)^2 r_{\text{cut}}^* \left( \frac{L}{L^*} \right)^{1/2 + \alpha}$$

here $L^*$ is the typical luminosity of a galaxy at the cluster redshift. The core radius and cut radius describe the limits between which the potential becomes
(a) Galaxies labeled 1 to 10 were included. (b) The choice of multiple-images are the in the potfile. SExtractor classifies galaxies same as Sharon et al. (2020) and are la-11 and 12 more as stars, therefore it was excluded in a similar manner. The third image included from the potfile. Information about pair for the 4th and 5th image families in the brightest cluster galaxy (BCG) is excluded in Sharon et al. (2020) was excluded implicitly entered into the potential section here since I was not able to see them by eye of the parameter file.

Figure 4: The images show the galaxies and multiple-images selected to be included in the potfile and multifile respectively.

a singular isothermal potential truncated at the cut radius (Kneib & Natarajan 2011).

The priors for the central galaxy causing the deflection are defined in the potential section. The ‘profil 81’ corresponds to the PIEMD mass profile. The x- and y- center are set to be 0 relative to the reference defined in the runmode section. The ellipticity and position angle (‘angle_pos’) were extracted from SE. The ‘cut_radius_kpc’ is far outside the lensing region and therefore cannot be used as a constraint for the mass distribution, therefore as seen in (Sharon ??), I have fixed the value to 1500. The redshift of the lens (‘z_lens’ was measured spectroscopically (cite sharon) to be 0.451. The ‘core_radius_kpc’ and and ‘v_disp’ were set to random values and were optimized in the limit section.

4.2.3 Potfile

Within the potfile section in the parameter file, the filename containing information about the positions and shapes of the remaining galaxies in the cluster are specified. The file containing information about these smaller scale mass clumps is an ASCII file with the following parameters from SExtractor: ID, RA, DEC, semi-major axis, semi-minor axis, position angle, and magnitude. The magnitude used here is isophotal magnitude from SExtractor. See Appendix 9.2.
Cluster member galaxies are determined using the red-sequence method. Since all galaxy clusters are observed to have a large population of elliptical and lenticular galaxies, a cluster can be found by observing an overdensity of red-sequence galaxies within a field of view. These galaxies exhibit a strong relationship between color and magnitude. For the galaxies in the red sequence, the redder a galaxy is the brighter it is. This method is used to primarily detect galaxies in a cluster which are then confirmed spectroscopically.

For this project, I have adopted a less scientific approach and included galaxies near arcs as cluster members instead of finding them using the red sequence method. See Figure 4a.

For the potfile section in the parameter file, I have again included some of the information extracted from SExtractor to optimize all the remaining galaxies together. Here I chose to vary the velocity dispersion of the galaxies as a whole given by ‘sigma’.

4.2.4 Multifile

The multifile or the multiple images file contain positional and photometric information about the arcs/multiple images in the HST image. Here again the ASCII file labeled in the parameter file contains the following parameters obtained from SE: ID, RA, DEC, semi-major, semi-minor, theta, z, mag. Here the theta and mag columns are left as zeroes since they are not used for optimization. The redshift for almost all the images are spectroscopically confirmed. Images are labeled in Figure 4b as ID x, where ID corresponds to the source the image belongs to, and x corresponds to how many times it is multiply imaged. For example, 1.3, represents the 3rd multiply-imaged pair of the first image family that correspond to source 1.

4.2.5 Lenstool Runs

Lenstool is run via an iterative process. Each change to the parameter file is done within a new directory to keep track of all the minor changes to the model. The model in this project was obtained via source plane optimization specified by the parameter ‘forme 0’ under the ‘image’ identifier. Source plane optimization is quick but not very accurate, but nevertheless produces a well constrained model, after which one can try the image plane optimization.

Initially the limits for the optimization are very broad and lenstool is allowed a great degree of freedom. The model is built up this way. Very few images in the multifile were used as constraints for the initial models. Over several runs, as the $\chi^2$ value decreased for each model, more images were added as constraints. Adding five image families takes a runtime of about five hours.
(a) Lenstool recreates the lensing field based on the inputs provided in the potential and potfile sections of the parameter file. The main mass clump in the caustic (yellow) lines are plotted over the lenser accounts for additional sources of mass IIST image of the lensing field. such as dark matter. The tangential critical (red) and caustic (yellow) lines are shown.

Figure 5: Lenstool mass distribution plots.

Lenstool produces many outputs, one of which is the ‘bestopt.par’ which includes better limits for optimization. Therefore, after each Lenstool run, I would update the optimization limits in the parameter file based on Lenstool’s recommendation in the ‘bestopt.par’ file. This helped constrain the model quickly and significantly lowered the $\chi^2$ value.

5 Results

The main objective of this project was to produce a mass map of the lensing cluster SDSS J1138+2754. Figure 5a shows Lenstool’s recreation of the finite number of mass clumps that represent the brightest cluster galaxy and cluster members as defined in the potfile and potential sections. In Figure 5a the red line represents the tangential critical curve which corresponds to the Einstein radius and constrains the mass to its interior. The yellow line (kite-shape referred to as an astroid) represents the tangential caustic line. A source that crosses the tangential caustic line in the source plane will be tangentially distorted near the tangential critical line in the image plane.
Figure 6: Lenstool model with five image families used as constraints (blue). Source predictions are given in green. The tangential critical line is shown in red, and the tangential caustic is shown in yellow. Additional multiple-image predictions can be seen by the overlap of 4.1,4.2 and 5.1,5.2.
Figure 7: Closer look at the caustic line for the cluster. The yellow line represents the tangential caustic. Sources (green) crossing the caustic at different locations will deform and multiply differently as observed in the image plane. This caustic is for a source redshift of 1.334.

Figure 5b shows the same tangential critical and caustic lines overlayed on the HST image. Figure 5b also shows the radial critical line (interior red ellipse) along which deformations are radial and the radial caustic line (interior yellow ellipse). The overall elliptical shape of the critical line mostly arises due to the elliptical potential at the center which is the brightest cluster galaxy. The remaining cluster member galaxies contribute to the unique shape of the critical lines.

Figure 7 offers a closer view of the sources near the tangential caustic line. In Figure 6, Lenstool's predictions of the source location is given in green and the corresponding image locations as observed in the data are given in blue. Upon reading the multilevel with the input image locations (blue), Lenstool identifies and accepts them as shown in magenta and can also predict any multiple image that the user may have missed. When source 1 crosses the cusp of the caustic in Figure 7, the image is magnified and can be seen near the critical line (1.1,1.2,1.3 in blue) and is multiply imaged 3 times (Figure 6). Source 2 is inside the caustic curve in the source plane, and is multiply imaged 2 times according to the magenta (2.1,2.2). Here for some reason Lenstool isn't predicting a third multiple image pair at the location 2.3 (blue) as observed in the data. Sources 4 and 5 are outside the caustic curve and are both multiply imaged 3 times. In my multilevel input, I excluded the third image pair for the 4th and 5th image families since I was not able to make them out by eye in the HST image. Nevertheless, based on the constraints, the model correctly predicts that there should be a third pair at the location given by the overlap between 4.1,4.2 and 5.1,5.2 as seen in Figure 6.
Figure 8: The model produced by Sharon et al. 2020 for the lensing field SDSS J1138+2754. The red line corresponds to the critical line for the cluster. The multiple images are circled and labeled.
Figure 8 shows the model produced by Sharon et al. (2020). While my model in Figure 6 looks similar to Figure 8, the noticeable differences in my model may be due to my selection of cluster member galaxies. Sharon et al. (2020) did use the third multiple-image pair as constraints in the model, therefore my model is correct in predicting images there.

5.1 Possible Improvements

There are many potential areas of improvement. Due to my limited understanding of the proper techniques for modeling gravitational lenses, I mostly took educated guesses to obtain the model I have so far. One option is to increase the number of models Lenstool samples from about 1000 to 2000, and to decreases the burn-in phase rate from 0.5 (inverse parameter in runmode section) to 0.05. Lowering the burn-in rate allows Lenstool to explore the parameter space more cautiously, and higher sampling allows for better model generation. Other than these, I would also consider running this model under image plane optimization which is often the final step in modeling via lenstool since it gives the best results. I refrained from taking this step since my model can be better constrained using source plane optimization and aslo due to the even greater runtime.

6 Conclusion

Modeling strong gravitational lenses using Lenstool produces the mass distribution of the lensing cluster, constraining the mass within the tangential critical line. Lenstool also predicts the positions of sources, and their relation near the source plane caustic and how that transforms their position in the image plane. The mass distribution constrained here includes the dark matter content of the cluster. Detailed studies of strong gravitational lenses allow astronomers probe distant sources that are otherwise way beyond our reach.

7 Software

Source Extractor (Bertin & Arnouts 1996), Lenstool (Jullo et al. 2007),

8 References


9. Appendix

9.1 SExtractor Configuration File

# Default configuration file for SExtractor 2.25.0
# EB 2022-11-03
#
#
#----------------------------------------- Catalog -----------------------------------------

CATALOG_NAME J1138_out_sexagesimal.cat  # name of the output catalog
CATALOG_TYPE ASCII_HEA            # NONE,ASCII,ASCII_HEAD, ASCII_SKYCAT,
                                  # ASCII_VOTABLE, FITS_1.0 or FITS_LDAC
PARAMETERS_NAME default.param # name of the file containing catalog contents

#----------------------------------------- Extraction -----------------------------------------

DETECT_TYPE CCD                  # CCD (linear) or PHOTO (with gamma correction)
DETECT_MINAREA 16   # min. # of pixels above threshold
DETECT_THRESH 1.5 # <sigmas> or <threshold>,<ZP> in mag.arcsec^-2
ANALYSIS_THRESH 1.5 # <sigmas> or <threshold>,<ZP> in mag.arcsec^-2
FILTER Y          # apply filter for detection (Y or N)?
FILTER_NAME default.conv # name of the file containing the filter

DEBLEND_NTHRESH 32  # Number of deblending sub-thresholds
DEBLEND_MINCONT 0.005 # Minimum contrast parameter for deblending
CLEAN Y         # Clean spurious detections? (Y or N)?
CLEAN_PARAM 1.0   # Cleaning efficiency
MASK_TYPE CORRECT # type of detection MASKing: can be one of
# NONE, BLANK or CORRECT

# Photometry

PHOT_APERTURES 5,10,20,30 # MAG_APER aperture diameter(s) in pixels
PHOT_AUTOPARAMS 2.5, 3.5 # MAG_AUTO parameters: <Kron_fact>,<min_radius>
PHOT_PETROPARAMS 2.0, 3.5 # MAG_PETRO parameters: <Petrosian_fact>,
                          # <min_radius>

SATUR_LEVEL 200000.0 # level (in ADUs) at which arises saturation
SATUR_KEY SATURATE # keyword for saturation level (in ADUs)

MAG_ZEROPOINT 24.88 # magnitude zero-point
MAG_GAMMA 4.0 # gamma of emulsion (for photographic scans)
GAIN 1.5 # detector gain in e-/ADU
GAIN_KEY GAIN # keyword for detector gain in e-/ADU
PIXEL_SCALE 0.04 # size of pixel in arcsec (0=use FITS WCS info)

# Star/Galaxy Separation

SEEING_FWHM 0.17 # stellar FWHM in arcsec
STARNNW_NAME default.nnw # Neural-Network_Weight table filename

# Background

BACK_SIZE 32 # Background mesh: <size> or <width>,<height>
BACK_FILTERSIZE 1 # Background filter: <size> or <width>,<height>
BACKPHOTO_TYPE LOCAL # can be GLOBAL or LOCAL
BACKPHOTO_THICK 32

# Check Image

CHECKIMAGE_TYPE NONE # can be NONE, BACKGROUND, BACKGROUND_RMS,
                    # MINIBACKGROUND, MINIBACK_RMS, -BACKGROUND,
                    # FILTERED, OBJECTS, -OBJECTS, SEGMENTATION,
                    # or APERTURES
CHECKIMAGE_NAME check.fits # Filename for the check-image

# Memory (change with caution!)

MEMORY_OBJSTACK 10000 # number of objects in stack
MEMORY_PIXSTACK 1500000 # number of pixels in stack
MEMORY_BUFSIZE 1024 # number of lines in buffer

# Miscellaneous

20
9.2  Potfile

#REFERENCE 0 174.5372604 +27.9085176
768  174.5406545 +27.9015745 7.862908e-05 6.0004e-05  290.95  20.0847  0
1278  174.5320543 +27.9125330 9.598521e-05 7.673486e-05  293.78  20.2386  0
743  174.5368101 +27.9005103 0.00010326 5.99204e-05  2.61  20.6182  0
843  174.5402956 +27.9043046 8.952206e-05 5.530064e-05  349.97  20.9278  0
805  174.5341908 +27.9032879 0.0001210699 7.408408e-05  57.35  21.1232  0
885  174.5424346 +27.9062640 0.0001088505 5.449395e-05  353.1  21.2370  0
766  174.5357859 +27.9006513 6.216292e-05 3.856416e-05  23.67  21.4004  0
976  174.5423335 +27.9101537 0.0001014324 4.248554e-05  274.2  21.4118  0
1357  174.5304425 +27.9142019 9.231922e-05 5.450899e-05  346.87  21.6320  0
884  174.5413378 +27.9069145 0.0001037939 4.007565e-05  279.03  22.2925  0
#892  174.5372604 +27.9085176 0.0001526839 8.951591e-05  87.24  20.0441  0

9.3  Multifile

#REFERENCE 0 174.5372604 +27.9085176
1.1  174.539460 27.912498 2.759492e-05 2.010432e-05 0.0 1.334 0.0
1.2  174.536280 27.912373 5.158645e-05 1.880667e-05 0.0 1.334 0.0
1.3  174.533020 27.910710 3.907239e-05 1.593675e-05 0.0 1.334 0.0
2.1  174.538084 27.910779 6.188633e-05 1.864282e-05 0.0 0.909 0.0
2.2  174.537286 27.910748 3.385148e-05 1.144173e-05 0.0 0.909 0.0
2.3  174.534447 27.909638 2.63957e-05 1.026602e-05 0.0 0.909 0.0
3.1  174.536764 27.914191 6.520378e-05 4.862624e-05 0.0 1.455 0.0
4.1  174.542160 27.903572 5.388843e-05 2.533873e-05 0.0 2.94 0.0
4.2  174.540960 27.903079 6.079834e-05 1.471146e-05 0.0 2.94 0.0
5.1  174.540057 27.912498 0.0001027477 4.292055e-05 0.0 2.0 0.0
5.2  174.538768 27.912722 2.33787e-05 8.825253e-06 0.0 2.0 0.0

9.4  Parameter file

runmode
reference 3 174.5372604 +27.9085176
inverse 3 0.5 999
image 1 J1138_mult0.cat
mass 3 2000 0.451 mass.fits
shearfield 0 2.0 shear.dat
verbose 1
end
grid
number 128
polar 0
nlen 11
nlen_opt 1
end
source
z_source 1.334
end
image
   multfile 1 J1138_mult0.cat
   mult_wcs 1
forme 0
   z_m_limit 0 5.0 3 2.0 1.0 0.1
sigposArcsec 0.30
end
potentiel BCG
profil 81
x_centre 0.
y_centre 0.
elipticity 0.414
angle_pos 87.24
core_radius_kpc 40
    cut_radius_kpc 1500
v_disp 881
z_lens 0.451
end
limit BCG
    cut_radius_kpc 0 1500
#core_radius_kpc 1 30 100 0.1
#v_disp 1 500 1500 0.1
#core_radius_kpc 3 42. 4.5 0.10000
#v_disp 3 895. 27.6 0.10000
core_radius_kpc 3 40. 5. 0.10000
    v_disp 3 885. 33. 0.10000
end
potfile
filein 3 J1138_pot0.cat
type 81
z_lens 0.451
core 0.027
#sigma 1 500 2000 0.5
    #cutkpc 1 1000 2500 0.5
#sigma 3 579. 70.
    #cutkpc 3 1985. 881.
sigma 3 557. 65. 0.1
cutkpc 0 1500
slope 0 4.0
vdslope 0 4
magO 16.00
end

cosmology
H0 70.0
omegaM 0.27
omegaX 0.73
wX -1.0
end

cline
	nplane 1 1.334
algorithm MARCHINGSQUARES

limitHigh 1.0
limitLow 0.05
end

field
dmax 400
end
finish

9.5 Best.par file

#Thu May 4 23:41:06 2023

#Source plane optimization
#Chi2tot(dof=11): 55.6652
#Chi2pos: 55.665164
#Chi2_vel: 0.000000
#Chi2_mass: 0.000000
#Chi2formx: 0.000000
#Chi2formy: 0.000000
#Chi2l: 0.000000
#log(Evidence): -0.264868
#n_Warning: 10
runmode

reference 3 174.537260 27.908518
image 1 J1138_mult0.cat
mass 3 2000 0.451000 mass.fits
end

grid
number 128
polar 0
nlens 11
end
potential 01
profile 81
x_centre 0.000000
y_centre 0.000000
ellipticity 0.414000
angle_pos 87.240000
core_radius 7.184421
core_radius_kpc 41.831838
cut_radius 257.617930
cut_radius_kpc 1500.000000
v_disp 895.321257
z_lens 0.4510
end
potential 768
profile 81
x_centre -10.797670
y_centre -24.995160
ellipticity 0.263932
angle_pos 290.950000
core_radius 0.004116
core_radius_kpc 0.023963
cut_radius 39.267756
cut_radius_kpc 228.639499
v_disp 219.503733
mag 20.084700
z_lens 0.4510
end
potential 1278
profile 81
x_centre 16.562196
y_centre 14.455440
ellipticity 0.220173
angle_pos 293.780000
core_radius 0.003834
core_radius_kpc 0.022323
cut_radius 36.581043
cut_radius_kpc 212.995908
v_disp 211.861436
mag 20.238600
z_lens 0.4510
end
potential 743
profile 81
x_centre 1.432542
y_centre -28.826280
ellipticity 0.496186
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core_radius_kpc  0.018743
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z_lens  0.4510
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y_centre  -15.166800
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y_centre  -18.826920
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angle_pos  57.350000
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cut_radius  24.340869
cut_radius_kpc  141.726564
v_disp  172.819168
mag  21.123200
z_lens  0.4510
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